For a monotonic likelihood $p(y|\beta)$ increasing in β_s, proper prior distribution $p(\beta|\sigma)$, and large β_s, the posterior distribution of β_s is proportional to the prior distribution for β_s, so that $p(\beta_s|y) \propto p(\beta_s|\sigma)$.

$$p(y_{new}) = \int_{-\infty}^{\infty} p(y_{new}|\beta)p(\beta)d(\beta)$$

$$p^*(y_{new}) = \int_{-\infty}^{0} p(y|\beta_s, \hat{\beta}_{-s}^{mle}) p(\beta_s|\beta_s \leq 0) d(\beta_s)$$