

tl;dr

Compression serves as a compelling theoretical rationale for interaction, but researchers should still include a product term in logit models to allow the model to represent non-interactive (i.e., non-compressive) relationships.

Compression and Conditional Effects

Prior work recognizes that logit models automatically include interaction, even without a product term. This has led these researchers to ask an important question:

Key Question: When it is important to include a product term in logit models to test an interactive hypothesis?

Literature: Include a Product Term... Sometimes

The table below summarizes the current advice that depends on the substantive claims of the researchers. I focus on the following advice from Berry, DeMeritt, and Esarey (2010, p. 261):

[A] researcher...may base his hypothesis that independent variables interact in influencing $\Pr(Y)$ strictly on an expectation of compression. In this case, there is *no need to include a product term* in the model.

Situation	Description	Include a Product Term?
Interaction in Influencing the Latent Outcome	Guided by a strong theory , the analyst hypothesizes that the explanatory variables interact in influencing the latent outcome (e.g., random utility framework).	Yes
Interaction Due to Compression Alone	Guided by a strong theory , the analyst hypothesis that the explanatory variables interact in influencing the probability of an event due to compression alone.	No
Specification Ambiguity	Guided only by weak intuition , the analyst hypothesizes that the explanatory variables interact in influencing the probability of an event , but have no strong rational for the functional form.	Yes

My Suggestion: Always Include a Product Term

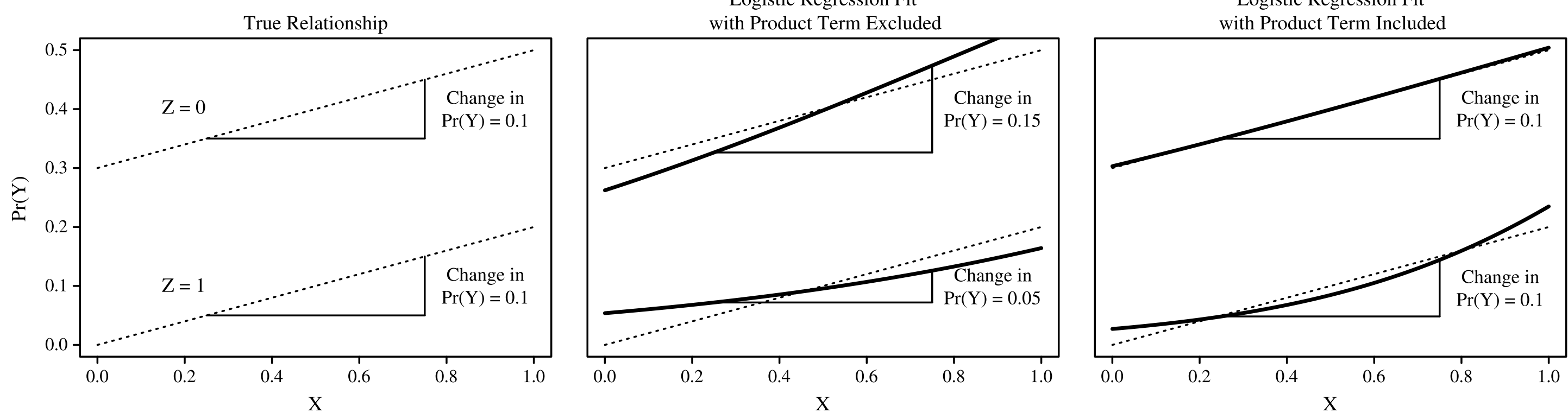
1. Researchers should use a statistical model that can represent (i) the theorized relationship and (ii) plausible alternatives (e.g., null hypothesis).
2. Logit models with no product term cannot represent non-interactive relationships.

$$\frac{\partial^2 \Pr(Y)}{\partial X \partial Z} = \Pr(Y)[1 - \Pr(Y)][1 - 2\Pr(Y)]\beta_x \beta_z$$

3. Logit models with a product term can represent non-interactive relationships.

$$\begin{aligned} \frac{\partial^2 \Pr(Y)}{\partial X \partial Z} &= \Pr(Y)[1 - \Pr(Y)]\beta_{xz} \\ &+ \Pr(Y)[1 - \Pr(Y)][1 - 2\Pr(Y)](\beta_x + \beta_{xz}Z)(\beta_z + \beta_{xz}X) \end{aligned}$$

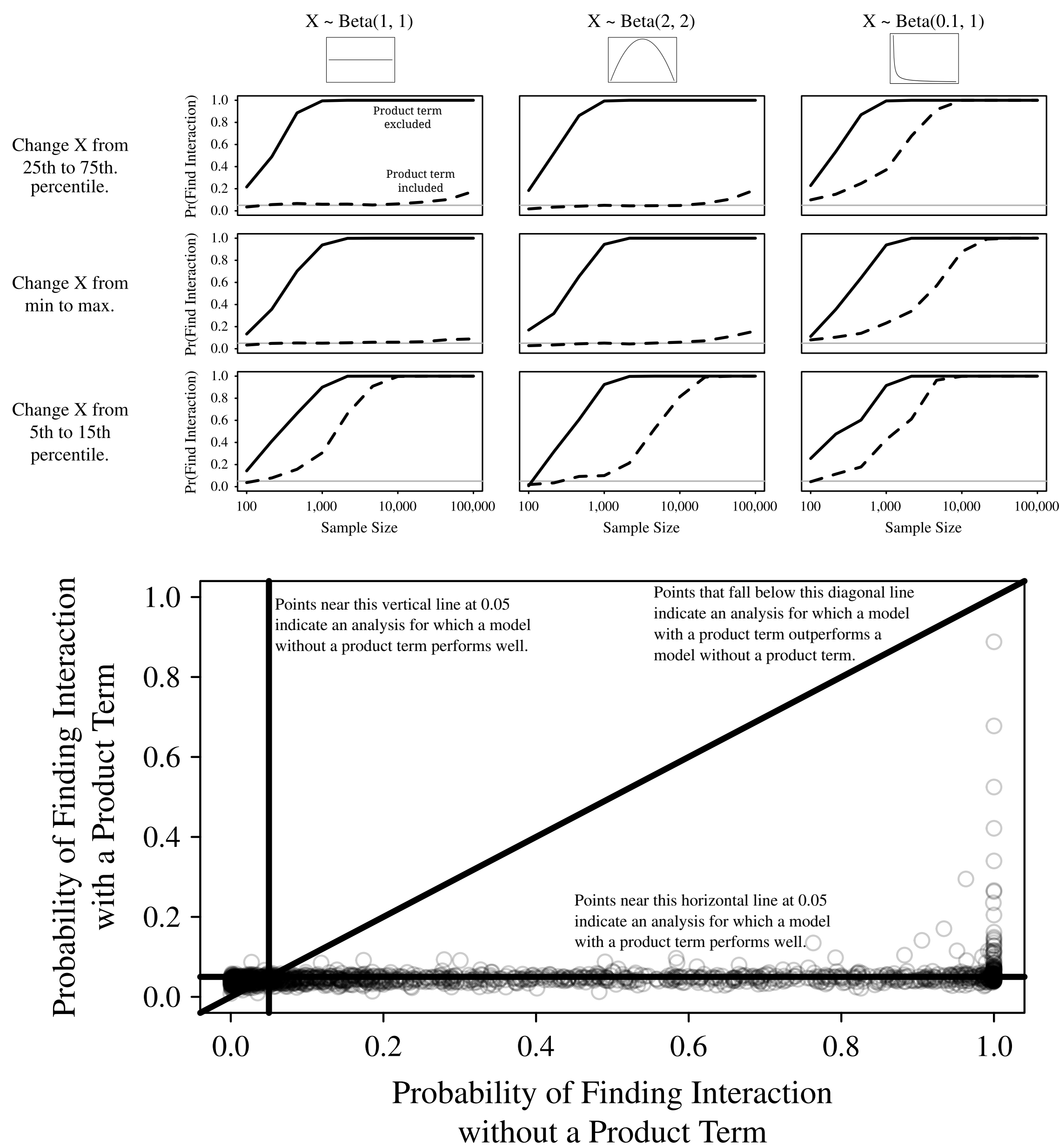
Key Point: Counter-intuitively, a product term allows the model to effectively represent non-interactive relationships in influencing $\Pr(Y)$.



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paper, code, and data at crain.co/compress

Simulations



Study 1

Fix the data-generating process (DGP) and vary the distribution of X , the sample size, and the quantity of interest (QI). Notice that except for unusual distributions of X and unusual QIs, including a product term removes almost all of the bias toward finding interaction.

Study 2

Vary the DGP along with the distribution of X , the sample size, and the QI. Notice that for a wide range of DGPs, including a product term removes almost all of the bias. Further, including a product term seems “never worse” than excluding the product term.

Democracy, Distance, and Conflict

Hypotheses

1. Regardless of the distance between the two countries in a dyad, as the joint level of democracy in the dyad increases, the probability of conflict in that dyad decreases.
2. Regardless of the level of democracy in the dyad, as the distance between the countries in the dyad increases, the probability of conflict in that dyad decreases.

Combining these two hypothesis leads to a third interactive hypothesis.

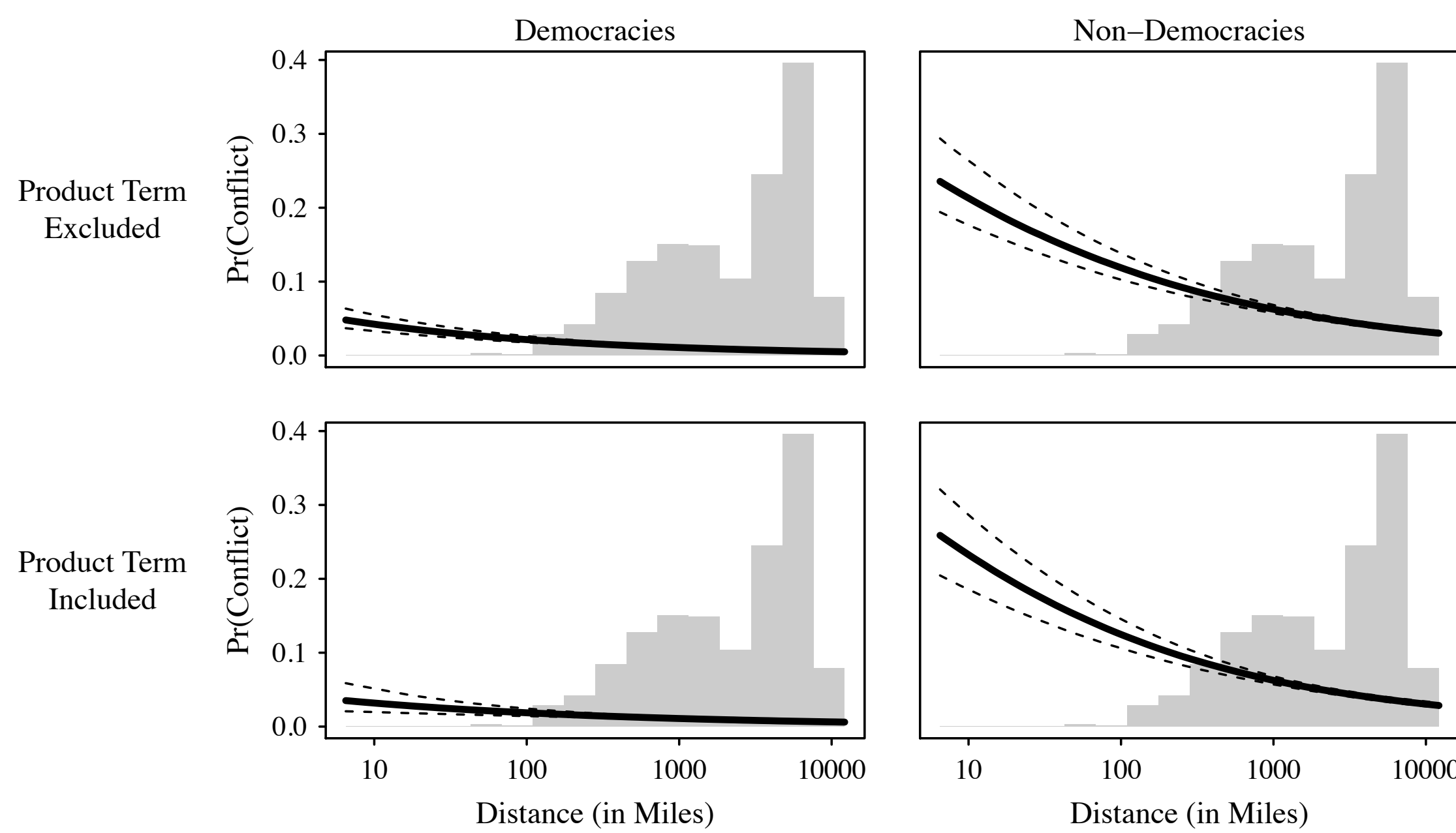
3. The negative effect of distance on the probability of conflict is larger (more negative) in non-democratic dyads than in democratic dyads.

The Temptation to Drop the Product Term

1. The theory suggests the product term is unnecessary.
2. The AIC suggests the models fit equally well and the BIC points toward the smaller model.
3. The product term is not statistically significant.

In this case, though, parsimony is not preferable.

Because the model with no product term would point toward interaction regardless of the data, the model with a product term provides more evidence for the hypothesis.



References

Berry, William, Jacqueline H. R. DeMeritt, and Justin Esarey. 2010. “Testing for Interaction in Binary Logit and Probit Models: Is a Product Term Essential?” *American Journal of Political Science* 54:248–266.