



UNIVERSITY OF  
CAMBRIDGE

# Machine Learning and the Physical World

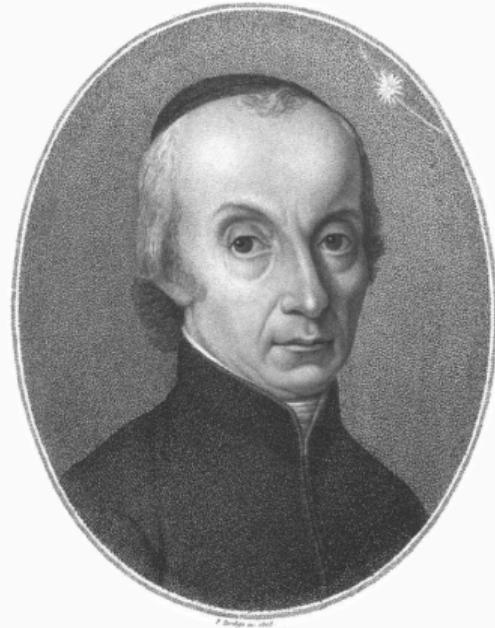
## Lecture 1 : Introduction

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Carl Henrik Ek - [che29@cam.ac.uk](mailto:che29@cam.ac.uk)

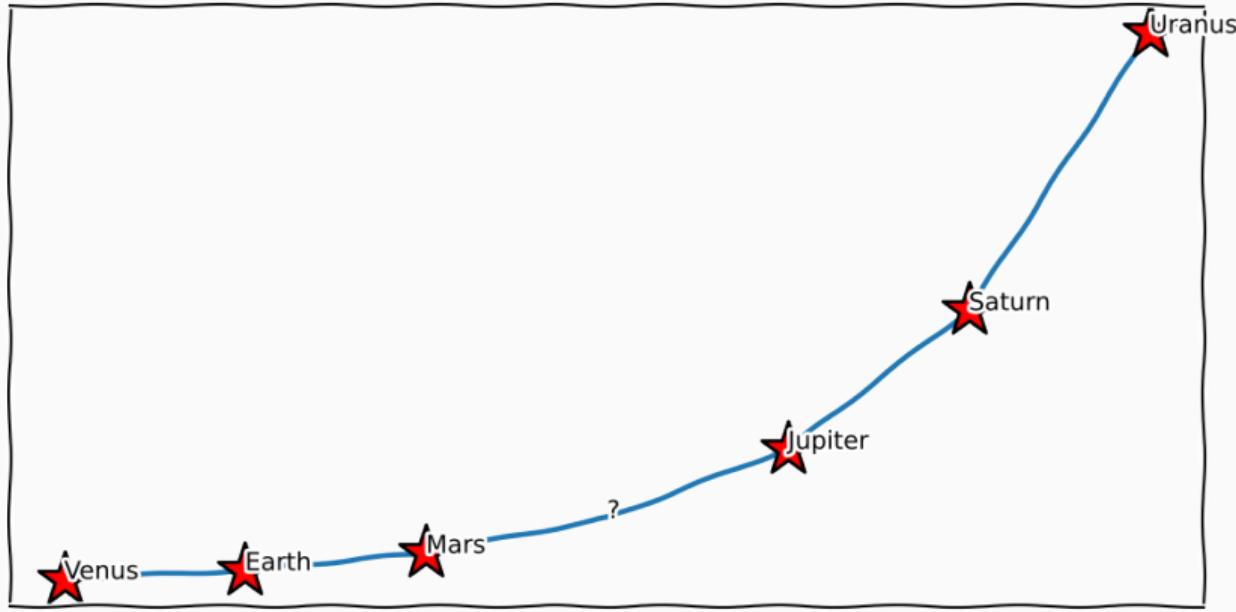
13th October, 2025

<http://carlhenrik.com>



P. Brugge sc. et del.





Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. Piazzi neu entdeckten Gestirns.

| 1801  | Mittlere<br>sonnen-<br>Zeit | Gedre-<br>Auffällig. in<br>Zeit | Gerade Auf-<br>steigung.<br>in Graden | Nördl.<br>Abweich. | Geozentri-<br>che Länge | Geozentri.<br>Breite | Ort der Sonne<br>+ 20°<br>Abstraktion | Logar.<br>d. Distanz<br>⊙ 3 |
|-------|-----------------------------|---------------------------------|---------------------------------------|--------------------|-------------------------|----------------------|---------------------------------------|-----------------------------|
|       | St                          | St                              | St                                    | St                 | Z                       | Z                    | Z                                     | St                          |
| Jan.  | 1 8 43 27,8                 | 3 27 11,25 51 47 48,8           | 15 37 45,5                            | 1 23 22 58,3       | 1 6 42,1                | 9 11 1 30,9          | 9, 9926156                            |                             |
|       | 2 8 39 24,6                 | 3 26 53,85 51 43 27,8           | 15 41 55,5                            | 1 23 19 44,3       | 1 2 24,9                | 9 12 2 18,6          | 9, 9926317                            |                             |
|       | 3 8 34 53,3                 | 3 26 38,41 51 39 36,0           | 15 44 31,6                            | 1 23 16 58,6       | 1 53 9,9                | 9 13 3 16,6          | 9, 9926324                            |                             |
|       | 4 8 30 42,1                 | 3 26 23,15 51 35 47,3           | 15 47 57,6                            | 1 23 14 35,5       | 1 53 55,6               | 9 14 4 14,9          | 9, 9926418                            |                             |
|       | 10 8 6 15,8                 | 3 25 32,11 51 28 1,5            | 16 10 31,0                            | 1 23 7 59,1        | 2 29 0,6                | 9 20 10 17,5         | 9, 9927641                            |                             |
|       | 11 8 2 17,5                 | 3 25 29,73 51 22 26,0           | .....                                 | .....              | .....                   | .....                | .....                                 |                             |
|       | 13 7 54 26,2                | 3 25 30,30 51 22 34,5           | 16 22 49,5                            | 1 23 10 37,6       | 2 16 59,7               | 9 23 12 13,8         | 9, 9928490                            |                             |
|       | 14 7 50 31,7                | 3 25 31,72 51 22 55,8           | 16 27 5,7                             | 1 23 12 1,2        | 2 12 56,7               | 9 24 14 13,5         | 9, 9928809                            |                             |
|       | 17 .....                    | .....                           | 16 40 13,0                            | .....              | .....                   | .....                | .....                                 |                             |
|       | 18 7 35 11,3                | 3 25 55,11 51 28 45,0           | .....                                 | .....              | .....                   | .....                | .....                                 |                             |
|       | 19 7 31 28,5                | 3 26 8,15 51 32 2/3             | 16 49 16,1                            | 1 23 25 59,2       | 1 53 38,2               | 9 29 19 53,8         | 9, 9930607                            |                             |
|       | 21 7 24 2,7                 | 3 26 34,27 51 38 34,1           | 16 58 35,9                            | 1 23 34 21,3       | 1 46 6,0                | 10 1 20 40,3         | 9, 9931434                            |                             |
|       | 22 7 20 21,7                | 3 26 49,42 51 41 21,3           | 17 3 18,5                             | 1 23 39 1,8        | 1 42 28,1               | 10 2 21 32,0         | 9, 9931886                            |                             |
|       | 23 7 16 43,6                | 3 27 6,90 51 46 43,5            | 17 8 5,5                              | 1 23 44 15,7       | 1 38 52,1               | 10 3 22 22,7         | 9, 9932348                            |                             |
|       | 28 6 58 51,3                | 3 28 54,55 52 13 38,3           | 17 32 54,1                            | 1 24 15 15,7       | 1 21 6,9                | 10 8 26 20,1         | 9, 9935061                            |                             |
|       | 30 6 51 52,9                | 3 29 48,14 52 27 2,1            | 17 43 11,0                            | 1 24 30 9,0        | 1 14 16,0               | 10 10 17 46,2        | 9, 9936332                            |                             |
|       | 31 6 48 26,4                | 3 30 17,25 52 34 18,8           | 17 48 21,5                            | 1 24 38 7,3        | 1 10 54,6               | 10 11 18 28,5        | 9, 9937007                            |                             |
| Febr. | 1 6 44 59,9                 | 3 30 47,21 52 41 48,0           | 17 53 36,3                            | 1 24 46 19,3       | 1 7 30,9                | 10 12 29 9,6         | 9, 9937703                            |                             |
|       | 2 6 41 35,8                 | 3 31 19,06 52 49 45,9           | 17 58 57,5                            | 1 24 54 57,9       | 1 4 10,5                | 10 13 29 49,9        | 9, 9938423                            |                             |
|       | 5 6 31 31,5                 | 3 33 2,70 53 15 40,5            | 18 15 1,0                             | 1 25 22 43,4       | 0 54 23,9               | 10 16 31 45,5        | 9, 9940751                            |                             |
|       | 8 6 21 39,2                 | 3 34 58,50 53 44 37,5           | 18 31 23,2                            | 1 25 53 29,5       | 0 45 5,0                | 10 19 33 33,3        | 9, 9943276                            |                             |
|       | 11 6 11 58,2                | 3 37 76,54 54 16 38,1           | 18 47 58,8                            | 1 26 26 40,0       | 0 36 2,9                | 10 22 35 13,4        | 9, 9945823                            |                             |

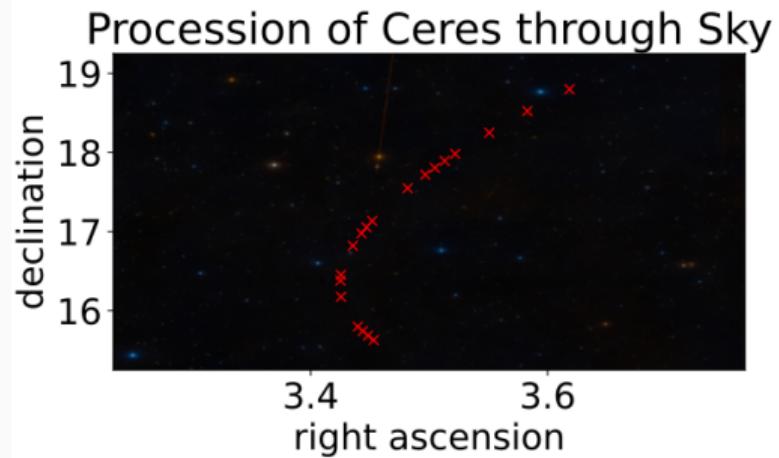
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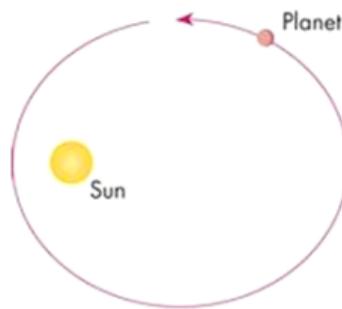
## A NEW PLANET.

An important circumstance in Astronomy has just occurred, no less than the Discovery of ANOTHER NEW PLANET!!! This celestial phenomenon moves between the orbits of Mars and Jupiter, and is an intermediate Planet between them. It was discovered by M. PIAZZI, an Italian Astronomer, on the 1<sup>st</sup> of January, 1801. He concealed the discovery, to preserve all the honour and observations to himself, till after six weeks close watching, he fell ill. It will not be in a situation, with regard to the Sun, to be observed again, till a month or two hence. It is but a small Planet, ranking only as a star of the eighth magnitude, and therefore not visible to the naked eye. Its motion is nearly parallel to the ecliptic, at present about  $4\frac{1}{2}^{\circ}$  to the north of it, and nearly entering the sign Leo. The distance from the Sun is about  $2\frac{1}{2}$  times that of the earth, and the periodical time nearly four years and two months.—Other particulars shall be given in our next.

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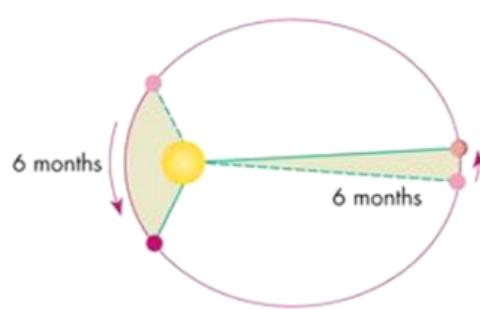


# Kepler's 3 Laws of Planetary Motion



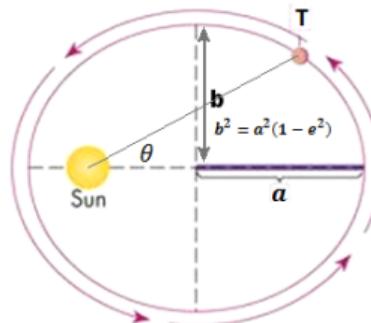
(1)

The orbits are ellipses



(2)

Equal areas in equal time



(3)  
 $T^2 \propto a^3$   $T$  = time to complete orbit  
 $a$  = semi-major axis



hier in der Nähe der Quadratur der Einfluss der Sonnen-Länge geringer ist, als in andern Lagen. Dr. Gauß glaubt daher, dass es nicht unfehllich wäre, wenn man die Fehler der Sonnentafeln aus sehr genauer Beobachtungen für diele Zeiten bestimme, und die Öter der Sonne hierauf verbessere. Diese vierzehn Elemente sind nun folgende:

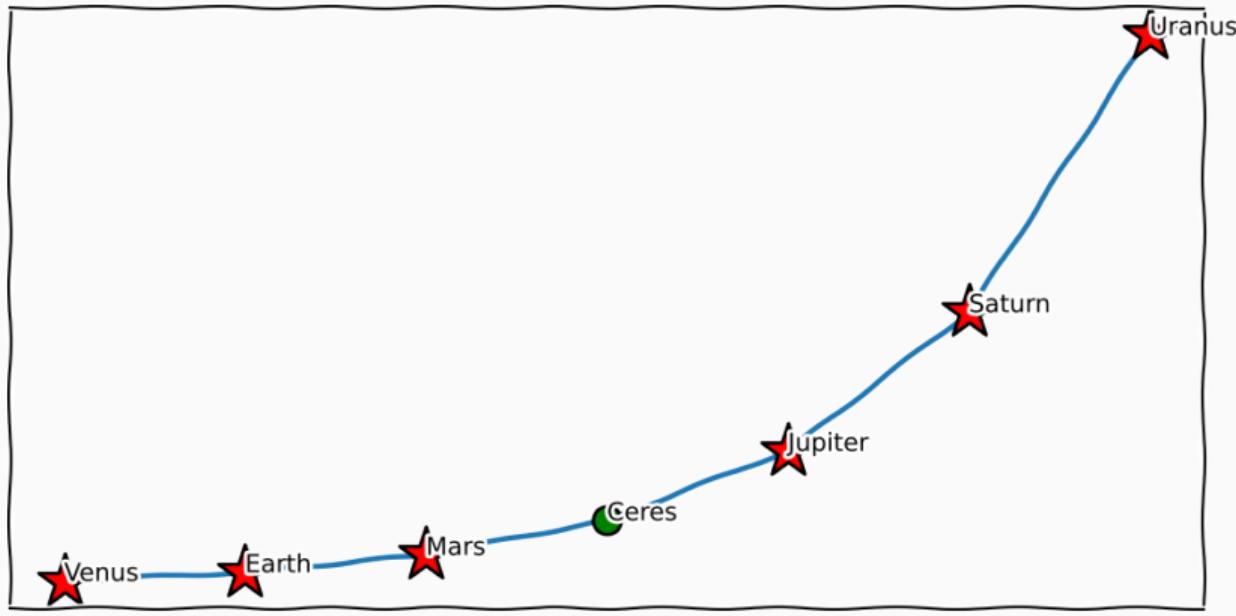
|                                 |              |                        |
|---------------------------------|--------------|------------------------|
| Sonneferne                      | 326° 07' 38" | Himmels-               |
| Ω                               | 0° 0 44'     | größte Mittelp. Gleit- |
| Neigung                         | 10° 36' 57"  | ebung                  |
| Log. halb. gr. Azo              | 0.4420327    | ugl. mittlere Längen.  |
| Exzentrizität                   | 0.0829017    | tropische Beweg.       |
| Epoche 1830 31 Dec. 77° 29' 34" |              | 77° 29' 34"            |

Aus diesen Elementen hat Dr. Gauß folgende  
Öter der Ceres Ferdinandea, im vorange berechnet.  
Die Zeit ist mittlere für Mitternacht in Palermo.

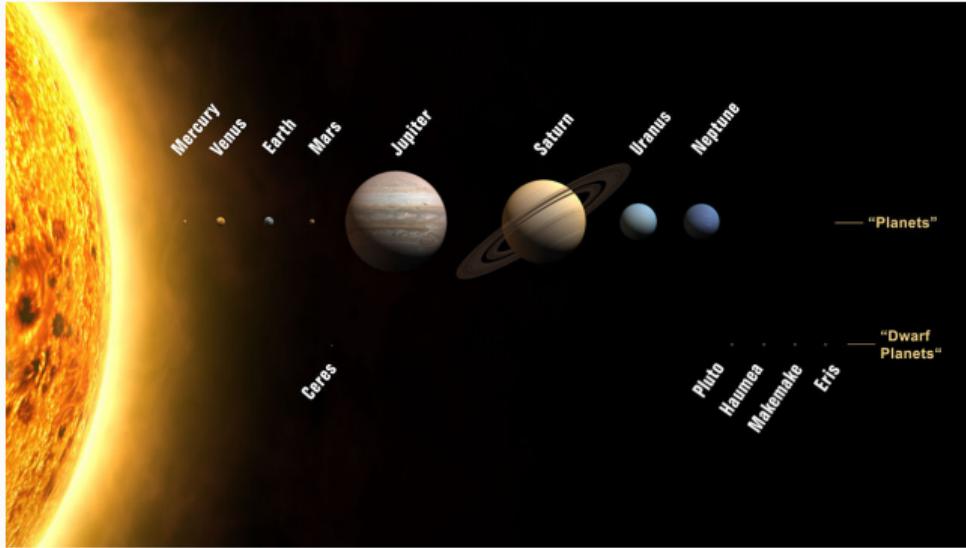
| Monat   | Gesuchte<br>örtliche<br>Länge | Geograph.<br>zentri-<br>sche Ab-<br>stand<br>nord. | Logarith.<br>der Ab-<br>stand<br>von der S | Logarith.<br>des Ab-<br>standes<br>von der C | Verhältnis<br>des Ab-<br>standes<br>von der C<br>zu dem<br>Himmels- |
|---------|-------------------------------|--|--|--|---|
| Z       | " "                           | " "  |  |  |   |
| Nov. 25 | 20 16                         | 9 25   | 0. 41181                                   | 0. 40468                                     | 0. 6102   |
| Dec. 1  | 22 15                         | 9 48   | 0. 40940                                   | 0. 40472                                     | 0. 6159   |
| 7       | 24 7                          | 10 17  | 0. 39544                                   | 0. 40479                                     | 0. 6815   |
| 13      | 25 53                         | 10 37  | 0. 38296                                   | 0. 40488                                     | 0. 7290   |
| 19      | 27 27                         | 11 4   | 0. 36902                                   | 0. 40499                                     | 0. 7779   |
| 25      | 28 53                         | 11 32  | 0. 35468                                   | 0. 40512                                     | 0. 8299   |
| 31      | 0 10 12                       | 11 0   | 0. 34000                                   | 0. 40528                                     | 0. 8869   |

Sollte man den Ort des Planeten nach diesen Elementen genauer, oder auf eine längere Zeit berechnen wollen: so setzen wir zu diesem Behufe noch folgende Formeln hierher:

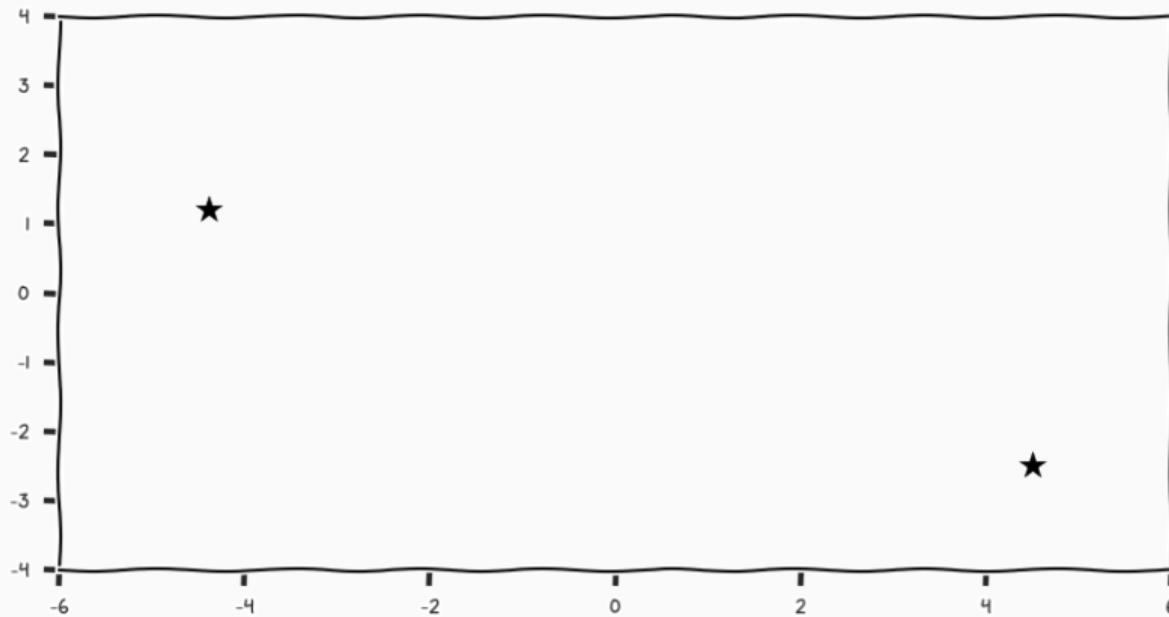
1) Zur



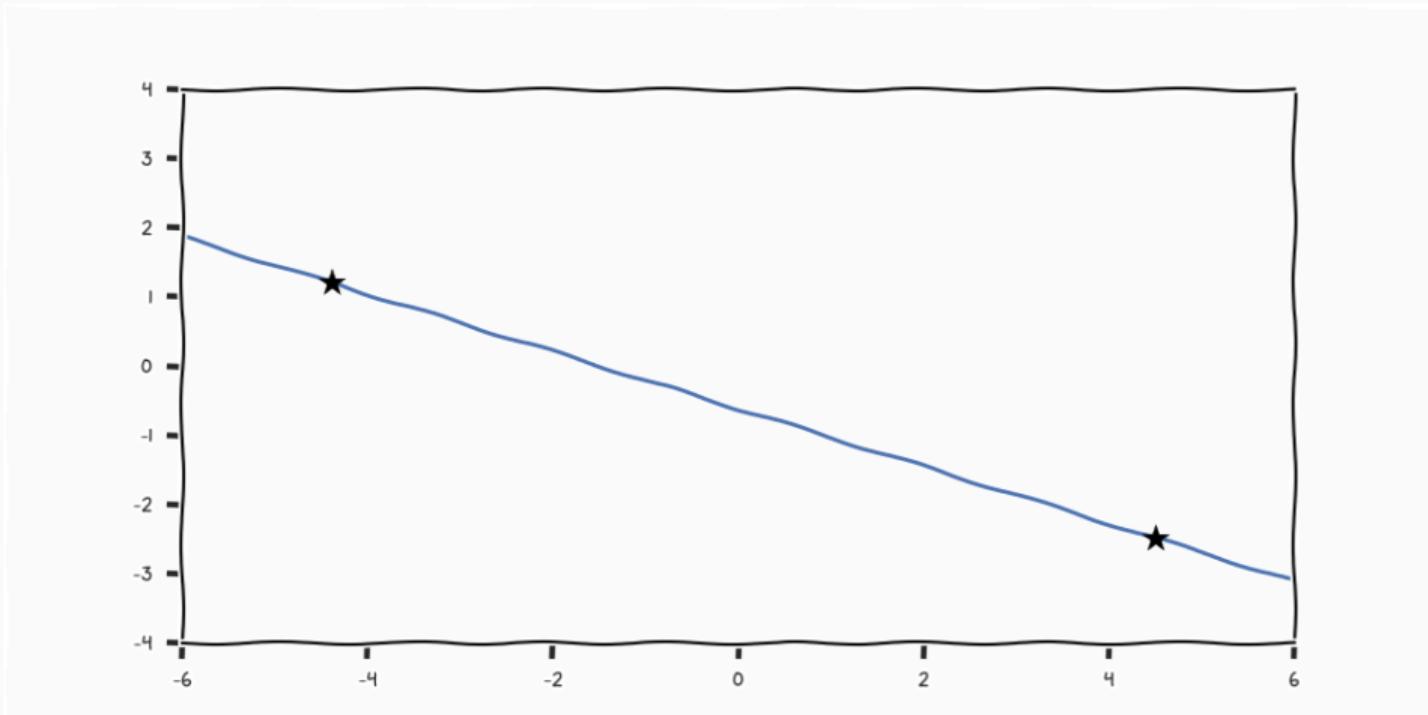




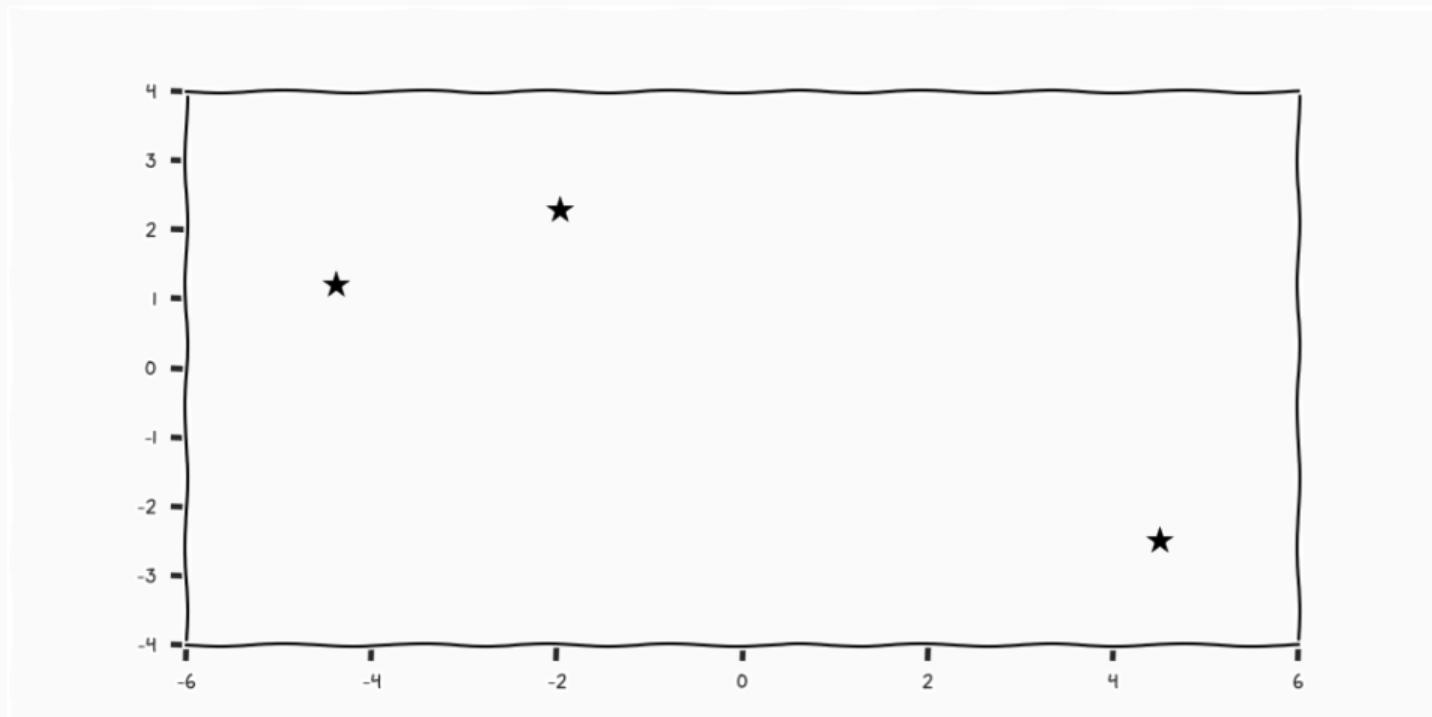
# Modelling



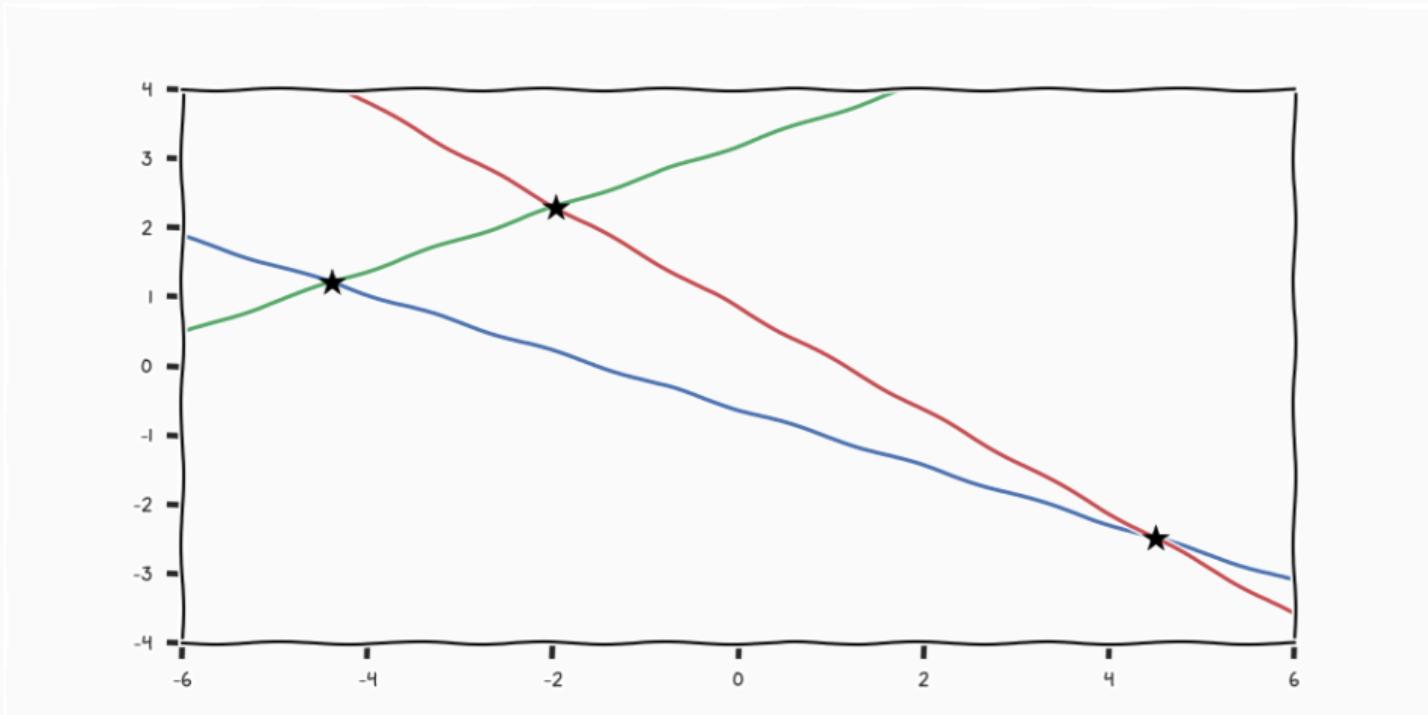
# Modelling



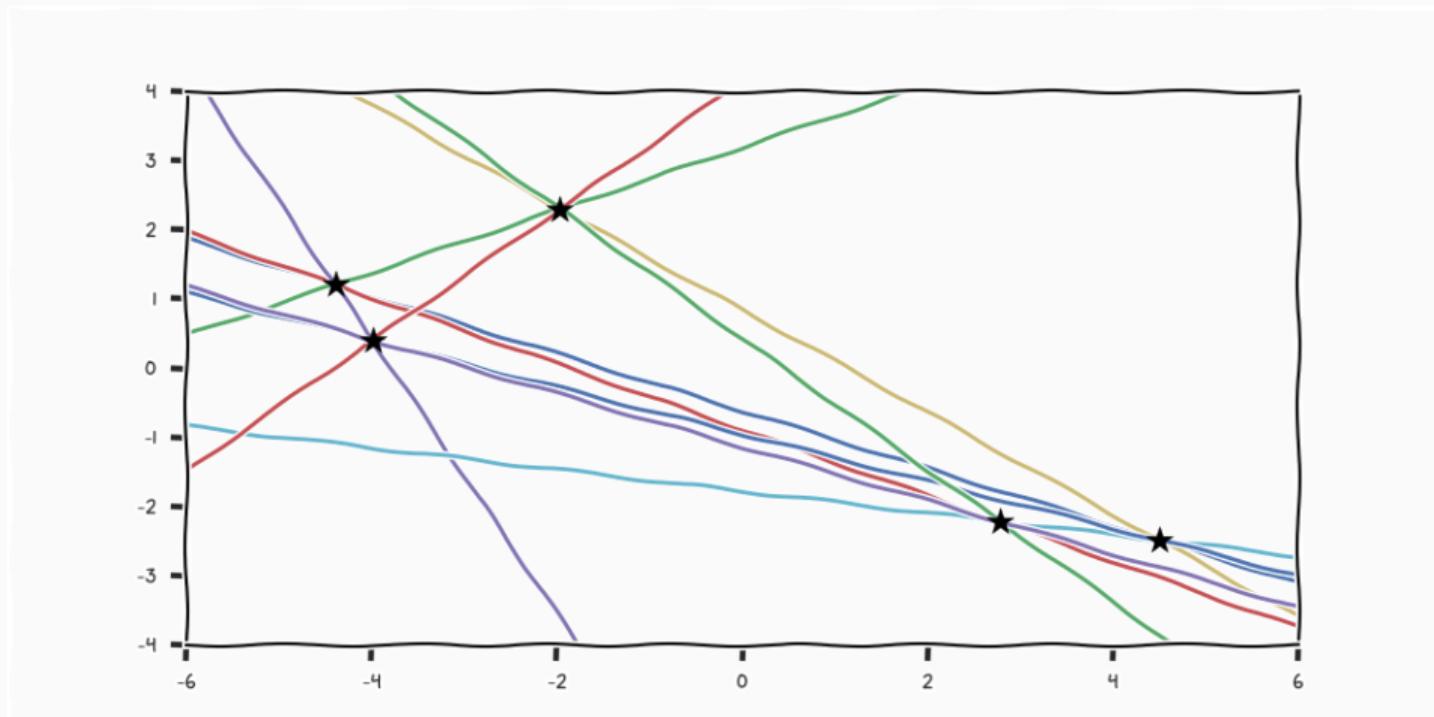
# An Over-determined System

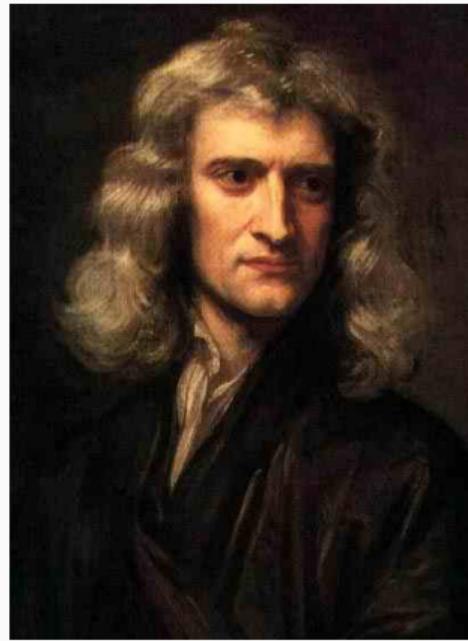


# An Over-determined System



# An Over-determined System





## An Over-determined System

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- An over-determined system means that we have more data than we need to determine our parameters.

## An Over-determined System

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- An over-determined system means that we have more data than we need to determine our parameters.
- The problem arises because the **model** is a simplification of the **world** and the data is therefore **inconsistent** with our model.



## Laplace Demon (Laplace, 1814)

We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Data + Model → Prediction

# Does God Play Dice?<sup>1</sup>

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*This led to the idea of scientific determinism, which seems first to have been publicly expressed by the French scientist, Laplace.*  
– Stephen Hawkins

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<sup>1</sup> Does God Play Dice? - Stephen Hawkins

*All these efforts in the search for truth tend to lead [the human mind] back continually to the vast intelligence which we have just mentioned, but from which it will always remain infinitely removed.*

*– Pierre Simon Laplace, A Philosophical Essay on Probabilities*  
*Laplace, 1814*

"The curve described by a simple molecule of air or vapor is regulated in a manner just as certain as the planetary orbits; the only difference between them is that which comes from our ignorance. Probability is relative, in part to this ignorance, in part to our knowledge. We know that of three or greater number of events a single one ought to occur; but nothing induces us to believe that one of them will occur rather than the others. In this state of indecision it is impossible for us to announce their occurrence with certainty. It is, however, probable that one of these events, chosen at will, will not occur because we see several cases equally possible which exclude its occurrence, while only a single one favors it."

$$p(\text{Data}) + p(\text{Model}) \rightarrow p(\text{Prediction})$$

## An Over-determined Sysatem

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- Over determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

## An Over-determined System

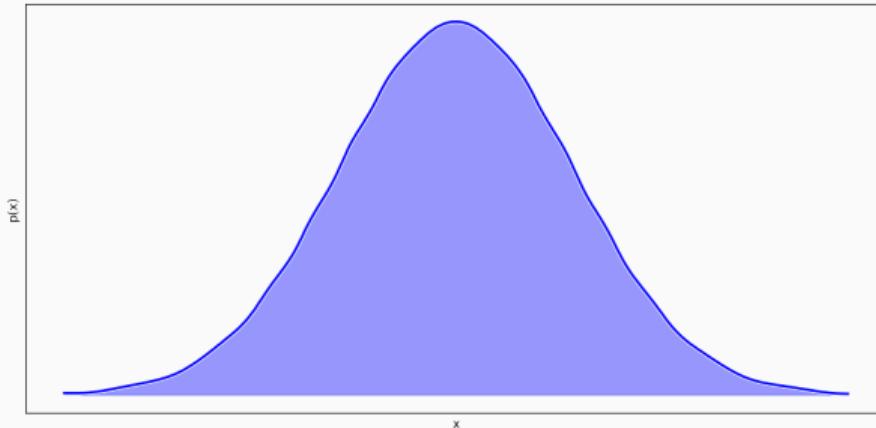
- Over determined system

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

- Parametrisation of ignorance

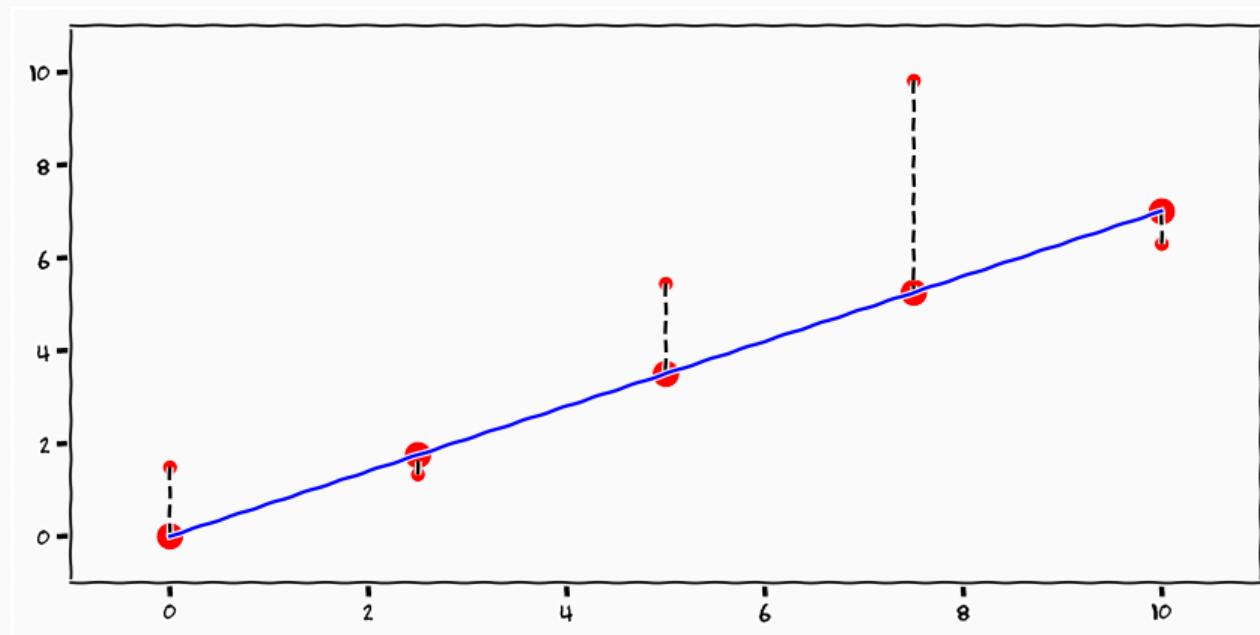
$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$

# The Gaussian Distribution



$$y_i = [a, b] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \cdot)$$

# Decomposition

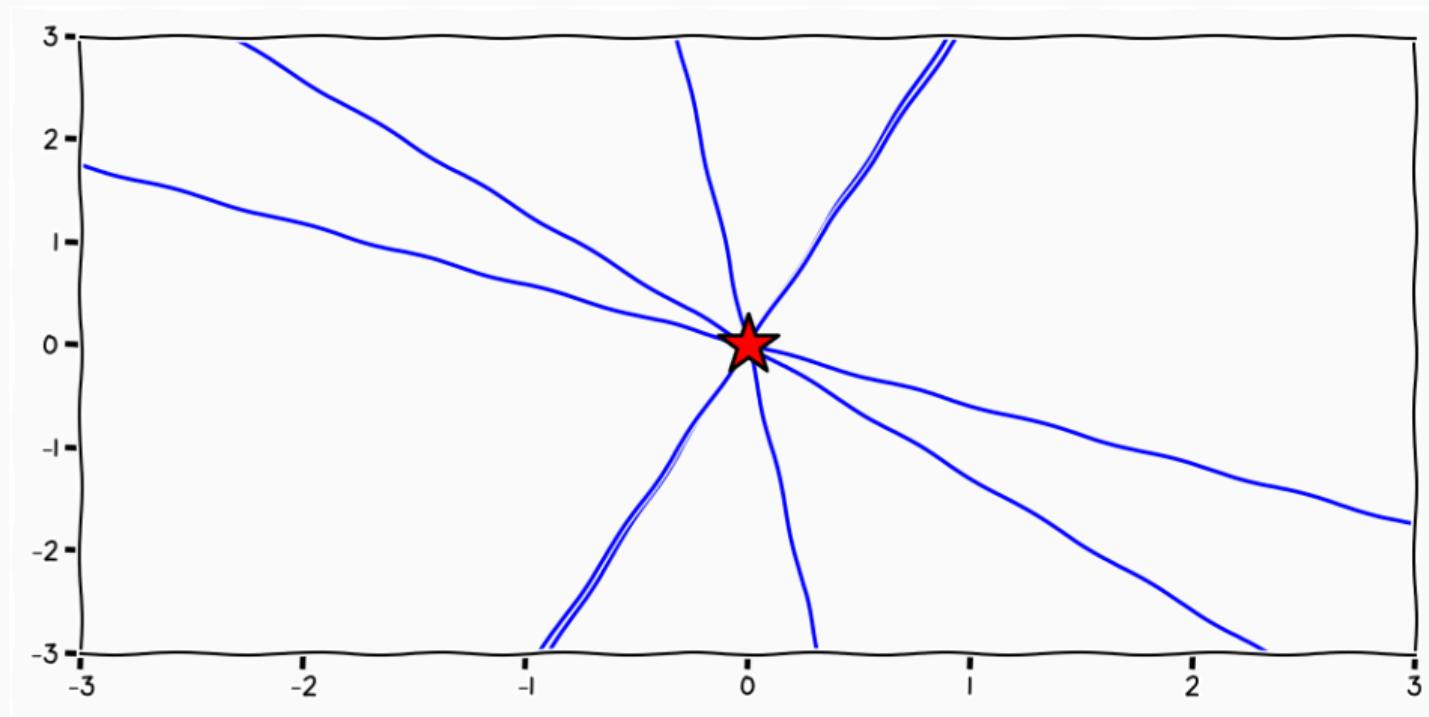


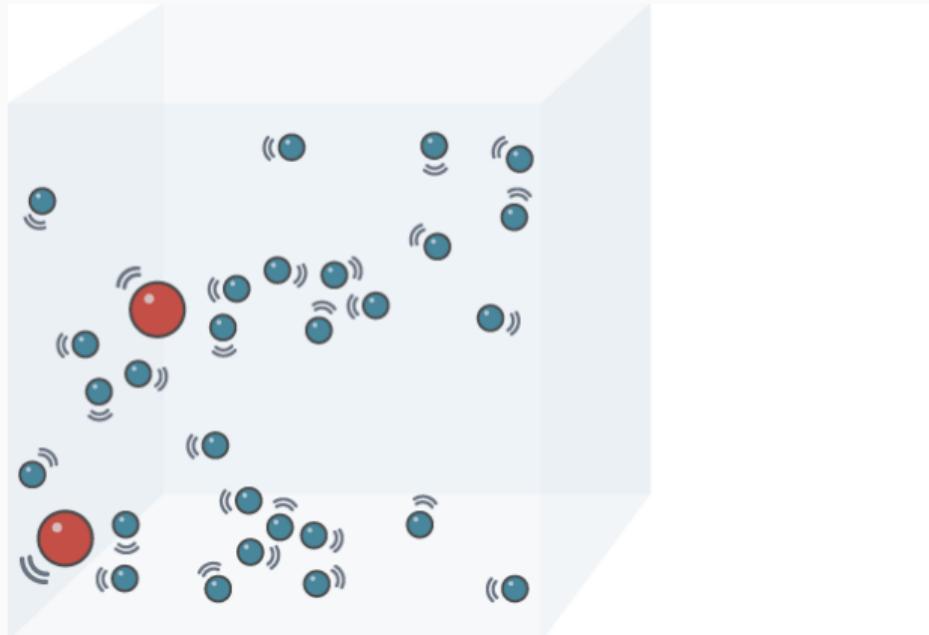
## Probability Model

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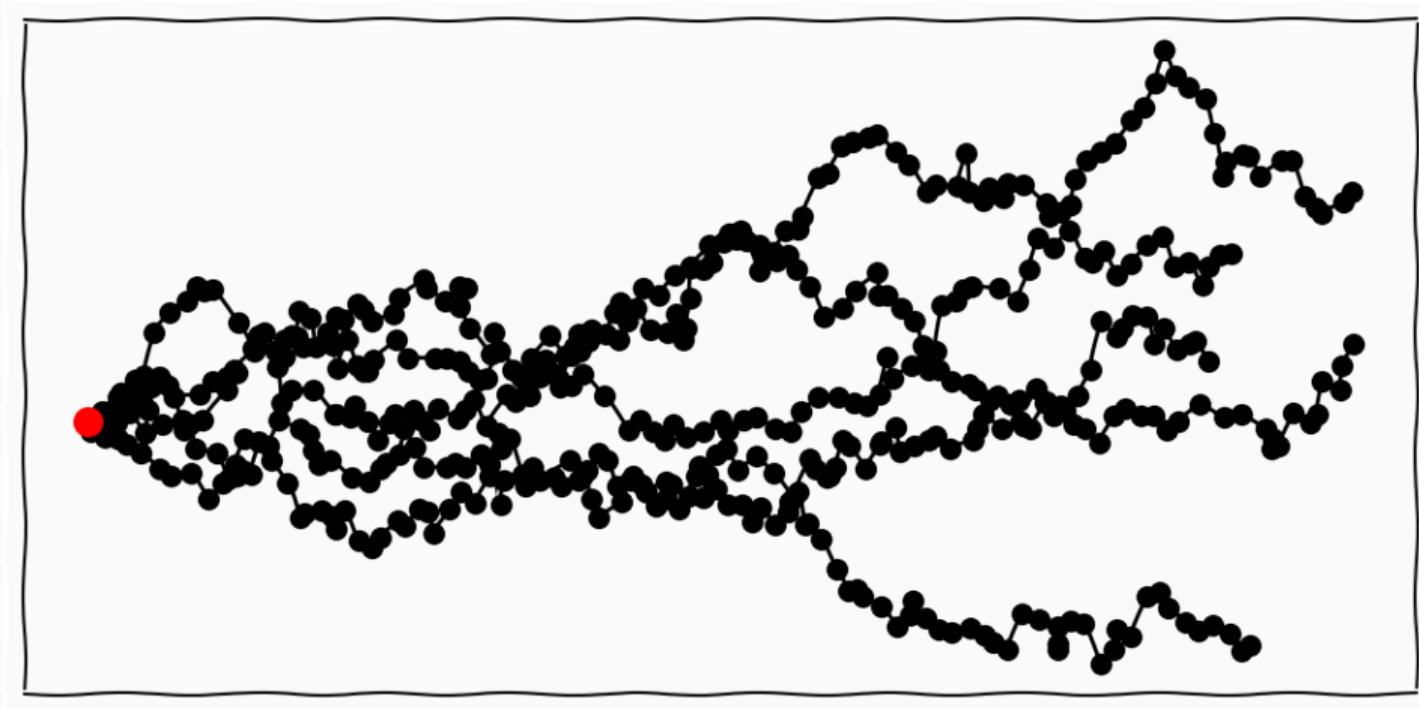
$$p(y_i \mid \mathbf{x}_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - \mathbf{w}^T \mathbf{x}_i)^2}{2\sigma^2}\right)$$

# An Under-determined System



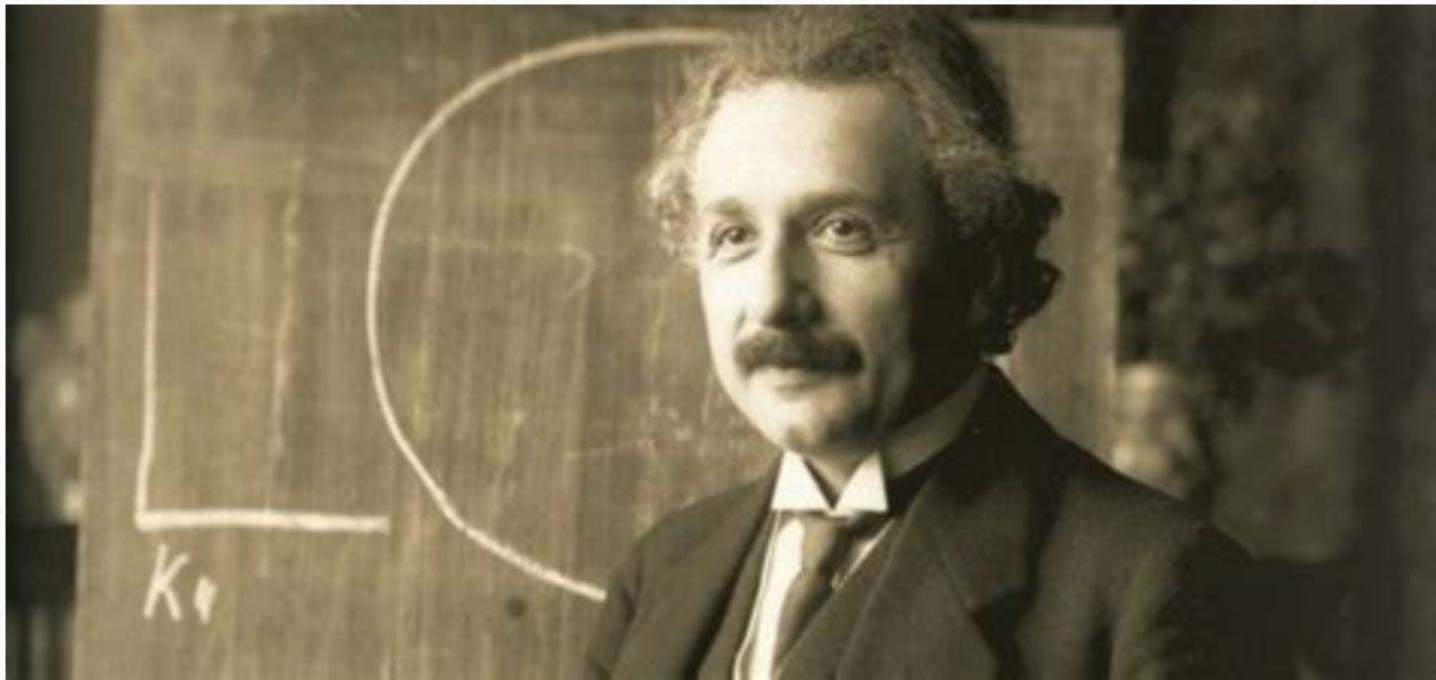


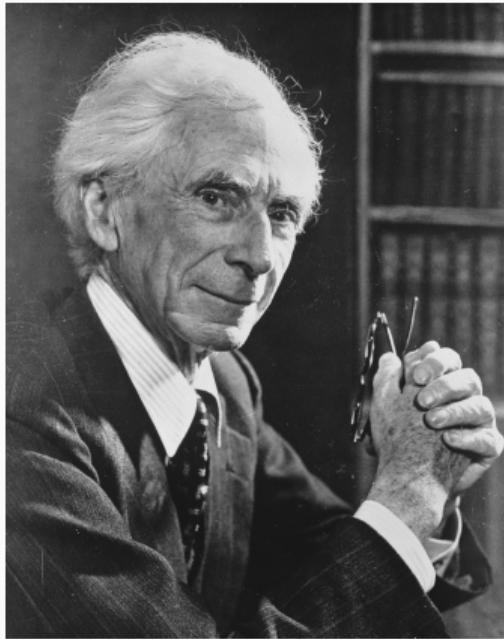
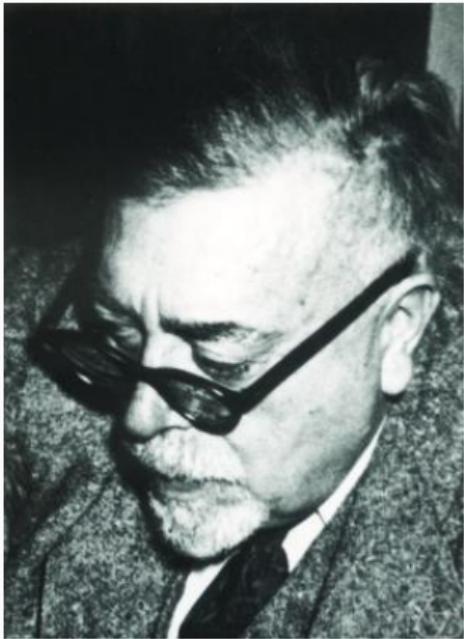




# Einstein

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## Under-determined System

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$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

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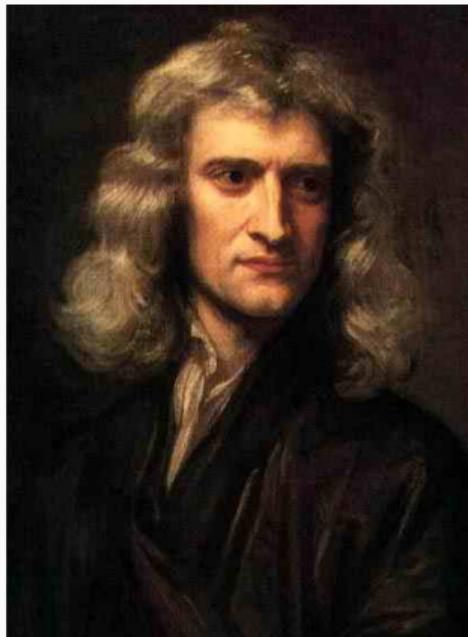
- Parametrisation of ignorance

$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \Sigma)$$

# Machine Learning in the Physical World

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$$y_i = [w_1, w_2] \begin{bmatrix} x_i \\ 1 \end{bmatrix} + \epsilon_i$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \Sigma)$$

$$\epsilon_i \sim p(\cdot)$$

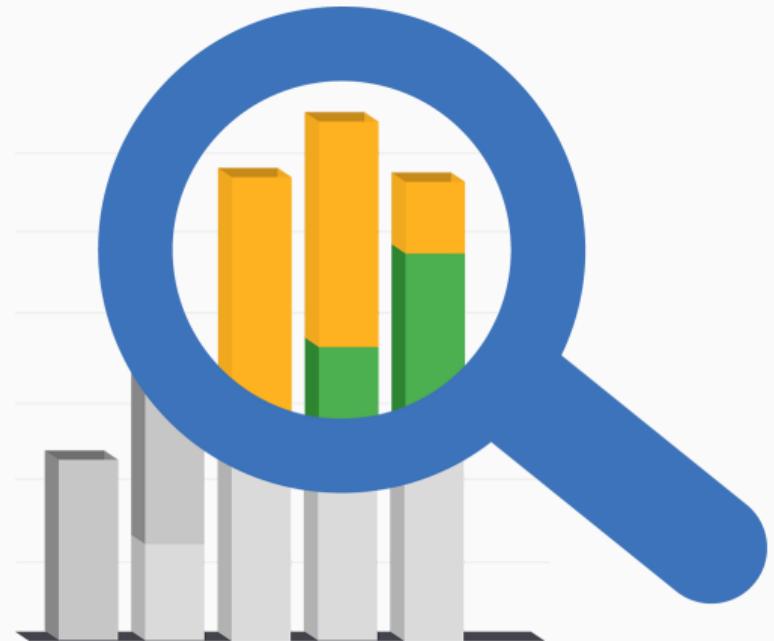
= ~?

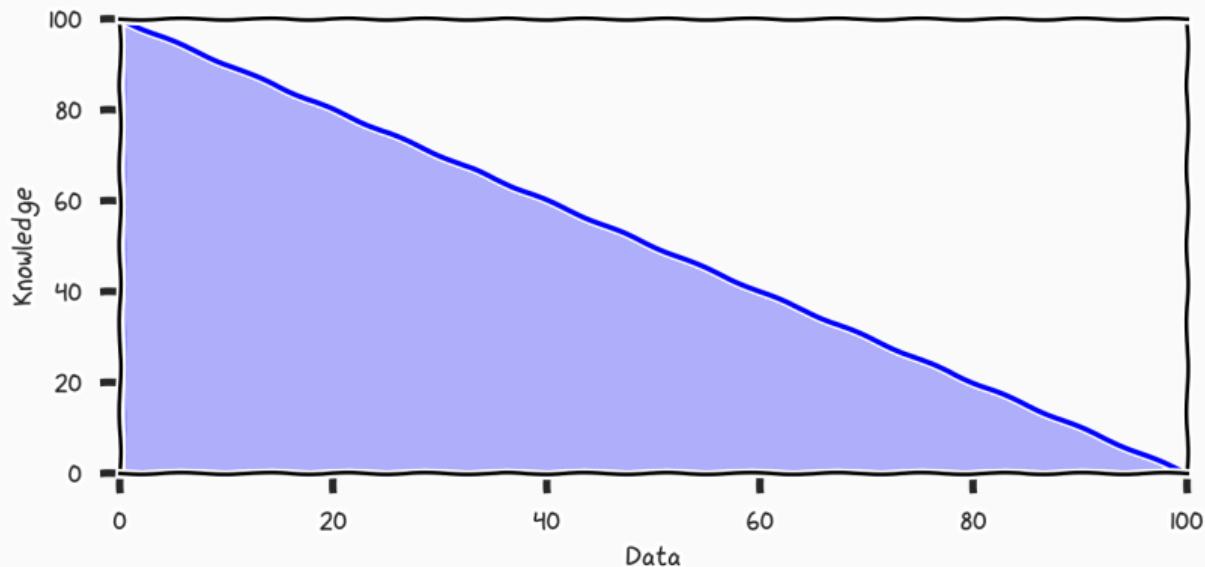
## Machine Learning and the Physical World

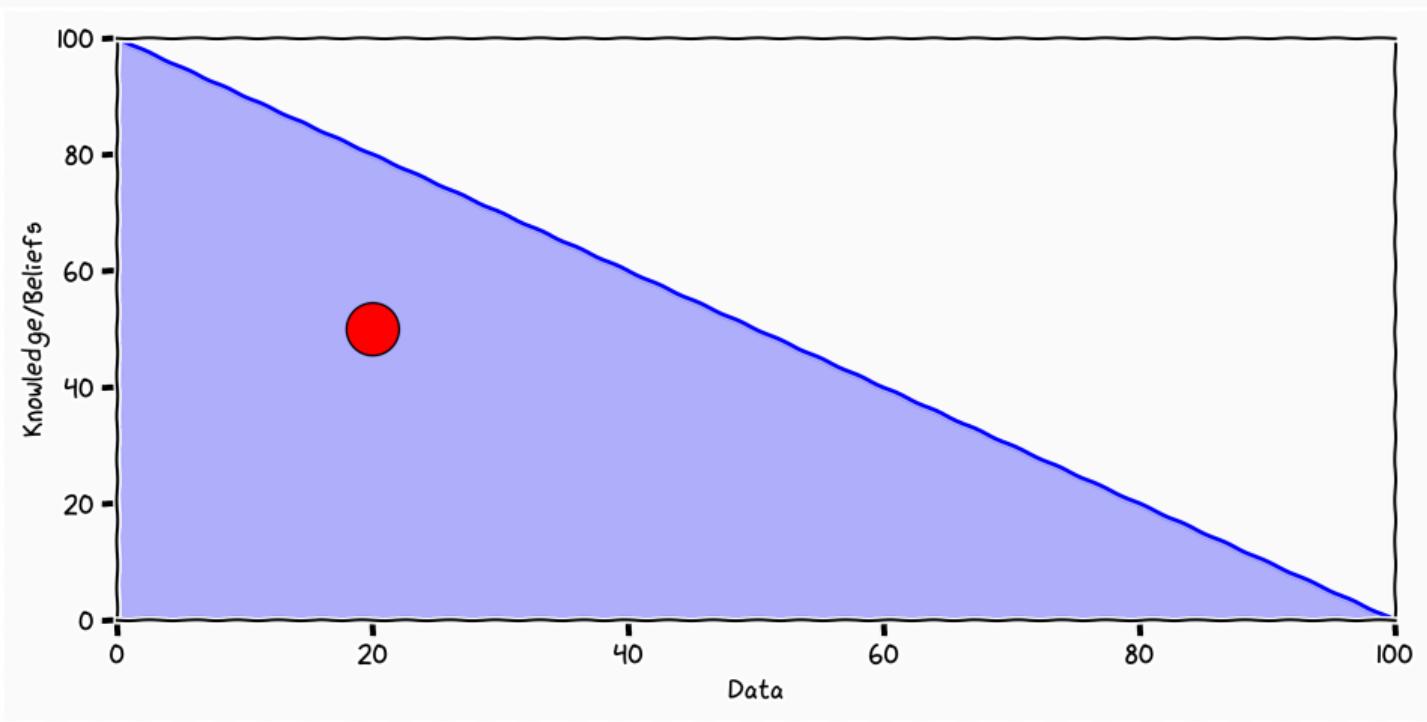
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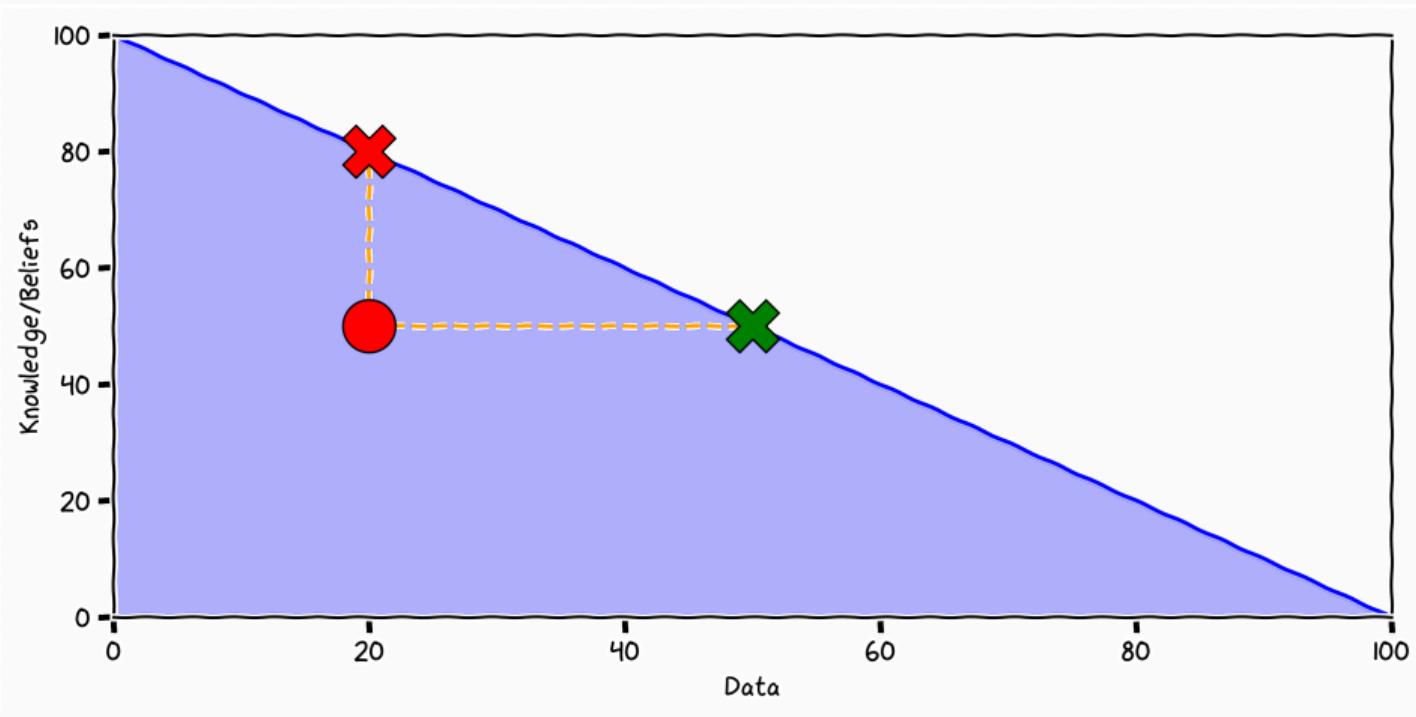
# Machine Learning

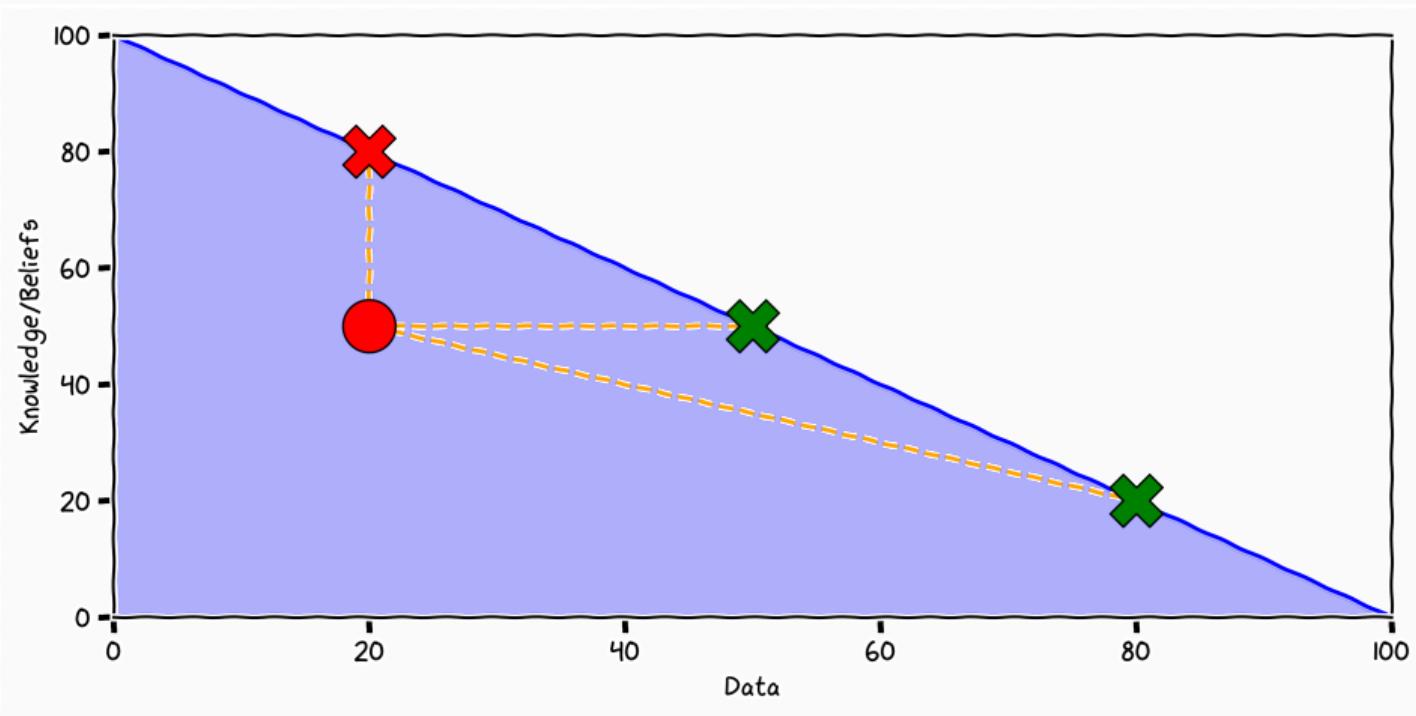
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## Aim of the course

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We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Compute  
Data + Model  $\overbrace{\rightarrow}^{\curvearrowright}$  Prediction

## What is this course not

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- Not about models

## What is this course not

---

- Not about models
- Not about inference

## What is this course not

---

- Not about models
- Not about inference
- Not about specific problems

# Lectures

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**Week 1** Introduction (che29)

**Week 1** Simulation (Nichola)

**Week 2** Quantification of Beliefs  
(che29)

**Week 2** Gaussian Processes  
(che29)

**Week 3** Practical Gaussian  
Processes (che29)

**Week 3** Emulation (che29)

**Week 4** Sequential Decision  
Making Under  
Uncertainty (che29)

**Week 4** Reinforcement Learning  
(che29)

**Week 5** Probabilistic Numeric  
(che29)

**Week 5** Experimental Design  
(Nichola)

**Week 6** Sensitivity Analysis  
(Nichola)

**Week 6** Multi-fidelity Modelling  
(che29)

<https://mlatcl.github.io/mlphysical/>

- Lecture Notes
- Worksheets

## Week 7 Project Guest Lecture

Week 7 Project Guest Lecture

Week 7 Generative Models

**Week 7** Project Guest Lecture

**Week 7** Generative Models

**Week 8** TBA

**Week 7** Project Guest Lecture

**Week 7** Generative Models

**Week 8** TBA

**Week 8** ?

## Assessment (30%)

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- Individual Assessment ( $2 \cdot 15\%$ )  
**Gaussian Processes** deadline 23-29/10  
**Sequential Decision Making** deadline 4-12/11

## Assessment (70%)

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- Group Project (70%)
  - pick your own simulation environment
  - **deadline** 20/1

## Summary

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  - model
  - data

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- The notion of probabilities allows us parametrise our ignorance
- Our ignorance comes from many sources
  - model
  - data
  - compute

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