Leading Order Neutralino Calculation

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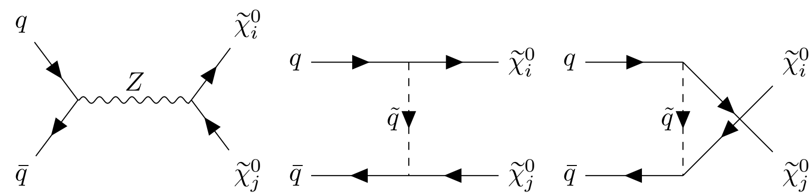

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = (k_1 + k_2)^2 && \text{(1a)}\\
t &= (p_1 - k_1)^2 = (p_2 - k_2)^2 && \text{(1b)}\\
u &= (p_1 - k_2)^2 = (p_2 - k_1)^2 && \text{(1c)}\\
s + t + u &= m_i^2 + m_j^2 && \text{(1d)}
\end{aligned}
$$

$$
\begin{aligned}
(p_1 \cdot p_2) &= \frac{s}{2} & (k_1 \cdot k_2) &= \frac{s - m_i^2 - m_j^2}{2} && \text{(2a)}\\
(p_1 \cdot k_1) &= \frac{m_i^2 - t}{2} & (p_2 \cdot k_2) &= \frac{m_j^2 - t}{2} && \text{(2b)}\\
(p_1 \cdot k_2) &= \frac{m_j^2 - u}{2} & (p_2 \cdot k_1) &= \frac{m_i^2 - u}{2} && \text{(2c)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_s &= -\frac{g^2}{2c_W^2} D_Z(s) \left[\bar{u}_i \gamma^\mu \left(O''^L_{ij} P_L + O''^R_{ij} P_R\right) v_j\right]\\
&\qquad \times \left[\bar{v}_2 \gamma_\mu \left(C^L_{Zqq} P_L + C^R_{Zqq} P_R\right) u_1\right] && \text{(3a)}\\
\mathcal{M}_t &= -2g^2 D_{\tilde{q}}(t) \left[\bar{u}_i \left(C_i^{L*} P_L + C_i^{R*} P_R\right) u_1\right]\\
&\qquad \times \left[\bar{v}_2 \left(C_j^R P_L + C_j^L P_R\right) v_j\right] && \text{(3b)}\\
\mathcal{M}_u &= -2g^2 D_{\tilde{q}}(u) \left[\bar{u}_j \left(C_j^{L*} P_L + C_j^{R*} P_R\right) u_1\right]\\
&\qquad \times \left[\bar{v}_2 \left(C_i^R P_L + C_i^L P_R\right) v_i\right] && \text{(3c)}
\end{aligned}
$$

$$
\begin{aligned}
I_{ss} = \sum_{\text{spins}} |\mathcal{M}_s|^2 = \frac{g^4}{c_W^2} |D_Z(s)|^2 \left((C_Z^L)^2 + (C_Z^R)^2\right) \bigg\{ \left|O_{ij}^L\right|^2 \left[(m_i^2 - t)^2 + (m_j^2 - t)^2\right]\\
- 2\,\mathrm{Re}\left\{\left(O_{ij}^L\right)^2\right\} m_i m_j s \bigg\} && \text{(4a)}
\end{aligned}
$$

1 Fierz identities

Introducing first the generalised gamma matrices Γ_I^r defined in the following way:

$$\Gamma_S^0 = \mathbb{1}, \tag{5a}$$
$$\Gamma_V^{0,\dots,3} = \gamma^\mu, \tag{5b}$$
$$\Gamma_T^{0,\dots,5} = \sigma^{\mu\nu}, \quad (\mu < \nu) \tag{5c}$$
$$\Gamma_A^{0,\dots,3} = \gamma^\mu\gamma^5, \tag{5d}$$
$$\Gamma_P^0 = \gamma^5, \tag{5e}$$

where $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The upper index r is understood to be summed over if it is repeated in an expression, while the index I is only summed over when explicitly stated. The complement $\Gamma_{I,r}$ is found by lowering any Lorentz index in the standard way. The generalised Fierz identity then tells us that for spinors $w_{1,\dots,4}$ that can either be positive energy spinors u or negative energy spinors v, we have that

$$\left(\bar{w}_1\Gamma_I^r w_2\right)\left(\bar{w}_3\Gamma_J^s w_4\right) = \sum_{M,N} {}^{IJ}_{rs}C^{MN}_{tu}\left(\bar{w}_1\Gamma_M^t w_4\right)\left(\bar{w}_3\Gamma_N^u w_2\right), \tag{6}$$

with numerical coefficients ${}^{IJ}_{rs}C^{MN}_{tu}$. The coefficients are found by

$${}^{IJ}_{rs}C^{MN}_{tu} = \frac{1}{16}\operatorname{Tr}\left[\Gamma_{M,t}\Gamma_I^r\Gamma_{N,u}\Gamma_J^s\right] \tag{7}$$

The Fierz transformation matrix F is given by[1]

$$F = \frac{1}{4}\begin{bmatrix} 1 & 1 & \frac{1}{2} & -1 & 1 \\ 4 & -2 & 0 & -2 & -4 \\ 12 & 0 & -2 & 0 & 12 \\ -4 & -2 & 0 & -2 & 4 \\ 1 & -1 & \frac{1}{2} & 1 & 1 \end{bmatrix} \tag{8}$$

in the bilinear product basis

$$q_S(1234) = \left(\bar{w}_1 w_2\right)\left(\bar{w}_3 w_4\right) \tag{9a}$$
$$q_V(1234) = \left(\bar{w}_1\gamma^\mu w_2\right)\left(\bar{w}_3\gamma_\mu w_4\right) \tag{9b}$$
$$q_T(1234) = \left(\bar{w}_1\sigma^{\mu\nu} w_2\right)\left(\bar{w}_3\sigma_{\mu\nu} w_4\right) \tag{9c}$$
$$q_A(1234) = \left(\bar{w}_1\gamma^\mu\gamma^5 w_2\right)\left(\bar{w}_3\gamma_\mu\gamma^5 w_4\right) \tag{9d}$$
$$q_P(1234) = \left(\bar{w}_1\gamma^5 w_2\right)\left(\bar{w}_3\gamma^5 w_4\right) \tag{9e}$$
$$\tag{9f}$$

where a vector $\boldsymbol{q}$ is given by

$$\boldsymbol{q}(abcd) = \sum_{i=1}^{5} n_i\boldsymbol{e}_i, \tag{10}$$

for some coefficients n_i and with the canonical unit vectors $\{\boldsymbol{e}_i\} = \left\{\begin{pmatrix}1\\0\\\vdots\end{pmatrix}, \begin{pmatrix}0\\1\\\vdots\end{pmatrix}, \dots\right\}$.

The Dirac quadrilinear q represents is found by the inner product

$$\boldsymbol{q}\cdot\sum_{i=1}^{5} q_{B_i}(abcd)\boldsymbol{e}_i, \tag{11}$$

[1] Jose F. Nieves and Palash B. Pal. "Generalized Fierz identities". In: *Am. J. Phys.* 72 (2004), pp. 1100–1108. DOI: 10.1119/1.1757445. arXiv: hep-ph/0306087.

where $B_i = S, V, T, A, P$.

The Fierz transformation swapping the indices $2 \leftrightarrow 4$ can be found by

$$\boldsymbol{q}(1234) = F\boldsymbol{q}(1432) \tag{12}$$

2 Factorisation

$$\begin{aligned}
&\left[\bar{u}_i \left(C_i^{L*} P_L + C_i^{R*} P_R\right) u_1\right] \left[\bar{v}_2 \left(C_j^R P_L + C_j^L P_R\right) v_j\right] \\
=&C_{SS} \left[\bar{u}_i u_1\right] \left[\bar{v}_2 v_j\right] + C_{SP} \left[\bar{u}_i u_1\right] \left[\bar{v}_2 \gamma^5 v_j\right] \\
&+ C_{PS} \left[\bar{u}_i \gamma^5 u_1\right] \left[\bar{v}_2 v_j\right] + C_{PP} \left[\bar{u}_i \gamma^5 u_1\right] \left[\bar{v}_2 \gamma^5 v_j\right],
\end{aligned} \tag{13}$$

where we have

$$C_{SS} = \frac{1}{4} \left(C_i^{L*} + C_i^{R*}\right) \left(C_j^L + C_j^R\right) \tag{14a}$$

$$C_{SP} = \frac{1}{4} \left(C_i^{L*} + C_i^{R*}\right) \left(C_j^L - C_j^R\right) \tag{14b}$$

$$C_{PS} = -\frac{1}{4} \left(C_i^{L*} - C_i^{R*}\right) \left(C_j^L + C_j^R\right) \tag{14c}$$

$$C_{PP} = -\frac{1}{4} \left(C_i^{L*} - C_i^{R*}\right) \left(C_j^L - C_j^R\right) \tag{14d}$$

$$\begin{aligned}
&\left[\bar{u}_i \left(C^{L*}_{\tilde{\chi}_i^0 \tilde{q}_A q} P_L + C^{R*}_{\tilde{\chi}_i^0 \tilde{q}_A q} P_R\right) u_1\right] \left[\bar{v}_2 \left(C^{R}_{\tilde{\chi}_j^0 \tilde{q}_A q} P_L + C^{L}_{\tilde{\chi}_j^0 \tilde{q}_A q} P_R\right) v_j\right] \\
=&C^{L*}_{\tilde{\chi}_i^0 \tilde{q}_A q} C^{R}_{\tilde{\chi}_j^0 \tilde{q}_A q} \left[\frac{1}{2} \left(\bar{u}_i P_L v_j\right) \left(\bar{v}_2 P_L u_1\right) + 3 \left(\bar{u}_i \sigma^{\mu\nu} v_j\right) \left(\bar{v}_2 \sigma_{\mu\nu} P_L u_1\right)\right] \\
&+ C^{L*}_{\tilde{\chi}_i^0 \tilde{q}_A q} C^{L}_{\tilde{\chi}_j^0 \tilde{q}_A q} \left[2 \left(\bar{u}_i \gamma^\mu P_L v_j\right) \left(\bar{v}_2 \gamma_\mu P_R u_1\right)\right] \\
&+ C^{R*}_{\tilde{\chi}_i^0 \tilde{q}_A q} C^{R}_{\tilde{\chi}_j^0 \tilde{q}_A q} \left[2 \left(\bar{u}_i \gamma^\mu P_R v_j\right) \left(\bar{v}_2 \gamma_\mu P_L u_1\right)\right] \\
&+ C^{R*}_{\tilde{\chi}_i^0 \tilde{q}_A q} C^{L}_{\tilde{\chi}_j^0 \tilde{q}_A q} \left[\frac{1}{2} \left(\bar{u}_i P_R v_j\right) \left(\bar{v}_2 P_R u_1\right) + 3 \left(\bar{u}_i \sigma^{\mu\nu} v_j\right) \left(\bar{v}_2 \sigma_{\mu\nu} P_R u_1\right)\right]
\end{aligned} \tag{15}$$