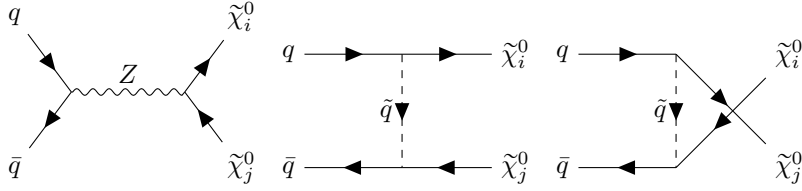


Leading Order Neutralino Calculation

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$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2 \quad (1a)$$

$$t = (p_1 - k_1)^2 = (p_2 - k_2)^2 \quad (1b)$$

$$u = (p_1 - k_2)^2 = (p_2 - k_1)^2 \quad (1c)$$

$$s + t + u = m_i^2 + m_j^2 \quad (1d)$$

$$(p_1 \cdot p_2) = \frac{s}{2} \quad (k_1 \cdot k_2) = \frac{s - m_i^2 - m_j^2}{2} \quad (2a)$$

$$(p_1 \cdot k_1) = \frac{m_i^2 - t}{2} \quad (p_2 \cdot k_2) = \frac{m_j^2 - t}{2} \quad (2b)$$

$$(p_1 \cdot k_2) = \frac{m_j^2 - u}{2} \quad (p_2 \cdot k_1) = \frac{m_i^2 - u}{2} \quad (2c)$$

$$\mathcal{M}_s = -\frac{g^2}{2c_W^2} D_Z(s) \left[\bar{u}_i \gamma^\mu \left(O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R \right) v_j \right] \times [\bar{v}_2 \gamma_\mu (C_{Zqq}^L P_L + C_{Zqq}^R P_R) u_1] \quad (3a)$$

$$\mathcal{M}_t = -2g^2 D_{\tilde{q}}(t) [\bar{u}_i (C_i^{L*} P_L + C_i^{R*} P_R) u_1] \times [\bar{v}_2 (C_j^R P_L + C_j^L P_R) v_j] \quad (3b)$$

$$\mathcal{M}_u = -2g^2 D_{\tilde{q}}(u) [\bar{u}_j (C_j^{L*} P_L + C_j^{R*} P_R) u_1] \times [\bar{v}_2 (C_i^R P_L + C_i^L P_R) v_i] \quad (3c)$$

$$I_{ss} = \sum_{\text{spins}} |\mathcal{M}_s|^2 = \frac{g^4}{c_W^2} |D_Z(s)|^2 \left((C_Z^L)^2 + (C_Z^R)^2 \right) \left\{ |O_{ij}^L|^2 [(m_i^2 - t)^2 + (m_j^2 - t)^2] - 2 \text{Re} \left\{ (O_{ij}^L)^2 \right\} m_i m_j s \right\} \quad (4a)$$

1 Fierz identities

Introducing first the generalised gamma matrices Γ_I^r defined in the following way:

$$\Gamma_S^0 = \mathbb{1}, \quad (5a)$$

$$\Gamma_V^{0,\dots,3} = \gamma^\mu, \quad (5b)$$

$$\Gamma_T^{0,\dots,5} = \sigma^{\mu\nu}, \quad (\mu < \nu) \quad (5c)$$

$$\Gamma_A^{0,\dots,3} = \gamma^\mu \gamma^5, \quad (5d)$$

$$\Gamma_P^0 = \gamma^5, \quad (5e)$$

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The upper index r is understood to be summed over if it is repeated in an expression, while the index I is only summed over when explicitly stated. The complement $\Gamma_{I,r}$ is found by lowering any Lorentz index in the standard way. The generalised Fierz identity then tells us that for spinors $w_{1,\dots,4}$ that can either be positive energy spinors u or negative energy spinors v , we have that

$$(\bar{w}_1 \Gamma_I^r w_2) (\bar{w}_3 \Gamma_J^s w_4) = \sum_{M,N} {}^{IJ}_{rs} C_{tu}^{MN} (\bar{w}_1 \Gamma_M^t w_4) (\bar{w}_3 \Gamma_N^u w_2), \quad (6)$$

with numerical coefficients ${}^{IJ}_{rs} C_{tu}^{MN}$. The coefficients are found by

$${}^{IJ}_{rs} C_{tu}^{MN} = \frac{1}{16} \text{Tr} [\Gamma_{M,t} \Gamma_I^r \Gamma_{N,u} \Gamma_J^s] \quad (7)$$

The Fierz transformation matrix F is given by¹

$$F = \frac{1}{4} \begin{bmatrix} 1 & 1 & \frac{1}{2} & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ 12 & 0 & -2 & 0 & 12 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & \frac{1}{2} & 1 & 1 \end{bmatrix} \quad (8)$$

in the bilinear product basis

$$q_S(1234) = (\bar{w}_1 w_2) (\bar{w}_3 w_4) \quad (9a)$$

$$q_V(1234) = (\bar{w}_1 \gamma^\mu w_2) (\bar{w}_3 \gamma_\mu w_4) \quad (9b)$$

$$q_T(1234) = (\bar{w}_1 \sigma^{\mu\nu} w_2) (\bar{w}_3 \sigma_{\mu\nu} w_4) \quad (9c)$$

$$q_A(1234) = (\bar{w}_1 \gamma^\mu \gamma^5 w_2) (\bar{w}_3 \gamma_\mu \gamma^5 w_4) \quad (9d)$$

$$q_P(1234) = (\bar{w}_1 \gamma^5 w_2) (\bar{w}_3 \gamma^5 w_4) \quad (9e)$$

$$(9f)$$

where a vector \mathbf{q} is given by

$$\mathbf{q}(abcd) = \sum_{i=1}^5 n_i \mathbf{e}_i, \quad (10)$$

for some coefficients n_i and with the canonical unit vectors $\{\mathbf{e}_i\} = \left\{ \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix}, \dots \right\}$.

The Dirac quadrilinear \mathbf{q} represents is found by the sum

$$\sum_{i=0}^5 n_i q_{B_i}(abcd), \quad (11)$$

¹Jose F. Nieves and Palash B. Pal. “Generalized Fierz identities”. In: *Am. J. Phys.* 72 (2004), pp. 1100–1108. DOI: 10.1119/1.1757445. arXiv: hep-ph/0306087.

where $B_i = S, V, T, A, P$.

The Fierz transformation swapping the indices $2 \leftrightarrow 4$ can be found by

$$\mathbf{q}(1234) = F\mathbf{q}(1432) \quad (12)$$

2 Factorisation

$$\begin{aligned} & [\bar{u}_i (C_i^{L*} P_L + C_i^{R*} P_R) u_1] [\bar{v}_2 (C_j^R P_L + C_j^L P_R) v_j] \\ &= C_{SS} [\bar{u}_i u_1] [\bar{v}_2 v_j] + C_{SP} [\bar{u}_i u_1] [\bar{v}_2 \gamma^5 v_j] \\ &+ C_{PS} [\bar{u}_i \gamma^5 u_1] [\bar{v}_2 v_j] + C_{PP} [\bar{u}_i \gamma^5 u_1] [\bar{v}_2 \gamma^5 v_j], \end{aligned} \quad (13)$$

where we have

$$C_{SS} = \frac{1}{4} (C_i^{L*} + C_i^{R*}) (C_j^L + C_j^R) \quad (14a)$$

$$C_{SP} = \frac{1}{4} (C_i^{L*} + C_i^{R*}) (C_j^L - C_j^R) \quad (14b)$$

$$C_{PS} = -\frac{1}{4} (C_i^{L*} - C_i^{R*}) (C_j^L + C_j^R) \quad (14c)$$

$$C_{PP} = -\frac{1}{4} (C_i^{L*} - C_i^{R*}) (C_j^L - C_j^R) \quad (14d)$$

$$\begin{aligned} & \left[\bar{u}_i \left(C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{L*} P_L + C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{R*} P_R \right) u_1 \right] \left[\bar{v}_2 \left(C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^R P_L + C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^L P_R \right) v_j \right] \\ &= C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{L*} C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^R \left[\frac{1}{2} (\bar{u}_i P_L v_j) (\bar{v}_2 P_L u_1) + \frac{1}{8} (\bar{u}_i \sigma^{\mu\nu} v_j) (\bar{v}_2 \sigma_{\mu\nu} P_L u_1) \right] \\ &+ C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{L*} C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^L \left[\frac{1}{2} (\bar{u}_i \gamma^\mu P_R v_j) (\bar{v}_2 \gamma_\mu P_L u_1) \right] \\ &+ C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{R*} C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^R \left[\frac{1}{2} (\bar{u}_i \gamma^\mu P_L v_j) (\bar{v}_2 \gamma_\mu P_R u_1) \right] \\ &+ C_{\tilde{\chi}_i^0 \tilde{q}_{Aq}}^{R*} C_{\tilde{\chi}_j^0 \tilde{q}_{Aq}}^L \left[\frac{1}{2} (\bar{u}_i P_R v_j) (\bar{v}_2 P_R u_1) + \frac{1}{8} (\bar{u}_i \sigma^{\mu\nu} v_j) (\bar{v}_2 \sigma_{\mu\nu} P_R u_1) \right] \end{aligned} \quad (15)$$