UNIVERSITY
OF OSLO

Master's thesis

My Master's Thesis

With Subtitle

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60 ECTS study points

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November 8, 2023

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Chapter 1

Introduction

This is where I introduce the master's thesis.

Chapter 2

Quantum Field Theory

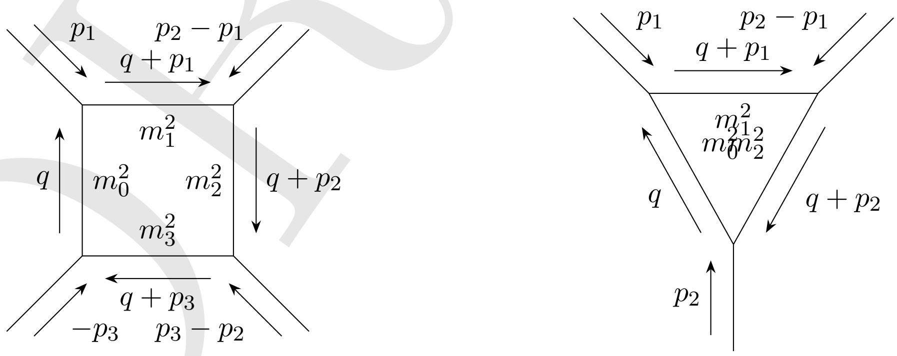
2.1 Passarino-Veltman Loop Integrals

By Lorentz invariance, there are a limited set of forms that loop integrals can take. Why is this?© These can be categorised according to the number of propagator terms they include, which corresponds to the number of externally connected points there are in the loop. A general scalar N-point loop integral takes the form

$$T_0^N\left(p_i^2, (p_i - p_j)^2; m_0^2, m_i^2\right) = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \mathrm{d}^d q\, \mathcal{D}_0 \prod_{i=1}^{N-1} \mathcal{D}_i, \tag{2.1}$$

where $\mathcal{D}_0 = \left[q^2 - m_0^2\right]^{-1}$ and $\mathcal{D}_i = \left[(q+p_i)^2 - m_i^2\right]^{-1}$ for $i \geq 0$. The first 4 scalar loop integrals are named accordingly

$$T_0^1 \equiv A_0(m_0^2) \tag{2.2}$$
$$T_0^2 \equiv B_0(p_1^2; m_0^2, m_1^2) \tag{2.3}$$
$$T_0^3 \equiv C_0(p_1^2, p_2^2, (p_1 - p_2)^2; m_0^2, m_1^2, m_2^2) \tag{2.4}$$
$$T_0^4 \equiv D_0(p_1^2, p_2^2, p_3^2, (p_1 - p_2)^2, (p_1 - p_3)^2, (p_2 - p_3)^2; m_0^2, m_1^2, m_2^2) \tag{2.5}$$



(a) (b)

Figure 2.1

Now, more complicated Lorentz structure can be obtained in loop integrals. However, these can still be related to the scalar integrals by exploiting the possible tensorial structure there are. Defining an arbitrary loop integral

$$T^N_{\mu_1\cdots\mu_P}\left(p_i^2, (p_i - p_j)^2; m_0^2, m_i^2\right) = \frac{(2\pi\mu)^{4-d}}{i\pi^2} \int \mathrm{d}^d q\, q_{\mu_1} \cdots q_{\mu_P} \mathcal{D}_0 \prod_{i=1}^{N-1} \mathcal{D}_i. \tag{2.6}$$

$$
\begin{aligned}
B^\mu &= p_1^\mu B_1, && \text{(2.7a)}\\
B^{\mu\nu} &= g^{\mu\nu} B_{00} + p_1^\mu p_1^\nu B_{11}, && \text{(2.7b)}\\
C^\mu &= \sum_{i=1}^{2} p_i^\mu C_i, && \text{(2.7c)}\\
C^{\mu\nu} &= g^{\mu\nu} C_{00} + \sum_{i,j=1}^{2} p_i^\mu p_j^\nu C_{ij}, && \text{(2.7d)}\\
C^{\mu\nu\rho} &= \sum_{i=1}^{2} (g^{\mu\nu} p_i^\rho + g^{\mu\rho} p_i^\nu + g^{\nu\rho} p_i^\mu) C_{00i} + \sum_{i,j,k=1}^{2} p_i^\mu p_j^\nu p_k^\rho C_{ijk}, && \text{(2.7e)}\\
D^\mu &= \sum_{i=1}^{3} p_i^\mu D_i, && \text{(2.7f)}\\
D^{\mu\nu} &= g^{\mu\nu} D_{00} + \sum_{i,j=1}^{3} p_i^\mu p_j^\nu D_{ij}, && \text{(2.7g)}\\
D^{\mu\nu\rho} &= \sum_{i=1}^{3} (g^{\mu\nu} p_i^\rho + g^{\mu\rho} p_i^\nu + g^{\nu\rho} p_i^\mu) D_{00i} + \sum_{i,j,k=1}^{3} p_i^\mu p_j^\nu p_k^\rho D_{ijk}, && \text{(2.7h)}\\
D^{\mu\nu\rho\sigma} &= (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) D_{0000} \\
&\quad + \sum_{i,j=1}^{3} (g_{\mu\nu} p_i^\rho p_j^\sigma + g_{\mu\nu} p_i^\sigma p_j^\rho + g_{\mu\rho} p_i^\nu p_j^\sigma + g_{\mu\rho} p_i^\sigma p_j^\nu + g_{\mu\sigma} p_i^\rho p_j^\nu + g_{\mu\nu} p_i^\nu p_j^\rho) D_{00ij} \\
&\quad + \sum_{i,j,k,l=1}^{3} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma D_{ijkl}, && \text{(2.7i)}
\end{aligned}
$$

where all coefficients must be completely symmetric in i, j, k, l.