Asset Purchases in Noisy Financial Markets

Gaetano Gaballo
HEC Paris & CEPR

Carlo Galli
UC3M & CEPR

UCLA, 8 May 2025

Price Elasticity to Asset Purchases

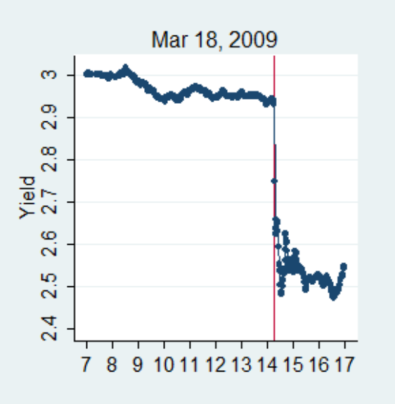


Figure: The impact of LSAP1 announcement ($300 billion of longer-term Treasury securities) on intra-day nominal yields on 10 year Treasury bonds. Krishnamurthy and Vissing-Jorgensen (2011).

Asset Purchases (APs) in Practice and in Theory

an overview

- Macro: focus on aggregates
  - debatable empirical evidence, hard identification
  - theory: future policy signalling, banks' balance sheet constraints, heterog. agents
    - break down of Wallace neutrality

Asset Purchases (APs) in Practice and in Theory

an overview

- Macro: focus on aggregates
  - debatable empirical evidence, hard identification
  - theory: future policy signalling, banks' balance sheet constraints, heterog. agents
    - break down of Wallace neutrality

- Finance: focus on financial markets
  - ample empirical evidence that APs lower yields, mostly narrow effects
  - theory: segmented markets (e.g. preferred habitat) and portfolio rebalancing

Asset Purchases (APs) in Practice and in Theory

an overview

- Macro: focus on aggregates
  - debatable empirical evidence, hard identification
  - theory: future policy signalling, banks' balance sheet constraints, heterog. agents
    - break down of Wallace neutrality

- Finance: focus on financial markets
  - ample empirical evidence that APs lower yields, mostly narrow effects
  - theory: segmented markets (e.g. preferred habitat) and portfolio rebalancing
    - $\rightarrow$ our starting point

“Narrow” Microfoundations?

"Narrow" Microfoundations?

Existing theories redux:

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**
  - expected impact on CB balance sheet: **gains**

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**
  - expected impact on CB balance sheet: **gains**

This paper: beliefs endogenous to APs

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**
  - expected impact on CB balance sheet: **gains**

This paper: beliefs endogenous to APs

- Heterogeneous beliefs on asset returns + learning from prices

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**
  - expected impact on CB balance sheet: **gains**

This paper: beliefs endogenous to APs

- Heterogeneous beliefs on asset returns + learning from prices
  - impact of LSAP on price: **non-monotone**

“Narrow” Microfoundations?

Existing theories redux:

- Heterogeneous term/risk premia (ex. preferred-habitat investors)
  - impact of LSAP on price: **monotone**
  - expected impact on CB balance sheet: **gains**
- Heterogeneous exogenous beliefs on asset returns
  - impact of LSAP: **monotone**
  - expected impact on CB balance sheet: **gains**

This paper: beliefs endogenous to APs

- Heterogeneous beliefs on asset returns + learning from prices
  - impact of LSAP on price: **non-monotone**
  - expected impact on CB balance sheet: **losses**

This Paper: some intuition

- Financial markets: rational investors with position bounds + dispersed information
  - private signals + learning from prices → asset under/over priced vs fundamentals
    (Albagli, Hellwig, Tsyvinski (2023))

This Paper: some intuition

- Financial markets: rational investors with position bounds + dispersed information
  - private signals + learning from prices $\rightarrow$ asset under/over priced vs fundamentals (Albagli, Hellwig, Tsyvinski (2023))

1. **APs** affect asset price (Q) and information it contains
   - crowding out pessimists ($\uparrow Q$) but **revealing crises** ($\downarrow Q$)
   - **non-monotonic** effects of APs on prices/yields

This Paper: some intuition

- Financial markets: rational investors with position bounds + dispersed information
  - private signals + learning from prices $\rightarrow$ asset under/over priced vs fundamentals (Albagli, Hellwig, Tsyvinski (2023))

1. **APs** affect asset price (Q) and information it contains
   - crowding out pessimists ($\uparrow Q$) but **revealing crises** ($\downarrow Q$)
   - **non-monotonic** effects of APs on prices/yields

$\hookrightarrow$ shutting down **learning-from-price externality** $\Rightarrow$ no price effect!

This Paper: some intuition

- Financial markets: rational investors with position bounds + dispersed information
  - private signals + learning from prices $\rightarrow$ asset under/over priced vs fundamentals (Albagli, Hellwig, Tsyvinski (2023))

1. **APs** affect asset price (Q) and information it contains
   - crowding out pessimists ($\uparrow Q$) but **revealing crises** ($\downarrow Q$)
   - **non-monotonic** effects of APs on prices/yields

   $\hookrightarrow$ shutting down **learning-from-price externality** $\Rightarrow$ no price effect!

2. **APs** imply redistribution between central bank, govt, investors
   - APs at market prices lead to **monetary financing** of the government

This Paper: some intuition

- Financial markets: rational investors with position bounds + dispersed information
  - private signals + learning from prices $\rightarrow$ asset under/over priced vs fundamentals (Albagli, Hellwig, Tsyvinski (2023))

1. **APs** affect asset price (Q) and information it contains
   - crowding out pessimists ($\uparrow Q$) but **revealing crises** ($\downarrow Q$)
   - **non-monotonic** effects of APs on prices/yields

   $\hookrightarrow$ shutting down **learning-from-price externality** $\Rightarrow$ no price effect!

2. **APs** imply redistribution between central bank, govt, investors
   - APs at market prices lead to **monetary financing** of the government

\+ **Optimality:** consumption-saving problem where APs undo externality from info frictions.

Literature

- Irrelevant under complete info & frictionless markets
  - Wallace (81), Backus Kehoe (89)
- Central bank replaces constrained banking sector
  - Curdia Woodford (11), Gertler Karadi (11), Chen et al. (12), Cui Sterk (21)
- Segmented markets and/or limits to arbitrage
  - Vayanos Vila (21), Costain et al. (22), Gourinchas et al. (22), Fanelli Straub (21), Itskhoki Mukhin (22)
- Commitment device
  - Mussa (81), Jeanne Svensson (07), Corsetti Dedola (16), Bhattarai et al. (22)
- Information frictions (signalling or behavioural agents)
  - Mussa (81), Iovino Sergeyev (21)

$\Rightarrow$ Dispersed info absent in existing macro theories

Outline

1. The impact of APs on prices/information/profits in financial mkts
   - quantity target
   - price target

2. Optimal APs? Dispersed info externality in consumption-saving problem.

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0, 1]$

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0,1]$
  - debt sold at market clearing price Q

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0,1]$
  - debt sold at market clearing price Q
  - stochastic repayment/default

$$\theta = \begin{cases} \theta_H = 1 & \text{with probability} \quad q \\ \theta_L \in (0,1) & \text{with probability} \quad 1-q \end{cases}$$

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0,1]$
  - debt sold at market clearing price Q
  - stochastic repayment/default

$$\theta = \begin{cases} \theta_H = 1 & \text{with probability} \quad q \\ \theta_L \in (0,1) & \text{with probability} \quad 1-q \end{cases}$$

  - govt 'profits/losses' in $(\theta, \widetilde{S})$ state: $\widetilde{S}\,(Q-\theta)$

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0,1]$
  - debt sold at market clearing price Q
  - stochastic repayment/default

$$\theta = \begin{cases} \theta_H = 1 & \text{with probability} \quad q \\ \theta_L \in (0,1) & \text{with probability} \quad 1-q \end{cases}$$

  - govt 'profits/losses' in $(\theta, \widetilde{S})$ state: $\widetilde{S}\,(Q - \theta)$
- Central bank
  - buys $b_{cb} \leq \mathrm{b}$ uncontingently, at prevailing market price Q
  - profits/losses in $(\theta, \widetilde{S} \geq \mathrm{b})$ state: $\mathrm{b}\,(\theta - Q)$

Public Sector

- Government
  - stochastic spending fully funded by debt issuance: $\widetilde{S} \sim U[0,1]$
  - debt sold at market clearing price Q
  - stochastic repayment/default

$$\theta = \begin{cases} \theta_H = 1 & \text{with probability} \quad q \\ \theta_L \in (0,1) & \text{with probability} \quad 1-q \end{cases}$$

  - govt 'profits/losses' in $(\theta, \widetilde{S})$ state: $\widetilde{S}\,(Q - \theta)$
- Central bank
  - buys $b_{cb} \leq \mathrm{b}$ uncontingently, at prevailing market price Q
  - profits/losses in $(\theta, \widetilde{S} \geq \mathrm{b})$ state: $\mathrm{b}\,(\theta - Q)$

    otherwise, $b_{cb} = \widetilde{S} < \mathrm{b}$, market is "passive", and we conservatively set $Q = \theta$

Investors

- Measure one of investors

- Portfolio allocation problem

$$\max_{b_i \in [0,1]} \quad \mathbb{E}\left[b_i(\theta - Q) \mid \Omega_i\right]$$

- Agent i's information set Ω_i

  1. Private signal: $x_i = \theta + \sigma_x \xi_i$, where $\xi_i \sim N(0,1)$ (define $x_i \sim \mathcal{N}$)
  2. Equilibrium bond price: Q
  3. Asset purchases: b_{cb}

Timing

1. Shocks $(\theta, \widetilde{S})$ realise, are not observed
2. Investors receive signals, submit *price-contingent* demand schedules
3. Walrasian auctioneer clears the market through equilibrium price Q
4. Payoffs are realised

Individual Strategies

- Agent i's strategy

$$\mathbb{E}\left[\theta - Q \,|\, x_i \sim \mathcal{N},\ Q,\ b_{cb}\right] \begin{cases} > 0 & \text{then} \quad b_i = 1 \\ < 0 & \text{then} \quad b_i = 0 \\ = 0 & \text{then} \quad b_i \in [0, 1] \end{cases}$$

Individual Strategies

- Agent i's strategy

$$\mathbb{E}\left[\theta - Q \mid x_i \sim \mathcal{N},\ Q,\ b_{cb}\right] \begin{cases} > 0 & \text{then} \quad b_i = 1 \\ < 0 & \text{then} \quad b_i = 0 \\ = 0 & \text{then} \quad b_i \in [0,1] \end{cases}$$

$\Rightarrow$ Monotone threshold strategies: investor i buys bonds iff $x_i \geq x_m$

Individual Strategies

- Agent i's strategy

$$\mathbb{E}\left[\theta - Q \,|\, x_i \sim \mathcal{N},\ Q,\ b_{cb}\right] \begin{cases} > 0 & \text{then} \quad b_i = 1 \\ < 0 & \text{then} \quad b_i = 0 \\ = 0 & \text{then} \quad b_i \in [0,1] \end{cases}$$

$\Rightarrow$ Monotone threshold strategies: investor i buys bonds iff $x_i \geq x_m$

- Discussion:
  - can extend to short-selling/leverage $b_i \in [-\underline{b}, \overline{b}]$
  - position bounds necessary, not sufficient, for non-neutrality
  - risk neutrality buys tractability, not essential

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S}$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \rightarrow \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \rightarrow \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \widetilde{S} - b_{cb} =: S \quad \text{(net supply per buyer)}$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \rightarrow \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \widetilde{S} - b_{cb} =: S \qquad \text{(net supply per buyer)}$$

- Marginal agent's private signal $=$ function of exogenous shocks $(\theta, \widetilde{S})$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - b_{cb}\right) \qquad\qquad \text{(define } x_m \sim \mathcal{M}_b\text{)}$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \rightarrow \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \widetilde{S} - b_{cb} =: S \quad \text{(net supply per buyer)}$$

- Marginal agent's private signal $=$ function of exogenous shocks $(\theta, \widetilde{S})$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - b_{cb}\right) \qquad \text{(define } x_m \sim \mathcal{M}_b\text{)}$$

- Marginal agent's indifference condition

$$Q = \mathbb{E}[\theta \mid x_m \sim \mathcal{N}, Q, b_{cb}]$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \rightarrow \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \widetilde{S} - b_{cb} =: S \quad \text{(net supply per buyer)}$$

- Marginal agent's private signal $=$ function of exogenous shocks $(\theta, \widetilde{S})$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - b_{cb}\right) \qquad \text{(define } x_m \sim \mathcal{M}_b\text{)}$$

- Marginal agent's indifference condition $\Leftrightarrow$ equilibrium price

$$Q = \mathbb{E}[\theta \mid x_m \sim \mathcal{N}, Q, b_{cb}] \qquad \Leftrightarrow \qquad Q \text{ is } x_m\text{-measurable}$$

Market Clearing and Price Signal

- Bond market clearing

$$\int_0^1 b_i \, \mathrm{d}i + b_{cb} = \widetilde{S} \qquad \to \qquad \mathrm{P}(x_i \geq x_m) + b_{cb} = \widetilde{S}$$

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \widetilde{S} - b_{cb} =: S \quad \text{(net supply per buyer)}$$

- Marginal agent's private signal $=$ function of exogenous shocks $(\theta, \widetilde{S})$

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - b_{cb}\right) \qquad\qquad \text{(define } x_m \sim \mathcal{M}_b\text{)}$$

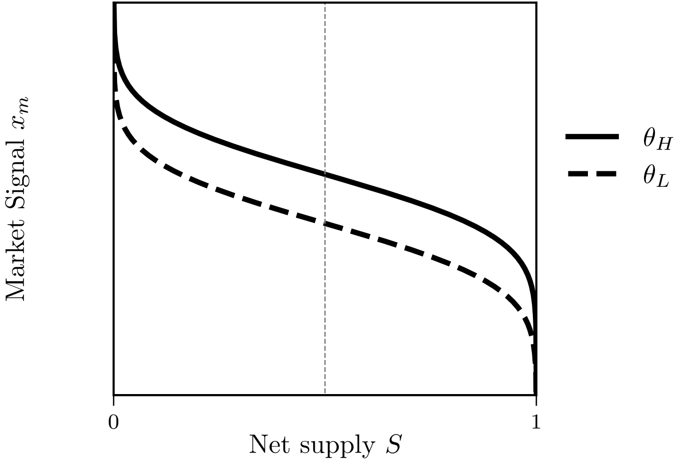
- Marginal agent's indifference condition $\Leftrightarrow$ equilibrium price

$$Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, Q, b_{cb}] \qquad \Leftrightarrow \qquad Q \text{ is } x_m\text{-measurable}$$

- $x_m(Q, \mathbf{b})$ is also the *price signal*. In equilibrium: $(\theta, \widetilde{S}) \xleftrightarrow{(b_{cb})} x_m \xleftrightarrow{(b_{cb})} Q$

Market Signal without APs (b = 0)

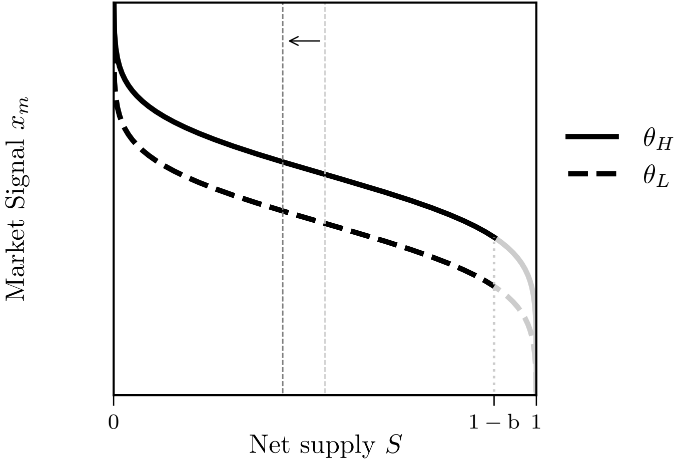
$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S}\right)$$



Market Signal with APs ($\mathrm{b} > 0$)

crowding out

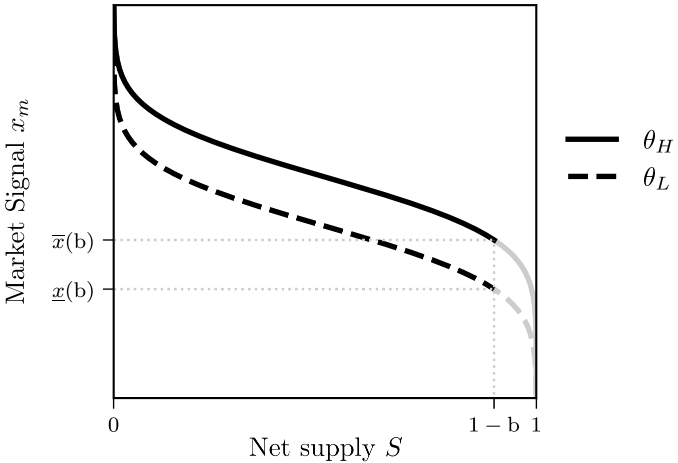
$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - \mathrm{b}\right)$$



Market Signal with APs ($\mathrm{b} > 0$)

information revelation

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\widetilde{S} - \mathrm{b}\right)$$



Posterior Beliefs and Equilibrium Price

- Repayment probability

$$p(x_i, x_m) := P(\theta_H \mid \; x_i \sim \mathcal{N} \;, x_m \sim \mathcal{M}_b) =$$

Posterior Beliefs and Equilibrium Price

- Repayment probability

$$p(x_i, x_m) := P(\theta_H \mid \; x_i \sim \mathcal{N} \;, x_m \sim \mathcal{M}_b) =$$

$$= \begin{cases} \dfrac{q\,\phi\left(\dfrac{\theta_H - (x_i + x_m)/2}{\sigma_x/\sqrt{2}}\right)}{\sum_j q_j\,\phi\left(\dfrac{\theta_j - (x_i + x_m)/2}{\sigma_x/\sqrt{2}}\right)} & \text{if } x_m \in [\overline{x}(\mathrm{b}), +\infty) \\ 0 & \text{if } x_m \in [\underline{x}(\mathrm{b}), \overline{x}(\mathrm{b})) \end{cases}$$

Posterior Beliefs and Equilibrium Price

- Repayment probability

$$p(x_i, x_m) := P(\theta_H \mid x_i \sim \mathcal{N}, x_m \sim \mathcal{M}_b) =$$

$$= \begin{cases} \dfrac{q\,\phi\left(\dfrac{\theta_H - (x_i + x_m)/2}{\sigma_x/\sqrt{2}}\right)}{\sum_j q_j\,\phi\left(\dfrac{\theta_j - (x_i + x_m)/2}{\sigma_x/\sqrt{2}}\right)} & \text{if } x_m \in [\overline{x}(\mathrm{b}), +\infty) \\ 0 & \text{if } x_m \in [\underline{x}(\mathrm{b}), \overline{x}(\mathrm{b})) \end{cases}$$

- Marginal investor m's indifference condition $\Leftrightarrow$ Equilibrium price

$$Q = \mathbb{E}[\theta \mid x_m \sim \mathcal{N}, x_m \sim \mathcal{M}_b] \quad = p(x_m)\,\theta_H + (1 - p(x_m))\,\theta_L$$

that is, $p(x_m) = p(x_i, x_m)|_{x_i = x_m}$.

'Bond Valuation' $\neq$ Equilibrium Price

- Condition only on public info: $x_m \sim \mathcal{M}_b$

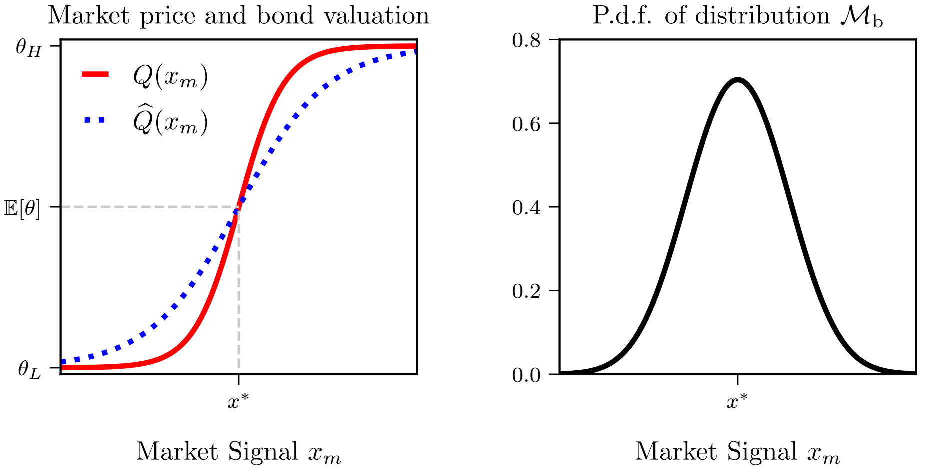
$$\widehat{p}(x_m) := P(\theta_H \,|\, x_m \sim \mathcal{M}_b) = \begin{cases} \dfrac{q\,\phi\left(\dfrac{\theta_H - x_m}{\sigma_x}\right)}{\sum_j q_j\,\phi\left(\dfrac{\theta_j - x_m}{\sigma_x}\right)} & \text{if } x_m \in [\overline{x}(\mathrm{b}), +\infty) \\ 0 & \text{if } x_m \in [\underline{x}(\mathrm{b}), \overline{x}(\mathrm{b})) \end{cases}$$

- Bond valuation

$$\widehat{Q}(x_m) = \widehat{p}(x_m)\,\theta_H + (1 - \widehat{p}(x_m))\,\theta_L$$

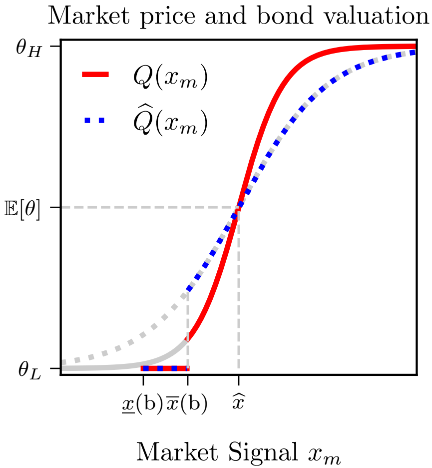
The Effect of APs

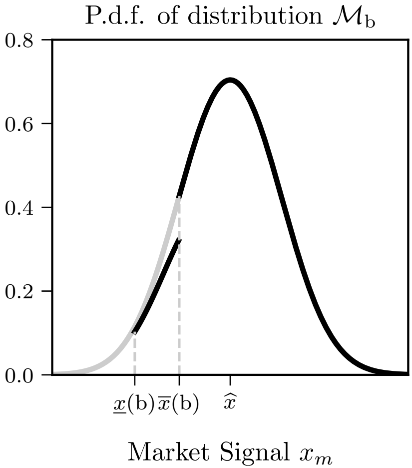
without APs ($\mathrm{b} = 0$)



The Effect of APs

with APs ($\mathrm{b} > 0$)





Average Prices and Returns

- The average bond valuation satisfies the L.I.E., its average is independent of APs

$$\mathbb{E}[\widehat{Q}] = \mathbb{E}[\,\mathbb{E}[\theta \,|\, x_m \sim \mathcal{M}_b]] = \mathbb{E}[\theta] \quad \forall\, \mathrm{b}$$

Average Prices and Returns

- The average bond valuation satisfies the L.I.E., its average is independent of APs

$$\mathbb{E}[\widehat{Q}] = \mathbb{E}[\,\mathbb{E}[\theta \,|\, x_m \sim \mathcal{M}_b]] = \mathbb{E}[\theta] \quad \forall\, \mathrm{b}$$

- The average bond price is an inverse U-shaped function of APs

$$\begin{aligned} \mathcal{Q} &= \mathbb{E}[Q(x_m)] \\ &= \mathbb{E}[\theta] + \int_{\overline{x}(\mathrm{b})} \big(Q(x_m) - \widehat{Q}(x_m)\big) \mathrm{d}\, F_{\mathcal{M}_b}(x_m) \end{aligned}$$

Interpretation and Magnitudes: LSAP1

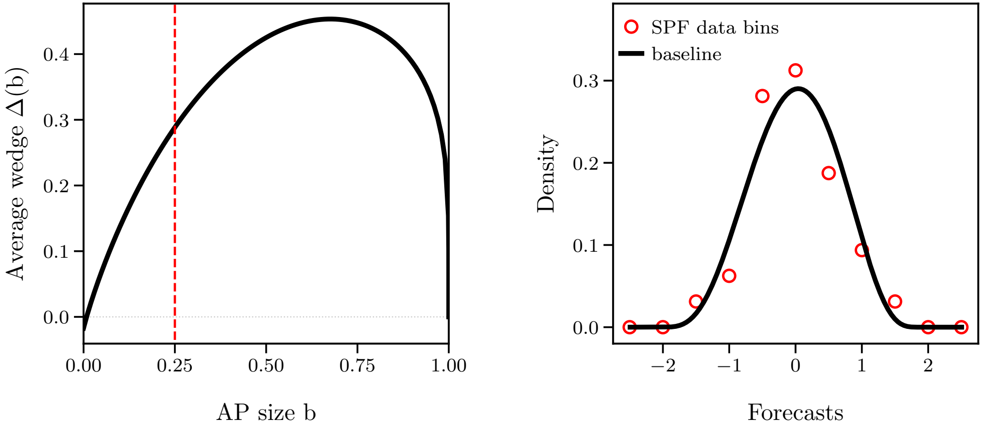


Figure: The left panel plots the average wedge as a function of the AP size; the right panel plots the probability density function of the distribution of investors' forecasts conditional on their private information x_i. Our baseline calibration is: $\theta_H - \theta_L = 5, q = 0.53, \sigma_x = 7.5$ matches the dispersion of expected real returns on 10 year US Treasuries from the Survey of Professional Forecasters in Q1-2009. The dashed line denotes the amount of treasury bonds purchases in LSAP relative to outstanding marketable stock.

APs & the Distribution of Profits

- Central bank profits

$$\mathbb{E}[\pi_{\mathsf{cb}}] = \mathrm{b}\ \left(\widehat{\mathcal{Q}} - \mathcal{Q}\right)$$

APs & the Distribution of Profits

- Central bank profits

$$\mathbb{E}[\pi_{\mathsf{cb}}] = \mathrm{b}\ \left(\widehat{\mathcal{Q}} - \mathcal{Q}\right)$$

- Investor profits

$$\mathbb{E}[\pi_{\mathsf{inv}}] = \mathbb{E}[\widetilde{S} - \mathrm{b}]\left(\widehat{\mathcal{Q}} - \mathcal{Q}\right) + \mathsf{Cov}\left[\widetilde{S} - \mathrm{b}, \left(\theta - Q(x_m)\right)\right]$$

APs & the Distribution of Profits

- Central bank profits

$$\mathbb{E}[\pi_{\mathsf{cb}}] = \mathrm{b}\ \left(\widehat{\mathcal{Q}} - \mathcal{Q}\right)$$

- Investor profits

$$\mathbb{E}[\pi_{\mathsf{inv}}] = \mathbb{E}[\widetilde{S} - \mathrm{b}]\left(\widehat{\mathcal{Q}} - \mathcal{Q}\right) + \mathsf{Cov}\left[\widetilde{S} - \mathrm{b}, \left(\theta - Q(x_m)\right)\right]$$

- Government profits

$$\mathbb{E}[\pi_{\mathsf{gov}}] = -\mathbb{E}[\pi_{\mathsf{inv}}] - \mathbb{E}[\pi_{\mathsf{cb}}]$$

APs & the Distribution of Profits

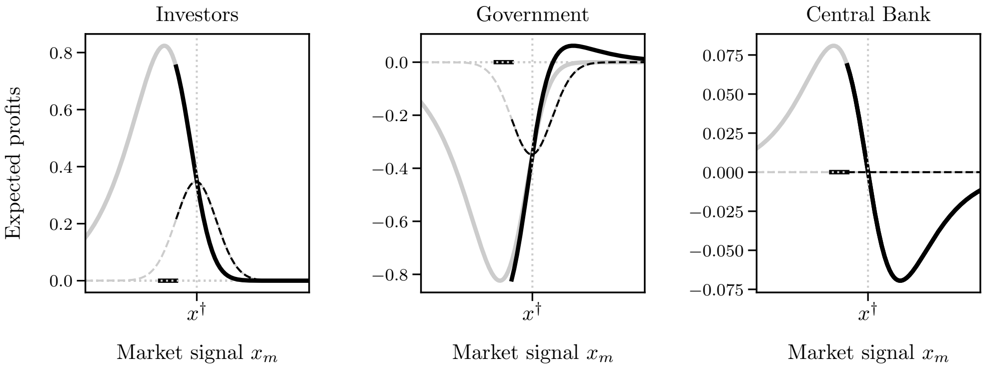


Figure: Expected gains conditional on x_m. Gray and black solid lines respectively denote the case without APs ($b = 0$) and with APs ($b = 0.15$). For the central bank, the gray line denotes gains with APs once we abstract from the crowding-out and revelation effects.

APs & the Distribution of Profits

unconditional

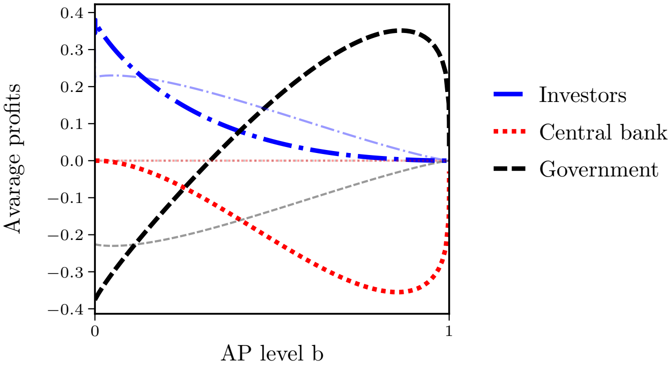


Figure: Average gains by investors, the central bank, and the government, as a function of the size of the AP program. Shaded lines represent average gains for each player in the absence of the wedge between bond prices and valuations.

Outline

1. The impact of APs on prices/information/profits in financial mkts
   - quantity target
   - price target

2. Optimal APs? Dispersed info externality in consumption-saving problem.

Price-Targeting APs

Under price-targeting APs, the central bank submits, simultaneously to investors, a limit order to buy up to a quantity $\overline{b}_n$ of bonds if the price is below a target Q_n, and nothing otherwise, that is

$$b_{cb} \begin{cases} = \bar{b}_n & \text{if} \quad Q < Q_n, \\ \in [0, \bar{b}_n] & \text{if} \quad Q = Q_n, \\ = 0 & \text{if} \quad Q > Q_n. \end{cases}$$

with $Q_n \in [\theta_L, \theta_H]$ being the announced price target.

Price-Targeting APs

- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$

Price-Targeting APs

- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$

- Targeted-price region ($Q = Q_n$)
  - CB intervenes and is unconstrained, $b_{cb} = \widetilde{S} - \Phi\left(\frac{\theta - x_n}{\sigma_x}\right) \in (0, \mathrm{b})$

Price-Targeting APs

- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$
- Targeted-price region ($Q = Q_n$)
  - CB intervenes and is unconstrained, $b_{cb} = \widetilde{S} - \Phi\left(\frac{\theta - x_n}{\sigma_x}\right) \in (0, \mathrm{b})$
  - price signal Q_n is uninformative

Price-Targeting APs

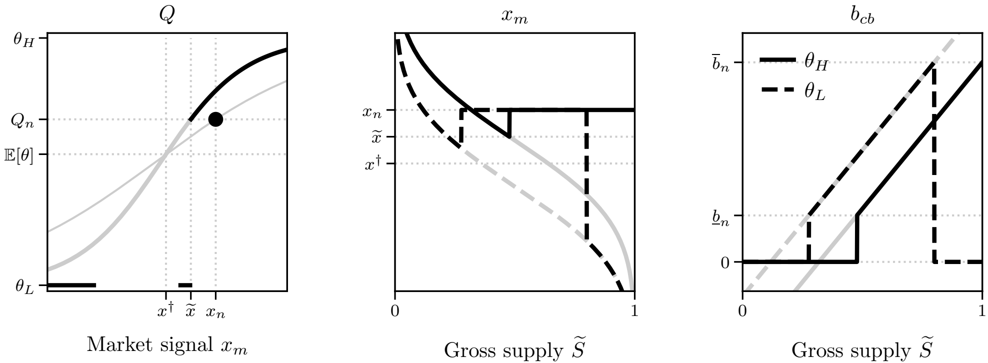
- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$
- Targeted-price region ($Q = Q_n$)
  - CB intervenes and is unconstrained, $b_{cb} = \widetilde{S} - \Phi\left(\frac{\theta - x_n}{\sigma_x}\right) \in (0, \mathrm{b})$
  - price signal Q_n is uninformative
    - CB becomes the marginal agent, Q_n inelastic to supply shocks
    - $b_{cb} \sim U$, independent from θ
    - $Q_n = \mathbb{E}[\theta \,|\, x_n \sim \mathcal{N}]$

Price-Targeting APs

- No-APs region ($Q > Q_n$)
  - CB does not intervene, $b_{cb} = 0$
  - $Q = \mathbb{E}[\theta \,|\, x_m \sim \mathcal{N}, x_m \sim \mathcal{M}]$
- Targeted-price region ($Q = Q_n$)
  - CB intervenes and is unconstrained, $b_{cb} = \widetilde{S} - \Phi\left(\frac{\theta - x_n}{\sigma_x}\right) \in (0, \mathrm{b})$
  - price signal Q_n is uninformative
    - CB becomes the marginal agent, Q_n inelastic to supply shocks
    - $b_{cb} \sim U$, independent from θ
    - $Q_n = \mathbb{E}[\theta \,|\, x_n \sim \mathcal{N}]$
- Residual region
  - $Q < Q_n$ even if $b_{cb} = \mathrm{b}$
  - fully revealing, we assume $b_{cb} = 0$

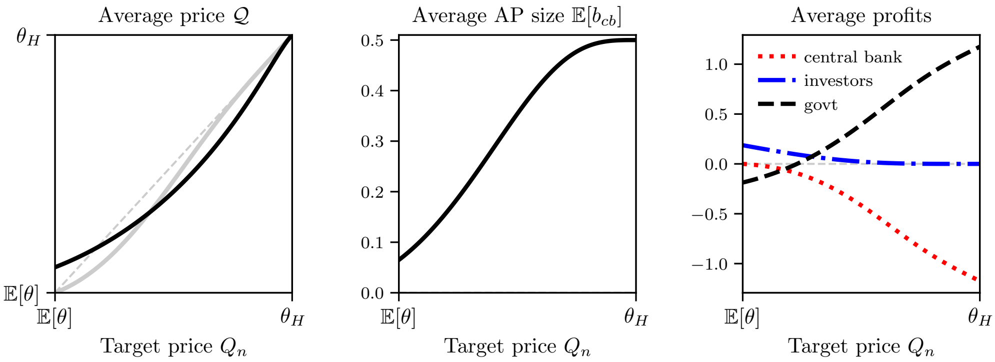
Price-Targeting APs

LSAP1 calibration



Price-Targeting APs

LSAP1 calibration



Outline

1. The impact of APs on prices/information/profits in financial mkts
   - quantity target
   - price target

2. Optimal APs? Dispersed info externality in consumption-saving problem.

A consumption-saving model with intermediaries

- Two periods, no production, agents face consumption-savings problem

A consumption-saving model with intermediaries

- Two periods, no production, agents face consumption-savings problem
- In the first period, household j solves:

$$\begin{aligned} &\max_{c_{j,0},c_{j,1},\{s_{j,i}\}_{i\in[0,1]}} \quad u(c_{j,0}) + u(c_{j,1}) \\ &\text{s.t.} \quad c_{j,0} = y - \int_0^1 s_{j,i}\,\mathrm{d}i \quad \text{and} \quad c_{j,1} = \int_0^1 \mathcal{R}_i s_{j,i}\,\mathrm{d}i + D - \tau. \end{aligned} \tag{1}$$

where:

- y is endowment,
- $s_{j,i}$ is lending of j to investor $i \in [0,1]$ at a rate $\mathcal{R}_i$
- D are dividends paid out by investors
- τ is a lump-sum tax.

A consumption-saving model with intermediaries

- Two periods, no production, agents face consumption-savings problem
- In the first period, household j solves:

$$\begin{aligned} &\max_{c_{j,0},c_{j,1},\{s_{j,i}\}_{i\in[0,1]}} \quad u(c_{j,0}) + u(c_{j,1}) \\ &\text{s.t.} \quad c_{j,0} = y - \int_0^1 s_{j,i}\,\mathrm{d}i \quad \text{and} \quad c_{j,1} = \int_0^1 \mathcal{R}_i s_{j,i}\,\mathrm{d}i + D - \tau. \end{aligned} \tag{1}$$

  where:
  - y is endowment,
  - $s_{j,i}$ is lending of j to investor $i \in [0,1]$ at a rate $\mathcal{R}_i$
  - D are dividends paid out by investors
  - τ is a lump-sum tax.
- Contrats are signed before any shock realize: $s_{j,i} = s$ and $\mathcal{R}_i = \mathcal{R}$.

Govenrment and CB

Govenrment and CB

- Government must consume total $\mathbf{G}$ in two periods:

$$t = 0 : g_0 = \tilde{S}Q \qquad\qquad t = 1 : \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

Govenrment and CB

- Government must consume total **G** in two periods:

$$t = 0 : g_0 = \tilde{S}Q \qquad\qquad t = 1 : \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

- Central Bank:

$$t = 0 : a_{cb} = b_{cb} + k_{cb} \qquad t = 1 : \theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$$

Govenrment and CB

- Government must consume total **G** in two periods:

$$t = 0 : g_0 = \tilde{S}Q \qquad\qquad t = 1 : \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

- Central Bank:

$$t = 0 : a_{cb} = b_{cb} + k_{cb} \qquad t = 1 : \theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$$

- Present value budget constraint:

$$\text{gov: } \tau = \tau_{cb} + \mathbf{G} + \underbrace{\tilde{S}(\theta - Q)}_{\pi_{gov}} \qquad \text{CB: } -\tau_{cb} = \underbrace{b_{cb}(\theta - Q)}_{\pi_{cb}}$$

Govenrment and CB

- Government must consume total **G** in two periods:

$$t = 0 : g_0 = \tilde{S}Q \qquad\qquad t = 1 : \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

- Central Bank:

$$t = 0 : a_{cb} = b_{cb} + k_{cb} \qquad t = 1 : \theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$$

- Present value budget constraint:

$$\text{gov: } \tau = \tau_{cb} + \mathbf{G} + \underbrace{\tilde{S}(\theta - Q)}_{\pi_{gov}} \qquad \text{CB: } -\tau_{cb} = \underbrace{b_{cb}(\theta - Q)}_{\pi_{cb}}$$

- Consolidated budget constraint: $\tau - \mathbf{G} = (\tilde{S} - b_{cb})(\theta - Q)$

Govenrment and CB

- Government must consume total $\mathbf{G}$ in two periods:

$$t = 0 : g_0 = \tilde{S}Q \qquad t = 1 : \mathbf{G} - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$$

- Central Bank:

$$t = 0 : a_{cb} = b_{cb} + k_{cb} \qquad t = 1 : \theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$$

- Present value budget constraint:

$$\text{gov: } \tau = \tau_{cb} + \mathbf{G} + \underbrace{\tilde{S}(\theta - Q)}_{\pi_{gov}} \qquad \text{CB: } -\tau_{cb} = \underbrace{b_{cb}(\theta - Q)}_{\pi_{cb}}$$

- Consolidated budget constraint: $\tau - \mathbf{G} = (\tilde{S} - b_{cb})(\theta - Q)$
  - with $Q = \theta$ the gov run a balanced budget, debt only serves for time mismatch!

Investors and Market Clearing

Investors and Market Clearing

- Investors maximize expected dividends:

$$\max_{s_i,\, b_i \in [0,1], d_i} \quad \mathbb{E}[d_i \,|\, \Omega_i]$$
$$\text{s.t.} \quad d_i = \underbrace{b_i(\theta - Q)}_{\pi_{cb}} - s_i(\mathcal{R} - 1)$$

- *Ex ante* zero-profit condition gives:

$$\mathcal{R} = 1 + \frac{1}{s}\mathbb{E}\left[\pi_{\text{inv}}\right],$$

where $\mathbb{E}\left[\pi_{\text{inv}}\right]$ denotes investors' average gains in the bond market as before.

Efficiency

Efficiency

- Euler Equation:

$$u'(c_0) = \mathcal{R}u'(c_1)$$

Efficiency

- Euler Equation:
$$u'(y - s) = \mathcal{R}\, u'(s - G).$$
  - after using market clearing and identities.

Efficiency

- Euler Equation:
$$u'(y-s) = \mathcal{R}\, u'(s-G).$$
  - after using market clearing and identities.

- Planner problem optimality:
$$u'(y-s) = u'(s-G).$$

Efficiency

- Euler Equation:
$$u'(y - s) = \mathcal{R}\, u'(s - G).$$
  - after using market clearing and identities.

- Planner problem optimality:
$$u'(y - s) = u'(s - G).$$

Proposition

Welfare is increasing in the central bank quantity-target b_{cb} or price-target Q_n insofar as

$$\mathcal{R} > 1 \quad \Leftrightarrow \quad \mathbb{E}\left[\pi_{inv}\right] > 0.$$

Conclusions

Conclusions

- A theory of APs with
  - dispersed info & learning from prices
  - limits to arbitrage
- Illustrate effects of (quantity/price-targeting) APs on
  - prices, and information contained therein
  - redistribution between govt, central bank and investors
- Optimality in a stylised consumption-saving model with intermediaries
  - heterogeneous beliefs creates inefficiency in saving choices
  - that APs can optimally handle because of the learning-from-prices externality