Homework/Programming Assignment #3 **Homework Due: 04/18/2023- 5:00PM**

Name/EID:
Email:
Signature (*required*)
I/We have followed the rules in completing this Assignment.

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Question	Points	Total
PA1	**80**	
PA2.1	**20**	
PA2. 2 (Bonus)	**10**	

Instruction:

1. Remember that this is a graded assignment. It is the equivalent of a **take-home exam**.
2. **For PA questions**, <u>you need to write a report showing how you derived your equations, describes your approach, test functions, and discusses the results</u>. You should show your test results for each function.

3. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.
4. It is open book, notes, and web. But you should cite any references you consult.
5. Unless I say otherwise in class, it is due before the start of class on the due date mentioned in the P/H Assignment.
6. **Sign and append** this score sheet as the first sheet of your assignment.
7. Remember to submit your assignment in the Canvas.

➢ Programming Assignment (PA)

PA1 [1]. This problem concerns calibration, simple registration, and tracking for a stereotactic navigation system that uses an electromagnetic positional tracking device shown in Fig. 2.

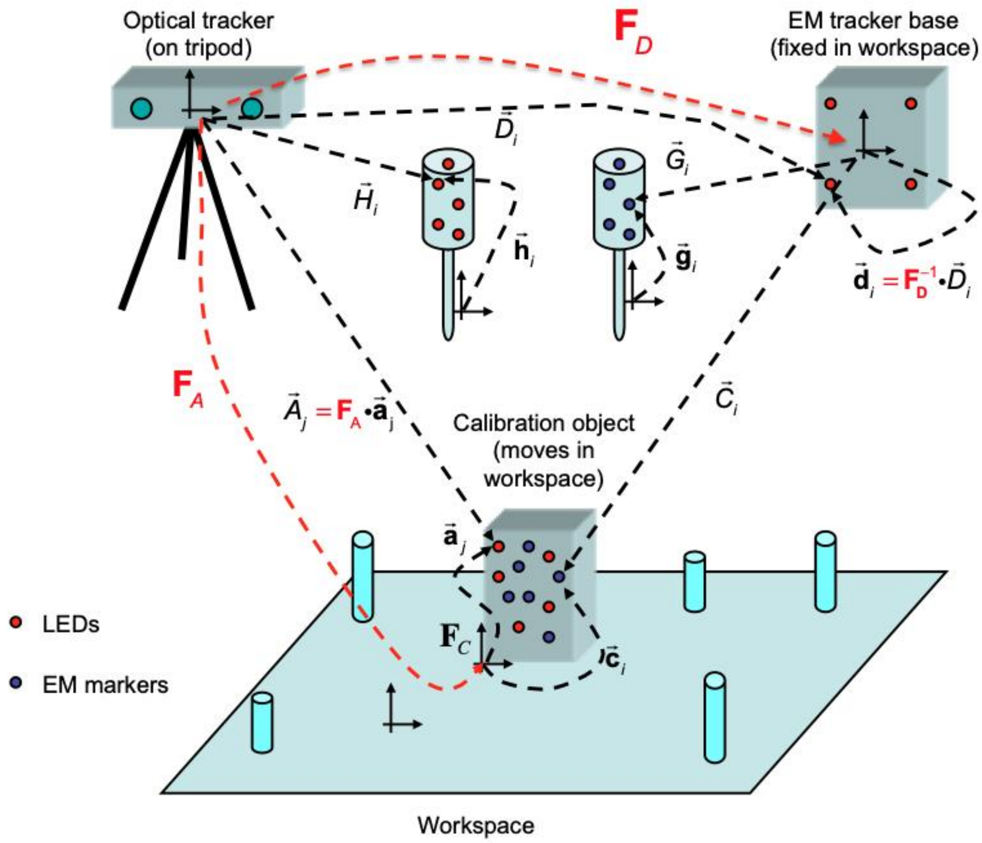


Figure 2. Stereotactic navigation system that uses an electromagnetic positional tracking device, an optical tracker, and a calibration object.

❖ Description of the Problem Scenario

✓ Equipment and Calibration

You have been given an **electromagnetic (EM) tracking system** that is capable of measuring the 3D position of small markers relative to a measuring base unit. The measurements are subject to some uncharacterized distortions, but you have been given a **calibration object** containing N_C **EM markers** at known positions c_i for $i=1\ldots N_C$ relative to the calibration object coordinate system $\mathbf{F_C}$. The calibration object also has N_A Optical LED markers at **known positions** $\mathbf{a_j}$ on the calibration object. N_D other optical markers are placed at known positions $\mathbf{d_j}$ on the base unit of the *electromagnetic tracking system*.

The *optical tracking system* is able to read the positions of the optical markers to very high accuracy. For the purpose of this exercise, we will assume no geometric error from the optical tracker. We will denote the positions of the calibration object LEDs relative to the optical tracker as $\mathbf{A_j}$ and those of the electromagnetic base markers as $\mathbf{D_j}$. We will designate the measured positions of the electromagnetic tracker markers on the calibration object, corresponding to the $\mathbf{c_i}$ as $\mathbf{C_i}$. In addition to its distortion, which is large (up to several mm) but very repeatable, the sensor system is subject to a certain amount of random noise ν, which varies from zero (on a good day) to a value up to about 0.3 mm.

The calibration object will be placed at various positions in the workspace, and the positions of the optical markers and the electromagnetic markers will be measured. Thus, **each "sample frame" of calibration data** will consist of the following information: **[, $D_1, \ldots ,D_N^D$, $A_1, \ldots ,$ A_N^A ,$C_1, \ldots ,C_N^C$].**

The workspace also has one or more **dimpled calibration posts** placed at fixed, though unknown, positions and orientations with respect to the electromagnetic tracking system. You have been given two pointer probes. One equipped with N_H LED markers at **unknown, though fixed positions** $\mathbf{h}_i$. The other is equipped with N_G electromagnetic markers, again at **unknown, though fixed locations** $\mathbf{g}_i$. You are given the data from "*pivot calibrations*" of each probe using a dimpled post. Each frame of data from the optical probe calibration consists of measurements **[$D_1, \ldots , D_N^D$,$H_1, \ldots ,H_N^H$]** and each frame of data from the electromagnetic probe calibration consists of values **[$G_1, \ldots ,G_N^G$].**

Note: To some extent, this represents an unrealistic scenario. One is not likely to have both an optically-tracked probe and an electromagnetically tracked probe, although it is perfectly plausible to have an optical tracking system for performing once-only calibrations. The main reason for including this element in our problem scenario is specifically to enable me to ask you to program a pivot calibration step with a non-distorted tracking system. Also, during system development it may indeed be plausible to have both optical and EM tracked probes. In fact, the calibration body itself may be a pointer containing both types of markers.

✓ Stereotactic Navigation after calibration

After the calibration steps described above are performed, the system is ready for use. We assume that N_B ***fiducial landmarks*** have been selected that can be located accurately in preoperative **CT images** and by pointing with the EM probe. Assume that the locations of these landmarks in the CT images are given by $\mathbf{b}_j$ for $j = 1,\ldots,N_B$.

The surgical procedure is as follows:

1. The patient's anatomy is fixed in space relative to the base of the EM tracking system.
2. The tip of the EM pointer probe is positioned on each of the fiducial landmarks $\mathbf{b}_j$ and a "frame" of EM data (i.e., [$G_1, \ldots , G_N^G$]) is taken. The corresponding position B_j of the pointer tip relative to the EM base coordinate system is computed.
3. The transformation between EM and CT coordinates is computed (i.e., $\mathbf{b}_j \cong \mathbf{F}_{reg} \bullet B_j$)
4. Subsequently, the system reads successive frames of EM data [$G_1, \ldots ,G_N^G$], computes the corresponding position of the tip in CT coordinates, and displays the corresponding CT data.

❖ Goals:

The specific goals of this assignment are:

1. Develop a 3D point set to 3D point set registration algorithm
2. Develop a "pivot" calibration method.
3. Given a distortion calibration data set, as described above, compute the "expected" values $C_i^{(\text{expected})}$ for the C_i :
   a. For each calibration data frame $[,D_1, \ldots ,D_{N^D} , A_1, .. , A_{N^A} ,C_1, \ldots ,C_{N^C}]$, compute the transformation $\mathbf{F}_D$ between optical tracker and EM tracker coordinates, i.e., compute a frame $\mathbf{F}_D$ such that $D_j = \mathbf{F}_D \bullet \mathbf{d}_j$.
   b. Similarly, compute a transformation $\mathbf{F}_A$ between calibration object and optical tracker. Coordinates, i.e., $A_j = \mathbf{F}_A \bullet \mathbf{a}_j$
   c. Given $\mathbf{F}_D$ and $\mathbf{F}_A$, compute: $C_i^{(\text{expected})} = \mathbf{F}_D^{-1} \bullet \mathbf{F}_A \bullet \mathbf{c}_i$.
   d. Output $C_i^{(\text{expected})}$ (see file formats below)

4. Apply the *EM tracking data* to perform a <u>pivot calibration</u> for the EM probe and determine the position relative to the EM tracker base coordinate system of the dimple in the calibration post. The suggested procedure is as follows.
   a. Use the first "frame" of pivot calibration data to define a local "probe" coordinate system and use this to compute $\mathbf{g}_j$. One simple method is as follows. First compute the midpoint of the observed points
   $$G_0 = \frac{1}{N_G} \sum G_j$$
   Then translate the observations relative to this midpoint. I.e., compute
   $$\mathbf{g}_j = G_j - G_0$$
   There are alternative methods, many of which involve rotating $\mathbf{g}_j$. But this isn't particularly critical. Your pivot calibration will determine a tip coordinates $\mathbf{t}_G$ defined in the same probe coordinate system. i.e., if $\mathbf{F}_G(t)$ gives the position and orientation of the pointer body at time t with respect to some tracker coordinate system, then $\mathbf{F}_G(t) \bullet \mathbf{t}_G$ gives the coordinates of the pointer tip with respect to the same tracker coordinate system.
   b. For each **"frame"** $\boldsymbol{k}$ of pivot data, compute a transformation $\mathbf{F}_G[k]$ such that
   $$\mathrm{G_j} = \mathbf{F}_\mathrm{G}[\mathrm{k}] \bullet \mathbf{g}_\mathrm{j}$$
   c. Now use the method discussed in class to solve the system
   $$\begin{matrix} \vdots \\ \mathrm{P_{dimple}} = \mathbf{F}_\mathrm{G}[\mathrm{k}] \bullet \mathbf{t}_\mathrm{G} \\ \vdots \end{matrix}$$

5. Apply the optical tracking data to perform a pivot calibration of the optical tracking probe. The suggested method is the same as above except that you should first use your value for $\mathbf{F}_D$ to transform the optical tracker beacon positions into EM tracker coordinates. Note that the optical tracker may not be in exactly the same position and orientation with respect to the EM tracker base for each observation frame of optical tracker data, so this is an important step.

❖ Data file formats

- "NAME-CALBODY.TXT" – input file which describes the calibration object

LINE	Data	Description
1	N_D,N_A,N_C,NAME-CALBODY.TXT	Number of optical markers on EM base, number of optical markers on calibration object, number EM markers on calibration object. Followed by ascii string giving file name
Next N_D records	$d_{x,i}$, $d_{y,i}$, $d_{z,i}$	Coordinates of $\mathbf{d}_i$
Next N_A records	$a_{x,i}$, $a_{y,i}$,, $a_{z,i}$	Coordinates of $\mathbf{a}_i$
Next N_C records	$c_{x,i}$, $c_{y,i}$,, $c_{z,i}$	Coordinates of $\mathbf{c}_i$

- "NAME-CALREADINGS.TXT" – input file which provides the values read by the sensor

LINE	Data	Description
1	N_D,N_A,N_C,N_{frames},NAME-CALREADINGS.TXT	Number of optical markers on EM base, number of optical markers on calibration object, number EM markers on calibration object, number of "data frames" of data, file name
Frame 1	$D_{x,i}$, $D_{y,i}$,$D_{z,i}$	Coordinates of Di
	…	
	$A_{x,i}$, $A_{y,i}$, $A_{z,i}$	Coordinates of A_i
	…	

	$C_{x,i}$, $C_{y,i}$, $C_{z,i}$	Coordinates of C_i
…	…	…
Frame N_{frames}	$D_{x,i}$, $D_{y,i}$, $D_{z,i}$	Coordinates of D_i
	…	
…	$A_{x,i}$, $A_{y,i}$, $A_{z,i}$	Coordinates of A_i
	…	
…	$C_{x,i}$, $C_{y,i}$, $C_{z,i}$	Coordinates of C_i

- “NAME-EMPIVOT.TXT” – input file which provides the values read by the sensor

LINE	Data	Description
1	N_G,N_{frames},NAME-EMPIVOT.TXT	Number of EM markers on probe, number of “data frames” of data, file name
Frame 1	$G_{x,1}$, $G_{y,1}$, $G_{z,1}$	Coordinates of G_1
…	…	…
…	$G_{x,NG}$,$Gy_{,NG}$, $G_{z,NG}$	Coordinates of G_NG
…	…	…
Frame N_{frames}	$G_{x,1}$, $G_{y,1}$, $G_{z,1}$	Coordinates of G_1
…	…	…
…	$G_{x,NG}$,$Gy_{,NG}$, $G_{z,NG}$	Coordinates of G_NG

- “NAME-OPTPIVOT.TXT” – input file which provides the values read by the sensor

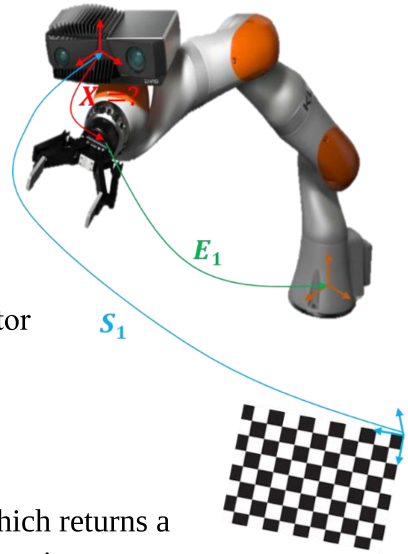
LINE	Data	Description
1	N_D,N_H,N_{frames},NAME-OPTPIVOT.TXT	Number of optical markers on EM base, number of optical markers on probe, number of “data frames” of data, file name
Frame 1	$D_{x,1}$,$D_{y,1}$,$D_{z,1}$	Coordinates of D_1

	$D_{x,N_D}, D_{y,N_D}, D_{z,N_D}$	Coordinates of D_{N_D}
…	$H_{x,1}, H_{y,1}, H_{z,1}$	Coordinates of H_1
…	…	…
	$H_{x,N_H}, H_{y,N_H}, H_{z,N_H}$	Coordinates of H_{N_H}
…	…	…
Frame N_{frames}	$D_{x,1}, D_{y,1}, D_{z,1}$	Coordinates of D_1
…		
…	$D_{x,N_D}, D_{y,N_D}, D_{z,N_D}$	Coordinates of D_{N_D}
	$H_{x,1}, H_{y,1}, H_{z,1}$	Coordinates of H_1
	…	…
	$H_{x,N_H}, H_{y,N_H}, H_{z,N_H}$	Coordinates of H_{N_H}

- "NAME-OUTPUT-1.TXT" – output file for problem 1

LINE	Data	Description
1	N_C, N_{frames},NAME-OUTPUT1.TXT	Number of EM markers on cal object, number of "data frames" of data, file name
2	P_x, P_y, P_z	Estimated post position with EM probe pivot calibration
3	P_x, P_y, P_z	Estimated post position with optical probe pivot calibration
Frame 1	$C_{x,1}, C_{y,1}, C_{z,1}$	Coordinates of $C_1^{(\text{expected})}$
…	…	…
…		Coordinates of $C_{N_C}^{(\text{expected})}$
…	…	…
Frame N_{frames}	$C_{x,1}, C_{y,1}, C_{z,1}$	Coordinates of $C_1^{(\text{expected})}$
…	…	…
…	$C_{x,N_C}, C_{y,N_C}, C_{z,N_C}$	Coordinates of $C_{N_C}^{(\text{expected})}$

PA2. For this part of the assignment you will implement a hand eye calibration algorithm to solve for the unknown transformation between the end effector frame on a Robot and the frame of the camera it is holding i.e., an Eye in Hand problem.



1) You are provided a script "*data_quaternion.m*", which returns collected data for **10 different robot (E_i) and sensor (S_i) configurations** (i.e., rotation in the quaternion form and a translation vector).

The goal is to write a generic function to solve for the unknown transformation X from the camera' frame to the robot's end effector frame using:

   a) **Quaternion analytical approach**.

2) You are also provided a script "*data_quaternion_noisy.m*", which returns a similar data to part 1 but including noisy data. For this part of assignment:

   a) Repeat part 1 and compare the results with the case of noise free data.
   b) Use half of the data sets (i.e., 5 sets of configurations) and compare the results with the case of noise free data and when you used the whole data set. Discuss your results.

➢ References:

[1] **PA1** has been adapted with permission from the *Computer Integrated Surgery* course, Dr. Russell H. Taylor, Johns Hopkins University