We can create two distinguishable particles
an EPR (Einstein-Podolsky-Rosen) pair
in a quantum mechanical superposition state
of the form of one of the Bell states
For example, for two photons 1 and 2
going in different directions
a state like $\left|\Phi^{+}\right\rangle_{12} = \frac{1}{\sqrt{2}}\left(\left|\mathrm{H}\right\rangle_1 \left|\mathrm{H}\right\rangle_2 + \left|\mathrm{V}\right\rangle_1 \left|\mathrm{V}\right\rangle_2\right)$

EPR Pairs

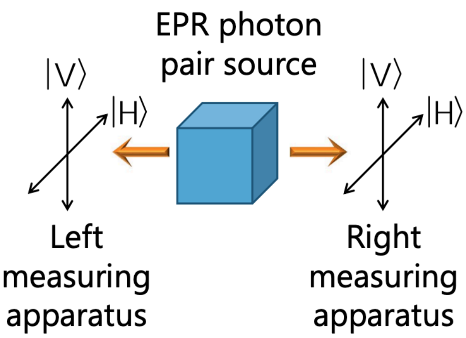
Such a state $\left|\Phi^{+}\right\rangle_{12} = \frac{1}{\sqrt{2}}\left(\left|\mathrm{H}\right\rangle_1 \left|\mathrm{H}\right\rangle_2 + \left|\mathrm{V}\right\rangle_1 \left|\mathrm{V}\right\rangle_2\right)$
is a linear superposition of
the state where the two photons
are both horizontally polarized
and the state where the two photons
are both vertically polarized

In this state $\left|\Phi^{+}\right\rangle_{12} = \frac{1}{\sqrt{2}}\left(\left|\mathrm{H}\right\rangle_1 \left|\mathrm{H}\right\rangle_2 + \left|\mathrm{V}\right\rangle_1 \left|\mathrm{V}\right\rangle_2\right)$
similarly, measuring the result $\left|V\right\rangle$ for one photon
will lead
according to quantum mechanics
to the inescapable conclusion that
the other photon will also be in the state $\left|V\right\rangle$

EPR experiment

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2\right)$$

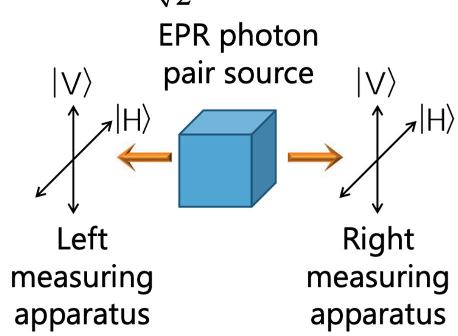
- An "EPR" pair of photons
  - in a $|\Phi^+\rangle_{12}$ Bell state
  - travel to two different measuring apparatuses
    - with their axes aligned
- Quantum mechanics predicts
  - if we measure one photon to be horizontal
    - then we will find the other photon is also horizontal

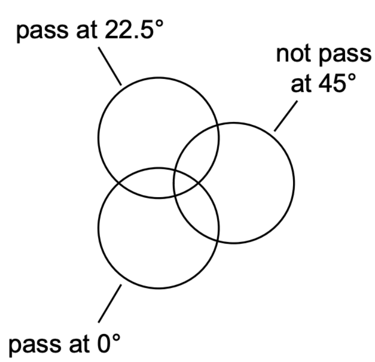


EPR experiment

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2\right)$$

- Similarly if we measure one photon to be vertical
  - the other photon will also be measured to be vertical
- This is the behavior we find also in experiments
- With aligned axes
  - both apparatuses measure the same polarization
    - for these Bell-state photons





Local variable theory

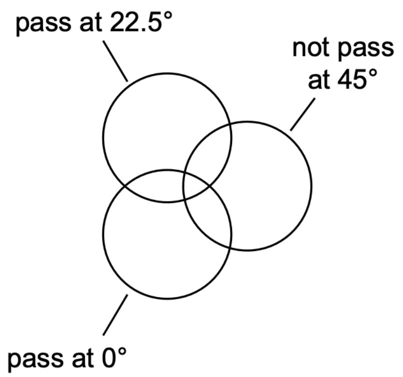
- We will be interested in three possible angles for a polarizer
  - 0°, 22.5°, or 45°
    - and three specific possible results for any one experiment
      - pass at 22.5°
      - pass at 0°
      - not pass at 45°

VENN Diagram for proof

All of these three regions can overlap and still be agreement on what happens with photons and polarizers.

We can only perform one test on a given photon with a polarizer set at 0,22.5,45 degrees.

Local variable theory



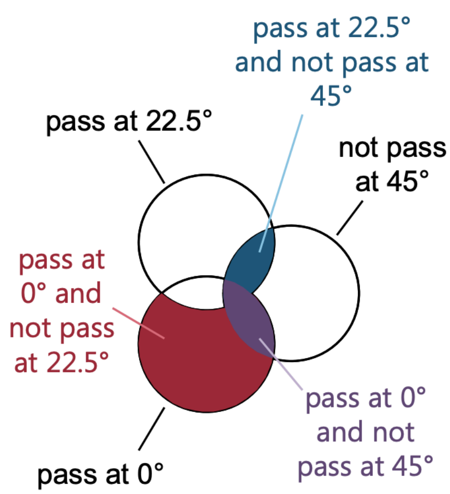
With our EPR photon pair source, we have

- two photons to use in two different experiments
  - one on the left, and one on the right

and we already know that photons prepared this way

- always behave identically
  - for identically set polarizers

A Bell's inequality



The probability that one photon will pass at 0° while the other will not pass at 45°

≤ the probability that one photon will pass at 0° and the other will not pass at 22.5°

+ the probability that one photon will pass at 22.5° and the other will not pass at 45°

Bell's inequalities and experiment

If we find an experiment that violates this inequality

- then we have to throw out deterministic local hidden variable theories
  - e.g., the idea that the photon has a variable that it carries with it that determines the result of the polarization measurement

Experiments do violate this inequality

- Therefore deterministic local hidden variable theories
  - cannot explain reality

This conclusion is independent of the correctness of quantum mechanics