The Aggregation Dilemma: How Best to Restructure Sovereign Bonds

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Sciences Po

Rethinking sovereign debt sustainability and crises
EUI, December 10th, 2024

Introduction

- Collective Action Clauses (CACs) key pillar of sovereign debt architecture

- In a bond restructuring, the sovereign makes an offer to bondholders
  - CACs allow qualified majority of consenting creditors to bind dissenting minority
    - $\Rightarrow$ alleviate ‘holdout’ problem in sovereign debt workouts

- Adoption
  - in NY/UK-law EM sovereign bonds since 2003
  - in euro area domestic law bonds since 2013

- Since 2015, new issues of EM sovereign bonds incorporate ‘enhanced’ version of CACs

Voting Rules under Enhanced CACs

When restructuring multiple bond series, sovereign can choose among two voting rules:

- **Two-limb aggregation:** restructuring binds all creditors if
  - approved by $\geq 2/3$ of face value *across* all series; and
  - approved by $\geq 1/2$ of face value *within* each series
- **Single-limb aggregation:** restructuring binds all creditors if
  - approved by $\geq 3/4$ of face value *across* all series; and
  - offer satisfies 'uniform applicability condition'

$\rightarrow$ no threshold *within* series

Motivation

- Single-limb aggregation introduced in 2015, viewed as key innovation
  - two-limb CACs adopted since Uruguay 2003
  - single-limb method used effectively (retroactively) in Greek 2012 PSI
  - no *within*-series thresholds $\rightarrow$ more robust defence against holdouts
- Enhanced CACs first tested in August 2020 by Argentina and Ecuador
  - both opted for two-limb aggregation, making $\neq$ offers to $\neq$ bond series Ecu Arg
- Meanwhile in euro area: ongoing project to replace two-limb with single-limb
- Goal: *"Bring theory as close as possible to policymarkers' questions"*

This Paper

- Part 1: Formulate problem & solve for government's optimal restructuring method
  - key ingredients: multiple bonds + creditor heterogeneity within & across bonds

- Part 2: Show how CACs design & expected use affect
  a) potential entry of large player with blocking capacity (vulture)
  b) equilibrium model of endogenous investor sorting into bonds

Related Literature

- Eichengreen and Portes (1995)
  "Loan contracts and bond covenants should specify that a majority of creditors be entitled to alter the terms of the debt agreement [...]"

- Theoretical: single bond restructurings
  - Haldane et al. (2005); Engelen and Lambsdorff (2009); Pitchford and Wright (2012, 2017); Bi, Chamon and Zettelmeyer (2016)

- Empirical: bond-level restructuring outcomes
  - Fang, Schumacher and Trebesch (2021); Asonuma, Niepelt and Ranciere (2023)

- Empirical: effects of CACs on bond prices
  - Becker et al. (2003); Eichengreen and Mody (2004); Carletti et al. (2016, 2021); Picarelli et al. (2019); Chung and Papaioannou (2020)

Outline

- Part 1: Optimal restructuring strategy, *given* creditor heterogeneity within & across bonds
- Part 2.a: CACs design and vulture funds
- Part 2.b: CACs design and endogenous investor sorting
- Conclusion

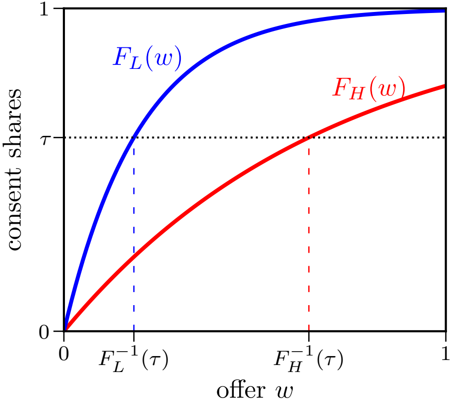
A Simple Two-Bond Model

- Two bonds to be restructured: bonds H and L, relative size λ and $1 - \lambda$
- Restructuring offer by sovereign $=$ recovery rates (w_H, w_L) per unit of face value
- Each bond is held by continuum of atomistic investors
  - holders of bond i have reservation values distributed according to cdf F_i
  - bondholder accepts if offer $>$ individual reservation value
  - given restructuring offer w_i, consent share within bond i is $F_i(w_i)$
- Heterogeneity
  - within bond: investors differ wrt discount rates, litigation skills, etc.
  - across bonds: different bond payment terms and bondholder bases
  - govt learns F_i's during preliminary talks

full problem

Heterogeneity Across Bonds

Assume holders of bond H have higher reservation values:



Government Problem

- The government wants to minimize the restructuring payout

$$\min_{w_H, w_L} \lambda\, w_H + (1-\lambda) w_L$$

under constraints that depend on the voting rule

  - Two-limb

$$\begin{aligned} \lambda F_H(w_H) + (1-\lambda) F_L(w_L) &\geq = \tau_2^{\mathrm{a}} && \text{(aggregate)} \\ F_i(w_i) &\geq \tau_2^{\mathrm{s}} \quad \text{for } i \in \{H, L\} && \text{(series-by-series)} \end{aligned}$$

  - Single-limb

$$\begin{aligned} \lambda F_H(u) + (1-\lambda) F_L(u) &\geq = \tau_1 && \text{(aggregate)} \\ w_H = w_L &= u && \text{(uniform applicability)} \end{aligned}$$

- Assume $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}} \leq \tau_1 \quad \Rightarrow \quad$ aggregate constraint is always binding

Single-Limb Aggregation

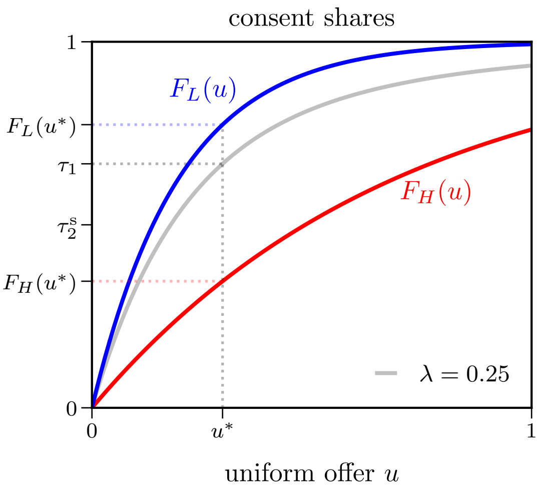
- Optimal uniform offer $u^\star$ is such that

$$\lambda F_H(u^*) + (1-\lambda) F_L(u^*) = \tau_1$$

- Remark

$$F_H(u^*) < \tau_1 < F_L(u^*)$$

- Low λ: $F_H(u^*) < \tau_2^{\mathrm{s}}$



Single-Limb Aggregation

- Optimal uniform offer $u^\star$ is such that

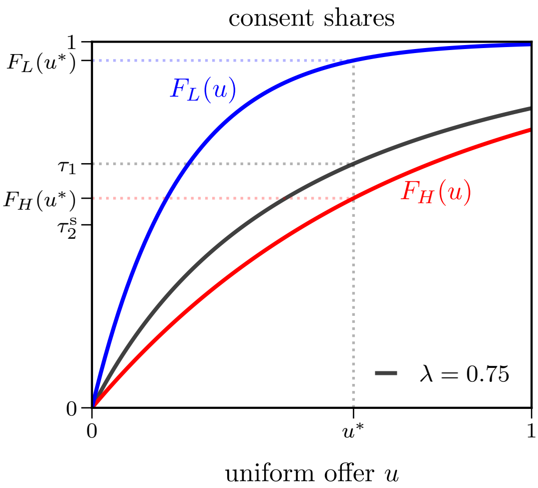
$$\lambda F_H(u^*) + (1-\lambda) F_L(u^*) = \tau_1$$

- Remark

$$F_H(u^*) < \tau_1 < F_L(u^*)$$

- Low λ: $F_H(u^*) < \tau_2^{\mathrm{s}}$

- High λ: $F_H(u^*) > \tau_2^{\mathrm{s}}$



Two-Limb Aggregation

- Consider auxiliary problem <u>*without*</u> series-by-series constraints

$$\min_{w_H, w_L} \lambda\, w_H + (1-\lambda) w_L \qquad \text{s.t.} \qquad \lambda F_H(w_H) + (1-\lambda) F_L(w_L) = \tau_2^{\mathrm{a}}$$

- Auxiliary solution $(\widehat{w}_H, \widehat{w}_L)$ pinned down (assuming convex problem) by

$$f_H(\widehat{w}_H) = f_L(\widehat{w}_L) \tag{FOC}$$

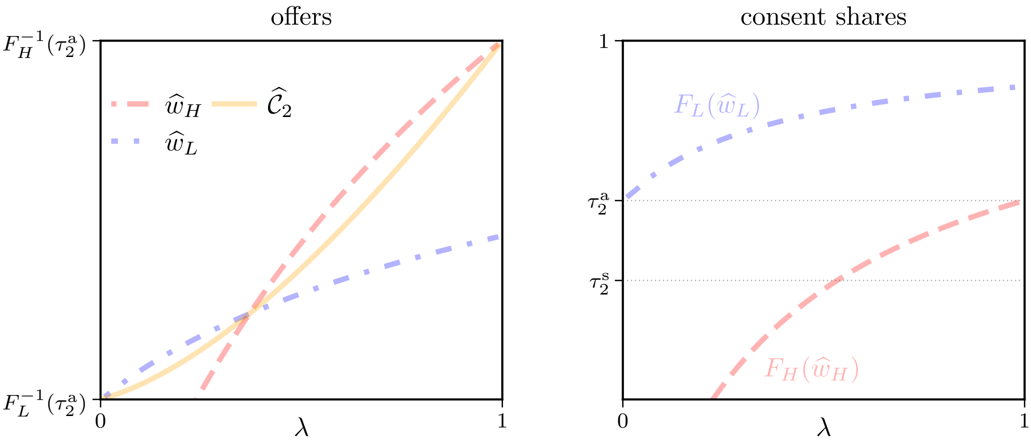
$$\lambda F_H(\widehat{w}_H) + (1-\lambda) F_L(\widehat{w}_L) = \tau_2^{\mathrm{a}} \tag{aggregate constraint}$$

- Check series-by-series constraints: $F_i(\widehat{w}_i) \geq \tau_2^{\mathrm{s}}$
  - if satisfied, optimal offer = auxiliary solution
  - if not satisfied, optimal offer s.t. *one* constraint binds, e.g. $\qquad w_H = F_H^{-1}(\tau_2^{\mathrm{s}}) > \widehat{w}_H$

- Optimal method (single- or two-limb) $\Leftrightarrow$ *closest* to auxiliary problem solution

Illustration

two-limb, auxiliary problem solution

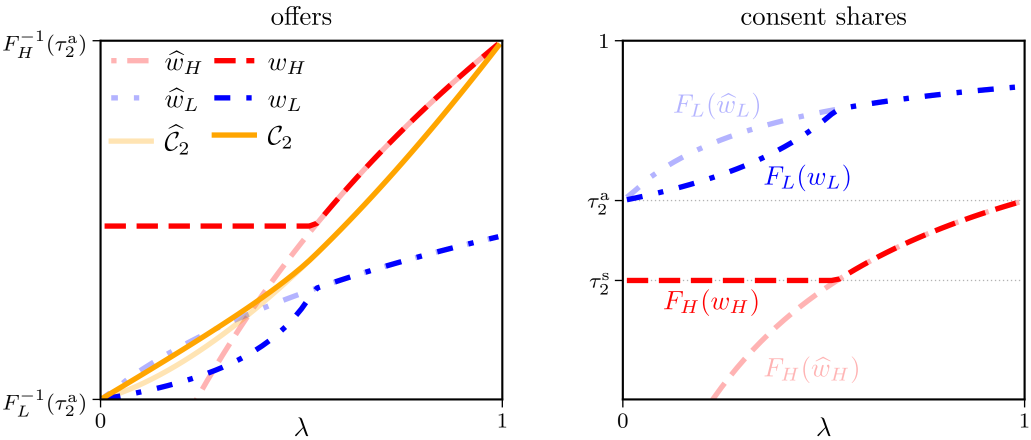


Assume $F_i(w) = 1 - e^{w/\phi_i}$, with $\phi_H = 0.7, \phi_L = 0.2$. Thresholds $\tau_1 = \tau_2^{\mathrm{a}} = 2/3, \tau_2^{\mathrm{s}} = 1/2$.

Lemma

Illustration

two-limb, full solution



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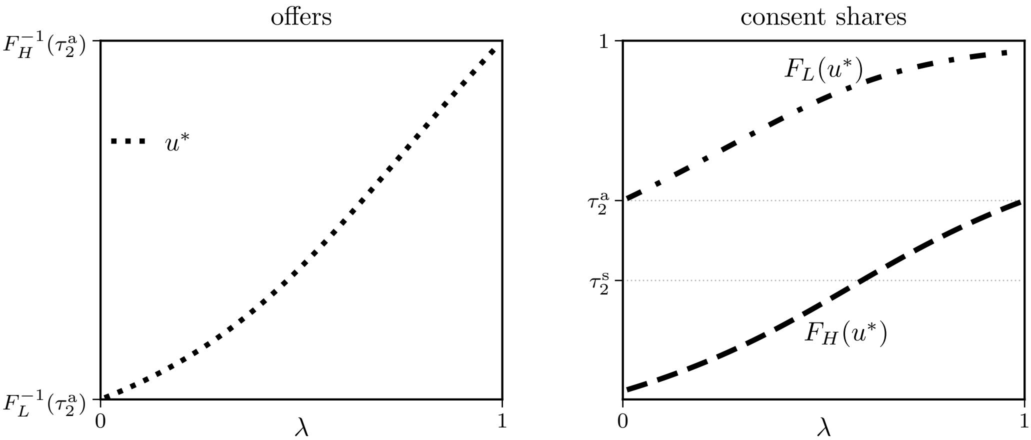
Lemma

Illustration

single-limb



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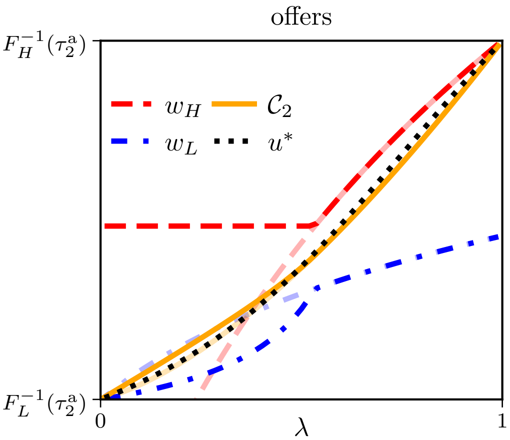
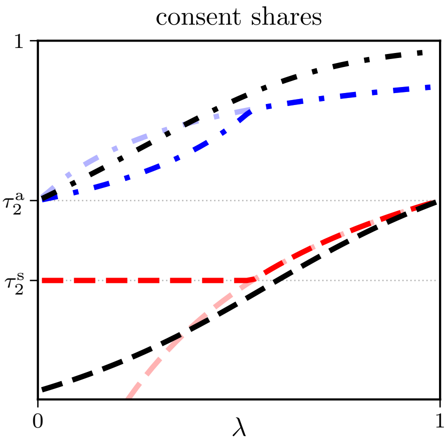
Lemma

Illustration

both methods





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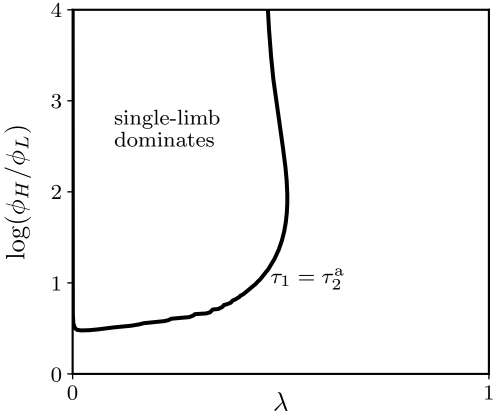
▸ Lemma

Optimal Voting Rule

parametric example



Distribution $F_i(w) = 1 - e^{w/\phi_i}$. Two-limb thresholds $\tau_2^{\mathrm{s}} = 1/2$ and $\tau_2^{\mathrm{a}} = 2/3$.

▸ more ▸ Prop.

Optimal Voting Rule

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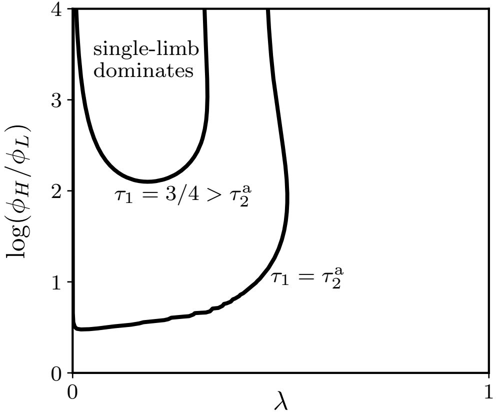

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▸ more ▸ Prop.

Outline

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Extended Setup

- Vulture fund (VF) may acquire share μ_i in bond i from bondholders
- VF holds blocking minority if
  - $\mu_i > 1 - \tau_2^{\mathrm{s}}$ under two-limb (*assume VF can't block both bonds*)
  - $\sum_i \lambda_i \mu_i > 1 - \tau_1$ under single-limb
- Costs and payoffs
  - Fixed entry cost ϵ_i if $\mu_i > 0$
  - If CACs are triggered: entry cost $q_i = w_i$, VF payoff w_i
  - If bond i is blocked: entry cost $q_i = F_i^{-1}(\mu_i)$, VF payoff h_i (reservation value, large)

$\Rightarrow$ VF entry only profitable if it blocks something, otherwise negative profits

Results Overview

- Assume:

  cost of blocking bond i (two-limb) $<$ VF's resources $<$ cost of blocking both bonds (single-limb)

- With both methods available, single-limb is a credible off-equilibrium threat

  VF blocks bond i $\Rightarrow$ govt uses single-limb $\Rightarrow$ VF makes a loss $\Rightarrow$

  $\Rightarrow$ VF does not enter $\Rightarrow$ govt chooses optimal method (Part 1), may be two-limb

- Takeaways
  - single-limb as an effective off-equilibrium threat, but maybe suboptimal absent VF
  - two-limb likely optimal absent VF, but less effective in preventing VF entry

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Equilibrium Model

setup

- Continuous-time stationary environment
- Continuum of risk-neutral investors with heterogeneous discount rates
  - Discount rate $r \sim G$ on $\mathcal{R} = [r_{\min}, r_{\max}]$.
- Two bonds S and L with exponentially decaying face values
  - decay rate δ_i, with $\delta_S > \delta_L$
  - coupon rate c_i
- Relative face values λ_S and $\lambda_L = 1 - \lambda_S$, constant over time
- A restructuring of both bonds may occur, with (exogenous) arrival rate η

Restructuring and Sorting Stages

- Restructuring stage
  - Holder of bond i with discount rate r has reservation value

$$h_i(r) = \frac{c_i}{r + \delta_i + \kappa}, \quad \kappa \geq 0$$

  and accepts restructuring offer if $w_i \geq h_i(r)$

- Sorting stage
  - Prior to restructuring, investor r values bond i at

$$Q_i(r, w_i) = \frac{c_i + \eta\, w_i}{r + \delta_i + \eta}$$

  - Assumption: each investors holds one unit of face value of either bond
  - The set of investors who sort into bond S is

$$\mathcal{R}_S(\Delta q, \mathbf{w}) = \big\{ r \in \mathcal{R} : \ Q_S(r, w_S) - Q_L(r, w_L) \geq \Delta q \big\}, \qquad \Delta q := q_S - q_L$$

Reservation Value Distributions and Equilibrium

- Given sorting partitions $\left(\mathcal{R}_S, \mathcal{R}_L\right)$, the distribution of reservation values for bond i is

$$F_i(w) = \mathrm{Prob}\Big(h_i(r) \leq w \mid r \in \mathcal{R}_i\Big)$$

- Equilibrium given by:
  (i) price differential Δq^*
  (ii) modification method and offers $\mathbf{w}^* = (w_S^*, w_L^*)$

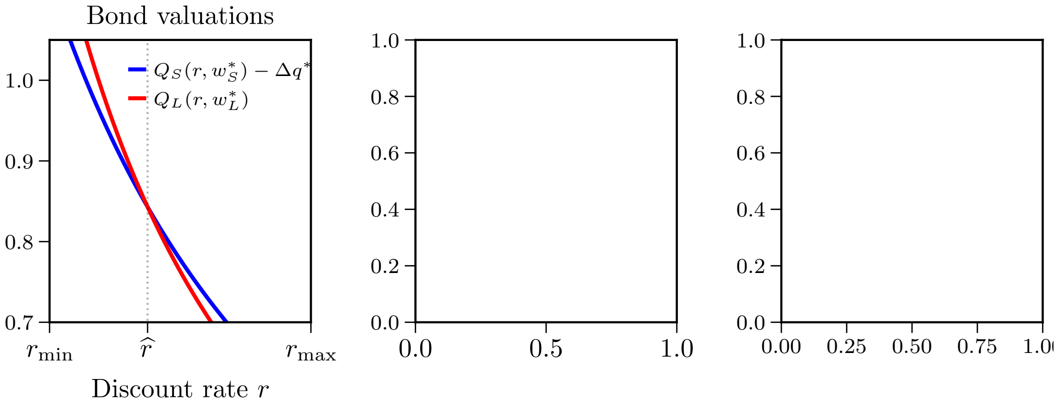
  such that
  - method and offers are optimal given F_i
  - investors sort optimally into bonds
  - the bond market clears

full def.

Equilibrium Example

- Consider parametrisation where more patient investors hold the long-term bond
- Market clearing requires $G(\widehat{r}) = \lambda_L$

▸ F_i



Assumptions: $r \sim U([0, 0.55])$, $(\delta_L, \delta_S) = (0.05, 0.25)$, $c_i = \mathbb{E}[r] + \delta_i$, $\lambda_L = 37\%$, $\eta = \kappa = 0.4$

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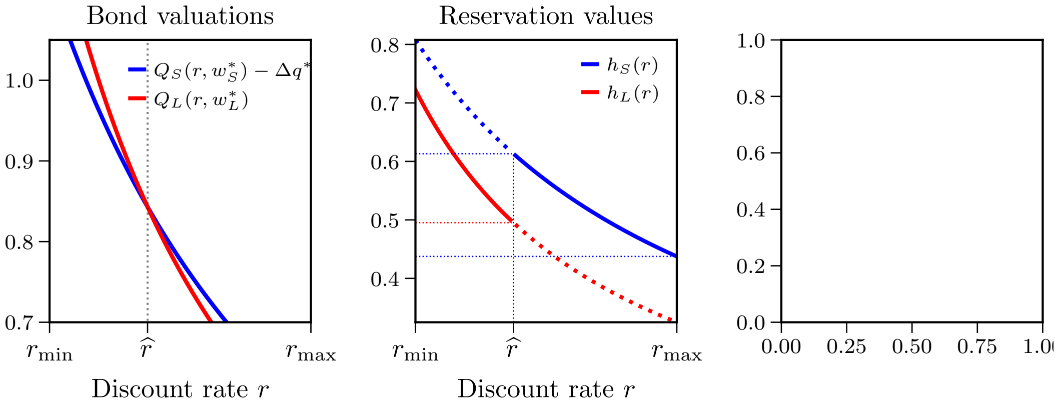
▸ F_i



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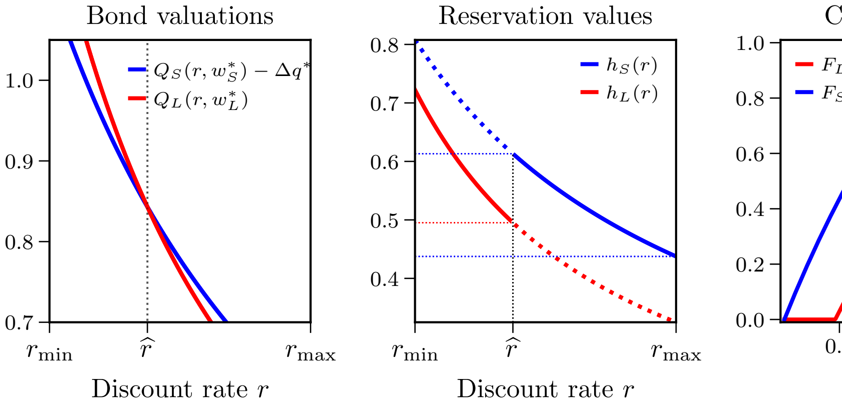

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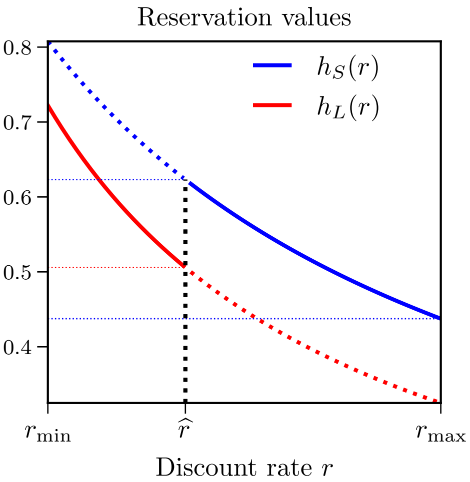
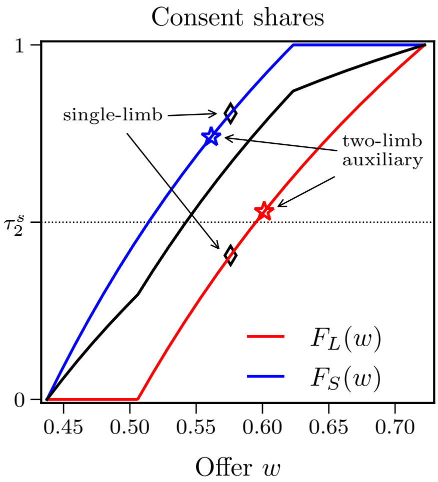
Comparative Statics wrt λ_L (1)

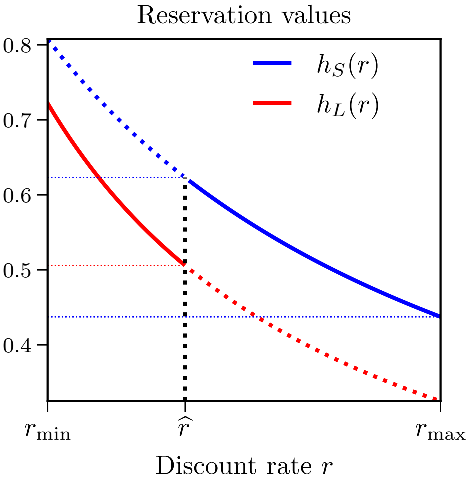
Baseline: auxiliary solution feasible $\Rightarrow$ two-limb dominates

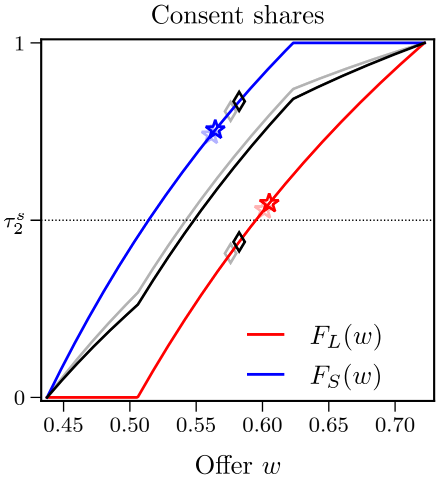




Comparative Statics wrt λ_L (2)

Size effect of $\uparrow \lambda_L$: offers increase, two-limb still optimal

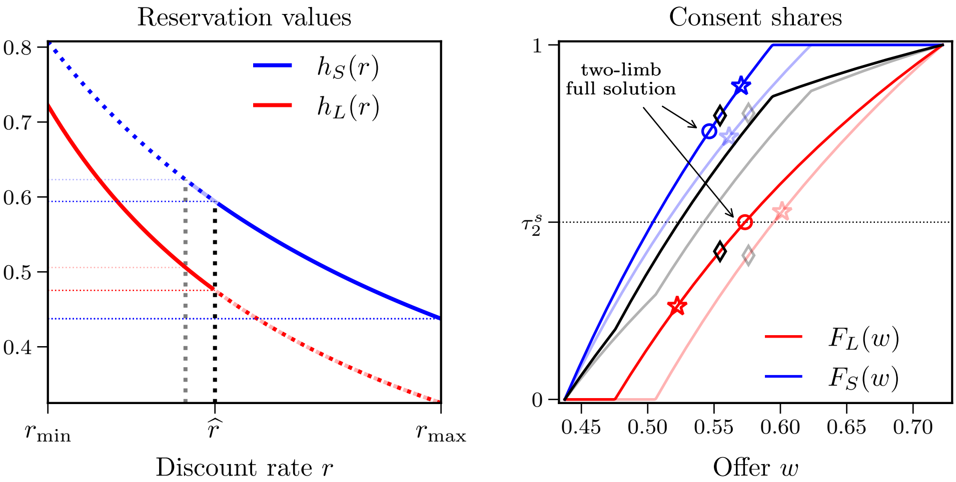




Comparative Statics wrt λ_L (3)

Sorting effect (new) of $\uparrow \lambda_L$: $\widehat{r}$ and F_i change, auxiliary offers diverge

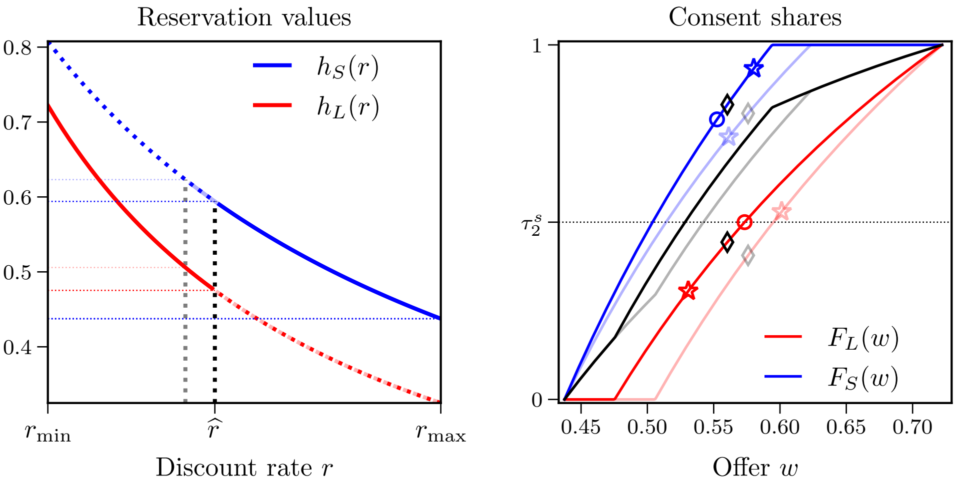
$\Rightarrow$ heterogeneity $\uparrow$ and single-limb dominates



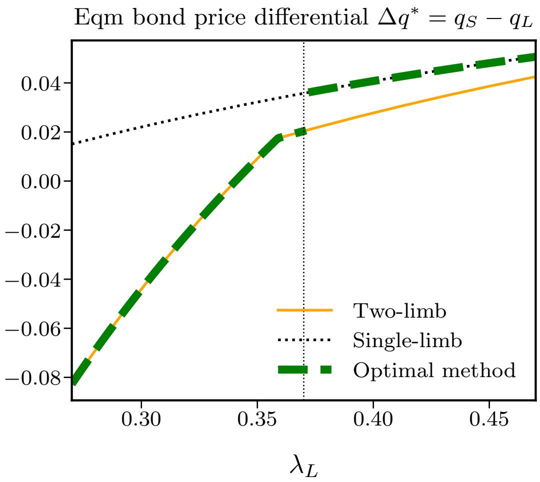
Comparative Statics wrt λ_L (4)

Total effect of $\uparrow \lambda_L$: sorting effect $>$ size effect

$\Rightarrow$ heterogeneity $\uparrow$ and single-limb dominates



Impact of CACs on Bond Market



▸ full graph

Outline

- Part 1: Optimal restructuring strategy, *given* creditor heterogeneity within & across bonds
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Conclusion

- Tradeoff between two aggregation methods
  - optimal procedure depends on bond heterogeneity, relative size, voting thresholds
  - single-limb can be optimal when small bond is held by tough creditors
- Off-equilibrium role of single-limb as deterrent against (non-atomistic) vulture
- Bond-specific reservation value distributions arise endogenously from investor sorting
  - anticipation of optimal use of CACs affects ex-ante bond market equilibrium
- Analysis of restructuring problem extends to N bonds + uncertainty
  - sub-aggregation, redesignation, bonds with different versions of CACs

Appendix

Uniform Applicability

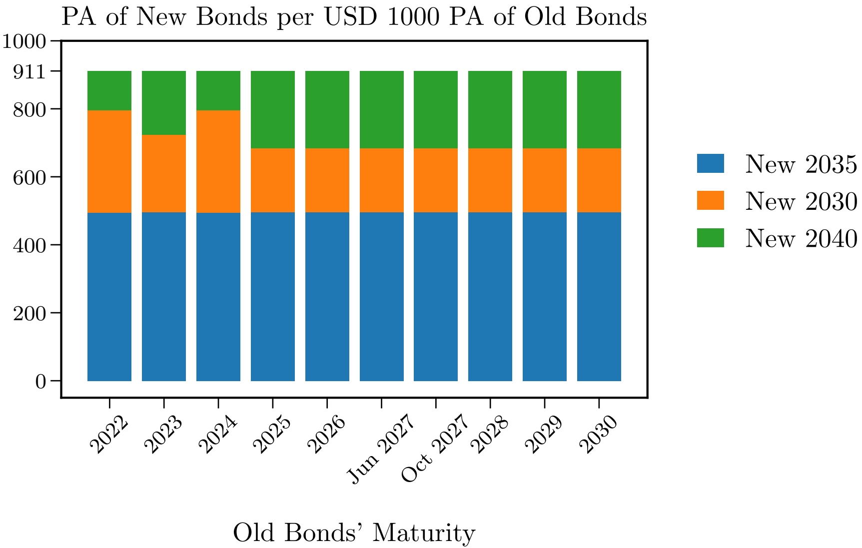
ICMA

- exchange, on the same terms, for the same (menu of) instrument(s)
- proposed amendments imply that new bonds have same provisions

Euro Area

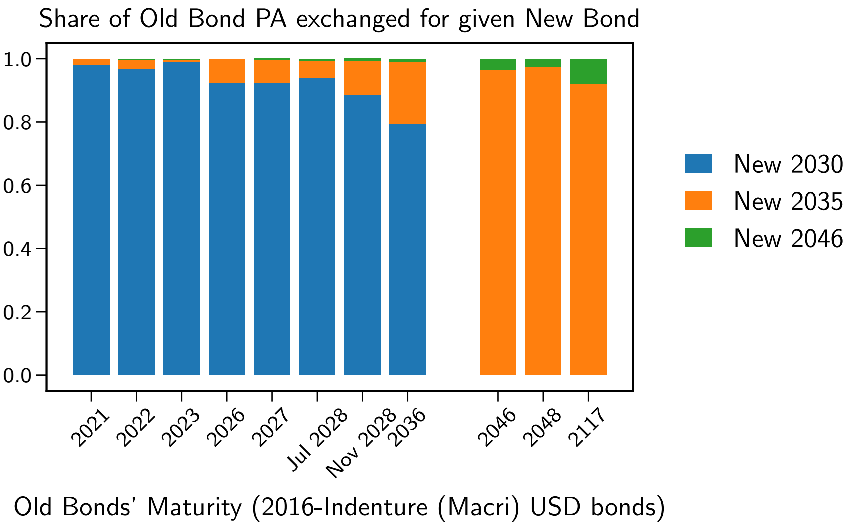
- exchange on the same terms, for the same (menu of) instrument(s)
- reduce face value by the same %
- extend maturity by the same time period or the same %

Ecuador 2020: Heterogeneous Offers



Argentina 2020: Heterogeneous Offers & Choices

before Priority Acceptance Procedure



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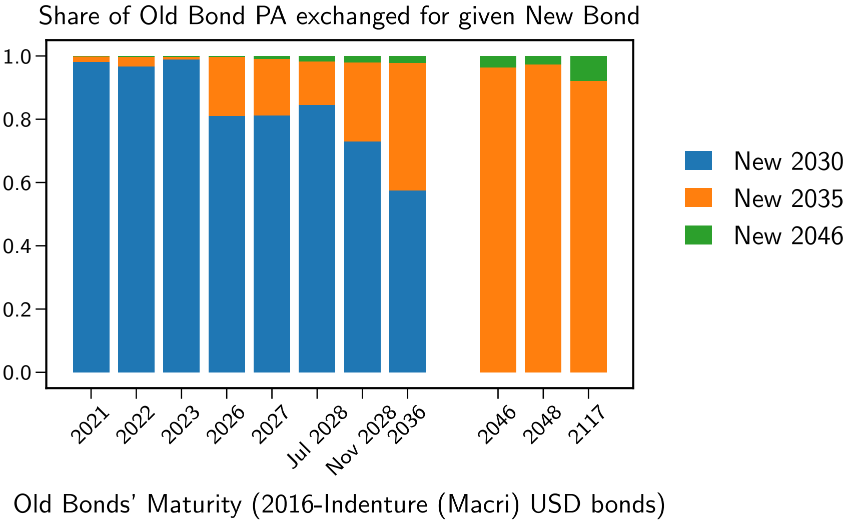

Optimal Voting Rule

Sufficient Conditions

Lemma

Two-limb dominates if *at least one of the following conditions holds*

(i) *the optimal single-limb offer* $u^\star$ *satisfies all s.b.s. constraints:* $F_i(u^\star) \geq \tau_2^{\mathrm{s}}$
$\rightarrow$ single-limb has no advantage vs two-limb

(ii) *the auxiliary solution* $(\widehat{w}_H, \widehat{w}_L)$ *satisfies all s.b.s. constraints:* $F_i(\widehat{w}_i) \geq \tau_2^{\mathrm{s}}$
$\rightarrow$ two-limb has no disadvantage vs single-limb

Back

Optimal Voting Rule

Comparative Statics

Proposition

- **Two-limb is optimal**
  - *if there is little heterogeneity across bonds* ▸ γs
  - *if bond H is relatively large, i.e., λ is high* ▸ λs
  - *when bond H is very small ($\lambda \approx 0$) and $\tau_1 > \tau_2^{\mathrm{a}}$.*

- *Under some technical conditions on densities f_H and f_L,* **single-limb is optimal**
  - *when $\lambda \approx 0$ and $\tau_1 = \tau_2^{\mathrm{a}}$*
  - *when $\lambda \approx \widetilde{\lambda}$, where $\widetilde{\lambda}$ is such that $\widehat{w}_L(\widetilde{\lambda}, \tau_2^a) = \widehat{w}_H(\widetilde{\lambda}, \tau_2^a) < F_H^{-1}(\tau_2^{\mathrm{s}})$, and $\tau_1 \approx \tau_2^a$.*

◂ Back

Comparative Statics in λ

Assuming $F_i(w) = 1 - e^{w/\phi_i}$, $\phi_H = 0.7, \phi_L = 0.2$ and $\tau_1 = \tau_2^{\mathrm{a}}$

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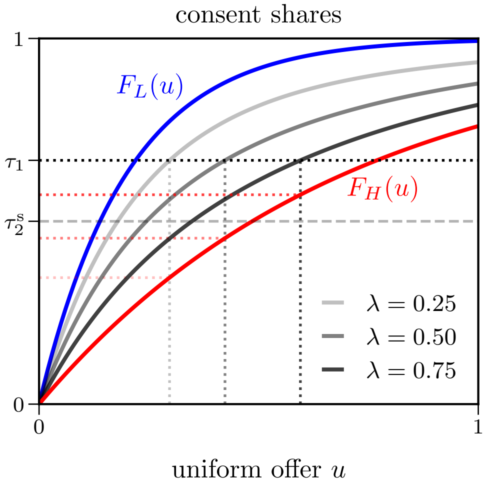

Comparative Statics: Role of Heterogeneity

Assuming $F_i(w) = 1 - e^{w/\phi_i}$ and $\tau_1 = \tau_2^{\mathrm{a}}$, for various ϕ_H/ϕ_L

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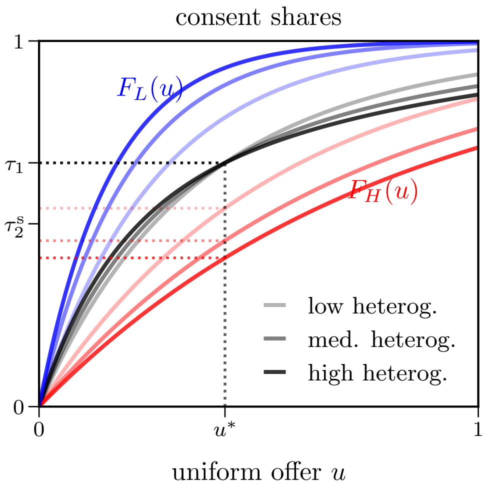
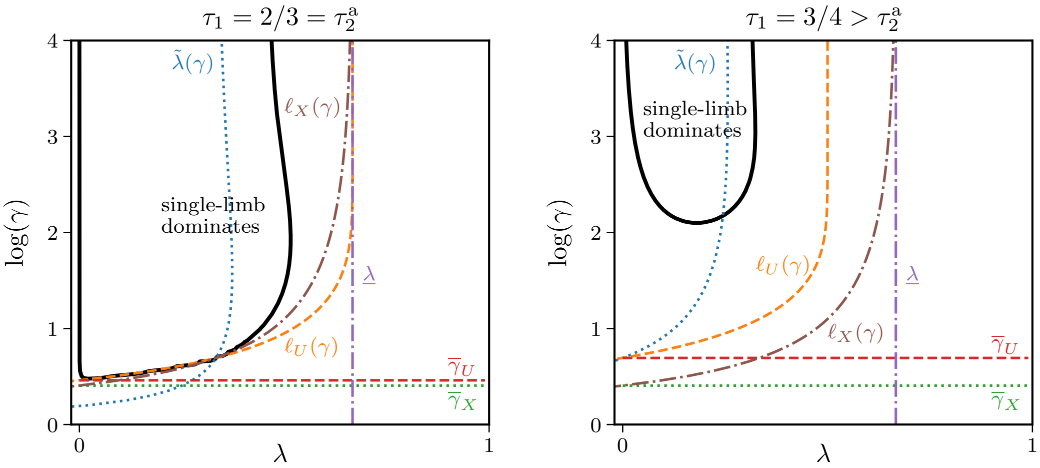


Optimal Voting Rule



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Equilibrium

Definition

Given distribution G on $\mathcal{R}$, bond characteristics (c_i, δ_i) and relative sizes λ_S, λ_L, default arrival rate η, and parameter κ, an equilibrium consists of

(i) *a price differential $\Delta q^\star$ and a partition $(\mathcal{R}_S, \mathcal{R}_L)$*

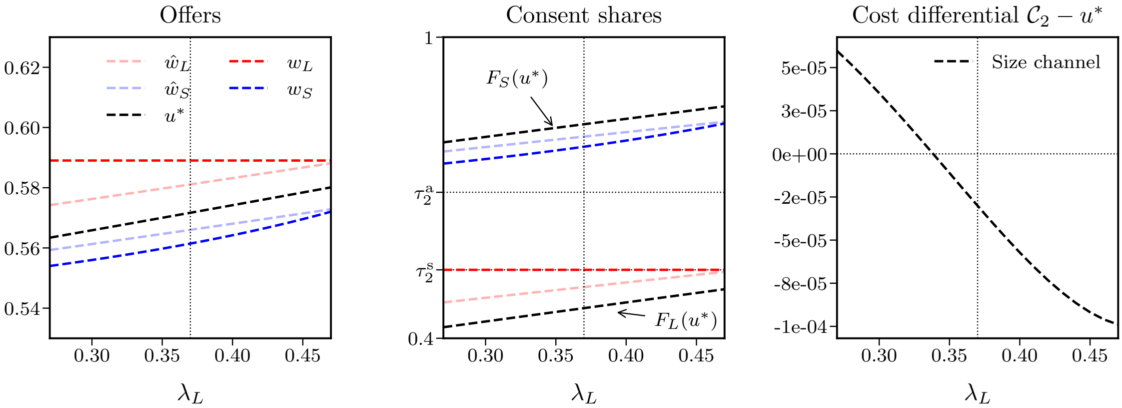
(ii) *a modification method and a pair of offers $\mathbf{w}^\star$*

such that

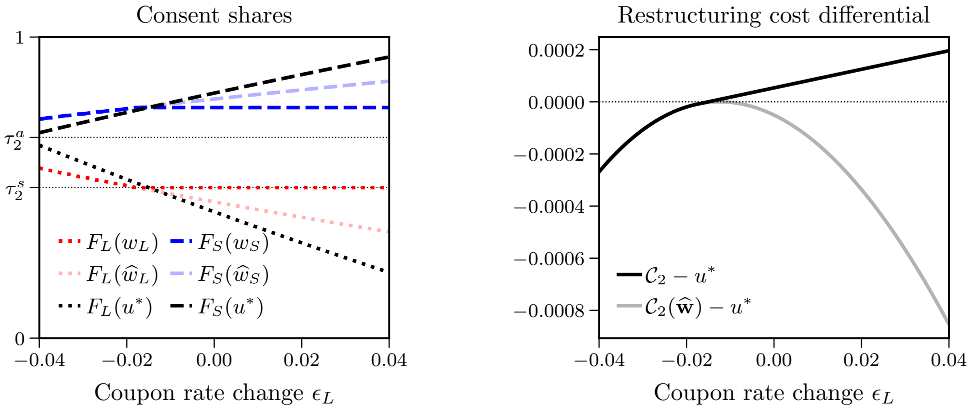
1. *the government chooses the modification method and restructuring offers optimally given the implied distributions F_S and F_L,*
2. *investors optimally choose which bond to hold: $\mathcal{R}_i = \mathcal{R}_i(\Delta q^\star, \mathbf{w}^\star)$,*
3. *the market clears for each bond, $\int_{\mathcal{R}_i} dG = \lambda_i$.*

back

Size Effect of $\uparrow \lambda_L$

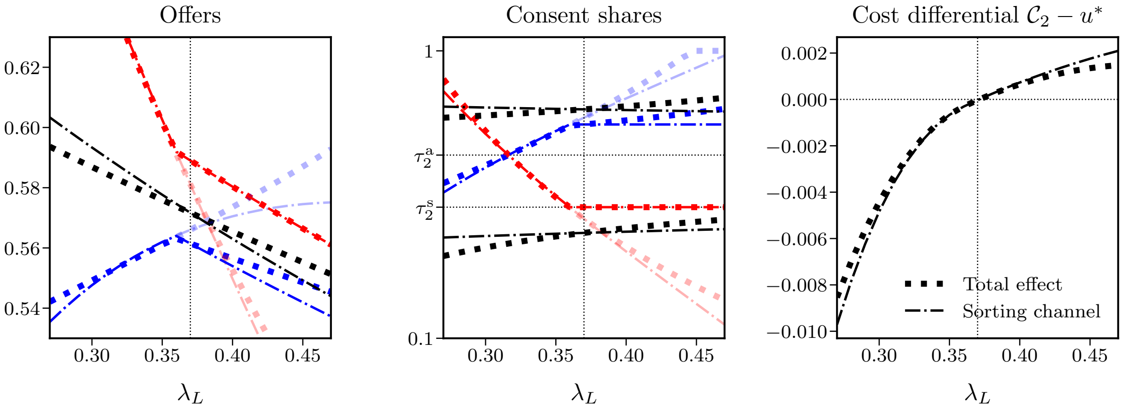


Role of Bond Heterogeneity

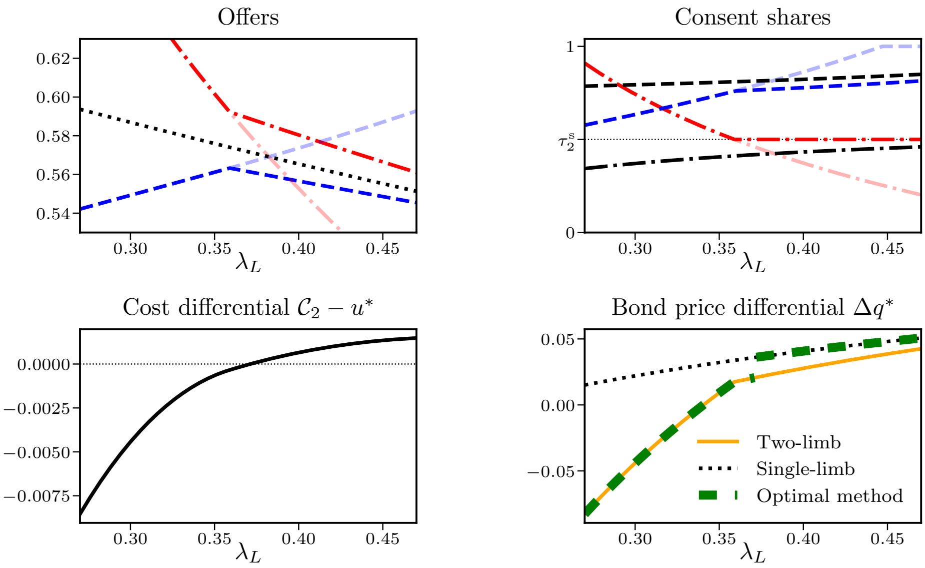


Assumptions: taking deviation around baseline $\widetilde{c}_L = c_L + \epsilon_L$

Sorting & Total Effect of $\uparrow \lambda_L$



Full Comparative Statics wrt λ_L



Equilibrium Example

- Investor r holding bond i accepts w_i iff $r \geq \bar{r}_i(w_i) := \frac{c_i}{w_i} - (\delta_i + \kappa_i)$

$$\underbrace{r_{min} \longrightarrow \bar{r}_L(w_L)}_{\text{hold bond } L,\ \text{prefer } h_L(r)} \underbrace{\longrightarrow \widehat{r}}_{\text{hold bond } L,\ \text{prefer } w_L} \underbrace{\longrightarrow \bar{r}_S(w_S)}_{\text{hold bond } S,\ \text{prefer } h_S(r)} \underbrace{\longrightarrow r_{max}}_{\text{hold bond } S,\ \text{prefer } w_S}$$

- With $r \sim \text{Uniform}[r_{\min}, r_{\max}]$, we get

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$$F_L = \frac{G(\widehat{r}) - G(\bar{r}_L(w_L))}{\lambda_L} = \left(\frac{\lambda_L(r_{\max} - r_{\min}) + \delta_L + \kappa + r_{\min}}{\lambda_L r_{\max} - r_{\min}} \right) - \left(\frac{c_L}{\lambda_L(r_{\max} - r_{\min})} \right) \frac{1}{w_L}$$

$$F_S = \frac{1 - G(\bar{r}_S(w_S))}{\lambda_S} = \left(\frac{(r_{\max} - r_{\min}) + \delta_S + \kappa + r_{\min}}{\lambda_S(r_{\max} - r_{\min})} \right) - \left(\frac{c_S}{\lambda_S(r_{\max} - r_{\min})} \right) \frac{1}{w_S}$$

Auxiliary Problem SOCs

The objective function is strictly convex if

$$\frac{d\log f_L(w_L)}{dw_L} < \frac{d\log f_H[g(w_L)]}{dw_L}.$$

A sufficient condition is that the two densities f_H and f_L are strictly decreasing

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