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Modelling of Actuator Dynamics for Spacecraft Attitude Control

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I. Introduction

SPACECRAFT mission success is often highly dependent on the performance and robustness of the attitude control system, which consists of different types of actuators, such as reaction control thrusters, reaction wheels, and magnetic actuators. Solutions to spacecraft attitude control problems often rely on inherent assumptions that the onboard actuators are able to deliver the exact torque desired by the attitude controller at a specified time. The actuators are thus assumed to have no dynamics, or the actuator dynamics are assumed to be fast enough to allow them to be neglected. Although this has shown to be a sufficient approach in the past, it is obvious that attitude actuators, as all electromechanical devices, have dynamics that might impact controller performance. With an increasing demand for high-precision attitude control for purposes such as formation control and optical intersatellite links, the necessity of including actuator dynamics within the control solution is increasingly evident.

Mathematical modelling of the complex and nonideal dynamical behavior of actuators and its influence on spacecraft attitude control is a cumbersome task. This dynamical behavior has traditionally been found through laboratory testing, and the subsequent controller design has been influenced by these considerations. One example in this direction is the requirement of reaction thruster response delays being less than the duration of the minimum activation pulse of the actuator. There exist, however, theoretical results from the modelling and analysis of actuator dynamics, such as in [1], in which the effect of unmodelled fast actuator dynamics on the output feedback stabilization of feedback linearizable systems is studied. Similarly, in [2,3], the robust stabilization of a class of nonlinear systems in the presence of unmodelled actuator and sensor dynamics is investigated. More applicable results for our purpose are found in [4,5], in which general multiple-input/multiple-output (MIMO) linear actuator models for underwater vehicle thrusters with dynamics are presented. Because underwater vehicle thrusters are essentially propellers connected to dc motors, analogous to, for example, reaction wheels for spacecraft attitude control, is it possible to describe other actuators with the same model subject to minor changes and tuning.

In this paper, we substantiate a unified mathematical model of various attitude control actuators for space applications, in particular, reaction thrusters, reaction wheels, and magnetic torquers. The general actuator dynamical model is based on the marine technology work of [5], appropriately fitted to the aforementioned actuator categories. To describe time delays in the response of actuators such as, for example, thrusters, an expansion of the general actuator model is suggested.

II. Mathematical Preliminaries

In this paper, reference coordinate frames are denoted by $\mathcal{F}$, and we denote by $\boldsymbol{\omega}_{b,a}^{c}$ the angular velocity of $\mathcal{F}_a$ relative to $\mathcal{F}_b$, referenced in $\mathcal{F}_c$. Matrices representing coordinate transformation between $\mathcal{F}_a$ and $\mathcal{F}_b$ are denoted by $\mathbf{R}_a^b$. When the context is sufficiently explicit, we may omit function arguments. To form the basis of our spacecraft attitude model, we use the standard definition of the Earth-centered inertial frame $\mathcal{F}_i$ (cf. Fig. 1), with the z axis toward celestial north, normal to the equatorial plane. The spacecraft is assumed to travel in a general elliptic Earth orbit, and we employ a standard local vertical/local horizontal definition of the spacecraft orbit reference frame $\mathcal{F}_o$, with unit vectors

$$\mathbf{x}_o = \mathbf{y}_o \times \mathbf{z}_o, \qquad \mathbf{y}_o = \mathbf{h}/h \quad \text{and} \quad \mathbf{z}_o = -(\mathbf{r}_c/r_c) \tag{1}$$

where $\mathbf{h} = \mathbf{r}_c \times \dot{\mathbf{r}}_c$ is the angular momentum vector of the orbit, and $h = |\mathbf{h}|$. We also define a spacecraft body frame $\mathcal{F}_b$ with an origin in the spacecraft center of mass and the axes fixed to the spacecraft body. With the assumptions of rigid body movement, the dynamical model of a spacecraft can be found from Euler's momentum equation [6]:

$$\mathbf{J}\,\dot{\boldsymbol{\omega}}_{i,b}^{b} = -\boldsymbol{\omega}_{i,b}^{b} \times (\mathbf{J}\boldsymbol{\omega}_{i,b}^{b}) + \boldsymbol{\tau}_d^b + \boldsymbol{\tau}_a^b \tag{2}$$

$$\boldsymbol{\omega}_{o,b}^{b} = \boldsymbol{\omega}_{i,b}^{b} + \mathbf{R}_i^b \boldsymbol{\omega}_{i,o}^{i} \tag{3}$$

where $\mathbf{J}$ is the spacecraft inertia matrix, and $\boldsymbol{\omega}_{i,b}^{b}$ is the angular velocity of the spacecraft body frame relative to the inertial frame, expressed in the body frame. The parameter $\boldsymbol{\omega}_{i,o}^{i}$ is the orbit angular velocity, $\boldsymbol{\tau}_d^b$ is the disturbance torque (e.g., gravity, atmospheric drag, etc.), and $\boldsymbol{\tau}_a^b$ is the actuator torque.

III. Actuator Torques

The actuator torque term $\boldsymbol{\tau}_a^b$ in Eq. (2) represents the input point for the desired actuator torque as designated by the attitude controller. This torque may be described in general for all actuators as

$$\boldsymbol{\tau}_a^b = \mathbf{B}_a(t)\mathbf{u} + \mathbf{D}_a(t)\boldsymbol{\omega}_{i,b}^{b} \tag{4}$$

where $\mathbf{u}$ is the control input, $\mathbf{B}_a(t)$ is the actuator matrix, and $\mathbf{D}_a(t)$ is the disturbance matrix representing cross-coupling effects on angular velocities. For generality, the torque equation is specified with time-varying matrices, although this property is dependent on the particular actuator in question.

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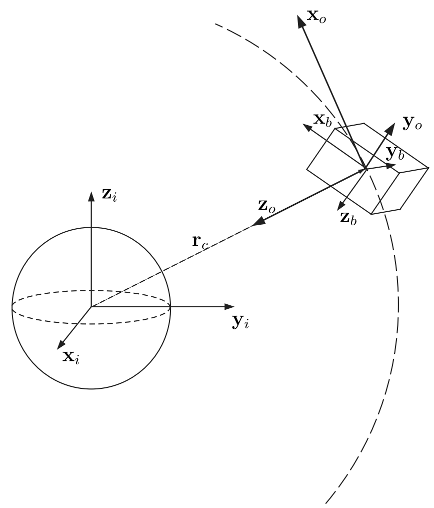

Fig. 1 Coordinate reference frames.

A. Reaction Thrusters

Reaction thrusters produce thrust by expelling propellant in the opposite direction, as a result of a chemical reaction or thermodynamic expansion [7]. One important term regarding thrusters is the minimum impulse bit, which is the integral of thrust over the minimum time the thruster is turned on for acceptably repeatable pulses. Thruster dynamics are nonlinear; however, they can be used in a quasi-linear mode by pulse-width modulation (PWM), that is, modulation of the width of the activated reaction pulse proportional to the control input. Torque components provided by the ith thruster can be written as follows [6]:

$$\begin{aligned}\boldsymbol{\tau}_{ti} &= \mathbf{r}_i \times \mathbf{F}_i = \mathbf{B}_{ti}\mathbf{F}_i \\ &= \begin{bmatrix} r_{yi}\sin(\beta_i)\cos(\alpha_i) - r_{zi}\sin(\alpha_i) \\ r_z\cos(\alpha_i)\cos(\beta_i) - r_{xi}\cos(\alpha_i)\sin(\beta_i) \\ r_{xi}\sin(\alpha_i) - r_y\cos(\alpha_i)\cos(\beta_i) \end{bmatrix}\mathbf{F}_i \end{aligned} \tag{5}$$

where the vector $\mathbf{r}_i = \mathbf{i}r_{xi} + \mathbf{j}r_{yi} + \mathbf{k}r_{zi}$ describes the placement of the ith reaction thruster in the spacecraft from the center of mass. The components of the thruster may be designated by

$$\mathbf{F}_i = \begin{bmatrix} F_{xi} \\ F_{yi} \\ F_{zi} \end{bmatrix} = F_i \begin{bmatrix} \cos(\alpha_i)\cos(\beta_i) \\ \sin(\alpha_i) \\ \cos(\alpha_i)\sin(\beta_i) \end{bmatrix} \tag{6}$$

where F_i is the thrust level, β_i is the elevation, and α_i is the azimuth (cf. Fig. 2). Total thrust from an arbitrary number of thrusters can be expressed on the general actuator form in Eq. (4) as the sum of Eq. (5), such that

$$\boldsymbol{\tau}^b_{a,t} = \sum_{i=1}^{N} \boldsymbol{\tau}_{ti} = \sum_{i=1}^{N} \mathbf{B}_{ti}\mathbf{F}_i = \mathbf{B}_t\mathbf{u} \tag{7}$$

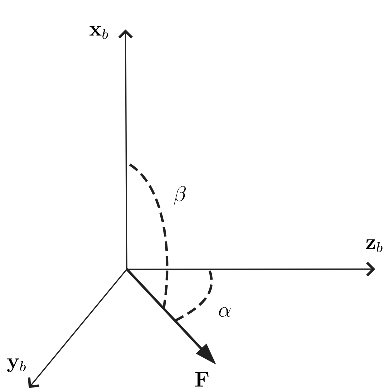

Fig. 2 Thruster azimuth and elevation.

where $i = 1, 2, 3, \ldots, N$, with N as the number of thrusters. Note that, in reference to Eq. (4), the term $\mathbf{D}(t) = 0$, because there are no disturbing cross-coupling effects from angular velocities when the reaction thrusters are appropriately mounted and the actuator matrix is not dependent on time.

B. Reaction Wheels

A reaction wheel is a flywheel with a vehicle-fixed axis designed to operate at zero bias [7], typically used in a spacecraft for storing momentum originating either from orbital perturbations or attitude maneuvers. In the case of attitude maneuvers, angular momentum is exchanged between the spacecraft and the wheel to provide angular velocities, such that rotating the wheel in one direction results in a rotation of the spacecraft in the opposite direction. One reaction wheel can only provide torque about one axis, which implies that at least three wheels are needed for rotation in 3 degrees of freedom. It should also be noted that the use of reaction wheels result in cross-coupled torques between spacecraft axes due to Coriolis forces and gyroscopic effects. Torque from reaction wheels can be written as follows [6]:

$$\boldsymbol{\tau}^b_{a,w} = \dot{\mathbf{h}}^b_w + \boldsymbol{\omega}^b_{i,b} \times \mathbf{h}^b_w - \boldsymbol{\tau}^b_{\text{friction}} \tag{8}$$

where $\mathbf{h}^b_w$ is the wheel angular momentum. The cross coupling in the second term of Eq. (8) arises from the gyroscopic effect of the spinning wheel, and the term $\boldsymbol{\tau}^b_{\text{friction}}$ denotes mechanical nonlinearities, such as stiction and friction. With an assumption of ideal behavior, we may neglect the friction in the reaction wheel, and the torque from the reaction wheels may hence be expressed on general form in Eq. (4) as

$$\boldsymbol{\tau}^b_{a,w} = \mathbf{B}_w\mathbf{u} + \mathbf{D}_w\boldsymbol{\omega}^b_{i,b} \tag{9}$$

where $\mathbf{u} = \dot{\mathbf{h}}^b_w$, $\mathbf{D} = -(\mathbf{h}^b_w\times)$ is the disturbance matrix due to cross-coupling effects, and $\mathbf{B}_w$ is the torque allocation matrix, which is equal to $\mathbf{I}_{3\times 3}$ if a configuration of three orthogonally placed reaction wheels is used. Note that both the disturbance matrix and the torque allocation matrix are time invariant.

C. Magnetic Torquers

Magnetic torquers (also known as torque rods or magnetic coils) are essentially electromagnets, constructed from a cylindrical rod of ferromagnetic material, typically an iron alloy, with distributed windings of electrically conductive material [7]. By mounting a magnetic torquer to the spacecraft body and applying current to the windings, a magnetic dipole field is induced around the magnetic torquer, which interacts with the surrounding magnetic field and thereby produces a torque on the spacecraft. Obviously, magnetic torquers are dependent on an external magnetic field, which makes them suitable for low-Earth-orbiting spacecraft but useless for, for example, lunar missions. Torque from magnetic torquers can be written as follows [6]:

$$\boldsymbol{\tau}^b_{a,m} = \mathbf{m}^b \times \mathbf{B}^b \tag{10}$$

where $\mathbf{m}^b$ is the generated magnetic moment inside the body, and $\mathbf{B}^b$ is the intensity of the surrounding magnetic field. By exploiting the skew symmetry operator, the torque from magnetic torquers can accordingly be written on the general form in Eq. (4) as

$$\boldsymbol{\tau}^b_{a,m} = \mathbf{m}^b \times \mathbf{B}^b(t) = -\mathbf{B}^b(t) \times \mathbf{m}^b = \mathbf{B}_m(t)\mathbf{u} \tag{11}$$

where $\mathbf{B}_m(t) = -(\mathbf{B}^b(t)\times)$, and $\mathbf{u} = \mathbf{m}^b$. The time-varying nature of the term $\mathbf{B}_m(t)$ is due to the varying strength and direction of the external magnetic field along the orbit trajectory.

IV. Actuator Dynamics

Equations (7), (9), and (11), show that torque from reaction thrusters, reaction wheels, and magnetic torquers can be written on the general actuator torque from Eq. (4). If $\mathbf{u}$ is equal to the desired

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control input $\mathbf{u}_d$, then the actuator is ideal and has no dynamics. This is seldom the case for practical actuators but is still neglected in most controller designs, justified by strict requirements for the actuator response. In the following subsections, we examine the case in which $\mathbf{u} \neq \mathbf{u}_d$ in the framework of the actuator dynamics and, subsequently, we present two general models of actuator dynamics that are able to describe the dynamics of all actuators.

A. Reaction Thruster Dynamics

Mathematical models of reaction thruster systems are used to predict the spacecraft response to a given set of inputs to the controller. Simple thruster models assume a thrust profile as a square pulse when, in reality, the response can be quite different, as shown in Fig. 3. Because it is practically impossible to describe the real thrust profile completely with a mathematical model, suitable approximations may be used to increase robustness and form a premise for making more precise maneuvers. If the square pulse approximation is not sufficiently representative for the thruster response, alternatives are trapezoidal and exponential approximations [7], shown in Fig. 4. The nomenclature used to describe thrust profiles is $t_{\text{delay}} = t_1 - t_0 \approx t_4 - t_3$, which is the time at which the electrical signals are sent from the controller until the time at which thrust is starting to build up or decay. This delay usually originates from the mechanical delay in the thrusters (e.g., opening or closing a valve). The time from 10 to 90% thrust level is referred to as the rise time of the thruster, t_{rise}, and is analogous to the fall time of the thruster, t_{fall}, from 90 to 10% thrust level. The term t_{on} indicates the time during which the thrust level is over 90%.

B. Reaction Wheel Dynamics

Dynamical models of reaction wheels have less uncertainty in their description as compared with reaction thrusters. The dynamical behavior of reaction wheels originates from the response of the flywheel and, when a control signal is input to a reaction wheel system, the largest limitation is caused by the dc motor. This motor is, in most cases, not as powerful as the desired controller input demands, because of strict requirements for weight in spacecraft missions; therefore, it is usually not capable of spinning the wheel as fast as commanded, which results in a maximum momentum change by the reaction wheel.

After attitude corrections with intrinsic momentum exchange, a reaction wheel can be left with a spin bias, which implies that the reaction wheel must maintain its spin to keep the spacecraft stabilized. In this case, the motor driver must compensate for friction and other disturbances that can affect the spin of the reaction wheel; if this is not completely corrected for, it becomes a dynamical aspect.

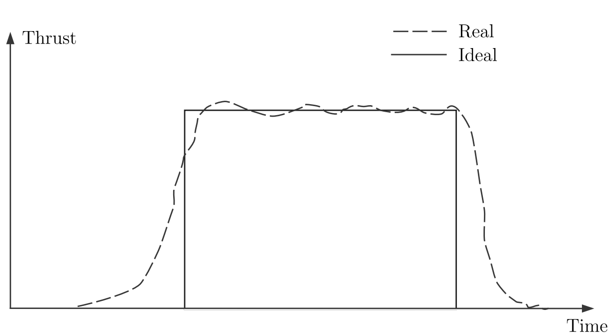

Fig. 3 Real thrust profile compared with square pulse profile.

C. Magnetic Torquer Dynamics

Magnetic torquers are electromagnetic devices with no mechanical parts, and the induced magnetic field that provides torque from the electromagnet is propagated almost instantly as the current flows through the coil. This, combined with the relative low torque provided by magnetic torquers, justify the neglect of actuator dynamics in most cases. One scenario, however, in which the magnetic torquer dynamics should be considered is when the input signal is pulse-width modulated with the lowest pulse width in the same order of magnitude as the propagation time of the magnetic field from the torquer. Another dynamical aspect can originate from the electrical driver circuits, which have to provide a constant current flowing in the coils. Any variations in this current will directly cause variations in the induced magnetic field.

D. General Dynamic Model

As we have seen in the previous subsections, all attitude actuators perform according to a dynamical behavior, although with different time constants, and this behavior may be included in the feedback control loop to increase controller performance. Because there are several similarities in the described attitude actuators, it is useful to define a general dynamical model that incorporates all actuators. In [5], a MIMO linear actuator model is presented as follows:

$$\mathbf{T}\,\dot{\mathbf{u}} + \mathbf{u} = \mathbf{v} \tag{12}$$

where $\mathbf{u} \in \mathbb{R}^M$ is a vector of actual control inputs with M as the number of actuators, $\mathbf{v} \in \mathbb{R}^M$ is a vector of desired actuator inputs, and $\mathbf{T}$ is a diagonal square matrix with positive time constants, hence, $\mathbf{T} = \mathbf{T}^T > 0$. An overview of this actuator model can be seen in Fig. 5. The PWM block is used for nonlinear actuators to give a linear connection between $\mathbf{v}$ and $\mathbf{v}_c$, and the actuator output multiplied with the actuator matrix $\mathbf{B}$ gives the torque $\boldsymbol{\tau}$ working on the spacecraft. If $\mathbf{v}$ is a square pulse with a duration of $t = 0.5$ s and the time constant of $\mathbf{T} = T = 0.05$, then Eq. (12) gives a response as depicted in Fig. 6. In the figure, the actuator dynamics are clearly exposed in comparison with the ideal instantaneous actuator response when $\mathbf{v} = \mathbf{u}$. This versatile approximation is used in a globally stable adaptive controller for marine vehicles and is also valid for other types of mechanical systems, such as spacecraft, aircraft, and robot manipulators [5]. Equation (12) may also be expanded to a more appropriate match of the actuators under consideration here. Reaction wheels and magnetic torquers are electrical systems and, consequently, their response starts to build up almost instantaneously. However, thrusters have a time delay from when the firing command is sent until the thrust starts to build up, due to the mechanical gas-flow system. One approach to including this delay in

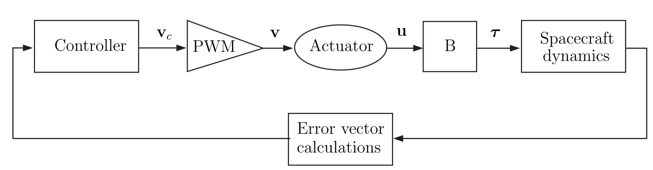

Fig. 5 Reaction thruster scheme.

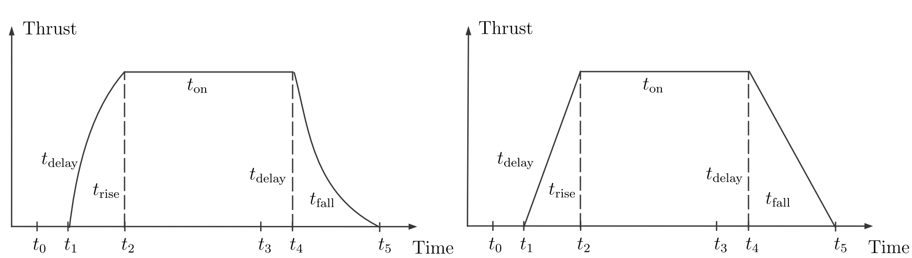

Fig. 4 Thrust profile approximation models: a) exponential, and b) trapezoidal.

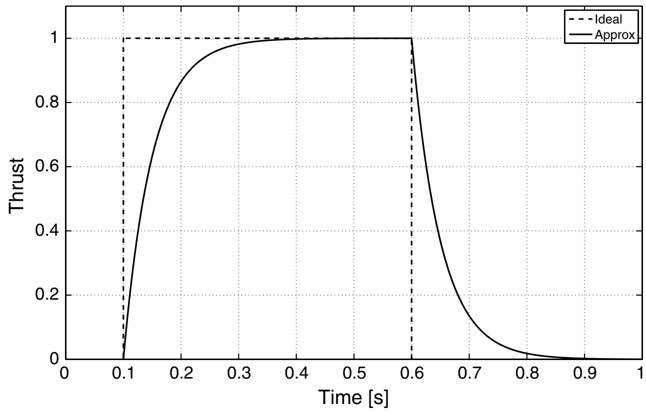

Fig. 6 Actuator response from Eq. (12) with $\mathbf{T} = T = 0.05$.

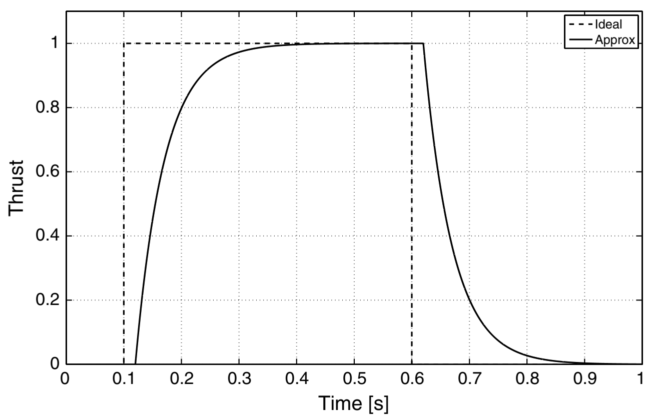

Fig. 7 Actuator response from Eq. (13) with $\mathbf{T} = T = 0.05$ and $t_d = 0.02$ second.

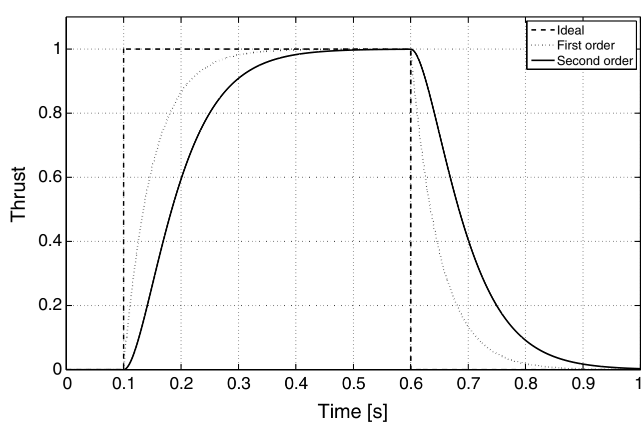

Fig. 8 Actuator response from Eq. (14) with $\mathbf{T}_u = \mathbf{T}_v = T = 0.05$.

an actuator dynamics model is to introduce a time displacement, t_d, in Eq. (12), that is (cf. Fig. 7),

$$\mathbf{T}\,\dot{\mathbf{u}}(t + t_d) + \mathbf{u}(t + t_d) = \mathbf{v}(t) \tag{13}$$

Although mathematically rigorous, the time-delay approximation introduces some difficulties for stability analysis. This specifically pertains to the analysis of uniformity of the solutions, because the time delay results in a dynamical shift in the total mathematical model.

E. General Second-Order Dynamics Model

A different and more mathematically suitable approach to including the mechanical time delay without introducing manual time delays in the model is an expansion of Eq. (12) with a second derivative term, for example,

$$\left.\begin{aligned}\mathbf{T}_u\dot{\mathbf{u}} + \mathbf{u} = \mathbf{v}\\ \mathbf{T}_v\dot{\mathbf{v}} + \mathbf{v} = \mathbf{w}\end{aligned}\right\} \Rightarrow \mathbf{T}_v\mathbf{T}_u\ddot{\mathbf{u}} + \mathbf{T}_v\dot{\mathbf{u}} + \mathbf{v} = \mathbf{w} \tag{14}$$

where $\mathbf{v}$ indicates the second derivative term. Equation (14) gives an actuator response as shown in Fig. 8, in which the original first-order actuator model in Eq. (12) is also included for comparison. The slower initial rise time in this actuator response may give a more accurate approximation for actuators with an explicit time delay. Thus, appropriate tuning of the second-order model in Eq. (14) will provide a suitable mathematical description of the actuator dynamics, which may be readily applied to the stability analysis of the closed-loop system. It is, however, important to note that the mathematical model is still an approximation of the actual behavior and, as such, it will not be a perfect representation of actuator behavior; it will, however, provide a significant improvement over solutions in which the actuator dynamics are neglected altogether.

V. Conclusions

In this paper, we have investigated the dynamical behavior of various actuators for spacecraft attitude control using functional descriptions and theoretical frameworks. Based on this investigation, we have provided an alternative way of describing the actuator dynamics on a general form, both in terms of a general theoretical model and a first- and second-order linear approximation. The presented models are suitable for direct inclusion within a controller design study to enable increased robustness and precision of control.

References

[1] Aldhaheri, R. W., and Khalil, H. K., "Effect of Unmodeled Actuator Dynamics on Output Feedback Stabilization of Nonlinear Systems," *Automatica*, Vol. 32, 1996, pp. 1323–1327. doi:10.1016/0005-1098(96)00077-5

[2] Mahmoud, M. S., and Khalil, H. K., "Robustness of High-Gain Observer-Based Nonlinear Controllers to Unmodeled Actuators and Sensors," *Automatica*, Vol. 38, 2002, pp. 361–369. doi:10.1016/S0005-1098(01)00253-9

[3] Khalil, H. K., "A Note on the Robustness of High-Gain-Observer-Based Controllers to Unmodeled Actuator and Sensor Dynamics," *Automatica*, Vol. 41, 2005, pp. 1821–1824. doi:10.1016/j.automatica.2005.04.008

[4] Fossen, T., *Guidance and Control of Ocean Vehicles*, Wiley, New York, 1994, pp. 309–311.

[5] Fossen, T. I., and Fjellstad, O.-E., "Cascaded Adaptive Control of Marine Vehicles With Significant Actuator Dynamics," *Proceedings of the IFAC World Congress*, International Federation of Automatic Control, Laxenburg, Austria, 1993, pp. 517–522.

[6] Sidi, M. J., *Spacecraft Dynamics and Control*, Cambridge Univ. Press, New York, 1997, pp. 95–262.

[7] Wertz, J. R. (ed.), *Spacecraft Attitude Determination and Control*, Astrophysics and Space Science Library, Kluwer Academic, London, U.K., 1978, pp. 200–275.

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