Weyl to go

Weyl spinor definitions

$$\psi_A = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad \bar\psi_A = \begin{pmatrix}\bar\psi_1\\ \bar\psi_2\end{pmatrix}$$

$$\psi^A = \epsilon^{AB}\psi_B = \begin{pmatrix}\psi_2\\ -\psi_1\end{pmatrix}^T$$
$$\bar\psi^{\dot A} = \epsilon^{\dot A\dot B}\bar\psi_B = \begin{pmatrix}\bar\psi_2\\ -\bar\psi_1\end{pmatrix}^T$$

$$\epsilon_{AB} = \epsilon_{\dot A\dot B} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$
$$\epsilon^{AB} = \epsilon^{\dot A\dot B} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$(\psi_A)^\dagger = \bar\psi_{\dot A}$$

$$\psi\chi \equiv \psi^A\chi_A = \psi_2\chi_1 - \psi_1\chi_2 = \psi^2\chi^1 - \psi^1\chi^2$$
$$\bar\psi\bar\chi \equiv \bar\psi_{\dot A}\bar\chi^{\dot A} = -\bar\psi_2\bar\chi_1 + \bar\psi_1\bar\chi_2 = -\bar\psi^2\bar\chi^1 + \bar\psi^1\bar\chi^2$$

$$\psi^2 = -2\psi_1\psi_2$$
$$\bar\psi^2 = 2\bar\psi_1\bar\psi_2$$

$$\sigma^\mu = (\mathbb{1}, \vec\sigma)$$
$$\bar\sigma^\mu = (\mathbb{1}, -\vec\sigma)$$
$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)$$

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Weyl spinor contractions

$$\eta\psi = \psi\eta$$
$$\bar\eta\bar\psi = \bar\psi\bar\eta$$
$$(\eta\psi)^\dagger = \bar\psi\bar\eta$$
$$(\eta\psi)\,(\eta\phi) = -\frac{1}{2}\,(\eta\eta)\,(\psi\phi)$$
$$\eta\sigma^\mu\bar\psi = -\bar\psi\bar\sigma^\mu\eta$$
$$(\sigma^\mu\bar\eta)_A(\eta\sigma^\nu\bar\eta) = \frac{1}{2}g^{\mu\nu}\eta_A(\bar\eta\bar\eta)$$
$$(\eta\sigma^\mu\bar\eta)(\eta\sigma^\nu\bar\eta) = \frac{1}{2}(\eta\eta)(\bar\eta\bar\eta)$$
$$(\eta\sigma^\mu\partial_\mu\bar\psi)(\eta\psi) = -\frac{1}{2}(\psi\sigma^\mu\partial_\mu\bar\psi)(\eta\eta)$$
$$(\partial_\mu\sigma^\mu\bar\eta)(\bar\eta\bar\psi) = -\frac{1}{2}(\partial_\mu\psi\sigma^\mu\bar\psi)(\bar\eta\bar\eta)$$
$$(\bar\eta\bar\psi)(\eta\sigma^\mu\bar\eta)(\eta\psi) = \frac{1}{4}(\eta\eta)(\bar\eta\bar\eta)(\psi\sigma^\mu\bar\psi)$$
$$\eta\sigma^{\mu\nu}\psi = -\psi\sigma^{\mu\nu}\eta$$

Weyl-Dirac translation

$$\Psi_D = \begin{pmatrix}\psi_L\\ \bar\psi_R\end{pmatrix}, \quad \chi_M = \begin{pmatrix}\chi\\ \bar\chi\end{pmatrix}$$

$$\gamma^\mu = \begin{bmatrix}0 & \sigma^\mu\\ \bar\sigma^\mu & 0\end{bmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$

$$\bar\Psi \equiv \Psi^\dagger\gamma^0 = \begin{pmatrix}\psi_R\\ \bar\psi_L\end{pmatrix}^T.$$

$$\bar\Psi_M\gamma^\mu P_{L/R}\Phi_M = -\bar\Phi_M\gamma^\mu P_{R/L}\Psi_M$$

Chiral identities

$$\Psi_{L/R} = \frac{1}{2}\left(\mathbb{1} \mp \gamma^5\right)$$
$$(\psi_R\phi_L) = \bar\Psi P_L\Phi$$
$$(\bar\psi_L\bar\phi_R) = \bar\Psi P_R\Phi$$
$$(\bar\psi_L\bar\sigma^\mu\phi_L) = \bar\Psi\gamma^\mu P_L\Phi$$
$$(\psi_R\sigma^\mu\bar\phi_R) = \bar\Psi\gamma^\mu P_R\Phi$$

Dirac algebra

(Both in 4 and $d = 4-\epsilon$ dimensions)

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_{4\times4}$$
$$\left\{\gamma^\mu, \gamma^5\right\} = 0$$

$$(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$
$$\left(\gamma^5\right)^\dagger = \gamma^5$$

$$\gamma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

$$M_n^{\mu_1\cdots\mu_n} \equiv \gamma^\nu\gamma^{\mu_1}\ldots\gamma^{\mu_n}\gamma_\nu$$
$$M_{n+1}^{\mu_1\cdots\mu_{n+1}} = 2\gamma^{\mu_{n+1}}\gamma^{\mu_1}\ldots\gamma^{\mu_n} - M_n^{\mu_1\cdots\mu_n}\gamma^{\mu_{n+1}}$$
$$M_0 = d$$
$$M_1^\mu = (2-d)\gamma^\mu$$
$$M_2^{\mu\nu} = 4g^{\mu\nu}\mathbb{1}_{4\times4} + (d-4)\gamma^\mu\gamma^\nu$$
$$M_3^{\mu\nu\rho} = (4-d)\gamma^\mu\gamma^\nu\gamma^\rho - 2\gamma^\rho\gamma^\nu\gamma^\mu$$

Traces

(Both in 4 and $d = 4-\epsilon$ dimensions.)

$$\mathrm{Tr}\left\{\gamma^5\right\} = \mathrm{Tr}\left\{\text{odd \# of }\gamma\text{-matrices}\right\} = 0$$
$$\mathrm{Tr}\{ab\} = 4g^{ab},$$
$$\mathrm{Tr}\{abcd\} = 4\left(g^{ab}\,g^{cd} - g^{ac}\,g^{cd} + g^{ad}\,g^{bc}\right),$$
$$\mathrm{Tr}\left\{\gamma^5 abcd\right\} = -4i\epsilon^{abcd},$$

$$\mathrm{Tr}\left\{\gamma^{\mu_1}\ldots\gamma^{\mu_{2n}}\right\} = \sum_{i=2}^{2n}(-1)^i g^{\mu_1\mu_i}\,\mathrm{Tr}\left\{\gamma^{\mu_2}\ldots\gamma^{\mu_{i-1}}\gamma^{\mu_{i-1}}\ldots\gamma^{2n}\right\}$$

$$\mathrm{Tr}\left\{P_{L/R}ab\right\} = \frac{1}{2}\,\mathrm{Tr}\{ab\},$$
$$\mathrm{Tr}\left\{P_{L/R}abcd\right\} = \frac{1}{2}\,\mathrm{Tr}\{abcd\} \pm 2i\epsilon^{abcd},$$