Inflation, Default Risk and Nominal Debt

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January 2020

Motivation

- Recent switch of many EM sovereigns to local-currency borrowing
- New issue arises
  - Strategic inflation as a way to alleviate debt burden
  - In addition to outright default
- Strategic inflation with nominal debt
  - Ex-post insurance benefits
  - Ex-ante time-consistency costs
- Joint behaviour of inflation and default spreads
  - Key for welfare implications of nominal debt
  - Linked to fiscal-monetary policy interaction in EM

Empirical Observations

- Asset price derivatives contain information on both risks, separately
- Common "printing press" argument does not hold
  - Default & inflation risks co-exist
- Default risk co-moves
  - With expected inflation
  - With realised inflation

...and this holds

  - Across countries, in long run
  - Within country, at short run frequencies

Theoretical Implications

Use facts to discipline quantitative sovereign default model

- Default as a binary choice
- Money (and inflation) as a continuous instrument
  1. dilutes real value of debt
  2. generates seignorage revenues

Dilution motive alone is counterfactual

- Inflation and default are substitutes
- Low incentive to inflate in bad times

Revenue motive reconciles model with data

- Seignorage flexible source of funding in bad times
- Inflation & default risks co-move

Takeaways

Default/inflation spreads drive government bond prices

- W/out commitment, determine costs of time-inconsistency
- Typically default spreads $\uparrow$ in bad times
- If inflation spreads co-move $\Rightarrow$ debt policy even more constrained

Framework can be used to study

1. Welfare properties of LC debt issuance
2. Optimal fiscal-monetary setup (central bank commitment vs flexibility)

Role of expectations: low credibility $\rightarrow$ LC debt issuance costly outside of crisis
Potential implications

- Monetary-fiscal framework crucial for LC debt issuance
- Trade-off insurance vs. extra time inconsistency source

Related Literature

Time-consistent policy with nominal debt & default

- Aguiar et al. (2014, 2015), Corsetti-Dedola (2016), Sunder-Plassman (2018), Na et al. (2018), Nuno-Thomas (2019), Roettger (2019)

Government debt currency denomination and "original sin"

- Eichengreen-Hausmann (1999, 2005), Du et al. (2016), Du-Schreger (2016, 2017), Engel-Park (2019), Ottonello-Perez (2018)

Time-consistent policy with default & nominal rigidities

- Na et al. (2018), Bianchi et al. (2019), Arellano et al. (2019)

Currency and balance of payment crises

- Krugman (1979), Obstfeld (1986), Burnside et al. (2001)

Empirical Facts

Data Description

- Period: Jan 2004 - Feb 2019, quarterly
- Countries: Brazil, Colombia, Indonesia, Mexico, Malaysia, Poland, Russia, Thailand, Turkey, South Africa
  - all with freely/managed floating exchange rates (Ilzetzki et al., 2019)

More Data

Asset Price Data: Default Risk

Instrument: 5y Credit Default Swaps (CDSs)

- USD denominated, no currency risk
- Insure against default losses on international law debt
- Correlated with foreign-currency bond spreads
- Back out implied, risk-neutral default probability

More Details Implied Default Probs

Asset Price Data: Inflation Risk

Proxy with currency risk

Instrument: 5y Cross-Currency Swaps (XCSs)

- No credit risk, fully collateralised OTC derivatives
- Long-term analogue of implied yields in exchange rate forwards

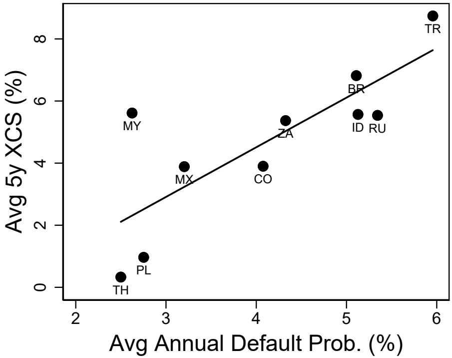
$$i - i^* = \frac{Fwd}{Spot}$$

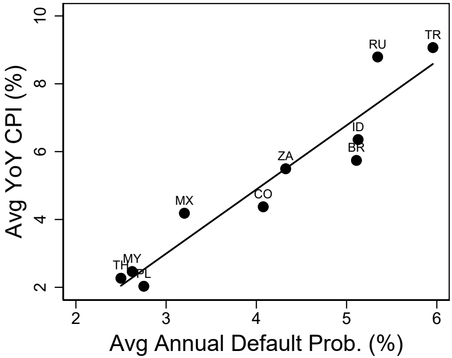
- Interpret $i - i^* \approx \mathbb{E}\pi - \mathbb{E}\pi^*$

More Details

Fact 1: Long-Run, Across Countries

Cross-country averages for the period 2004q1-2018q4

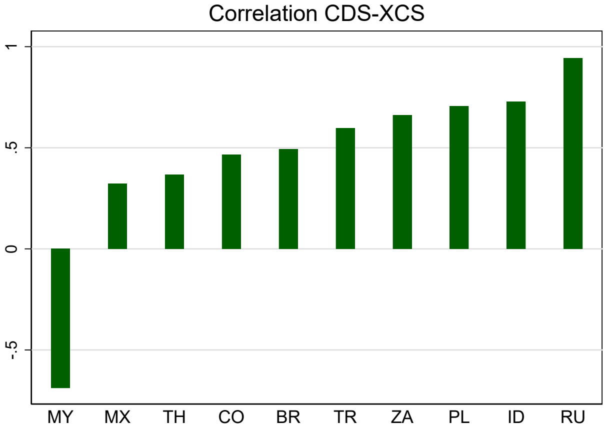




Implied Default Prob. Post GFC IRS

Fact 2: Asset Price Correlation, Within Country

Time-series correlation between 5y default risk (CDS) & 5y currency risk (XCS)



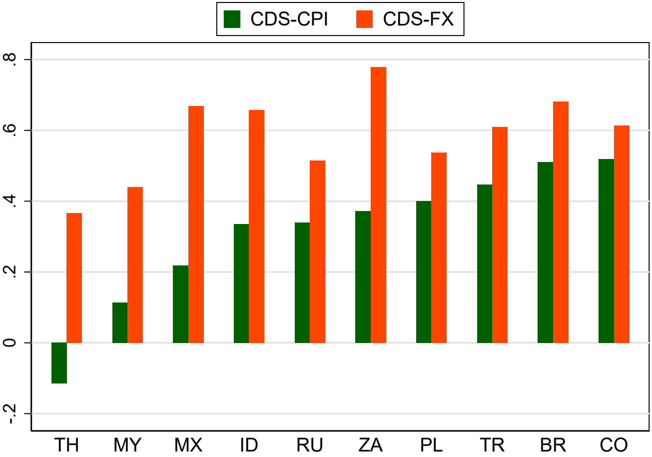
Panel: $\widehat{DP}_{i,t} = 0.437\ XCS_{i,t}$ (two-way FE, SE 0.096)

More Panel

Fact 3: Macro Correlations, Within Country

Time series correlation between

- 5y default risk (CDS) & nominal exchange rate (FX) yoy changes
- 5y default risk (CDS) & consumer price index (CPI) yoy changes



Taking Stock

Document co-movement

- Among asset prices: default risk and currency risk
- With macro variables: default risk and inflation/exchange rate depreciation
- In short & long run

Model

Environment

Quantitative, sovereign default model with

- Nominal debt
- Money
- Endogenous government spending

Players

- Benevolent government
- Domestic households
- Foreign lenders

Households

- Preferences: utility from real money balances (from $t-1$) and public good g_t

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t U^h \left(c_t, \frac{M_t}{P_t}, g_t \right) = \frac{c_t^{1-\gamma}}{1-\gamma} + \alpha_m \frac{(M_t/P_t)^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta}$$

- Receive exogenous, stochastic income $y_t \sim AR(1)$
- Consume, pay taxes, hold money, save in domestic (zero net supply) bonds

$$c_t + \frac{M_{t+1}}{P_t} + \frac{1}{R_t} \frac{B_{t+1}^d}{P_t} = \frac{M_t}{P_t} + \frac{B_t^d}{P_t} + y_t(1-\tau_t)$$

- Euler equations for domestic bonds

$$\frac{1}{R_t} = \mathbb{E}_t \beta_h \left[\frac{U_{c,t+1}^h}{U_{c,t}^h} \frac{P_t}{P_{t+1}} \right]$$

- Money demand equation

$$R_t - 1 = \mathbb{E}_t \left[\frac{U_{m,t+1}^h}{U_{c,t+1}^h} \right]$$

Government

- Benevolent, maximises households' utility, own discount factor β, MIU wedge

$$U\left(c_t, \frac{M_t}{P_t}, g_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma} + (\alpha_m + \alpha_\nu)\frac{(M_t/P_t)^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta}$$

- No commitment to default & monetary policy

- Borrows externally, issues money domestically, chooses spending
  - **"Benchmark" model**: can also choose taxes freely
  - **"Reduced" model**: taxes are fixed

- Default implies
  - Exclusion from debt markets: receive offer to repay $B_t(1-h)$ & re-enter w.p. θ
  - Reduced output $y^d(y_t) \leq y_t$

MIU wedge

Timing

1) Start period with B_t, M_t, y_t

2) Government default/repay decision

3) Government fiscal/monetary policy decisions

- Repay
  - issue B_{t+1} to lenders at price q_t, choose g_t, τ_t, M_{t+1}

$$\tau_t y_t + q_t \frac{B_{t+1}}{P_t} + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + \frac{B_t}{P_t} + g_t$$

- Default
  - Choose g_t, τ_t, M_{t+1}

$$\tau_t y^d(y_t) + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + g_t.$$

4) Households consumption/saving decisions

Lenders

- Risk-neutral, perfectly competitive, deep pockets
- Opportunity cost of funds R^*
- Zero-profit price of a unit of **new** government debt

$$q_t = \frac{1}{R^*}\mathbb{E}_t\Big[\underbrace{\frac{1-\delta_{t+1}}{1+\pi_{R,t+1}}}_{\text{repay}} + \underbrace{\frac{\delta_{t+1}\ q_{D,t+1}}{1+\pi_{D,t+1}}}_{\text{default}}\Big]$$

- Zero-profit price of a unit of **defaulted** government debt

$$q_{D,t} = \frac{1}{R^*}\mathbb{E}_t\Big[\underbrace{(1-\theta)\frac{q^n_{D,t+1}}{1+\pi^n_{D,t+1}}}_{\text{no offer}} + \underbrace{\theta\delta_{t+1}\frac{(1-h)q^o_{D,t+1}}{1+\pi^o_{D,t+1}}}_{\text{reject offer}} + \underbrace{\theta(1-\delta_{t+1})\frac{1-h}{1+\pi_{R,t+1}}}_{\text{accept offer}}\Big]$$

Implied expected default and inflation:

- Default probability $DP_t = \mathbb{E}_t\delta_{t+1}$
- Expected inflation $XCS_t = \mathbb{E}_t[\delta_{t+1}\pi_{D,t+1} + (1-\delta_{t+1})\pi_{R,t+1}]$

Private Sector Equilibrium

Focus on time-consistent, Markov-perfect equilibrium

- Gov't internalises effect of policy on future policies, prices and hhs' allocations

Recursive formulation

- Denote current, future variables with (x, x')
- Make problem stationary $\rightarrow$ normalise **nominal** variables: $\tilde{X} = X/M$

$\Rightarrow$ Aggregate state variables $(y, \tilde{B})$

Given $\mathcal{S} := (\tilde{B}, y; \delta, g, \tau, \mu, \tilde{B}')$, Private Sector Equilibrium (PSE) is

- Household consumption policy $c(\mathcal{S})$
- Prices $R(\mathcal{S})$ and $m(\mathcal{S})$
- Market clearing: money balances ($\tilde{M}'^{d} = 1$), domestic bonds ($\tilde{B}'^{d} = 0$).

Equilibrium Definition

Government Recursive Problem

- Default choice

$$V(\tilde{B}, y) = \max_{\delta \in \{0,1\}} (1-\delta) V^R(\tilde{B}, y) + \delta V^D(\tilde{B}, y)$$

- Repayment value

$$\begin{aligned} V^R(\tilde{B}, y) = \max_{g,\tau,\mu,\tilde{B}'} \; & U(c(\mathcal{S}), m(\mathcal{S}), g) + \beta \mathbb{E}_{y'|y} V(\tilde{B}', y') \\ \text{s.t.} \quad & y + \underbrace{q(\mathcal{S})\tilde{B}'(1+\mu)m(\mathcal{S})}_{qB'/P} = \underbrace{\tilde{B}m(\mathcal{S})}_{B/P} + c(\mathcal{S}) + g \end{aligned}$$

- Default value

$$\begin{aligned} V^D(\tilde{B}, y) = \max_{g,\mu} \; & U(c(\mathcal{S}), m(\mathcal{S}), g) + \beta \mathbb{E}\left[\theta V\left(\frac{\tilde{B}(1-h)}{1+\mu}, y'\right) + (1-\theta) V^D\left(\frac{\tilde{B}}{1+\mu}, y'\right)\right] \\ \text{s.t.} \quad & y^D(y) = c(\mathcal{S}) + g \end{aligned}$$

Lenders' Recursive Problem

Equilibrium

Definition (Markov-Perfect Equilibrium)

Given the aggregate state $\{\tilde{B}, y\}$, a recursive equilibrium consists of

- Government value functions $V(\tilde{B}, y), V^R(\tilde{B}, y), V^D(\tilde{B}, y)$,
- Associated policy functions $\delta(\tilde{B}, y)$, $g(\tilde{B}, y)$, $\tau(\tilde{B}, y)$, $\mu(\tilde{B}, y)$ and $\tilde{B}'(\tilde{B}, y)$
- Private sector equilibrium $\mathcal{P}$

such that:

1. Value and policy functions solve the government problem, given $\mathcal{P}$ and debt price functions q, q_D
2. $\mathcal{P}$ is the PSE associated with government value and policy functions

Optimality: Repayment

Can summarise policy with $(c, \tilde{B}')$

- back out (g, τ, μ, m) from (RC) and PSE conditions

Inflation

- Benefit: $\downarrow$ real value of debt due $(\tilde{B}m)$ $+ \uparrow$ tax revenues to finance g
- Cost: $\downarrow$ utility (U_m)

Two first-order conditions:

Private-public consumption

$$\underbrace{U_g - U_c}_{\text{MC redistribution}} = \underbrace{m_{(c)}(U_m - U_g\tilde{B})}_{\text{MB} \uparrow \text{ real balances}}$$

Euler equation

$$\underbrace{U_g dr_{(\tilde{B}')}(\tilde{B}')}_{\text{MR debt issuance}} + \underbrace{m_{(\tilde{B}')}\left(U_m - U_g\tilde{B}\right)}_{\text{MB} \uparrow \text{ real balances}} = \underbrace{\beta \mathbb{E} U'_g m'}_{\text{MC higher debt tmr}}$$

Backup MIU wedge

Optimality: Default

Can summarise policy with μ

- back out (c, τ, g, m) from (RC) and PSE conditions

Inflation

- Benefit: $\downarrow$ real debt due at re-entry $+ \uparrow$ tax revenues to finance g
- Cost: $\downarrow$ utility (U_m)

First-order condition for μ

$$\underbrace{\frac{\partial}{\partial \mu} \beta \mathbb{E}\left[(1-\theta) V^D\left(y', \frac{\tilde{B}}{1+\mu}\right) + \theta V\left(y', \frac{\tilde{B}(1-h)}{1+\mu}\right)\right]}_{\downarrow \text{ future debt burden}} \underbrace{-c_{(\mu)}(U_g - U_c)}_{\text{MB redistribution}} = \underbrace{-m_{(\mu)} U_m}_{\text{MC } \downarrow \text{ real balances}}$$

Backup

Computation with Taste Shocks 1/2

Government recursive problem

- Default choice

$$V(\tilde{B}, y, \{\epsilon_R, \epsilon_D\}) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta)[V^R(\tilde{B}, y) + \rho_\delta \epsilon_R] + \delta[V^D(\tilde{B}, y) + \rho_\delta \epsilon_D] \right\}$$

- Repayment value

$$V^R(\tilde{B}, y, \{\epsilon_{\tilde{B}'}\}) = \max_{\tilde{B}'} \left\{ W^R(\tilde{B}, y;\ \tilde{B}') + \rho_{\tilde{B}'} \epsilon_{\tilde{B}'} \right\}$$

where $\quad W^R(\tilde{B}, y;\ \tilde{B}') = U(c(\tilde{B}'), m(\tilde{B}'), g(\tilde{B}')) + \beta \mathbb{E}_{y'|y} V(\tilde{B}', y')$

- Default value

$$V^D(\tilde{B}, y, \{\epsilon_\mu\}) = \max_{\mu} \left\{ W^D(\tilde{B}, y;\ \mu) + \rho_\mu \epsilon_\mu \right\}$$

where

$$W^D(\tilde{B}, y;\ \mu) = U(c(\mu), m(\mu), g(\mu)) + \beta \mathbb{E} \left[\theta V \left(\frac{\tilde{B}(1-h)}{1+\mu}, y' \right) + (1-\theta) V^D \left(\frac{\tilde{B}}{1+\mu}, y' \right) \right]$$

Computation with Taste Shocks 2/2

- $\left\{\epsilon_R, \epsilon_D, \epsilon_{\tilde{B}'}, \epsilon_\mu\right\} \sim^{iid} Gumbel(-\bar{\mu}, 1)$
- Choice probabilities:

$$\mathbb{P}(x|\tilde{B}, y) = \frac{\exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]}{\sum_x \exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]}$$

- Expected values:

$$V^i(\tilde{B}, y) = \rho_x \log\left\{\sum_x \exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]\right\}$$

Magnitudes

- Consider choice x'' such that $\log\frac{W^i(\tilde{B}, y;\ x'')}{\max_x W^i(\tilde{B}, y; x)} = -.05\%$
- $\rho_{\tilde{B}'} = 1e-3$

$$\mathbb{P}[\tilde{B}''_{(-.05\% V^R}|\tilde{B}, y] = 1e-12$$

- $\rho_\mu = 5e-3$

$$\mathbb{P}[\mu_{(-.05\% V^D)}|\tilde{B}, y] = .001$$

- $\rho_{R,D} = 5e-3$

$$\mathbb{P}[\delta_{(-.05\% V^R)}|\tilde{B}, y] = .057$$

Quantitative Evaluation

- Recall preferences $U(c, m, g) = \frac{c^{1-\gamma}}{1-\gamma} + (\alpha_m + \alpha_\nu)\frac{m^{1-\eta}}{1-\eta} + \alpha_g \frac{g^{1-\zeta}}{1-\zeta}$
- Output process

$$\log(y_t) = \rho_y \log(y_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$$

- Default costs

$$y^d(y) = y - \max\{0, d_0 y + d_1 y^2\}$$

- External parameters:

Variable		Value	Source
Private good utility curvature	γ	2	Conventional value
Money in utility curvature	η	3	Money demand rate-elasticity
International risk-free rate	$R^* - 1$	0.00598	US Treasury rate
Log-output autocorrelation	ρ	0.9293	estimated
Log-output innovation st. dev.	σ_ϵ	0.0115	estimated
Re-entry probability	θ	0.282	Arellano (2008)
Recovery upon default	$1 - h$	0.63	Cruces-Trebesch (2013)

Money Demand Elasticity Money Growth and Seignorage

Benchmark Model

Assume lump-sum taxation available to the government

- Policy not constrained by Private Sector Equilibrium
- Govt can use τ to finance g
- Inflation not distorting $U_c - U_g$ margin, no wedges
- Public good utility curvature equal to private ($\zeta = 2$)

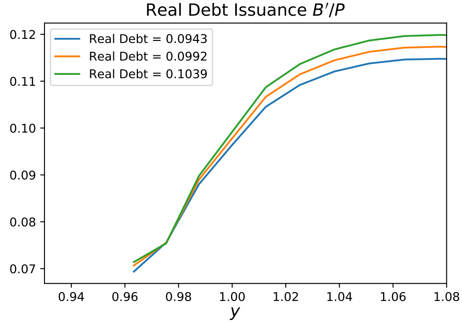
Parameters selected to match targets

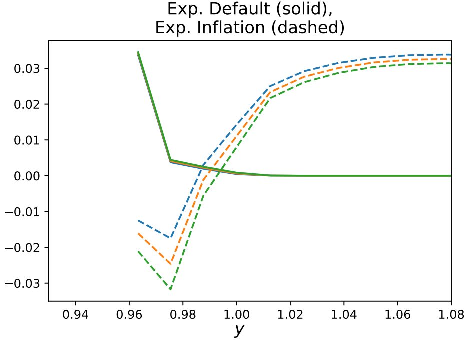
Variable		Value	Target	Data	Model
Govt discount factor	β	0.83	Debt service/GDP	0.058	0.099
Household discount factor	β_h	0.99	Risk-free rate	0.073	0.064
MIU constant	α_m	2.7e-5	Monetary base/GDP	0.098	0.112
MIU constant (govt)	α_ν	1.5e-3	CPI Inflation	0.049	0.038
Public good utility constant	α_g	0.07	c/g ratio	3.67	3.66
Default cost parameter	d_0	-0.3	Default prob. (mean)	0.045	0.029
Default cost parameter	d_1	0.325	Default prob. (st. dev.)	0.020	0.052

Benchmark Model: Equilibrium Policy and Prices

Non-targeted moments

Moment	Model	Data
$\rho(DP_t, XCS_t)$	-0.25	0.46
$\rho(y_t, XCS_t)$	0.43	0.02
$\rho(y_t, DP_t)$	-0.55	-0.2
$\rho(DP_t, \pi_t)$	0.02	0.31





Reduced Model

Assume taxation is exogenous

- Fiscal capacity in EM typically low, hard to adjust
- Seignorage as a flexible source of funding
- Inflation tax distorts $U_c - U_g$ margin, wedges
- Public good utility curvature larger than private ($\zeta = 5$)

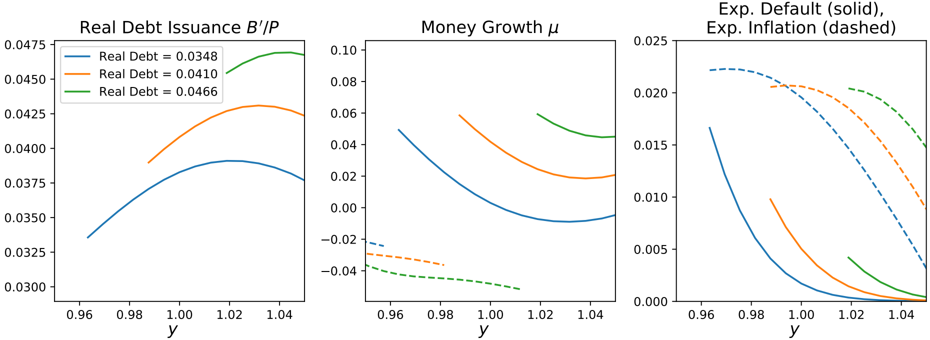
Parameters selected to match targets

Variable		Value	Target	Data	Model
Govt discount factor	β	0.65	Debt service/GDP	0.058	0.041
Household discount factor	β_h	0.997	Risk-free rate	0.073	0.067
MIU constant	α_m	2e-5	Monetary base/GDP	0.098	0.103
MIU constant (govt)	α_ν	8e-4	CPI Inflation	0.049	0.057
Public good utility constant	α_g	8e-4	c/g ratio	3.67	3.64
Default cost parameter	d_0	-0.07	Default prob. (mean)	0.045	0.033
Default cost parameter	d_1	0.0975	Default prob. (st. dev.)	0.020	0.027
Tax rate	τ	0.215	CV(Seignorage)		10

Equilibrium Policy and Prices

Non-targeted moments

Moment	Model	Data
$\rho(DP_t, XCS_t)$	0.43	0.46
$\rho(y_t, XCS_t)$	-0.73	0.02
$\rho(y_t, DP_t)$	-0.53	-0.2
$\rho(DP_t, \pi_t)$	0.34	0.31



Takeaways

Counter-cyclical inflation

- Consistent with evidence in emerging market economies
- In bad times, strong motive to finance g with inflation tax
  - not there with lump-sum taxation
- Matches co-movement btw default risk - inflation risk - realised inflation

Co-movement of inflation & default spreads

- Exacerbates time inconsistency $\rightarrow$ debt is costly when most needed
- Trade-off: insurance benefit vs. time-consistency costs relevant
  - Debt denomination
  - Central bank independence vs. fiscal flexibility

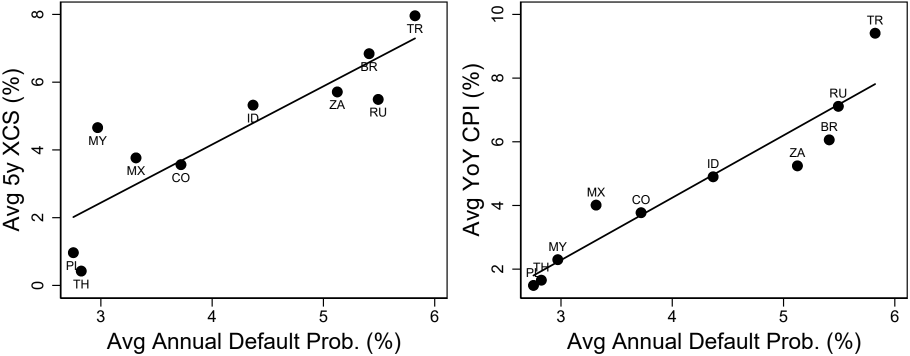
Conclusion

- Default risk co-moves with inflation risk, realised inflation and exchange rates
- Theory of monetary financing to match the data, debt dilution alone not enough
- Implications for debt currency denomination and fiscal-monetary interactions in economies with default risk

Appendix

Fact 1: Long-Run, Cross-Country

Cross-country averages for the period 2010q1-2018q4



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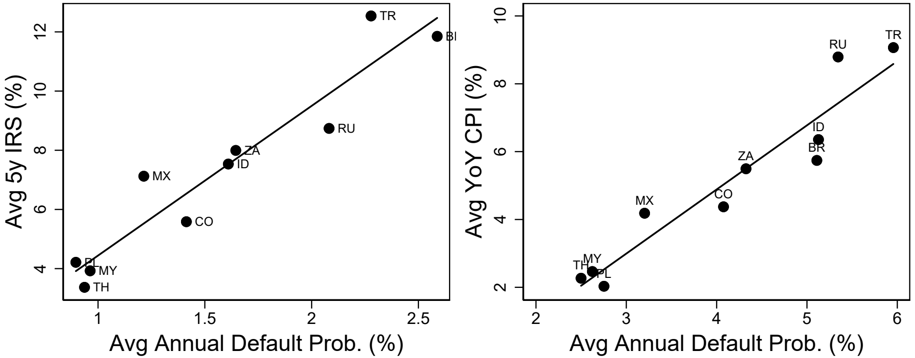
Fact 1: Long-Run, Cross-Country

Cross-country averages for the period 2004q1-2018q4



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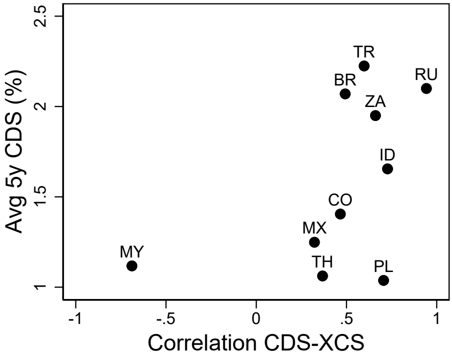
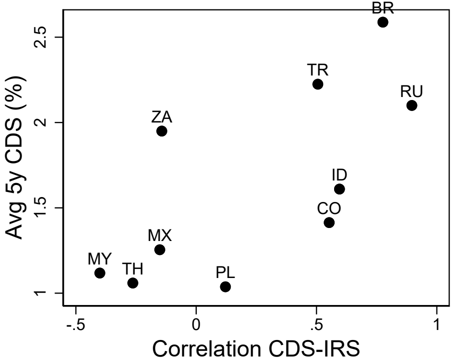
Fact 2: More Time-Series Correlation





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Data: Local-Currency Debt Focus

	Total Debt (% of GDP)	Foreign-Currency Debt (% of Total)
Brazil	66.4	5.5
Colombia	39.2	28.6
Indonesia	33.2	41.0
Mexico	33.8	27.4
Malaysia	48.1	6.6
Poland	50.2	25.5
Russia	13.9	30.4
Thailand	27.3	2.3
Turkey	38.4	34.2
South Africa	38.7	11.4

Source: World Bank Quarterly Public Sector Debt database.

- LC defaults as frequent as FC defaults
  - (post'97: 40 events, 35% FC, 25% LC, 32% both)
  - (post'75: 63 events, 43% FC, 33% LC, 24% both)
- Same credit ratings on LC & FC debt

(source: Moody's sector in-depth (02/04/2019)) Back

Descriptive Statistics (2004m1-2019m2)

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	CPI yoy	FX yoy	IRS 5y	CDS 5y	Debt/GDP (%)	FC Debt Share (%)	Ext Debt Share (%)
BR	5.7 (1.8)	3.1 (19.3)	9.2 (1.9)	2.2 (1.3)	66.4	5.5	13.3
CO	4.4 (1.7)	1.3 (15.1)	6.5 (1.8)	1.8 (1)	39.2	28.6	37.7
ID	6.4 (3.4)	3.9 (9.8)	8.4 (2.3)	2.0 (1.2)	33.2	41.0	55.1
MX	4.2 (1)	4.4 (11)	7.1 (1.6)	1.2 (0.6)	33.8	27.4	30.6
MY	2.5 (1.6)	0.8 (8.2)	3.8 (0.4)	1.1 (0.4)	48.1	6.6	27.1
PL	2.0 (1.7)	0.7 (15.4)	4.2 (1.6)	1.1 (0.6)	50.2	25.5	44.7
RU	8.8 (3.7)	6.6 (20.3)	8.0 (3.2)	2.2 (1.3)	13.9	30.4	29.2
TH	2.3 (2.2)	-1.5 (6)	3.0 (1)	1.1 (0.5)	27.3	2.3	11.0
TR	9.1 (3)	9.6 (16.5)	11.3 (3.8)	2.4 (0.9)	38.4	34.2	30.2
ZA	5.5 (2.3)	5.1 (14.8)	8.0 (1.1)	1.6 (0.8)	38.7	11.4	27.7

Variance Decompositions

Country	R^2	IRS %	CDS %	Covariance %
BR	0.68	64	14	22
CO	0.50	78	6	15
ID	0.71	72	4	24
MX	0.86	100	0	0
MY	0.54	91	6	3
PL	0.82	85	7	8
RU	0.20	12	50	38
TH	0.73	98	1	1
TR	0.78	59	10	31
ZA	0.91	93	1	6

Table: Time series regression and variance-covariance decomposition of 5y LC bond yields monthly changes, for the period Jan 2004 - Feb 2019. HAC robust standard errors used in all regressions, significance levels indicated by *** ($p<0.01$), ** ($p<0.05$), * ($p<0.1$).

Asset Price Details: Default Risk

CDSs:

- Pay a periodic premium (spread) in exchange for default "insurance"
- Credit event: change in interest, principal, postponement of interest/principal, change in currency or seniority
- Upon credit event: protection buyer has option to deliver to seller an **acceptable** bond in a **permitted** currency
- Deliverable currencies typically include USD, EUR, YEN; GBP, CHF, CAD, AUD

Moody's sector in-depth (2019)

- LC defaults as frequent as FC defaults
  - post'97: 40 events, 35% FC, 25% LC, 32% both
  - post'75: 63 events, 43% FC, 33% LC, 24% both
- Same credit ratings on LC & FC debt

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CDS-Implied Default Probabilities

- Survival prob. with default intensity $\lambda(t)$: $S(t) = Pe^{-\int_0^t \lambda(u)du}$
- Premium leg: PV of all premium payments

$$PV_{prem} = \mathbb{E}\int_0^T DF(t)U_{par}\mathbb{1}[T_1 > t] = U_{par}\int_0^T DF(t)S(t)dt.$$

- Protection leg: PV of LGD, at random time $T_1 | T_1 < T^{expiry}$

$$PV_{prot} = \mathbb{E}\left\{DF(T_1) \times LGD \times \mathbb{1}[T_1 \leq T]\right\} = LGD\int_0^T DF(t)S(t)\lambda(t)dt.$$

- Par spread is given by

$$U_{par} = \frac{LGD \ \int_0^T DF(t) \ S(t)\lambda(t)dt}{\int_0^T DF(t)S(t)dt}.$$

- Assume: constant hazard rate $(\lambda(t) = \lambda)$: $\lambda = \frac{U_{par}}{LGD}$
- Default probability thus given by

$$\text{DefProb}_t = 1 - S(t) = 1 - e^{-\lambda t} = 1 - e^{-\frac{U_{par}}{LGD}t}.$$

Back to LR Facts Back to CDS

Asset Price Details: Inflation Risk

IRSs:

- pay/receive periodic fixed rate for local LIBOR ($\approx$ key CB rate)
- constant maturity, fully collateralised OTC derivatives

Fixed-for-Fixed Cross-Currency Swaps (Du-Schreger, 2016):

- when Non-Deliverable Cross-Currency Swaps are available
  - NDS fixed-for-floating: LC fixed $\leftrightarrow$ USD LIBOR
  - Plain USD IRS: USD LIBOR $\leftrightarrow$ USD fixed
- when Cross-Currency Swap Basis is available
  - Plain LC IRS: LC fixed $\leftrightarrow$ LC LIBOR
  - XC Basis: LC LIBOR $\leftrightarrow$ USD LIBOR
  - Plain USD IRS: USD LIBOR $\leftrightarrow$ USD fixed

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Repayment Problem

- Plugging in $q(y, \tilde{B}')$ and $m := 1/\tilde{P}$ simplifies the resource constraint to

$$y + \underbrace{\frac{\mathbb{E}^q(\tilde{B}')\tilde{B}'}{R^*}}_{dr(\tilde{B}')} - \tilde{B}m = c + g$$

where

$$\mathbb{E}^q(\tilde{B}') = \mathbb{E}\left[(1-\delta')m'_R + \delta' q_D(y', \tilde{B}')m'_D\right]$$

- Households' real money demand (omitting y)

$$(1+\mu)m = \mathcal{M}^d(c, \tilde{B}') := \frac{\beta_h}{U_c}\mathbb{E}\left[(U'_c + U'_m)m'\right]$$

- Plug $\mathcal{M}^d$ into hh BC yields $m(c, \tilde{B}')$

$$c + \mathcal{M}^d(c, \tilde{B}') = m + y(1-\tau)$$

- Get $\mu(c, \tilde{B}')$ from either money demand or hh BC

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Default Problem

- Households' real money demand

$$(1+\mu)m = \frac{\beta_h}{u'(c)}\mathbb{E}\left[(U'_c + U'_m)m'\right]$$

- Combining it with the hh BC, get

$$c(\mu) = \left\{c: \ u'(c)[y_D(1-\tau)-c] = \beta_h \frac{\mu}{1+\mu}\mathbb{E}\left[(U'_c + U'_m)m'\right]\right\}$$

- Get $m(\mu)$ from either money demand or hh BC

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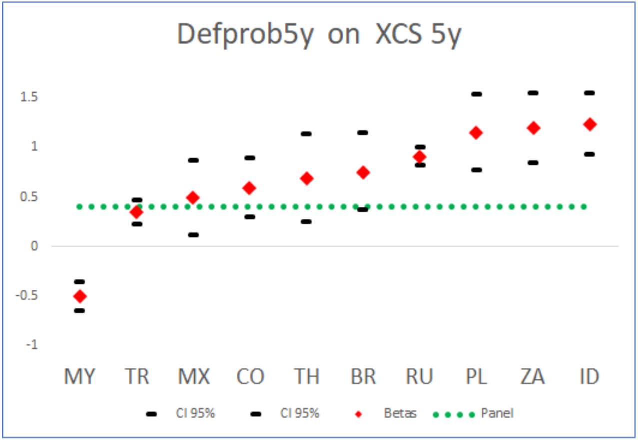
Controlling for a Global Factor



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Private Sector Equilibrium

Definition (Private Sector Equilibrium (PSE))

Given $\mathcal{S} := (\tilde{B}, y; \delta, g, \tau, \mu, \tilde{B}')$, a symmetric PSE consists of

- Household policies $c(\mathcal{S})$, $\tilde{M}'^{d}(\mathcal{S})$ and $\tilde{B}'^{d}(\mathcal{S})$,
- The risk-free rate $R(\mathcal{S})$ and the inverse of the price level $m(\mathcal{S})$

such that:

1. Policies solve the household problem;
2. Market clearing: money balances ($\tilde{M}'^{d} = 1$), domestic bonds ($\tilde{B}'^{d} = 0$).

Back

Lenders conditions, recursive formulation

- Inflation $1+\pi' := \frac{\tilde{P}'(1+\mu)}{\tilde{P}}$
- Price of new debt, upon repayment (omitting y)

$$q(\mathcal{S}) = \frac{1}{R^*}\frac{\tilde{P}_R(\mathcal{S})}{1+\mu}\mathbb{E}\left[\frac{1-\mathcal{H}_D(y',\tilde{B}')}{\tilde{P}'_R(y',\tilde{B}')} + \mathcal{H}_D(y',\tilde{B}')\frac{q_D(y',\tilde{B}')}{\tilde{P}'_D(y',\tilde{B}')}\right]$$

- Price of defaulted debt

$$q_D(\mathcal{S}) = \frac{1}{R^*}\frac{\tilde{P}_D(\mathcal{S})}{1+\mu}\mathbb{E}\Big\{(1-\theta)\frac{q'_D(y',\tilde{B}^n)}{\tilde{P}'_D(y',\tilde{B}^n)} + \theta(1-h)\left[\mathcal{H}_D(y',\tilde{B}^o)\frac{q'_D(y',\tilde{B}^o)}{\tilde{P}'_D(y',\tilde{B}^o)} + \frac{1-\mathcal{H}_D(y',\tilde{B}^o)}{\tilde{P}'_R(y',\tilde{B}^o)}\right]\Big\}$$

where $\tilde{B}^n := \tilde{B}/(1+\mu), \quad \tilde{B}^o := (1-h)\tilde{B}/(1+\mu)$

- Default probability $DP(y,\tilde{B}') = \mathbb{E}_{y'|y}\mathcal{H}_D(y',\tilde{B}')$
- Expected inflation

$$XCS(\mathcal{S}) = \frac{1+\mu}{\tilde{P}(\mathcal{S})}\mathbb{E}_{y'|y}\{\mathcal{H}_D(y',\tilde{B}')\tilde{P}'_D(y',\tilde{B}') + [1-\mathcal{H}_D(y',\tilde{B}')]\tilde{P}'_R(y',\tilde{B}')\}$$

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Money Demand Elasticity

Taking the money demand equation

$$i_{d,t+1} = \mathbb{E}_t \frac{\alpha_m (m_{t+1})^{-\eta}}{c_{t+1}^{-\gamma}}$$

and linearising

$$\mathbb{E} \log(M_{t+1}/P_{t+1}) = \frac{const}{\eta} + \frac{\gamma}{\eta} \mathbb{E} \log c_{t+1} - \frac{1}{i_d \eta} i_{d,t+1}$$

which implies

- Semi-elasticity $= \frac{1}{i_d \eta}$
- Elasticity $= \frac{1}{\eta}$

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Inflation Expectations Cyclicality

- Defined as

$$\frac{\partial XCS(y,B')}{\partial y} = \frac{\partial}{\partial y}\int [\delta'\pi'_D + (1-\delta')\pi'_R] f(y',y)dy'$$

$\rightarrow$ to co-move with default risk, need counter-cyclical XCS

Decompose

$$= \overbrace{\frac{\partial \tilde{B}'}{\partial y}}^{>0} \int \overbrace{\frac{\delta'\partial\pi'_D + (1-\delta')\partial\pi'_R}{\partial \tilde{B}'}}^{>0} dF(y'|y) \qquad (a)\ \frac{\partial \pi'}{\partial \tilde{B}'} \text{ effect: } > 0$$

$$\overbrace{+\int^{\hat{y}} \pi'_D \frac{\partial f(y'|y)}{\partial y} dy' + \int_{\hat{y}}^{y} \pi'_R \frac{\partial f(y'|y)}{\partial y} dy'}^{\text{P-mass}\uparrow} + \overbrace{\int_{y} \pi'_R \frac{\partial f(y'|y)}{\partial y} dy'}^{\text{P-mass}\downarrow} \qquad (b)\ \frac{\partial \pi'}{\partial y'} \text{ effect:} < 0?$$

$$- \underbrace{\frac{\partial \tilde{B}'}{\partial y}\frac{\partial \hat{y}}{\partial \tilde{B}'}}_{>0} [\pi'_R(\tilde{B}',\hat{y}) - \pi'_D(\tilde{B}',\hat{y})] f(\hat{y}|y) \qquad (c) \text{ cutoff effect}$$

Back

MIU Wedge

The benchmark model FOC yield

$$U_c = U_g$$
$$U_m = U_g \tilde{B}$$

Household money demand

$$R - 1 = \mathbb{E} \frac{U_m'^{hh}}{U_c'^{hh}}$$

Combining the two equations

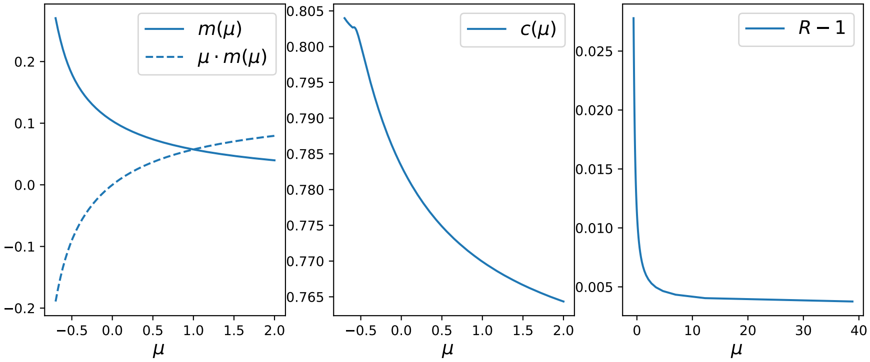
$$R - 1 = \tilde{B}' \mathbb{E} \frac{U_m'^{hh}}{U_m'}$$

Back to Govt Problem Back to FOCs

Money Growth and Seignorage

- Recall HH money demand $R - 1 = \mathbb{E}\frac{\alpha_m (M'/P')^{-\eta}}{c'^{-\gamma}}$
- An increase in μ or in M'
  - $\downarrow R, c \Rightarrow \downarrow$ real money demand
  - $\downarrow$ m, $\uparrow$ seignorage $m\mu$

Changes in Money Growth μ, at $(y = 1, \tilde{B} = 0.4)$



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