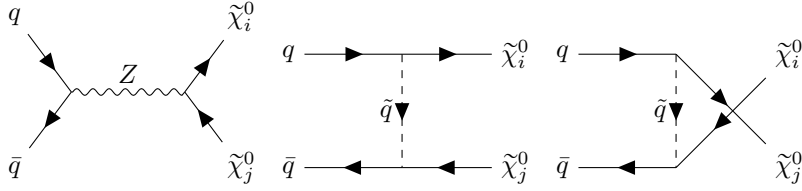


Leading Order Neutralino Calculation

Carl Martin Fevang

June 11, 2023



$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2 \quad (1a)$$

$$t = (p_1 - k_1)^2 = (p_2 - k_2)^2 \quad (1b)$$

$$u = (p_1 - k_2)^2 = (p_2 - k_1)^2 \quad (1c)$$

$$s + t + u = m_i^2 + m_j^2 \quad (1d)$$

$$(p_1 \cdot p_2) = \frac{s}{2} \quad (k_1 \cdot k_2) = \frac{s - m_i^2 - m_j^2}{2} \quad (2a)$$

$$(p_1 \cdot k_1) = \frac{m_i^2 - t}{2} \quad (p_2 \cdot k_2) = \frac{m_j^2 - t}{2} \quad (2b)$$

$$(p_1 \cdot k_2) = \frac{m_j^2 - u}{2} \quad (p_2 \cdot k_1) = \frac{m_i^2 - u}{2} \quad (2c)$$

$$\begin{aligned} \mathcal{M}_s = & -\frac{g^2}{2c_W^2} D_Z(s) \left[\bar{u}_i \gamma^\mu \left(O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R \right) v_j \right] \\ & \times \left[\bar{v}_2 \gamma_\mu \left(C_{Zqq}^L P_L + C_{Zqq}^R P_R \right) u_1 \right] \end{aligned} \quad (3a)$$

$$\begin{aligned} \mathcal{M}_t = & -2g^2 D_{\tilde{q}}(t) \left[\bar{u}_i \left(C_i^{L*} P_L + C_i^{R*} P_R \right) u_1 \right] \\ & \times \left[\bar{v}_2 \left(C_j^R P_L + C_j^L P_R \right) v_j \right] \end{aligned} \quad (3b)$$

$$\begin{aligned} \mathcal{M}_u = & -2g^2 D_{\tilde{q}}(u) \left[\bar{u}_j \left(C_j^{L*} P_L + C_j^{R*} P_R \right) u_1 \right] \\ & \times \left[\bar{v}_2 \left(C_i^R P_L + C_i^L P_R \right) v_i \right] \end{aligned} \quad (3c)$$

$$\begin{aligned} I_{ss} = \sum_{\text{spins}} |\mathcal{M}_s|^2 = & \frac{g^4}{c_W^2} |D_Z(s)|^2 \left((C_Z^L)^2 + (C_Z^R)^2 \right) \left\{ |O_{ij}^L|^2 \left[(m_i^2 - t)^2 + (m_j^2 - t)^2 \right] \right. \\ & \left. - 2 \operatorname{Re} \left\{ (O_{ij}^L)^2 \right\} m_i m_j s \right\} \end{aligned} \quad (4a)$$

1 Fierz identities

Introducing first the generalised gamma matrices Γ_I^r defined in the following way:

$$\Gamma_S^0 = \mathbb{1}, \quad (5a)$$

$$\Gamma_V^{0,\dots,3} = \gamma^\mu, \quad (5b)$$

$$\Gamma_T^{0,\dots,5} = \sigma^{\mu\nu}, \quad (\mu < \nu) \quad (5c)$$

$$\Gamma_A^{0,\dots,3} = \gamma^\mu \gamma^5, \quad (5d)$$

$$\Gamma_P^0 = \gamma^5, \quad (5e)$$

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The upper index r is understood to be summed over if it is repeated in an expression, while the index I is only summed over when explicitly stated. The complement $\Gamma_{I,r}$ is found by lowering any Lorentz index in the standard way. The generalised Fierz identity then tells us that for spinors w_1, \dots, w_4 that can either be positive energy spinors u or negative energy spinors v , we have that

$$(\bar{w}_1 \Gamma_I^r w_2) (\bar{w}_3 \Gamma_J^s w_4) = \sum_{M,N} {}^{IJ}C_{tu}^{MN} (\bar{w}_1 \Gamma_M^t w_4) (\bar{w}_3 \Gamma_N^u w_2), \quad (6)$$

with numerical coefficients ${}^{IJ}C_{tu}^{MN}$. The coefficients are found by

$${}^{IJ}C_{tu}^{MN} = \frac{1}{16} \text{Tr} [\Gamma_{M,t} \Gamma_I^r \Gamma_{N,u} \Gamma_J^s] \quad (7)$$

2 Factorisation

$$\begin{aligned} & [\bar{u}_i (C_i^{L*} P_L + C_i^{R*} P_R) u_i] [\bar{v}_2 (C_j^L P_L + C_j^R P_R) v_j] \\ &= C_{SS} [\bar{u}_i u_1] [\bar{v}_2 v_j] + C_{SP} [\bar{u}_i u_1] [\bar{v}_2 \gamma^5 v_j] \\ &+ C_{PS} [\bar{u}_i \gamma^5 u_1] [\bar{v}_2 v_j] + C_{PP} [\bar{u}_i \gamma^5 u_1] [\bar{v}_2 \gamma^5 v_j], \end{aligned} \quad (8)$$

where we have

$$C_{SS} = \frac{1}{4} (C_i^{L*} + C_i^{R*}) (C_j^L + C_j^R) \quad (9a)$$

$$C_{SP} = -\frac{1}{4} (C_i^{L*} + C_i^{R*}) (C_j^L - C_j^R) \quad (9b)$$

$$C_{PS} = -\frac{1}{4} (C_i^{L*} - C_i^{R*}) (C_j^L + C_j^R) \quad (9c)$$

$$C_{PP} = \frac{1}{4} (C_i^{L*} - C_i^{R*}) (C_j^L - C_j^R) \quad (9d)$$