Introduction to Supersymmetry

Carl Martin Fevang

September 4, 2023

1 Supersymmetry

Introduce the notion of supersymmetry and some terms that will be necessary.

2 The Super-Poincaré group

Introduce the Poincaré group and outline its supersymmetric extension algebraically.

3 Superspace

In order to make fields that transform appropriately under super-Poincaré transformations, it is useful to embed them in what is called *superspace*. This is an extension of the space-time on which the coordinate transformations of the Poincaré group work. The additional operators (supercharges) of the Super-Poincaré group will be analogous to these coordinate transformations, but on superspace.

We extend spacetime with four *Grassmann number*-valued coordinates, $\theta_A, \bar{\theta}^{\dot{A}}$ where $A, \dot{A} \in 1, 2$. Grassmann numbers are similar to ordinary numbers, but are *anti*-commutative under multiplication, i.e. $\theta_1\theta_2 = -\theta_2\theta_1$. As a consequence of this, any repeated Grassmann number is zero, since $\theta_1\theta_1 = -\theta_1\theta_1 = 0$.[1] These numbers arise naturally in structures that have anti-commuting properties, such as fermionic spinors, and the Grassmann dimensions of superspace mirror the anti-commuting properties of the supercharges Q_A.

[1] I note that 0 is itself a (trivial) Grassmann number, and it constitutes the intersection between the real number line and the Grassmann number line.