Asset Purchases and Default-Inflation Risks in Noisy Financial Markets

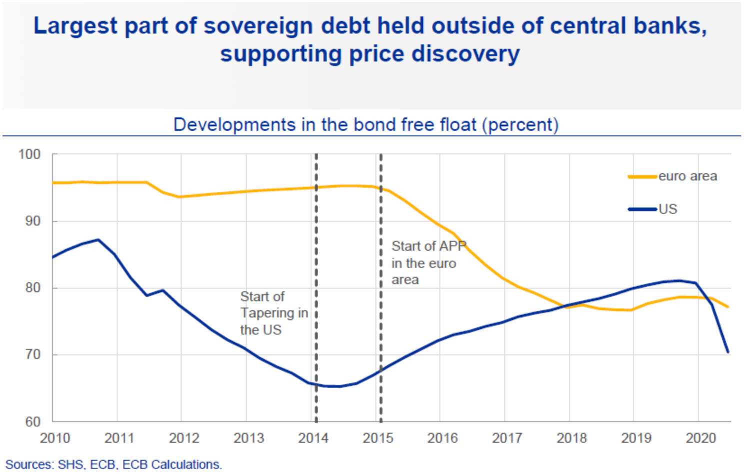
Gaetano Gaballo
HEC Paris and CEPR

Carlo Galli
UC3M

Expectations in Dynamic Macroeconomic Models

BSE Summer Forum, June 20th, 2023

Motivation



"The shadow of fiscal dominance: misconceptions, perceptions and perspectives"
Isabel Schnabel, September 11th 2020

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- APs and monetary-fiscal interactions in General Equilibrium
- Imperfect financial markets generate inefficiently high returns
- APs work through a **dispersed info channel** (w/ learning from prices)

Our story

- Heterogeneous agents:
  - **Investors** save either in nominal defaultable **bonds** or a safe asset
  - **Savers** save in **money**

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- CB can do better! Price-targeting APs lower bond returns and are inflation-neutral.

Outline

- OLG Model
  - Financial Market

- Equilibrium & Welfare in Monetary Dominance
  - without APs
  - with non-contingent APs

- Equilibrium & Welfare in Fiscal Dominance
  - with non-contingent APs
  - with price-targeting APs

- Final Discussion

Model

Government Bonds

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- $R_t\,\theta_t$ is the **ex-post nominal return** on bonds

Government and Central Bank

- Gov't budget

$$\widetilde{S}_t + \tau_t + \frac{R_{t-1}\theta_{t-1}}{\Pi_t} \frac{B_{t-1}}{P_{t-1}} = \frac{B_t}{P_t} + 2\, T_{o,t},$$

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$$\frac{R_{t-1}\theta_{t-1}}{\Pi_t}\frac{B_{cb,t-1}}{P_{t-1}} - \frac{B_{cb,t}}{P_t} + \tau_t + \frac{M_t}{P_t} = \frac{1}{\Pi_t}\frac{M_{t-1}}{P_{t-1}}$$

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Households: Savers

Agent $s \in [0,1]$, born at time t, has utility:

$$U_{s,t} = \frac{C_{s,y,t}^{1-\sigma}}{1-\sigma} + C_{s,o,t+1}$$

and budget constraints:

$$\begin{aligned} \text{young:} \quad & C_{s,y,t} && = w - \bar{b}_{s,t} \\ \text{old:} \quad & C_{s,o,t+1} && = \Pi_t^{-1} \bar{b}_{s,t} - T_{o,t+1} \end{aligned}$$

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Savings $\bar{b}_{s,t}$ chosen before any shock happens

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- Investor i enters the market with funds $\overline{b}_{i,t}$, cannot sell short ($\underline{b} = 0$)
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- Normalise $\rho = 1$

Welfare

- Welfare is the ex-ante utility of agents

$$W := \mathbb{E}\left[\frac{(C_{i,y,t})^{1-\sigma}}{1-\sigma} + \frac{(C_{s,y,t})^{1-\sigma}}{1-\sigma} + \underbrace{\overline{b}_{i,t} + \overline{b}_{s,t} - \widetilde{S}_t}_{=C_{i,o,t+1}+C_{s,o,t+1}}\right]$$

$\Rightarrow$ plugging in lump-sum taxes, the return on savings is always $= 1$!

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- **Social Optimum:** $E[Q(b_{i,t})]=1$ & $E\left[\frac{1}{\Pi_{t+1}}\right]=1$
  - Can APs lower financial returns without increasing inflation?

Equilibrium & Welfare

with Monetary Dominance

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- Monotone threshold strategies: investor i buys bonds iff $x_i \geq x_m$

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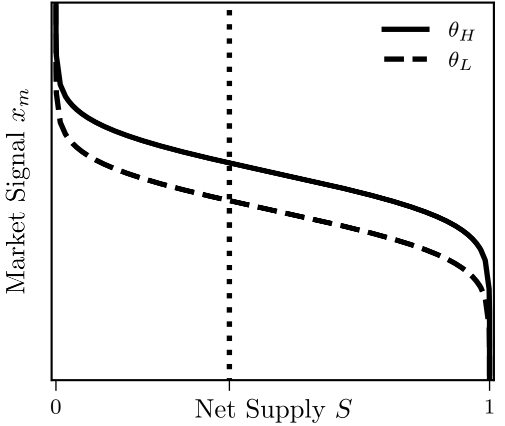
- Solving for the **equilibrium** cutoff signal

$$x_m(R, b_{cb}) = \theta - \sigma_x \Phi^{-1}\left(\frac{\widetilde{S} - b_{cb}}{\bar{b}}\right)$$

marginal agent's private signal $\Leftrightarrow$ price signal = exogenous fn of shocks $(\theta, \widetilde{S})$

Market Signal: no APs, unit bounds

$$x_m = \theta - \sigma_x \Phi^{-1}\left(\frac{\widetilde{S} - b_{cb}}{\bar{b}}\right)$$



Market Signal: no APs, $\bar{b} > 1$

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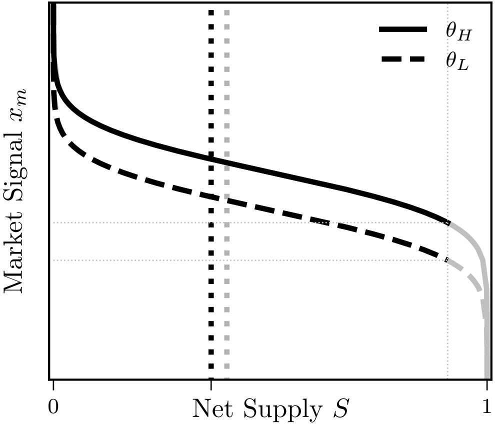

Market Signal: Information Revelation

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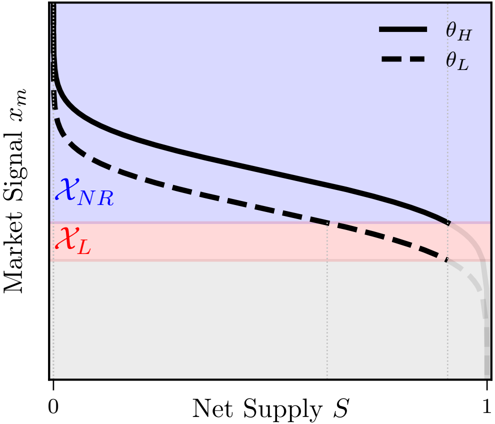

Non-Contingent AP Policy

- "Non-contingent" (on θ)

$$b_{cb}(\widetilde{S}) = \begin{cases} \bar{b}_{cb} & \text{if} \quad \widetilde{S} \geq \bar{b}_{cb} \\ \widetilde{S} & \text{if} \quad \widetilde{S} < \bar{b}_{cb} \end{cases}$$

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$\Rightarrow$ with probability $P_0 := \overline{b}_{cb}$

- the market is *passive*
- CB buys at $R = \frac{1}{\theta_H}$

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$\Rightarrow$ with probability $P_0 := \bar{b}_{cb}$

- the market is *passive*
- CB buys at $R = \frac{1}{\theta_H}$

- APs $\approx$ as if investors could individually buy more

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \frac{\widetilde{S} - b_{cb}}{\bar{b}}$$

Market Signal: Non-Contingent APs

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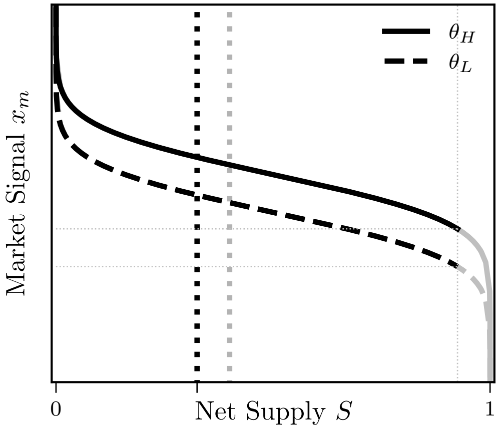

Market Signal: Non-Contingent APs, $\bar{b} > 1$

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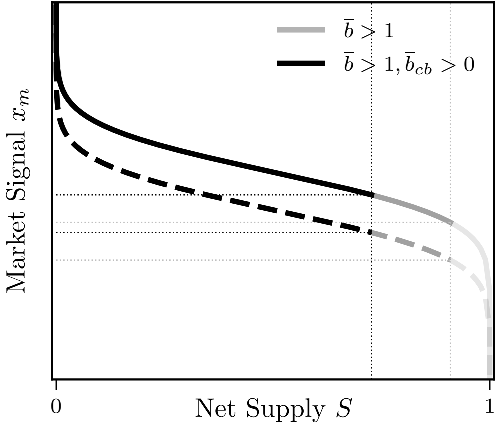

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$$f(x_m \mid \theta) = \begin{cases} \dfrac{1}{(S_{\max} - S_{\min})} \dfrac{1}{\sigma_x} \phi\left(\dfrac{x_m - \theta}{\sigma_x}\right) & \text{for } x_m \in \text{Supp}(x_m \mid \theta) \\ 0 & \text{otherwise} \end{cases}$$

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- Now focus on states where the market is *active*

Posterior Beliefs

- Observing $R \Leftrightarrow$ observing x_m

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$$\text{Prob}(\theta_H \,|\, x_i,\, x_m,\, b_{cb}) = \begin{cases} \dfrac{q\, f(x_i, x_m|\theta_H)}{q\, f(x_i, x_m|\theta_H) + (1-q)\, f(x_i, x_m|\theta_L)} & \text{if} \quad x_m \in \mathcal{X}_{NR}, \\ 0 & \text{if} \quad x_m \in \mathcal{X}_L \end{cases}$$

where

$$f(x_i, x_m \mid \theta) = \phi\left(\frac{\theta - \frac{x_i + x_m}{2}}{\sigma_x/\sqrt{2}}\right)$$

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$$f(x_i, x_m \mid \theta) = \phi\left(\frac{\theta - \frac{x_i + x_m}{2}}{\sigma_x/\sqrt{2}}\right)$$

- An external observer w/out private information (public) instead uses

$$\text{Prob}(\theta_H \,|\, x_m,\, b_{cb}) \qquad \text{and} \qquad f(x_m \mid \theta) = \phi\left(\frac{\theta - x_m}{\sigma_x}\right)$$

Equilibrium Prices and Market vs Public Beliefs

- Expected payoff for the marginal agent (market)

$$\mathbb{E}[\theta \mid x_i, x_m, b_{cb}]_{\boldsymbol{x_i = x_m}}$$

Equilibrium Prices and Market vs Public Beliefs

- Expected payoff for the marginal agent (market) pins down equilibrium $R(x_m, b_{cb})$

$$R\,\mathbb{E}[\theta \mid x_i, x_m, b_{cb}]_{\boldsymbol{x_i = x_m}} = 1$$

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- Market **weights the price signal** information more than the public
  - For large $\boldsymbol{x_m}$ the market **over**-values the asset $\Rightarrow$ $\mathbb{E}[R\,\theta] < 1$
  - For small $\boldsymbol{x_m}$ the market **under**-values the asset $\Rightarrow$ $\mathbb{E}[R\,\theta] > 1$

(Albagli, Hellwig, Tsyvinski (2023))

Individual Profits

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\begin{aligned}
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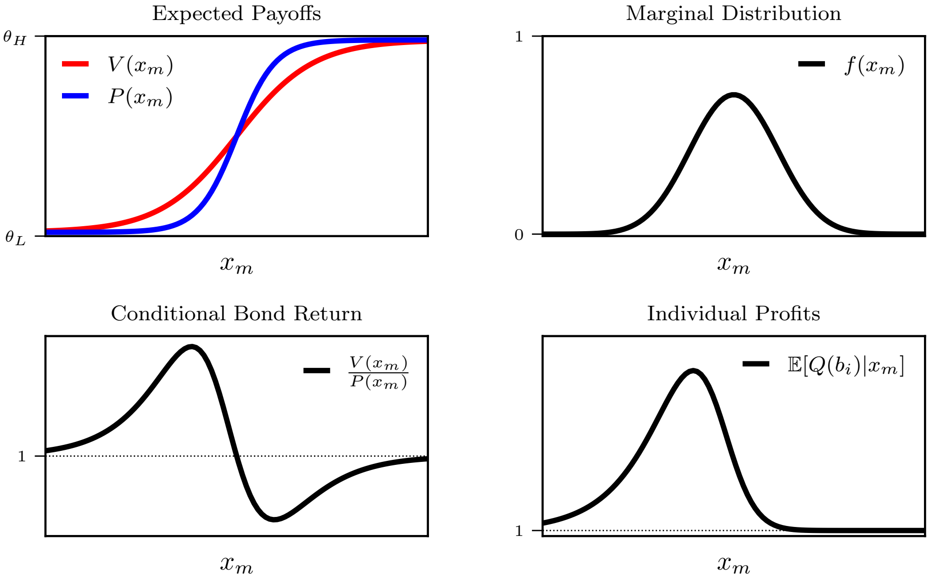
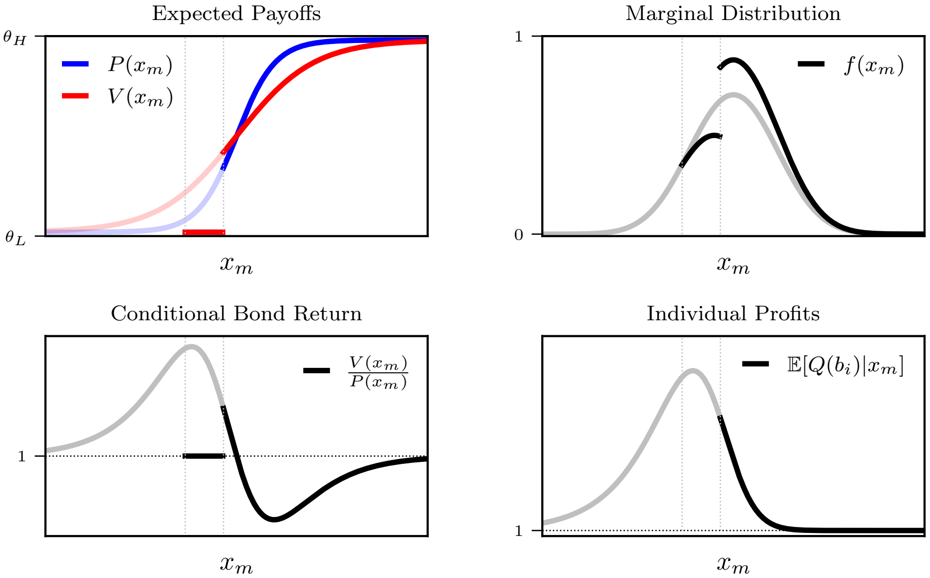
- Expected individual payoff (before receiving x_i), unconditional

$$\bar{b}\,\mathbb{E}\left[Q(b_i)\right] = \bar{b}\int_{\mathbb{R}} \mathbb{E}\left[Q(b_i)\,|\,x_m\right]\,\mathrm{d}F(x_m)$$

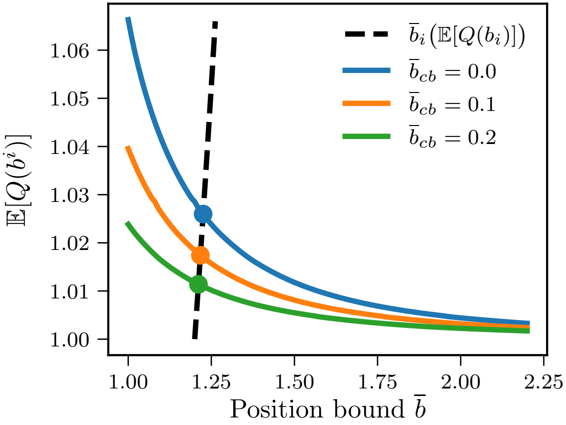
Equilibrium: no APs



Equilibrium: APs ($\bar{b}_{cb} > 0$)

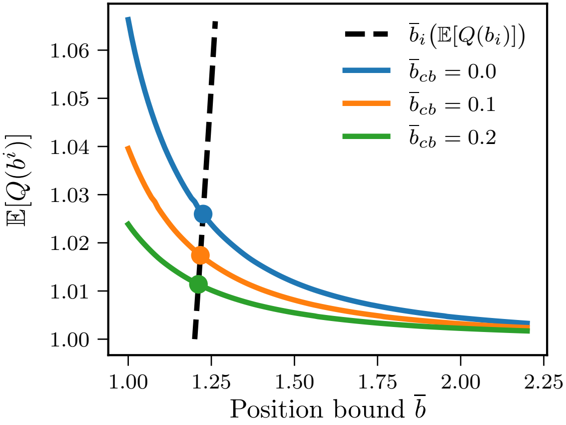


Welfare



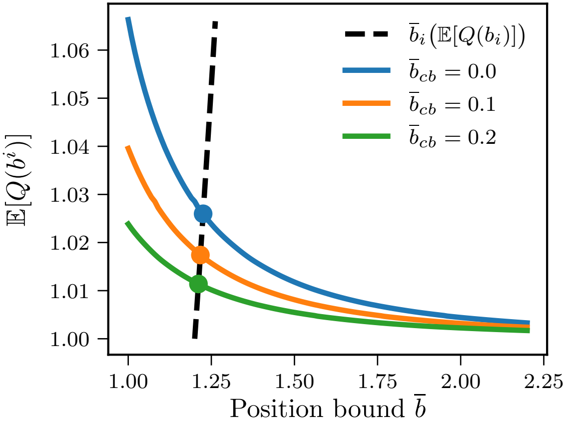
$\overline{b}_i\left(\mathbb{E}[Q(b_i)]\right)$
$\overline{b}_{cb} = 0.0$
$\overline{b}_{cb} = 0.1$
$\overline{b}_{cb} = 0.2$
1.06
1.05
1.04
1.03
1.02
1.01
1.00
$\mathbb{E}[Q(b^i)]$
1.00
1.25
1.50
1.75
2.00
2.25
Position bound $\overline{b}$

Welfare



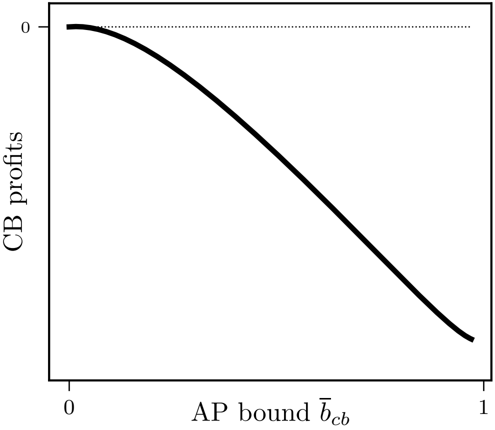
- market solution $\neq$ first best
- limits to arbitrage $\Rightarrow$ indiv. incentive to participate in bond market
- investors do not internalise effect of aggregate demand on R, T_o

Welfare



- market solution $\neq$ first best
- limits to arbitrage $\Rightarrow$ indiv. incentive to participate in bond market
- investors do not internalise effect of aggregate demand on R, T_o
- APs $\downarrow$ expected individual profits $\Rightarrow$ consumption and welfare increase

What About Central Bank Profits?



- When supply is small, very costly for CB to buy all of it

▸ algebra

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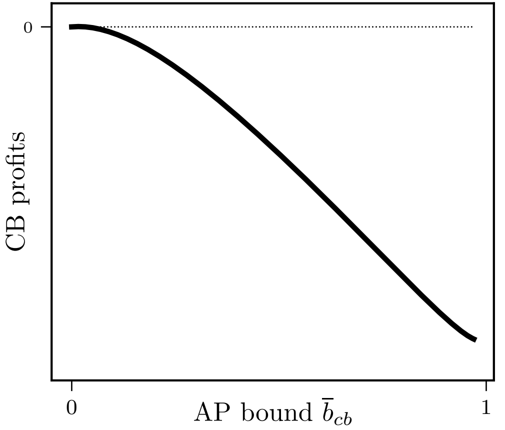


- When supply is small, very costly for CB to buy all of it

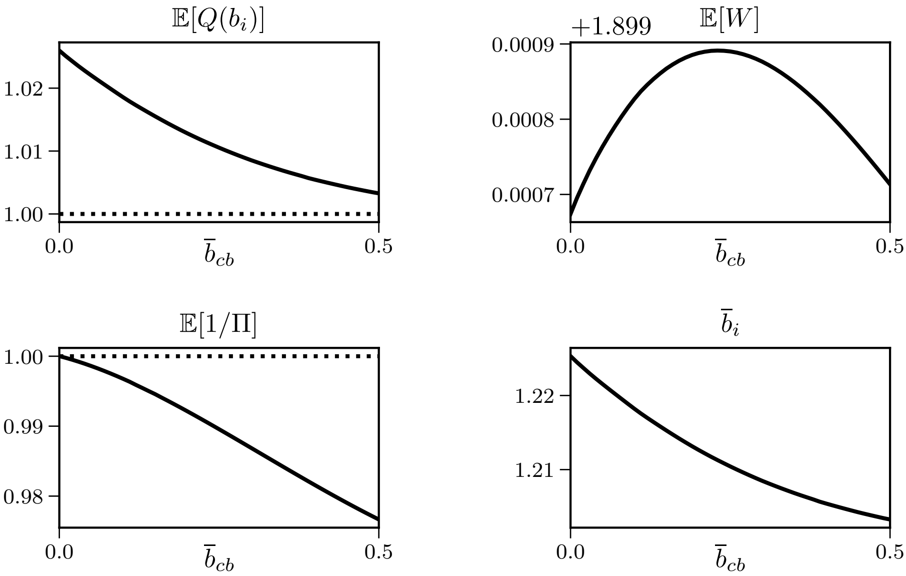
  ⇒ **with Fiscal Dominance, CB losses are a problem**

▸ algebra
▸ welfare

Equilibrium & Welfare

with Fiscal Dominance

Non-Contingent APs with Fiscal Dominance



Price-Targeting AP Policy

- CB buys $b_{cb} \in [0, \bar{b}_{cb}]$ to target a marginal agent x^* $\Leftrightarrow$ price R^*

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- Price-targeting policies are 'non-informative' if

$$\mathbb{E}[\theta | x_i = x^*, x_m = x^*, b_{cb}] = \mathbb{E}[\theta | x_i = x^*]$$

Beliefs- & Balance Sheet-Neutrality

- Choose target x^* such that, without APs,

$$\mathbb{E}[\theta|x_i = x^*, x_m = x^*] = \mathbb{E}[\theta|x_i = x^*]$$

Beliefs- & Balance Sheet-Neutrality

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graph

$$\mathbb{E}[\theta|x_i = x^*, x_m = x^*] = \mathbb{E}[\theta|x_i = x^*] = \mathbb{E}[\theta] =: \frac{1}{R^*}$$

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$$\mathbb{E}[R^* \, \theta] = \frac{\mathbb{E}[\theta]}{\mathbb{E}[\theta]} = 1$$

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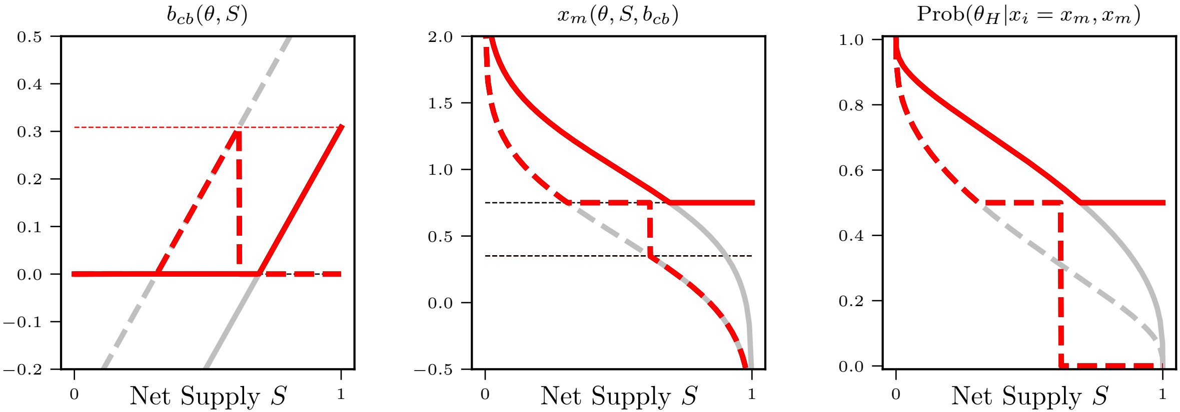
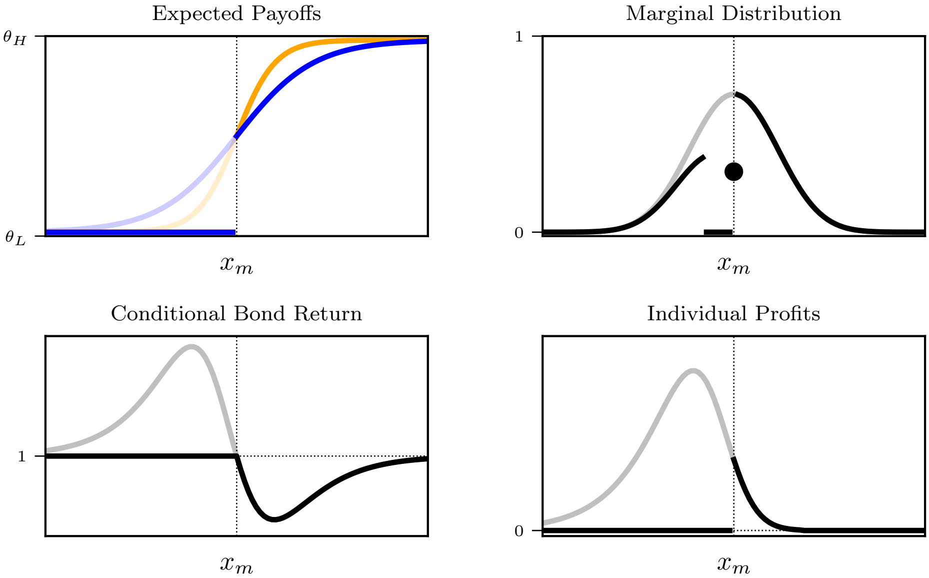
$\Rightarrow \ \mathbb{E}[\Pi] = 1$

Price-Targeting APs



Equilibrium: Price-Targeting APs



Conclusion

Takeaways

- A GE theory of APs with
  - dispersed info & learning from prices
  - market segmentation (Investors & Savers)
  - limits to arbitrage
- With common/perfect information: agent heterogeneity irrelevant, APs are neutral
- With dispersed information
  - Investors save too much
  - APs effective in reducing inefficiency
- Fiscal-monetary regimes
  - Monetary dominance: non-contingent APs work, but create CB losses
  - Fiscal dominance: inflation cost of APs via CB losses & Savers
- Price-targeting APs $\uparrow$ welfare, are beliefs- & inflation-neutral

Neutrality

- Consider problem of agent $i \in (0,1)$

$$\max_{c_i, b_i} \mathbb{E}[u(c_i)|\Omega_i] \quad \text{s.t.} \quad c_i = b_i R\,\theta + (1-b_i)1 + \tau$$

- Asset market clearing: $\int b_i \mathrm{d}i + b_{cb} = S$
- Profits of AP authority: $\tau = b_{cb}(R\theta - 1)$

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(a) Limits to arbitrage ($b_i \in [\underline{b}, \overline{b}]$) + No info frictions ($\Omega_i = \Omega$)

- RA market clearing, $c_i = c$, all agents on EE $\quad \rightarrow \quad \mathbb{E}[u'(c)(R\,\theta - 1) \mid \Omega] = 0$

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   - RA market clearing, $c_i = c$, all agents on EE $\quad \rightarrow \quad \mathbb{E}[u'(c)(R\,\theta - 1) \mid \Omega] = 0$

(b) No limits to arbitrage + Info frictions
   - Each i on own EE, interior solution for each i $\quad \rightarrow \quad \mathbb{E}[u'(c_i)(R\,\theta - 1) \mid \Omega_i] = 0$

Neutrality

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- Profits of AP authority: $\tau = b_{cb}(R\theta - 1)$

$\Rightarrow$ Problem becomes

$$\max_{c_i, b_i} \mathbb{E}[u(c_i)|\Omega_i] \quad \text{s.t.} \quad c_i = (b_i + b_{cb})R\,\theta + (1 - b_i - b_{cb})$$

(a) Limits to arbitrage ($b_i \in [\underline{b}, \overline{b}]$) + No info frictions ($\Omega_i = \Omega$)
   - RA market clearing, $c_i = c$, all agents on EE $\quad\rightarrow\quad \mathbb{E}[u'(c)(R\,\theta - 1) \mid \Omega] = 0$

(b) No limits to arbitrage + Info frictions
   - Each i on own EE, interior solution for each i $\quad\rightarrow\quad \mathbb{E}[u'(c_i)(R\,\theta - 1) \mid \Omega_i] = 0$

$\Rightarrow$ **Homogeneous crowding out, APs irrelevant**

intro

AP Policies

- "Non-contingent" (on θ)

$$b_{cb}(\widetilde{S}) = \begin{cases} \overline{b}_{cb} & \text{if} \quad \widetilde{S} + \underline{b} \geq \overline{b}_{cb} \\ \widetilde{S} + \underline{b} & \text{if} \quad \widetilde{S} + \underline{b} < \overline{b}_{cb} \end{cases}$$

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  - CB buys at $R = \frac{1}{\theta_H}$

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- Price-targeting (*later*)

$$b_{cb}(\theta, \widetilde{S}, x^*) = \begin{cases} \widetilde{S} - \overline{b}\,\Phi\left(\dfrac{\theta - x^*}{\sigma_x}\right) & \text{if} \quad \widetilde{S} \in \widetilde{\mathcal{S}}(\theta, x^*) \\ 0 & \text{otherwise} \end{cases}$$

Central Bank Profits

$$\mathbb{E}[\Pi_{cb} - 1] = \int_0^{\bar{b}_{cb}} \widetilde{S} \left(\mathbb{E}[\theta] \frac{1}{\theta_H} - 1 \right) \mathrm{d}\widetilde{S} + \int_{\mathcal{X}_{\mathrm{NR}}} \bar{b}_{cb} \left(\mathbb{E}[R\,\theta \mid x_m] - 1 \right) \mathrm{d}F(x_m)$$