Statistical Science
1989, Vol. 4, No. 2, 73–86

Francis Galton's Account of the Invention of Correlation

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Abstract. Francis Galton's invention of correlation dates from late in the year 1888, and it arose when he recognized a common thread in three different scientific problems he was studying. Galton's own 1890 account of the moment of discovery is discussed and contrasted with Karl Pearson's widely known association of correlation with a retreat into a recess at Naworth Castle. The circumstances that led Galton to write the account are reviewed.

Key words and phrases: Correlation, Galton, Pearson, regression.

1. GALTON'S INVENTION OF CORRELATION

Francis Galton discovered the concept of correlation in the late fall of 1888. It may seem surprising that such a fundamental and ubiquitous idea is only a century old, but unlike most major conceptual innovations in statistics, its moment of discovery is particularly well-documented. It is rare, not only in statistics but in all of science, for a pathbreaking scientist to leave a frank, detailed, contemporary record of the steps leading to discovery and the flush of excitement at the moment of realization that a discovery had been made. Accounts such as James D. Watson's *The Double Helix* are exceptions to the rule that scientists tend to write of their work long after the event, when the excitement has died away and only ego and the distorted vision of hindsight are available to guide the author and hence the reader. Galton was also an exception.

Galton has long been thought to have given an account of the moment of discovery of correlation in the autobiography he published late in life, *Memories of My Life*. This charming and still readable account, mostly of his early life and adventures, contains a short passage that Karl Pearson fastened onto as describing the moment in question. It comes in a section where Galton is discussing his early use of a "statistical scale" to relate measurements taken under different conditions; his phrase described essentially our now common practice of expressing measurements in terms of the number of standard deviation units from the mean (although Galton used median deviation units from the median). Galton wrote:

> As these lines are being written, the circumstances under which I first clearly grasped the important generalization that the laws of Heredity were solely concerned with deviations expressed in statistical units, are vividly recalled to my memory. It was in the grounds of Naworth Castle, where an invitation had been given to ramble freely. A temporary shower drove me to seek refuge in a reddish recess in the rock by the side of the pathway. There the idea flashed across me, and I forgot everything else for a moment in my great delight.
>
> Galton, 1908, page 300

In his mammoth biography of Galton, Karl Pearson seized on this passage, writing "That 'recess' deserves a commemorative tablet as the birthplace of the true conception of correlation!" (Pearson, 1914–1930, volume 2, page 393), and he dated the incident at Naworth Castle as 1888 or 1889. Pearson's enthusiasm is understandable, but his inference was incorrect. It has been clear for some time (e.g., Hilts, 1973, page 233) that the passage described a different event (probably the simple but equally fundamental idea of a statistical scale), and that it dated from the early or middle 1870s, when Galton began on the quantitative trail that was to lead him through "reversion" to "regression" to "correlation." But if internal evidence in Galton's writings has long permitted the discrediting of the assignment of correlation to Naworth Castle, the true story has remained hidden. In the early 1980s, historian Ted Porter (1986) made a discovery

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while researching for his book on the history of Nineteenth Century statistical ideas. In a massive bibliography of that century's periodical literature (Fletcher, 1893, page 101), Porter found reference to an article of Galton's that had escaped Pearson's notice, perhaps because Galton had omitted it from the bibliography he had included in his autobiography (Galton, 1908, pages 325–331; it is also omitted from that given more recently by Forrest, 1974). Because Galton wrote and published a huge amount, and no existing bibliography is close to complete, such omission is not surprising in some respects. But unlike most of the items I have encountered that these lists overlook, such as Galton (1894), published in the *Homing News and Pigeon Fanciers' Journal*, the article Porter found was not in an obscure place and it was not on an obscure topic. Entitled "Kinship and correlation," it appeared in *The North American Review* in 1890, one of the most prestigious and widely read American periodicals of its time. The article gave nothing less than a detailed account of the exciting discovery Galton had just made, together with a lucid explanation of the nature of the concept and the promise it held for the future of statistics.

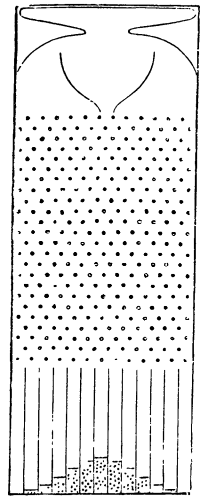

FIG. 1. *A schematic rendition of the design for Galton's first Quincunx built in* 1873. *Shot are dropped down the funnel at the top, and they are deflected by the offset rows of pins before falling into one of the compartments at the bottom. The tendency to produce a normal-like histogram at the bottom illustrates the central limit theorem. From Galton* (1889, *page* 63); *a photograph of the original Quincunx is given in Stigler* (1986, *page* 277).

2. THE BACKGROUND OF GALTON'S WORK

Like all major scientific discoveries, correlation did not appear in a vacuum. It was a concluding step in a 20-year research project. I have described the context and stages of that project rather fully elsewhere (Stigler, 1986, Chapter 8), and I shall only sketch an outline here.

Galton's interest in heredity dates at least from the 1860s, but in the early 1870s he had intensified his quantitative study of the topic. He was faced with a problem; how to reconcile an empirical fact with a mathematical theorem. The fact was that most physical measurements (such as height for men or diameter for seeds) were approximately normally distributed in the populations he studied. The theorem was the central limit theorem, which stated that the normal distribution should arise when an object is subjected to a large number of independent disturbances, no few of them dominant. The problem of reconciliation that confronted Galton was that he believed his physical measurements to be subject to important, even dominant influences in the process of heredity. How could, for example, the dominant factor of father's height be reconciled with the appearance of normality in the offspring that seemed to belie the existence of such a single dominant factor?

Galton resolved this dilemma with the help of an ingenious analogue computer, the Quincunx. In 1873 he had one built along the lines of Figure 1, the form in which it is best known today. In this form it is a device for illustrating the central limit theorem: shot are poured in at the top, cascade over the successive rows of pins (each in principle imparting an independent disturbance to the shot's course), and they arrive at the bottom forming (if all goes well) a heap that resembles a normal curve. The name "Quincunx" was borrowed from agriculture, where it had been used to describe a favored arrangement of planting trees, usually fruit trees, in successively offset equally spaced rows. Galton's Quincunx was a brilliant conception, and it is still a marvelous tool for illustrating lectures. In fact, when I delivered the Fisher Memorial Lecture at University College London in 1986, I was privileged to demonstrate the use of Galton's original 1873 Quincunx to the audience, and it worked perfectly!

The original Quincunx was seriously limited, though, in what it could do, and in 1877 Galton conceived of a variation that was more than a tool for illustration, it was a tool for discovery (Figure 2). I know of no evidence that Galton ever built this variation; it seems likely that it is simply one of the greatest mental experiments in the history of science. The idea he had was this: What if the shot were interrupted at an intermediate level of the Quincunx and held there in compartments? They should display a normal-like outline, simply less disperse than if they were allowed to run the full course. And what if the shot in a single one of these intermediate compartments were released and allowed to fall? They would form a small normal hillock (in Galton's term) directly below the compartment. Now what if all shot in all compartments were released? Each would produce a

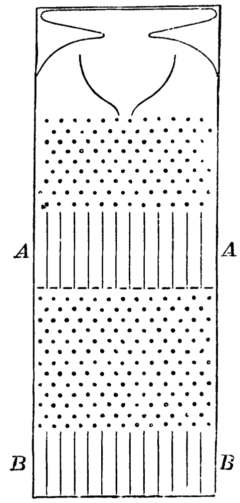

FIG. 2. *A drawing of Galton's two stage Quincunx from Galton (1889, page 63). This device may have never been built by Galton, but it served as a powerful conceptual experiment for the development of regression. Galton described such a device in correspondence as early as January 12, 1877 (Stigler, 1986, pages 278 and 279).*

small normal hillock, and the aggregate of all these normal hillocks should form an outline that would be indistinguishable from that which would have occurred if the shot had not been interrupted at all!

At one level, this may be taken as a proof-by-analogue of a nontrivial mathematical theorem—"A normal mixture of normal distributions is itself normally distributed." But Galton's use was at a deeper conceptual level—it showed how a normal population (the bottom level or a set of human heights) could be taken apart, dissected into smaller normal populations, each of which could be associated with another measurement (the index of the midlevel compartment or the height of a parent). And this association of the (bottom) normal values with the dominant (intermediate) causes could be accomplished without violating the central limit theorem! From this point of view, a table such as Table 1 can be thought of as giving the dissection of a normal population. The normal population is the children's heights; the column totals give essentially the totals for the bottom level of the Quincunx. Each row of the body of the table gives the frequency distribution for a small normal hillock, and the row labels (height of midparent) indicate which intermediate level compartment had given rise to the hillock. This type of table has come to be called a correlation table, but of course it predates the idea of correlation. What then *is* the idea of correlation, what discovery did Galton make in late 1888?

The major components of what we take to be correlation were in place by 1886. The two-stage Quincunx was described by Galton in 1877 (Stigler, 1986, pages 276–281), and by 1886 this conceptual apparatus and a body of empirical work such as that described in Table 1 had led to a rather full development of the ideas of regression: Galton summarized all this work in his book *Natural Inheritance*, published in 1889. By smoothing tables such as Table 1 (adding counts in groups of four adjacent cells, a sort of two-dimensional kernel density estimate), Galton was led to describe the table by a bivariate normal distribution. He also took note of the two regression lines and their relationship to the constants of the normal surface (described in terms of the marginal distribution of one variable and the conditional distribution of the other). But if correlation was not far away, it was still not there, and the word does not appear in *Natural Inheritance*.

What Galton was missing at that stage were three realizations: that the two regression lines had the same slopes (if the axes are interchanged and standard deviation units are used), that the common slope could be used as a single numerical summary of the strength of the relationship between the variables and that the idea was applicable much more generally than simply in problems of heredity. The first two of these would have been easily accessible to Galton in 1886 (indeed the first is implicit in his calculations), but they would have not been more than numerical or algebraic curiosities without the third. It was that third realization, the generality of the problem he had essentially solved, that dawned on Galton late in 1888. As he himself tells us,

> Few intellectual pleasures are more keen than those enjoyed by a person who, while he is occupied in some special inquiry, suddenly perceives that it admits of a wide generalization and that his results hold good in previously unsuspected directions.

3. GALTON'S 1890 ACCOUNT

The story is told in this 1890 article of how, in late 1888, after Galton had parted with the final revision of the page proofs of *Natural Inheritance*, he was simultaneously pursuing two superficially unrelated investigations. One was a question in anthropology: If a single thigh bone is recovered from an ancient grave, what does its length tell the anthropologist about the total height or stature of the individual to whom it had belonged? The other was a question in forensic science: What, for the purposes of criminal identification, could be said about the relationship between measurements taken of different parts of the same person (the lengths of different limbs surely did not constitute independent bits of data for purposes of identification)? Galton recognized these problems were identical, and he set to work on them with a data

TABLE 1
Galton's correlation table

Height of the midparent in inches	Height of the adult child <61.7	62.2	63.2	64.2	65.2	66.2	67.2	68.2	69.2	70.2	71.2	72.2	73.2	>73.7	Total no. of adult children
>73.0	—	—	—	—	—	—	—	—	—	—	—	1	3	—	4
72.5	—	—	—	—	—	—	—	1	2	1	2	7	2	4	19
71.5	—	—	—	—	1	3	4	3	5	10	4	9	2	2	43
70.5	1	—	1	—	1	1	3	12	18	14	7	4	3	3	68
69.5	—	—	1	16	4	17	27	20	33	25	20	11	4	5	183
68.5	1	—	7	11	16	25	31	34	48	21	18	4	3	—	219
67.5	—	3	5	14	15	36	38	28	38	19	11	4	—	—	211
66.5	—	3	3	5	2	17	17	14	13	4	—	—	—	—	78
65.5	1	—	9	5	7	11	11	7	7	5	2	1	—	—	66
64.5	1	1	4	4	1	5	5	—	2	—	—	—	—	—	23
<64.0	1	—	2	4	1	2	2	1	1	—	—	—	—	—	14
Totals	5	7	32	59	48	117	138	120	167	99	64	41	17	14	928

This cross-tabulation was compiled by Galton in 1885 and published in 1886 and again in 1889. It gives the heights of 928 adult children, classified by height of "midparents." All female heights were rescaled by multiplying by 1.08, and midparent heights were computed by averaging the height of the father and the rescaled height of the mother. For more information, see Stigler (1986, especially Table 8.1, page 286).

set he had on measures made on 348 adult males, of their stature and of their left cubit (the distance from elbow to fingertips). The data and Galton's plot of the two regression lines are given in my book (Stigler, 1986, pages 319–320). In this 1890 article Galton described how, while plotting these data, it suddenly came to him that the problem was the same as that he had considered in studying heredity, "that not only were the two new problems identical in principle with the old one of kinship which I had already solved, but that all three of them were no more than special cases of a much more general problem—namely that of Correlation."

The earlier studies of kinship had involved either generational relationships (parent-child) or lateral ones (two brothers). The conceptual apparatus of the two-stage Quincunx had been marvelously liberating in one respect, in freeing Galton from the confinement of the most narrowly construed central limit theorem, but it was limiting in another. It, like most of the kinship studies, involved a directional relationship of quantities of essentially the same kind. The two new problems allowed Galton to view the matter afresh, without such a constraint, and the generality became obvious (at least to a Galton).

There is a breathless quality to part of this narrative.

> Fearing that this idea, which had become so evident to myself, would strike many others as soon as *Natural Inheritance* was published, and that I should be justly reproached for having overlooked it, I made all haste to prepare a paper for the Royal Society with the title of Correlation. It was read some time before the book was published, and it even made its appearance in print a few days the earlier of the two.

Actually, the title of the article was "Co-relations and their measurements, chiefly from anthropometric data." The spelling "correlation" was common at the time (and used by Galton in subsequent writings); indeed I have conjectured that the choice of terms was due to the appeal it held for Galton when he came upon it in a book of Jevons with that spelling, and that the brief usage of "co-relation" was to emphasize the novelty of the concept to which Galton attached the term (Stigler, 1986, pages 297 and 298).

After the account of the discovery, Galton moved on to give a tutorial on the ideas involved. The tutorial is for the most part marvelously clear, and still a model for the exposition of difficult concepts, as well as being quite revealing about how Galton viewed his own creation. To Galton, correlation meant what we might call today intraclass correlation—two variables are correlated because they share a common set of influences. He described the effect of correlation on the dispersion of differences (the difference in heights of two random Englishmen is said to have median 2.4″, the difference in heights of two brothers had median 1.4″). Galton seems to have only conceived of correlation as a positive relationship; negative correlations play no role in his discussion.

Galton gave three examples to illustrate the concept of correlation, examples where he could make concrete the common factors behind the relationship. The first of these, on kinship, seems hazy and unsatisfactory to modern eyes, but perhaps that is because we view the problem through a clarifying lens, Mendelian genetics, that was not available to Galton. The other two examples are superb—the trip time for two clerks travelling home taking the same bus over part of the journey, and the stock portfolios of two investors who hold some shares in the same commercial ventures.

Galton was able to use his examples to underscore the fact that correlation did not in any way depend upon the choice of origin. At first glance he might seem to have faltered on the question of dependence on the scaling of measurements; because of the difference of scales, he tells us, "There is relation between stature and length of finger, but no real correlation." But he quickly recovers and explains that a simple multiplication (to measure the quantities in units of "probable error," where this is a term that denotes a median deviation for a symmetric distribution) will turn the relationship into correlation, and that he will henceforth tacitly assume that has been done.

He also tells us that the concepts only apply to variables that have at least a "quasi-normal" (approximately normal) distribution. Here, as elsewhere in his writings, he is enchanted by this "singularly beautiful law," and we might even accuse him of over-enthusiasm. "Now, when a series of measures are submitted to a competent statistician, it is a very simple matter for him to discover whether they vary normally or not." But in the end he is cautious, and his insistence upon a check of distributional assumptions is too rarely imitated by his descendants.

The article includes a definition of the coefficient of correlation (called an index of correlation here, the term coefficient was applied only in 1892 by Edgeworth, when he introduced the "Pearson" product moment estimator (Stigler, 1986, pages 319–325)). The definition is explained in terms of an example. After explaining the tricky notion of regression toward the mean and how when the variables are measured on the same scale the "ratio of regression measures correlation," Galton goes to the more complicated situation where the scales of dispersion differ. Suppose that in a population of men, we consider the relationship between the length of left middle finger and height. We find that those men whose finger lengths deviate from the average by 1″ have heights that deviate from the average height (in the same direction as the deviation of finger length) by an average of 8.19″. Also, those whose heights are 1″ from the average have finger lengths that deviate (on the average) by 0.06″ from the population average. Galton noted that these, the two regression lines, were quite different relationships; we might write

$$E(Y - \mu_Y \mid X) \doteq 8.19(X - \mu_X),$$

$$E(X - \mu_X \mid Y) = 0.06(Y - \mu_Y).$$

To determine the "index of correlation" it is necessary to take account of the fact that the two dispersions (essentially standard deviations) are in the ratio of 15 to 175. The index then is

$$(0.06) \times \frac{175}{15} = (8.19) \times \frac{15}{175} = 0.7.$$

Returning to his anthropological example, Galton is able to explain how his discovery reveals that the then (and possibly still) current practice of proportional rescaling is erroneous, because it ignores the regression effect. If a thigh bone is 5% longer than an average thigh bone, we should *not* infer that the man was 5% taller than average! Such a practice would tend to overestimate by an amount that is greater, the lower the correlation of the measures.

The article ends with a claim that the methods discussed will be particularly useful for the study of social problems, such as the relationship of poverty and crime, and with an implicit challenge to the reader:

> There is a vast field of topics that fall under the laws of correlation, which lies quite open to the research of any competent person who cares to investigate it.

4. HOW GALTON CAME TO WRITE THE ARTICLE

Galton wrote this article at the age of 68, and although he remained active to the end of his life, 21 years later, this remained his last substantive discussion of correlation. He soon became occupied with the study of fingerprints, and the methods he created were soon taken over and developed much further by Francis Edgeworth, Karl Pearson and G. Udny Yule.

The article "Kinship and correlation" owes its existence to the persistence of the Editor-in-Chief of *The North American Review*, Thorndike Rice. Rice had first approached Galton more than a decade earlier, writing on January 7, 1879,

> Dear Sir,
>
> Your name and your work are so well known and so highly esteemed in this country, that a direct contribution from your pen to the American people would, it is needless to say, be considered by them a great compliment and obligation and command a large and appreciative audience. The growing interest, too, that is manifested here in every species of knowledge can be satisfied only with the views of the authorities in each line of inquiry. We would, therefore, feel honored and gratified, if you would consent to write for *The North American Review* an article on any subject of your choice.

The *Review* is widely known as the medium of the best thought in America, and, as you are probably aware, many distinguished Europeans, among them Mr. Gladstone, have of late contributed to it papers of the first importance.

Hoping that you will grant us an early and favorable reply, I remain, Dear Sir,

Very truly yours,
A. Thorndike Rice

Galton's negative reply of February 7 does not survive. Rice wrote again in March, trying to get Galton interested in writing a paper of a nature similar to his "recent admirable paper on psychometric facts, which I found to be of extraordinary interest." But Rice was unsuccessful, and the matter was dropped for some time.

Rice did not forget about Galton—perhaps Galton's missing replies left a door open—and a decade later he directed his assistant to try again. On March 26, 1889, William H. Rideing wrote, raising the ante:

My dear Sir,

In the absence of A. Rice, Editor-in-chief of the *Review*, I am desired to inquire whether your time and inclinations will allow you to write an article popularizing the results of your most recent researches in reference to hereditary tendencies in man. The article might be called *New Light on Heredity* and the style of it should not be too abstruse for the general reader.

As to length 5000 words could be enough, and the *honorarium* would be twenty-five guineas.

Trusting that this proposition will be agreeable to you, I am faithfully yours,

William H. Rideing

This time Galton took the bait. His draft reply, dated April 28, 1889, survives, although it is not entirely legible. Galton acknowledged Rideing's letter requesting a popular treatment of his recent results and was quite encouraging:

I should not be at all disinclined to do so because I know that my last book could not be properly understood except by those who read it very carefully indeed, and who also have some special knowledge of more than one kind as well. But now that I have solidly discussed the subject I feel able to write more freely; being made secure from the charge of superficiality. What I should be happy to do is to let you have an article with the same title as the book, *Natural Inheritance*—this would adequately convey what I want to express . . .

Galton did not care for the suggested title; he wrote "I do not like the title you suggest of New Light on Heredity—it sounds to me flashy and too ambitious." He continued,

As regards length, I think I could hardly give so much as 5000 words. The article would be more effective if not so long. Anyhow I would wait until I have said my say, letting the length depend on what is really wanted to be said as I hate padding. I understand the honorarium in any case to be 25 guineas.

On hearing the reply I will begin the article.

Rideing replied eagerly on May 15, 1889, but the title remained a sticking point.

Though not insisting upon it, however, we should like a fresher title than that which you propose. *All* our readers are not scholars and philosophers, and some of them need to be attracted by the promise of entertainment in the captions of the articles.

Regarding the *honorarium*, it will still be twenty-five guineas, even though the article measures less than 5000 words.

But the sailing was not to be so smooth. In November, Galton wrote that he had tried to write the promised article but failed. But the door was not closed—a note of Galton's indicates he wrote that he "might write an article on variability." Rideing was agreeable, writing in December,

We are glad to accept your suggestion of an article on variability and do not doubt that you will be able to deal with the subject in a way which will enlighten and at the same time amuse even superficial and lazy readers, of whom the world has more than a few.

Evidently Galton changed his mind one final time, for although there are no further letters in this file at the Galton Archives, the publication in early 1890 would have left little time for vacillation. Once he started, "Kinship and correlation" must have flowed out easily. We tend these days to speak of Editors in curses with our teeth clenched, but we are in debt to the Editors of *The North American Review* for this splendid exposition of a subtle and difficult topic. One such success can forgive a multitude of sins.

ACKNOWLEDGMENTS

I am grateful to the Library, University College London, for permission to reprint correspondence from the Galton Papers and to the University College Archivist, G. M. Furlong, for assistance in deciphering Galton's pencil notes of a century ago. This research was supported in part by National Science Foundation

Grant DMS-86-01732. This manuscript was prepared using computer facilities supported in part by National Science Foundation Grants DMS-86-01732 and DMS-87-03942 to the Department of Statistics at The University of Chicago.

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