Uncertainty Propagation in Orbital Rendezvous Manouvers

Project Report

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Carl Martin Krokeide
Department of Engineering Cybernetics
Norwegian University of Science and Technology

Abstract

This report covers a study investigating positioning uncertainties when applying orbital rendezvous maneuvers. Model assumptions, nonlinearities and measurement uncertainties were identified, and their contribution to the uncertainty of the final position was modeled.

The main result of the study is the development of a formation positioning uncertainty model for the scenario of multiple chaser satellites rendezvousing a target object. The model is based on the parameters of the XX satellite. The inputs to the model are: 1) The current orbit parameter observations and corresponding uncertainties of the chaser and target objects, and 2) The planned thruster firings of the chaser satellites. The model outputs the spatial and timing uncertainties of the final chaser formation.

Supervisor 1: Prof. Damiano Varagnolo
Supervisor 2: Dr. Leonard Felicetti

NTNU
Norwegian University of Science and Technology

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1 Introduction

1.1 Problem Formulation

The aim of this study is to investigate spatial and timing uncertainties associated with orbital rendezvous maneuvers. This is of interest because of the limited observation possibilities of orbiting objects, thus making accurate location difficult to retrieve. In addition, the large scales in time and distance of orbital maneuvers, nonlinearities in the mechanics and the presence of disturbances make the uncertainty propagation interesting and non-trivial.

The theory of uncertainty propagation is an important aspect of many space operations. Long-duration, high-precision uncertainty propagation is central to orbital trajectory estimation and optimization.

The concept of uncertainty propagation in rendezvous maneuvers is a key aspect to solving the issue of space debris, which is a major challenge of today's space industry. The number of space debris objects is expected to steadily increase over time, due to chain-reactions of collisions between existing debris objects. To prevent a runaway fragmentation of space debris, about 4 to 5 high risk objects must be actively removed from the Low Earth Orbit each year [2]. This study aims to contribute to this growing area of research by exploring the possibility of using a formation of low-cost satellites to perform Active Debris Removal.

1.2 Previous Work

In the section that follows, selected previous works on the uncertainty propagation in orbital mechanics are presented.

2 Theory

This section provides a short introduction to relevant theory in orbital mechanics and propagation of uncertainties. The intended reader is someone with a technical background other than orbital mechanics or statistics, and the goal is to give a sufficient understanding of necessary concepts.

2.1 Orbital Mechanics

The sections on orbital mechanics are mostly based on the book by Curtis [1].

2.1.1 Orbital Parameters

The following is an introduction to a parametric description of orbits, using the definitions from Curtis [1]. Under the assumption that an orbit is an ideal Keplerian orbit, it can be uniquely identified using six orbital elements. Three parameters are required to define the orbit on a plane, and three additional parameters are needed to further place the orbit in three dimensional space.

The three parameters used to describe an orbit on a plane are the *specific angular momentum*, *eccentricity*, and *true anomaly*. The specific angular momentum is, as the name suggests, the angular momentum of the orbiting object. This value is constant at any point on a given orbit. The eccentricity value describes how much the shape of the orbit deviates from a circle, and can be found by dividing the distance between the center of the orbit and one of the foci by the *semi-major axis* (longest distance to the central body). The shape of a orbit corresponding to different eccentricity values is given in Table 1. The true anomaly is the angle between the position of the orbiting object and *periapsis* (the shortest distance to the central body).

$e = 0$	Circle
$0 < e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

Table 1: Orbital shapes for different eccentricity values

To place the orbit in three dimensional space, three axis rotations are required. The orbital elements corresponding to these rotations are the *inclination*, *right ascension of the ascending node* and the *argument of perigee*. The inclination describes the angle between the orbital plane and the *equatorial* (Earth XY) plane. The right ascension of the ascending node is the angle between the equatorial X-axis and the point on the equatorial plane where the orbit passes through from below. The third angle, argument of perigee, is the angle in the orbital plane between the point the orbit passes above the equatorial plane and the point of perigee.

A summary of the six orbital elements is given in Table 2.

Name	Symbol	Range	Unit
Specific angular momentum	h	-	$\frac{kgm}{s^2}$
Inclination	i	[0, 180]	Deg
Right ascension of the ascending node	Ω	[0, 360]	Deg
Eccentricity	e	-	-
Argument of perigee	ω	[0, 360]	Deg
True anomaly	Θ	[0, 360]	Deg

Table 2: Overview of the six orbital parameters

2.1.2 Coordinate Systems

2.1.2.1 Perifocal Coordinate System

The center of the perifocal coordinate system is at the focus of the orbit. The perifocal frame is defined with the unit vectors $\hat{\mathbf{p}}$, $\hat{\mathbf{q}}$ and $\hat{\mathbf{w}}$. The unit vectors $\hat{\mathbf{p}}$ and $\hat{\mathbf{q}}$ lie in the orbital plane, with $\hat{\mathbf{p}}$ pointing towards the *periapsis* of the orbit and $\hat{\mathbf{q}}$ towards the point at which *true anomaly* is equal to 90 deg. The unit vector $\hat{\mathbf{w}}$ is the cross product of the unit vectors in the orbital plane, $\hat{\mathbf{w}} = \hat{\mathbf{p}} \times \hat{\mathbf{q}}$, and thus points in the same direction as the angular momentum vector $\vec{\mathbf{h}}$ of the orbiting object.

The position of the orbiting object described in the perifocal plane is given in Eq. 1, and velocity in Eq. 2.

$$\vec{\mathbf{r}} = \bar{x}\hat{\mathbf{p}} + \bar{y}\hat{\mathbf{q}} \tag{1}$$

$$\vec{\mathbf{v}} = \dot{\vec{\mathbf{r}}} = \dot{\bar{x}}\hat{\mathbf{p}} + \dot{\bar{y}}\hat{\mathbf{q}} \tag{2}$$

2.1.2.2 Local Vertical Local Horizontal Coordinate System

Define the Local Vertical Local Horizontal (LVLH) coordinate frame.

Fill in

2.1.2.3 Earth Centered Intertial Coordinate System

Define the Earth Centered Intertial (ECI) coordinate frame.

Fill in

2.1.3 Lagrange Coefficients

Using the Lagrange Coefficients, the functions f and g, from the French mathematical physicist Joseph-Louis Lagrange (1736-1813), we are able to describe the position and velocity of an orbiting object at any point in time if the initial conditions are known, as shown in Eq. 3. The Lagrange Coefficients are derived based on the fact that the angular momentum is constant throughout the orbit.

$$\vec{\mathbf{r}} = f\vec{\mathbf{r_0}} + g\vec{\mathbf{v_0}} \tag{3}$$

The Lagrange Coefficients f and g, and their time derivatives $\dot{f}$ and $\dot{g}$, are defined below.

$$f = \frac{\bar{x}\dot{\bar{y}}_0 - \bar{y}\dot{\bar{x}}_0}{h} \tag{4}$$
$$g = \frac{-\bar{x}\bar{y}_0 + \bar{y}\bar{x}_0}{h} \tag{5}$$
$$\dot{f} = \frac{\dot{\bar{x}}\dot{\bar{y}}_0 - \dot{\bar{y}}\dot{\bar{x}}_0}{h} \tag{6}$$
$$\dot{g} = \frac{-\dot{\bar{x}}\bar{y}_0 + \dot{\bar{y}}\bar{x}_0}{h} \tag{7}$$

Using the change in true anomaly $\Delta\Theta$, or the change in universal anomaly χ, the Lagrange Coefficients and their time derivatives have the following expressions. The left column in the functions below are the expressions based on true anomaly $\Delta\Theta$, and the right column are the expressions based on universal anomaly χ.

$$f = 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\Theta) \qquad = 1 - \frac{\chi^2}{r}C(z) \tag{8}$$
$$g = \frac{r_0 r}{h}\sin\Delta\Theta \qquad = \Delta t - \frac{1}{\sqrt{\mu}\chi^3 S(z)} \tag{9}$$
$$\dot{f} = \frac{\mu}{h}\frac{1 - \cos\Delta\Theta}{\sin\Delta\Theta}[\frac{\mu}{h^2}(1 - \cos\Delta\Theta - \frac{1}{r_0} - \frac{1}{r}] \qquad = \frac{\sqrt{\mu}}{r_0 r}\chi[zS(z) - 1] \tag{10}$$
$$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\Theta) \qquad = 1 - \frac{\chi^2}{r}C(z) \tag{11}$$

For the functions above, $z = \alpha\chi^2$, $\alpha = \frac{1}{a}$ is the reciprocal of the semimajor axis, and $C(z)$ and $S(z)$ are *Stumpff functions* defined below.

$$C(z) = \sum_{k=0}^{\infty}(-1)^k\frac{z^k}{(2k+2)!} \tag{12}$$
$$S(z) = \sum_{k=0}^{\infty}(-1)^k\frac{z^k}{(2k+3)!} \tag{13}$$

2.1.4 Lambert's problem

Lambert's problem, from the French-born German astronomer J. H. Lambert (1728-1777), concerns finding the trajectory joining two orbital points P_0 and P, given the transfer time Δt.

Algorithm 1 presents an approach using the Lagrange Coefficients to solve Lambert's problem. The methos presented is based on *Algorithm 5.2* from the book by Curtis [1].

Algorithm 5.2 Solve Lambert's problem. A MATLAB implementation appears in Appendix D.25.
Given $\mathbf{r}_1$, $\mathbf{r}_2$ and Δt, the steps are as follows:

1. Calculate r_1 and r_2 using Equation 5.24.
2. Choose either a prograde or a retrograde trajectory and calculate $\Delta\theta$ using Equation 5.26.
3. Calculate A in Equation 5.35.
4. By iteration, using Equations 5.40, 5.43 and 5.45, solve Equation 5.39 for z. The sign of z tells us whether the orbit is a hyperbola ($z < 0$), parabola ($z = 0$) or ellipse ($z > 0$).
5. Calculate y using Equation 5.38.
6. Calculate the Lagrange f, g and $\dot{g}$ functions using Equations 5.46.
7. Calculate $\mathbf{v}_1$ and $\mathbf{v}_2$ from Equations 5.28 and 5.29.
8. Use $\mathbf{r}_1$ and $\mathbf{v}_1$ (or $\mathbf{r}_2$ and $\mathbf{v}_2$) in Algorithm 4.2 to obtain the orbital elements.

Algorithm 1: Solve Lambert's Problem Using Lagrange Coefficients

Input: Starting point, end point, direction and time frame of desired trajectory

Output: Initial velocity required to follow desired trajectory and velocity at end point

Calculate the change in true anomaly, $\Delta\Theta$, based on starting point, end point and direction of orbit (prograde or retrograde)

return *velocityStart, velocityEnd*

Insert Algorithm from Curtis

2.2 Uncertainty Propagation

In the following section different methods of uncertainty propagation are presented. The term *uncertainty propagation* refers to the method of predicting a systems state and associated uncertainty at a certain point in time, given information about the initial state and uncertainty of the system. A propagator can be either linear or nonlinear, with several approaches existing within both classes.

2.2.1 Monte Carlo Simulations

Monte Carlo simulation relies on repeated random sampling to obtain numerical results. MC simulation is computationally heavy, but has the advantage that when the number of samples approaches infinity, the result converges towards the true probability distribution of the system.

3 Methods

3.1 Uncertainty Analysis

3.1.1 Error Sources

Sources of uncertainty in space operations will be introduced in the following section. Uncertainties in orbital estimations can originate from phenomena in the physical world, or come from approximations and deviations in the models used.

Based on the review by Luo and Yang [4], who follow the opinion of Fehse [3], the uncertainties related to space operation can be divided into three categories: *Dynamic model errors*, *actuation errors* and *navigation errors*. Dynamic model errors concern the deviations in the model parameters to the real world values, such as the gravitational parameters, drag and radiation pressure. Actuation errors concern the difference between the correct actuation values and the values produced by the actuators and control system. Navigation errors are the deviations in the perceived state of the system from the actual state. An overview of the types of uncertainty in orbital mechanics is given in Tabel 2.

Classification	**Parameter**
Dynamic Model Errors	Gravitational Field Drag Radiation Pressure
Actuation Errors	Direction Timing Force
Navigation Errors	Atmospheric Effects Instrument Modeling Clock Accuracy

Table 3: Error Sources in Orbital Mechanics

3.2 Simulations

4 Results

5 Discussion

5.1 Recommendations for Further Work

6 Conclusion

References

[1] Howard D. Curtis. *Orbital Mechanics for Engineering Students*. 2nd ed. Elsevier, 2009.

[2] European Space Agency. *About Space Debris*. 2018. URL: `https://www.esa.int/Our%5C_Activities/Space%5C_Safety/Space%5C_Debris/About%5C_space%5C_debris` (visited on 10/03/2019).

[3] Wigbert Fehse. *Automated Rendezvous and Docking of Spacecraft*. eng. Vol. 16. Cambridge University Press, 2003. ISBN: 9780521824927.

[4] Ya-zhong Luo and Zhen Yang. "A review of uncertainty propagation in orbital mechanics". In: *Progress in Aerospace Sciences* 89 (2017), pp. 23–39. DOI: `https://doi.org/10.1016/j.paerosci.2016.12.002`.

Appendices