Neutralino Interactions in the MSSM

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June 8, 2023

1 Superfields

A left-handed scalar superfield Φ can be written out in terms of component fields as[1]

$$\Phi = A + i(\theta\sigma^\mu\bar{\theta})\partial_\mu A - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box A + \sqrt{2}(\theta\psi) - \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi\sigma^\mu\bar{\theta}) + (\theta\theta)F, \tag{1}$$

where A, F are complex scalar fields and ψ is a left-handed Weyl spinor. Ψ has a right-handed scalar superfield compliment found by conjugating it:

$$\Phi^\dagger = A^* - i(\theta\sigma^\mu\bar{\theta})\partial_\mu A^* - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box A^* + \sqrt{2}(\bar{\theta}\bar{\psi}) + \frac{i}{\sqrt{2}}(\bar{\theta}\bar{\theta})(\theta\sigma^\mu\partial_\mu\bar{\psi}) + (\bar{\theta}\bar{\theta})F^*, \tag{2}$$

where $\bar{\psi}$ is the right-handed compliment of ψ such that $\bar{\psi}^{\dot{A}} = \delta^{\dot{A}A}(\psi_A)^*$.

A vector superfield V can be written in Wess-Zumino gauge as

$$V_{\mathrm{WZ}} = (\theta\sigma^\mu\bar{\theta})\left[V_\mu + i\partial_\mu(A - A^*)\right] + (\theta\theta)(\overline{\theta\lambda}) + (\bar{\theta}\bar{\theta})(\theta\lambda) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})D, \tag{3}$$

where V_μ is a real vector field, λ is a left-handed Weyl spinor and D is a (auxiliary) complex scalar field. The $\partial_\mu(A - A^*)$-term represents the gauge freedom remaining in the choice of supergauge after choosing Wess-Zumino gauge, and can be ignored when working out the interaction terms.

2 MSSM Superfields

For completeness, I list here the relevant superfields containing the neutralinos and fields that couple to them. These include the $SU(2)_L$ superfield doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$, $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$, $L_i = \begin{pmatrix} l_i \\ \nu_i \end{pmatrix}$ and $Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$, and $SU(2)_L$ singlet superfields E_i, U_i and D_i, where $i = 1, 2, 3$ enumerates the three generations of leptons/quarks. There are also the vector superfields B^0 for the $U(1)_Y$ gauge group, and W^0 and $W^\pm$ for the $SU(2)_L$ gauge group.

2.1 Neutralino Fields

Letting $\Psi_{\tilde{\chi}^0} = \left(\widetilde{B}^0, \widetilde{W}^0, \widetilde{H}_u^0, \widetilde{H}_d^0\right)$ denote a vector[2] of the fermion field superpartners to the B and W vector fields and the $H_{u/d}$ scalar fields — the neutralinos are given

[1] Parentheses are used to clarify Weyl spinor contraction.

[2] I use row vector notation here for convenience. In equations this is understood to be a column vector.

Superfield	Boson field	Fermion field	Auxiliary field
$H^0_{u/d}$	$H^0_{u/d}$	$\widetilde{H}^0_{u/d}$	$F_{H_{u/d}}$
$H^\pm_{u/d}$	$H^\pm_{u/d}$	$\widetilde{H}^\pm_{u/d}$	$F_{H_{u/d}}$
l_i	$\widetilde{l}_{iL}$	l_{iL}	$F_{l_{iL}}$
E_i	$\widetilde{l}^*_{iR}$	l_{iR}	$F^*_{l_{iR}}$
ν_i	$\widetilde{\nu}_{iL}$	ν_{iL}	$F_{\nu_{iL}}$
u_i	$\widetilde{u}_{iL}$	u_{iL}	$F_{u_{iL}}$
U_i	$\widetilde{u}^*_{iR}$	u_{iR}	$F^*_{u_{iR}}$
d_i	$\widetilde{d}_{iL}$	d_{iL}	$F_{d_{iL}}$
D_i	$\widetilde{d}^*_{iR}$	d_{iR}	$F^*_{d_{iR}}$
B^0	B^0_μ	$\widetilde{B}^0$	D_{B^0}
W^0	W^0_μ	$\widetilde{W}^0$	D_{W^0}
$W^\pm$	$W^\pm_\mu$	$\widetilde{W}^\pm$	$D_{W^\pm}$

Table 1: Table of the MSSM superfields and their component field names. Not that all fermion fields are left-handed Weyl spinors, in spite of any L or R in the subscript. The conjugate superfields changes these to right-handed Weyl spinors.

by

$$\widetilde{\chi}^0 = \left(\widetilde{\chi}^0_1, \widetilde{\chi}^0_2, \widetilde{\chi}^0_3, \widetilde{\chi}^0_4\right) = N\Psi_{\widetilde{\chi}^0}, \tag{4}$$

where N is a unitary matrix diagonalising the neutralino mass matrix $M_{\widetilde{\chi}^0\text{-mass}}$. Thus, the neutralino interactions are found in terms in the superlagrangian that include the vector superfields B^0 and W^0, and the scalar superfields H^0_u and H^0_d. Translating from the $\Psi_{\widetilde{\chi}^0}$-basis to the $\widetilde{\chi}^0$-basis, we have that

$$\widetilde{B}^0 = N^*_{i1}\widetilde{\chi}^0_i, \quad \widetilde{W}^0 = N^*_{i2}\widetilde{\chi}^0_i, \quad \widetilde{H}^0_d = N^*_{i3}\widetilde{\chi}^0_i, \quad \widetilde{H}^0_u = N^*_{i4}\widetilde{\chi}^0_i, \tag{5}$$

where a sum over i is implied.

3 MSSM Superlagrangian

3.1 Neutralino interactions

The neutralino interactions appear in the kinetic terms of all the superfields that couple to the $U(1)_Y$ and $SU(2)_L$ gauge groups, and the Yukawa terms that include the Higgs fields in the superpotential. The exception is neutralino interaction with the charginos, which is also found in the supersymmetric field strength term.

4 Interaction Lagrangian

The ordinary interaction Lagrangian is found from integrating over the Grassmann variables of the superlagrangian. Only terms containing all four Grassmann variables survive this, so we only need to look for the superlagrangian terms that include all of $(\theta\theta)(\bar{\theta}\bar{\theta})$.[3] Looking first at a general kinetic term of a superfield coupled to a $U(1)$ vector field, it has the form[4]

$$\mathcal{L}_{\text{kin}} = \Phi^\dagger e^{2qV}\Phi. \tag{6}$$

[3]Terms with an insufficient amount of θs are ignored in the following.

[4]P. Binetruy. *Supersymmetry: Theory, experiment and cosmology.* 2006.

To find interaction that include the λ-fields of the vector superfield, we find

$$\begin{aligned}\mathcal{L}_{\text{kin}} &\overset{\lambda,\bar{\lambda}}{\supset} 2q\left\{A^*(\overline{\theta\theta})(\theta\lambda)\sqrt{2}(\theta\psi)+\sqrt{2}(\bar{\theta}\bar{\psi})(\theta\theta)(\overline{\theta\lambda})A\right\}\\ &\overset{\text{Eq. 30}}{=} -\sqrt{2}q(\theta\theta)(\overline{\theta\theta})\left\{(\lambda\psi)A^*+\text{c. c.}\right\}.\end{aligned} \tag{7}$$

Ignoring ordinary kinetic terms, the interactions that include the ψ-fields of the scalar superfields include

$$\begin{aligned}\mathcal{L}_{\text{kin}} &\overset{\psi,\bar{\psi}}{\supset} 2q\left\{A^*(\overline{\theta\theta})(\theta\lambda)\sqrt{2}(\theta\psi)+\sqrt{2}(\bar{\theta}\bar{\psi})(\theta\sigma^\mu\bar{\theta})V_\mu\sqrt{2}(\theta\psi)+\sqrt{2}(\bar{\theta}\bar{\psi})(\theta\theta)(\overline{\theta\lambda})A\right\}\\ &\overset{\text{Eq. 30 and 32}}{=} q(\theta\theta)(\overline{\theta\theta})\left\{-\sqrt{2}(\lambda\psi)A^*+(\psi\sigma^\mu\bar{\psi})V_\mu-\sqrt{2}(\overline{\psi\lambda})A\right\},\end{aligned} \tag{8}$$

where we notice that the $\lambda\psi$-interactions are the same as the ones we found in Eq. 7. With a Yukawa superpotential on the form

$$W = y_{ij}\Phi_i\Phi\Phi_j, \tag{9}$$

the superlagrangian looks like

$$\mathcal{L}_{\text{Yukawa}} = y_{ij}(\overline{\theta\theta})\Phi_i\Phi\Phi_j + \text{c. c.} \tag{10}$$

Extracting the ψ fermion interactions from the Φ superfield, we have

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &\overset{\psi,\bar{\psi}}{\supset} y_{ij}(\overline{\theta\theta})\sqrt{2}(\theta\psi)\left\{A_i\sqrt{2}(\theta\psi_i)+\sqrt{2}(\theta\psi_j)A_j\right\}+\text{c. c.}\\ &\overset{\text{Eq. 30}}{=} -y_{ij}(\theta\theta)(\overline{\theta\theta})\left\{A_i(\psi\psi_j)+(\psi_i\psi)A_j+\text{c. c.}\right\}\end{aligned} \tag{11}$$

4.1 Neutralino interaction terms

4.1.1 Gaugino parts

First, I will look at the bino and wino interactions. From electroweak unification, we have that the coupling g and g' of the W^a and B^0 superfields respectively are related by

$$g' = gt_W, \tag{12}$$

where $t_W = s_W/c_W \equiv \sin\theta_W/\cos\theta_W$ where θ_W is the Weinberg mixing angle. Rewriting using $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, and defining $W^\pm = W^1 \mp iW^2$ and $W^0 \equiv W^3$, we have

$$Yg'B^0 + \frac{1}{2}g\sigma^aW^a = g\left\{Yt_WB^0+\frac{1}{2}\sigma_3W^0+\frac{1}{2}\sigma_+W^++\frac{1}{2}\sigma_-W^-\right\} \tag{13}$$

So an MSSM superfield doublet $\Phi = \begin{pmatrix}\Phi^+\\ \Phi^-\end{pmatrix}$ charged under $U(1)_Y$ with charge Y and $SU(2)_L$ will have a kinetic term

$$\mathcal{L}_{\Phi\text{-kin}} = \Phi^\dagger e^{2g\left[Yt_WB^0+\frac{1}{2}\sigma_3W^0+\frac{1}{2}\sigma_+W^++\frac{1}{2}\sigma_-W^-\right]}\Phi. \tag{14}$$

We can extract the fermion interactions from the vector superfields B^0 and W^0 using Eq. 7 to be

$$\begin{aligned}\mathcal{L}_{\Phi\text{-kin}} \overset{\widetilde{B}^0,\widetilde{W}^0}{\supset} -\sqrt{2}g(\theta\theta)(\overline{\theta\theta})\Big\{&Yt_W(\widetilde{B}^0\psi^+)A^{+*}+I^3_+(\widetilde{W}^0\psi^+)A^{+*}\\ &+Yt_W(\widetilde{B}^0\psi^-)A^{-*}+I^3_-(\widetilde{W}^0\psi^-)A^{-*}+\text{c. c.}\Big\},\end{aligned} \tag{15}$$

where we recognise $I^3_\pm = \pm\frac{1}{2}$ as the eigenvalues of $\frac{1}{2}\sigma_3$.

In the MSSM, the Dirac fermions are made up from two scalar superfields, supplying the left- and right-handed components separately. Both superfields couple to the $U(1)_Y$ gauge group with charge $Q - I^3$,[5] where Q is the electric charge of the fermion, and I^3 is the weak isospin; either $\pm\frac{1}{2}$ for the superfields supplying left-handed fermions and 0 for the superfields supplying the right-handed ones. Only the left-handed field couples to the $SU(2)_L$ gauge group. Thus, the bino and wino interaction with a pair of MSSM fermions formed from a superfield doublet $F = \begin{pmatrix} f_L^+ \\ f_L^- \end{pmatrix}$ and the superfields $f_R^\pm$ are

$$\begin{aligned}\mathcal{L}_{\text{EW-kin}} \overset{\widetilde{B}^0,\widetilde{W}^0}{\supset} -\sqrt{2}g(\theta\theta)(\bar{\theta}\bar{\theta})\Big\{ &\left(Q_+ - \frac{1}{2}\right) t_W (\widetilde{B}^0 f_L^+)\widetilde{f}_L^{+*} + \frac{1}{2}(\widetilde{W}^0 f_L^+)\widetilde{f}_L^{+*} \\ &+ \left(Q_- + \frac{1}{2}\right) t_W (\widetilde{B}^0 f_L^-)\widetilde{f}_L^{-*} - \frac{1}{2}(\widetilde{W}^0 f_L^-)\widetilde{f}_L^{-*} \\ &- Q_+ t_W (\bar{\widetilde{B}}^0 \bar{f}_R^+)\widetilde{f}_R^{+*} - Q_- t_W (\bar{\widetilde{B}}^0 \bar{f}_R^-)\widetilde{f}_R^{-*} + \text{c. c.}\Big\}. \end{aligned} \tag{16}$$

To get the Lagrangian on a familiar form in terms of Dirac spinors, I will define the following fields, built from left- and right-handed Weyl spinors:

$$f = \begin{pmatrix} f_L \\ \bar{f}_R \end{pmatrix}, \quad \widetilde{B}^0_D = \begin{pmatrix} \widetilde{B}^0 \\ \bar{\widetilde{B}}^0 \end{pmatrix}, \quad \widetilde{W}^0_D = \begin{pmatrix} \widetilde{W}^0 \\ \bar{\widetilde{W}}^0 \end{pmatrix}, \tag{17}$$

with conjugates

$$\bar{f} = \begin{pmatrix} f_R \\ \bar{f}_L \end{pmatrix}^T, \quad \bar{\widetilde{B}}^0_D = \begin{pmatrix} \widetilde{B}^0 \\ \bar{\widetilde{B}}^0 \end{pmatrix}^T, \quad \bar{\widetilde{W}}^0_D = \begin{pmatrix} \widetilde{W}^0 \\ \bar{\widetilde{W}}^0 \end{pmatrix}^T, \tag{18}$$

Suppressing $\pm$ in the field names and inserting the isospin I^3 for the factors of $\pm\frac{1}{2}$, we have the ordinary Lagrangian term

$$\begin{aligned}\mathcal{L}_{f\widetilde{B}^0\widetilde{W}^0} = -\sqrt{2}g\Big\{ &\bar{\widetilde{B}}^0_D \Big[\left(Q_f - I^3_f\right) t_W \tilde{f}_L^* P_L - Q_f t_W \tilde{f}_R^* P_R \Big] f \\ &+ \bar{\widetilde{W}}^0_D \left(I^3_f \tilde{f}_L^* P_L \right) f + \text{c. c.} \Big\}, \end{aligned} \tag{19}$$

after integrating over the Grassmann variables. Changing to the $\widetilde{\chi}^0_i$-basis we have

$$\mathcal{L}_{\widetilde{\chi}^0_i \tilde{f} f} = -\sqrt{2}g \sum_i \bar{\widetilde{\chi}}^0_i \Big\{ \Big[\underbrace{\left(Q_f - I^3_f\right) t_W N_{i1} + I^3_f N_{i2}}_{\equiv C^{L*}_{\widetilde{\chi}^0_i \tilde{f} f}} \Big] \tilde{f}_L^* P_L \underbrace{- Q_f t_W N_{i1}}_{\equiv C^{R*}_{\widetilde{\chi}^0_i \tilde{f} f}} \tilde{f}_R^* P_R \Big\} f + \text{c. c.} \tag{20}$$

Now we can generalise this to include squark mixing between the left- and right-handed squarks into mass eigenstates $\widetilde{f}_{1,2}$, where

$$\begin{pmatrix} \widetilde{f}_1 \\ \widetilde{f}_2 \end{pmatrix} = \begin{bmatrix} c_{\tilde{f}} & -s^*_{\tilde{f}} \\ s_{\tilde{f}} & c^*_{\tilde{f}} \end{bmatrix} \begin{pmatrix} \widetilde{f}_L \\ \widetilde{f}_R \end{pmatrix}. \tag{21}$$

[5] The field supplying the right-handed part has the opposite sign charge such that $\Phi_R^\dagger$ and Φ_L have the same sign.

This leaves us with the Lagrangian terms

$$\mathcal{L}_{\widetilde{\chi}_i^0 \tilde{f}_1 f} = -\sqrt{2} g \sum_i \bar{\widetilde{\chi}}_i^0 \Big\{ \underbrace{c_{\tilde{f}} C^{L*}_{\widetilde{\chi}_i^0 \tilde{f} f}}_{\equiv C^{L*}_{\widetilde{\chi}_i^0 \tilde{f}_1 f}} P_L \underbrace{-s^*_{\tilde{f}} C^{R*}_{\widetilde{\chi}_i^0 \tilde{f} f}}_{\equiv C^{R*}_{\widetilde{\chi}_i^0 \tilde{f}_1 f}} P_R \Big\} \tilde{f}_1^* f + \text{c. c.} \tag{22}$$

$$\mathcal{L}_{\widetilde{\chi}_i^0 \tilde{f}_2 f} = -\sqrt{2} g \sum_i \bar{\widetilde{\chi}}_i^0 \Big\{ \underbrace{s_{\tilde{f}} C^{L*}_{\widetilde{\chi}_i^0 \tilde{f} f}}_{\equiv C^{L*}_{\widetilde{\chi}_i^0 \tilde{f}_2 f}} P_L + \underbrace{c^*_{\tilde{f}} C^{R*}_{\widetilde{\chi}_i^0 \tilde{f} f}}_{\equiv C^{R*}_{\widetilde{\chi}_i^0 \tilde{f}_2 f}} P_R \Big\} \tilde{f}_2^* f + \text{c. c.} \tag{23}$$

4.1.2 Higgsino parts

The Higgs superfield doublets $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ have kinetic terms

$$\mathcal{L}_{H\text{-kin}} = H_u^\dagger e^{g\left[\frac{1}{2}\sigma^a W^a + \frac{1}{2} t_W B^0\right]} H_u + H_d^\dagger e^{g\left[\frac{1}{2}\sigma^a W^a - \frac{1}{2} t_W B^0\right]} H_d. \tag{24}$$

These give rise to multiple neutralino interaction terms from the neutral Higgs superfields

$$\mathcal{L}_{H^0\text{-kin}} = {H_u^0}^\dagger e^{g\left[-\frac{1}{2} W^0 + \frac{1}{2} t_W B^0\right]} H_u^0 + {H_d^0}^\dagger e^{g\left[\frac{1}{2} W^0 - \frac{1}{2} t_W B^0\right]} H_d^0. \tag{25}$$

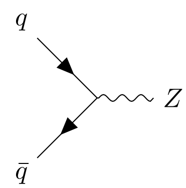
Using Eq. 8 we have the higgsino interaction terms (upper signs correspond to u, and lower signs to d)

$$\begin{aligned} \mathcal{L}_{\tilde{H}^0 \dots} &= \mp \frac{g}{2} (\widetilde{H}^0_{u/d} \sigma^\mu \overline{\widetilde{H}}{}^0_{u/d}) \left(W^0_\mu - t_W B^0_\mu \right) \\ &\pm \frac{g}{\sqrt{2}} \left[(\widetilde{W}^0 \widetilde{H}^0_{u/d}) H^{0\,*}_{u/d} - t_W (\widetilde{B}^0 \widetilde{H}^0_{u/d}) H^{0\,*}_{u/d} + \text{c. c.} \right]. \end{aligned} \tag{26}$$

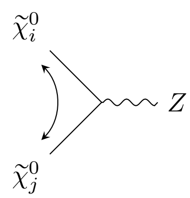
$$\begin{aligned} \mathcal{L}_{\widetilde{H}^0 Z} &= -\frac{g}{2c_W} Z_\mu \left[(\widetilde{H}^0_u \sigma^\mu \overline{\widetilde{H}}{}^0_u) - (u \leftrightarrow d) \right] \\ &= \frac{g}{2c_W} Z_\mu \left[(\overline{\widetilde{H}}{}^0_u \bar{\sigma}^\mu \widetilde{H}^0_u) - (u \leftrightarrow d) \right] \\ &= \frac{g}{2c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}_i^0 \gamma^\mu P_L \widetilde{\chi}_j^0 - (4 \leftrightarrow 3) \right] \\ &= \frac{g}{4c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}_i^0 \gamma^\mu P_L \widetilde{\chi}_j^0 - N_{i4} N^*_{j4} \bar{\widetilde{\chi}}_j^0 \gamma^\mu P_R \widetilde{\chi}_i^0 - (4 \leftrightarrow 3) \right] \\ &= \frac{g}{4c_W} Z_\mu \sum_{ij} \left[N_{i4} N^*_{j4} \bar{\widetilde{\chi}}_i^0 \gamma^\mu P_L \widetilde{\chi}_j^0 - N^*_{i4} N_{j4} \bar{\widetilde{\chi}}_i^0 \gamma^\mu P_R \widetilde{\chi}_j^0 - (4 \leftrightarrow 3) \right] \\ &= \frac{g}{2c_W} Z_\mu \sum_{ij} \bar{\widetilde{\chi}}_i^0 \gamma^\mu \Big[\underbrace{\frac{1}{2} \left(N_{i4} N^*_{j4} - N_{i3} N^*_{j3} \right)}_{\equiv O''^L_{ij}} P_L \underbrace{- \frac{1}{2} \left(N^*_{i4} N_{j4} - N^*_{i3} N_{j3} \right)}_{\equiv O''^R_{ij}} P_R \Big] \widetilde{\chi}_j^0 \end{aligned} \tag{27}$$

$$\begin{aligned} \mathcal{L}_{\widetilde{H}^0 \widetilde{W}^0 \tilde{B}^0 H^0} &= \frac{g}{\sqrt{2}} \left[\left((\widetilde{W}^0 \widetilde{H}^0_u) - t_W (\widetilde{B}^0 \widetilde{H}^0_u) \right) H^{0\,*}_u + \text{c. c.} - (u \leftrightarrow d) \right] \\ &= \frac{g}{2\sqrt{2}} \sum_{ij} \left[(N_{i2} - t_W N_{i1}) \, N^*_{j4} (\bar{\widetilde{\chi}}_i^0 P_L \widetilde{\chi}_j^0) H^{0\,*}_u + \text{c. c.} - (u, 4 \leftrightarrow d, 3) \right] \\ &= \frac{g}{2\sqrt{2}} \sum_{ij} \left[(N_{i2} - t_W N_{i1}) \, (N^*_{j4} H^{0\,*}_u - N^*_{j3} H^{0\,*}_d) (\bar{\widetilde{\chi}}_i^0 P_L \widetilde{\chi}_j^0) + \text{c. c.} \right] \end{aligned} \tag{28}$$

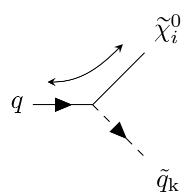
Variable	Value
$C^{L*}_{\tilde{\chi}^0_i\tilde{f}_1 f}$	$c_{\tilde{f}}\left[\left(Q_f - I^3_f\right)t_W N_{i1} + I^3_f N_{i2}\right]$
$C^{R*}_{\tilde{\chi}^0_i\tilde{f}_1 f}$	$s^*_{\tilde{f}} Q_f t_W N_{i1}$
$C^{L*}_{\tilde{\chi}^0_i\tilde{f}_2 f}$	$s_{\tilde{f}}\left[\left(Q_f - I^3_f\right)t_W N_{i1} + I^3_f N_{i2}\right]$
$C^{R*}_{\tilde{\chi}^0_i\tilde{f}_2 f}$	$-c^*_{\tilde{f}} Q_f t_W N_{i1}$
O''^L_{ij}	$\frac{1}{2}\left(N_{i4}N^*_{j4} - N_{i3}N^*_{N_3}\right)$
O''^R_{ij}	$-\frac{1}{2}\left(N^*_{i4}N_{j4} - N^*_{i3}N_{N_3}\right)$



$$= -\frac{ig}{c_W}\gamma^\mu\left[C^L_{Zqq}P_L + C^R_{Zqq}P_R\right] \tag{29a}$$



$$= \frac{ig}{c_W}\gamma^\mu\left[O''^L_{ij}P_L + O''^R_{ij}P_R\right] \tag{29b}$$



$$= -i\sqrt{2}g\left[C^{L*}_{\tilde{\chi}^0_i\tilde{q}_k q}P_L + C^{R*}_{\tilde{\chi}^0_i\tilde{q}_k q}P_R\right] \tag{29c}$$

A Weyl identities

$$\begin{aligned}(\theta\psi)(\theta\chi) &= (\theta_2\psi_1 - \theta_1\psi_2)(\theta_2\chi_1 - \theta_1\chi_2) = -\theta_2\psi_1\theta_1\chi_2 - \theta_1\psi_2\theta_2\chi_1 \\ &= -\theta_1\theta_2\psi_1\chi_2 + \theta_1\theta_2\psi_2\chi_1 = -\theta_1\theta_2(\psi_1\chi_2 - \psi_2\chi_1) \\ &= -\frac{1}{2}(\theta\theta)(\psi\chi)\end{aligned} \tag{30}$$

$$(\psi\sigma^\mu\bar{\chi}) = -(\bar{\chi}\bar{\sigma}^\mu\psi) \tag{31}$$

$$(\bar{\theta}\bar{\psi})(\theta\sigma^\mu\bar{\theta})(\theta\psi) = \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})(\psi\sigma^\mu\bar{\psi}) \tag{32}$$

B Weyl spinors to Dirac spinors

We can build a Dirac spinor Ψ using a left-handed Weyl spinor ψ_L and a right-handed Weyl spinor $\bar{\psi}_R$ such that

$$\Psi = \begin{pmatrix} \psi_L \\ \bar{\psi}_R \end{pmatrix}. \tag{33}$$

I note that the labels L/R, perhaps confusingly, do not label whether the Weyl spinor is left- or right-handed, but rather what they were originally *intended* to be — $\bar{\psi}_L$ would still be a right-handed Weyl spinor. The γ-matrices in the Weyl representation are

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix}, \tag{34}$$

so we define the conjugate Dirac spinor

$$\bar{\Psi} \equiv \Psi^\dagger \gamma^0 = \begin{pmatrix} \psi_R \\ \bar{\psi}_L \end{pmatrix}^T. \tag{35}$$

A Majorana fermion is constructed from just one Weyl spinor, such that $\psi_L = \psi_R \equiv \psi$. The projection operators $P_{L/R}$ project out the left-handed or right-handed Weyl spinors from the Dirac spinor. The following Weyl spinor products can then be rewritten in terms of Dirac spinors:

$$(\psi_R \phi_L) = \bar{\Psi} P_L \Phi \tag{36a}$$
$$(\bar{\psi}_L \bar{\phi}_R) = \bar{\Psi} P_R \Phi \tag{36b}$$
$$(\bar{\psi}_L \bar{\sigma}^\mu \phi_L) = \bar{\Psi} \gamma^\mu P_L \Phi \tag{36c}$$
$$(\psi_R \sigma^\mu \bar{\phi}_R) = \bar{\Psi} \gamma^\mu P_R \Phi \tag{36d}$$

Using equation Eq. 31, we get the Majorana Dirac spinor relation

$$\bar{\Psi}_M \gamma^\mu P_{L/R} \Phi_M = -\bar{\Phi}_M \gamma^\mu P_{R/L} \Psi_M \tag{37}$$

C Electroweak theory

$$B^0_\mu = c_W A^0_\mu - s_W Z^0_\mu \tag{38a}$$
$$W^0_\mu = s_W A^0_\mu + c_W Z^0_\mu \tag{38b}$$