Power series

1. Solve $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 0$ by assuming the solution takes the form of a power series centered at x=0. You will not be able to find numerical values for the coefficients, you just need to find equations that relate the coefficients of higher powers of x to coefficients for lower powers of x. (Find what c_{n+2} is in terms of n and c_n, for example.)

2. Solve $\frac{dy}{dx} - 2y + \cos(x) = 0$ by assuming the solution takes the form of a power series centered at x=0. You will not be able to find numerical values for the coefficients, you just need to find equations that relate the coefficients of higher powers of x to coefficients for lower powers of x. (Find what c_{n+1} is in terms of n and c_n, for example.)

Fourier Series

3. Find the solution to $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 2y = \frac{1}{2}\sin(2x)$ over the region $0 \leq x \leq 2\pi$ by assuming your solution can take the form of a Fourier series. This problem was taken from this differential equations resource.

4. By assuming your solution can take the form of a Fourier series over the region $0 \leq x < 2$ find the solution to

$$\frac{d^2y}{dx^2} + \omega^2 y = \begin{cases} 1 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \end{cases}$$

   ω is a constant, you may leave your solution in terms of this value.

Thought exercises

5. Try solving problem 2 by assuming the solution instead takes the form of a Fourier series over the region 0 to 2π. What differences do you notice? Why could we get numerical value for the coefficients for one case and not the other? In cases like this where you could assume either a Fourier or power series to find a solution for the ODE, what other clues could you use to determine which guess is better (beyond the problem telling you to use one or the other)?