- 1. If we are given z = z(x, y) and y = y(x) solve for $\frac{dz}{dx}$.
- 2. Use the equality below

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx = \int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

to compute the following integral

$$\int_{-\infty}^{\infty} e^{-ax^2 + bx} dx.$$

3. Suppose that $f(x,y) = \frac{\cos x}{\ln(y)}$. Prove the statement below and interpret it geometrically

$$\left(\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y}\right)_x\right)_y = \left(\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x}\right)_y\right)_x.$$

- 4. Set up integrals for the following values:
 - (a) The **mass** of a three dimensional ball of radius R with density equal to the distance from its surface.
 - (b) The **volume** of the earth with a latitude greater than 60 (e.g. within 30 degrees or $\frac{\pi}{6}$ radians of the north or south pole.) Assume the earth has radius r.
 - (c) The **mass** of a cylinder with radius R and height H with density equal to the square root it's height. We haven't discussed cylindrical coordinates, but see if you can reason through it. (Hint: start with polar coordinates).
- 5. Suppose it is a very sunny day and the incidence of UV radiation on a beach is given by $U(x,y)=x^2e^y-xy^3$. Naturally, you are enjoying your beach day by running in a circle at constant speed: $x(t)=\cos t$ and $y(t)=\sin t$. Find $\frac{dU}{dt}$ in two ways:
 - (a) By the chain rule.
 - (b) By finding U explicitly as a function of t and differentiating.
- 6. **challenge:** Integrate

$$\int e^x \cos x \, dx$$

7. **challenge:** Show that $\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$. Assume that x, y, and z are interdependent variables, e.g., they are functions of each other. If it makes it easier to conceptualize, imagine there exists some function f(x,y,z), though you won't need to invoke f. (Hint: consider the total derivative of dx and dy and substitute.)

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