UNIVERSIDAD NACIONAL DE COLOMBIA

Master's Thesis Proposal

Anisotropy-driven instabilities in collisionless plasmas with non-Maxwellian distributions

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Research Group:
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Contents

1 Summary

Collisionless plasmas, which constitute most of the visible universe, exhibit complex dynamics governed by collective electromagnetic interactions. Under low-collisionality conditions, temperature anisotropies naturally emerge through mechanisms such as magnetic compression and particle acceleration, providing free energy that drives microinstabilities including firehose, mirror, and whistler modes.

While classical models typically assume Maxwellian velocity distributions, recent observations from *Parker Solar Probe*, *MMS*, and *Cluster* have revealed that space plasmas frequently display non-Maxwellian suprathermal tails characterized by κ and bi-κ distributions. These populations modify the dispersion properties and growth rates of kinetic instabilities, altering how plasma energy is redistributed.

This project proposes a systematic comparison between Maxwellian and non-Maxwellian plasmas using fully kinetic Particle-in-Cell (PIC) simulations. By quantifying the thresholds, growth rates, and nonlinear evolution of anisotropy-driven instabilities, this study aims to elucidate how suprathermal populations influence plasma self-organization, energy conversion, and the relaxation of anisotropies relevant to solar wind and magnetosheath conditions.

2 Theoretical Framework

2.1. Introduction

Collisionless plasmas, which comprise the vast majority of the visible universe, are characterized by complex dynamics governed fundamentally by collective electromagnetic interactions rather than binary collisions [1]. In these low-collisionality regimes, temperature anisotropies ($T_\perp \neq T_\parallel$) emerge naturally through various kinetic mechanisms... These mechanisms include adiabatic magnetic compression, plasma expansion, and particle acceleration occurring at shock fronts and within magnetic reconnection regions [2, 3]. The solar wind is a nearly collisionless plasma that originates in the solar corona and exhibits these mechanisms [4, 5]. The anisotropies represent a source of free energy capable of driving kinetic microinstabilities, most notably the **firehose**, **mirror**, and **whistler** modes [6, 7].

While canonical plasma models have traditionally relied on the assumption of Maxwellian velocity distribution functions (VDFs), in-situ observations from cutting-edge space missions—such as *Cluster*, the *Magnetospheric Multiscale (MMS)* mission, and the *Parker Solar Probe*—have conclusively revealed that space plasmas frequently exhibit non-Maxwellian features. Specifically, they display suprathermal tails best described by κ and **bi-**κ distributions. Such deviations from thermodynamic equilibrium have profound implications for the plasma's response to perturbations, significantly modifying wave dispersion relations, particle diffusion coefficients, and the macroscopic transport of energy and momentum.

Consequently, understanding these non-thermal effects is imperative not only for the advancement of kinetic plasma theory but also for accurate space-weather modeling. The evolution of anisotropy-driven instabilities directly dictates the stability and relaxation processes in the solar wind, the magnetosheath, and other heliospheric environments. In particular, elucidating the role of suprathermal populations in regulating stability thresholds and saturation mechanisms remains a pivotal open problem in space plasma physics.

2.2. Literature Review and Kinetic Models

2.3. Foundations of Anisotropy-Driven Instabilities

The generation of kinetic instabilities by temperature anisotropies in collisionless plasmas is a well-established phenomenon in the literature. [6] laid the theoretical groundwork for analyzing these microinstabilities, deriving the linear dispersion relations and stability criteria assuming Maxwellian VDFs. These theoretical predictions were subsequently validated against observational data. In a landmark study, utilized extensive datasets from the *Wind* spacecraft to confirm that the solar wind proton temperature anisotropy is constrained by the thresholds of the firehose and mirror instabilities, demonstrating an excellent agreement with linear theory predictions for Maxwellian cores.

2.3.1. The Kappa Distribution Paradigm

Despite the success of Maxwellian models, the ubiquity of suprathermal particles necessitates a more generalized approach. Observations indicate that VDFs frequently exhibit high-energy tails that follow a power-law decay, —meaning the particle population decreases algebraically ($v^{-\alpha}$) rather than exponentially (e^{-v^2}) at high velocities. This behavior is better captured by the use of the **Kappa (κ)** distribution instead of the standard Maxwellian distribution. As extensively reviewed by **Lazar et al. (2017)** [8], the index κ serves as a quantitative measure of non-Maxwellianity, where lower values correspond to significant deviations from thermal equilibrium and harder spectral tails.

The generalized κ-distribution function for a particle species s in a 2D velocity space is mathematically defined as [8]:

$$f_{\kappa,s}(v_x, v_y) = \frac{n_s}{\pi\kappa\theta_s^2}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa)}\left[1 + \frac{v_x^2 + v_y^2}{\kappa\theta_s^2}\right]^{-(\kappa+1)} \tag{2-1}$$

where $\theta_s = \sqrt{\frac{2k_B T_s}{m_s}\left(1 - \frac{1}{\kappa}\right)}$ represents the effective thermal speed, and Γ denotes the Gamma function. The condition $\kappa > 1$ is strictly required to ensure the convergence of the distribution moments.

To explicitly model the coupling between temperature anisotropy and suprathermal populations, we employ the **bi-κ** distribution, which extends the formalism to anisotropic temperatures ($T_\perp, T_\parallel$):

$$f_{\mathrm{bi}-\kappa,s}(v_\perp, v_\parallel) \propto \left[1 + \frac{v_\perp^2}{\kappa\theta_{\perp s}^2} + \frac{v_\parallel^2}{\kappa\theta_{\parallel s}^2}\right]^{-(\kappa+1)}. \tag{2-2}$$

Analytical studies have demonstrated that these non-Maxwellian structures significantly alter the linear stability landscape. For instance, [9] investigated the specific role of suprathermal populations, showing that low-κ indices typically amplify the growth rates of firehose and mirror instabilities and modify their stability boundaries. Furthermore, these distributions are expected to modify the nonlinear saturation phase of the instability and the anisotropy relaxation rate through complex resonant and non-resonant wave-particle interactions.

However, a critical gap remains: comparative fully kinetic simulations between Maxwellian and non-Maxwellian plasmas in 2D configurations are limited. This project seeks to bridge this gap by performing a systematic analysis of the **linear growth rates, fluctuating magnetic energy density, and particle heating rates** using the Particle-in-Cell (PIC) method to quantify how suprathermal particles reshape the onset, nonlinear evolution, and energy conversion processes of these fundamental microinstabilities.

3 Objectives

3.1. General Objective

To investigate how non-Maxwellian distributions influence the onset and evolution of anisotropy-driven instabilities in collisionless plasmas, including their role in dynamic plasma processes *such as* magnetic reconnection, shocks, or self-organized structures.

3.2. Specific Objectives

1. Study the instabilities driven by temperature anisotropy in plasmas with Maxwellian distributions, identifying the dominant modes and the conditions governing their development.

2. Analyze how non-Maxwellian distributions (e.g., κ or bi-κ types) modify stability criteria, growth rates, and particle dynamics.

3. Explore the manifestation and impact of these instabilities in dynamic plasma contexts such as magnetic reconnection regions, shocks, or self-organized structures, considering their relevance to the overall plasma evolution.

4 Methodology

This research adopts a fully kinetic approach based on the PIC method, solving the self-consistent coupling of charged particle motion and electromagnetic field evolution through the Vlasov-Maxwell system in 2D geometry.

Simulation Framework

Simulations will be performed using the **PSC (Plasma Simulation Code)**, an open-source high-performance PIC code [10]. The code integrates particle dynamics using a standard relativistic leapfrog scheme and **evolves the electromagnetic fields** via a Finite-Difference Time-Domain (FDTD) Maxwell solver.

We adopt a **2.5D simulation configuration** (2 spatial dimensions, 3 velocity dimensions). In this geometry, spatial variations are resolved in the (x, y) plane, while the electromagnetic fields and particle velocities retain all three vector components (v_x, v_y, v_z). This 2.5D setup is chosen because it captures the essential physics of anisotropy-driven instabilities—including out-of-plane scattering and cyclotron dynamics—while maintaining computational efficiency for extensive parameter surveys compared to fully 3D runs.

Benchmark cases will validate numerical accuracy against analytical predictions where available. Equation normalization will use proton cyclotron frequencies ($\Omega_p = eB_0/m_p$) and ion inertial lengths ($d_i = c/\omega_{pi}$) as characteristic units.

Benchmark cases will validate numerical accuracy against analytical predictions where available. Equation normalization will use proton cyclotron frequencies (Ω_p) and ion inertial lengths (d_i) as characteristic units.

Initial Conditions and Parameters

The plasma will be initialized with uniform density in the 2D simulation domain. The magnetic field will be oriented in-plane or with an out-of-plane guide field component depending on the instability under study. Temperature anisotropy $A = T_y/T_x$ will be systematically varied from 0.1 to 10 to probe different stability regimes. Plasma beta values $\beta = 0.1 - 10$ and κ indices in the range $2 \leq \kappa \leq 10$ will replicate typical space plasma conditions [11].

Non-Maxwellian distributions will be implemented using appropriate particle loading algorithms for 2D velocity space, ensuring physically consistent representation of suprathermal tails. Periodic boundary conditions in both spatial dimensions will ensure global energy conservation.

Diagnostics and Analysis

The diagnostics will include:

- **2D Fast Fourier Transform (FFT)** of electromagnetic fields to identify dominant wave modes and their propagation directions in the simulation plane.
- **Velocity Distribution Function (VDF) analysis** computed from particle data to track the evolution of distribution functions in 2D velocity space (v_x, v_y).
- **Magnetic and electric fluctuation analysis** including computation of $\delta B/B_0$ and $\delta E/E_0$ ratios to characterize wave activity.
- **Mean anisotropy evolution** tracking T_y/T_x and its correlation with instability development and saturation.
- **Wave growth rates** extracted from temporal evolution of Fourier modes to quantify instability development.
- **Energy partitioning** tracking between magnetic, electric, and particle kinetic energy components.

Post-processing will be conducted using Python libraries (NumPy, SciPy, Matplotlib) with custom-developed analysis routines specifically designed for 2D PIC simulation data.

Validation and Comparison

Simulations of Maxwellian plasmas will first be validated against known linear theory results for 2D configurations. Subsequently, identical configurations will be executed for κ distributions to quantify deviations in threshold and nonlinear evolution. A verification protocol will include checks for: energy conservation, numerical stability, convergence with respect to grid resolution and particle number, and reproducibility of known linear growth rates where analytical solutions are available.

5 Schedule

The project will be developed over two academic semesters. The first period (2026-I) focuses on theoretical preparation, model setup, and preliminary tests. The second period (2026-II) will concentrate on the execution of the PIC simulations, data analysis, and thesis writing.

Semester 2026-I

Activity per week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Literature review on anisotropy-driven instabilities and non-Maxwellian plasmas																
Study of kinetic theory and dispersion relations for Maxwellian and κ distributions																
Design and setup of initial simulation parameters and test cases																
Execution of preliminary PIC simulations (Maxwellian)																
Writing of theoretical framework and objectives																

Table 5-1: Activities planned for Semester 2026-I

Semester 2026-II

Activity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Execution of main PIC simulations (Non-Maxwellian)																
Analysis of simulation outputs: spectra, growth rates, and anisotropy relaxation																
Comparison with theoretical models and literature																
Writing and refinement of the thesis document																
Preparation for thesis defense and presentation																

Table 5-2: Activities planned for Semester 2026-II

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This project will develop using the CECC Cluster Federation of the National University of Colombia. All the simulations will executed on this high-performace computing infrastructure, employibg the Particle-in-Cell (PIC) method to model. The computational resources provided by the CECC cluster will allow the analysis of multiple scenarios. All required computational expenses and resources will be covered by the student.

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