Mission Planning Optimization

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1 Problem Description

We have an agent that should visit a set of positions (goals) and collect the price associated to each one of them, while avoiding the obstacles present in the environment; it can move for a maximum of T seconds.

An example of map is available in figure 1.

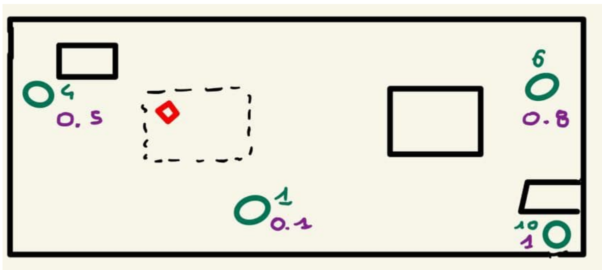

Figure 1: In red can be seen the agent, which can start from any position inside the dotted rectangle; in green are shown the goals with their reward (green number) and their difficulty (purple number); the black lines represent obstacles.

At each goal x_i will be assigned a difficulty d_i based on its position with respect to the nearest obstacles in order to weight its price r_i with the risks associated to it.

2 Integer Programming Formulation

Given n goals, lets consider n+1 goals, with start position as first goal, and define:
Matrix $C^{(n+1)x(n+1)}$, where c_{ij} is equal to the changes of direction required to go from goal j to node i.
Vector $D^{(n+1)x1}$, where d_i is equal to the difficulty associated to the node i.
Vector $R^{(n+1)x1}$, where r_i is equal to the reward associated to the node i.
Matrix $T^{(n+1)x(n+1)}$, where t_{ij} is the time required to go from node j to node i.
Matrix $E^{(n+1)x(n+1)}$, where e_{ij} is the energy required to go from node j to node i.

The variables will be organized in a (n+1)x(n+1) matrix called X, where $x_{ij} = 1$ if trajectory from goal j to goal i is used, 0 otherwise.

Thus the problem and its constraints can be written as follow:

$$\min_{\boldsymbol{X}} \sum_{ij} (c_{ij} - d_i r_i) x_{ij} \tag{1}$$

subject to constraints

$$\sum_{(i,j)\in S, i\neq j} x_{ij} \leq |S| - 1,\ S \subseteq Goals \text{ and } |S| \geq 2 \tag{2}$$

$$\sum_{j=0}^{n+1} x_{kj} \geq \sum_{i=0}^{n+1} x_{ik},\ \forall k \in 1..n+1 \tag{3}$$

$$\sum_{i=0}^{n+1} x_{i0} = 1 \tag{4}$$

$$\sum_{j=0}^{n+1} x_{0j} = 0 \tag{5}$$

$$\sum_{j=0}^{n+1} x_{ij} \leq 1,\ \forall i \in 1..n+1 \tag{6}$$

$$\sum_{i=0}^{n+1} x_{ij} \leq 1,\ \forall j \in 0..n+1 \tag{7}$$

$$\sum_{i,\ j=0}^{n+1} t_{ij} x_{ij} \leq ThreshT \tag{8}$$

$$\sum_{i,\ j=0}^{n+1} e_{ij} x_{ij} \leq ThreshE \tag{9}$$

$$0 \leq x_{ij} \leq 1,\ \forall i, j \in 0..n+1 \tag{10}$$

$$x_{ij} \in N,\ \forall i, j \in 0..n+1 \tag{11}$$

Where constraints (2) and (3) are used respectively to avoid cycles and to impose contiguity, while constraints (4) and (5) force the use of goal 0, i.e. starting position, and avoid visiting the starting position from other goals.
These last two constraints are special cases of constraints (6) and (7), that are used to avoid moving to or from a node more then once, i.e. only one variable per column and row is allowed to be 1.

Constraints (8), (9), (10) are used to impose time and energy constraints and as upper and lower bound for the variables, whereas constraint (11) is used to force each variable to be a natural number.

This last constraint will be removed to create the relaxed problems, created in the branch and bound algorithm, to be solved by the revised simplex algorithm.