Weyl to go

Weyl spinor definitions

$$\psi_A = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad \bar\psi_A = \begin{pmatrix}\bar\psi_1\\ \bar\psi_2\end{pmatrix}$$

$$\psi^A = \epsilon^{AB}\psi_B = \begin{pmatrix}\psi_2\\ -\psi_1\end{pmatrix}^T$$

$$\bar\psi^{\dot A} = \epsilon^{\dot A\dot B}\bar\psi_B = \begin{pmatrix}\bar\psi_2\\ -\bar\psi_1\end{pmatrix}^T$$

$$\epsilon_{AB} = \epsilon_{\dot A\dot B} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$

$$\epsilon^{AB} = \epsilon^{\dot A\dot B} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$(\psi_A)^\dagger = \bar\psi_{\dot A}$$

$$\psi\chi \equiv \psi^A\chi_A = \psi_2\chi_1 - \psi_1\chi_2 = \psi^2\chi^1 - \psi^1\chi^2$$

$$\bar\psi\bar\chi \equiv \bar\psi_{\dot A}\bar\chi^{\dot A} = -\bar\psi_2\bar\chi_1 + \bar\psi_1\bar\chi_2 = -\bar\psi^2\bar\chi^1 + \bar\psi^1\bar\chi^2$$

$$\psi^2 = -2\psi_1\psi_2$$

$$\bar\psi^2 = 2\bar\psi_1\bar\psi_2$$

$$\sigma^\mu = (\mathbb{1}, \vec\sigma)$$

$$\bar\sigma^\mu = (\mathbb{1}, -\vec\sigma)$$

$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)$$

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Weyl spinor contractions

$$\eta\psi = \psi\eta$$

$$\bar\eta\bar\psi = \bar\psi\bar\eta$$

$$(\eta\psi)^\dagger = \bar\psi\bar\eta$$

$$(\eta\psi)(\eta\phi) = -\frac{1}{2}(\eta\eta)(\psi\phi)$$

$$\eta\sigma^\mu\bar\psi = -\bar\psi\bar\sigma^\mu\eta$$

$$(\sigma^\mu\bar\eta)_A(\eta\sigma^\nu\bar\eta) = \frac{1}{2}g^{\mu\nu}\eta_A(\bar\eta\bar\eta)$$

$$(\eta\sigma^\mu\bar\eta)(\eta\sigma^\nu\bar\eta) = \frac{1}{2}(\eta\eta)(\bar\eta\bar\eta)$$

$$(\eta\sigma^\mu\partial_\mu\bar\psi)(\eta\psi) = -\frac{1}{2}(\psi\sigma^\mu\partial_\mu\bar\psi)(\eta\eta)$$

$$(\partial_\mu\sigma^\mu\bar\eta)(\bar\eta\bar\psi) = -\frac{1}{2}(\partial_\mu\psi\sigma^\mu\bar\psi)(\bar\eta\bar\eta)$$

$$(\bar\eta\bar\psi)(\eta\sigma^\mu\bar\eta)(\eta\psi) = \frac{1}{4}(\eta\eta)(\bar\eta\bar\eta)(\psi\sigma^\mu\bar\psi)$$

$$\eta\sigma^{\mu\nu}\psi = -\psi\sigma^{\mu\nu}\eta$$

Charge conjugation

$$\Gamma^r = \{\mathbb{1}, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \gamma^{\mu\nu}\}$$

$$C^\dagger = C^{-1}$$
$$C^T = -C$$

$$C^{-1}\Gamma^r C = \eta^r\Gamma^{rT}$$
$$\eta^r = \begin{cases}1 & 1\le r\le 6\\ -1 & 7\le r\le 16\end{cases}$$

For $w = u, v$ and $w' = v, u$

$$w = C\bar w'^T$$

$$\bar w_1\Gamma^r w_2 = -\eta^r\bar w_2'\Gamma^r w_1'$$

Weyl-Dirac translation

$$\Psi_D = \begin{pmatrix}\psi_L\\ \bar\psi_R\end{pmatrix}, \quad \chi_M = \begin{pmatrix}\chi\\ \bar\chi\end{pmatrix}$$

$$\gamma^\mu = \begin{bmatrix}0 & \sigma^\mu\\ \bar\sigma^\mu & 0\end{bmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$

$$\bar\Psi \equiv \Psi^\dagger\gamma^0 = \begin{pmatrix}\psi_R\\ \bar\psi_L\end{pmatrix}^T.$$

$$\bar\Psi_M\gamma^\mu P_{L/R}\Phi_M = -\bar\Phi_M\gamma^\mu P_{R/L}\Psi_M$$

Chiral identities

$$\Psi_{L/R} = \frac{1}{2}\left(\mathbb{1}\mp\gamma^5\right)$$

$$(\psi_R\phi_L) = \bar\Psi P_L\Phi$$
$$(\bar\psi_L\bar\phi_R) = \bar\Psi P_R\Phi$$
$$(\bar\psi_L\bar\sigma^\mu\phi_L) = \bar\Psi\gamma^\mu P_L\Phi$$
$$(\psi_R\sigma^\mu\bar\phi_R) = \bar\Psi\gamma^\mu P_R\Phi$$

Dirac algebra

(Both in 4 and $d = 4-\epsilon$ dimensions)

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_{4\times 4}$$
$$\{\gamma^\mu, \gamma^5\} = 0$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$
$$(\gamma^5)^\dagger = \gamma^5$$

$$\gamma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

$$\begin{aligned}
M_n^{\mu_1\cdots\mu_n} &\equiv \gamma^\nu\gamma^{\mu_1}\dots\gamma^{\mu_n}\gamma_\nu\\
M_{n+1}^{\mu_1\cdots\mu_{n+1}} &= 2\gamma^{\mu_{n+1}}\gamma^{\mu_1}\dots\gamma^{\mu_n} - M_n^{\mu_1\cdots\mu_n}\gamma^{\mu_{n+1}}\\
M_0 &= d\\
M_1^\mu &= (2-d)\gamma^\mu\\
M_2^{\mu\nu} &= 4g^{\mu\nu}\mathbb{1}_{4\times4} + (d-4)\gamma^\mu\gamma^\nu\\
M_3^{\mu\nu\rho} &= (4-d)\gamma^\mu\gamma^\nu\gamma^\rho - 2\gamma^\rho\gamma^\nu\gamma^\mu
\end{aligned}$$

Traces

(Both in 4 and $d = 4-\epsilon$ dimensions.)

$$\begin{aligned}
T_n^{\mu_1\cdots\mu_n} &= \mathrm{Tr}\left\{\gamma^{\mu_1}\cdots\gamma^{\mu_n}\right\}\\
T_{2n+1}^{\mu_1\cdots\mu_{2n+1}} &= 0\\
T_{2n}^{\mu_1\cdots\mu_{2n}} &= \sum_{i=2}^{2n}(-1)^i g^{\mu_1\mu_i}T_{2n-2}^{\mu_2\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_{2n}} \qquad n\ge 1\\
T_0 &= 4\\
T_2^{\mu\nu} &= 4g^{\mu\nu}\\
T_4^{\mu\nu\rho\sigma} &= 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right)
\end{aligned}$$

$$\begin{aligned}
\tilde T_n^{\mu_1\cdots\mu_n} &= \mathrm{Tr}\left\{\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_n}\right\}\\
\tilde T_{2n+1}^{\mu_1\cdots\mu_{2n+1}} &= 0\\
\tilde T_{2n}^{\mu_1\cdots\mu_{2n}} &= \sum_{i=2}^{2n}(-1)^i g^{\mu_1\mu_i}\tilde T_{2n-2}^{\mu_2\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_{2n}} \qquad n\ge 3\\
\tilde T_0 &= 0\\
\tilde T_2^{\mu\nu} &= 0\\
\tilde T_4^{\mu\nu\rho\sigma} &= -i\epsilon^{\mu\nu\rho\sigma}
\end{aligned}$$