

# Asset Purchases and Heterogeneous Beliefs on Default Risk

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# Motivation

APs typically a monetary policy tool when at the ZLB

The “Securities Markets Programme” (SMP), which was announced by the Governing Council on 10 May 2010, is intended to ensure depth and liquidity in malfunctioning segments of the debt securities markets and to restore an appropriate functioning of the monetary policy transmission mechanism.<sup>1</sup>

APs as a “fiscal” tool, to prevent sovereign debt crises and support govt debt service...

Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.

(26th July 2012)



**European Central Bank** ✓

@ecb

Lagarde: We are not here to close spreads, there are other tools and other actors to deal with these issues

3:10 PM · Mar 12, 2020 · Twitter Web App

...or not?

1. Should APs target the real risk-free rate and/or default spreads?
2. What is the impact of APs on the information embedded in prices?

We answer this with a model with

- fiscal-monetary interactions
- sovereign default
- noisy aggregation of heterogeneous information

## Asset purchases

- expose the CB balance sheet (hence inflation) to default risk (−)
  - crowd out private investors  $\Rightarrow$  relevant if beliefs are heterogeneous
    - reduce nominal and real sovereign yields (+)
    - affect the informational content of market prices, asymmetrically (−)
  - net welfare effect  $> 0$  under some conditions
- $\Rightarrow$  information frictions as a rationale for why APs may work “in theory”

## Implications

- degree of belief heterogeneity key for AP elasticity of the interest rate

1. Model setup
2. Homogeneous information
3. Heterogeneous information

## Model: Government

Two periods,  $t = 1, 2$

First period  $t = 1$

- fund stochastic spending by issuing nominal+defaultable debt

$$g = b \quad \text{where } g = \gamma y(\epsilon)$$

Second period  $t = 2$

- raise taxes, can default ( $\delta \in \{0, 1\}$ ) with haircut  $h$  and deadweight loss  $\theta$

$$b \frac{R(1 - \delta h)}{\Pi} = \tau y(\epsilon) \quad \rightarrow \quad \underbrace{\frac{R(1 - \delta h)}{\Pi(\delta)}}_{\psi(\delta)} = \tau$$

- default decision

$$\zeta(\psi(0)) > \zeta(\psi(1)) + \theta$$

but for today, default iff  $\theta < \hat{\theta}$

normalisation

## Model: Households

Continuum of risk-neutral agents  $i \in [0, 1]$

First period  $t = 1$

- receive information on  $(\theta, \epsilon, R)$  and APs
- receive endowment  $e_1$ , save it in 3 assets

$$e_1 \geq b^i + m^i + s^i$$

Second period  $t = 2$

$$c^i = b^i \frac{R(1 - \delta h)}{\Pi} + \frac{m^i}{\Pi} + \rho s^i - \tau y(\epsilon) - \zeta(\tau)$$

- pay taxes, consume
- tax distortions  $\zeta(\tau)$  create deadweight losses

## Model: Central Bank

First period  $t = 1$ : issue money, save via storage (real + risk-free) or bonds

$$s^{cb} + b^{cb} = m \frac{s^{cb}}{m} = 1 - \alpha$$

Second period  $t = 2$ : reimburse money with returns from saving

$$\rho s^{cb} + \frac{b^{cb} R(1 - \delta h)}{\Pi} = \frac{m}{\Pi} (1 - \alpha) \rho + \alpha \frac{R(1 - \delta h)}{\Pi} = \frac{1}{\Pi}$$

Let share of money invested in bonds be  $\alpha := \frac{b}{m}$

- return of money as  $\alpha$ -weighted average
- $\alpha \rightarrow$  degree of fiscal dominance

Real bond returns

$$\psi(R, \delta, \alpha) = \frac{R(1 - \delta h)}{\Pi(R, \delta, \alpha)} \longrightarrow \psi_{\alpha} \begin{cases} > 0 & \text{if repay} \\ < 0 & \text{if default} \end{cases}$$



Bonds market clearing

$$b^{cb} + \int b^i di = b$$

Goods market clearing

$$c = \rho[e_1 - g(\epsilon)] - \zeta(\tau)$$

Timing assumptions

- 1.1 asset purchases  $\alpha$  are unconditional, CB does not observe shocks
- 1.2 shocks  $(\theta, \epsilon)$  realise
- 1.3 agents receive information and make portfolio decisions
- 2.1 government observes shocks perfectly, takes default decision
- 2.2 payoffs realise & agents consume

# Two illustrative cases

## Case 1: homogeneous information + no uncertainty

- inflation is anchored, money, bonds and storage are perfect substitutes
- ⇒ **irrelevance result**

## Case 2: homogeneous information + exogenous uncertainty

- agents and CB share same uncertainty:  $\text{Prob}(\text{Repay}) = p$
- eqm  $R$  solves no-arbitrage

$$p\psi(0) + (1 - p)\psi(1) = \rho$$

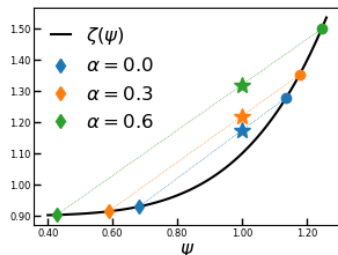
- expected welfare loss

$$p\zeta(\psi(0)) + (1 - p)\zeta(\psi(1))$$

Effect of asset purchases  $\alpha$ ?

- ↑ CB exposure to default risk
- $\mathbb{V}(\Pi) \uparrow$
- $\mathbb{V}(\psi) \uparrow$
- but  $\mathbb{E}(\psi) \leftrightarrow$  because of no-arbitrage!

⇒ **no APs w/ convex distortions,  $\alpha^* = 0$**



### Case 3: heterogeneous information + learning

Agent  $i$  observes: [private signal  $x_i$  of fundamental  $\theta$ ] + [endogenous price signal]

Distribution of subjective repayment probabilities  $p_i$

$$\left\{ \begin{array}{lll} x_i > \hat{x} & \mathbb{E}_i[\psi(\delta)] > \rho & b^i = \bar{b}, m^i = e_1 - \bar{b} \\ x_i < \hat{x} & \mathbb{E}_i[\psi(\delta)] < \rho & s^i = e_1 \\ x_i = \hat{x} & \mathbb{E}_i[\psi(\delta)] = \rho & b^i + m^i + s^i = e_1 \end{array} \right.$$

Now:  $p^m \psi(0) + (1 - p^m) \psi(1) = \rho$  (no-arb)

$p^e \zeta(\psi(0)) + (1 - p^e) \zeta(\psi(1))$  (ex-ante welfare loss)

$p^m \neq p^e$  (Hellwig-Mukherji-Tsyvinski)

Effect of  $\alpha$  through bond market clearing  $\rightarrow$  “move” market beliefs

$$\underbrace{P(x_i > \hat{x}(R))}_{\text{mass of optimists}} \left[ \underbrace{\bar{b}}_{\text{direct bond demand}} + \underbrace{(e_1 - \bar{b})}_{\text{money demand } m} \underbrace{\alpha}_{\text{AP ratio } b^{cb}/m} \right] = \underbrace{b(\epsilon)}_{\text{random bond supply}}$$

APs create upper bound to agents' bond demand:  $P(x_i > \hat{x}(R)) \leq \frac{\max_{\epsilon} b(\epsilon)}{b + (e_1 - b)\alpha}$

## Features

- risk-neutrality
- binary action

## Properties

- non-linear payoffs (step function here)
- no mean-variance problem (e.g. CARA)

## With APs (our contribution)

- truncation of posterior beliefs distribution

## Model details

- Agents observe  $x_i = \theta + \sigma_x \xi_i$ ,  $\xi \sim N(0, 1)$
- Bonds market clearing

$$\Phi\left(\frac{\theta - \hat{x}}{\sigma_x}\right) [1 + \alpha(e_1 - 1)] = \Phi(\epsilon) \quad \Rightarrow \quad \hat{x}(R, \alpha) = \underbrace{\theta - \sigma_x \underbrace{\Phi^{-1}\left(\frac{\Phi(\epsilon)}{1 + \alpha(e_1 - 1)}\right)}_{:=\nu(\epsilon, \alpha)}}_{:=z(\theta, \epsilon, \alpha)}$$

- the marginal agent is  $\hat{x} = z$
- the error term  $\nu(\epsilon, \alpha) \sim \text{TruncN}$  with  $\text{Supp}_\nu = (-\infty, \bar{\nu}(\alpha))$ , so that  $\theta < z + \bar{\nu}(\alpha)$
- Marginal agent's posterior beliefs

$$f(\theta \mid x_i = z, z) = \frac{1}{\sigma_{post}} \phi\left(\frac{\theta - \mu_{post}}{\sigma_{post}}\right) / \Phi\left(\frac{z + \bar{\nu} - \mu_{post}}{\sigma_{post}}\right)$$

where  $\mu_{post} := \frac{\tau_\theta \mu_\theta + (\tau_x + \tau_z)z}{\tau_\theta + \tau_x + \tau_z}$

- “Market” probability of default

$$p^m = P(\theta < \hat{\theta} \mid x_i = z, z) = F_{\theta \mid x_i = z, z}(\hat{\theta})$$

- “External” probability of default

$$p^e = P(\theta < \hat{\theta} \mid z) = F_{\theta \mid z}(\hat{\theta})$$

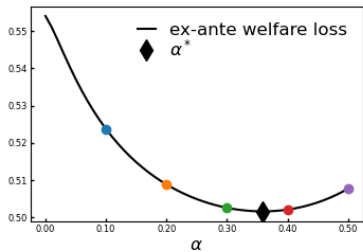
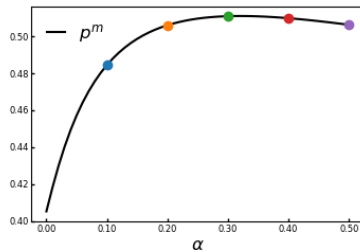
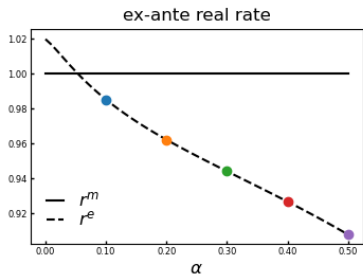
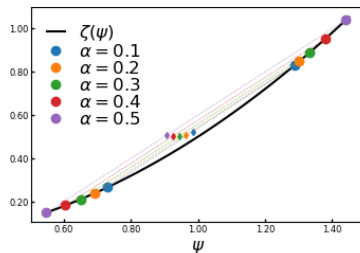
Marginal trader's identity & subjective default probability

- location effect: select more optimistic agent:  $p^m \uparrow$
- scale effect: truncation of posterior right tail:  $p^m \downarrow$

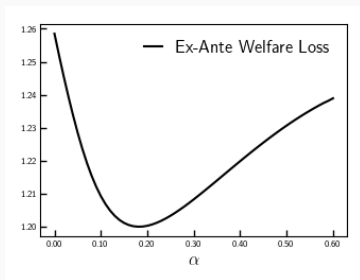
Welfare loss

- real repayment *mean*  $\downarrow$
- real repayment *variance* still  $\uparrow$

# Illustration for a given $(\theta, \epsilon)$ pair



Integrating over all  $(\theta, \epsilon)$  realisations





Thank You!

$$\gamma y(\epsilon) = b$$

$$\frac{B_1}{P_2} R(1 - \delta h) = \hat{\tau}$$

which becomes

$$b \frac{P_1}{P_2} R(1 - \delta h) = \tau y(\epsilon)$$

$$\gamma y(\epsilon) \frac{P_1}{P_2} R(1 - \delta h) = \tau y(\epsilon)$$

$$\gamma \frac{P_1}{P_2} R(1 - \delta h) = \tau$$

# Central Bank Balance Sheet

Central bank balance sheet at  $t = 1$

Assets	Liabilities
bonds $b^{cb}$	money $m$
storage $s^{cb}$	

Central bank balance sheet at  $t = 2$

Assets	Liabilities
bonds $b^{cb} \frac{R(1-\delta h)}{\Pi}$	money $\frac{m}{\Pi}$
storage $\rho s^{cb}$	

## Price level determination and real bond returns

Solving for the real return on money

$$\frac{1}{\bar{\Pi}} = \rho \frac{1 - \alpha}{1 - \alpha R(1 - \delta h)}$$

Plug into real bond returns

$$\psi(R, \alpha, \delta) = \rho \frac{1 - \alpha}{\frac{1}{R(1 - \delta h)} - \alpha}$$

Since  $R \in \left[1, \frac{1}{1-h}\right]$

- in repayment  $\delta = 0$  and  $R > 1$ 
  - central bank makes profits
  - there is deflation:  $\frac{1}{\bar{\Pi}} > \rho$
  - larger APs imply larger deflation and debt service:  $\uparrow \alpha \Rightarrow \uparrow \psi(0)$
- in default  $\delta = 1$  and  $R(1 - h) < 1$ 
  - central bank makes losses
  - there is inflation:  $\frac{1}{\bar{\Pi}} < \rho$
  - larger APs imply larger inflation and lower debt service:  $\uparrow \alpha \Rightarrow \downarrow \psi(1)$