Recitation 3

Carlo Cavicchia

carlo.cavicchia@uniroma1.it ✉

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

What proportion of children has an MDI of at least 120?

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

$$P(X > 120) = P(\frac{X - 100}{16} > \frac{120 - 100}{16})$$

$$= P(Z > 1.25) = P(Z < -1.25) = 0.1056$$

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

What proportion of children has an MDI of at least 80?

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

What proportion of children has an MDI of at least 80?

up to you -> https://forms.gle/ByPXKYDxrq2b2oNh9

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

$$P(X > 80) = P(\frac{X - 100}{16} > \frac{80 - 100}{16})$$

$$= P(Z > -1.25) = 1 - P(Z < -1.25) = 1 - 0.1056 = 0.894$$

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

Compute the probability that in a sample of 25 babies, the proportion of children with an MDI is greater than 90

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

$$P(\bar{X} > 90) = P(\frac{\bar{X} - 100}{16/\sqrt{25}} > \frac{90 - 100}{16/\sqrt{25}})$$

$$= P(Z > -3.13) = 1 - P(Z < -3.13) = 0.9991$$

Exercise 1

The Mental Development Index (MDI) of the Bayley Scales of Infant Development is a standardized measure used in observing infants over time. It is approximately normal with a mean of 100 and a standard deviation of 16.

Compute the probability that in a sample of 4 babies, only one has an MDI value less than 80 years old.

FOR HOME

Exercise 2

The average monthly rental rate for one-bedroom apartments in Rome is 650 euros. Suppose rental rates across all one-bedroom apartments in Rome follow approximately a normal distribution, with a standard deviation of 100 euros.

Find the approximate proportion of one-bedroom apartments for which the rental rate is between 500 euros and 900 euros a month.

The average monthly rental rate for one-bedroom apartments in Rome is 650 euros. Suppose rental rates across all one-bedroom apartments in Rome follow approximately a normal distribution, with a standard deviation of 100 euros.

$$P(500 < X < 900) = P(\frac{500 - 650}{100} < \frac{X - 650}{100} < \frac{900 - 650}{100})$$

$$= P(-1.5 < Z < 2.5) = \Phi(2.5) - \Phi(-1.5) = 0.9938 - 0.0668 = 0.927$$

The average monthly rental rate for one-bedroom apartments in Rome is 650 euros. Suppose rental rates across all one-bedroom apartments in Rome follow approximately a normal distribution, with a standard deviation of 100 euros.

Compute the probability that in a sample of 5 one-bedroom apartments, the proportion of one-bedroom apartments for which the rental rate is less than 500 euros a month.

Exercise 2

The average monthly rental rate for one-bedroom apartments in Rome is 650 euros. Suppose rental rates across all one-bedroom apartments in Rome follow approximately a normal distribution, with a standard deviation of 100 euros.

$$P(\bar{X} < 500) = P(\frac{\bar{X} - 650}{100/\sqrt{5}} < \frac{500 - 650}{100/\sqrt{5}}) = P(Z < -3.35) = 0.0004$$

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

What is the point estimate of the population mean?

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

up to you -> https://forms.gle/PBVVTjrFNj6maaRSA

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

The sample mean.

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

Is the estimator used in (a) unbiased?

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

It is unbiased. Its standard error tends to 0 when the sample size goes to inifinity.

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

Find the standard error of the sample mean.

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

$$1.99/\sqrt{50} = 0.2814$$

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

Test the null hypothesis $H_0 : \mu = 3$ vs the alternative $H_1 : \mu > 3$ at the significance level $\alpha = 0.05$.

Exercise 3

It has been asked "What do you think is the ideal number of children for a family to have?". The 50 females who responded had a median of 2, mean of 3.22, and standard deviation of 1.99.

Step 1: Assumptions: The variable is quantitative, the sample size is large enough.

Step 2: Hypotheses $H_0 : \mu = 3$ vs the alternative $H_1 : \mu > 3$

Step 3: Test statistic $z_{oss} = \sqrt{n} \times (\bar{x} - \mu_0)/\sigma = \sqrt{50} \times (3.22 - 3)/1.99 = 0.7817$

Step 4: p-value = $P(Z > z_{oss}) = P(Z > 0.7817) = 0.2172$

Step 5: Conclusion: p-value > α. Thus we do not reject H_0

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

State the null and alternative hypotheses for this investigation.

Exercise 4

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

up to you -> https://forms.gle/CXz5EVgh9JuMVbAg7

Exercise 4

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

$H_0 : \mu = 3.05$ vs the alternative $H_1 : \mu \neq 3.05$

Exercise 4

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

Test the null hypothesis H_0 vs the alternative H_1 at the significance level $\alpha = 0.05$.

Exercise 4

Suppose the mean GPA of all students graduating from Penn State University in 2005 was 3.05. The registrar plans to look at records of 100 students graduating in 2015 to see if mean GPA has changed. The sample mean is 2.15, while standard deviation is 1.5.

Step 1: Assumptions: The variable is quantitative, the sample size is large enough.

Step 2: Hypotheses $H_0 : \mu = 3.05$ vs the alternative $H_1 : \mu \neq 3.05$

Step 3: Test statistic $z_{oss} = \sqrt{n} \times (\bar{x} - \mu_0)/\sigma = \sqrt{100} \times (2.15 - 3.05)/1.5 = -6$

Step 4: p-value = $P(Z > |z_{oss}|) = 2P(Z > 6) \approx 0$

Step 5: Conclusion: p-value < α. Thus we do reject H_0

Exercise 5

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

What type of statistical variable is it?

Exercise 5

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

Quantitative continuous.

Exercise 5

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

Find mean and median.

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

https://forms.gle/wtmyYdTBrkoT1gaWA

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

Mean$= 5.7$; Median$= 5$.

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

Is the distribution of these data symmetric?

Exercise 5

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

The mean is different from the median. The mean is equal to 5.7, while the median is equal to 5. It follows that the distribution is skewed to the right, since the median is smaller than the mean (in other words the distribution has a long right tail).

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

Compute the variance.

Here it is the distribution of family year income (ten thousands of dollars) in a small village: $4, 5, 6, 5, 3, 10, 5, 8, 4, 7$.

The variance is 4.01

$$\sigma^2 = (2 \times (4 - 5.7)^2 + 3 \times (5 - 5.7)^2 +$$

$$+(6 - 5.7)^2 + (3 - 5.7)^2 + (7 - 5.7)^2 + (8 - 5.7)^2 + (10 - 5.7)^2)/10 = 4.01$$

The following table contains income (X) and spending (Y) amounts for 5 families:

x	y
6	5
7	6
9	7
5	4
8	5

Are the variable correlated? Compute the correlation coefficient and interpret it.

Exercise 6

$\bar{x} = 7, \bar{y} = 5.4$

x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x}) \times (y - \bar{y})$	$(x - \bar{x})^2$	$(y - \bar{y})^2$
6	5	-1	-0.4	0.4	1	0.16
7	6	0	0.6	0	0	0.36
9	7	2	1.6	3.2	4	2.56
5	4	-2	-1.4	2.8	4	1.96
8	5	1	-0.4	-0.4	1	0.16
				6	**10**	**5.2**

It follows that $r = 6/\sqrt{(10 \times 5.2)} = 0.83$.

The correlation is positive and quite strong. It follows that income and spending are lineraly dependent.

In a sample of 402 students, 174 are registred to Political Sciences.

Find the sample proportion

Exercise 7

In a sample of 402 students, 174 are registred to Political Sciences.

$$\hat{p} = \frac{174}{402} = 0.433$$

In a sample of 402 students, 174 are registred to Political Sciences.

Determine if the propotion of students registred to Political Sciences over the whole population of students is different to 0.5 for $1 - \alpha = 0.95$

Exercise 7

In a sample of 402 students, 174 are registred to Political Sciences.

Step 1: Assumptions: The variable is quantitative, the sample size is large enough.

Step 2: Hypotheses $H_0 : p = 0.5$ vs the alternative $H_1 : p \neq 0.5$

Step 3: Test statistic
$z_{oss} = \sqrt{n} \times (\hat{p} - p_0)/\sqrt{p_0(1 - p_0)} = \sqrt{402} \times (0.433 - 0.5)/\sqrt{0.246} = 2.711$

Step 4: p-value = $P(Z > |z_{oss}|) = 2P(Z > 2.711) = 0.0068$

Step 5: Conclusion: p-value < α. Thus we do reject H_0