Partonic cross section of gluino pair-production to leading order

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Abstract

This essay investigates to leading order the production of gluino-gluino pairs from the partonic channel $q\bar{q} \to \tilde{g}\tilde{g}$. Gluino production in hadron colliders dominately occurs via strong interactions, and the differential partonic cross section is calculated using Feynman rules from supersymmetric Quantum Chromodynamics under the assumption of massless initial state quarks. We are pleased that the results are consistent with already established research.

1 Background

The Standard Model (SM) is a tremendously successful theory, predicting outcomes of electromagnetic, weak and strong interactions of elementary particles to great accuracy. However, it falls short in other areas within well established physics. For instance, it has no viable dark matter candidates and it does not explain the accelerating expansion of space. This incompletion of particle physics calls for investigation of new theories that extend the SM. One of the most promising theoretical frameworks for beyond-the-SM theories is supersymmetry (SUSY). In particular, a popular supersymmetric extension to the SM is the Minimal Supersymmetric Standard Model (MSSM), which could account for the unresolved issues within the SM. The MSSM is the simplest supersymmetric extension of the SM, in the sense that it introduces the least number of new fields and gauges while being consistent with the known SM fields.

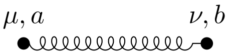
In supersymmetric models such as the MSSM, all SM particles have so-called superpartners, thought of as supersymmetric versions of the SM particles. The gluino, which is of principal importance to this paper, is the superpartner of the gluon. The gluino is a Majorana fermion, unlike the gluon which is a vector gauge boson, and it interacts through the strong force as a color octet. These strong interactions are described by supersymmetric theory of Quantum Chromodynamics (SUSY-QCD), where particle cross sections are calculated perturbatively in the coupling constant α_s. As the coupling in SUSY-QCD, and in QCD generally, is of significant magnitude, the higher order contributions in the perturbative expansion of the cross section cannot be neglected. However, calculating these contributions is beyond the scope of this paper, as we will consider the tree level production of gluino pairs from quark-anti-quark collisions.

2 Amplitudes and Feynman rules in SUSY-QCD

To compute the cross section for the interaction considered, we must first regard the needed Feynman rules to find the averaged squared amplitude $\langle|\mathcal{M}_{tot}|^2\rangle$ of the process. From [1] we have the relevant rules for our purpose which are listed below.

2.1 Propagators

In Feynman gauge ($\xi = 1$), we have the gluon propagator given by equation 1. Here, k^2 is the squared four-momentum of the propagator.



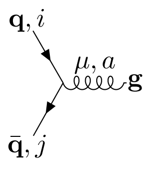
$$= -\frac{i\delta^{ab}g_{\mu\nu}}{k^2}. \qquad (1)$$

Equation 2 gives us the squark propagator, where $m_{\tilde{q}}$ is the squark mass and k^2 is the squared four-momentum of the virtual squark.

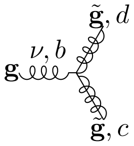
$$= \frac{i\delta^{jl}}{k^2 - m_{\tilde{q}}^2} \qquad (2)$$

2.2 Vertex rules

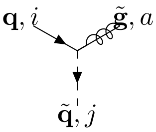
In the following, T_a are the generators of the gauge group $SU(3)$, f^{abc} are the affiliated structure constants, and $P_L = (1-\gamma^5)/2$, $P_R = (1+\gamma^5)/2$ are the left- and right-handed chiral projection operators respectively.



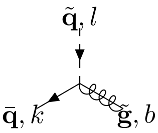
$$= -ig_s(T_a)^{ij}\gamma^\mu P_L \qquad (3)$$



$$= -g_s f^{bcd}\gamma^{\nu} \tag{4}$$



$$= -i\sqrt{2}g_s P_L (T_a)^{ji} \tag{5}$$



$$= -i\sqrt{2}g_s P_R (T_b)^{kl} \tag{6}$$

2.3 Amplitudes

The vertex rules above are listed only for particles of left-handed chirality, and we need to account for the right-handed contributions to the cross section as well. By assuming massless initial state quarks, we are able to consider their chirality as an invariant property, such that amplitude interference terms of the form $\mathcal{M}_L^{\dagger}\mathcal{M}_R$ are zero. As long as the process is parity conserving, we thus know that

$$|\mathcal{M}_{tot}|^2 = |\mathcal{M}_L|^2 + |\mathcal{M}_R|^2 = 2|\mathcal{M}_L|^2. \tag{7}$$

As we know that the incoming quarks has spin $s = \pm 1/2$, we will need to average over the four possible combinations of initial spin states in order to find the averaged total amplitude squared $\langle |\mathcal{M}_{tot}|^2 \rangle$. Moreover, since the quarks also have color, denoted by $a = r$ (red), g (green), b (blue), we must additionally average over these nine different combinations of initial color states. Thus,

$$\langle |\mathcal{M}_{tot}|^2 \rangle = \frac{1}{4} \cdot \frac{1}{9} \sum_{s,a} |\mathcal{M}_{tot}|^2. \tag{8}$$

We can further calculate the partonic differential cross section $d\hat{\sigma}/dt$ of the process by

$$\frac{d\hat{\sigma}_{q\bar{q}\to\tilde{g}\tilde{g}}}{dt} = \frac{1}{64\pi\hat{s}} \frac{1}{\vec{p}_i^{\,2}} \langle |\mathcal{M}_{tot}|^2 \rangle , \tag{9}$$

where $\vec{p}_i^{\,2}$ is the squared three-momentum of each of the initial state quarks in the partonic centre of mass-frame and $\hat{s}$ is the Mandelstam variable.

3 Contributing Feynman diagrams

The process $q\bar{q} \to \tilde{g}\tilde{g}$ has three contributing Feynman diagrams at tree level, each of which can be seen in figure 1. The relationship between the contributions to the total amplitude can be seen in equation 10, where the u-channel contribution picks up a relative negative sign due to anticommutation of fermions when interchanging the final state momenta, as is displayed in figure 1.

$$\mathcal{M}_L = \mathcal{M}_{s,L} + \mathcal{M}_{t,L} - \mathcal{M}_{u,L} \tag{10}$$

Utilizing the relationship between the squared total amplitude $|\mathcal{M}_{tot}|^2$ and $|\mathcal{M}_L|^2$ in equation 7, and the expression for the left-handed contributions in equation 10, we get the following expression for the total squared amplitude,

$$\begin{aligned} |\mathcal{M}_{tot}|^2 = 2\big(&|\mathcal{M}_{s,L}|^2 + |\mathcal{M}_{t,L}|^2 + |\mathcal{M}_{u,L}|^2 \\ &+ 2\mathfrak{Re}\{\mathcal{M}_{s,L}\mathcal{M}_{t,L}^{\dagger}\} - 2\mathfrak{Re}\{\mathcal{M}_{s,L}\mathcal{M}_{u,L}^{\dagger}\} \\ &- 2\mathfrak{Re}\{\mathcal{M}_{t,L}\mathcal{M}_{u,L}^{\dagger}\}\big). \end{aligned} \tag{11}$$

3.1 The s-channel

Regarding the s-channel amplitude $\mathcal{M}_{s,L}$, by applying the Feynman rules of equations 3, 4 and 1 we have,

$$\begin{aligned} i\mathcal{M}_{s,L} = {} & \bar{v}^{s_2}(p_2)(-ig_s(T_a)^{ij}\gamma^{\mu}P_L)u^{s_1}(p_1)\frac{i\delta^{ab}g_{\mu\nu}}{k^2} \\ & \times \bar{u}^{s_3}(p_3)(-g_s f^{bcd}\gamma^{\nu})v^{s_4}(p_4). \end{aligned}$$

Here, $u^{s_1}(p_1)$ and $\bar{v}^{s_2}(p_2)$ are the Dirac spinors of the incoming quark and anti-quark respectively, while $\bar{u}^{s_3}(p_3)$ and $v^{s_4}(p_4)$ are Majorana spinors of the outcoming pair of gluinos. Since we are considering the s-channel contribution, we have the following relation between k and the Mandelstam variable $\hat{s}$,

$$k^2 = (p_1 + p_2)^2 = \hat{s}.$$

We will in the following use the fact that $\alpha_s = g_s^2/(4\pi)$. By squaring and summing over spin and color, we find

$$\begin{aligned} \sum_{s,a} |\mathcal{M}_{s,L}|^2 = {} & C_s \frac{4\pi^2\alpha_s^2}{\hat{s}^2} Tr\{\gamma^{\mu}(1-\gamma^5)\not{p}_1\gamma^{\nu}(1-\gamma^5)\not{p}_2\} \\ & \times Tr\{\gamma_{\mu}(\not{p}_3 + m_{\tilde{g}})\gamma_{\nu}(\not{p}_4 - m_{\tilde{g}})\}. \end{aligned}$$

After use of trace identities of the gamma matrices and insertion of Mandelstam variables $\hat{s}$,

$$\begin{aligned} \hat{t} &= (p_1 - p_3)^2 = (p_2 - p_4)^2, \\ \hat{u} &= (p_1 - p_4)^2 = (p_2 - p_3)^2, \end{aligned}$$

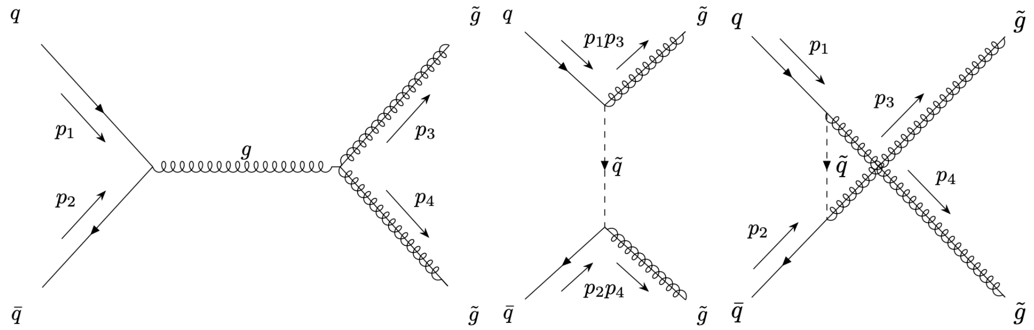


Figure 1: The figure shows the three Feynman diagrams for the process $q\bar{q} \to \tilde{g}\tilde{g}$. From left to right we have the s-channel, the t-channel and the u-channel diagrams.

we find

$$\sum_{s,a} |\mathcal{M}_{s,L}|^2 = C_s \frac{64\pi^2\alpha_s^2}{\hat{s}^2}\left(2m_{\tilde{g}}^2\hat{s}+(m_{\tilde{g}}^2-\hat{t})^2+(m_{\tilde{g}}^2-\hat{u})^2\right). \tag{12}$$

The color factor C_s can be calculated by using the $\mathfrak{su}(3)$ properties from equations 20, 21 and 22,

$$\begin{aligned} C_s &= (T_a)^{ij} f^{acd} (T_b)^{ji} f^{bcd} \\ &= 12. \end{aligned}$$

3.2 The t- and u-channels

For the t-channel contribution to the total amplitude, we apply the Feynman rules from equations 5, 6 and 2.

$$\begin{aligned} i\mathcal{M}_{t,L} &= \bar{u}^{s_3}(p_3)(-i\sqrt{2}g_s P_L (T_a)^{ji}) u^{s_1}(p_1) \frac{i\delta^{jl}}{k^2 - m_{\tilde{q}}^2} \\ &\times \bar{v}^{s_2}(p_2)(-i\sqrt{2}g_s P_R (T_b)^{kl}) v^{s_4}(p_4) \end{aligned}$$

Here, the squared propagator momentum $k^2 = \hat{t}$. By squaring and summation we get the following,

$$\begin{aligned} \sum_{s,a} |\mathcal{M}_{t,L}|^2 = &\frac{4\pi^2\alpha_s^2}{(\hat{t} - m_{\tilde{q}}^2)^2} C_t \\ &\times Tr\{(\not{p}_3 + m_{\tilde{g}})(1-\gamma^5)\not{p}_1(1+\gamma^5)\} \\ &\times Tr\{\not{p}_2(1+\gamma^5)(\not{P}_4 - m_{\tilde{g}})(1-\gamma^5)\}, \end{aligned}$$

which after using trace identities and inserting the Mandelstam variables results in

$$\sum_{s,a} |\mathcal{M}_{t,L}|^2 = \frac{64\pi^2\alpha_s^2}{(\hat{t} - m_{\tilde{q}}^2)^2} C_t (m_{\tilde{g}}^2 - \hat{t})^2. \tag{13}$$

The only difference from the t-channel to the u-channel is the interchanging of momenta $p_3 \leftrightarrow p_4$, which gives the squared propagator momentum $k^2 = \hat{u}$. Thus, we find the u-channel contribution to be

$$\sum_{s,a} |\mathcal{M}_{u,L}|^2 = \frac{64\pi^2\alpha_s^2}{(\hat{u} - m_{\tilde{q}}^2)^2} C_u (m_{\tilde{g}}^2 - \hat{u})^2, \tag{14}$$

where the color factors $C_t = C_u$ as the diagrams share the same vertices. Calculating the color factors using properties from equations 21 and 23, we find

$$\begin{aligned} C_t &= (T_b T_a)^{ki} (T_a T_b)^{ik} \\ &= \frac{16}{3}. \end{aligned}$$

3.3 Interference terms

Regarding the interference terms, we have

$$\begin{aligned} \mathcal{M}_{s,L}\mathcal{M}_{t,L}^\dagger = &\frac{ig_s^2}{\hat{s}} (T_a)^{ij} f^{acd} \bar{v}^{s_2}(p_2)\gamma^\mu P_L u^{s_1}(p_1) \\ &\times \bar{u}^{s_3}(p_3)\gamma_\mu v^{s_4}(p_4) \times \frac{-2g_s^2}{\hat{t} - m_{\tilde{q}}^2}(T_c T_d)^{ji} \\ &\bar{u}^{s_1}(p_1) P_R u^{s_3}(p_3) \bar{v}^{s_4}(p_4) P_L v^{s_2}(p_2). \end{aligned}$$

By summing over spin and color, we then find

$$\begin{aligned} \sum_{s,a} \mathcal{M}_{s,L}\mathcal{M}_{t,L}^\dagger &= \frac{4\pi^2\alpha_s^2}{\hat{s}(\hat{t} - m_{\tilde{q}}^2)} C_{st} Tr\{(\not{p}_2 \gamma^\mu (1-\gamma^5)\not{p}_1 \\ &\times (1+\gamma^5)(\not{p}_3 + m_{\tilde{g}})\gamma_\mu(\not{p}_4 - m_{\tilde{g}})(1+\gamma^5)\} \\ &= \frac{64\pi^2\alpha_s^2}{\hat{s}(\hat{t} - m_{\tilde{q}}^2)} C_{st}\left((m_{\tilde{g}}^2 - \hat{t})^2 + m_{\tilde{g}}^2\hat{s}\right), \end{aligned} \tag{15}$$

where the color factor C_{st} can be found to be

$$C_{st} = -i(T_a)^{ij} f^{acd} (T_c T_d)^{ji} = 6,$$

using equations 22 and 25. In order to find the $\mathcal{M}_{s,L}\mathcal{M}^\dagger_{u,L}$ term, we need only replace $\hat{t}$ with $\hat{u}$ in equation 15. Thus,

$$\sum_{s,a}\mathcal{M}_{s,L}\mathcal{M}^\dagger_{u,L} = \frac{64\pi^2\alpha_s^2}{\hat{s}(\hat{u}-m_{\tilde{q}}^2)}C_{su}\big((m_{\tilde{g}}^2-\hat{u})^2+m_{\tilde{g}}^2\hat{s}\big). \tag{16}$$

Here, $C_{su} = -C_{st}$. We further calculate the last interference term,

$$\begin{aligned}\mathcal{M}_{t,L}\mathcal{M}^\dagger_{u,L} = &\frac{(-2g_s^2)^2}{(\hat{t}-m_{\tilde{q}}^2)(\hat{u}-m_{\tilde{q}}^2)}(T_bT_a)^{ki}(T_bT_a)^{ik}\\ &\times \bar{v}^{s_2}(p_2)P_Rv^{s_4}(p_4)\bar{u}^{s_3}(p_3)P_Lu^{s_1}(p_1)\\ &\times \bar{v}^{s_3}(p_3)P_Lv^{s_2}(p_2)\bar{u}^{s_1}(p_1)P_Ru^{s_4}(p_4).\end{aligned}$$

By the properties of equations 26, 27, 28, we find

$$\begin{aligned}\sum_{s,a}\mathcal{M}_{t,L}\mathcal{M}^\dagger_{u,L} &= \frac{64\pi^2\alpha_s^2}{(\hat{t}-m_{\tilde{q}}^2)(\hat{u}-m_{\tilde{q}}^2)}C_{tu}Tr\{(\not{p}_3+m_{\tilde{g}})\\ &\quad\times \not{p}_1P_R(\not{p}_4+m_{\tilde{g}})\not{p}_2P_L\}\\ &= \frac{64\pi^2\alpha_s^2}{(\hat{t}-m_{\tilde{q}}^2)(\hat{u}-m_{\tilde{q}}^2)}C_{tu}m_{\tilde{g}}^2\hat{s},\end{aligned} \tag{17}$$

where the color factor C_{tu} is computed by applying equation 24,

$$\begin{aligned}C_{tu} &= Tr\{T_bT_aT_bT_a\}\\ &= -2/3.\end{aligned}$$

4 The differential cross section

With contributions from equations 12, 13, 14, 15, 16, 17 inserted into 11 with the appropriate color factors, and all combined into equation 8, we find the total averaged squared amplitude for the process $q\bar{q}\to\tilde{g}\tilde{g}$ to be

$$\begin{aligned}\langle|\mathcal{M}_{tot}|^2\rangle = &\frac{128\pi^2\alpha_s^2}{9}\Bigg[\frac{3}{\hat{s}^2}\big(2m_{\tilde{g}}\hat{s}^2+(m_{\tilde{g}}^2-\hat{t})^2\\ &+(m_{\tilde{g}}^2-\hat{u})^2\big)+\frac{4}{3}\left(\frac{(m_{\tilde{g}}^2-\hat{t})^2}{(\hat{t}-m_{\tilde{q}}^2)^2}+\frac{(m_{\tilde{g}}^2-\hat{u})^2}{(\hat{u}-m_{\tilde{q}}^2)^2}\right)\\ &+\frac{3}{\hat{s}}\left(\frac{m_{\tilde{g}}^2\hat{s}+(m_{\tilde{g}}^2-\hat{t})^2}{\hat{t}-m_{\tilde{q}}^2}+\frac{m_{\tilde{g}}^2\hat{s}+(m_{\tilde{g}}^2-\hat{u})^2}{\hat{u}-m_{\tilde{q}}^2}\right)\\ &+\frac{1}{3}\frac{m_{\tilde{g}}^2\hat{s}}{(\hat{t}-m_{\tilde{q}}^2)(\hat{u}-m_{\tilde{q}}^2)}\Bigg].\end{aligned} \tag{18}$$

Because the initial state quarks are assumed to be massless in these calculations, we know that the initial squared three-momentum of each of the two quarks is $\vec{p}_i^{\,2} = s/4$ in the partonic centre of mass-frame. With equation 18 inserted into equation 9, we finally find

$$\begin{aligned}\frac{d\hat{\sigma}_{q\bar{q}\to\tilde{g}\tilde{g}}}{dt} = &\frac{8\pi\alpha_s^2}{9s^2}\Bigg[\frac{3}{\hat{s}^2}\big(2m_{\tilde{g}}\hat{s}^2+(m_{\tilde{g}}^2-\hat{t})^2+(m_{\tilde{g}}^2-\hat{u})^2\big)\\ &+\frac{4}{3}\left(\frac{(m_{\tilde{g}}^2-\hat{t})^2}{(\hat{t}-m_{\tilde{q}}^2)^2}+\frac{(m_{\tilde{g}}^2-\hat{u})^2}{(\hat{u}-m_{\tilde{q}}^2)^2}\right)\\ &+\frac{3}{\hat{s}}\left(\frac{m_{\tilde{g}}^2\hat{s}+(m_{\tilde{g}}^2-\hat{t})^2}{\hat{t}-m_{\tilde{q}}^2}+\frac{m_{\tilde{g}}^2\hat{s}+(m_{\tilde{g}}^2-\hat{u})^2}{\hat{u}-m_{\tilde{q}}^2}\right)\\ &+\frac{1}{3}\frac{m_{\tilde{g}}^2\hat{s}}{(\hat{t}-m_{\tilde{q}}^2)(\hat{u}-m_{\tilde{q}}^2)}\Bigg].\end{aligned} \tag{19}$$

The resulting cross section is consistent with published results [2].

5 Afterword

The origins of the quark and anti-quark are not discussed in this paper, however we can envision that the particles originate from colliding hadrons, e.g. a proton and an anti-proton. In this case, the reasonable continuation of these calculations would be to compute the cross section of $gg\to\tilde{g}\tilde{g}$, which is the other process responsible for gluino pair production. With $\hat{\sigma}_{gg\to\tilde{g}\tilde{g}}$ and $\hat{\sigma}_{q\bar{q}\to\tilde{g}\tilde{g}}$ we are able to retrieve the total hadronic cross section

$$\sigma = \sum_{i,j=g,q,\bar{q}}\int dx_1\int dx_2\ \ f_i^p(x_1)f_j^{\bar{p}}(x_2)\hat{\sigma}_{ij\to\tilde{g}\tilde{g}}$$

and further calculate number of expected events to compare with experimental results at hadron colliders. However, as earlier stated, the tree level cross section alone is not of much interest, due to the large higher order corrections. These corrections tend to diverge, hence, the calculation is generally not as straight forward as the calculations presented in this paper. Accounting for higher order corrections improves our theoretical predictions, and is accordingly of prime importance to particle physicists.

References

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6 The undesirable fifth page: Appendix

6.1 Properties of the Lie algebra of *SU(3)*

Quarks, gluons and their respective superpartners all transforms under the gauge group of $SU(3)$. In the fundamental representation of the Lie algebra $\mathfrak{su}(3)$, we have that the antisymmetric structure constants f^{abc} and generators T_a fulfill the following identities [3],

$$f^{acd}f^{bcd} = 3\delta^{ab}, \tag{20}$$

$$Tr\{T_aT_b\} = \frac{1}{2}\delta_{ab}, \tag{21}$$

$$\delta_{aa} = \delta^{ab}\delta_{ab} = 8, \tag{22}$$

$$(T_aT_a)^{ij} = \frac{4}{3}\delta^{ij}, \tag{23}$$

$$Tr\{T_aT_bT_aT_c\} = -\frac{1}{12}\delta_{bc}, \tag{24}$$

$$Tr\{T_aT_bT_c\} = \frac{1}{4}(d^{abc} + if^{abc}), \tag{25}$$

where in the last equation d^{abc} is a symmetric structure constant of $SU(3)$ yielding

$$\{T^a, T^b\} = \frac{1}{3}\delta_{ab} + d^{abc}T^c.$$

6.2 Properties of charge conjugation matrices and Majorana spinors

In order to calculate the interference term $\sum_{s,a} \mathcal{M}_{t,L}\mathcal{M}^{\dagger}_{u,L}$, we make use of the following identities [4] involving charge conjugation matrices C,

$$\Gamma^T = C^{-1}\Gamma C, \tag{26}$$

$$C^T = -C, \tag{27}$$

$$(v^s(p))^T = \bar{u}^s(p)C^T. \tag{28}$$

In the last equation, $v^s(p)$ and $\bar{u}^s(p)$ can be spinors of Majorana or Dirac particles.