Transformations

- 1. Determine which (if any) of the following operators correspond to linear transformations on some arbitrary function f(x):
 - (a) $\hat{A} = \frac{d}{dx}$
 - (b) $\hat{B} = \frac{d}{dx}x$
 - (c) $\hat{C} = \int dx$
- 2. Consider an operator \hat{O} that acts on a four-dimensional vector space with basis functions $|\phi_k\rangle$ for k=1,2,3, and 4. Somehow you know the operator acts on each of the basis functions according to

$$\hat{O}|\phi_1\rangle = i|\phi_2\rangle + 3|\phi_3\rangle \tag{1}$$

$$\hat{O} |\phi_2\rangle = -i |\phi_1\rangle - 2i |\phi_3\rangle \tag{2}$$

$$\hat{O} |\phi_3\rangle = 3 |\phi_1\rangle + 2i |\phi_2\rangle + |\phi_3\rangle \tag{3}$$

$$\hat{O}|\phi_4\rangle = \frac{\pi}{2}|\phi_4\rangle \tag{4}$$

- (a) Assuming these four basis functions form an orthonormal set $(\langle \phi_i | \phi_j \rangle = \delta_{ij})$, write the matrix representation for the operator \hat{O} in this basis.
- (b) Determine the action of this operator on the vector

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\phi_1\rangle - |\phi_3\rangle \right) \tag{5}$$

Classes of Operators

3. In the notes (Example 5.1) we found the following form for a matrix that rotates a vector by some angle θ about the y-axis:

$$\mathcal{R}(\theta) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}.$$
 (6)

Show that this matrix is a unitary matrix.

4. Let $|a\rangle$, $|b\rangle$, and $|c\rangle$ be vectors defined in terms of the Cartesian basis vectors according to

$$|a\rangle = \frac{1}{\sqrt{2}}|x\rangle + \frac{1}{\sqrt{2}}|y\rangle \tag{7}$$

$$|b\rangle = \frac{1}{\sqrt{2}}|x\rangle - \frac{1}{\sqrt{2}}|y\rangle \tag{8}$$

$$|c\rangle = |z\rangle$$
. (9)

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③ ③ ⑤

Let the vector space V consist of the span of $|a\rangle$ and $|b\rangle$ and the vector space W of the span of $|a\rangle$ and $|c\rangle$.

- (a) Write the operators for the projection onto each of these two vector spaces, V and W.
- (b) Determine the matrix representation for each of these two operators.
- (c) Using either the operator you obtained in (a) or the matrix you obtained in (b), obtain the projection of

$$|\omega\rangle = \frac{1}{\sqrt{3}} (|x\rangle + |y\rangle + |z\rangle)$$

onto each of the vector spaces V and W.