1. Consider the function
$$f(x) = 2\cos(x) + \sin(2x)\,. \tag{1}$$
Use Taylor expansions to derive three different quadratic approximations to this function. Think about using important points of the function (i.e. intercepts or stationary points) as starting points for Taylor expansion. How does the approximation differ based on the point chosen around which to Taylor expand? Feel free to use an online graphing calculator to help.

2. Consider the differential equation
$$\dot{f}(t) = f(t) - t \tag{2}$$
with the initial condition $f(t = 0) = \frac{1}{2}$.
   (a) Use the Euler method to approximate the function $f(t)$ from $t = 0$ to $t = 1$ using time steps of $\Delta t = 0.2$ and $\Delta t = 0.5$.
   (b) Plot both approximations as well as the analytical function, $f(t) = -\frac{1}{2}e^t + t + 1$. Comment on the qualities of the approximations. How could we improve our approximation?

3. Solve the self-consistent equation
$$x = \tanh(ax)\,, \tag{3}$$
where a is a parameter that is greater than 0. How do the solutions (and number of solutions) change with different values of a?

4. Repeat the first exercise with the function
$$f(x) = \frac{1}{4}x^4 - \frac{1}{6}x^3 - \frac{5}{4}x^2 - x + 1\,. \tag{4}$$