UNIVERSITY
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Master's thesis

My Master's Thesis

With Subtitle

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DRAFT

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Chapter 1

Introduction

This is where I introduce the master's thesis.

Chapter 2

Quantum Field Theory

2.1 Lagrangian Formalism and the Path Integral

PHANTOM PARAGRAPH: Here I want to introduce the Lagrangian formulation of quantum field theory, and the types of fields we will be working with. Furthermore, I want to introduce the perturbation series through the path integral, and talk about the nature of quantum effects. Also derive Feynman rules from path integral.

2.2 Renormalised Quantum Field Theory

PHANTOM PARAGRAPH: Talk about loop integrals, divergences, regularisation and renormalisation.

2.3 The Standard Model

PHANTOM PARAGRAPH: Introduce the *relevant* fields of the Standard Model and its construction. Maybe mention the Higgs mechanism?

2.4 Loop Integrals and Regularisation

PHANTOM PARAGRAPH: Introduce loop integrals, how to calculate them, where divergences appear and how to regularise them.

2.4.1 Dimensional Regularisation

2.4.2 Passarino-Veltman Loop Integrals

By Lorentz invariance, there are a limited set of forms that loop integrals can take.Why is this?© These can be categorised according to the number of propagator terms they include, which corresponds to the number of externally connected points there are in the loop. A general scalar N-point loop integral takes the form

$$T_0^N\left(p_i^2,(p_i-p_j)^2;m_0^2,m_i^2\right)=\frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \mathrm{d}^d q\,\mathcal{D}_0\prod_{i=1}^{N-1}\mathcal{D}_i, \tag{2.1}$$

where $\mathcal{D}_0=\left[q^2-m_0^2\right]^{-1}$ and $\mathcal{D}_i=\left[(q+p_i)^2-m_i^2\right]^{-1}$. The first 4 scalar loop integrals are named accordingly

$$T_0^1\equiv A_0(m_0^2) \tag{2.2}$$
$$T_0^2\equiv B_0(p_1^2;m_0^2,m_1^2) \tag{2.3}$$
$$T_0^3\equiv C_0(p_1^2,p_2^2,(p_1-p_2)^2;m_0^2,m_1^2,m_2^2) \tag{2.4}$$
$$T_0^4\equiv D_0(p_1^2,p_2^2,p_3^2,(p_1-p_2)^2,(p_1-p_3)^2,(p_2-p_3)^2;m_0^2,m_1^2,m_2^2) \tag{2.5}$$

More complicated Lorentz structure can be obtained in loop integrals, however, these can still be related to the scalar integrals by exploiting the possible tensorial structure they can have. Defining an arbitrary loop integral

$$T^N_{\mu_1\cdots\mu_P}\left(p_i^2,(p_i-p_j)^2;m_0^2,m_i^2\right)=\frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \mathrm{d}^d q\,q_{\mu_1}\cdots q_{\mu_P}\mathcal{D}_0\prod_{i=1}^{N-1}\mathcal{D}_i. \tag{2.6}$$

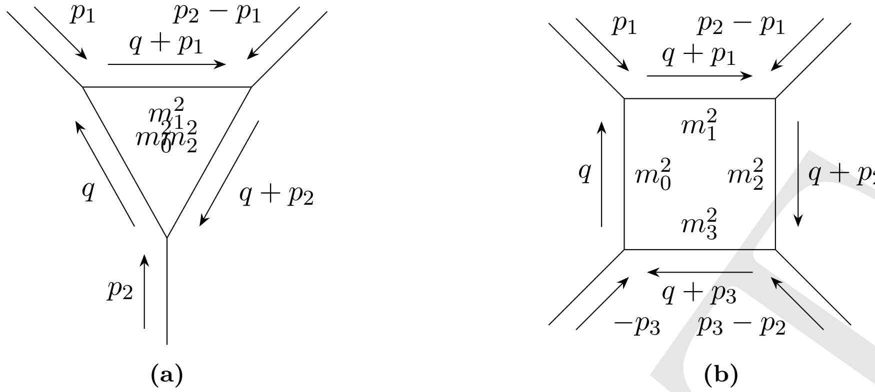


Figure 2.1: Illustration of the momentum conventions for loop diagrams used in the Passarino-Veltman functions.

These tensors can only depend on the metric $g^{\mu\nu}$ and the external momenta p_i. The possible structures up to four-point loops are as following:

$$B^{\mu} = p_1^{\mu} B_1, \tag{2.7a}$$

$$B^{\mu\nu} = g^{\mu\nu} B_{00} + p_1^{\mu} p_1^{\nu} B_{11}, \tag{2.7b}$$

$$C^{\mu} = \sum_{i=1}^{2} p_i^{\mu} C_i, \tag{2.7c}$$

$$C^{\mu\nu} = g^{\mu\nu} C_{00} + \sum_{i,j=1}^{2} p_i^{\mu} p_j^{\nu} C_{ij}, \tag{2.7d}$$

$$C^{\mu\nu\rho} = \sum_{i=1}^{2} (g^{\mu\nu} p_i^{\rho} + g^{\mu\rho} p_i^{\nu} + g^{\nu\rho} p_i^{\mu}) C_{00i} + \sum_{i,j,k=1}^{2} p_i^{\mu} p_j^{\nu} p_k^{\rho} C_{ijk}, \tag{2.7e}$$

$$D^{\mu} = \sum_{i=1}^{3} p_i^{\mu} D_i, \tag{2.7f}$$

$$D^{\mu\nu} = g^{\mu\nu} D_{00} + \sum_{i,j=1}^{3} p_i^{\mu} p_j^{\nu} D_{ij}, \tag{2.7g}$$

$$D^{\mu\nu\rho} = \sum_{i=1}^{3} (g^{\mu\nu} p_i^{\rho} + g^{\mu\rho} p_i^{\nu} + g^{\nu\rho} p_i^{\mu}) D_{00i} + \sum_{i,j,k=1}^{3} p_i^{\mu} p_j^{\nu} p_k^{\rho} D_{ijk}, \tag{2.7h}$$

$$\begin{aligned} D^{\mu\nu\rho\sigma} &= (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) D_{0000} \\ &+ \sum_{i,j=1}^{3} (g_{\mu\nu} p_i^{\rho} p_j^{\sigma} + g_{\mu\nu} p_i^{\sigma} p_j^{\rho} + g_{\mu\rho} p_i^{\nu} p_j^{\sigma} + g_{\mu\rho} p_i^{\sigma} p_j^{\nu} + g_{\mu\sigma} p_i^{\rho} p_j^{\nu} + g_{\mu\nu} p_i^{\nu} p_j^{\rho}) D_{00ij} \\ &+ \sum_{i,j,k,l=1}^{3} p_i^{\mu} p_j^{\nu} p_k^{\rho} p_l^{\sigma} D_{ijkl}, \end{aligned} \tag{2.7i}$$

where all coefficients must be completely symmetric in i, j, k, l.

Chapter 3

Supersymmetry

3.1 Introduction to Supersymmetry

PHANTOM PARAGRAPH: INTRODUCE SUPERSYMMETRY, WHAT THE SYMMETRY IS AND HOW IT TRANSFORMS FERMIONIC AND BOSONIC FIELDS THROUGH EACH OTHER. INTRODUCE SUPERSPACE, GRASSMANN CALCULUS, SUPERFIELDS AND SUPERLAGRANGIANS.

3.1.1 A Simple Supersymmetric Theory

To illustrate what supersymmetry looks like in practice, it can be helpful to look at a simple example. Take a Lagrangian for a massive complex scalar field ϕ and a massive Weyl spinor field ψ

$$\mathcal{L} = (\partial_\mu\phi)(\partial^\mu\phi^*) + i\psi\sigma^\mu\partial_\mu\psi^\dagger - |m_B|^2\,\phi\phi^* - \frac{1}{2}m_F(\psi\psi) - \frac{1}{2}m_F^*(\psi\psi)^\dagger. \tag{3.1}$$

To impose some symmetry between the bosonic and fermionic degrees of freedom, we want to examine a transformation of the scalar field through the spinor field and vice versa. A general, infinitesimal such transformation can be parametrised by

$$\delta\phi = \epsilon a(\theta\psi), \tag{3.2a}$$
$$\delta\phi^* = \epsilon a^*(\theta\psi)^\dagger, \tag{3.2b}$$
$$\delta\psi_\alpha = \epsilon\left(c(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu\phi + F(\phi,\phi^*)\theta_\alpha\right), \tag{3.2c}$$
$$\delta\psi^\dagger{}_{\dot\alpha} = \epsilon\left(c^*(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* + F^*(\phi,\phi^*)\theta^\dagger_{\dot\alpha}\right), \tag{3.2d}$$

where ϵ is some infinitesimal parameter for the transformation, θ is some Grassmann-valued Weyl spinor, a, c are complex coefficients of the transformation and $F(\phi,\phi^*)$ is some linear function ϕ and ϕ^*. The change in the scalar field part of the Lagrangian is

$$\delta\mathcal{L}_\phi/\epsilon = a(\theta\partial_\mu\psi)\,(\partial^\mu\phi^*) - a\,|m_B|^2\,(\theta\psi)\phi^* + \text{c. c.}, \tag{3.3}$$

and likewise for the spinor field part

$$\delta\mathcal{L}_\psi/\epsilon = -ic^*(\psi\sigma^\mu\bar\sigma^\nu\theta)\partial_\mu\partial_\nu\phi^* + i(\psi\sigma^\mu\theta^\dagger)\partial_\mu F^* + m_F\left[c(\psi\sigma^\mu\theta^\dagger)\partial_\mu\phi + (\psi\theta)F\right] + \text{c. c.} \tag{3.4}$$

The first term in Eq. (3.4) can be rewritten using the commutativity of partial derivatives and the identity $(\sigma^\mu\bar\sigma^\nu + \sigma^\mu\bar\sigma^\nu)_\alpha{}^\beta = -2g^{\mu\nu}\delta_\alpha^\beta$ to get $ic^*\,(\theta\psi)\,\partial_\mu\partial^\mu\phi^*$. Up to a total derivative, we can then write the change in the spinor part as

$$\delta\mathcal{L}_\psi/\epsilon = -ic^*(\theta\partial_\mu\psi)\partial^\mu\phi^* + (\psi\sigma^\mu\theta^\dagger)\partial_\mu\,(iF^* + m_F c\phi) + m_F\theta\psi F + \text{c. c.}\,. \tag{3.5}$$

The total change of the Lagrangian (again up to a total derivate) can then be grouped as

$$\begin{aligned}\delta\mathcal{L}/\epsilon = {} & (a - ic^*)\,(\theta\partial_\mu\psi)\,(\partial^\mu\phi^*) + (\psi\sigma^\mu\theta^\dagger)\partial_\mu\,(iF^* + m_F c\phi) \\ & + (\theta\psi)\left(a\,|m_B|^2\,\phi^* + m_F F\right) + \text{c. c.},\end{aligned} \tag{3.6}$$

giving us three different conditions for the action to be invariant:

$$a - ic^* = 0, \tag{3.7a}$$
$$iF^* + m_F c\phi = 0, \tag{3.7b}$$
$$a\,|m_B|^2\,\phi^* + m_F F = 0. \tag{3.7c}$$

This is fulfilled if

$$
\begin{aligned}
c &= ia^*, && (3.8a)\\
F &= -am_F^*\phi^*, && (3.8b)\\
a\left|m_B\right|^2 &= a^*\left|m_F\right|^2. && (3.8c)
\end{aligned}
$$

What is interesting is the last condition, because it requires a to be real, as both $|m_B|^2$ and $|m_F|^2$ are real, but also requires $|m_B|^2 = |m_F|^2$. For the theory to be supersymmetric in this sense, the masses of the boson and fermion must be the same!

Revisiting F, it can be introduced as an auxiliary field to bookkeep the supersymmetry transformation. By including the non-dynamical term to the Lagrangian $\mathcal{L}_F = F^*F + mF\phi + m^*F^*\phi^*$, we make sure F takes the correct value in the transformation from its equation of motion $\frac{\partial\mathcal{L}}{\partial F} = F^* + m\phi \stackrel{!}{=} 0$. Inserting F back into the Lagrangian reproduces the mass term of the scalar field, allowing us to write the original Lagrangian as

$$
\mathcal{L} = (\partial_\mu\phi)(\partial^\mu\phi^*) + i\psi\sigma^\mu\partial_\mu\psi^\dagger + F^*F + mF\phi + m^*F^*\phi^* - \frac{1}{2}m(\psi\psi) - \frac{1}{2}m^*(\psi\psi)^\dagger, \tag{3.9}
$$

with the *supersymmetry transformation* rules

$$
\begin{aligned}
\delta\phi &= \epsilon(\theta\psi), & \delta\phi^* &= \epsilon(\theta\psi)^\dagger, && (3.10a)\\
\delta\psi_\alpha &= \epsilon\left(-i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu\phi + F\theta_\alpha\right), & \delta\psi^\dagger_{\dot\alpha} &= \epsilon\left(i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* + F^*\theta^\dagger_{\dot\alpha}\right), && (3.10b)\\
\delta F &= i\epsilon\left(\partial_\mu\psi\sigma^\mu\theta^\dagger\right), & \delta F^* &= -i\epsilon\left(\theta\sigma^\mu\partial_\mu\psi^\dagger\right), && (3.10c)
\end{aligned}
$$

where I have set $a = 1$ without loss of generality, and found the appropriate transformation law for F such that the Lagrangian is invariant up to total derivatives. The dynamics of this Lagrangian are the same as before, but the supersymmetry is now made manifest, i.e. the transformation is free of any dependence on the contents of the Lagrangian.

In fact, one can show that a general supersymmetric Lagrangian consisting of a scalar field and a fermion field can be written

$$
\mathcal{L} = (\partial_\mu\phi)(\partial^\mu\phi^*) + i\psi\sigma^\mu\partial_\mu\psi^\dagger + F^*F + \left\{mF\phi - \frac{1}{2}m(\psi\psi) - \lambda\phi(\psi\psi) + \text{c. c.}\right\} \tag{3.11}
$$

up to renormalisable interactions.

3.2 The Super-Poincaré Group

PHANTOM PARAGRAPH: INTRODUCE THE SUPER-POINCARÉ ALGEBRA, AND SUPERSPACE AS A VESSEL FOR MANIFESTLY SUPERSYMMETRIC THEORIES. LEAD INTO SUPERFIELDS, AND GENERAL SUPERSYMMETRIC SUPERLAGRANGIANS.

3.2.1 The Poincaré and Super-Poincaré Algebras

3.2.2 Superspace

To facilitate the construction of supersymmetric theories, it will be helpful to construct a space such that the generators of the Super-Poincaré group manifest as simple coordinate transformations, as in the case of the ordinary Poincaré group. Starting with a general element $g \in SP$, it can be parametrised through the exponential map as

$$g = \exp\left(ix^\mu P_\mu + i(\theta Q) + i(\theta Q)^\dagger + \frac{i}{2}\omega_{\mu\nu}M^{\mu\nu}\right), \tag{3.12}$$

where $x^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha}, \omega_{\mu\nu}$ parametrise the group, and $P_\mu, Q_\alpha, Q^{\dagger\dot\alpha}, M^{\mu\nu}$ are the generators of the group. Since the parameters $x^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha}$ live in a representation of the Lorentz algebra generated by $M^{\mu\nu}$, the effect of this part of the Super-Poincaré group on the parameters can be determined easily. Likewise, the parameters $\omega_{\mu\nu}$ are in a trivial representation of the algebra generated by $P_\mu, Q_\alpha, Q^{\dagger\dot\alpha}$, and need not then be considered. It is therefore poignant to create a space with $x^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha}$ as the coordinates, modding out the Lorentz algebra part. We create superspace as a coordinate system with coordinates $z^\pi = (x^\mu, \theta^\alpha, \theta^\dagger_{\dot\alpha})$, and look at how they transform under Super-Poincaré group transformations. A function $F(z)$ on superspace can then be written using the generators $K_\pi = (P_\mu, Q_\alpha, Q^{\dagger\dot\alpha})$ as $F(z) = \exp(iz^\pi K_\pi)\, F(0)$. Applying a Super-Poincaré group element without the Lorentz generators $\bar g(a,\eta) = \exp\left(ia^\mu P_\mu + i(\eta Q) + i(\eta Q)^\dagger\right)$ we have

$$F(z') = \exp(iz'^\pi K_\pi)\, F(0) = \exp\left(ia^\mu P_\mu + i(\eta Q) + i(\eta Q)^\dagger\right)\exp(iz^\pi K_\pi)\, F(0), \tag{3.13}$$

which by the Baker-Campbell-Hausdorff© formula gives

$$z'^\pi K_\pi = (x^\mu + a^\mu)P_\mu + (\theta^\alpha + \eta^\alpha)Q_\alpha + (\theta^\dagger_{\dot\alpha} + \eta^\dagger_{\dot\alpha})Q^{\dagger\dot\alpha} + \frac{i}{2}\left[a^\mu P_\mu + (\eta Q) + (\eta Q)^\dagger, z^\pi K_\pi\right] + \dots \tag{3.14}$$

Now, P_μ commutes with all of K_π, and Q_α ($Q^{\dagger\dot\alpha}$) anti-commute with themselves, for every combination of different α ($\dot\alpha$), so the only relevant part of the commutator is

$$\left[(\eta Q), (\theta Q)^\dagger\right] + \left[(\eta Q)^\dagger, (\theta Q)\right] = -\eta^\alpha \left\{Q_\alpha, Q^\dagger_{\dot\alpha}\right\}\theta^{\dagger\dot\alpha} + (\eta \leftrightarrow \theta) = -2(\eta\sigma^\mu\theta^\dagger)P_\mu + (\eta \leftrightarrow \theta). \tag{3.15}$$

Since this commutator is proportional to P_μ which in turn commutes with everything, all higher order commutators vanish, and we can conclude that the transformed coordinates z'^π are given by

$$z'^\pi = \left(x^\mu + a^\mu + i(\theta\sigma^\mu\eta^\dagger) - i(\eta\sigma^\mu\theta^\dagger), \theta^\alpha + \eta^\alpha, \theta^\dagger_{\dot\alpha} + \eta^\dagger_{\dot\alpha}\right). \tag{3.16}$$

This gives us a differential representation of the K_π generators as

$$P_\mu = -i\partial_\mu, \tag{3.17a}$$
$$Q_\alpha = -(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu - i\partial_\alpha, \tag{3.17b}$$
$$Q^\dagger_{\dot\alpha} = -(\theta\bar\sigma^\mu)_{\dot\alpha}\partial_\mu - i\partial_{\dot\alpha}. \tag{3.17c}$$

Now, to look into what the these functions of superspace look like, we can expand $F(z)$ in terms of the coordinates $\theta^\alpha, \theta^\dagger_{\dot\alpha}$, as these expansions are finite due to the fact that none of these coordinates can appear more than once. Demanding that the function $F(z)$ be invariant under Lorentz transformations, the x^μ-dependendt coefficients of the expansion must transform such that each term is a scalar (or fully contracted Lorentz structure). This limits a general such function of superspace to be written as

$$\begin{aligned} F(z) =& f(x) + \theta^\alpha\phi_\alpha(x) + \theta^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha}(x) + (\theta\theta)m(x) + (\theta\theta)^\dagger n(x) \\ &+ (\theta\sigma^\mu\theta^\dagger)V_\mu(x) + (\theta\theta)\theta^\dagger_{\dot\alpha}\lambda^{\dagger\dot\alpha}(x) + (\theta\theta)^\dagger\theta^\alpha\psi_\alpha(x) + (\theta\theta)(\theta\theta)^\dagger d(x). \end{aligned} \tag{3.18}$$

3.2.3 Superfields

To construct a manifestly supersymmetric theory, it will then be useful to start with finding representations of the Super-Poincaré group. This is exactly what we have already done; the functions on superspace find themselves in a representation space of a differential representation of the K_π generators of the Super-Poincaré group, and a scalar representation of the remaining Lorentz generators. Inside the general function on superspace Eq. (3.18), we find many component functions in different representation spaces of the Lorentz group. Furthermore, supersymmetry transformations transform these fields into one another. This seems like an ideal vessel for constructing supersymmetric fields theories.

We define the *superfield* Φ as an operator valued function on superspace. The general one Eq. (3.18) is in a reducible representation space of the Super-Poincaré group, so we define three *irreducible* representations that will be useful going forward:

$$\text{\textit{Left-handed scalar superfield:}} \qquad \bar{D}_{\dot{\alpha}}\Phi = 0 \tag{3.19}$$
$$\text{\textit{Right-handed scalar superfield:}} \qquad D_\alpha \Phi^\dagger = 0 \tag{3.20}$$
$$\text{\textit{Vector superfield:}} \qquad \Phi^\dagger = \Phi \tag{3.21}$$

Here the dagger operation refers to complex conjugation, and the differential operators $D_\alpha, \bar{D}_{\dot{\alpha}}$ are defined as

$$D_\alpha = \partial_\alpha + i(\sigma^\mu \theta^\dagger)_\alpha \partial_\mu, \tag{3.22a}$$
$$\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i(\theta\sigma^\mu)_{\dot{\alpha}} \partial_\mu. \tag{3.22b}$$

These differential operators are covariant differentials in the sense that the commute with supersymmetry transformations, i.e. $D_\alpha F(z) \to D'_\alpha(\bar{g}F(z)) = \bar{g}\,(D_\alpha F(z))$

3.3 Minimal Supersymmetric Standard Model

PHANTOM PARAGRAPH: INTRODUCE THE FIELD CONTENT OF THE MSSM AND EXPLAIN CONVENTIONS AND NAMES. TALK ABOUT THE RELEVANT PARTS OF THE SUPERLAGRANGIAN.

3.4 Electroweakinos

PHANTOM PARAGRAPH: INTRODUCE THE ELECTROWEAKINOS AND HOW THEY ARE DERIVED FROM THE VARIOUS FERMIONS PARTNERS OF ELECTROWEAK BOSONS. TALK ABOUT MASS MATRICES AND MASS EIGENSTATES

3.4.1 Mass mixing

3.5 Feynman Rules of Neutralinos

PHANTOM PARAGRAPH: DERIVE THE ORDINARY LAGRANGIAN FOR NEUTRALINOS FROM THE SUPERLAGRANGIAN.

Chapter 4

Neutralino Pair Production at Parton Level

TODO:

- Formulate a section on the dipole formalism used in Debove et al. and make a comparison.

4.1 Kinematics

To start off, it will be useful to introduce some procedure for going forward in the phase space of an inclusive $2 \to 2(+1)$ cross-section process. The phase space of 2-body and 3-body final states are quite different as there are more degrees of freedom in the 3-body final state. In the end, these extra degrees of freedom will be need to be integrated over to make an additive comparison between the 2-body and 3-body processes, however, exactly how we choose to parametrise and subsequently integrate over the extra degrees of freedom can matter quite a bit. To start out, let us count the degrees of freedom of a scattering problem involving N four-momenta $p_{i=1,\ldots,N}$. Assuming our end result to be Lorentz invariant, there are $N(N+1)/2$ different scalar products that can be produced using N different four-momenta. Momentum conservation allows us to eliminate one momentum, such that we have $N(N-1)/2$ possible scalar products. Denoting the scalar products by $m_{ij}^2 \equiv (p_i + p_j)^2$ for $j \neq i$, and $m_i^2 \equiv p_i^2$, we can find a relation between scalar products by using momentum conservation.

$$
\begin{aligned}
m_{ij}^2 &= \left(p_i - \sum_{k\neq j} p_k\right)^2 = \left(\sum_{k\neq i,j} p_k\right)^2 = \sum_{k\neq i,j}\sum_{l\neq i,j} p_k \cdot p_l \\
&= \sum_{k\neq i,j}\sum_{l\neq i,j,k} \frac{m_{kl}^2 - m_k^2 - m_l^2}{2} + \sum_{k\neq i,j} m_k^2 \\
&= \sum_{k\neq i,j}\sum_{\substack{l\neq i,j\\ l>k}} m_{kl}^2 - \frac{1}{2}\sum_{k\neq i,j}(N-3)m_k^2 - \frac{1}{2}\sum_{l\neq i,j}(N-3)m_l^2 + \sum_{k\neq i,j} m_k^2 \\
&= \sum_{k\neq i,j}\sum_{\substack{l\neq i,j\\ l>k}} m_{kl}^2 - (N-4)\sum_{k\neq i,j} m_k^2.
\end{aligned} \tag{4.1}
$$

⌜This little generalised relation might not be immediately necessary...⌟ Furthermore, we assign the N scalar products m_i^2 to the invariant masses of the incoming and outgoing particles, thus not counting them as degrees of freedom, leaving us with $n_{\mathrm{dof}} = \frac{N(N-3)}{2}$ degrees of freedom.[1] This means that in a $2 \to 2$ process, we have 2 degrees of freedom, and in a $2 \to 3$ process we have 5.

4.1.1 2-body Phase Space

The Lorentz invariant phase space differential for a 2-body final state with four-momenta p_i, p_j in d dimensions is

$$
\mathrm{d}\Pi_{2\to 2} = (2\pi)^d\, \delta^d\left(P - p_i - p_j\right) \frac{\mathrm{d}^{d-1}\boldsymbol{p}_i}{(2\pi)^{d-1}} \frac{1}{2E_i} \frac{\mathrm{d}^{d-1}\boldsymbol{p}_j}{(2\pi)^{d-1}} \frac{1}{2E_j}. \tag{4.2}
$$

[1] I note that we often consider the invariant mass of the incoming bodies to be fixed, which would reduce our degrees of freedom by one.

Going to the centre-of-mass frame of the incoming partons, we have $P^\mu = (\sqrt{s}, 0, 0, 0)$, allowing us to integrate over the spatial part of Dirac delta-function to arrive at

$$\mathrm{d}\Pi_{2\to2} = \frac{1}{(2\pi)^{d-2}} \mathrm{d}^{d-1}\boldsymbol{p} \frac{1}{4E_i E_j} \delta\left(\sqrt{s} - E(p, m_i) - E(p, m_j)\right), \tag{4.3}$$

where the $E(p, m) = \sqrt{p^2 + m^2}$. We can write out the differential of the spatial component of p_i in spherical coordinates as $\mathrm{d}^{d-1}\boldsymbol{p} = \mathrm{d}\Omega_{d-1} \mathrm{d}p\, p^{d-2} = \mathrm{d}\Omega_{d-2} \sin^{d-3}\theta\, \mathrm{d}\theta\, \mathrm{d}p\, p^{d-2}$. As a $2 \to 2$ process is restricted to planar motion, we can always go to a frame of reference such that any amplitude we calculate will not be dependent on the spatial angles $\mathrm{d}\Omega_{d-2}$, allowing us to integrate over them using that $\int \mathrm{d}\Omega_{d-2} = 2\pi^{\frac{d-2}{2}} \frac{1}{\Gamma\left(\frac{d-2}{2}\right)}$ to get

$$\mathrm{d}\Pi_{2\to2} = \frac{1}{(4\pi)^{\frac{d-2}{2}}} \frac{1}{\Gamma\left(\frac{d-2}{2}\right)} \frac{p^{d-3}}{2\sqrt{s}} \sin^{d-3}\theta\, \mathrm{d}\theta, \tag{4.4}$$

where we understand the momentum to be given by $p = \frac{\sqrt{\lambda(s, m_i^2, m_j^2)}}{2\sqrt{s}}$. In $d = 4$ dimensions, it is often convenient to change to the Mandelstam variable t, which for massless initial state particles becomes $t = \frac{1}{2}\left(-s + m_i^2 + m_j^2 + \sqrt{\lambda(s, m_i^2, m_j^2)} \cos\theta\right)$. Making the change of variable, the differential phase space reduces to

$$\mathrm{d}\Pi_{2\to2}|_{d=4} = \frac{1}{8\pi s} \mathrm{d}t \tag{4.5}$$

4.1.2 3-body Phase Space

⌜Fill out an introduction here.⌟

The differential Lorentz invariant phase space for a 3-body final state with four-momenta p_i, p_j, k in d dimensions is

$$\mathrm{d}\Pi_{2\to3} = (2\pi)^d \delta^d(P - p_i - p_j - k) \frac{\mathrm{d}^{d-1}\boldsymbol{p}_i}{(2\pi)^{d-1}} \frac{1}{2E_i} \frac{\mathrm{d}^{d-1}\boldsymbol{p}_j}{(2\pi)^{d-1}} \frac{1}{2E_j} \frac{\mathrm{d}^{d-1}\boldsymbol{k}}{(2\pi)^{d-1}} \frac{1}{2\omega}. \tag{4.6}$$

First, it will be useful to write out the differential in $\boldsymbol{k}$ in spherical coordinates where it reads $\mathrm{d}^{d-1}\boldsymbol{k} = \omega^{d-2} \mathrm{d}\Omega_{d-1} \mathrm{d}\omega$. Then we can go to the centre-of-mass frame of the neutralinos $\boldsymbol{p}_i, \boldsymbol{p}_j$ where we have $P - k = (Q, 0, 0, 0)$. This leaves

$$\mathrm{d}\Pi_{2\to3} = \frac{1}{8} \frac{1}{(2\pi)^{2d-3}} \delta(Q - E_i - E_j) \delta^{d-1}(\boldsymbol{p}_i + \boldsymbol{p}_j) \frac{\omega^{d-3}}{E_i E_j} \mathrm{d}^{d-1}\boldsymbol{p}_i\, \mathrm{d}^{d-1}\boldsymbol{p}_j\, \mathrm{d}\Omega_{d-2}\, \mathrm{d}\omega. \tag{4.7}$$

⌜Complete this calculation.⌟

It is important to note that there is an interdependence between Q^2 and ω, which can be found using that $P^2 = s$. In the centre-of-mass frame of the neutralinos, the magnitude of the three-momentum of P must be ω, so we have the relation $P_0^2 = s + \omega^2$, which together with momentum conservation $P_0 - \omega = Q$ yields

$$\omega = \frac{s - Q^2}{2Q}. \tag{4.8}$$

Switching integration variables to Q^2 and $y = \frac{1}{2}(1+\cos\theta)$, we finally get

$$\mathrm{d}\Pi_{2\to 3} = \frac{1}{8}\frac{1}{(4\pi)^{\frac{3d-4}{2}}}\frac{1}{\Gamma\left(\frac{d-2}{2}\right)}\frac{\left(\lambda\left(Q^2, m_i^2, m_j^2\right)\right)^{\frac{d-3}{2}}}{Q^{2d-2}}\left(s^2 - Q^4\right)\left(s - Q^2\right)^{d-4}$$
$$\left(y(1-y)\right)^{\frac{d-4}{2}}\,\mathrm{d}y\,\mathrm{d}Q^2\,\sin^{d-3}\theta^*\mathrm{d}\theta^*\,\mathrm{d}\Omega^*_{d-2}. \tag{4.9}$$

Parametrising the free variables in a $2 \to 3$ process can be tricky. I will define some natural variables in two different frames of reference, and rediscover the Lorentz transformation between them to parametrise all scalar products in terms of the variables in these reference frames. First, we will consider the lab frame, or the centre-of-mass frame of the incoming partons with momenta $\boldsymbol{k}_{i,j}$. We can reduce this to an ordinary $2 \to 2$ scattering by considering the outgoing neutralinos with momenta $\boldsymbol{p}_{i,j}$ as a single system. This lets us write the momenta as

$$k_i^\mu = \frac{\sqrt{s}}{2}\left(1,0,0,1\right), \tag{4.10a}$$
$$k_j^\mu = \frac{\sqrt{s}}{2}\left(1,0,0,-1\right), \tag{4.10b}$$
$$k^\mu = \frac{\sqrt{s}}{2}(1-z)\left(1,\sin\theta,0,\cos\theta\right), \tag{4.10c}$$
$$(p_i+p_j)^\mu = \frac{\sqrt{s}}{2}\left((1+z), -(1-z)\sin\theta, 0, -(1-z)\cos\theta\right). \tag{4.10d}$$

The centre-of-mass frame of the neutralinos is defined by $(p_i^* + p_k^*)^\mu = (\sqrt{zs}, 0, 0, 0)$.[2] We find the transformation to this frame then by making appropriate boosts and rotations of this four-vector. Let us start by rotating the 3-momentum to lie along the positive z-direction. As the y-component is already zero in the lab-frame, we only require a rotation around the y-axis, we can be parametrised by the following matrix

$$\mathrm{Rot}_y(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & 0 & \sin\alpha \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\alpha & 0 & \cos\alpha \end{pmatrix}. \tag{4.11}$$

Using $\alpha = -\theta - \pi$ we get that $\mathrm{Rot}_y(-\theta-\pi)(p_i+p_j)^\mu = \frac{\sqrt{s}}{2}\left((1+z), 0, 0, (1-z)\right)$. We can subsequently boost along the z-axis to eliminate the z-component. Such a boost can be parametrised by

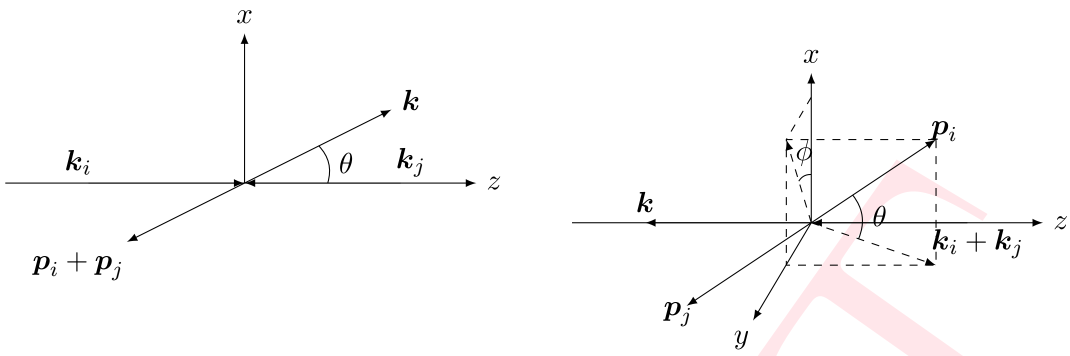
$$\mathrm{Boost}_z(\beta) = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix}, \tag{4.12}$$

where $\gamma = \left(1-\beta^2\right)^{-1/2}$. The z-component is eliminated using $\beta = -\frac{1-z}{1+z}$, such that we end up with

$$(p_i^* + p_j^*)^\mu \equiv \mathrm{Boost}_z\left(-\frac{1-z}{1+z}\right)\mathrm{Rot}_y\left(-\theta-\pi\right)(p_i+p_j)^\mu = \left(\sqrt{zs}, 0, 0, 0\right)$$

as we expected.

[2] I will from now on always put a star on quantities pertaining to the centre-of-mass frame of the neutralinos.



(a) Angular definition in the centre-of-mass frame of the initial particles with momenta $\boldsymbol{k}_{i,j}$.

(b) Angular definitions in the centre-of-mass frame of the outgoing particles with momenta $\boldsymbol{p}_{i,j}$.

Figure 4.1

Now we can parametrise ${p^*_{i,j}}^\mu$ in this frame using two angular variables θ^*, ϕ^*, knowing that $\boldsymbol{p}_i + \boldsymbol{p}_j = 0$,

$$p_i^{*\mu} = (E_i, p\sin\theta^*\cos\phi^*, p\sin\theta^*\sin\phi^*, p\cos\theta^*)\,, \tag{4.13a}$$
$$p_j^{*\mu} = (E_j, -p\sin\theta^*\cos\phi^*, -p\sin\theta^*\sin\phi^*, -p\cos\theta^*)\,. \tag{4.13b}$$

To find what $E_{i,j}$ and p need to be, we can transform k^μ and $k^\mu_{i,j}$ to this frame of reference, finding

$$k^{*\mu} = \frac{\sqrt{s}}{2}\frac{1-z}{\sqrt{z}}\,(1, 0, 0, -1)\,, \tag{4.14a}$$
$$\left(k_i^* + k_j^*\right)^\mu = \frac{s}{2\sqrt{z}}\,(1+z, 0, 0, -(1-z))\,, \tag{4.14b}$$

and use conservation of momentum and the fact that ${p^*_{i,j}}^2 = m^2_{i,j}$ to get that

$$E_{i,j}(z) = \frac{zs + m^2_{i,j} - m^2_{j,i}}{2\sqrt{zs}}, \tag{4.15a}$$
$$p(z) = \frac{\sqrt{\lambda\left(zs, m_i^2, m_j^2\right)}}{2\sqrt{zs}}. \tag{4.15b}$$

Now to get all momenta in the lab frame, we can apply the reverse transformations on ${p^*_{i,j}}^\mu$ using that $\mathrm{Rot}_y^{-1}(\alpha) = \mathrm{Rot}_y(-\alpha)$ and $\mathrm{Boost}_z^{-1}(\beta) = \mathrm{Boost}_z(-\beta)$:

$$p^\mu_{i,j} = \mathrm{Rot}_y\left(\theta + \pi\right)\mathrm{Boost}_z\left(\frac{1-z}{1+z}\right){p^*_{i,j}}^\mu. \tag{4.16}$$

4.1.3 Differential Cross-Section

$$\mathrm{d}\hat{\sigma}^d = \frac{1}{(4\pi)^{\frac{d-2}{2}}}\frac{1}{\Gamma\left(\frac{d-2}{2}\right)}\frac{p^{d-3}}{4\hat{s}\sqrt{\hat{s}}}\,|\mathcal{M}|^2\sin^{d-3}\theta\,\mathrm{d}\theta \tag{4.17}$$

$$\mathrm{d}\hat{\sigma} = \frac{1}{16\pi}\frac{1}{\hat{s}^2}\,|\mathcal{M}|^2\,\mathrm{d}t \tag{4.18}$$

4.2 Leading Order Cross-Section

4.3 NLO Corrections

TODO:

- Discuss supersymmetry breaking in dimensional regularisation and its effect on the cross-section.

4.3.1 Self-Energy Contributions

4.3.2 Vertex Corrections

4.3.3 Box Diagrams

4.3.4 Real Emission

Chapter 5

Proton—Proton Neutralino Pair Production