Inflation, Default Risk and Nominal Debt*

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Abstract

This paper explores the trade-off between strategic inflation and default for a set of large emerging market economies that borrow mostly in their local currency. Using over-the-counter derivatives data, I find a robust, positive correlation between default risk, inflation risk, and realised inflation. I use these facts to discipline a quantitative sovereign default model where a government issues debt in domestic currency and lacks commitment to both fiscal and monetary policy. I show that simple models of debt dilution via default and inflation have counterfactual implications, as default and inflation are substitutes and co-move negatively. I highlight the role that monetary financing plays to match the data, allowing inflation to serve a second purpose: in bad times, seignorage is especially useful as a flexible source of funding when other margins may be hard to adjust. The model matches the positive correlation between inflation and default risk, and allows to quantitatively evaluate its implication for the trade-off between the insurance benefits of nominal debt and the ex-ante cost of a further source of time inconsistency.

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1 Introduction

In the last two decades, many emerging market (EM) governments significantly tilted the currency composition of their public debt from foreign to local currency.[1] Borrowing in local currency makes inflation an additional instrument for public debt management, on top of repayment through fiscal surpluses and outright default. This raises the questions of what is the interplay between the temptations to default on sovereign debt or to inflate it away, and how these incentives shape macroeconomic policies in EM. The inflation and default spreads embedded in government bond interest rates have a critical role in determining the trade-off between the ex-post benefits and the ex-ante costs of these policies, in the presence of time inconsistencies. A key empirical regularity in the sovereign default literature is the counter-cyclicality of default spreads, which constrains borrowing in situations where the government needs it the most. Whether inflation spreads display the same or the opposite feature has crucial implications for the ability of the issuer to use debt policy as a way to smooth shocks over time.

This paper studies in detail the relationship between strategic inflation, default and inflation risk for a set of large EM sovereigns. A common argument regarding countries that borrow in their own currency is that they need not default on their debt, because they can always resort to the printing press in case of need. I show that in the data, despite the shift to local-currency debt, default risk for these countries remains non-negligible and displays a robust, positive relationship with realised and expected inflation. I use these facts to discipline the behaviour of default and inflation spreads in a quantitative sovereign default model where a government issues debt in domestic currency and lacks commitment to both fiscal and monetary policy. I find that, to reconcile the model with the data, it is important to account for the role of inflation as a tool to raise fiscal revenues, especially in periods when other margins may be hard to adjust. The model allows to quantitatively evaluate the trade-off between the insurance benefits of nominal debt and the cost of a further source of time inconsistency, when inflation and default risks co-move.

In Section 2, I document a number of stylised facts on the relationship between default risk, inflation risk, realised inflation and exchange rate depreciation for a set of ten large EM countries. I exploit the availability of over-the-counter derivatives that price default and currency risks separately: I use Credit Default Swaps (CDSs) as an indicator of default risk, and fixed-for-

[1] See Du and Schreger (2016b) and Ottonello and Perez (2019).

fixed Cross-Currency Swaps (XCSs) as an indicator of the expected depreciation of a currency against the US dollar, which I use as a proxy for expected inflation. Using different assets has the advantage of avoiding an econometric decomposition of local-currency sovereign spreads into default and currency premia, and addresses liquidity problems in government debt markets as the derivatives I analyse are standardised and liquid.

I highlight three novel facts that emerge from the data. First, I look at long-run averages across countries: I find that countries with high default risk display high inflation levels, both realised and in expectation. Second, I show that, within each country, inflation and default risk are positively correlated at quarterly frequencies. This relationship is robust to controlling for global risk factors that may drive investors' risk premia. Third, I find that default risk also co-moves, within each country, with realised inflation and exchange rate depreciation.

Based on this evidence, I develop a quantitative sovereign default model with nominal debt to study the joint behaviour of default risk, expected and realised inflation. The model is a version of Arellano (2008) where external debt is denominated in domestic currency and the government lacks commitment to both fiscal and monetary policy. I follow the literature in assuming that inflation is a continuous instrument with convex costs, while default is a binary choice that entails a fixed output cost and temporary exclusion from debt markets.

First, I test the simplest version of the model, where inflation only serves the purpose of diluting the real value of debt, an assumption common to both sovereign default and monetary models. A priori, it is not obvious whether inflation and default risks co-move positively. After a bad shock, the temptation to inflate and reduce the debt burden is stronger, as long as default does not occur, but at the same time the government gets closer to a default, after which debt is reduced via a haircut and inflation incentives are weaker. I evaluate these mechanisms quantitatively, and find that the model predicts a negative correlation between inflation and default risks, since inflation and default are substitute instruments. Moreover, the model generates low levels of inflation upon default. These results are at odds with my empirical findings, and with the fact that sovereign defaults are generally followed by periods of abnormally high inflation.[2]

Second, in Section 3 I highlight the role that monetary financing (or seignorage) has in reconciling the model with the data, by allowing inflation to serve a second purpose. I augment

[2]See Na et al. (2018), Reinhart and Rogoff (2009, 2011) and references therein.

the model with endogenous government spending and money in the utility function: the latter creates a role for money and allows to model inflation costs explicitly, the former generates a motive for the government to use seignorage as a tax instrument to transfer resources from the private to the public sector.

In the model, inflation therefore serves a dual purpose: it is both a tax on foreign lenders, as it dilutes the real value of external debt when it is unanticipated, and a tax on domestic money holdings, which is especially useful when other margins may be hard to adjust, such as during the periods of autarky following a default. The relative importance of these two functions, as well as the way in which they are embedded into expectations and sovereign bond spreads, are crucial for the ability of this framework to generate the co-movement between inflation and default risk that I observe in the data. This in turn hinges on two fundamental properties of the model: the correlation between inflation and the cycle, and the behaviour of inflation upon default. Inflation cyclicality is driven on one hand by the cyclical properties of external debt, and on the other hand by the strength of the incentive to use the inflation tax to smooth public spending over the cycle. The behaviour of inflation upon default depends, as in the benchmark model described previously, on the relative importance of the debt dilution and the tax motives. Default happens in bad times, where the need to smooth spending via the inflation tax is high; at the same time, a default wipes off a substantial fraction of debt, which reduces the incentives to further dilute it with inflation.

I quantitatively evaluate the importance of these forces with a numerical example, and I find that, when public good demand is sufficiently inelastic, the tax motive is strong, reduces the repayment-default inflation differential, and allows the model to match the co-movement of inflation and default risks that I observe in the data. Although an exact calibration of the model is still in progress at the moment, I provide an analytical decomposition of the mechanism that drives the asset price co-movement, and highlight in detail the conditions under which its behaviour mirrors the data.

The sovereign default literature has mostly studied the implications of default risk in real models. I complement this analysis by studying the way in which default premia interact with inflation premia, when the government cannot commit to monetary policy. I use my empirical findings to discipline this relationship within a framework where there exists a single policymaker.[3] The model allows to quantitatively evaluate the trade-off between the benefits of nominal debt, via the use of inflation as a way to obtain state-contingent real returns, and the cost of a further source of time inconsistency when inflation and default risks co-move. My findings suggest that the expectation of inflation as a source of fiscal revenues, both during repayment and in default periods, has important implications for the conduct of monetary policy. This is a natural starting point to think about the institutional relationship between the fiscal and the monetary authority and its credibility, and to discipline time-consistent models of fiscal-monetary interactions.

Relation to the Literature. This paper relates to several strands of the literature on sovereign default, monetary and exchange rate policy.

A literature that dates back to the seminal work of Calvo (1988) analyses time-consistent monetary and fiscal policy with sovereign default, considering the role of inflation and exchange rate devaluation as an implicit way to default on local-currency debt, and their interplay with explicit default. A number of recent papers have addressed this issue by embedding a monetary side into real sovereign default models in the tradition of Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006) and subsequent work. Nuño and Thomas (2015) and Aguiar et al. (2014) consider a planning problem and study the trade-off between the ex-post benefits and the ex-ante costs of discretionary inflation as a way to dilute the real value of debt. Closest to my work are Roettger (2019) and Sunder-Plassmann (2018), who exclude lump-sum taxation and consider the distortions created by monetary policy in an optimal policy framework. I contribute to the literature in a number of ways. First, I show that the simple planner version of this framework is not well-suited to the study of EM economies, because it does not match the asset price facts observed in the data. Second, I model the monetary side of the economy in a more flexible way, to allow for realistic money demand elasticities. Third, I analyse in detail the asset pricing implications of the model and their real effects, isolating the mechanisms that are key to reconcile the model with the data.

There are other papers that consider the relationship between inflation and default when debt is nominal, but take quite different approaches. Araujo et al. (2013), Aguiar et al. (2013, 2014),

[3] Or where the objective function of the fiscal and monetary authority coincide.

Corsetti and Dedola (2016) and Bassetto and Galli (2019) examine the role of inflation as partial default within the context of self-fulfilling runs on government debt. By contrast, I abstract from belief-driven crises and follow most of the quantitative literature in focusing on defaults driven by fundamentals. Engel and Park (2018), Ottonello and Perez (2019) and Du et al. (2016) study the currency composition of debt when the government lacks commitment to repay and to inflate, in order to rationalise the recent surge in local-currency borrowing. I abstract from this margin for reasons of tractability, and focus on countries that have self-selected into issuing most of their debt in local currency.

A recent body of work studies optimal default and monetary policy in economies with nominal rigidities. Na et al. (2018) show how downward wage rigidities can rationalise the joint occurrence of defaults and large exchange rate devaluations: during a default, optimal exchange rate policy calls for a reduction in the real value of wages that stimulates employment. Bianchi et al. (2019) and Bianchi and Mondragon (2018) consider a similar framework to respectively study optimal fiscal policy and self-fulfilling debt runs. Arellano et al. (2019) embed an external default model within a New Keynesian open economy framework with commitment on the monetary side, and study the co-movement of default spreads with realised inflation and short-term nominal rates. In these papers, inflation either creates deadweight losses (in the case of price rigidities) or reduces involuntary unemployment (in the case of wage rigidities), while it does not provide debt relief since debt is assumed to be denominated in foreign currency. By contrast, I study a framework where inflation has the different functions of debt dilution and source of revenues for the fiscal authority.

Finally, the treatment of inflation as a source of fiscal revenues in this paper is also related to the literature on currency and balance-of-payment crisis, dating back to the seminal contribution of Krugman (1979) and the large body of subsequent work. With respect to this class of models, I consider default, and I model endogenously the reason behind the use of seignorage revenues to fund fiscal deficits.

The paper proceeds as follows: Section 2 presents a number of stylised facts about the relationship between default risk, currency risk, and realised inflation; Section 3 presents the model environment; Section 4 illustrates the main mechanisms of the model; Section 5 analyses the quantitative performance of the model; Section 6 concludes.

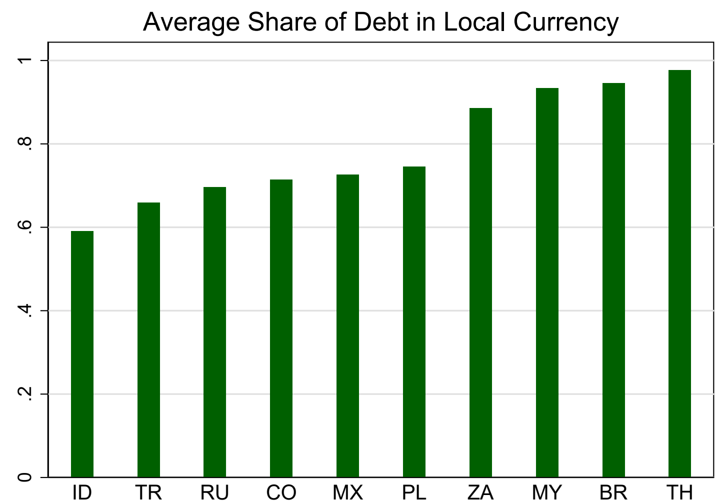

Figure 1: Average share of local-currency total public debt by country over the period 2004-2018. Country labels: Brazil (BR), Colombia (CO), Indonesia (ID), Mexico (MX), Malaysia (MY), Poland (PO), Russia (RU), Thailand (TH), Turkey (TR), South Africa (ZA).

2 Empirical Observations

This section documents a number of facts about the relationship between default risk, currency risk, realised inflation and exchange rate depreciation. Data are quarterly series. Most of the data is for the period 2004q1-2018q4, although some data series for some countries start later. The countries I consider are: Brazil, Colombia, Indonesia, Mexico, Malaysia, Poland, Russia, Thailand, Turkey, South Africa. These countries are chosen on the basis of data availability, but they also share two important features: first, they all have freely floating or managed floating exchange rates, according to the categorisation of Ilzetzki et al. (2019); second, a large share of their debt is denominated in local currency, as illustrated by Figure 1 which plots the average share of local-currency total public debt over the period 2004-2018.

The data sources are the following. All data on derivatives prices, government bond interest rates, inflation and exchange rates is taken from Bloomberg. Inflation is defined as the annual change in the national consumer price index. Output and debt data are taken from the World Bank, the IMF, and where necessary from national statistical offices.

To measure default risk, I use credit default swaps. These are OTC derivatives that quote the premium (commonly called spread) that investors can pay in order to fully insure themselves against a credit event on a country's government debt. Credit events include a set of circumstances that are normally associated with default and debt restructuring, such as postponements or cancellation of interest or principal payments. A number of features make these derivatives a particularly compelling measure of default risk: first, they are denominated in US dollars, which means that the payoff of the instrument is insulated from the value of the issuer's currency and its expected correlation with a default episode; second, they are based on bonds issued under international law, which shields them from country-specific idiosyncracies and capital control legislation;[4] third, they are standardised instruments, which gives them a constant maturity and makes them more liquid than foreign currency government debt; fourth, for those countries where sufficiently good data is available, I show in the appendix that CDSs are highly correlated with foreign currency bond spreads. Lastly, because mark-to-market positions in over-the-counter derivatives are collateralised on a daily basis, counterparty risk is not a concern for any of the derivatives data used in this paper. For the purpose of this analysis, I use CDS spreads for the five year maturity, which is the most liquid. To help interpret the data, I back out risk-neutral implied default probabilities assuming a constant default hazard rate function.[5]

To measure currency risk, I use the price of fixed-for-fixed cross-currency swaps (XCSs henceforth). For reasons of liquidity and consistency with the measure of default risk, I look a five year maturities. The XCS rate is essentially the long-term equivalent of the interest rate differential implied by exchange rate forwards. Assuming risk neutrality, I use this implied rate as a measure of the expected depreciation of a country's currency against the US dollar. Since these instruments are not directly quoted in financial markets, I follow Du and Schreger (2016a) and construct them by combining fixed-for-floating cross-currency swaps and local currency interest rate swaps. I use XCSs rather than exchange rate forwards because the latter are only generally liquid up to 12 month maturities, while the former are liquid up to 10 year maturities.

I now uncover a number of stylised facts regarding the relationship between default risk, currency risk and inflation. First, I document the long-run property of these variables, across

[4]The downside of this feature is that, in the case of a selective default only on domestic-law debt, these CDSs would not get triggered. However, default on international debt is widely believed to have a higher, if at all different, likelihood than default on domestic debt, so I consider the default risk embedded in these assets a lower bound.

[5]Explicit derivations are provided in the appendix.

countries. Second, I document their short-run properties, over time and within each country.

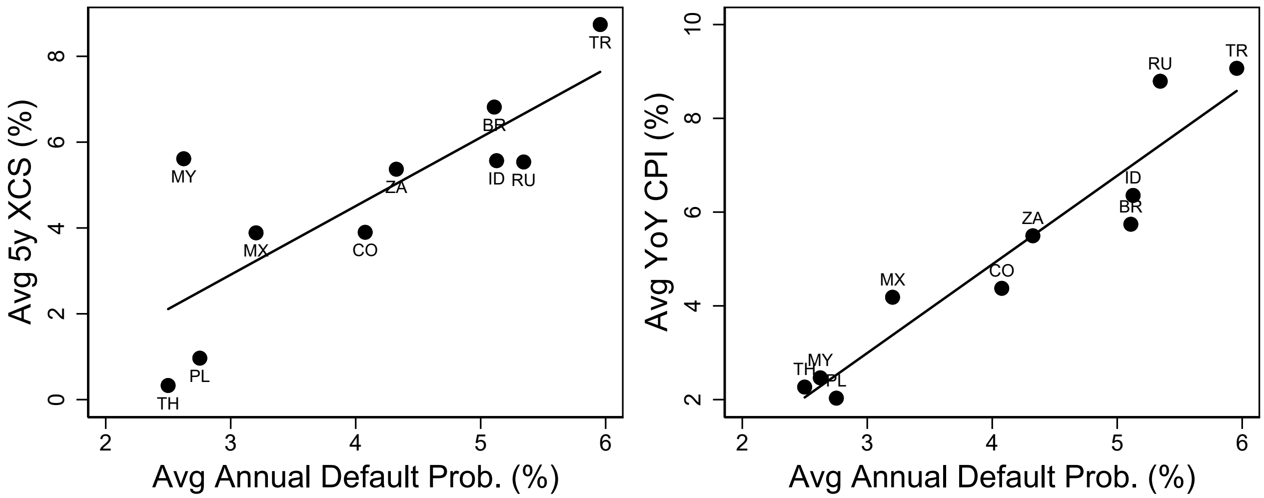

Figure 2: Long-term averages for the period 2004q1-2018q4. The left panel plots average default probabilities against average XCS rates. The right panel plots average default probabilities against year-on-year consumer price inflation.

Long-Run Facts. Figure 2 highlights two cross-country relationships: long-run default risk is positively correlated with long-run currency risk, as proxied by XCS rates (left panel), as well as with long-run realised CPI inflation (right panel). This implies that countries with historically high default probabilities tend to have historically high inflation and exchange rate depreciation, both realised and in expectation.

This finding is robust to the time interval considered, as long as it is of sufficient length: Figure 7 in the appendix shows the same picture for the period 2010q1-2018q4.

Short-Run Facts. The first short-run fact concerns the relationship between default and currency risk. Figure 3 shows, for each country, the quarterly correlation of CDSs and XCSs. Except for Malaysia, these correlations all suggesting that default and currency risks co-move not only at long-run frequencies, but also in the short-run, within each country. As the data on these asset prices is available at virtually any frequency, it is also possible to show that the positive correlations are there also at shorter time frequencies.

An obvious concern that could arise regarding this fact, is that this short-run co-movement is

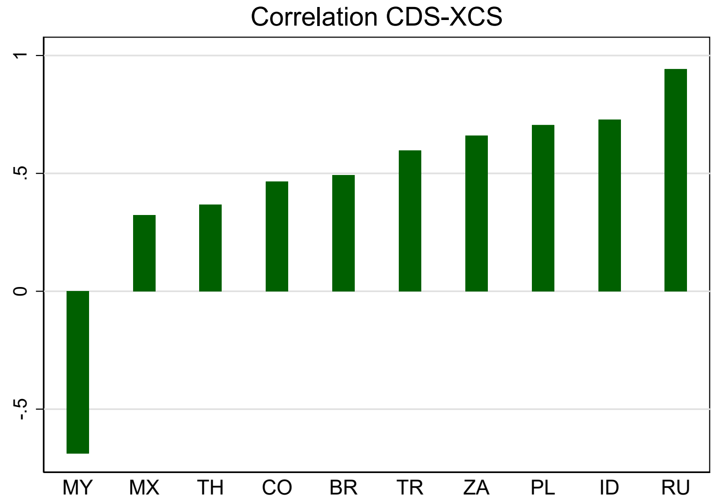

Figure 3: Short-run correlation of five year CDSs with five year XCSs, ordered by country. Quarterly data for the period 2004q1-2018q4.

driven by global risk factors that affect the discount factor of foreign investors trading both assets, rather than by country-specific variables. I check that this is not the case by running a panel regression of five year CDS-implied default probabilities against five year XCS rates, controlling for both country and time fixed effects. Controlling for time fixed effects should accounts for any common, time-varying component affecting the relationship between our measures of default and currency risk. The resulting linear coefficient on XCS rates is equal to 0.437, with a standard deviation of 0.096.[6] This lets us conclude that the relationship between default and currency risk still stands even after controlling for a global factor, and that a one percentage point increase in expected exchange rate depreciation is linked, on average, with an increase in the probability of default just below half a percentage point.

The second short-run fact concerns the relationship between default risk and nominal variables. Figure 4 shows, for each country, the quarterly correlation between one year absolute changes in CDSs, one year percentage changes in the nominal exchange rate (left panel), and CPI inflation (right panel). The figure highlights that not only default risk is associated with currency *risk*, as highlighted in the previous paragraphs, but also with the rate of change of the nominal exchange rate and the price level.

[6]Standard errors are clustered at the country level.

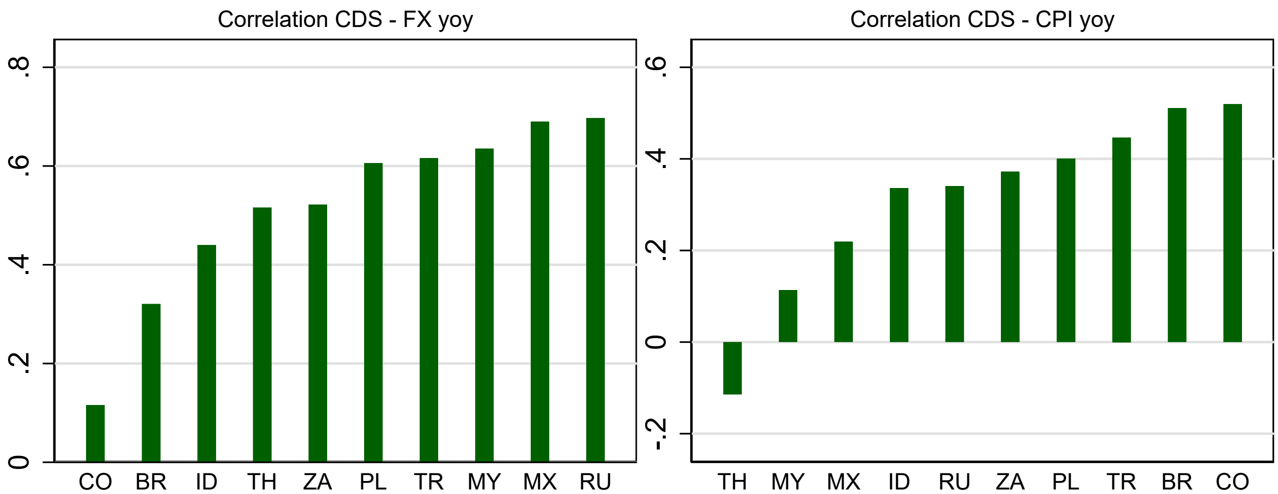

Figure 4: Short run correlations of one year absolute changes in CDSs (of five year maturity) with one year percentage changes in the nominal exchange rate (left panel), and CPI inflation (right panel). Quarterly data for the period 2004q1-2018q4, ordered by country.

These facts call for a joint analysis of the behaviour of inherently fiscal, such as default risk, and monetary issues, such as expected and realised inflation and exchange rate depreciation, as data for this sample of emerging market economies suggests these variables exhibit a significant co-movement both in the short and long run.

3 Model

I now consider a infinite-horizon, quantitative sovereign default model with money in the utility function, endogenous government spending, and where the government engages in strategic default and inflation. As will be explained more in detail later on, I will consider two versions of this framework, one in which the government can use lump-sum taxation (the "benchmark" model), and another one in which it cannot (the "restricted" model).

The environment I study is given by a small open economy that is populated by a benevolent government, a continuum of measure one of atomistic households, and where the government can trade bonds with a continuum of foreign, risk-neutral lenders. I now present in detail the problem of each of these players.

3.1 Households

Households get utility from consumption of the private good c_t, from the ratio M_t/P_t of nominal money balances M_t to the aggregate price level P_t, and from public good consumption g_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t U^h(c_t, M_t/P_t, g_t)$$

where the period-t utility function displays strong separability and is given by

$$U(c_t, m_t, g_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + \alpha_m \frac{m_t^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta}. \tag{1}$$

In each period, households take prices and government policy as given, and choose consumption c_t, money holdings M_{t+1}, and domestic bond holdings B_{t+1}^d. Domestic bonds are risk-free, pay the gross risk-free rate R_f, and are only traded among households. Household income is given by an exogenous stochastic endowment y_t that follows the AR(1) process

$$\log(y_t) = \rho_y \log(y_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right), \tag{2}$$

a fraction τ_t of which is paid in taxes to the government. The household budget constraint is given by

$$c_t + \frac{M_{t+1}}{P_t} + \frac{1}{R_t}\frac{B_{t+1}^d}{P_t} = \frac{M_t}{P_t} + \frac{B_t^d}{P_t} + y_t(1-\tau_t). \tag{3}$$

In their decision problem, households take government monetary, debt and spending policy as given. Combining the first-order conditions for consumption, money and bond holdings yields two standard Euler equations for money and domestic bonds

$$\frac{U_{c,t}^h}{P_t} = \beta \mathbb{E}_t \left[\frac{U_{c,t+1}^h + U_{m,t+1}^h}{P_{t+1}} \right] \tag{4}$$

$$\frac{1}{R_t}\frac{U_{c,t}^h}{P_t} = \beta \mathbb{E}_t \left[\frac{U_{c,t+1}^h}{P_{t+1}} \right] \tag{5}$$

where $U_{x,t}^h$ denotes marginal utility with respect to variable x in period t. Equation (4) is the households' money demand equation, and will have a crucial role in determining the equilibrium price level as a function of the government money supply.

Combining the two Euler equations we can express the domestic risk-free rate as a function of the expected future marginal rate of substitution between consumption and real money balances

$$R_t - 1 = \mathbb{E}_t \left[\frac{U_{m,t+1}^h}{U_{c,t+1}^h} \right]. \tag{6}$$

Under the current preferences specification there is no satiation point for real money balances, and therefore the Friedman rule only holds in the limiting case where $U^h_{m,t+1} \to +\infty$.

3.2 Government Problem

I study the problem of a single policymaker that encompasses both the fiscal and the monetary authority; in other words, government policy here can be thought of as the union of fiscal and monetary policy when the respective authorities act in a coordinated way and share the same objective.

It can borrow externally using debt instruments that are short-term, non-contingent, defaultable and denominated in local-currency. The government is benevolent and maximises the utility of households. Its objective function is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, g_t)$$

where the period-t utility function is given by

$$\frac{c_t^{1-\gamma}}{1-\gamma} + (\alpha_m + \alpha_\nu)\frac{m_t^{1-\eta}}{1-\eta} + \alpha_g \frac{g_t^{1-\zeta}}{1-\zeta}, \tag{7}$$

where c, m, g are the aggregate values of private good consumption, real money balances and public good consumption respectively.

There are two differences between equation (7) and the objective function of households in equation (1): first, I assume the government has a discount factor that is different from that of households; second, I assume that the government gets additional utility from the aggregate level of real money balances, which is an externality that agents do not take into account in their individual decision-making process, as they are atomistic and there is a continuum of them. The first assumption is needed in order for the model to be able to target both average government debt service and the domestic risk-free rate. The former depends crucially on the degree of impatience of the government, and the latter depends the rate of time preference of households. While it is not common to target domestic risk-free rates in sovereign default models, it is important to do so here: as is shown in Appendix B.2, the semi-elasticity of household money demand to the interest rate depends crucially on the level of such rate, which I discipline by targeting its empirical equivalent. The second assumption adds an additional cost of surprise inflation to the problem of the government. This represents an additional channel that corrects the government lack of commitment to inflation, and allows to drive a wedge between the domestic risk-free rate and the average debt-money ratio. These objects are otherwise identical in the benchmark model, as I show in detail in Section 4.

Timing. At the beginning of each period, the government can be either in good credit standing or in default, depending on its default history.

When it is in good standing, it first chooses whether to default or repay its debt due B_t. If it repays, it then chooses spending g_t, the money supply M_{t+1}, the tax rate τ_t and new debt B_{t+1}, which is issued to foreign lenders at a price of q_t. If the government decides not to repay, it switches to a default standing. When it is in default, the government is temporarily excluded from international debt markets, and the domestic economy incurs an output loss that reduces output to $y^d(y_t) \leq y_t$.

I assume that default is partial and the length of the exclusion period is random: in all periods that follow a decision to default, with probability θ the government gets a chance to re-enter debt markets at the condition of repaying a fraction $(1-h)$ of its outstanding nominal debt obligation. If it accepts, it repays the debt and re-enters debt markets; if it declines, it keeps its default standing, and its outstanding debt remains equal to a fraction $(1-h)$ of the amount due prior to the re-entry offer. This assumption has two implications: first, upon default debt effectively becomes long-term; second, the government has always the chance to remain in default for a period long enough that its debt obligations become arbitrarily small as it receives a sufficient number or haircuts. The latter implication is important to ensure that eventually the government always re-enters credit market: it could otherwise be possible that, depending on the size of the defaulted debt stock and on the level of inflation upon default, the government never finds it optimal to re-enter credit markets.

During default periods, the government still chooses spending, the money supply and the tax rate.

The government budget constraint during periods of repayment is given by

$$\tau_t y_t + q_t \frac{B_{t+1}}{P_t} + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + \frac{B_t}{P_t} + g_t \tag{8}$$

where all variables have been introduced in the previous paragraphs. The budget constraint of the government during periods of default is instead given by

$$\tau_t y^d(y_t) + \frac{M_{t+1}}{P_t} = \frac{M_t}{P_t} + g_t. \tag{9}$$

3.3 Lenders

Foreign lenders are risk-neutral, perfectly competitive, and have an opportunity cost of capital equal to the international gross risk-free rate R^*, which I assume constant for simplicity. They are indifferent with respect to the amount of government bonds they buy, as long as they make zero profits in expectation. The zero-profit price of a unit of government debt is given by

$$q_t = \frac{1}{R^*}\mathbb{E}_t\left[\frac{1-\delta_{t+1}}{1+\pi_{R,t+1}} + \frac{\delta_{t+1}\ q_{D,t+1}}{1+\pi_{D,t+1}}\right]$$

where δ_t is a default indicator that takes the value of 1 if the government chooses to default at t and zero otherwise, $\pi_{R,t+1}, \pi_{D,t+1}$ denote the net inflation rate $P_{t+1}/P_t - 1$ conditional on repayment and default respectively, $q_{D,t+1}$ denotes the price of debt upon a default in period $t+1$.

The value of debt upon default in some period t is in turn given by

$$q_{D,t} = \frac{1}{R^*}\mathbb{E}_t\left[(1-\theta)\frac{q^n_{D,t+1}}{1+\pi^n_{D,t+1}} + \theta\delta_{t+1}\frac{(1-h)q^o_{D,t+1}}{1+\pi^o_{D,t+1}} + \theta(1-\delta_{t+1})\frac{1-h}{1+\pi_{R,t+1}}\right].$$

The first term inside square brackets denotes the case, which I denote with superscript n, where the government does not receive an offer to re-enter credit markets. The second term denotes the case, which I denote with superscript o, where the government receives an offer to re-enter markets but decides to reject it, so it remains in default and its debt receives a haircut h. The third term denotes the case where an offer to re-enter is received and accepted.

From the debt price of new debt, it is easy to derive the model counterparts of the default and inflation risks I analysed in the empirical section. Expected default is given by

$$DP_t := \mathbb{E}_t\delta_{t+1}, \tag{10}$$

while expected inflation (or exchange rate depreciation, which are identical in the model) are given by

$$XCS_t := \mathbb{E}_t[\delta_{t+1}\pi_{D,t+1} + (1-\delta_{t+1})\pi_{R,t+1}]. \tag{11}$$

3.4 Equilibrium

I consider the time-consistent Markov-perfect equilibrium where the government internalises the effect of its policies on household allocations, current and future equilibrium prices, and future government policies.[7]

I drop time subscripts and move to a recursive formulation where x and x' respectively indicate the current and future value of variable x.

The only exogenous state variable in the model is given by the output shock y. The aggregate endogenous state variables are the stocks of government debt B and money M. A well known fact in the time consistent policy literature is that, in this class of models, the ratio of government debt to the money stock is a sufficient statistic for the government endogenous state.

Accordingly, I normalise all *nominal* variables by the aggregate stock of money M, and denote the normalised version of nominal variable X with $\tilde{X} := X/M$.[8] The current aggregate state is then given by the pair $(\tilde{B}, y)$.

In a Markov equilibrium, government policies only depend on the value of the current aggregate state variables. As households are atomistic, they take as given current and future private and government policies.

To describe the equilibrium, it is necessary to consider *current* government policy actions as well as its future policy functions.

Current government policy is the set of default, spending, money growth and future debt-to-money choices $s := (\delta, g, \mu, \tilde{B}')$. Government policy functions are given by a mapping from the aggregate state to policy choices $\mathcal{H} : (\tilde{B}, y) \to s$.

Households move after the government, and their actions depend on the aggregate state as well as on government current and future policies. Let $\mathcal{S} := (\tilde{B}, y, s)$ summarise the aggregate state and current government policy, i.e. the variables that are relevant for the current equilibrium in the private sector. Analogously, $\mathcal{S}' := (\tilde{B}', y', s' = \mathcal{H}(\tilde{B}', y'))$ will summarise the future aggregate state and future government policy.

Definition 1 (Private Sector Equilibrium)**.** *Given aggregate state and current government poli-*

[7]In time-consistent Markov-perfect equilibria, the government takes as given the policies of its future self.

[8] It is worth noting that, under this normalisation, the gross inflation rate will now be given by $1 + \pi' = m(1 + \mu)/m'$, where μ is the net growth rate of money and $m := M/P$ is the real stock of money.

cies $\mathcal{S}$, and future government policies $\mathcal{H}$, a symmetric private sector equilibrium (PSE) consists of

- *Households' policies for consumption $c(\mathcal{S})$, money demand $\tilde{M}'^d(\mathcal{S})$ and domestic bond holdings $\tilde{B}'^d(\mathcal{S})$,*
- *The risk-free interest rate on domestic bonds $R(\mathcal{S})$ and the inverse of the price level $m(\mathcal{S})$,*[9]

such that:

1. *Households' policies are optimal, i.e. satisfy their budget constraint (3) and the Euler equations for money (4) and domestic bonds (5);*
2. *The markets for money balances and domestic bonds clear.*

Money market clearing simply requires that $\tilde{M}'^d = 1$. Domestic bonds are in zero net supply, so market clearing requires that domestic bond holdings are zero at all times.

Combining its conditions, we can summarise the PSE with the household budget constraint

$$c(\mathcal{S}) + (1+\mu)m(\mathcal{S}) = m(\mathcal{S}) + y(1-\tau) \tag{12}$$

and the money demand equation

$$(1+\mu)m(\mathcal{S}) = \frac{\beta_h}{U_c(\mathcal{S})}\mathbb{E}[(U_c'(\mathcal{S}') + U_m'(\mathcal{S}'))m'(\mathcal{S}')]. \tag{13}$$

Government Problem. I now characterise the recursive problem of the government. The government is benevolent, and chooses debt, monetary and spending policy to maximise households' utility, internalising the effect of its policies on the private sector equilibrium and on the price of debt. At this point it is worth recalling that, while the government does maximise households' static utility, it has a discount factor that is different from that of the households.

Let us first specify the bond price functions conditional on repayment and default, which are two endogenous object that the government takes into account when formulating its debt issuance decision.

[9] Aggregate real money balances m are effectively the inverse of the price level, normalised by the aggregate stock of money. Mathematically, $m = 1/\tilde{P} = M/P$.

The debt price for newly issued debt must satisfy the zero-profit condition for lenders given by

$$q(\mathcal{S}) = \frac{1}{1+r_f}\mathbb{E}_{y'|y}\left[\frac{1-\mathcal{H}_\delta(\mathcal{S}')}{1+\pi_R'(\mathcal{S},\mathcal{S}')} + \frac{\mathcal{H}_\delta(\mathcal{S}')q_D(\mathcal{S}')}{1+\pi_D'(\mathcal{S},\mathcal{S}')}\right] \tag{14}$$

where $\mathcal{H}_\delta$ denotes the future government default policy, and $1+\pi_i'(\mathcal{S},\mathcal{S}') = m(\mathcal{S})(1+\mu)/m'(\mathcal{S}')$ where $i = R$ in repayment and $i = D$ in default. The price of debt depends on the current state (through y, because of the persistence of output), current government policy (which determines the current price level and future default incentives), future states (which also determine future default incentives) and future policy (which affect the default decision as well as the price level and in turn future inflation).

Analogously, the price of defaulted debt is given by

$$\begin{aligned} q_D(\mathcal{S}) = \frac{1}{R^*}\mathbb{E}\Bigg\{&(1-\theta)\frac{q_D(\mathcal{S}_n')}{1+\pi_D'(\mathcal{S},\mathcal{S}_n')} \\ &+\theta(1-h)\left[\frac{\mathcal{H}_\delta(\mathcal{S}_o')q_D(\mathcal{S}_o')}{1+\pi_D'(\mathcal{S},\mathcal{S}')} + \frac{1-\mathcal{H}_\delta(\mathcal{S}_o')}{1+\pi_R'(\mathcal{S},\mathcal{S}_o')}\right]\Bigg\} \end{aligned} \tag{15}$$

where the $\{o, n\}$ subscripts respectively denote the case where the government receives or not a haircut on debt together with the offer to re-enter credit markets.

Let $V^R(\tilde{B}, y), V^D(\tilde{B}, y)$ denote respectively the value of repayment and default for the government. The value of the option to default is given by

$$V(\tilde{B}, y) = \max_\delta \left\{(1-\delta)V^R(\tilde{B}, y) + \delta V^D(\tilde{B}, y)\right\}.$$

During periods of repayment, the value of the government is given by

$$V^R(\tilde{B}, y) = \max_{g,\mu,\tilde{B}'} \; U(c(\mathcal{S}), m(\mathcal{S}), g) + \beta\mathbb{E}_{y'|y}V(\tilde{B}', y')$$

subject to the PSE conditions with $\mathcal{S} = (\tilde{B}, y, \delta = 0, g, \mu, \tilde{B}')$ and the small open economy resource constraint[10]

$$y + q(\mathcal{S})\tilde{B}'(1+\mu)m(\mathcal{S}) = c(\mathcal{S}) + g + \tilde{B}\; m(\mathcal{S}). \tag{16}$$

[10]The non-normalised equivalent of the resource constraint is given by the (perhaps more familiar)

$$y + q\frac{B'}{P} = \frac{B}{P} + c + g.$$

The value of default is given by

$$V^D(\tilde{B}, y) = \max_{g,\mu} \ U(c(\mathcal{S}), m(\mathcal{S}), g) + \beta \mathbb{E}_{y'|y} \left[\theta V \left(\frac{\tilde{B}(1-h)}{1+\mu}, y' \right) + (1-\theta) V^D \left(\frac{\tilde{B}}{1+\mu}, y' \right) \right]$$

subject to the PSE conditions with $\mathcal{S} = (\tilde{B}, y, \delta = 1, g, \mu, \tilde{B}' = \tilde{B}/(1+\mu))$ and the autarky resource constraint upon default

$$y^d(y) = c(\mathcal{S}) + g.$$

I can now define the recursive equilibrium of the economy.

Definition 2 (Markov-Perfect Equilibrium). *A recursive equilibrium consists of*

- *government value functions* $V(\tilde{B}, y), V^R(\tilde{B}, y), V^D(\tilde{B}, y)$,
- *associated current government policies for default* $\delta(\tilde{B}, y)$, *spending* $g(\tilde{B}, y)$, *money growth* $\mu(\tilde{B}, y)$ *and borrowing* $\tilde{B}'(\tilde{B}, y)$
- *a PSE denoted by* $\mathcal{P}$

such that:

1. *Value and policy functions solve the government problem, given the aggregate state* $\{\tilde{B}, y\}$, *the debt price functions (14)-(15) and the PSE* $\mathcal{P}$.
2. $\mathcal{P}$ *is the PSE associated with government value and policy functions.*
3. *Current policy and value functions are consistent with future policy and value functions.*[11]

4 Model Analysis

In this section I characterise optimal policy for the government. As explained earlier, the government chooses borrowing and spending internalising the effect of these policies on household consumption and real money balances, via the price level, both in the current and in future periods.

[11] That is, $\delta = \mathcal{H}_\delta$ and likewise for $g, \mu, \tilde{B}'$.

4.1 Benchmark Model

Here I characterise optimality within the benchmark model where the government is assumed to be free to set the tax rate on households' endowment in every period. When evaluating the quantitative performance of this model, I will also assume that the curvature of the utility from public good consumption is equal to that of private good consumption. These two assumptions are made in order to make the model behave in a way that resembles existing papers in the literature on sovereign default and strategic inflation, such as Aguiar et al. (2014) and Nuño and Thomas (2015). I thus use this framework to assess how well the existing literature performs with respect to the facts uncovered in the previous empirical section.

When the government can set the tax rate, it has lump-sum taxation available. Lump-sum taxes are a source of domestic revenues that is preferable to money issuance, as the latter creates inflation which hurts private money holdings. In this case, the only purpose of issuing money domestically then becomes that of affecting the price level and in turn the real value of the government external debt obligations. It is of course true that creating inflation through the money supply will affect transfer resources from the private to the public sector, but the government will simply use lump-sum taxes or transfers to offset such effect, which allows the domestic economy to be at its first best.

Intra-temporal optimality, repayment. The static first-order conditions of the government problem with respect to private good consumption, public good consumption and real money balances can be summarised as

$$U_c = U_g \tag{17}$$

$$U_m = \tilde{B} U_c. \tag{18}$$

The first condition shows that, with lump-sum taxes, there is never a wedge between the marginal utility of the public and private good consumption. The second condition highlights the incentives to use inflation as a way to implicitly default on government external debt. The cost of generating surprise inflation is represented by the left-hand side, and is given by the presence of money in the utility function. The benefit of surprise inflation is given by a reduction in the real value of external debt repayment, which the government values at the marginal utility of consumption.

Comparing equations (18) and (6) highlights the rationale behind the introduction of α_ν in the government objective function: without a wedge between the cost of inflation for households and the government, the risk-free interest rate would equate at all times the debt-money ratio, which would not allow to obtain a realistic value for the semi-elasticity of money demand to the interest rate.[12]

Inter-temporal optimality, repayment. The inter-temporal optimality condition for the debt-to-money ratio is given by

$$q_{\tilde{B}'}\tilde{B}' + q = \beta \mathbb{E} \frac{U_c'}{U_c} \frac{m'}{m(1+\mu)}, \tag{19}$$

where $q_{\tilde{B}'}$ is the partial derivative of the price of government debt with respect to $\tilde{B}'$, and $\frac{m'}{m(1+\mu)}$ is the inverse of the gross inflation rate (cfr. footnote 8). Equation (19) is a Euler equation for defaultable debt that is standard in sovereign default models.

Optimality, Default. The first-order conditions of the government problem with respect to public good consumption, private good consumption, real money balances and the growth rate of the stock of money are given by

$$U_c = U_g \tag{20}$$

$$-m_{(\mu)} U_m = \frac{\partial \ \beta \mathbb{E}\left[(1-\theta)V^D\left(y', \frac{\tilde{B}}{1+\mu}\right) + \theta V\left(y', \frac{\tilde{B}(1-h)}{1+\mu}\right)\right]}{\partial \mu}. \tag{21}$$

As in the case of repayment, the condition (20) shows that the marginal utilities of private and public good consumption are equated at all times, when lump-sum taxation is available. During default periods, the government is in autarky and the resource constraint is only a function of real variables, so there is no direct relationship between the real and the monetary sides of the economy.

Condition (21) highlights the effects of changes in the growth rate of the money supply. First, an increase in money growth is given by a reduction in the future debt-money ratio, which will be valued by the government upon re-entry into international debt markets. This is represented by the term on the right-hand side. Second, money supply affects the price level, hence current

[12]See Section 3.2 for a discussion of this issue.

real money balances, through the household money demand equation. This is represented by the term on the left-hand side.

The reason why households' money demand enters in the default problem but not the repayment one, is that in repayment, the government effectively has two instruments to affect $\tilde{B}'$: debt issuance and money issuance. In default, the former is not available, so the money demand equation in (13) determines a one-to-one relationship between money supply and the value of money.

Discussion The static and dynamic optimality conditions highlight the similarities between the benchmark model and a part of the literature: first, the government is effectively facing a planner problem domestically; second, it uses external, defaultable debt to smooth household consumption over time; third, it sets inflation in order to reduce the real value of debt, at a cost which in this model is given by households money in the utility.[13] All these are common features of a number of sovereign default models that analyse the role of inflation as a substitute of default. As I show below, this class of models has troubles in matching the empirical facts I uncover in Section 2.

4.2 Restricted Model

In the restricted model, I assume that the tax rate is exogenous. This is an extreme assumption that greatly simplifies the analysis; however, what is important is that taxation cannot respond quickly to output shocks. My results would therefore still carry through in any setting where there exists some adjustment friction that constrains the speed at which taxes can respond to shocks.

When lump-sum taxes are not available, the need to finance a desired level of public good provision introduces an additional motive for the government to use inflation: the government can recur to seignorage as a way to raise domestically an amount of funds larger or smaller than the flow it receives exogenously from taxes. In this setting inflation thus acquires a second function, on top of its role as a way to make the real value of external debt state-contingent

[13] In Nuño and Thomas (2015) and Aguiar et al. (2014), the cost of inflation is given by a quadratic utility cost for the government.

ex-post. The main result of this paper derives from the fact that, once this additional function of inflation is taken into account, the co-movement between default risk, inflation risk and realised inflation goes back to being positive, and consistent with my empirical findings.

In this context, the Private Sector Equilibrium becomes a constraint to the government problem. As I show in Appendix B.3, the repayment problem boils down to the choice of consumption of the private good and the new debt-money ratio, while the default problem reduces to the only choice of the growth rate of money. All other government policies and equilibrium prices can be then backed out from the Private Sector Equilibrium conditions or the lenders' break-even conditions.

Intra-temporal optimality, repayment. Combining the first-order conditions for consumption of the private and public good, we get

$$U_c - U_g = \frac{\partial m}{\partial c}(U_g \tilde{B} - U_m).$$

The left- and right-hand side respectively represent the marginal benefit and cost of re-allocating a unit of resources from public spending to private consumption. The term $\frac{\partial m}{\partial c}$ on the right-hand side represents the effect of an increase in private consumption on money demand, which reduces the price level (and increases real money balances); that in turn has an effect which depends on the marginal utility from real money balances (i.e. the cost of surprise inflation) and the effect of inflation on the government budget constraint, evaluated at the marginal utility of spending.

Thus changes to the price level and in turn to inflation (i) allow to transfer resources between the government and the private sector, (ii) reduce the debt burden by diluting the real value of external debt, and (iii) reduce the real value of money balances, thus hurting household through the utility they derive from them.

In the reduced model, when the exogenous stream of taxes is too low,[14] public good consumption is below its first best level ($U_g > U_c$) and at the same time the cost of generating further revenues through seignorage is too large ($U_m > U_g \tilde{B}$). The opposite is true when the exogenous stream of taxes is too high. In the benchmark model, lump-sum taxation does not distort any margin and there is no wedge between U_g and U_c. At the same time, the only purpose of surprise inflation is to default implicitly on debt, and the value of such action depends on the marginal

[14] Compared to the optimal tax rate implied by the planner allocation in the benchmark model.

utility of public and private consumption, so also the wedge between U_m and $U_g\tilde{B}$ is equal to zero at all times.

Inter-temporal optimality, repayment. The inter-temporal optimality condition for the debt-to-money ratio is given by

$$\hat{q}_{\tilde{B}'}\tilde{B}' + q + \frac{\partial log\mathcal{M}}{\partial \tilde{B}'}\left(\frac{U_m}{U_g} - \tilde{B}\right) = \beta\mathbb{E}\frac{U_g'}{U_g}\frac{m'}{(1+\mu)m}, \tag{22}$$

where $\hat{q}_{\tilde{B}'}$ is the partial derivative of the expectation term of q with respect to $\tilde{B}'$,[15] and $\mathcal{M}$ represents household demand for real money balances and is defined as the right-hand side of equation (6).

Looking at condition (22), the terms $\hat{q}_{\tilde{B}'}\tilde{B}' + q$ on the left-hand, as well as the whole right-hand side, are identical to the Euler equation (19) of the benchmark model. The last term on the right-hand side represents the impact, via the money demand equation, of the future debt-money ratio on real money balances, which in turn have a net effect which is analogous to that described in the previous paragraph. The incentives of the government to borrow will thus be intrinsically dependent on the elasticity of money demand to $\tilde{B}'$.

Optimality, Default. In periods of repayment, the government can work on two margins: the consumption-saving decision of the households, and the resource constraint with the rest of the world. During default periods instead, only the former margin is available, so I focus on the choice of the money growth rate. Again, Appendix B.3 illustrates the simplified problem and explains how to back out the other equilibrium variables from the choice of μ.

The first-order condition for μ is given by

$$\frac{\partial\ \beta\mathbb{E}\left[(1-\theta)V^D\left(y', \frac{\tilde{B}}{1+\mu}\right) + \theta V\left(y', \frac{\tilde{B}(1-h)}{1+\mu}\right)\right]}{\partial\mu} - c_{(\mu)}(U_g - U_c) = -m_{(\mu)}U_m. \tag{23}$$

Equation (23) displays the effects of an increase in the growth rate of money. First, it reduces the future debt-money ratio that determines the government incentive to re-enter in credit markets once it has a chance to do so. This is represented by the first term on the left-hand side. Second, it reduces real money balances through the money demand equation, which is represented by the

[15] That is, not deriving $(1+\mu)m$ with respect to $\tilde{B}'$. $\hat{q}_{\tilde{B}'}$ here is thus identical to $q_{\tilde{B}'}$ in the benchmark model.

term at the right-hand side. Third, it reduces households' wealth and in turn their consumption, by increasing the amount of resources the government is collecting from the private sector through seignorage. This channel is represented by the second term on the left-hand side, and is only present in the full model where τ is fixed. In the model with lump-sum taxes, the marginal utilities of private and public consumption are equalised, and the resources taken away from households with seignorage are rebated to them via lump-sum taxation.

5 Quantitative Evaluation

Although an exact calibration of the model is currently work in progress, I here display a numerical example aimed at displaying the key properties of each of the two models. First, I describe the parametrisation. Second, I analyse the model mechanics , and in particular how well each model does in matching the empirical results of Section 2.

Parametrisation. A period is a quarter. Table 1 show the parameters that are chosen externally. Unless otherwise specified, I use data for the period 2004q1-2018q4 and average are computes over this period. Household and government preferences are as specified in Sections 3.1 and 3.2. The curvature γ of the utility from private consumption is set equal to 2, a standard value in the quantitative sovereign default literature. The curvature η of the utility from real money balances is set to 3 and is chosen to match the elasticity of money demand[16] as reported in the empirical studies of Benati et al. (2019) and Ball (2001). The curvature ζ of the public good utility is an important parameter in the model, as it determines the relative volatilities of private and public consumption, and in turn the incentive for the government to use inflation to collect fiscal revenues. The value I choose for this parameter are different in the two models, so I postpone its discussion to where I discuss the quantitative performance separately.

The process for output is given by

$$\log(y_t) = \rho_y \log(y_{t-1}) + \epsilon_t, \quad \epsilon_t$$

where ϵ_t is normally distributed with mean zero and variance σ_ϵ^2. I estimate the output process

[16] Appendix B.2 explains the derivation in detail.

parameters on de-trended quarterly real GDP data for each country.[17] I then take the median estimate for the autoregressive coefficient and the innovation standard deviation, which are 0.9293 and 0.0115 respectively. The costs of default are assumed to be non-linear and are modelled, following several works in the literature, as

$$y^d(y) = y - \max\{0, d_0 y + d_1 y^2\}.$$

The international risk-free rate is set to 0.00598, which is equal to the average annualised nominal rate on 5-year US Treasuries. The probability of re-entry is taken from Arellano (2008) and is set to 0.282, which implies an average exclusion from credit markets of about 3.5 quarters. This is admittedly a short period of time, but is chosen among the available estimates to give the benchmark model the best possible chance to generate high inflation upon default. The default recovery rate is taken from Cruces and Trebesch (2013) and is set to 0.63.[18]

Table 1: Parameters selected directly.

Variable	Symbol	Value	Source
Risk-aversion coefficient	γ	2	Conventional value
International risk-free rate	r_f	0.00598	US Treasury rate
Log-output autocorrelation coefficient	ρ	0.9293	estimated
Log-output innovation standard deviation	σ_ϵ	0.0115	estimated
Re-entry probability	θ	0.282	Arellano (2008)
Debt recovery rate upon default	$1-h$	0.63	Cruces and Trebesch (2013)
Money in utility curvature	η	3	Prior literature

Solution Method. I solve the model numerically on Julia using value function iteration. I follow Gordon (2019), Dvorkin et al. (2018) and Arellano et al. (2019) and use taste shocks

[17] GDP data for each country is seasonally adjusted and de-trended using a linear filter. Although longer time series are available for most countries, I restrict the period length to 2004q1-2018q4 in order to match the asset price data.

[18] It is worth noting that this differs from the convention in the CDS industry, which is to assume lower recovery rates: 40 percent for senior unsecured credit in advanced economies, and 25 percent for emerging markets.

to render the probability distribution of some of the government future policy choices non-degenerate. This substantially improves the convergence properties of the model, which otherwise struggles to converge due to the presence of money, which is effectively a very long-term asset. Appendix B.4 explains how this approach is implemented in detail.

5.1 Benchmark Model

I now present the quantitative performance of the benchmark model. I set $\zeta = \gamma$ in order to nest the standard model of the quantitative sovereign default literature where there is no distinction between private and public consumption. The remaining parameters are chosen to match a number of targets, as illustrated by Table 2 in the following way. The discount factor β to match the average debt service ($\tilde{B}m$ in the model), the household discount factor β_h to match the average domestic risk-free rate ($R-1$ in the model), the money in the utility constant α_m to match the long-term average of the monetary base (m in the model), the government additional money in the utility constant to match average CPI inflation ($\tilde{P}'(1+\mu)/\tilde{P}$ in the model), the public good utility constant α_g to match the average private to public good consumption ratio (c/g in the model). Finally, the default cost parameters d_0 and d_1 are chosen to match the mean and standard deviation of 5 year CDS-implied annual default probabilities. These risk-neutral probabilities are backed out from CDS par spreads assuming a constant hazard rate of default. The detailed derivation can be found in Appendix B.1.[19]

Non-Targeted Moments. Table 3 shows the performance of the model with respect to a number of non-targeted moments of interest. The first line displays the correlation between CDS-implied default probabilities, DP_t, and expected inflation implied in the price of government debt,

[19] Real default models typically target risk-neutral default spreads rather than probabilities. Using this model's notation, the price of a hypothetical foreign-currency (i.e. real) bond would be

$$q(y, b') = \frac{1}{1 + r_f} \mathbb{E}[1 - \delta' + \delta'(1 - h)].$$

Defining the default spread as $s := \frac{1}{q} - (1 + r_f)$, the one-to-one relationship between default probabilities and spreads is given by

$$\mathbb{E}(\delta) = \frac{1}{(1 + r_f)} \frac{s}{h}.$$

Variable		Value	Target	Data	Model
Govt discount factor	β	0.83	Debt service/GDP	0.058	0.088
Household discount factor	β_h	0.99	Risk-free rate	0.073	0.064
MIU constant	α_m	2.7e-5	Monetary base/GDP	0.098	0.112
MIU constant (govt)	α_ν	1.5e-3	CPI Inflation	0.049	0.038
Public good utility constant	α_g	0.07	c/g ratio	3.67	3.66
Default cost parameter	d_0	-0.3	Default prob. (mean)	0.045	0.029
Default cost parameter	d_1	0.325	Default prob. (st. dev.)	0.020	0.052

Table 2: Parameters selected to match targets.

XCS_t, which is the model equivalent of the cross-currency swap rates analysed in the empirical section of the paper.[20] Clearly, the benchmark model delivers a correlation between these two asset prices of opposite sign with respect to what we observe in the data. The reason for this is that, in this model, inflation expectations are pro-cyclical (as highlighted in the second row of the table), while default spreads are counter-cyclcal (as highlighted in the third row). Finally, there is essentially no relationship between realised inflation and default spreads, which is also at odds with the empirical evidence. The next paragraph provides an intuitive explanation of these moments.

Moment	Model	Data
$\rho(DP_t, XCS_t)$	-0.25	0.46
$\rho(y_t, XCS_t)$	0.43	0.02
$\rho(y_t, DP_t)$	-0.55	-0.2
$\rho(DP_t, \pi_t)$	0.02	0.31

Table 3: Non-targeted moments, benchmark model.

Equilibrium Policy and Asset Prices Figure 5 illustrates the behaviour of three important equilibrium variables. The left panel plots the policy function for new real debt issuance, as

[20] DP_t and XCS_t are explicitly defined in equations (10) and (11) respectively.

a function of output (on the horizontal axis) and of three levels of initial real debt (I pick the average level of debt and two other values that are one standard above/below the mean). The picture clearly shows that the model displays a common feature of sovereign default models: debt is strongly procyclical, which means that the government on average experiences capital inflows in good times (when output is high), and outflows in bad times (when output is low). This is consistent with empirical findings on the cyclicality of the trade balance in emerging market economies.

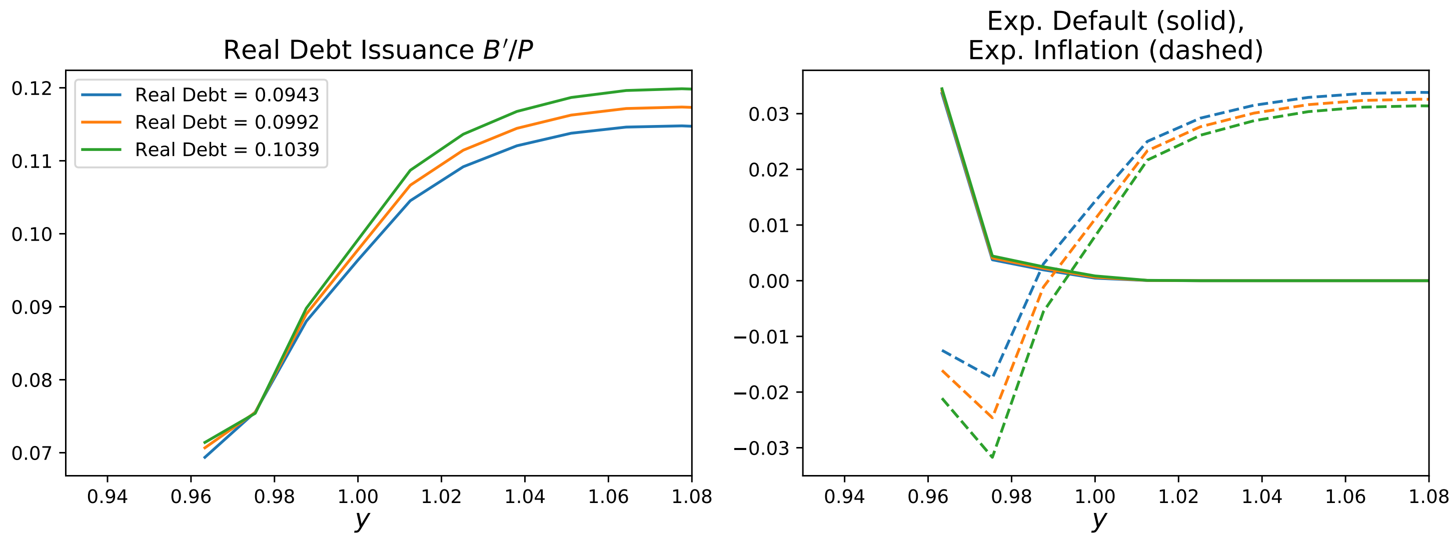

Figure 5: Equilibrium Debt Policy and Expected Default/Inflation.

A direct implication of this is that inflation incentives are also pro-cyclical. The reason for this is that, in this model, inflation only serves the purpose of manipulating the real value of debt. Clearly then the incentive to do so will be stronger, the larger is the stock of debt to be repaid. As explained previously, larger debt stocks are more likely during periods of high output. There is however a possible counteracting force: the incentive to inflate will also be higher in periods of low output, because that is when a lower debt burden is most valued by the government.

This is a force that is quantitatively weaker than that derived from the size of the debt stock. The right panel of Figure 5 plots the expected default (solid lines) and expected inflation (dashed lines) associated with equilibrium debt policy. The picture shows that, as is common in default models, default spreads are counter-cyclical. This is driven by the strong incentive of the government to borrow in bad times, and by the fact that output persistence makes the debt price schedule for debt less favourable as a future default is more likely. Inflation expectations are instead pro-cyclical, since the behaviour of debt is the prevalent force driving inflation incentives.

In sum, the quantitative performance of the benchmark model allows us to conclude that, when the only purpose of inflation is to serve as an implicit default instrument, the very features at the core of sovereign default models are those that imply the model is at odds with the data along a number of important real and financial dimensions. The following section highlights how a small modification to the model allows to reconcile the model with the empirical evidence.

5.2 Reduced Model

I now consider the quantitative implication of the reduced model, where I assume that the tax rate at which the government collects taxes from the private sector is exogenous and constant.

The parametrization of this model is as follows. First, I set the curvature of public good utility $\zeta = 5$, to strengthen incentive of the government to use inflation as a source of revenues to finance spending. It is important to note that this parametric assumption alone would not change the qualitative properties of the benchmark model analysed in the previous subsection. As in the benchmark model, the remaining parameters are chosen to match a number of targets, as illustrated by Table 4. It is worth discussing the role of the tax rate τ: the difference between tax revenues τy and the desired level of public good consumption g determines the desired amount of deficit, which must be financed with either new debt or inflation. It can of course happen that the government aims to run a surplus, in which case debt policy and seignorage will be used to transfer resources to households. I thus aim to to discipline the model by calibrating τ to the coefficient of variation of seignorage (i.e. the ratio between the standard deviation and the mean).

Non-Targeted Moments. Table 5 displays the model performance with respect to the same non-targeted moments against which we evaluated the benchmark model in the previous section. The table clearly shows that the model succeeds in matching a number of important features of the data: default and inflation risks co-move (first line), default spreads remain counter-cyclical (third line), and default risk is positively correlated with realised CPI inflation (fourth line). The main driver of this substantial change in the model performance is highlighted in the second line:

Variable		Value	Target	Data	Model
Govt discount factor	β	0.65	Debt service/GDP	0.058	0.041
Household discount factor	β_h	0.997	Risk-free rate	0.073	0.067
MIU constant	α_m	2e-5	Monetary base/GDP	0.098	0.103
MIU constant (govt)	α_ν	8e-4	CPI Inflation	0.049	0.057
Public good utility constant	α_g	8e-4	c/g ratio	3.67	3.64
Default cost parameter	d_0	-0.07	Default prob. (mean)	0.045	0.033
Default cost parameter	d_1	0.0975	Default prob. (st. dev.)	0.020	0.027
Tax rate	τ	0.215	CV(Seignorage)		10

Table 4: Parameters selected to match targets.

realised and expected inflation have now becomes strongly counter-cyclical. This is consistent with the empirical evidence of realised inflation cyclicality in emerging market economies.

Moment	Model	Data
$\rho(DP_t, XCS_t)$	0.43	0.46
$\rho(y_t, XCS_t)$	-0.73	0.02
$\rho(y_t, DP_t)$	-0.53	-0.2
$\rho(DP_t, \pi_t)$	0.34	0.31

Table 5: Non-targeted moments, reduced model.

Equilibrium Policy and Asset Prices Figure 6 plots equilibrium debt policy in the left panel, money supply policy in the middle panel, and equilibrium expected default (solid lines) and inflation (dashed lines) in the right panel. The three coloured lines indicate three different initial levels of real debt (equal to, above and below the simulated mean by one standard deviation). As the graph shows, debt policy is moderately pro-cyclical, especially for low values of output. As in the benchmark model, the reason for this is that the government would like to borrow more in bad times, but it does not because it is costlier to do: output is persistent, a future default is more likely, and therefore lenders charge higher interest rates on government debt. This implies that, when output is low, the government needs to use seignorage as an alternative source of revenues to fund levels of public spending above its exogenous tax revenues. The strength of this motive is stronger, the higher the curvature of the utility from public good consumption, and the higher the deficit the government would like to run. In the current calibration, this "tax" motive behind money supply, seignorage and inflation becomes stronger than the "default" motive of inflation which was the only force present in the benchmark model. The relationship between inflation and the cycle thus dominates that between inflation and debt, making realised and expected inflation rise in bad times (as highlighted by the right panel), as default spreads do.

Another aspect worth noting is that money growth, and in turn inflation, is significantly higher of repayment than in times of default, which suggests that the debt dilution motive is still the dominant force in determining contrasts between default and repayment periods.

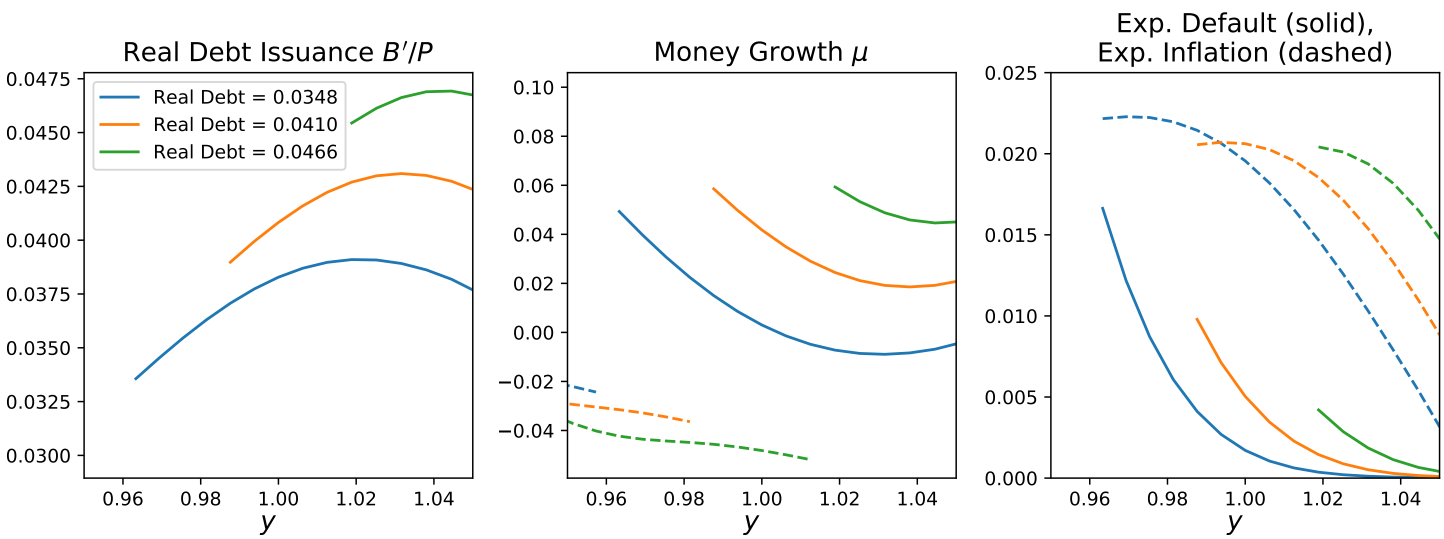

Figure 6: Equilibrium Debt Policies and Asset Prices.

Analytical Decomposition of Asset Prices. I now analyse the behaviour of asset prices more in depth, proposing an analytical framework to assess the relative importance of the forces highlighted in the previous paragraph. Let default risk DP_t and inflation risk XCS_t be given by (10) and (11) respectively.

The key factor in determining the co-movement of these risks is given by the relationship between inflation expectations XCS and the cycle. The reason is that a key feature of the model is given by the fact that default spreads are counter-cyclical, as can be seen in the figure.[21] If inflation spreads are also, at least moderately, counter-cyclical, then the model has a chance of matching the data. It is therefore instructive to explore what are the drivers behind the cyclicality of inflation spreads, looking at how they change with output shocks.

We can decompose the derivative of inflation spreads with respect to output (assuming a continuous output distribution as well as differentiability in the debt and inflation policy functions)

$$\frac{\partial XCS(y, \tilde{B}')}{\partial y} = \frac{\partial}{\partial y} \int [\delta(\tilde{B}', y')\pi_D(\tilde{B}', y') + (1 - \delta(\tilde{B}', y'))\pi_R(\tilde{B}', y')] f(y', y) dy'$$

in the following components

$$\begin{aligned}
&= \frac{\partial \tilde{B}'}{\partial y} \int \frac{\delta' \partial \pi'_D + (1 - \delta')\partial \pi'_R}{\partial \tilde{B}'} dF(y'|y) \\
&+ \int^{\hat{y}} \pi'_D \frac{\partial f(y'|y)}{\partial y} dy' + \int_{\hat{y}}^{y} \pi'_R \frac{\partial f(y'|y)}{\partial y} dy' + \int_{y} \pi'_R \frac{\partial f(y'|y)}{\partial y} dy' \\
&- \frac{\partial \tilde{B}'}{\partial y} \frac{\partial \hat{y}}{\partial \tilde{B}'} [\pi'_R(\tilde{B}', \hat{y}) - \pi'_D(\tilde{B}', \hat{y})] f(\hat{y}|y).
\end{aligned}$$

Let us consider this decomposition in light of a *drop* in output, which as said previously tends to correspond to a rise in default spreads.

- The first component, represented in the first row, shows the effect on expected inflation through debt: since debt is pro-cyclical in the model, a drop in output corresponds to a lower future debt-to-money ratio, which brings about less expected inflation, since inflation is increasing in $\tilde{B}$. This effect make XCS pro-cyclical.

- The second component, represented in the second row, shows the effect on expected inflation through a shift in the distribution of y' due to the persistence of the output process: a drop in output implies lower expected output. This has an ambiguous effect: in the repayment region, this shifts probability mass to states where inflation is higher, since realised inflation is counter-cyclical; on the other hand, this channel also shifts mass to the default region, where inflation will be lower. The net effect depends on the slope of π_R as a function of output, and on the size of the $\pi_R - \pi_D$ differential.

[21] This is consistent with the data, and a fundamental feature of quantitative sovereign default models.

- The third component, represented in the third row, isolates the effect on expected inflation through a change in the default cutoff $\hat{y}$: a drop in output implies a drop in debt issuance, which means the cutoff decreases, i.e. the default region is smaller. This increases expected inflation, because of the sign of the $\pi_R - \pi_D$ differential. This effect make XCS counter-cyclical, but is likely to be small as it depends on the output distribution density at $\hat{y}$.

As explained previously, the calibrated version of the model shows that the second component is the key driver of the counter-cyclicality of expected inflation.

6 Conclusion

In this paper, I have studied in detail the relationship between strategic inflation, default and inflation risk. In the data, default risk for a set of EM sovereigns is sizeable and positively related to realised and expected inflation. A simple model of default and debt dilution via inflation has a hard time in matching these facts, because inflation and default are essentially substitutes. To reconcile the model with the data, it is important that inflation also serves a second purpose: that of generating fiscal revenues, which is especially useful in bad times and during periods of autarky.

The model I develop allows to quantitatively evaluate the trade-off between the insurance benefits and the time-inconsistency costs of issuing debt in domestic currency, showing that the way in which default and inflation risks move is crucial in this regard. In light of this, the paper offers a natural starting point to study the interplay between fiscal-monetary interactions and the welfare benefits of local-currency debt.

Appendix A Empirical Appendix

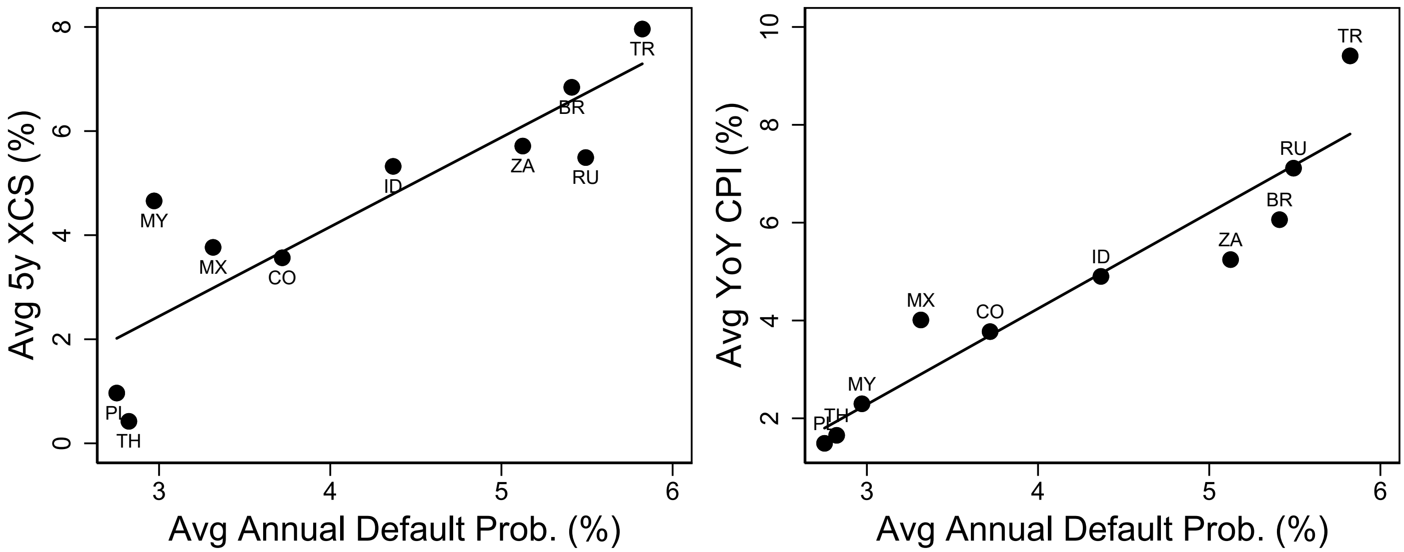

Figure 7: Long-term averages for the period 2010q1-2018q4. The left panel plots average default probabilities against average XCS rates. The right panel plots average default probabilities against realised CPI inflation.

Appendix B Theory Appendix

B.1 CDS-Implied Default Probabilities

To extract default probabilities from CDS spreads, I follow the finance and asset pricing literature and model default as the first jump of a (potentially inhomogeneous) Poisson process, with $\lambda(t)$ denoting the default intensity, or hazard rate function. $\lambda(t)$ thus represents the probability that default happens at time t, conditional on not having happened before. In turn, the survival probability is given by

$$S(t) = Pe^{-\int_0^t \lambda(u)du} \tag{24}$$

which becomes $S(t) = e^{-\lambda t}$ if the hazard rate is assumed constant.

A CDS contract is composed of two legs, the premium leg and the protection leg. The premium leg consists of periodic payments of a premium expressed in percentage terms of the notional, also called par spread, until maturity or the default event, whichever comes first. The protection leg consists of a one-off repayment of the notional, if default occurs before maturity, or nothing otherwise.

I now write down the pricing formulas for both legs. In doing so, I adopt the following simplifying assumptions: interest rates, default intensity and recovery rate are independent, and the premium leg pays the spread continuously until default (otherwise we would need to consider premium arrears to be paid upon default). Let U_{par} represent the par spread, $DF(t)$ the risk-free discount factor used to discount a period-t cash-flow back to time 0, T_1 the time of default (i.e. the first jump of the Poisson process), and $S(t)$ the survival probability up to t.

The PV of the premium leg is given by the present value of all premium payments, discounted by the risk-free rate and the survival probability:

$$PV_{prem} = \mathbb{E}\left\{\int_0^T DF(t)U_{par}\mathbb{1}[T_1 > t]\right\} = U_{par}\int_0^T DF(t)S(t)dt. \tag{25}$$

The PV of the protection leg is given by the present value of the random payment of the notional loss given default, denoted LGD, at default time T_1, if such time is before expiry T, and zero otherwise:

$$PV_{prot} = \mathbb{E}\left\{DF(T_1) \times LGD \times \mathbb{1}[T_1 \leq T]\right\} = LGD\int_0^T DF(t)S(t)\lambda(t)dt. \tag{26}$$

It follows that the par spread is given by

$$U_{par} = \frac{LGD\ \int_0^T DF(t)\ S(t)\lambda(t)dt}{\int_0^T DF(t)S(t)dt}. \tag{27}$$

Assuming that the hazard rate is constant ($\lambda(t) = \lambda$) simplifies the expression to

$$\lambda = \frac{U_{par}}{LGD}. \tag{28}$$

The probability of default in $(0, t)$ is thus given by

$$\text{DefProb}_t = 1 - S(t) = 1 - e^{-\lambda t} = 1 - e^{-\frac{U_{par}}{LGD}t}. \tag{29}$$

B.2 Money Demand Elasticities

Under the parametric assumptions of the model, the money demand equation (4) is given by

$$R_t - 1 = \mathbb{E}\frac{\alpha_m(M_{t+1}/P_{t+1})^{-\eta}}{c_{t+1}^{-\sigma}}.$$

Linearising this equation around the stochastic steady state we get

$$\mathbb{E}\log(M_{t+1}/P_{t+1}) = \frac{const}{\eta} + \frac{\gamma}{\eta}\mathbb{E}\log c_{t+1} - \frac{1}{i\eta}i_t \tag{30}$$

where i is the steady state interest rate and $const$ is a constant. It follows that the semi-elasticity of future real money balances to the interest rate is given by $(i\eta)^{-1}$, which under my calibration targets is equal to a value of -5.21: for a 100 basis points increase in i_t, future real money balances are on average 5.21% lower. The elasticity of future real money balances to the interest rate is instead given simply by $1/\eta$, which is equal to $1/3$ at the chosen level of the curvature of money in the utility.

B.3 Policy Implementation

For a given aggregate state $(\tilde{B}, y)$, consider an arbitrary choice of private consumption c and future debt-money ratio $\tilde{B}'$. The right-hand side of the money demand equation (13) is thus pinned down, as $\mathcal{S}'$ is given by the choice of $\tilde{B}'$ and future equilibrium policies. This in turn pins down the left-hand side of the household budget constraint (12), determining real money balances m. The value of μ can then be backed out from (13), while the value of g can be obtained through the resource constraint (16).

B.4 Numerical Solution Method

The government recursive problem after the addition of taste shocks is as follows. All of the shocks introduced below $(\epsilon_R, \epsilon_D, \epsilon_{\tilde{B}'}, \epsilon_\mu)$ are assumed to be identically and independently distributed according to a Gumbel distribution with a mean of $-\bar{\mu}$, where $\bar{\mu}$ is the Euler-Mascheroni constant, and a standard deviation of one.

The value of the option to default is

$$V(\tilde{B}, y) = \max_{\delta\in\{0,1\}} \left\{(1-\delta)[V^R(\tilde{B}, y) + \rho_\delta\epsilon_R] + \delta[V^D(\tilde{B}, y) + \rho_\delta\epsilon_D]\right\}.$$

The value function of the government upon repayment is

$$V^R(\tilde{B}, y, \{\epsilon_{\tilde{B}'}\}) = \max_{\tilde{B}'} \left\{W^R(\tilde{B}, y;\ \tilde{B}') + \rho_{\tilde{B}'}\epsilon_{\tilde{B}'}\right\}$$

where

$$W^R(\tilde{B}, y;\ \tilde{B}') = U(c(\tilde{B}'), m(\tilde{B}'), g(\tilde{B}')) + \beta \mathbb{E}_{y'|y} V(\tilde{B}', y').$$

Each $\tilde{B}'$ choice is thus associated with an element of the taste shock vector $\{\epsilon_{\tilde{B}'}\}$. The value function of the government upon default is

$$V^D(\tilde{B}, y, \{\epsilon_\mu\}) = \max_\mu \left\{ W^D(\tilde{B}, y;\ \mu) + \rho_\mu \epsilon_\mu \right\}$$

where

$$W^D(\tilde{B}, y;\ \mu) = U(c(\mu), m(\mu), g(\mu)) + \beta \mathbb{E}\left[\theta V\left(\frac{\tilde{B}(1-h)}{1+\mu}, y'\right) + (1-\theta) V^D\left(\frac{\tilde{B}}{1+\mu}, y'\right)\right].$$

Each μ choice is thus associated with an element of the taste shock vector $\{\epsilon_\mu\}$.

The above assumptions imply that for each choice $x = \tilde{B}', \mu, \delta = 1, \delta = 0$, the probability of observing such choice is given by

$$\mathbb{P}(x|\tilde{B}, y) = \frac{\exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]}{\sum_x \exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]}.$$

Furthermore, the expected value of each of the three value functions described above can be written as

$$V^i(\tilde{B}, y) = \rho_x \log\left\{\sum_x \exp\left[W^i(\tilde{B}, y, x)/\rho_x\right]\right\}.$$

In the calibration of the model, I choose the smallest values of $\rho_{\tilde{B}'}, \rho_\mu, \rho_R, \rho_D$ such that the model converges. The magnitude of each of these parameters can be illustrated as follows. Consider some choice x'' that yields a 0.05% drop in utility when compared to the optimal choice (in the absence of taste shocks), that is $\log \frac{W^i(\tilde{B}, y;\ x'')}{\max_x W^i(\tilde{B}, y; x)} = -.05\%$. I now compute the probability of making choice x'', i.e. to make a "suboptimal" choice that delivers a lower utility than what would be the optimal choice in the absence of taste shocks.

- I set $\rho_{\tilde{B}'} = 1e - 3$. The probability of a suboptimal (as defined above) choice is

$$\mathbb{P}[\tilde{B}''_{(-.05\% V^R)}|\tilde{B}, y] = 1e - 12.$$

- I set $\rho_\mu = 5e - 3$. The probability of a suboptimal (as defined above) choice is

$$\mathbb{P}[\mu_{(-.05\% V^D)}|\tilde{B}, y] = .001.$$

- I set $\rho_{R,D} = 5e - 3$. The probability of a suboptimal (as defined above) choice is

$$\mathbb{P}[\delta_{(-.05\% V^R)}|\tilde{B}, y] = .057.$$

References