Neutralino Interactions in the MSSM

Carl Martin Fevang

January 8, 2024

Abstract

In this document, I derive how to construct the fermion interaction Lagrangian from kinetic and Yukawa terms of the superlagrangian. This is then applied to the MSSM superlagrangian to get the Feynman rules for neutralino interaction with the SM Z-boson and quark/squark pairs.

1 Superfields

Here I list some general expansions of fields over superspace, *superfield*. The fields are expanded in the superspace coordinates $\theta_{A=1,2}, \theta^{\dagger\dot{A}=1,2}$ that are four Grassmann coordinates imposed in a spinor structure with one left-handed Weyl spinor and a right-handed Weyl spinor.

A left-handed scalar superfield Φ can be written out in terms of component fields as[1]

$$\Phi = A + i(\theta\sigma^\mu\theta^\dagger)\partial_\mu A - \frac{1}{4}(\theta\theta)(\theta\theta)^\dagger \Box A + \sqrt{2}(\theta\psi) - \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi\sigma^\mu\theta^\dagger) + (\theta\theta)F, \quad (1)$$

where A, F are complex scalar fields and ψ is a left-handed Weyl spinor field. Φ has a right-handed scalar superfield compliment found by conjugating it:

$$\Phi^\dagger = A^* - i(\theta\sigma^\mu\theta^\dagger)\partial_\mu A^* - \frac{1}{4}(\theta\theta)(\theta\theta)^\dagger \Box A^* + \sqrt{2}(\theta\psi)^\dagger + \frac{i}{\sqrt{2}}(\theta\theta)^\dagger(\theta\sigma^\mu\partial_\mu\psi^\dagger) + (\theta\theta)^\dagger F^*, \quad (2)$$

where $\psi^\dagger$ is the right-handed compliment of ψ such that ${\psi^\dagger}^{\dot{A}} = \delta^{\dot{A}A}(\psi_A)^*$.

A vector superfield V can be written in Wess-Zumino gauge as

$$V_{\mathrm{WZ}} = (\theta\sigma^\mu\theta^\dagger)\left[V_\mu + i\partial_\mu(A - A^*)\right] + (\theta\theta)(\theta\lambda)^\dagger + (\theta\theta)^\dagger(\theta\lambda) + \frac{1}{2}(\theta\theta)(\theta\theta)^\dagger D, \quad (3)$$

where V_μ is a real vector field, λ is a left-handed Weyl spinor field and D is a (auxiliary) complex scalar field. The $\partial_\mu(A - A^*)$-term represents the gauge freedom remaining in the choice of supergauge after choosing Wess-Zumino gauge, and can be ignored when working out the interaction terms. I note that this gauge implies that no powers of the vector superfield above 2 are non-zero because of the Grassmann content. For the remainder of this document, vector superfields will be assumed to be in Wess-Zumino gauge.

2 MSSM fields

For completeness, I list here the relevant superfields containing the neutralinos and the superfields that couple directly to them.

[1] Parentheses are used to clarify Weyl spinor contraction.

2.1 The superfields

The neutralino fields are found in the scalar superfield $SU(2)_L$ doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$, $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$, and the vector superfields B^0 for the $U(1)_Y$ gauge group, and W^0 for the $SU(2)_L$ gauge group. The fields that couple directly to them are found in these same superfields, the remaining $SU(2)_L$ vector superfields $W^\pm$, the scalar superfield $SU(2)_L$ doublets $L_i = \begin{pmatrix} l_i \\ \nu_i \end{pmatrix}$ and $Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$, and the $SU(2)_L$ singlet superfields $\bar{E}_i$, $\bar{U}_i$ and $\bar{D}_i$, where $i = 1, 2, 3$ enumerates the three generations of leptons/quarks.

Superfield	Boson field	Fermion field	Auxiliary field
$H_{u/d}^0$	$H_{u/d}^0$	$\widetilde{H}_{u/d}^0$	$F_{H_{u/d}^0}$
H_u^+	H_u^+	$\widetilde{H}_u^+$	$F_{H_u^+}$
H_d^-	H_d^-	$\widetilde{H}_d^-$	$F_{H_d^-}$
l_i	$\widetilde{l}_{iL}$	l_i	F_{l_i}
$\bar{E}_i$	$\widetilde{l}_{iR}^*$	$\bar{e}_i$	$F_{\bar{E}_i}^*$
ν_i	$\widetilde{\nu}_{iL}$	ν_i	F_{ν_i}
u_i	$\widetilde{u}_{iL}$	u_i	F_{u_i}
$\bar{U}_i$	$\widetilde{u}_{iR}^*$	$\bar{u}_i$	$F_{\bar{U}_i}^*$
d_i	$\widetilde{d}_{iL}$	d_i	F_{d_i}
$\bar{D}_i$	$\widetilde{d}_{iR}^*$	$\bar{d}_i$	$F_{\bar{D}_i}^*$
B^0	B_μ^0	$\widetilde{B}^0$	D_{B^0}
W^0	W_μ^0	$\widetilde{W}^0$	D_{W^0}
$W^\pm$	$W_\mu^\pm$	$\widetilde{W}^\pm$	$D_{W^\pm}$

Table 1: Table of the MSSM superfields and their component field names.Note that the fermion fields are left-handed Weyl spinors, in spite of any L or R in the subscript. The conjugate superfields changes these to right-handed Weyl spinors.

2.2 The neutralino component fields

Letting $\Psi_{\widetilde{\chi}^0} = \left(\widetilde{B}^0, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0\right)$ denote a vector[2] of the fermion component fields in the B^0 and W^0 vector superfields and the $H_{u/d}^0$ scalar superfields — the neutralino mass eigenstates are given by

$$\widetilde{\chi}^0 = \left(\widetilde{\chi}_1^0, \widetilde{\chi}_2^0, \widetilde{\chi}_3^0, \widetilde{\chi}_4^0\right) = N\Psi_{\widetilde{\chi}^0}, \tag{4}$$

where N is a unitary matrix diagonalising the neutralino mass matrix $M_{\widetilde{\chi}^0\text{-mass}}$. N can be chosen such that the neutralino masses are real and positive. The neutralino interactions terms are then found in terms in the superlagrangian that include the vector superfields B^0 and W^0, and the scalar superfields $H_{u/d}^0$. For later use, I note that translating from the $\Psi_{\widetilde{\chi}^0}$-basis to the $\widetilde{\chi}^0$-basis, we have that

$$\widetilde{B}^0 = N_{i1}^* \widetilde{\chi}_i^0, \quad \widetilde{W}^0 = N_{i2}^* \widetilde{\chi}_i^0, \quad \widetilde{H}_d^0 = N_{i3}^* \widetilde{\chi}_i^0, \quad \widetilde{H}_u^0 = N_{i4}^* \widetilde{\chi}_i^0, \tag{5}$$

where a sum over i is implied.

[2] I use row vector notation here for convenience. In equations this is understood to be a column vector.

3 MSSM Superlagrangian

Here, I summarise the relevant MSSM superlagrangian terms in which the interactions are found.

3.1 Neutralino interactions

The neutralino interactions appear in the kinetic terms of all the superfields that couple to the $U(1)_Y$ and $SU(2)_L$ gauge groups, and the Yukawa terms that include the Higgs fields in the superpotential. The exception is neutralino interaction with the charginos, which is found in the supersymmetric field strength term.

4 Interaction Lagrangian

In the following, I go through how ordinary Lagrangian interaction terms are found from superfield terms in the superlagrangian.

4.1 General superlagrangian terms

The ordinary interaction Lagrangian is found from integrating over the Grassmann variables of the superlagrangian. Only terms containing all four Grassmann variables survive this, so we only need to look for the superlagrangian terms that include all of $(\theta\theta)(\theta\theta)^\dagger$.[3] Looking first at a general kinetic term of a left-handed scalar superfield Φ coupled to a $U(1)$ vector superfield V, it has the form[4]

$$\mathcal{L}_{\text{kin}} = \Phi^\dagger e^{2qV}\Phi. \tag{6}$$

Using Weyl identities from App. A, the interactions that include the λ-fields of the vector superfield can be found as

$$\begin{aligned}\mathcal{L}_{\text{kin}} &\overset{\lambda,\lambda^\dagger}{\supset} 2q\left\{A^*(\theta\theta)^\dagger(\theta\lambda)\sqrt{2}(\theta\psi) + \sqrt{2}(\theta\psi)^\dagger(\theta\theta)(\theta\lambda)^\dagger A\right\} \\ &\overset{\text{Eq. 31a}}{=} -\sqrt{2}q(\theta\theta)(\theta\theta)^\dagger\left\{(\lambda\psi)A^* + (\lambda\psi)^\dagger A\right\}. \end{aligned}\tag{7}$$

Ignoring ordinary kinetic terms, the interactions that include the ψ-fields of the scalar superfields include

$$\begin{aligned}\mathcal{L}_{\text{kin}} &\overset{\psi,\psi^\dagger}{\supset} 2q\left\{A^*(\theta\theta)^\dagger(\theta\lambda)\sqrt{2}(\theta\psi) + \sqrt{2}(\theta\psi)^\dagger(\theta\sigma^\mu\theta^\dagger)V_\mu\sqrt{2}(\theta\psi) + \sqrt{2}(\theta\psi)^\dagger(\theta\theta)(\theta\lambda)^\dagger A\right\} \\ &\overset{\text{Eq. 31a and 31c}}{=} q(\theta\theta)(\theta\theta)^\dagger\left\{-\sqrt{2}(\lambda\psi)A^* + (\psi\sigma^\mu\psi^\dagger)V_\mu - \sqrt{2}(\psi\lambda)^\dagger A\right\}, \end{aligned}\tag{8}$$

where we notice that the $\lambda\psi A$-interactions are the same as the ones we found in Eq. 7. With a Yukawa superpotential on the form

$$W = y_{ij}\Phi_i\Phi\Phi_j, \tag{9}$$

the superlagrangian looks like

$$\mathcal{L}_{\text{Yukawa}} = y_{ij}(\theta\theta)^\dagger\Phi_i\Phi\Phi_j + \text{c. c.} \tag{10}$$

Extracting the ψ fermion interactions from the Φ superfield, we have

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &\overset{\psi,\psi^\dagger}{\supset} y_{ij}(\theta\theta)^\dagger\sqrt{2}(\theta\psi)\left\{A_i\sqrt{2}(\theta\psi_i) + \sqrt{2}(\theta\psi_j)A_j\right\} + \text{c. c.} \\ &\overset{\text{Eq. 31a}}{=} -y_{ij}(\theta\theta)(\theta\theta)^\dagger\left\{A_i(\psi\psi_j) + (\psi_i\psi)A_j + \text{c. c.}\right\} \end{aligned}\tag{11}$$

[3]Terms with an insufficient amount of θs are ignored in the following.

[4]P. Binétruy. *Supersymmetry: Theory, experiment and cosmology.* 2006.

4.2 Neutralino interaction terms

Now to relate this to the neutralino fields in the MSSM.

4.2.1 Gaugino parts

First, I will look at the bino and wino interactions. From electroweak unification, we have that the coupling g and g' of the W^a and B^0 superfields respectively are related by

$$g' = g t_W, \tag{12}$$

where $t_W = s_W/c_W \equiv \sin\theta_W/\cos\theta_W$ where θ_W is the Weinberg mixing angle. Rewriting using $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, and defining $W^\pm = W^1 \mp iW^2$ and $W^0 \equiv W^3$, we have

$$Yg'B^0 + \frac{1}{2}g\sigma^a W^a = g\left\{Yt_W B^0 + \frac{1}{2}\sigma_3 W^0 + \frac{1}{2}\sigma_+ W^+ + \frac{1}{2}\sigma_- W^-\right\} \tag{13}$$

So an MSSM superfield doublet $\Phi = \begin{pmatrix}\Phi_+ \\ \Phi_-\end{pmatrix}$ charged under $U(1)_Y$ with charge Y and $SU(2)_L$ will have a kinetic term

$$\mathcal{L}_{\Phi\text{-kin}} = \Phi^\dagger e^{2g\left[Yt_W B^0 + \frac{1}{2}\sigma_3 W^0 + \frac{1}{2}\sigma_+ W^+ + \frac{1}{2}\sigma_- W^-\right]}\Phi. \tag{14}$$

We can extract the fermion interactions from the vector superfields B^0 and W^0 using Eq. 7 to be

$$\begin{aligned}\mathcal{L}_{\Phi\text{-kin}} \overset{\widetilde{B}^0,\widetilde{W}^0}{\supset} -\sqrt{2}g(\theta\theta)(\theta\theta)^\dagger\Big\{&Yt_W(\widetilde{B}^0\psi_+)A_+^* + \frac{1}{2}(\widetilde{W}^0\psi_+)A_+^* \\ &+ Yt_W(\widetilde{B}^0\psi^-)A_-^* - \frac{1}{2}(\widetilde{W}^0\psi_-)A_-^* + \text{c. c.}\Big\}.\end{aligned} \tag{15}$$

In the MSSM, the Dirac fermions are made up from two scalar superfields, supplying the left- and right-handed components separately. Both superfields couple to the $U(1)_Y$ gauge group with charge $Q - I^3$,[5] where Q is the electric charge of the fermion, and I^3 is the weak isospin; either $\pm\frac{1}{2}$ for the superfields supplying left-handed fermions or 0 for the superfields supplying the right-handed ones. Only the left-handed field couples to the $SU(2)_L$ gauge group.

$\widetilde{f}_L, f \in f$ and $\widetilde{f}_R^*, \bar{f} \in \bar{F}$

Thus, using Eq. (15) and Eq. 7, the bino and wino interaction with a pair of MSSM fermions formed from a superfield doublet $F = \begin{pmatrix} f_+ \\ f_- \end{pmatrix}$ and the superfields $\bar{F}_\pm$ are

$$\begin{aligned}\mathcal{L}_{\text{EW-kin}} \overset{\widetilde{B}^0,\widetilde{W}^0}{\supset} -\sqrt{2}g(\theta\theta)(\theta\theta)^\dagger\Big\{&\left(Q_+ - \frac{1}{2}\right)t_W(\widetilde{B}^0 f_+)\widetilde{f}_{L+}^* + \frac{1}{2}(\widetilde{W}^0 f_+)\widetilde{f}_{L+}^* \\ &+ \left(Q_- + \frac{1}{2}\right)t_W(\widetilde{B}^0 f_-)\widetilde{f}_{L-}^* - \frac{1}{2}(\widetilde{W}^0 f_-)\widetilde{f}_{L-}^* \\ &- Q_+ t_W(\widetilde{B}^0\bar{f}_+)^\dagger\widetilde{f}_{R-}^* - Q_- t_W(\widetilde{B}^0\bar{f}_-)^\dagger\widetilde{f}_{R-}^* + \text{c. c.}\Big\}.\end{aligned} \tag{16}$$

To get the Lagrangian on a familiar form in terms of Dirac spinors, I will define the following fields in a familiar way. For clarity, I suppress the $\pm$ in the fields, and rather

[5] The field supplying the right-handed part has the opposite sign charge such that $\Phi_R^\dagger$ and Φ_L have the same sign.

write the final Lagrangian on a form which generalises to both of them using $I_f^3 = \pm\frac{1}{2}$ where appropriate.

$$f_D = \begin{pmatrix} f \\ f^\dagger \end{pmatrix}, \quad \widetilde{B}_D^0 = \begin{pmatrix} \widetilde{B}^0 \\ \widetilde{B}^{0\dagger} \end{pmatrix}, \quad \widetilde{W}_D^0 = \begin{pmatrix} \widetilde{W}^0 \\ \widetilde{W}^{0\dagger} \end{pmatrix}, \tag{17}$$

with conjugates

$$\bar{f}_D = \begin{pmatrix} \bar{f} \\ f^\dagger \end{pmatrix}^T, \quad \bar{\widetilde{B}}_D^0 = \begin{pmatrix} \widetilde{B}^0 \\ \widetilde{B}^{0\dagger} \end{pmatrix}^T, \quad \bar{\widetilde{W}}_D^0 = \begin{pmatrix} \widetilde{W}^0 \\ \widetilde{W}^{0\dagger} \end{pmatrix}^T. \tag{18}$$

Using App. B and integrating (trivially) over the Grassmann coordinates using $\int \mathrm{d}^4\theta\,(\theta\theta)(\theta\theta)^\dagger = 1$, we are left with the ordinary Lagrangian term

$$\begin{aligned}\mathcal{L}_{f\widetilde{B}^0\widetilde{W}^0} = -\sqrt{2}g\Big\{&\bar{\widetilde{B}}_D^0\left[\left(Q_f - I_f^3\right)t_W\widetilde{f}_L^*P_L - Q_f t_W\widetilde{f}_R^*P_R\right]f_D \\ &+ \bar{\widetilde{W}}_D^0\left(I_f^3\widetilde{f}_L^*P_L\right)f_D + \text{c. c.}\Big\}.\end{aligned} \tag{19}$$

Changing to the $\widetilde{\chi}^0$-basis we have

$$\mathcal{L}_{\widetilde{\chi}^0\widetilde{f}f} = -\sqrt{2}g\sum_i \bar{\widetilde{\chi}}_i^0\Big\{\big[\underbrace{\left(Q_f - I_f^3\right)t_W N_{i1}^* + I_f^3 N_{i2}^*}_{\equiv C^{L*}_{\widetilde{\chi}_i^0\widetilde{f}f}}\big]\widetilde{f}_L^*P_L \underbrace{-Q_f t_W N_{i1}}_{\equiv C^{R*}_{\widetilde{\chi}_i^0\widetilde{f}f}}\widetilde{f}_R^*P_R\Big\}f_D + \text{c. c.} \tag{20}$$

We can generalise this to include squark mixing between the left- and right-handed squarks into mass eigenstates $\widetilde{f}_{A=1,2}$, where

$$\begin{pmatrix} \widetilde{f}_1 \\ \widetilde{f}_2 \end{pmatrix} = V_{\widetilde{f}}\begin{pmatrix} \widetilde{f}_L \\ \widetilde{f}_R \end{pmatrix} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}\begin{pmatrix} \widetilde{f}_L \\ \widetilde{f}_R \end{pmatrix}. \tag{21}$$

This leaves us with the Lagrangian terms

$$\mathcal{L}_{\widetilde{\chi}^0\widetilde{f}f} = -\sqrt{2}g\sum_i\sum_A \bar{\widetilde{\chi}}_i^0\Big\{\underbrace{R_{A1}C^L_{\widetilde{\chi}_i^0\widetilde{f}f}}_{\equiv C^{L*}_{\widetilde{\chi}_i^0\widetilde{f}_A f}}P_L + \underbrace{R_{A2}C^R_{\widetilde{\chi}_i^0\widetilde{f}f}}_{\equiv C^{R*}_{\widetilde{\chi}_i^0\widetilde{f}_A f}}P_R\Big\}\widetilde{f}_A^* f_D + \text{c. c.} \tag{22}$$

$$\tag{23}$$

4.2.2 Higgsino parts

The Higgs superfield doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ have kinetic terms

$$\mathcal{L}_{H\text{-kin}} = H_u^\dagger e^{g\left[\frac{1}{2}\sigma^a W^a + \frac{1}{2}t_W B^0\right]}H_u + H_d^\dagger e^{g\left[\frac{1}{2}\sigma^a W^a - \frac{1}{2}t_W B^0\right]}H_d. \tag{24}$$

These give rise to multiple neutralino interaction terms from the neutral Higgs superfields

$$\mathcal{L}_{H^0\text{-kin}} = {H_u^0}^\dagger e^{g\left[-\frac{1}{2}W^0 + \frac{1}{2}t_W B^0\right]}H_u^0 + {H_d^0}^\dagger e^{g\left[\frac{1}{2}W^0 - \frac{1}{2}t_W B^0\right]}H_d^0. \tag{25}$$

Using Eq. 8 we have the higgsino interaction terms (upper signs correspond to u, and lower signs to d)

$$\begin{aligned}\mathcal{L}_{\widetilde{H}^0-\text{int}} = &\mp\frac{g}{2}(\widetilde{H}_{u/d}^0\sigma^\mu\bar{\widetilde{H}}_{u/d}^0)\left(W_\mu^0 - t_W B_\mu^0\right) \\ &\pm\frac{g}{\sqrt{2}}\left[(\widetilde{W}^0\widetilde{H}_{u/d}^0)H_{u/d}^{0\,*} - t_W(\widetilde{B}^0\widetilde{H}_{u/d}^0)H_{u/d}^{0\,*} + \text{c. c.}\right].\end{aligned} \tag{26}$$

From electroweak symmetry breaking, we have that the Z-boson vector field is given by $Z_\mu = c_W W^0_\mu - s_W B^0_\mu$, meaning we can extract a Z-boson interaction from Eq. 26 as

$$
\begin{aligned}
\mathcal{L}_{\widetilde{H}^0 Z} &= -\frac{g}{2c_W} Z_\mu \left[(\widetilde{H}^0_u \sigma^\mu \overline{\widetilde{H}}{}^0_u) - (u \leftrightarrow d) \right] \\
&\stackrel{\text{Eq. 31b}}{=} \frac{g}{2c_W} Z_\mu \left[(\overline{\widetilde{H}}{}^0_u \bar{\sigma}^\mu \widetilde{H}^0_u) - (u \leftrightarrow d) \right] && (27) \\
& && (28)
\end{aligned}
$$

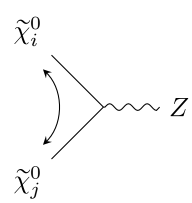
Converting the Weyl spinors into Dirac spinors in the usual way (see App. B), and changing to the $\widetilde{\chi}^0$-basis according to Eq. 5, we get

$$
\begin{aligned}
\mathcal{L}_{\widetilde{H}^0 Z} &= \frac{g}{2c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}{}^0_i \gamma^\mu P_L \widetilde{\chi}^0_j - (4 \leftrightarrow 3) \right] \\
&\stackrel{\text{Eq. 36}}{=} \frac{g}{4c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}{}^0_i \gamma^\mu P_L \widetilde{\chi}^0_j - N_{i4} N^*_{j4} \bar{\widetilde{\chi}}{}^0_j \gamma^\mu P_R \widetilde{\chi}^0_i - (4 \leftrightarrow 3) \right] \\
&= \frac{g}{4c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}{}^0_i \gamma^\mu P_L \widetilde{\chi}^0_j - N^*_{i4} N_{j4} \bar{\widetilde{\chi}}{}^0_i \gamma^\mu P_R \widetilde{\chi}^0_j - (4 \leftrightarrow 3) \right] \\
&= \frac{g}{2} Z_\mu \sum_{ij} \bar{\widetilde{\chi}}{}^0_i \gamma^\mu \Big[\underbrace{\frac{1}{2c_W} \left(N_{i4} N^*_{j4} - N_{i3} N^*_{j3} \right)}_{\equiv O''^L_{ij}} P_L - \underbrace{\frac{1}{2c_W} \left(N^*_{i4} N_{j4} - N^*_{i3} N_{j3} \right)}_{\equiv O''^R_{ij}} P_R \Big] \widetilde{\chi}^0_j ,
\end{aligned}
\tag{29}
$$

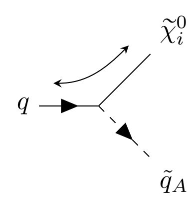
where in the third line I relabelled the indices of the second term.

4.2.3 Neutralino Feynman rules

The neutralino interactions with a quark/squark pair, and with a Z-boson, can be summarised in the following Feynman rules. Due to the symmetry of Majorana particles, we can match either index i, j with the external neutralinos, such that the corresponding Feynman rule is multiplied by two. In the case of identical neutralino interaction, symmetry dictates that the factor of two be removed. The external lines drawn indicate that the Feynman rule is identical either way you choose to read it.[6]



$$= ig\gamma^\mu \left[O''^L_{ij} P_L + O''^R_{ij} P_R \right] \tag{30a}$$



$$= -i\sqrt{2} g \left[C^{L*}_{\widetilde{\chi}^0_i \tilde{q}_A q} P_L + C^{R*}_{\widetilde{\chi}^0_i \tilde{q}_A q} P_R \right] \tag{30b}$$

[6] I go more into detail on how to read Feynman diagrams with neutralinos elsewhere.

Variable	Value
$C^L_{\widetilde{\chi}^0_i \widetilde{f}_A f}$	$R^*_{A1}\left[\left(Q_f - I^3_f\right) t_W N_{i1} + I^3_f N_{i2}\right]$
$C^R_{\widetilde{\chi}^0_i \widetilde{f}_A f}$	$-R^*_{A2} Q_f t_W N^*_{i1}$
O''^L_{ij}	$\frac{1}{2c_W}\left(N_{i4}N^*_{j4} - N_{i3}N^*_{j3}\right)$
O''^R_{ij}	$-\frac{1}{2c_W}\left(N^*_{i4}N_{j4} - N^*_{i3}N_{j3}\right)$

Table 2: A summary of the variables used in the derived Feynman rules and their definitions.

A Weyl identities

A brief summary of the Weyl identities that have been used in the derivations made in this document. Given two arbitrary left-handed Weyl spinors ψ, χ, with right-handed compliments $\bar{\psi}, \bar{\chi}$, the following identities hold.

$$(\theta\psi)(\theta\chi) = -\frac{1}{2}(\theta\theta)(\psi\chi) \tag{31a}$$

$$(\psi\sigma^\mu\bar{\chi}) = -(\bar{\chi}\bar{\sigma}^\mu\psi) \tag{31b}$$

$$(\theta\psi)^\dagger(\theta\sigma^\mu\theta^\dagger)(\theta\psi) = \frac{1}{4}(\theta\theta)(\theta\theta)^\dagger(\psi\sigma^\mu\psi^\dagger) \tag{31c}$$

B Weyl spinors to Dirac spinors

We can build a Dirac spinor Ψ using a left-handed Weyl spinor ψ and a right-handed Weyl spinor $\bar{\psi}^\dagger$ such that

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^\dagger \end{pmatrix}. \tag{32}$$

The γ-matrices in the Weyl representation are

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix}, \tag{33}$$

so we define the conjugate Dirac spinor

$$\bar{\Psi} \equiv \Psi^\dagger\gamma^0 = \begin{pmatrix} \bar{\psi} \\ \psi^\dagger \end{pmatrix}^T. \tag{34}$$

A Majorana fermion is constructed from just one Weyl spinor, such that $\psi_L = \psi_R \equiv \psi$. The projection operators $P_{L/R}$ project out the left-handed or right-handed Weyl spinors from the Dirac spinor. The following Weyl spinor products can then be rewritten in terms of Dirac spinors:

$$(\bar{\psi}\phi_L) = \bar{\Psi}P_L\Phi \tag{35a}$$

$$(\psi\bar{\phi})^\dagger = \bar{\Psi}P_R\Phi \tag{35b}$$

$$(\psi^\dagger\bar{\sigma}^\mu\phi) = \bar{\Psi}\gamma^\mu P_L\Phi \tag{35c}$$

$$(\bar{\psi}\sigma^\mu\bar{\phi}^\dagger) = \bar{\Psi}\gamma^\mu P_R\Phi \tag{35d}$$

Using equation Eq. 31b, we get the Dirac spinor relation between two Majorana spinors Ψ_M, Φ_M

$$\bar{\Psi}_M \gamma^\mu P_{L/R} \Phi_M = -\bar{\Phi}_M \gamma^\mu P_{R/L} \Psi_M \tag{36}$$