Dealing with Heterogeneous Bondholders in Sovereign Restructurings: The Aggregation Dilemma

Carlo Galli
UC3M and CEPR

Stéphane Guibaud
Sciences Po

April 1, 2025*

Abstract

Most sovereign bonds today feature "aggregated" collective action clauses (CACs), which allow the government to restructure *multiple* bond series at once through a vote of bondholders. We characterize how the optimal voting procedure and restructuring offers depend on the heterogeneity within and across bonds, and on their relative size. We then analyze how the design of CACs affects bondholder bases, relative bond valuations, and the entry of vulture funds. Our results help rationalize the aggregation method employed by Argentina and Ecuador to restructure their international bonds in 2020, and shed a critical light on the ongoing reform of Euro-Area CACs.

JEL Codes: F34; H63

Keywords: Sovereign Debt Restructurings, Sovereign Bonds, Collective Action Clauses, Voting Rules, Holdout Creditors

*For helpful comments and suggestions we thank Tamon Asonuma, Marco Bassetto, Bruno Biais, Fernando Broner, Satyajit Chatterjee, Nicolas Coeurdacier, Jean-Edouard Colliard, Giancarlo Corsetti, Aitor Erce, Sebastian Fanelli, Paul Fontanier, Maria Fayos Herrera, Anna Gelpern, Tim Kehoe, Guido Lorenzoni, Alberto Martin, Dmitry Mukhin, Dirk Niepelt, Hélene Rey, Cesar Sosa-Padilla, David Sraer, David Thesmar, Christoph Trebesch, Emircan Yurdagul, and conference and seminar participants at DebtCon, CEPR ESSIM, the BSE Summer Forum, HEC Paris, the IMF Sovereign Debt Workshop, SED, SAEe, the Public Debt Management Conference, and the EUI. Galli gratefully acknowledges financial support from Fundación Ramón Areces, from MICIN/AEI/10.13039/501100011033 with grants CEX2021-001181-M and PID2021-122931NB-I00, and from the Comunidad de Madrid with grant EPUC3M11 (V PRICIT). Contact details: `cgalli@eco.uc3m.es`; `stephane.guibaud@sciencespo.fr`.

Introduction

Collective action clauses (CACs) constitute a key pillar of the sovereign debt architecture and determine the fate of several trillion dollars of government debt securities worldwide, should they be subject to a restructuring. These contractual provisions enable the affirmative vote of a qualified majority of bondholders to bind a dissenting minority to the terms of a restructuring proposal put forth by the government—emulating the majority voting provisions of corporate insolvency laws.[1] For over two decades, CACs have been systematically inserted in the international bonds of major emerging market sovereign issuers with a view to alleviating the "holdout" problem in sovereign debt workouts, whereby a few creditors can jeopardize a restructuring and increase the cost of debt distress.[2] Since 2013, Euro-Area government bonds also incorporate majority voting provisions.

This paper is concerned with the different voting rules that a government can employ to restructure *multiple* bond series under CACs. While first-generation CACs operated through separate votes in each bond series, most sovereign bonds today include clauses that make the restructuring outcome depend (at least partly) on the *aggregate* vote across all series. In particular, under the so-called "enhanced" formulation of CACs introduced in 2014 and widely adopted since by emerging market issuers, the sovereign can choose—at the time it seeks approval for a restructuring of its bonds—between two aggregated voting procedures.[3] Under the "two-limb" procedure, the government's proposal goes through and affects all bond series if the consent share *across* series is above some aggregate threshold (typically 66.7%) and if the approval rate *within* each series is sufficiently high (typically above 50%). Instead, the "single-limb" procedure only relies on the aggregate vote—with a supermajority threshold typically set at 75%—but comes with the additional constraint, known as *uniform applicability* condition, that all bond series should receive the same restructuring terms (see ICMA (2014)).

Despite their widespread adoption, the workings and efficacy of enhanced CACs are still not well understood. While two-limb CACs have been in use since 2003, the addition of the single-limb procedure constituted the real innovation in the drafting of the ICMA model. The presumption in policy circles was that this procedure would become the method of choice to conduct distressed bond exchanges.[4] This view was severely challenged when Argentina and

[1]In 2002, IMF's First Deputy Managing Director Anne Krueger envisioned a statutory bankruptcy regime for countries, but the idea of a "sovereign debt restructuring mechanism" failed to gather political support.

[2]For further background information on sovereign CACs, see among others Eichengreen and Portes (1995), Buchheit and Gulati (2002), IMF (2014), and Gelpern et al. (2016). The coordination problem among creditors was first analyzed in the context of corporate workouts by Roe (1987) and Gertner and Scharfstein (1991).

[3]Since their introduction under the auspices of the International Capital Market Association (ICMA) and the International Monetary Fund (IMF), enhanced CACs have been inserted in 91% of new international sovereign bonds issued by emerging markets and low-income countries (IMF (2020)).

[4]For a rendition of that view, see IMF (2014) and Sobel (2016). In hindsight, de la Cruz and Lagos (2021) reflect: "The ICMA CACs included an innovation expected to enhance the ability of a sovereign to restructure its outstanding bonds [...]. The single-limb, uniformly applicable, modification was the star of the day."

Ecuador restructured their international bonds in 2020: then, in the first test of enhanced CACs, both governments opted for *two-limb* aggregation, taking many commentators by surprise.[5] Yet, on the other side of the Atlantic, whereas all Euro-Area public debt securities issued after January 2013 are subject to two-limb CACs, Eurozone governments recently agreed to equip their future bond issues with CACs that *only* allow for *single-limb* aggregation.[6]

Motivated by their ubiquity and by these puzzling developments, this paper offers the first formal analysis of aggregated CACs. We address the following set of questions: For a government willing to obtain the best restructuring deal, what is the optimal voting procedure? Can one rationalize the choice made by Argentina and Ecuador? Is the decision by Eurozone policymakers to reform the design of Euro CACs well-advised? And more broadly, how does the design of CACs affect allocations and prices in the bond market ahead of a restructuring? To analyze these issues, we adopt a framework that features multiple bond series and heterogeneous bondholders, allowing for heterogeneity within and across series. Whereas the former type of heterogeneity may arise from differences in discount rates, balance sheets, regulation, information, or litigation skills, the latter can be due to differences across bonds in terms of maturity or coupon rate, as well as in their investor bases.[7]

The paper makes four main contributions. First, we establish that, contrary to the conventional view, single-limb aggregation is not always optimal, and we clarify the circumstances under which each voting procedure dominates. Second, we provide detailed case studies documenting the concrete use of aggregated CACs in sovereign restructurings and apply the theory to these cases. Third, we show that the *option* to resort to single-limb voting, although it may not be used, deters the entry of holdouts—thereby providing a theoretical underpinning for the design of enhanced CACs. Fourth, we propose a dynamic equilibrium model of the sovereign bond market under aggregated CACs that can account for the joint behavior of restructuring outcomes and pre-restructuring valuations as seen in the data.

In the first part of our analysis in Sections 1-3, we consider the problem of a sovereign that seeks to restructure multiple bonds at the best possible terms under enhanced CACs. When deciding whether to accept a restructuring offer, bondholders have heterogeneous reservation values. We initially assume that all investors are atomistic, and we take the distribution of reservation values as given for each bond. The government's *aggregation dilemma*—i.e., the choice between the two restructuring methods—involves a clear tradeoff: relative to the

[5]See, e.g., Setser (2020) ("Ironically, Argentina is not actually using the most innovative feature of the new ICMA collective action clauses—so called single limb voting") and de la Cruz and Lagos (2021) ("Argentina chose not to use ICMA's prodigious child, the single-limb, uniformly applicable, aggregation method").

[6]The decision, taken in November 2021 as part of a reform of the Treaty establishing the European Stability Mechanism, will enter into force once the parliaments of all member states have ratified the Amending Agreement. At the time of writing, Italy is the only member state that has not yet ratified the agreement.

[7]As put by Setser (2020) in the context of the 2020 Argentine restructuring: "Technically, all of the international sovereign bonds are of equal legal rank. But that does not mean all the bondholders want the same deal. Some think their bonds have a claim to be treated better than other equally ranked bonds."

two-limb procedure, single-limb aggregation comes with a benefit arising from the removal of per-series constraints, but also with a cost as it prevents differentiated offers across bonds.

In a two-bond case, we characterize how the cost-minimizing aggregation procedure and restructuring offers depend on the heterogeneity across bonds, their relative sizes, and the voting thresholds. We show that single-limb aggregation is optimal only under specific circumstances, namely, when one bond is held by investors who are especially demanding in terms of recovery value and this bond is small in the restructuring pool. Under such conditions—which somewhat counterintuitively require sufficient *heterogeneity* across bonds—it can be valuable to use the single-limb procedure and make a uniform offer that attracts a low consent share from the more demanding bondholders. However, when the bonds are relatively homogeneous, or when the "expensive" bond—i.e., the bond whose holders tend to have higher reservation values—is large, the per-series constraints are not binding, hence single-limb voting only comes at a cost due to the uniform applicability restriction. Besides, the single-limb method typically entails a higher aggregate threshold, which contributes to making it less appealing.

Section 4 confronts the theory to the data. We delve into the specific details of the first three instances of sovereign bond restructurings in which aggregated voting was actually used—starting with the 2012 Greek "CAC retrofit"—and map these cases into the theory. In particular, considering the particular circumstances of the 2020 Argentine and Ecuadorian workouts under enhanced CACs, we find that our results help account for the use of two-limb voting by both governments. In the case of the Argentine "Macri bonds", there was substantial heterogeneity across series, but the more demanding ones comprised a large fraction of the restructuring pool. In the case of Ecuador, there was little heterogeneity across bonds. In both configurations, the unique advantage of single-limb aggregation was worthless.

To speak to the main issue that they seek to address—namely, the holdout problem—we analyze in Section 5 how the design of CACs affects the potential entry of (non-atomistic) vulture funds, that may be able to block a restructuring by acquiring sufficiently large positions. We show that, even though it may not be employed on the equilibrium path under enhanced CACs, the single-limb procedure plays an important role as an *off-equilibrium* deterrent to holdout attempts: vultures refrain from taking positions that would block the restructuring of one bond under two-limb voting because they anticipate that the government would then resort to single-limb aggregation to bypass the need to obtain majority approval in each bond. In that sense, the hype that surrounded the introduction of the single-limb procedure in 2014 was quite justified. Taken together, our results imply that single-limb and two-limb aggregation should be viewed as complementary tools in the restructuring arsenal: whereas the former discourages the entry of potential holdouts, the latter often turns out to be the optimal restructuring method once vultures have been kept at bay. The ongoing reform of Euro-Area CACs, which replaces two-limb by single-limb aggregation, fails to take advantage of this complementarity.

In Section 6, we provide a microfoundation for the reservation value distributions that we take as given in the first part of our analysis. Bonds explicitly differ in their maturities and coupon rates, and the bond market is populated by investors with heterogeneous discount rates. Investors sort into bonds in an initial trading stage and, in the event of a restructuring, the government optimally chooses the recovery values and aggregation method. In equilibrium, the government's restructuring approach depends on the bond-specific reservation value distributions and on relative face values in the same way as in the reduced-form model. However, the impact of a change in the bonds' relative size on the optimal voting procedure and payouts is more subtle in this setting, because a change in composition does affect the equilibrium reservation value distributions through the endogenous sorting of investors. In a numerical application, we show that the model is sufficiently rich to account for a number of quantitative features of the Argentine restructuring of Macri bonds under enhanced CACs, including differences in recovery values and pre-restructuring prices across maturities. Finally, we illustrate how the relative valuation of bonds ahead of a restructuring depends on the design of CACs in our setup.

Related Literature. Legal considerations lie at the heart of the sovereign debt literature. The seminal paper by Eaton and Gersovitz (1981) puts at center stage the lack of enforceability stemming from the absence of an international bankruptcy court and from the legal doctrine of sovereign immunity.[8] Bolton and Jeanne (2007) analyze how the weak contractual environment and lack of a bankruptcy regime affect the types of debt claims used in international borrowing and lending. Another important feature of the sovereign debt market—the absence of a seniority structure across debt claims—can be traced back to the common use of pari passu and negative pledge clauses in sovereign debt contracts, giving rise to the debt dilution problem analyzed in Bolton and Jeanne (2009), Chatterjee and Eyigungor (2015), and Hatchondo et al. (2016) among others. Our contribution consists in analyzing how some recent and widely adopted contractual innovations shape sovereign debt workouts and the sovereign bond market ahead of restructurings.

Following earlier work by Bulow and Rogoff (1989), a recent strand of the sovereign debt literature—e.g., Yue (2010), Benjamin and Wright (2013), Hatchondo et al. (2014), Dvorkin et al. (2021) and Arellano et al. (2023)—focuses on the restructuring process, studying its quantitative importance and examining the determinants of the level of haircuts and length of negotiations. In particular, Pitchford and Wright (2012) analyze how the type of settlement process affects delays in an environment where the government cannot commit to settling on worse terms with holdouts. Instead, we zoom in on the role of contractually specified voting procedures in determining restructuring payouts, and analyze their broader impact on the bond market equilibrium.

[8] Although sovereigns are no longer immune from suits in key jurisdictions such as the U.S. and the U.K., they are still effectively immune from attachment attempts by judgment creditors.

Existing theoretical work on majority voting provisions in sovereign bonds—see, e.g., Bi et al. (2016), Engelen and Lambsdorff (2009), and Haldane et al. (2005)—concentrates on *series-by-series* CACs.[9] Considering setups featuring one bond and a single set of creditors, these papers study how strategic interactions and restructuring outcomes are affected by the introduction of a supermajority rule in place of a unanimity requirement.[10] Closer to our paper, Bond and Eraslan (2010) analyze the optimal choice of voting rule by the debtor government. By design, this prior work is silent on cross-bond heterogeneity and aggregation. Instead, we adopt a setting with multiple bond series to address issues specifically related to the new enhanced CAC standard and to the design of aggregated Euro CACs.[11]

On the empirical front, while the literature on private debt workouts—such as, e.g., Cruces and Trebesch (2013)—mostly focuses on aggregate restructuring outcomes, Sturzenegger and Zettelmeyer (2006, 2008) and Zettelmeyer et al. (2013) document within-deal variation in haircuts for selected episodes, Fang et al. (2021) analyze the combined impact of CACs and haircuts on participation rates within restructuring episodes, and Asonuma et al. (2023) systematically explore the relationship between haircuts and maturity at the bond level. None of these studies investigates the impact of *aggregated* CACs on restructuring outcomes. Our contribution consists in documenting how enhanced CACs were used when they were first tested and in mapping these cases into the theory. As more evidence on the restructuring of bonds with aggregated CACs accumulates in the future, our analysis can provide guidance for further empirical work, especially with regard to the choice of aggregation procedure.

Finally, our analysis is connected to a series of empirical studies that investigate how the inclusion of CACs affects sovereign bond prices, starting with the early contribution by Eichengreen and Mody (2004). Among recent work on the topic, Picarelli et al. (2019) and Carletti et al. (2021) focus on the pricing of Euro CACs, while Chung and Papaioannou (2021) document the impact of enhanced CACs in normal times and during distress episodes. To our knowledge, no study has yet explored the differential price impact of various forms of aggregated voting provisions, and how it varies across bonds. Our framework could be used to derive theoretical predictions on these effects and guide such investigations.

Outline. The paper proceeds as follows. Section 1 describes the tradeoff that the government faces in choosing its restructuring approach under enhanced CACs and formulates general sufficient optimality conditions. Section 2 offers an analytical characterization of the optimal aggregation procedure and restructuring proposal in the two-bond case, while Section 3 gives closed-form results and illustrations in a parametric example. Section 4 provides detailed

[9] This original (non-aggregated) version of CACs has long been inserted in English-law governed sovereign issues and became commonly adopted under New York law in the early 2000s.

[10] Ghosal and Thampanishvong (2013) analyze the impact of a change in voting threshold on interim and ex ante efficiency in the presence of debtor moral hazard and incomplete information.

[11] Early analyses of CACs in a one-bond setting are still relevant in practice in situations where a single bond is being restructured, or when the bonds being restructured only feature old-style series-by-series CACs.

information on the concrete use of aggregated CACs and discusses the relevant cases in light of the theory. Section 5 analyzes how the design of CACs affects the potential entry of vulture funds. Section 6 embeds the government's problem in a setting where bond-specific reservation value distributions arise endogenously from the sorting of heterogeneous investors into bonds. Section 7 concludes. All proofs are in the appendix.

1 The Restructuring Problem

This section lays out a general framework to analyze the government's optimal use of aggregated voting provisions in the restructuring of multiple bonds under enhanced CACs, allowing for creditor heterogeneity both within and across bond series.[12]

Bonds, Restructuring Proposal, and Creditor Heterogeneity. There is a countable set $\mathcal{B}$ of bond series to be restructured, with $|\mathcal{B}| \geq 2$. The bonds are unsecured obligations, and are of equal legal rank. The relative size of bond series i, expressed as a fraction of the face value of the entire restructuring pool, is given by λ_i, with $\sum_{i \in \mathcal{B}} \lambda_i = 1$. A restructuring proposal $\mathbf{w} = \{w_i\}_{i \in \mathcal{B}}$ made by the government consists of series-specific recovery values w_i per unit of face value. Upon receiving an offer from the government, a bondholder accepts if the proposed recovery value w_i is at least as high as her own idiosyncratic reservation value.[13] To capture within-series creditor heterogeneity as well as cross-series heterogeneity arising from differences in creditor base and/or bond characteristics (e.g., payment terms), we assume that the reservation values of holders of bond series i are distributed according to the cumulative distribution function F_i, known by the government.[14] The share of holders of series i that give their consent to an offer w is thus equal to $F_i(w)$. For expositional simplicity, we assume that all investors are atomistic and that the CDFs are continuous. Section 5 analyzes an extension where a large investor can take blocking positions.

Aggregated Voting Rules. The sovereign can choose among two voting procedures—or "modification methods"—at the time of the restructuring. According to the two-limb procedure, all bond series are restructured if the weighted consent share over the entire pool is greater than or equal to the aggregate threshold τ_2^{a}, and if the consent share within each

[12]In practice, a bond series typically corresponds to a unique ISIN, although it may sometimes include bonds with different ISINs but same payment terms.

[13]We take these reservation values as exogenous, abstracting from explicit strategic considerations that may affect the expected value of holding out. There exists little evidence on holdout payoffs, apart from well-publicized cases such as the Argentine settlement following the 2001 default—see Cruces and Samples (2016). Schumacher et al. (2021) provide empirical evidence on the incidence of sovereign debt litigation.

[14]In practice, the government learns about reservation values during preliminary talks with bondholder committees. In Appendix B, we consider an environment where the government faces some uncertainty over the consent shares that a restructuring proposal will attract. We show that the presence of uncertainty does not alter the main insights from our analysis.

series is no smaller than $\tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}}$. Instead, under single-limb voting, the uniform applicability condition requires that the same offer be made to all bond series,[15] and CACs are triggered as long as the aggregate threshold τ_1 is reached.

1.1 Government's Problem

The government seeks to restructure all bonds series $i \in \mathcal{B}$ at the best possible terms. The set of constraints that need to be satisfied by the restructuring proposal to achieve this objective depends on the elected modification method. Under two-limb voting, the restructuring offer $\mathbf{w} = \{w_i\}_{i\in\mathcal{B}}$ must satisfy the aggregate constraint

$$\sum_{i\in\mathcal{B}} \lambda_i F_i(w_i) \geq \tau_2^{\mathrm{a}}, \tag{1}$$

along with the per-series consent requirements

$$F_i(w_i) \geq \tau_2^{\mathrm{s}} \quad \text{for all } i \in \mathcal{B}. \tag{2}$$

Instead, under the single-limb procedure, the "uniform" offer w must be such that

$$\sum_{i\in\mathcal{B}} \lambda_i F_i(w) \geq \tau_1.$$

In choosing the modification method and the offer w_i made to each bond series $i \in \mathcal{B}$, the government wishes to minimize the total payout

$$C = \boldsymbol{\lambda}.\mathbf{w} = \sum_{i\in\mathcal{B}} \lambda_i w_i.$$

We proceed under the realistic assumption that $1/2 \leq \tau_2^{\mathrm{s}} < \tau_2^{\mathrm{a}} \leq \tau_1 < 1$.[16]

1.2 Optimal Restructuring Offers

A preliminary step towards comparing two-limb vs single-limb aggregation consists in characterizing the optimal restructuring proposal under each procedure. The optimal uniform offer u^* under single-limb voting is such that the average consent share, weighted by the bond face

[15] This condition is meant to provide a safeguard to ensure inter-creditor equity, by avoiding that holders of large bond series dictate terms that are discriminatory against smaller series (see IMF (2014)).

[16] In the ICMA template, the thresholds are $\tau_1 = 3/4$, $\tau_2^{\mathrm{s}} = 1/2$, and $\tau_2^{\mathrm{a}} = 2/3$. The ICMA standard includes series-by-series voting as a third option, with a supermajority threshold $\tau_0 = 3/4$. Since two-limb aggregation dominates series-by-series voting, we focus on the optimal aggregation procedure.

values, is equal to the aggregate threshold τ_1, i.e.,

$$\sum_{\mathcal{B}} \lambda_i F_i(u^*) = \tau_1, \tag{3}$$

and the minimum restructuring cost C_1 under single-limb voting is equal to u^*.

Instead, the optimal two-limb offer is the solution to the constrained problem

$$C_2 \equiv \min_{\{w_i\}} \sum_{\mathcal{B}} \lambda_i w_i$$

subject to

$$\sum_{\mathcal{B}} \lambda_i F_i(w_i) = \tau_2^{\mathrm{a}}, \qquad \text{and} \qquad F_i(w_i) \geq \tau_2^{\mathrm{s}}, \quad i \in \mathcal{B}.$$

Since $\tau_2^{\mathrm{a}} > \tau_2^{\mathrm{s}}$, the aggregate constraint (1) is binding and holds as an equality. However, it is a priori unclear which of the per-series constraints, if any, may be binding.

Auxiliary Problem. It will be useful in our analysis to consider the auxiliary problem

$$\min_{\{w_i\}} \sum_{\mathcal{B}} \lambda_i w_i \qquad \text{subject to} \qquad \sum_{\mathcal{B}} \lambda_i F_i(w_i) = \tau, \tag{4}$$

for a given generic aggregate threshold τ. The government's cost-minimization problem under single-limb and two-limb voting can be construed by reference to this problem. Under single-limb voting, the aggregate threshold is τ_1 and the offer needs to satisfy the additional "uniform applicability" restriction, which simplifies the problem into (3). Instead, under two-limb voting, the aggregate threshold is τ_2^{a} and the government needs to take into account the additional series-by-series constraints (2). Whenever the solution to the auxiliary problem for $\tau = \tau_2^{\mathrm{a}}$ satisfies the latter constraints, it therefore coincides with the optimal two-limb offer. The Lagrangian for the auxiliary problem is

$$\mathcal{L} = \sum_{\mathcal{B}} \lambda_i w_i + \xi \Big(\tau - \sum_{\mathcal{B}} \lambda_i F_i(w_i) \Big).$$

Assuming differentiability, the first-order condition is

$$F_i'(w_i) = \xi^{-1}, \qquad \text{for all } i \in \mathcal{B}. \tag{5}$$

A marginal increase dw in the offer to series i raises the aggregate consent share by $\lambda_i F_i'(w_i) dw$ at a cost of $\lambda_i dw$. The optimality condition (5) requires that the "bang for the buck" $F_i'(w_i)$ be equalized across all bonds.

1.3 The Aggregation Dilemma: Key Tradeoff

Comparing across procedures, the unique appeal of single-limb voting comes from the fact that it removes the need to satisfy the per-series constraints; however, resorting to this method does entail a cost arising from the uniform applicability restriction—let alone the higher aggregate consent threshold when $\tau_1 > \tau_2^{\mathrm{a}}$. These observations immediately deliver sufficient conditions under which two-limb aggregation is optimal.

Proposition 1. *Two-limb aggregation is (at least weakly) optimal if one of the following conditions holds:*

(i) The optimal single-limb uniform offer, given by the unique solution u^* to (3), is such that $F_i(u^*) \geq \tau_2^{\mathrm{s}}$ for all $i \in \mathcal{B}$;

(ii) The solution $\widehat{\mathbf{w}} = \{\widehat{w}_i\}$ to the auxiliary problem (4) for $\tau = \tau_2^{\mathrm{a}}$ is such that $F_i(\widehat{w}_i) \geq \tau_2^{\mathrm{s}}$ for all $i \in \mathcal{B}$.

Furthermore, if condition (ii) holds, the optimal two-limb offer coincides with $\widehat{\mathbf{w}}$.

Indeed, the unique advantage of single-limb aggregation is worthless if any of the two conditions holds. Hence, two-limb voting dominates in these configurations. Going beyond these general statements requires making further assumptions on the environment—that is, on the number of bonds and their relative sizes λ_i, on the reservation value distributions F_i, and on the various voting thresholds.

2 Two-Bond Case

We now focus on the case where there are only two bond series outstanding, H and L, with relative weights $\lambda_H = \lambda \in (0, 1)$ and $\lambda_L = 1 - \lambda$. We denote by $F_i : \mathbb{R}^+ \to [0, 1]$ the cumulative distribution function of reservation values for bond $i \in \{H, L\}$, which we assume to be strictly increasing and twice differentiable on $\mathbb{R}^+$ with $F_i(0) = 0$, and we denote by f_i the corresponding density function.[17] We assume that holders of bond H tend to have higher reservation values, so that

$$F_H(w) < F_L(w) \quad \text{for all } w > 0, \tag{6}$$

or equivalently

$$F_L^{-1}(\tau) < F_H^{-1}(\tau) \quad \text{for all } \tau \in (0, 1).$$

[17] Section 3 illustrates the results derived in this section in the context of a parametric example.

Because it takes a more generous offer to reach a certain approval rate for bond H than for bond L, we will sometimes loosely refer to bond H as the "expensive" bond.

2.1 Single-Limb Aggregation

Under single-limb voting, the government's restructuring proposal must satisfy the uniform applicability condition, requiring the offer to be the same across series. As per (3), the cost-minimizing offer is given by $u^* = u(\lambda, \tau_1)$, where $u(\lambda, \tau)$ is implicitly defined as the unique solution to the equation

$$\lambda F_H(u) + (1-\lambda)F_L(u) = \tau. \tag{7}$$

The following observations obtain immediately.

Lemma 1. *The consent shares under the optimal uniform offer u^* are such that*

$$F_H(u^*) < \tau_1 < F_L(u^*).$$

Moreover, u^* is strictly increasing in τ_1 and in the relative size λ of the expensive bond, with

$$\lim_{\lambda \downarrow 0} u^* = F_L^{-1}(\tau_1) \qquad \text{and} \qquad \lim_{\lambda \uparrow 1} u^* = F_H^{-1}(\tau_1).$$

Next, we establish conditions under which $F_H(u^*) \geq \tau_2^{\mathrm{s}}$, in which case the optimal single-limb offer u^* satisfies both of the per-series constraints imposed under two-limb voting.

Lemma 2. *If $F_L^{-1}(\tau_1) \geq F_H^{-1}(\tau_2^{\mathrm{s}})$, the optimal uniform offer u^* is such that $F_H(u^*) \geq \tau_2^{\mathrm{s}}$ for any $\lambda \in (0,1)$. Otherwise, $F_H(u^*) \geq \tau_2^{\mathrm{s}}$ if $\lambda \geq \lambda_\dagger$, where $\lambda_\dagger$ is such that $u(\lambda_\dagger, \tau_1) = F_H^{-1}(\tau_2^{\mathrm{s}})$.*

Naturally, the optimal uniform offer $u^* = u(\lambda, \tau_1)$ is more likely to satisfy the per-series constraints for high values of τ_1 and low values of τ_2^{s}. Besides, since the offer u^* induces an average consent share τ_1, greater homogeneity across bonds makes it more likely that $F_H(u^*)$ is close to τ_1 and thus greater than τ_2^{s}. Finally, as captured by the second part of the lemma, a higher relative size λ of the expensive bond H makes it more likely that the constraint on this bond be satisfied by increasing the amount of the uniform offer u^*.

2.2 Two-Limb Aggregation

Under two-limb aggregation, the optimal offer is the solution to

$$\min_{w_H,\, w_L} \; \lambda w_H + (1-\lambda) w_L$$

subject to

$$\lambda F_H(w_H) + (1-\lambda)F_L(w_L) = \tau_2^{\mathrm{a}}, \tag{8}$$

$$F_i(w_i) \geq \tau_2^{\mathrm{s}}, \qquad i = H, L. \tag{9}$$

Let us first focus on the aggregate constraint (8), and let $\mathcal{W}$ denote the set of offers w_L to the holders of bond L such that this constraint can be met for some offer w_H to bond H.[18] Given an offer $w_L \in \mathcal{W}$ to bond L, the offer w_H that the sovereign needs to make to holders of bond H in order to satisfy the aggregate condition (8) is given by

$$w_H = F_H^{-1}\left(\frac{\tau_2^{\mathrm{a}} - (1-\lambda)F_L(w_L)}{\lambda}\right) \equiv g(w_L). \tag{10}$$

The function g is strictly decreasing in w_L, reflecting the fact that if a more generous offer is made to bond L—thus increasing the consent share for this series—a less appealing offer can be made to the other bond. Furthermore, one can see that for any pair of offers (w_H, w_L) satisfying the aggregate requirement (8), the following equivalence relationships must hold:

$$F_H(w_H) < F_L(w_L) \quad \Leftrightarrow \quad F_L(w_L) > \tau_2^{\mathrm{a}} \quad \Leftrightarrow \quad F_H(w_H) < \tau_2^{\mathrm{a}}. \tag{11}$$

Auxiliary Problem. To characterize the optimal offer under two-limb aggregation, we consider the corresponding auxiliary problem

$$\min_{w_H,\, w_L} \; \lambda w_H + (1-\lambda)w_L \quad \text{subject to} \quad \lambda F_H(w_H) + (1-\lambda)F_L(w_L) = \tau_2^{\mathrm{a}},$$

and we denote its solution by $\widehat{\mathbf{w}} = (\widehat{w}_H, \widehat{w}_L)$. This problem can be stated more concisely as

$$\min_{w_L \in \mathcal{W}} \; \lambda g(w_L) + (1-\lambda)w_L, \tag{12}$$

and one can show that the objective function in (12) is strictly convex when

$$\frac{d \log f_L(w_L)}{dw_L} < \frac{d \log f_H\big(g(w_L)\big)}{dw_L}. \tag{13}$$

In particular, (13) is satisfied if the two densities f_H and f_L are strictly decreasing. Assuming convexity, an interior solution to (12) is pinned down by the first-order condition

$$f_L(w_L) = f_H\big(g(w_L)\big). \tag{14}$$

[18]See Remark A-1 in the appendix for an explicit definition of $\mathcal{W} \equiv \mathcal{W}(\lambda, \tau_2^{\mathrm{a}})$.

Optimal Two-Limb Offer. The optimal two-limb offer coincides with the auxiliary solution $\widehat{\mathbf{w}}$ as long as the latter satisfies the two per-series constraints (9), i.e., when

$$\tau_2^{\mathrm{s}} \leq F_L(\widehat{w}_L) \leq \frac{\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}}}{1-\lambda}. \tag{15}$$

The second inequality in (15) guarantees that $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$, and is trivially satisfied if $\lambda \geq (1-\tau_2^{\mathrm{a}})/(1-\tau_2^{\mathrm{s}})$. On the other hand, the inequality $F_L(\widehat{w}_L) \geq \tau_2^{\mathrm{s}}$ is necessarily satisfied when λ is sufficiently small. Intuitively, when a bond is large relative to the total restructuring pool, the need to reach the high aggregate threshold τ_2^{a} makes it likely that the series-by-series requirement is met for this bond. When these sufficient conditions on λ fail, the following lemma provides an alternative set of conditions under which the auxiliary solution satisfies the constraints (9).

Lemma 3. *Suppose that the auxiliary problem (12) is strictly convex with an interior solution. The solution $\widehat{\mathbf{w}}$ to this problem satisfies the series-by-series constraint on bond H when $\lambda \geq (1-\tau_2^{\mathrm{a}})/(1-\tau_2^{\mathrm{s}})$ or if the inequality (A.6) given in the Appendix holds. It satisfies the series-by-series constraint on bond L when $\lambda \leq (\tau_2^{\mathrm{a}}-\tau_2^{\mathrm{s}})/(1-\tau_2^{\mathrm{s}})$ or if inequality (A.7) holds.*

When instead $\widehat{w}_i < F_i^{-1}(\tau_2^{\mathrm{s}})$, the individual constraint on bond i is binding. The optimal two-limb offer then sets $w_i = F_i^{-1}(\tau_2^{\mathrm{s}})$ and adjusts the recovery value on the other bond downwards to satisfy the aggregate consent condition.

2.3 Optimal Restructuring Procedure

In view of Proposition 1, we therefore obtain the following result.

Proposition 2. *If either the conditions stated in Lemma 2 or those provided in Lemma 3 hold, two-limb aggregation is optimal.*

Indeed, the conditions stated in Lemmas 2 and 3 guarantee that the optimal uniform offer and the auxiliary solution satisfy all per-series constraints, respectively—the unique advantage of the single-limb procedure being worthless in either case. In view of our discussion of Lemma 2, two-limb voting is more likely to dominate for high values of τ_1 and low values of τ_2^{s}, for low levels of heterogeneity across bonds, and for a high relative size λ of the expensive bond. To complement Proposition 2, it is worth noting that

$$\tau_1 > \tau_2^{\mathrm{a}} \qquad \Rightarrow \qquad \lim_{\lambda\downarrow 0} C_1 = F_L^{-1}(\tau_1) > F_L^{-1}(\tau_2^{\mathrm{a}}) = \lim_{\lambda\downarrow 0} C_2, \tag{16}$$

hence when $\tau_1 > \tau_2^{\mathrm{a}}$, two-limb voting also dominates for λ sufficiently small.

The last result of this section exhibits sufficient conditions under which single-limb voting dominates.

Proposition 3. *Suppose that* $F_L^{-1}(\tau_1) < F_H^{-1}(\tau_2^{\rm s})$ *and the densities* f_H *and* f_L *are strictly decreasing and intersect only once, at a point* $\tilde{w} \in \big(F_L^{-1}(\tau_1), F_H^{-1}(\tau_2^{\rm s})\big)$. *Then single-limb voting is optimal (i) for* λ *sufficiently small when* $\tau_2^{\rm a} = \tau_1$, *and (ii) in the neighborhood of the point* $\tilde{\lambda} > 0$ *such that* $u(\tilde{\lambda}, \tau_1) = \tilde{w}$ *when* $\tau_1 - \tau_2^{\rm a}$ *is not too large.*

The heterogeneity across bonds must be sufficiently large for the premise of Proposition 3 to be satisfied. Along with (16), Part (i) of the proposition reveals that, for low values of the relative weight λ of the expensive bond, the optimal voting procedure depends crucially (in an intuitive way) on whether $\tau_1 > \tau_2^{\rm a}$ or $\tau_1 = \tau_2^{\rm a}$. Part (ii) of the proposition points to a parameter configuration in which the auxiliary solution entails making the *same* offer $\tilde{w} < F_H^{-1}(\tau_2^{\rm s})$ to the two bonds, thereby violating the per-series constraint for bond H. Single-limb voting is clearly optimal in such a configuration, as the uniform applicability restriction comes at no cost while the unique advantage of single-limb aggregation is valuable.

3 Parametric Example

We now consider a particular specification of the two-bond case where reservation values for each bond are exponentially distributed, that is,

$$F_i(w) = 1 - e^{-\frac{w}{\phi_i}}, \qquad \text{for } w \geq 0, \tag{17}$$

implying that $F_i^{-1}(\tau) = -\phi_i \log(1-\tau)$, for all $\tau \in (0,1)$. As before, we suppose that holders of bond H tend to have higher holdout values. We thus proceed under the assumption that

$$\gamma \equiv \frac{\phi_H}{\phi_L} > 1.$$

3.1 Closed-Form Results

Under the exponential specification, the auxiliary problem is strictly convex and the consent shares under the auxiliary solution are given by[19]

$$F_L(\widehat{w}_L) = \frac{\lambda(\gamma-1) + \tau_2^{\rm a}}{\lambda(\gamma-1)+1} > \tau_2^{\rm a}, \tag{18}$$

and

$$F_H(\widehat{w}_H) = \frac{\tau_2^{\rm a} - (1-\lambda)F_L(\widehat{w}_L)}{\lambda} = \frac{1 + \lambda(\gamma-1) - \gamma(1-\tau_2^{\rm a})}{1+\lambda(\gamma-1)} < \tau_2^{\rm a}. \tag{19}$$

From (18)-(19), one can see that only the individual constraint on bond H may ever be

[19]The analytical derivations for this section are provided in Appendix A.3.

binding. The auxiliary consent shares (and corresponding offers $\widehat{w}_L$ and $\widehat{w}_H$) are increasing in the relative size λ of the more demanding bond. Moreover, one can check that $F_L(\widehat{w}_L)$ is increasing in γ while $F_H(\widehat{w}_H)$ is decreasing in γ—i.e., the spread in consent shares under the auxiliary solution is increasing in the degree of bond heterogeneity.

Optimal Two-Limb Offer. The optimal offers under two-limb aggregation coincide with the auxiliary solution as long as $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$. Since $F_H(\widehat{w}_H)$ is increasing in λ and decreasing in γ, the inequality is more likely to hold for high values of λ and low values of γ. Indeed, one can show that $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$

- if $\gamma \leq (1-\tau_2^{\mathrm{s}})/(1-\tau_2^{\mathrm{a}}) \equiv \overline{\gamma}_X$, for all values of λ;
- if $\lambda \geq (1-\tau_2^{\mathrm{a}})/(1-\tau_2^{\mathrm{s}}) \equiv \underline{\lambda} \in (0,1)$, for all values of γ;
- in the remainder of the parameter space for γ sufficiently small or λ sufficiently large.[20]

Conversely, the consent requirement on the expensive bond H is binding when there is sufficient heterogeneity across the two bonds and the relative size of bond H is small. Analytical expressions for the optimal two-limb offers in this case are provided in the appendix.

Optimal Voting Procedure. When parameter values are such that $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$, two-limb aggregation dominates since the per-series constraints have no bite. The other sufficient condition for two-limb optimality is that the optimal uniform offer u^* satisfies the per-series constraints (i.e., $F_H(u^*) \geq \tau_2^{\mathrm{s}}$), in which case the unique advantage of single-limb aggregation is worthless. In the appendix, we show that this condition holds if the heterogeneity parameter γ is below some threshold $\overline{\gamma}_U$, or else if λ is sufficiently large.[21]

Proposition 3 allows us to establish instead a sufficient condition for single-limb optimality. We show in the appendix that the premise of the proposition is satisfied for sufficiently high values of γ. Then, single-limb voting is optimal for λ sufficiently close to zero when $\tau_1 = \tau_2^{\mathrm{a}}$. If instead $\tau_1 > \tau_2^{\mathrm{a}}$, as long as $\tau_1 - \tau_2^{\mathrm{a}}$ is not too large, single-limb aggregation is optimal when the relative size λ of bond H is close to the value $\tilde{\lambda}$—given by (A.23) in the appendix—such that the auxiliary solution would require giving the same recovery value to the two bonds, with a consent share for bond H below τ_2^{s}.

When none of the sufficient conditions holds, one can still compare the restructuring costs across the two aggregation procedures. Under two-limb voting, the constraint on bond H must then be binding, and the restructuring cost is given by

$$C_2 = \lambda \phi_H \log\left(\frac{1}{\zeta_2^s}\right) + (1-\lambda)\phi_L \log\left(\frac{1-\lambda}{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}\right),$$

[20] An explicit condition of the form $\lambda \geq \ell_X(\gamma)$ for $\gamma > \overline{\gamma}_X$ is provided in the appendix, see Remark A-3.

[21] The appendix gives an explicit condition of the form $\lambda \geq \ell_U(\gamma) \in (0,1)$ for $\gamma > \overline{\gamma}_U$.

where $\zeta_2^j \equiv 1 - \tau_2^j$ for $j \in \{\mathrm{s}, \mathrm{a}\}$. Noting that the optimal single-limb offer u^* is such that

$$F(u^*) \equiv \lambda F_H(u^*) + (1-\lambda)F_L(u^*) = 1 - \lambda e^{-u^*/\phi_H} - (1-\lambda)e^{-u^*/\phi_L} = \tau_1,$$

one can see that single-limb dominates if and only if $F(C_2) > \tau_1$. We thus obtain a necessary and sufficient condition for single-limb optimality in terms of parameter values, namely:

$$\lambda \left(\zeta_2^{\mathrm{s}}\right)^{\lambda} \left(\frac{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}{1-\lambda}\right)^{\frac{1-\lambda}{\gamma}} + (1-\lambda)\left(\zeta_2^{\mathrm{s}}\right)^{\lambda\gamma} \left(\frac{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}{1-\lambda}\right)^{1-\lambda} < 1 - \tau_1. \tag{20}$$

3.2 Illustration

For the sake of illustration, we first assume that $\tau_1 = \tau_2^{\mathrm{a}} = 2/3$ and $\tau_2^{\mathrm{s}} = 1/2$, and we set the distributional parameters to $\phi_H = 0.7$ and $\phi_L = 0.2$ (i.e., the average reservation values for the two bonds are 70 and 20 cents on the dollar, respectively). Figure 1 depicts optimal offers (left panel) and consent shares (right panel) under the two aggregation methods as a function of the relative size λ of the expensive bond. Consider the two-limb modification method first. When λ is large enough, the auxiliary solution is feasible, both offers are strictly increasing in λ and consent shares are larger than the series-by-series threshold τ_2^{s}. When instead λ is low, the series-by-series constraint for the expensive bond binds: w_H is flat, the consent share for bond H equals τ_2^{s}, and the government sets $w_L < \widehat{w}_L$, so as to satisfy the aggregate consent requirement. Under single-limb voting, the optimal uniform offer and the associated

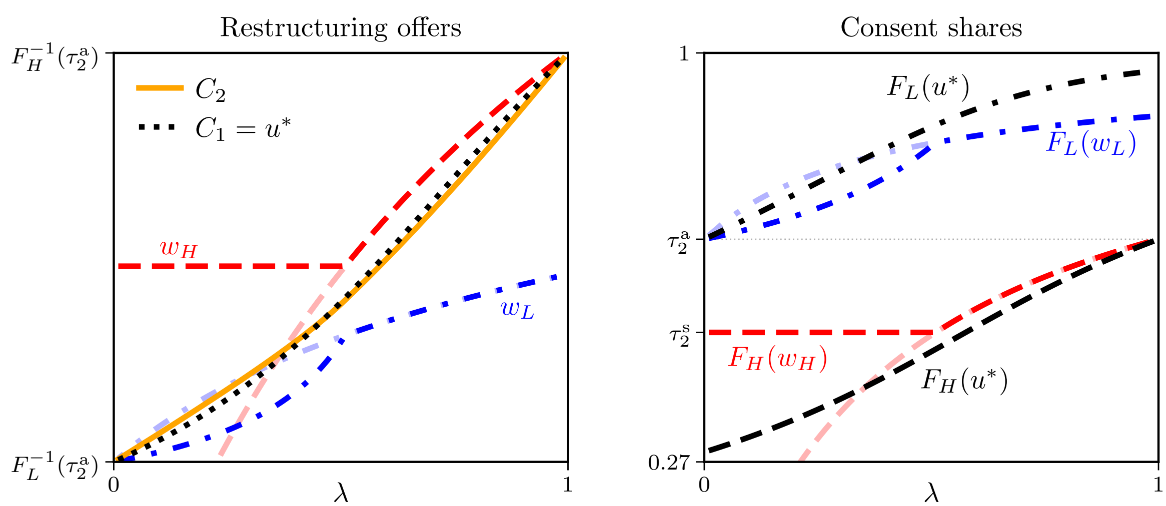

Figure 1: Optimal Restructuring Offers and Consent Shares.

Notes: Optimal offers and consent shares under two-limb voting are depicted in both panels as dash-dotted blue and dashed red lines for bonds L and H, respectively, with shaded lines corresponding to the auxiliary solution when it violates the constraint for bond H. The optimal uniform offer under single-limb aggregation is depicted as a dotted black line in the left panel, and the corresponding consent shares for bonds L and H appear as dash-dotted and dashed black lines in the right panel. The left panel also represents as a solid orange line the restructuring cost C_2 under two-limb voting. Parameter values are given in the main text.

consent shares are smoothly increasing in λ. The left panel of Figure 1 also displays the total restructuring cost as a function of λ for each of the two modification methods—showing that the two-limb method dominates for sufficiently high values of λ, while single-limb aggregation dominates when the share of the expensive bond is low.

Optimal Aggregation Method. Figure 2 represents the regions of the parameter space in which each of the two aggregation methods dominates, when $\tau_1 = \tau_2^{\mathrm{a}} = 2/3$ (left panel) and when the thresholds are set as in the standard ICMA CACs, with $\tau_1 = 3/4$ and $\tau_2^{\mathrm{a}} = 2/3$ (right panel). In both panels, the optimal restructuring method is determined according to the necessary and sufficient condition (20) as a function of the relative size λ of the expensive bond and the degree of heterogeneity across bonds—captured on the y-axis by $\log(\gamma) > 0$. Consistent with the rest of our analysis, the figure illustrates that single-limb aggregation dominates when the heterogeneity across bonds is substantial and the relative size of the expensive bond is not too large.[22] Naturally, the region where single-limb voting dominates shrinks when τ_1 increases. The right panel of the figure also illustrates that when $\tau_1 > \tau_2^{\mathrm{a}}$, two-limb voting dominates in the neighborhood of $\lambda = 0$, in line with the observation (16).

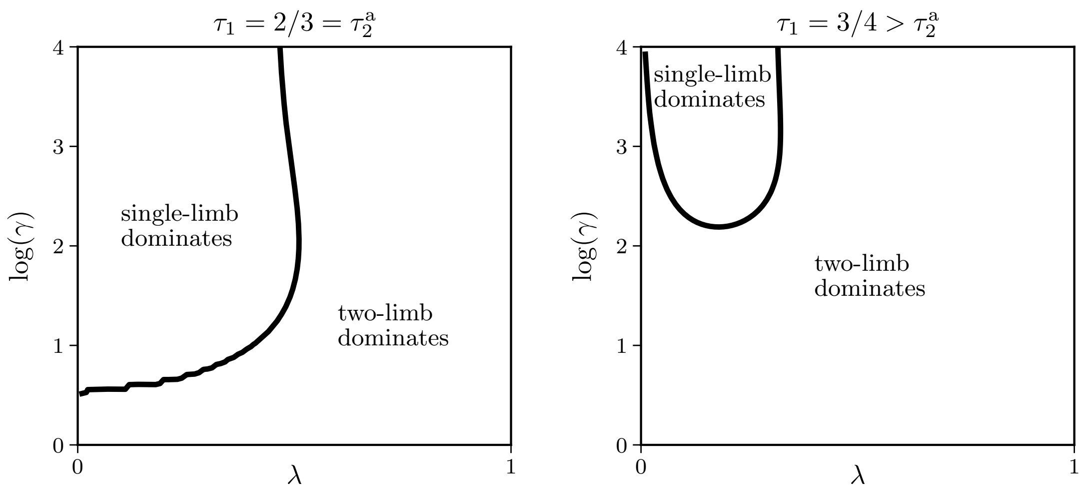

Figure 2: Optimal Voting Rule as a function of Relative Size (λ) and Bond Heterogeneity (γ).

4 Aggregated CACs in Action: Case Studies

Turning to the data, this section provides detailed information on the use of aggregate voting in sovereign bond restructurings. To start with, we review the very first use of aggregation in the context of the Greek restructuring of 2012. We then document the approach taken by Argentina and Ecuador to restructure their bonded debt under enhanced CACs in the summer of 2020. We discuss these three cases in light of the insights derived in the previous sections.

[22]Figure B.4 in Appendix B generalizes this result to an environment with stochastic consent shares.

4.1 Greece 2012

The private debt workout conducted by the Greek authorities in March 2012 (euphemistically referred to as "Private Sector Involvement", or PSI) is the largest such operation in history.[23] The restructuring pool was comprised of 117 bonds, with a total face value of €205.6 billion. Although all bonds received the same exchange offer, the voting procedure that determined their restructuring outcomes very much differed across categories of bonds. While Greece handled its foreign-law bonds according to their contractual provisions, it resorted to an innovative legal procedure—a CAC retrofit—to deal with its bonds governed by domestic law.

The Restructuring of Greek Foreign-Law Bonds. About 10% of the public debt subject to the PSI consisted of bonds governed by foreign law. The vast majority of these instruments were governed by English law and featured *series-by-series* CACs. A few other bonds—governed by Japanese, Swiss, or Italian law—did not incorporate any majority voting provision. In the latter, the exchange offer was opposed by a large fraction of bondholders (69% on average). Among the 35 instruments governed by English law, CACs were triggered in 17 series, taking the final participation rate for these bonds to 100%. In other English-law governed bonds, holdouts had acquired a blocking minority that prevented the activation of CACs, illustrating the limitations of series-by-series CACs as a defense against free riders.[24]

The Aggregated CAC Retrofit. The other 90% of the restructuring pool consisted of bonds governed by domestic law—including Greek government bonds (GGBs) and bonds issued by state-owned enterprises with a government guarantee. None of these instruments originally incorporated CACs. However, since they were issued under local law, it only took a change of statute to restructure them. Concretely, in February 2012, the Greek authorities passed a law, known as the Greek Bondholder Act, to the effect of inserting retroactively a majority voting provision in all local law GGBs—which, with a face value of €177 billion, accounted for the largest share (about 86%) of the entire restructuring pool.[25] The retrofitted CAC was of the single-limb type, with an aggregate majority threshold set at 66.7% of the face value taking part in the vote. It would thus have cost a wannabe holdout about €59 billion to secure a blocking position.

Payment terms differed significantly across the instruments subject to the CAC retrofit, with maturities ranging from 2012-2040, and coupon rates on medium- and long-term bonds between 2.30% and 6.50%. Shorty before the release of the exchange offer, shorter-maturity bonds traded about 50% higher than long-term bonds. Nonetheless, although it was not

[23] This section borrows extensively from the detailed analysis of the Greek PSI by Zettelmeyer et al. (2013).

[24] The share of holdouts in some of these bonds was as high as 100%. A single investor, Dart Management, reportedly owned one English-law bond almost entirely. Interestingly, holdouts did not block the activation of CACs in the largest series (e.g., in the €5.5 billion government bond with final payment due 11 April 2016).

[25] The authorities used moral suasion to ensure an orderly restructuring of the 22 series of local-law guaranteed bonds. Holdouts troubled the restructuring of one series, while other series obtained 100% participation.

bound by any "uniform applicability" constraint, Greece offered the same exchange package to all bonds.[26] Overall, the required majority was largely met and all local-law GGBs were entirely restructured, ensuring the large success of the PSI.[27]

Discussion. The aggregated CAC retrofit contributed considerably to a swift and orderly operation that resulted in the restructuring of approximately €199 billion of bonded debt. Nonetheless, one may wonder why Greece did not avail itself of the possibility to make differentiated offers across bonds. According to Zettelmeyer et al. (2013), the main reason why Greece opted for a single offer was to streamline the deal as much as possible to complete it before the amortization of a €9.7 billion GGB due 20 March 2012. In theory, nothing prevents the solution to the auxiliary problem (4) from entailing a uniform offer across bonds. An extra consideration that may be relevant in this case is that long-term bonds were primarily held by institutional investors who took part in negotiations with the authorities, whereas short-dated instruments had recently been bought by distressed debt funds at large discounts in the hope of a quick full repayment. The government's objective in the auxiliary problem may have involved distorted weights reflecting the higher perceived cost of payouts made to vultures.

Could another Euro-Area government exploit its "local law advantage" in the same way as Greece did? Although the answer is uncertain, this is rather unlikely. According to Grund (2017), the jurisprudence of European courts in the wake of the Greek PSI sets important limits to future potential interference with bondholders' property rights. Another important difference is that the original documentation of local-law GGBs did not specify any restructuring procedure, so that the Greek Bondholder Act was actually filling a void, whereas Euro-Area government bonds now incorporate majority voting provisions. Any attempt at changing such provisions ex post to further facilitate a restructuring would probably be met with serious legal challenges.

4.2 Argentina 2020

The restructuring of Argentine international bonds conducted by the Fernandez administration between April and August 2020 involved two distinct sets of bonds.[28] The "Kirchner bonds", comprised of 8 series, had been issued as part of the 2005-2010 exchange operations following Argentina's 2001 default. Instead, the 17 series that constituted the set of "Macri bonds" had been issued between 2016-2019 under the Macri administration—once Argentina

[26] Extending a single offer when attempting to restructure multiple sovereign bonds is not the norm. According to Asonuma et al. (2023), over the 1998-2019 period, 19 restructurings of international sovereign bonds (out of 27 in total) involved multiple bonds, 11 of which entailed differentiated offers across series.

[27] Holders of the more expensive short-term bonds were probably more reluctant to accept the uniform exchange offer. Unfortunately, instrument-level data on participation rates is not publicly available for GGBs.

[28] The material of this section is mostly based on the prospectus of the exchange invitation and on the press release (dated 31 August 2020) announcing the exchange results.

had finally settled with tenacious holdouts 15 years after its default, and thereby regained access to the international bond market. The total face value of these eligible bonds amounted to about \$65 billion, of which the Macri bonds accounted for almost two thirds (\$42 billion).

While all 25 series had been issued under English or New York law, the two sets of bonds were subject to different contractual frameworks (or "indentures") and contained two distinct versions of aggregated CACs. On the one hand, under the 2005 indenture, the Kirchner bonds included a two-limb version of CACs, with an aggregate threshold of 85% and a per-series threshold set at 66.7%. On the other hand, the 2016 indenture governing Macri bonds featured enhanced ICMA CACs, leaving the sovereign the choice between two-limb aggregation (with aggregate and per-series thresholds 66.7% and 50%, respectively) and the "uniformly applicable" single-limb voting procedure, with a 75% aggregate threshold.

Both sets of bonds were quite heterogeneous.[29] Among the Kirchner bonds, the most substantial difference was between the four series of so-called "Discount Bonds", which had high coupon rates (between 7.82%-8.28%), and the four series of "Par Bonds", which had lower coupon rates (3.38-3.75%, increasing to 4.74-5.25% from 2029 onward).[30] Instead, Macri bonds differed considerably in their maturities, which ranged from 2020 all the way to 2117. Since Argentine bonds were not accelerated following the payment default of May 2020, differences in coupon rates and maturities remained material at the time of the restructuring.

Two-Limb CACs—The Restructuring of Kirchner Bonds. In dealing with Kirchner bonds, Argentina had no choice with respect to the mode of aggregation, which had to be two-limb.[31] The Republic made differentiated exchange proposals, extending a more generous offer (by a factor of about 1.4) to the holders of Discount Bonds—which were more valuable due to their higher coupon rates. Wary of not meeting the 66.7% threshold in all 8 series, Argentina controversially announced that, should one or more series fail to reach that threshold, it would allow itself to "redesignate" ex post the aggregation pool.[32] Holdouts did obtain blocking minorities (about 37% and 40%) in two of the Par Bonds. But thanks to redesignation, CACs were triggered and the other six series of Kirchner Bonds were entirely restructured according to the proposal.

Enhanced CACs—The Restructuring of Macri Bonds. In dealing with the Macri bonds, the Republic could choose between two-limb and single-limb aggregation. One subtlety of the 2016 indenture—referred to as the "carry-over" clause—was that, if opting for the two-

[29]See Setser (2020) for a discussion of this aspect and its implications for the best restructuring approach.

[30]The names "Discount" and "Par" refer to the terms of the bond exchange at the time of the 2005-2010 restructurings. Discount and Par bonds had maturities in 2033 and 2038, respectively. Within each category, series differed in their currency denomination (USD or EUR) and issue date (2005 or 2010).

[31]Alternatively, Argentina could have decided to use series-by-series voting with a 75% threshold. Given the aggregate threshold of 85% under two-limb voting, the choice of modification method was not trivial, but the lower per-series threshold (66.7%) made it more difficult for a potential holdout to acquire a blocking minority.

[32]The analysis conducted in Appendix B in a setting with stochastic consent shares allows for redesignation.

limb method, the Republic could decide, in assessing the aggregate criterion (which required an average approval rate above 66.7%), to also take into account the votes of the Kirchner series. Given that Macri bonds accounted for about 2/3 of the face value of all eligible bonds, assuming an 85% approval rate among Kirchner bonds, the carry-over clause effectively reduced the average consent share that had to be met across Macri bonds to 57.5%—indeed, $(1/3) \times 85 + (2/3) \times 57.5 = 66.7$. The government did select the two-limb procedure and made differentiated offers across series, extending a more attractive offer to the 2020-2036 maturities than to the 2046-2117 bonds. The aggregate requirement was met and all 17 Macri series obtained more than 50% per-series approval, so that all of them were fully restructured.

Discussion. The presence of aggregated CACs made the 2020 Argentine debt workout much smoother than the 2005-2010 restructuring, with about $63.9 billion of bonded debt successfully restructured in just a few months. The blocking of CACs in two of the Kirchner series, however, demonstrates that the two-limb voting procedure is still vulnerable to holdouts—a point that we further elaborate in Section 5.

Did Argentina use enhanced CACs optimally? The analytical insights derived in the previous sections suggest so. There was clearly a lot of heterogeneity across Macri series, with very long-term bonds (including the famous "century bond" expiring in 2117) trading at higher discounts ahead of the restructuring.[33] However, the more expensive bonds with maturities up to 2036 accounted for a large share (about 78%) of the face value of the Macri pool. Given the large weight of these more demanding series, a uniform offer would have had to attract a consent share of more than 50% from these bonds in order to obtain 75% average approval under single-limb aggregation.[34] Hence, the unique advantage of the single-limb procedure (i.e., removing the need to satisfy per-series constraints) was worthless. Moreover, the carry-over clause could be invoked to reduce the aggregate consent requirement on Macri series under two-limb voting, which further increased the appeal of the latter procedure.

4.3 Ecuador 2020

The Ecuadorian restructuring of 2020 involved 10 bond series, with a total face value of $17.4 billion. One of the bonds, issued in 2014, featured the old-style, within-series version of majority voting provisions. The other 9 bonds instead incorporated the aggregated ICMA model of CACs with standard threshold values (as in the Macri bonds). The latter bonds had maturities from 2022-2030, and coupon rates between 7.875% and 10.750%.

Like Argentina, Ecuador opted in favor of the two-limb procedure to restructure the bonds subject to enhanced CACs. According to the government's proposal extended on 20 July 2020,

[33] Prior to the restructuring, shorter-maturity Macri bonds traded 19% higher than longer-maturity ones.

[34] Even assuming 100% consent among the longer-term bonds, it would still have taken a 68% approval rate among the other bonds to reach the single-limb aggregate threshold ($0.78 \times 68 + 0.22 \times 100 = 75$).

each instrument could be exchanged against a basket of new bonds with the same reduction in face value. The composition of the exchange basket differed across series, with the duration of the exchange basket increasing in the maturity of the original instrument, but the exchange offers were overall quite similar across series.[35] On 28 August 2020, the government announced the completion of the restructuring: the conditions for the activation of CACs had been met and all bond series were to be fully restructured.[36]

Discussion. How does the Ecuadorian case map into theory? In this case, the degree of heterogeneity across the series comprised in the restructuring pool was arguably mild. The range of maturities was indeed much smaller than in the case of Macri bonds. As a matter of fact, the approval rates induced by rather similar exchange offers were very homogeneous across bonds, varying within a range of about 1%. With little heterogeneity across series, none of the per-series constraints is binding given the need to reach the (higher) aggregate threshold. Hence, two-limb aggregation is indeed optimal in such a configuration.

5 Aggregated CACs and Vultures

Taking one step back, this section analyzes how the design of CACs and their optimal use by the government affect the potential entry of vulture funds who, by acquiring large positions, may be able to block bond restructurings.

5.1 Setup with Vulture Entry

We consider an extension of the setup of Section 2 in which, prior to the restructuring episode, a vulture fund may potentially acquire bonds H and L from the continua of heterogeneous investors who hold them. We denote by μ_i the share of bond i acquired by the fund, and we denote by q_i the price paid by the fund per unit of face value. We assume that the fund needs to pay a fixed cost ε_i to enter the market for bond i, which can be interpreted as the search cost of finding a counterparty in a highly decentralized marketplace.

Blocking Positions. In a restructuring, holding a sufficiently large position effectively allows the fund to prevent the triggering of CACs. We assume that the fund systematically opposes a restructuring when it is able to do so, and that it derives h_i per unit of face value held from blocking the restructuring of bond i. In particular, single-limb aggregation can be blocked if $\sum_i \lambda_i \mu_i > 1 - \tau_1$, and the fund can block the restructuring of bond i under the two-limb

[35] As of 31 August 2020, at market prices, the exchange baskets for the 2022, 2023, and 2025-2030 series were worth 66.40, 64.82, and 63.93 cents on the dollar, respectively.

[36] Investor heterogeneity in litigation proclivity was manifested in late July 2020, when two funds, Contrarian and GMO, initiated (short-lived) legal action against Ecuador in a New York district court.

procedure when $\mu_i > 1 - \tau_2^{\mathrm{s}}$. We assume that if CACs are not triggered for a bond, then this bond is left entirely unrestructured.[37]

Bond Market Prices. The price q_i at which the fund may acquire a position in bond i depends on the payoff that atomistic investors expect at the subsequent restructuring stage. If the market understands that the fund will be unable to prevent the restructuring of bond i, the price q_i coincides with the government's optimal restructuring offer w_i. If instead the fund's acquired positions later enable it to block the restructuring of bond i, investors would ultimately be left with their own reservation values for this bond. In that case, each investor is thus willing to sell as long as the market price q_i is above her reservation value, implying that the fund can acquire a share μ_i of bond i at a cost of $F_i^{-1}(\mu_i)$ per unit of face value.

Cost of Holdout Strategies. Since $\tau_2^{\mathrm{a}} \leq \tau_1$, the minimum cost at which the fund can acquire a blocking position in the two bonds under either aggregation method is given by

$$\kappa_2 \equiv \inf_{\{\mu_i\}} \sum_i \mathbf{1}_{\{\mu_i>0\}} \Big(\lambda_i \mu_i F_i^{-1}(\mu_i) + \varepsilon_i \Big) \quad \text{s.t.} \quad \sum_i \lambda_i \mu_i > 1 - \tau_1.$$

Likewise, the minimum cost at which the fund may be able to prevent the restructuring of bond i via two-limb aggregation is

$$\kappa_i \equiv \lambda_i (1 - \tau_2^{\mathrm{s}}) F_i^{-1}(1 - \tau_2^{\mathrm{s}}) + \varepsilon_i, \qquad i = H, L.$$

We assume, without loss of generality, that bond L is the cheapest to block, i.e., $\kappa_L < \kappa_H$.[38]

Limited Financial Resources. We assume that the fund has limited resources relative to the size of the outstanding bond series, which restricts its potential ambitions as a holdout. Namely, we assume that the fund's resources e are such that

$$\kappa_L < e < \min\{\kappa_H, \kappa_2\}.$$

We thus focus on the case where the only holdout strategy that the fund may consider is to acquire a blocking position in one bond (namely, bond L) that would prevent the activation of CACs for this bond under two-limb aggregation.

5.2 Equilibrium

In equilibrium, the outcome of the entry stage in which the fund decides on (μ_H, μ_L) must be consistent with the outcome of the restructuring stage. To start with, it is straightforward to

[37] In other words, minimum participation thresholds are greater than or equal to the CACs thresholds.

[38] Under the maintained assumption (6), we know that $F_L^{-1}(1 - \tau_2^{\mathrm{s}}) < F_H^{-1}(1 - \tau_2^{\mathrm{s}})$. However, the blocking costs also depend on the size of the bonds (λ_i) and entry costs (ε_i).

see that, under the previously stated assumptions, the fund would set $\mu_H = 0$, i.e., it would refrain entirely from entering the market for bond H. Indeed, since any position that the fund may feasibly acquire in that bond would be non-blocking, the price q_H would be equal to the restructuring payout w_H, and the fixed entry cost would thus be enough to keep the fund at bay. By the same logic, one can see that any position $\mu_L > 0$ that is non-blocking is suboptimal. The only remaining question, therefore, is: would the fund decide to acquire a blocking position in bond L or would it rather stay away altogether?

Two-Limb CACs. Consider first the case where CACs only allow for two-limb aggregation. If the fund contemplates acquiring a blocking position in bond L, the optimal blocking share μ_L^* is given by the solution to the constrained problem

$$B \equiv \sup_{\mu_L > 1-\tau_2^{\mathrm{s}}} \lambda_L \mu_L \Big(h_L - F_L^{-1}(\mu_L) \Big) - \varepsilon_L \quad \text{s.t.} \quad \lambda_L \mu_L F_L^{-1}(\mu_L) + \varepsilon_L \leq e.$$

As long as $B > 0$, it is optimal for the fund to purchase a share μ_L^* of bond L and then block the restructuring of the bond in order to get the holdout payoff h_L, with net benefit B.

Enhanced CACs. Suppose now that the government can choose between the two aggregation methods. How would it respond if the fund builds a blocking position in bond L? Under the two-limb method, bond L would have to be left unrestructured. Instead, under single limb, the government would pick the uniform offer u^* that attracts an aggregate consent share τ_1. As long as the cost of leaving one bond unrestructured is sufficiently high,[39] the government would optimally respond using single-limb aggregation, thus defeating any attempt to block bond L. Hence, the fund would optimally refrain from entering, and the government would choose its restructuring approach according to the logic outlined in Section 2.

5.3 Vulture Entry and the Design of CACs

The analysis of this section establishes that, although single-limb aggregation may not be used in equilibrium under enhanced CACs, it does serve as an *off-equilibrium* threat that deters potential holdouts from acquiring a blocking position in a bond. Indeed, in the equilibrium under enhanced CACs described above, the vulture fund optimally decides to stay at bay. Off equilibrium, the fund considers building a blocking position $\mu_L > 1 - \tau_2^{\mathrm{s}}$ at a cost $q_L = u^*$ per unit of face value, where the market price q_L reflects the (correct) expectation that the government would defeat the holdout attempt using single-limb aggregation with uniform offer u^*—so that the fund would ultimately make a loss due to the fixed entry cost. Hence in the restructuring, the government ends up selecting the aggregation method according to the relative size of the bonds and the cross-bond heterogeneity in the reservation values of

[39] In practice, this cost may arise from not being able to re-access the international bond market.

non-vulture investors, as illustrated in Section 3. Unless a relatively small bond is held by highly demanding creditors, the government would thus be using two-limb aggregation in equilibrium. However, if the option to use single-limb aggregation were to be removed, the equilibrium outcome would be dramatically different: the vulture fund would optimally decide to acquire a blocking position in one bond, making its restructuring impossible or more costly.

6 Sorting Equilibrium under CACs

This section embeds the government's restructuring problem in a dynamic, continuous-time model where bonds differ in their maturities and the creditor base of each bond is determined endogenously. By providing an explicit treatment of the heterogeneity within and across bonds, we thus microfound the reduced-form formulation of Section 2. We assess the model's quantitative performance by taking it to the 2020 Argentine restructuring data.

6.1 Stationary Environment

The market is populated by a continuum of risk-neutral investors who differ in their discount rates r, distributed according to the cumulative distribution function G on $\mathcal{R} = [r_{\min}, r_{\max}]$.

Bonds. There are two bonds S and L. The face value of bond $i \in \{S, L\}$ decays exponentially at rate δ_i. We shall assume that $\delta_S > \delta_L$, hence bonds S and L can be thought of as short-term and long-term bonds, respectively. The government may default on its bonds, and the arrival time of default is exponentially distributed with parameter η. While there is no default, bond i pays out coupons continuously, at rate c_i. Upon occurrence of a default, bondholders receive a bond-specific recovery rate w_i per unit of face value, as further specified below. We assume that the government continuously issues new bonds, so that the relative face values of short- and long-term bonds remain constant over time. We denote by $\lambda_S \in (0,1)$ and $\lambda_L = 1 - \lambda_S$ the relative face values of the two bonds. Moreover, we denote by $\Delta q \equiv q_S - q_L$ the price differential (per unit of face value) between the two bonds.

Restructuring. Upon default, the government offers recovery rates $\mathbf{w} = (w_S, w_L)$ and selects one of the contractually defined modification methods to implement the restructuring. We denote by $h_i(r)$ the reservation value of an investor with discount rate r holding bond i, the exact specification of which depends on the details of the microfoundation. In what follows, we use the functional form

$$h_i(r) = \frac{c_i}{r + \delta_i + \chi}, \qquad \chi \geq 0.$$

Intuitively, the parameter χ captures the extent to which the investor's reservation value is discounted relative to her subjective valuation $c_i/(r+\delta_i)$ of the bond's promised cashflows. An investor accepts the restructuring offer w_i if and only if $r \geq c_i/w_i - (\delta_i + \chi)$.

Sorting Stage. Prior to the occurrence of a restructuring with anticipated recovery rate ω_i, the valuation of bond i (per unit of face value) by investor r is given by

$$Q_i(r, w_i) = \frac{c_i + \eta\, w_i}{r + \delta_i + \eta}.$$

Given anticipated recovery rates $\mathbf{w}$, investors choose which bond to hold based on their subjective valuations and the price differential Δq. The set of investors sorting into bond S is

$$\mathcal{R}_S(\Delta q, \mathbf{w}) = \big\{ r \in \mathcal{R} \,:\, Q_S(r, w_S) - Q_L(r, w_L) \geq \Delta q \big\},$$

while investors with discount rates in $\mathcal{R}_L(\Delta q, \mathbf{w}) = \mathcal{R} \setminus \mathcal{R}_S(\Delta q, \mathbf{w})$ hold bond L. We proceed under the assumption that investors take a unit position in either bond.

Equilibrium. Given a partition $\big(\mathcal{R}_S, \mathcal{R}_L\big)$, the mass of investors who hold bond i is

$$\mu_i = \int_{r \in \mathcal{R}_i} dG(r),$$

and the CDF of reservation values for bond i is

$$F_i(w) = \frac{1}{\mu_i} \int_{r \in \mathcal{R}_i} \mathbf{1}_{\big\{ h_i(r) \leq w \big\}} dG(r). \tag{21}$$

The government takes these distributions as given when choosing its restructuring approach.

Definition 1. *Given the distribution G of discount rates on $\mathcal{R}$, bond characteristics (c_S, δ_S) and (c_L, δ_L), relative face values (λ_S, λ_L), default arrival rate η, and discount parameter χ, an equilibrium consists of*

(i) a price differential Δq^* and a partition $\big(\mathcal{R}_S, \mathcal{R}_L\big)$,

(ii) a modification method (two-limb or single-limb) and a pair of recovery rates $\mathbf{w}^*$,

such that

1. *the government chooses the modification method and restructuring offers optimally given the implied distributions F_S and F_L given by (21);*
2. *investors optimally choose which bond to hold: $\mathcal{R}_i = \mathcal{R}_i(\Delta q^*, \mathbf{w}^*)$;*
3. *the market clears for each bond, $\mu_i = \lambda_i$.*

In what follows, we restrict our attention to parametric configurations that give rise to a threshold-type equilibrium in which long-term bonds are held by the more patient investors, with discount rates below some threshold $\widehat{r}$, i.e.,

$$\mathcal{R}_L = \left[r_{\min}, \widehat{r}\right] \qquad \text{and} \qquad \mathcal{R}_S = \left[\widehat{r}, r_{\max}\right].$$

Market clearing requires that the threshold $\widehat{r}$ be such that $G(\widehat{r}) = \lambda_L$, and the price differential is pinned down as $\Delta q^* = Q_S\big(\widehat{r}, w_S^*\big) - Q_L\big(\widehat{r}, w_L^*\big)$, where w_i^* denotes the optimal offer to bond i. Reservation values in the restructuring lie in the intervals

$$\mathcal{V}_S = \left[h_S(r_{\max}), h_S(\widehat{r})\right] \quad \text{and} \quad \mathcal{V}_L = \left[h_L(\widehat{r}), h_L(r_{\min})\right].$$

6.2 Numerical Application

We now assume that G is uniform on $\mathcal{R}$, in which case the reservation value CDFs and their supports can be written analytically as

$$F_i(w) = a_i - b_i/w, \qquad \text{for} \;\; w \in \mathcal{V}_i = \left[\frac{b_i}{a_i}, \frac{b_i}{a_i - 1}\right], \qquad i \in \{S, L\}, \tag{22}$$

where

$$\begin{aligned} a_S &= \frac{r_{\max} + \delta_S + \chi}{\lambda_S(r_{\max} - r_{\min})}, & b_S &= \frac{c_S}{\lambda_S(r_{\max} - r_{\min})}, \\ a_L &= \frac{\lambda_L r_{\max} + (1-\lambda_L) r_{\min} + \delta_L + \chi}{\lambda_L(r_{\max} - r_{\min})}, & b_L &= \frac{c_L}{\lambda_L(r_{\max} - r_{\min})}. \end{aligned} \tag{23}$$

Thanks to the simple functional form taken by the CDFs in this case, one can solve for the optimal restructuring offers under each aggregation method in closed form (see Appendix A.4).

For illustration, we take $[r_{\min}, r_{\max}] = [0.0025, 0.2775]$ and $(\delta_S, \delta_L) = (0.18, 0.006)$ for the bonds' decay rates, setting the coupon rates at $c_i = \mathbb{E}(r) + \delta_i$.[40] We fix the relative size of the short-term bond at $\lambda_S = 0.65$, and take $\eta = 0.3$ and $\chi = 0.275$ for the default intensity and discount parameters, respectively. These parameter values imply that the marginal investor has discount rate $\widehat{r} = 9.875\%$, while the durations of bonds S and L evaluated at this rate are 3.6 and 9.5 years, respectively—roughly in line with the durations of the shorter- and longer-maturity Argentine Macri bonds.[41] To provide for a clean comparison of aggregation methods, the voting thresholds are set at $\tau_2^{\mathrm{s}} = 1/2$ and $\tau_2^{\mathrm{a}} = \tau_1 = 2/3$. In the context of the restructuring of Macri bonds by Argentina, the difference in aggregate thresholds across procedures only reinforced the appeal of two-limb voting.

[40] This normalization implies that, absent default risk, the average investor would value the bonds at par.

[41] The principal-weighted average duration of the 2020-2036 bonds was 3.8 years, while that of the 2046-2117 bonds was 9.2 years. These two groups of bonds received different offers in the restructuring (see Section 4.2).

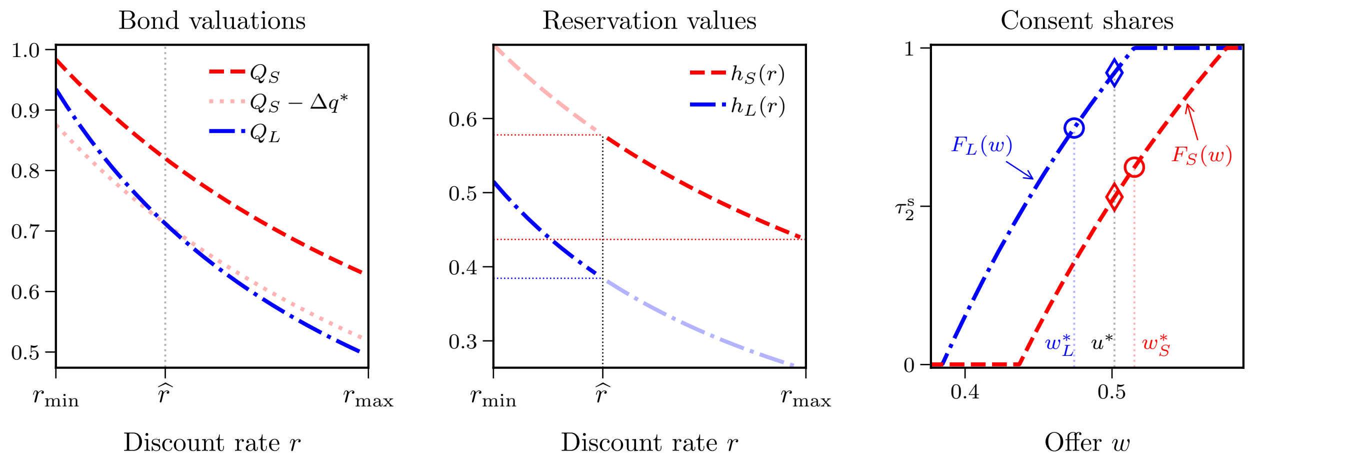

Figure 3: Equilibrium Outcomes in the Sorting and Restructuring Stages.

Figure 3 illustrates the equilibrium objects graphically for these parameter values. The left panel of the figure depicts the bond valuation functions $Q_i(.\,, w_i^*)$ and the threshold $\widehat{r}$ in the sorting stage; the center panel plots the reservation value functions $h_i(.)$, using shaded lines in the regions of $\mathcal{R}$ where they are not relevant; and the right panel illustrates the reservation value CDFs and optimal offers in the restructuring stage.

In this example, as in the case of the Argentine Macri bonds, the short-term bond commands a higher market price in the trading stage ($\Delta q^* > 0$). The holders of these bonds also tend to be more demanding in the restructuring, as captured by the relative position of the CDFs in the right panel. Optimal offers (and consent shares) under two-limb and single-limb aggregation can be read off the circle and diamond markers, respectively. In this parametrization, the auxiliary solution is feasible and two-limb voting dominates: the government makes differentiated offers across series, extending a more generous recovery value to the short-term bond ($w_S^* > w_L^*$)—as did Argentina in dealing with its Macri bonds. Under the alternative single-limb approach, due to the high weight λ_S, the uniform offer u^* attracts a consent share from bond S above τ_2^s, making the unique advantage of this procedure worthless.

Recovery Values: Model v Data. In the Argentine restructuring, the average recovery value received by Macri bonds, expressed as a fraction of face value, was about 46 cents on the dollar, and the recovery on short-term bonds was larger than that on long-term bonds by a factor of 1.12.[42] Our numerical application generates recovery values w_i around 50 cents on the dollar, with $w_S^*/w_L^* = 1.09$. One may instead want to consider recovery values expressed as a fraction of the pre-restructuring bond prices, as Asonuma et al. (2023) do in their empirical study of bond-level haircuts in sovereign workouts. In the data, the recovery values expressed as a fraction of the Macri bond prices observed as of the close of April 10, 2020 (i.e., one week before the initial terms of the restructuring proposal were announced) are 1.53 and

[42]This calculation relies on the close prices of exchange bonds as of 09/15/2020, a few days after settlement.

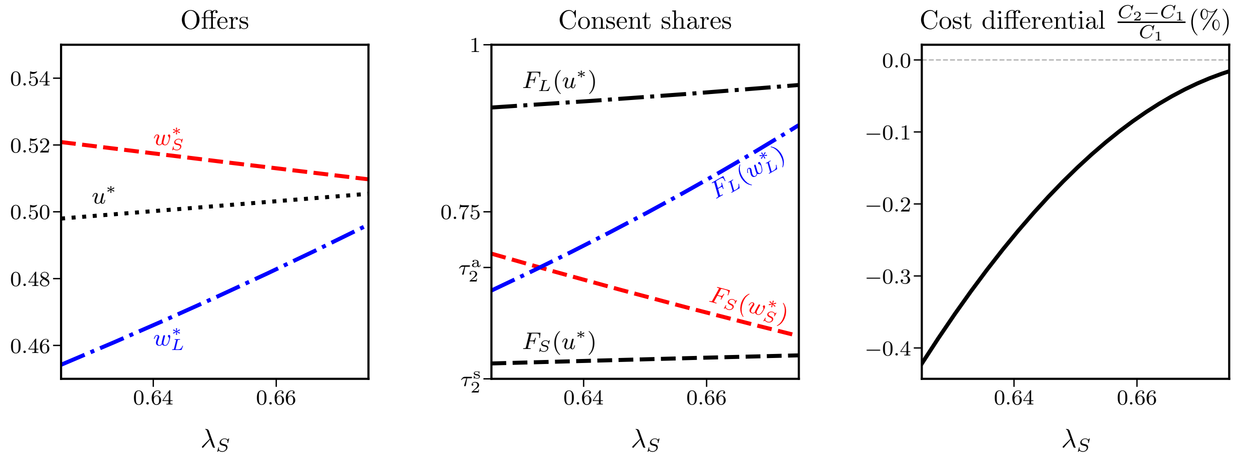

Figure 4: Comparative Statics with Respect to Relative Bond Size.

1.62 for short- and long-term bonds, consistent with the empirical regularity documented by Asonuma et al. (2023) that short-term bonds tend to receive lower recovery rates. The model counterparts to these quantities, w_i^*/q_i^*, are indeterminate in our setup: indeed, in our numerical example, any prices (q_S, q_L) such that $q_S - q_L = \Delta q^*$ and $q_S \leq Q_S(r_{\max}, w_S^*)$ could be observed in equilibrium.[43] Focusing on the equilibrium in which $q_S^* = 0.382$ and $q_L^* = 0.274$, the model-implied recovery rates are 1.35 and 1.73, respectively, roughly in line with the data.

6.3 Impact of Relative Size on Restructuring Outcomes

We now briefly revisit the comparative statics with respect to the relative size of the bonds. In the extended setup with sorting, a change in relative face values affects the equilibrium restructuring outcome through two distinct channels: the pure size effect discussed in Section 2, and an *additional* channel driven by the endogenous change in reservation value distributions.

For instance, consider a change in the relative size λ_S of the short-term bond in the previous numerical example, keeping other parameter values fixed. The first channel through which this change affects the restructuring outcome goes along the logic of Figure 1: holding the reservation value CDFs fixed, as the share of the more demanding short-term bond increases, all offers and consent shares increase. However, as captured by (22)–(23), a higher share λ_S of the short-term bond, by reducing the threshold $\widehat{r}$, raises the upper bound of $\mathcal{V}_S$ and the lower bound of $\mathcal{V}_L$, without affecting the other bounds: F_S becomes flatter and F_L steeper. We refer to the change in restructuring outcomes driven by this change in reservation value CDFs—keeping the weight of the short-term bond at its baseline value—as the "sorting channel".

Figure 4 depicts the total impact of a local change in λ_S, plotting recovery values, consent shares, and the difference in restructuring cost across the two-limb and single-limb procedures.

[43]The restriction $q_S \leq Q_S(r_{\max}, w_S^*)$ ensures that all investors in $\mathcal{R}_S$ are happy to buy bond S. It is easy to show that $q_L \leq Q_L(\widehat{r}, w_L^*)$ for any such q_S, ensuring that all investors in $\mathcal{R}_L$ are happy to buy bond L.

In this example, the pure size effect is dwarfed by the sorting channel. Due to the change in the slopes of the CDFs caused by an increase in λ_S, the latter channel causes a decrease in the two-limb offer for bond S and an increase for bond L, so as to keep the optimality condition (14) satisfied. As the offers become less differentiated across bonds (and closer to u^*) under two-limb voting, and as the per-series constraint on the more demanding short-term bond comes closer to being binding, the relative appeal of the two-limb procedure vanishes.

6.4 Impact of CACs on Bond Prices

Finally, we discuss how the design of CACs affects bond valuations, focusing on the equilibrium price differential (Δq^*) between the short- and long-term bonds. The specification of CACs affects the price differential in the trading stage through its impact on the equilibrium offers in the restructuring stage.

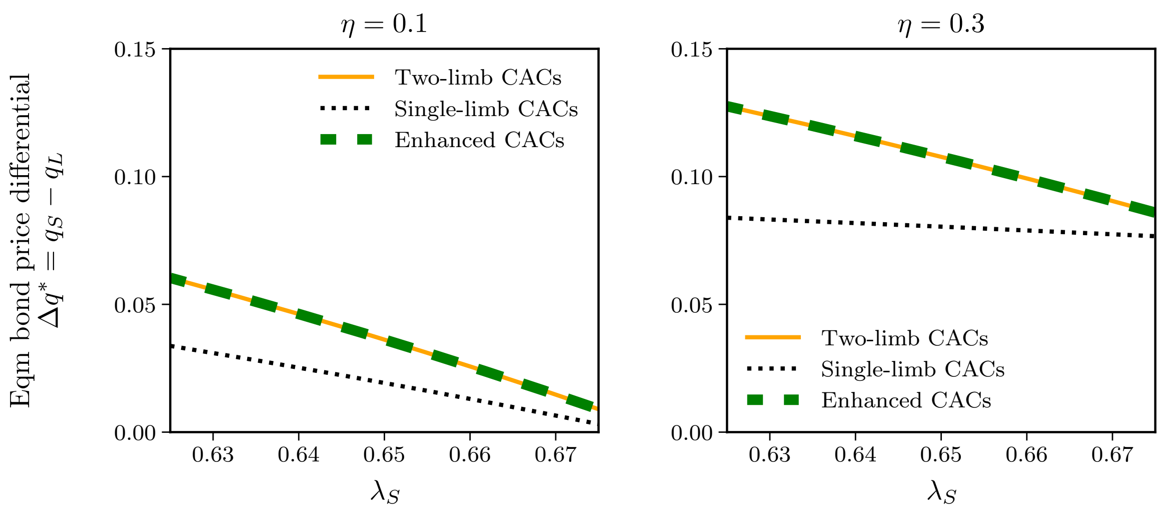

Figure 5: Bond Price Differential as Function of CACs Type, Relative Size, and Default Rate.

Figure 5 depicts Δq^* as a function of the relative size λ_S of the short-term bond under either two-limb CACs (solid orange), single-limb CACs (dotted black), or enhanced CACs (thick dashed green). The two panels are obtained for different values of the likelihood of default, η. Under any type of CACs, the relative price of the short-term bond is decreasing in its relative size and increasing in the arrival rate of default.[44] Moreover, in this example, the price differential Δq^* under single-limb CACs is lower than under two-limb CACs, as $w_L^* < u^* < w_S^*$, and the outcome under enhanced CACs coincides with that under two-limb CACs, as single-limb voting is never optimal. Finally, comparing across the two panels, the figure illustrates that the specification of CACs has a greater impact on relative prices when the probability of a future restructuring is larger.

[44]Bond valuations themselves are decreasing in the default likelihood η. In general, the impact of η on the price differential Δq^* may be non-monotonous.

7 Conclusion

For over two decades, policy efforts aimed at ensuring the timely and orderly resolution of sovereign debt distress have revolved almost entirely around CACs. This paper offers the first formal analysis of the now widespread *aggregated* CACs, which allow the government to restructure multiple bonds at the same time on the basis of the aggregate consent share.

Focusing first on the best choice of aggregation procedure by a sovereign seeking to minimize the total restructuring payout under enhanced CACs, we solve for the optimal restructuring approach as a function of the heterogeneity across bonds, their relative size, and the various voting thresholds. The resolution of the "aggregation dilemma" amounts to a non-trivial comparison between two non-linear constrained minimization problems. Our analysis reveals that resorting to the novel single-limb procedure, making a uniform offer to all series, is optimal only under specific circumstances—namely, when one bond is held by investors who are particularly reluctant to take a haircut and this bond is relatively small in the restructuring pool. Our results help rationalize the two-limb approach taken by Argentina and Ecuador in August 2020, when enhanced CACs were tested for the first time.

Our work also elucidates how aggregated CACs and the anticipation of their optimal use affect vulture entry, the sorting of investors across bonds, and relative bond valuations. In particular, we show that single-limb aggregation can serve as an *off-equilibrium* deterrent when potential holdouts are looming, even though it may not be used in equilibrium. We thus provide a theoretical justification for the design of enhanced CACs, which take full advantage of the complementarity between the two aggregation procedures.

Our analysis could be extended to $N > 2$ bonds. We conjecture that the method we have developed for the two-bond case can be applied recursively to solve for the government's optimal restructuring approach with many bonds, including the optimal use of sub-aggregation.

References

Arellano, Cristina, Xavier Mateos-Planas, and José-Víctor Ríos-Rull, "Partial Default," *Journal of Political Economy*, 2023, *131* (6), 1385–1439.

Asonuma, Tamon, Dirk Niepelt, and Romain Ranciere, "Sovereign Bond Prices, Haircuts and Maturity," *Journal of International Economics*, 2023, *140*, 103689.

Benjamin, David and Mark L.J. Wright, "Recovery before Redemption: A Theory of Delays in Sovereign Debt Renegotiations," Working Paper, 2013.

Bi, Ran, Marcos Chamon, and Jeromin Zettelmeyer, "The Problem that Wasn't: Coordination Failures in Sovereign Debt Restructurings," *IMF Economic Review*, 2016, *64* (3), 471–501.

Bolton, Patrick and Olivier Jeanne, "Structuring and Restructuring Sovereign Debt: The Role of a Bankruptcy Regime," *Journal of Political Economy*, 2007, *115* (6), 901–924.

_ and _ , "Structuring and Restructuring Sovereign Debt: The Role of Seniority," *The Review of Economic Studies*, 2009, *76* (3), 879–902.

Bond, Philip and Hülya Eraslan, "Strategic Voting over Strategic Proposals," *Review of Economic Studies*, 2010, *77* (2), 459–490.

Buchheit, Lee C and G Mitu Gulati, "Sovereign Bonds and the Collective Will," *Emory LJ*, 2002, *51*, 1317.

Bulow, Jeremy and Kenneth Rogoff, "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, 1989, *97* (1), 155–178.

Carletti, Elena, Paolo Colla, Mitu Gulati, and Steven Ongena, "The Price of Law: The Case of the Eurozone Collective Action Clauses," *Review of Financial Studies*, 2021, *34* (12), 5933–5976.

Chatterjee, Satyajit and Burcu Eyigungor, "A Seniority Arrangement for Sovereign Debt," *American Economic Review*, December 2015, *105* (12), 3740–65.

Chung, Kay and Michael G. Papaioannou, "Do Enhanced Collective Action Clauses Affect Sovereign Borrowing Costs?," *Journal of Banking and Financial Economics*, October 2021, *1* (15), 59–87.

Cruces, Juan J. and Christoph Trebesch, "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, July 2013, *5* (3), 85–117.

Cruces, Juan J and Tim R Samples, "Settling Sovereign Debt's Trial of the Century," *Emory Int'l L. Rev.*, 2016, *31*, 5.

de la Cruz, Andrés and Ignacio Lagos, "CACs at Work: What Next?," *Capital Markets Law Journal*, 2021.

Dvorkin, Maximiliano, Juan M. Sánchez, Horacio Sapriza, and Emircan Yurdagul, "Sovereign Debt Restructurings," *American Economic Journal: Macroeconomics*, April 2021, *13* (2), 26–77.

Eaton, Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 1981, *48* (2), 289–309.

Eichengreen, Barry and Ashoka Mody, "Do Collective Action Clauses Raise Borrowing Costs?," *The Economic Journal*, 04 2004, *114* (495), 247–264.

— **and Richard Portes**, *Crisis? What Crisis? Orderly Workouts for Sovereign Debtors*, CEPR Press, September 1995.

Engelen, Christian and Johann Graf Lambsdorff, "Hares and Stags in Argentinean Debt Restructuring," *Journal of International Economics*, June 2009, *78* (1), 141–148.

Fang, Chuck, Julian Schumacher, and Christoph Trebesch, "Restructuring Sovereign Bonds: Holdouts, Haircuts and the Effectiveness of CACs," *IMF Economic Review*, March 2021, *69* (1), 155–196.

Gelpern, Anna, Ben A. Heller, and Brad W. Setser, "Count the Limbs: Designing Robust Aggregation Clauses in Sovereign Bonds," in M. Guzman, J. A. Ocampo, and J. E. Stiglitz, eds., *Too Little, Too Late: The Quest to Resolve Sovereign Debt Crises*, Columbia University Press, 2016, pp. 109–143.

Gertner, Robert and David Scharfstein, "A Theory of Workouts and the Effects of Reorganization Law," *Journal of Finance*, 1991, *46* (4), 1189–1222.

Ghosal, Sayantan and Kannika Thampanishvong, "Does Strengthening Collective Action Clauses (CACs) Help?," *Journal of International Economics*, 2013, *89* (1), 68–78.

Grund, Sebastian, "Restructuring Government Debt under Local Law: the Greek Case and Implications for Investor Protection in Europe," *Capital Markets Law Journal*, 2017, *12* (2), 253–273.

Haldane, Andrew G., Adrian Penalver, Victoria Saporta, and Hyun Song Shin, "Analytics of Sovereign Debt Restructuring," *Journal of International Economics*, 2005, *65* (2), 315–333.

Hatchondo, Juan Carlos, Leonardo Martinez, and César Sosa Padilla, "Voluntary Sovereign Debt Exchanges," *Journal of Monetary Economics*, 2014, *61*, 32–50. Carnegie-Rochester-NYU Conference Series on "Fiscal Policy in the Presence of Debt Crises" held at the Stern School of Business, New York University on April 19-20, 2013.

— , — , **and César Sosa-Padilla**, "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 2016, *124* (5), 1383–1422.

ICMA, "Standard Aggregated Collective Action Clauses (CACs) for the Terms and Conditions of Sovereign Notes," August 2014.

IMF, "Strengthening the Contractual Framework to Address Collective Action Problems in Sovereign Debt Restructuring," September 2014.

_ , "The International Architecture for Resolving Sovereign Debt Involving Private-Sector Creditors—Recent Developments, Challenges, And Reform Options," October 2020.

Picarelli, Mattia Osvaldo, Aitor Erce, and Xu Jiang, "The Benefits of Reducing Hold-out Risk: Evidence from the Euro CAC Experiment, 2013–2018," *Capital Markets Law Journal*, 2019, *14* (2), 155–177.

Pitchford, Rohan and Mark L. J. Wright, "Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment," *Review of Economic Studies*, 2012, *79* (2), 812–837.

Roe, Mark J., "The Voting Prohibition in Bond Workouts," *Yale Law Journal*, April 1987, *97*, 232–79.

Schumacher, Julian, Christoph Trebesch, and Henrik Enderlein, "Sovereign Defaults in Court," *Journal of International Economics*, 2021, *131*, 103388.

Setser, Brad, "The State of Argentina's Debt Restructuring," `https://www.cfr.org/blog/state-argentinas-debt-restructuring`, June 24, 2020.

Sobel, Mark, "Strengthening Collective Action Clauses: Catalysing Change—the Back Story," *Capital Markets Law Journal*, 2016, *11* (1), 3–11.

Sturzenegger, Federico and Jeromin Zettelmeyer, *Debt Defaults and Lessons from a Decade of Crises* MIT Press, The MIT Press, 2006.

_ **and** _ , "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998-2005," *Journal of International Money and Finance*, September 2008, *27* (5), 780–805.

Yue, Vivian Z., "Sovereign Default and Debt Renegotiation," *Journal of International Economics*, 2010, *80* (2), 176–187.

Zettelmeyer, Jeromin, Christoph Trebesch, and Mitu Gulati, "The Greek Debt Restructuring: an Autopsy," *Economic Policy*, 2013, *28* (75), 513–563.

A Technical Appendix

This appendix contains the proofs and derivations of all the results stated in the main text.

A.1 Proof of Proposition 1

Define the set of offers that satisfy the aggregate constraint

$$\mathcal{A}(\tau) = \Big\{ \mathbf{w} \,:\, \sum_{i \in \mathcal{B}} \lambda_i F_i(w_i) \geq \tau \Big\},$$

the set of offers that satisfy the series-by-series constraints

$$\mathcal{S}(\tau) = \Big\{ \mathbf{w} \,:\, F_i(w_i) \geq \tau \quad \text{for all } i \in \mathcal{B} \Big\},$$

and the set of offers that satisfy the uniform applicability condition

$$\mathcal{U} = \Big\{ \mathbf{w} \,:\, w_i = w_j \quad \text{for all } i, j \in \mathcal{B} \Big\}.$$

The restructuring cost associated with the optimal single-limb offer is

$$C_1 \equiv \min_{\mathbf{w} \in \mathcal{A}(\tau_1) \cap \mathcal{U}} \boldsymbol{\lambda}.\mathbf{w} =: \boldsymbol{\lambda}.\mathbf{w}^*, \tag{A.1}$$

while the restructuring cost associated with the optimal two-limb offer is

$$C_2 \equiv \min_{\mathbf{w} \in \mathcal{A}(\tau_2^{\mathrm{a}}) \cap \mathcal{S}(\tau_2^{\mathrm{s}})} \boldsymbol{\lambda}.\mathbf{w} =: \boldsymbol{\lambda}.\mathbf{w}^{**}, \tag{A.2}$$

and the auxiliary problem can be formulated as

$$\widehat{C} \equiv \min_{\mathbf{w} \in \mathcal{A}(\tau_2^{\mathrm{a}})} \boldsymbol{\lambda}.\mathbf{w} =: \boldsymbol{\lambda}.\widehat{\mathbf{w}}.$$

Noting that $\mathcal{A}(\tau_1) \subseteq \mathcal{A}(\tau_2^{\mathrm{a}})$ since $\tau_2^{\mathrm{a}} \leq \tau_1$, condition (i) follows from the fact that

$$\mathbf{w}^* \in \mathcal{S}(\tau_2^{\mathrm{s}}) \quad \Rightarrow \quad C_1 = \min_{\mathbf{w} \in \mathcal{A}(\tau_1) \cap \mathcal{U} \cap \mathcal{S}(\tau_2^{\mathrm{s}})} \boldsymbol{\lambda}.\mathbf{w} \geq C_2, \tag{A.3}$$

while condition (ii) follows from the observation that

$$\widehat{\mathbf{w}} \in \mathcal{S}(\tau_2^{\mathrm{s}}) \quad \Rightarrow \quad \mathbf{w}^{**} = \widehat{\mathbf{w}} \quad \text{and} \quad C_2 = \widehat{C} \leq C_1. \tag{A.4}$$

Furthermore, (A.3) and (A.4) involve a strict inequality $C_2 < C_1$ if $\mathbf{w}^{**} \notin \mathcal{U}$. ∎

A.2 Proofs and Derivations for Section 2

Proof of Lemma 1. The properties of $u^* = u(\lambda, \tau_1)$ stated in the lemma follow immediately from the implicit definition (7), using the stochastic ordering assumption (6). ∎

Proof of Lemma 2. Using Lemma 1, it is immediate to see that if $F_H^{-1}(\tau_2^{\mathrm{s}}) \leq F_L^{-1}(\tau_1)$, then $u^* \geq F_H^{-1}(\tau_2^{\mathrm{s}})$ for any value of $\lambda \in (0,1)$. If instead $F_L^{-1}(\tau_1) < F_H^{-1}(\tau_2^{\mathrm{s}})$, there exists a unique $\lambda_\dagger \in (0,1)$ such that $u(\lambda_\dagger, \tau_1) = F_H^{-1}(\tau_2^{\mathrm{s}})$, and $u^* = u(\lambda, \tau_1) \geq F_H^{-1}(\tau_2^{\mathrm{s}})$ for all $\lambda \geq \lambda_\dagger$. ∎

Remark A-1. When considering two-limb aggregation in the two-bond case, the set $\mathcal{W}$ of possible offers w_L to the holders of bond L such that the aggregate consent requirement (8) can be met for some value of w_H is defined as follows

$$\mathcal{W} = \left\{ \begin{array}{ll} \mathbb{R}^+, & \text{if } \lambda \geq \tau_2^{\mathrm{a}} \\ \left(F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}}-\lambda}{1-\lambda}\right), \infty\right), & \text{if } 1-\tau_2^{\mathrm{a}} \leq \lambda < \tau_2^{\mathrm{a}} \\ \left(F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}}-\lambda}{1-\lambda}\right), F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}}}{1-\lambda}\right)\right], & \text{if } \lambda < 1-\tau_2^{\mathrm{a}} \end{array} \right\}. \tag{A.5}$$

Remark A-2. When the auxiliary solution satisfies (14), taking into account the fact that the function g defined by (10) depends on λ, one can show that

$$\frac{d\widehat{w}_L}{d\lambda} = \left(\frac{f_L'(\widehat{w}_L)}{f_H'(\widehat{w}_H)} + \frac{1-\lambda}{\lambda}\right)^{-1} \frac{F_L(\widehat{w}_L) - \tau_2^{\mathrm{a}}}{\lambda^2 f_L(\widehat{w}_L)},$$

while

$$\frac{d\widehat{w}_H}{d\lambda} = \left(1 + \frac{1-\lambda}{\lambda}\frac{f_H'(\widehat{w}_H)}{f_L'(\widehat{w}_L)}\right)^{-1} \frac{F_L(\widehat{w}_L) - \tau_2^{\mathrm{a}}}{\lambda^2 f_L(\widehat{w}_L)}.$$

In particular, in the special case where f_H and f_L are decreasing, one can see that whenever the auxiliary solution is such that $F_L(\widehat{w}_L) > \tau_2^{\mathrm{a}}$ (or equivalently, in view of (11), such that the consent share of the cheap bond is higher than for the expensive bond), a marginal increase in the relative size λ of bond H is accompanied by an improvement in the exchange offers made to both bonds.

Proof of Lemma 3. Define $J(w) = \lambda g(w) + (1-\lambda)w$, which is assumed to be strictly convex with an interior minimum $\widehat{w}_L \in \mathcal{W}$, and let $\widehat{w}_H = g(\widehat{w}_L)$. The inequality $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$ is equivalent to $\widehat{w}_L \leq \overline{w} \equiv g^{-1}\big(F_H^{-1}(\tau_2^{\mathrm{s}})\big) = F_L^{-1}\big((\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}})/(1-\lambda)\big)$, which is guaranteed to hold if $J'(\overline{w}) \geq 0$. Differentiating with respect to w, one can show that the sign of $J'(w)$ coincides with the sign of $f_H\big(g(w)\big) - f_L(w)$. Hence the requirement $J'(\overline{w}) \geq 0$ is equivalent to

$$f_L\left(F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}} - \lambda\tau_2^{\mathrm{s}}}{1-\lambda}\right)\right) \leq f_H\left(F_H^{-1}(\tau_2^{\mathrm{s}})\right). \tag{A.6}$$

On the other hand, $F_L(\widehat{w}_L) \geq \tau_2^{\mathrm{s}}$ is equivalent to $\widehat{w}_L \geq \underline{w} = F_L^{-1}(\tau_2^{\mathrm{s}})$ and is guaranteed by the the requirement that $J'(\underline{w}) \leq 0$, which is itself equivalent to

$$f_L\left(F_L^{-1}(\tau_2^{\mathrm{s}})\right) \geq f_H\left(F_H^{-1}\left(\frac{\tau_2^{\mathrm{a}} - (1-\lambda)\tau_2^{\mathrm{s}}}{\lambda}\right)\right). \tag{A.7}$$

∎

Proof of Proposition 2. This is an immediate corollary of Proposition1: indeed Lemmas 2 and 3 provide conditions such that (i) and (ii) hold, respectively. ∎

Proof of Proposition 3. We prove parts (i) and (ii) of the proposition separately.

Proof of Part (i). Throughout this part, we denote by $C_1(\lambda)$ and $C_2(\lambda)$ the restructuring cost under single-limb and two-limb voting, and we assume that the aggregate thresholds are identical under the two modification methods, namely, $\tau_1 = \tau_2^{\mathrm{a}} = \tau^{\mathrm{a}}$. It follows immediately that $\lim_{\lambda \downarrow 0} C_1(\lambda) = \lim_{\lambda \downarrow 0} C_2(\lambda) = F_L^{-1}(\tau^{\mathrm{a}})$. In the remainder of the proof, we focus on the slope of the functions C_1 and C_2 for λ close to zero. Noting that $C_1(\lambda) = u(\lambda, \tau^{\mathrm{a}})$, we apply the implicit function theorem to obtain

$$\frac{dC_1(\lambda)}{d\lambda} = \frac{\partial u(\lambda, \tau^{\mathrm{a}})}{\partial \lambda} = \frac{F_L(u^*) - F_H(u^*)}{\lambda f_H(u^*) + (1-\lambda) f_L(u^*)} > 0.$$

In particular

$$C_1'(0) = \lim_{\lambda \downarrow 0} \frac{dC_1(\lambda)}{d\lambda} = \frac{\tau^{\mathrm{a}} - F_H \circ F_L^{-1}(\tau^{\mathrm{a}})}{f_L \circ F_L^{-1}(\tau^{\mathrm{a}})}. \tag{A.8}$$

Next, we analyze the restructuring cost under two-limb voting, $C_2(\lambda)$. First, we establish that under the assumptions of Proposition 3 and for λ small, the individual consent requirement on bond H is binding. To see this, we start from the identity

$$f_L\big(F_L^{-1}(\tau^{\mathrm{a}})\big) - f_H\big(F_H^{-1}(\tau_2^{\mathrm{s}})\big) = \Big[f_L\big(F_L^{-1}(\tau^{\mathrm{a}})\big) - f_H\big(F_L^{-1}(\tau^{\mathrm{a}})\big)\Big] + \Big[f_H\big(F_L^{-1}(\tau^{\mathrm{a}})\big) - f_H\big(F_H^{-1}(\tau_2^{\mathrm{s}})\big)\Big].$$

By assumption, $F_L^{-1}(\tau^{\mathrm{a}}) < \tilde{w}$ and $f_L(w) > f_H(w)$ for all $w < \tilde{w}$, implying that the first term is strictly positive. Noting that the second term is also strictly positive under the assumptions of the proposition, we thus conclude that

$$f_L\big(F_L^{-1}(\tau^{\mathrm{a}})\big) > f_H\big(F_H^{-1}(\tau_2^{\mathrm{s}})\big). \tag{A.9}$$

In turn, the inequality (A.9) implies that (A.6) is violated for λ close to zero. Hence for λ close to zero, the two-limb offer is constrained by the individual consent requirement on bond H.

Therefore, in this neighborhood,

$$C_2(\lambda) = \lambda F_H^{-1}(\tau_2^{\mathrm{s}}) + (1-\lambda) F_L^{-1}\left(\frac{\tau^{\mathrm{a}} - \lambda \tau_2^{\mathrm{s}}}{1-\lambda}\right),$$

and

$$\frac{dC_2(\lambda)}{d\lambda} = F_H^{-1}(\tau_2^{\mathrm{s}}) - F_L^{-1}\left(\frac{\tau^{\mathrm{a}} - \lambda \tau_2^{\mathrm{s}}}{1-\lambda}\right) + \frac{\tau_2^{\mathrm{a}} - \tau_2^{\mathrm{s}}}{1-\lambda}\left[f_L \circ F_L^{-1}\left(\frac{\tau^{\mathrm{a}} - \lambda \tau_2^{\mathrm{s}}}{1-\lambda}\right)\right]^{-1}.$$

In particular,

$$C_2'(0) = \lim_{\lambda \downarrow 0} \frac{dC_2(\lambda)}{d\lambda} = F_H^{-1}(\tau_2^{\mathrm{s}}) - F_L^{-1}(\tau^{\mathrm{a}}) + \frac{\tau^{\mathrm{a}} - \tau_2^{\mathrm{s}}}{f_L \circ F_L^{-1}(\tau^{\mathrm{a}})}. \tag{A.10}$$

Combining (A.8) and (A.10), we obtain that

$$C_1'(0) < C_2'(0) \quad \Leftrightarrow \quad \big(F_H^{-1}(\tau_2^{\mathrm{s}}) - F_L^{-1}(\tau^{\mathrm{a}})\big) f_L\big(F_L^{-1}(\tau^{\mathrm{a}})\big) > \tau_2^{\mathrm{s}} - F_H\left(F_L^{-1}(\tau^{\mathrm{a}})\right).$$

To show that this inequality holds, we note that f_H being strictly decreasing implies that F_H is strictly concave, which in turn implies that

$$\big(F_H^{-1}(\tau_2^{\mathrm{s}}) - F_L^{-1}(\tau^{\mathrm{a}})\big) f_H\left(F_L^{-1}(\tau^{\mathrm{a}})\right) > F_H\big(F_H^{-1}(\tau_2^{\mathrm{s}})\big) - F_H\left(F_L^{-1}(\tau^{\mathrm{a}})\right) = \tau_2^{\mathrm{s}} - F_H\left(F_L^{-1}(\tau^{\mathrm{a}})\right),$$

and the desired inequality follows from the fact that $f_L(w_L) > f_H(w_L)$ since $w_L < \tilde{w}$. By continuity, we conclude that $C_1 < C_2$ for λ close to 0.

Proof of Part (ii). Noting that $F_H^{-1}(\tau_2^{\mathrm{s}}) < F_H^{-1}(\tau_1)$, we know by assumption that the location of the crossing point $\tilde{w}$ is such that $\tilde{w} \in \big(F_L^{-1}(\tau_1), F_H^{-1}(\tau_1)\big)$. In view of Lemma 1, the intermediate value theorem implies that there exists $\tilde{\lambda} \in (0,1)$ such that $u(\tilde{\lambda}, \tau_1) = \tilde{w}$. By construction, the single-limb uniform offer u^* is equal to $\tilde{w}$ for $\lambda = \tilde{\lambda}$. Moreover, $\tilde{\lambda} F_H(\tilde{w}) + (1-\tilde{\lambda}) F_L(\tilde{w}) = \tau_1$ is equivalent to $\tilde{w} = g(\tilde{w}; \tilde{\lambda}, \tau_1)$, where $g(w; \lambda, \tau)$ denotes the unique value of w_H such that $\lambda F_H(w_H) + (1-\lambda) F_L(w) = \tau$. Therefore we can write

$$f_L(\tilde{w}) = f_H(\tilde{w}) = f_H(g(\tilde{w}; \tilde{\lambda}, \tau_1)). \tag{A.11}$$

Setting $\lambda = \tilde{\lambda}$, we next consider the restructuring cost under two-limb voting for $\tau_2^{\mathrm{a}} = \tau_1$. The assumption that the densities are decreasing guarantees convexity of the corresponding auxiliary problem, and (A.11) implies that the auxiliary solution is given by $\widehat{\mathbf{w}} = (\tilde{w}, \tilde{w})$. Yet since $F_H(\tilde{w}) < \tau_2^{\mathrm{s}}$ by assumption, the auxiliary solution violates the individual consent requirement on bond H, implying that $C_2 > \tilde{w}$. Therefore, for $\lambda = \tilde{\lambda}$ and $\tau_2^{\mathrm{a}} = \tau_1$, we have $C_1 = \tilde{w} < C_2$ and by continuity, we conclude that the single-limb method is optimal when the parameters $(\lambda, \tau_2^{\mathrm{a}})$ are close to $(\tilde{\lambda}, \tau_1)$. ∎

A.3 Derivations and Additional Results for Section 3

With a view towards applying some of the results derived in Section 2, we first note that under the parametric specification (17), the two densities are strictly decreasing, with

$$f_i(w) = \frac{1}{\phi_i} e^{-\frac{w}{\phi_i}}.$$

Moreover, the two densities cross at a single point $\tilde{w}$ given by

$$\tilde{w} = \frac{\gamma \log \gamma}{\gamma - 1} \phi_L. \tag{A.12}$$

Holding ϕ_L fixed, $\tilde{w}$ is increasing in the heterogeneity parameter γ. We also note that

$$F_L^{-1}(\tau_1) < F_H^{-1}(\tau_2^{\mathrm{s}}) \quad \Leftrightarrow \quad \gamma > \frac{\log(1-\tau_1)}{\log(1-\tau_2^{\mathrm{s}})} \equiv \overline{\gamma}_U \quad (> 1). \tag{A.13}$$

Auxiliary Solution. Under the exponential specification, condition (13) for strict convexity of the auxiliary problem is satisfied and the mapping g that captures the aggregate consent requirement becomes

$$g(w_L) = -\phi_H \log\left(\frac{1 - \tau_2^{\mathrm{a}} - (1-\lambda)e^{-w_L/\phi_L}}{\lambda}\right), \qquad w_L \in \mathcal{W}.$$

The first-order condition (14) to the auxiliary problem gives[45]

$$\widehat{w}_L = \phi_L \log\left(\frac{1 + \lambda(\gamma - 1)}{1 - \tau_2^{\mathrm{a}}}\right), \tag{A.14}$$

and the induced consent share for bond L is given by (18). Since $F_L(\widehat{w}_H) > \tau_2^{\mathrm{a}}$, it follows from (11) that $F_H(\widehat{w}_H) < \tau_2^{\mathrm{a}}$. Hence, in the context of this parametric example, only the individual constraint on bond H may ever be binding. We compute

$$\widehat{w}_H = g(\widehat{w}_L) = \phi_H \log\left(\frac{1 + \lambda(\gamma - 1)}{\gamma(1 - \tau_2^{\mathrm{a}})}\right), \tag{A.15}$$

and the consent share for bond H is given by (19). The auxiliary offers $\widehat{w}_L$ and $\widehat{w}_H$, and the corresponding consent shares, are increasing in λ—in line with the more general result stated in Remark A-2 in Appendix A.2. Moreover, one can check that $F_L(\widehat{w}_L)$ is increasing in γ

[45] The solution to the auxiliary problem is pinned down by this condition as long as the problem's solution is interior. In view of (A.5), one can see that the auxiliary solution is non-interior if and only if $\lambda < 1 - \tau_2^{\mathrm{a}}$ and $\gamma > (1-\lambda)/(1 - \lambda - \tau_2^{\mathrm{a}})$, in which case $\widehat{w}_L = \sup \mathcal{W}$ and the consent share for bond H is zero, thus violating the individual constraint for this bond. Hence, whenever the optimal two-limb offers coincide with the auxiliary solution, the recovery values on the two bonds are given by (A.14) and (A.15).

while $F_H(\widehat{w}_H)$ is decreasing in γ—that is, the spread in consent shares under the auxiliary solution is increasing in the degree of bond heterogeneity.

Optimal Two-Limb Offer. The optimal offers under two-limb voting coincide with the auxiliary solution (A.14)–(A.15) as long as $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$. Using (19), one can see that the latter inequality is equivalent to

$$\frac{1+\lambda(\gamma-1)}{\gamma} \geq \frac{1-\tau_2^{\mathrm{a}}}{1-\tau_2^{\mathrm{s}}}. \tag{A.16}$$

For given values of the voting thresholds τ_2^{a} and τ_2^{s}, one can characterize more explicitly the set of values for λ and γ such that (A.16) holds. Since $F_H(\widehat{w}_H)$ is increasing in λ and decreasing in γ, the inequality is more likely to hold for high values of λ and low values of γ. Indeed, it is easy to see that (A.16) holds

- if $\gamma \leq (1-\tau_2^{\mathrm{s}})/(1-\tau_2^{\mathrm{a}}) \equiv \overline{\gamma}_X$, for all values of λ;
- if $\lambda \geq (1-\tau_2^{\mathrm{a}})/(1-\tau_2^{\mathrm{s}}) \equiv \underline{\lambda} \in (0,1)$, for all values of γ;
- in the remainder of the parameter space for γ sufficiently small or λ sufficiently large (see Remark A-3 below for an explicit condition).

Conversely, the consent requirement on the expensive bond H is binding when there is sufficient heterogeneity across the two bonds ($\gamma > \overline{\gamma}_X$) and the relative size of bond H is small. For such parameter values, the optimal two-limb offers are

$$w_H = F_H^{-1}(\tau_2^{\mathrm{s}}) = \phi_H \log\left(\frac{1}{1-\tau_2^{\mathrm{s}}}\right), \tag{A.17}$$

$$w_L = F_L^{-1}\left(\frac{\tau_2^{\mathrm{a}}-\lambda\tau_2^{\mathrm{s}}}{1-\lambda}\right) = \phi_L \log\left(\frac{1-\lambda}{1-\tau_2^{\mathrm{a}}-\lambda(1-\tau_2^{\mathrm{s}})}\right). \tag{A.18}$$

Remark A-3. When $\gamma > \overline{\gamma}_X = \underline{\lambda}^{-1}$, the condition on λ such that (A.16) holds is

$$\lambda \geq \frac{\underline{\lambda}\gamma - 1}{\gamma - 1} \equiv \ell_X(\gamma) \in (0,1), \tag{A.19}$$

where ℓ_X is increasing in γ and converges to $\underline{\lambda}$ in the limit as γ goes to infinity. When $\lambda < \underline{\lambda}$, the condition on γ can be expressed as

$$\gamma \leq \frac{1-\lambda}{\underline{\lambda}-\lambda}, \tag{A.20}$$

where the right-hand side is increasing in λ, starting at $\overline{\gamma}_X$ for $\lambda = 0$ and going to infinity in the limit as $\lambda \uparrow \underline{\lambda}$.

Optimal Voting Procedure. When parameter values are such that (A.16) holds, two-limb aggregation is optimal since the series-by-series constraints have no bite. Yet another sufficient condition for two-limb optimality is that the optimal uniform offer u^* satisfies the series-by-series constraints, in which case the unique advantage of single-limb aggregation is worthless. Applying Lemma 2 (see also Proposition 2), one can see that $F_H(u^*) \geq \tau_2^{\mathrm{s}}$ if $\gamma \leq \overline{\gamma}_U$, where $\overline{\gamma}_U$ is given by (A.13), or alternatively if

$$\lambda \geq \frac{1-\tau_1-(1-\tau_2^{\mathrm{s}})^\gamma}{1-\tau_2^{\mathrm{s}}-(1-\tau_2^{\mathrm{s}})^\gamma} \equiv \ell_U(\gamma) \in (0,1) \qquad \text{for } \gamma > \overline{\gamma}_U, \tag{A.21}$$

where one can check that the definition of ℓ_U ensures that $u\big(\ell_U(\gamma),\tau_1\big) = F_H^{-1}(\tau_2^{\mathrm{s}})$. It is worth noting that Condition (A.21) can also be viewed as setting an upper bound on γ that is an increasing function of λ, starting at $\overline{\gamma}_U$ for $\lambda = 0$ and going to infinity as $\lambda \uparrow (1-\tau_1)/(1-\tau_2^{\mathrm{s}})$. One can see that when $\tau_1 = \tau_2^{\mathrm{a}}$, the functions ℓ_U^{-1} and ℓ_X^{-1} both have an asymptote at $\lambda = \underline{\lambda}$, whereas if $\tau_1 > \tau_2^{\mathrm{a}}$ the asymptote for ℓ_U^{-1} is at a value of λ strictly below $\underline{\lambda}$.

In order to exploit Proposition 3 and establish instead a sufficient condition for single-limb optimality, we first need to find restrictions that ensure that $\tilde{w} \in \big(F_L^{-1}(\tau_1), F_H^{-1}(\tau_2^{\mathrm{s}})\big)$. It is immediate to see from (A.12) that this amounts to finding $\gamma > \overline{\gamma}_U$ such that

$$-\log(1-\tau_1) < \frac{\gamma \log \gamma}{\gamma - 1} < -\gamma \log(1-\tau_2^{\mathrm{s}}), \tag{A.22}$$

and one can show (see Remark A-4 below) that these inequalities are satisfied for high values of γ. For such values of γ, we know from Proposition 3 that single-limb voting is optimal for λ sufficiently close to zero when $\tau_1 = \tau_2^{\mathrm{a}}$, and in the neighborhood of

$$\tilde{\lambda} = \frac{1-\tau_1-\gamma^{-\frac{\gamma}{\gamma-1}}}{(\gamma-1)\gamma^{-\frac{\gamma}{\gamma-1}}}, \tag{A.23}$$

when $\tau_1 - \tau_2^{\mathrm{a}}$ is small, where one can check that the point $\tilde{\lambda} > 0$ is such that $u(\tilde{\lambda},\tau_1) = \tilde{w}$.

Remark A-4. To see that the inequalities in (A.22) are satisfied for high values of γ, first note that $-\log(1-\tau) > 0$ is increasing in τ, with $-\log(1-\tau) > 1$ for $\tau > 1 - e^{-1} \approx 0.63$. Since $\gamma \log \gamma/(\gamma-1)$ is strictly increasing in γ, with limit 1 as $\gamma \downarrow 1$ and going to infinity as $\gamma \to \infty$, the first inequality is satisfied for γ sufficiently large. Likewise, since $\log \gamma/(\gamma-1)$ is strictly decreasing in γ, with limit 1 as $\gamma \downarrow 1$ and going to zero as $\gamma \to \infty$, the second inequality is also satisfied for γ sufficiently large.

Necessary and Sufficient Condition. When none of the sufficient conditions holds, the participation constraint on bond H must be binding under two-limb voting, in which case the optimal two-limb offers are given by (A.17)-(A.18). The restructuring cost is then given by

$$C_2 = \lambda\phi_H \log\left(\frac{1}{\zeta_2^s}\right) + (1-\lambda)\phi_L \log\left(\frac{1-\lambda}{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}\right),$$

where $\zeta_2^j = 1 - \tau_2^j$ for $j \in \{\mathrm{s}, \mathrm{a}\}$. Under single-limb voting, the optimal uniform offer u^* is implicitly defined by

$$F(u^*) \equiv \lambda F_H(u^*) + (1-\lambda)F_L(u^*) = 1 - \lambda e^{-u^*/\phi_H} - (1-\lambda)e^{-u^*/\phi_L} = \tau_1,$$

and the total cost for the government is $C_1 = u^* = F^{-1}(\tau_1)$. Therefore, single-limb aggregation is optimal if and only if $F^{-1}(\tau_1) < C_2$, which is equivalent to $F(C_2) > \tau_1$. We thus obtain a necessary and sufficient condition for single-limb optimality in terms of parameter values:

$$\lambda\,(\zeta_2^{\mathrm{s}})^{\lambda}\left(\frac{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}{1-\lambda}\right)^{\frac{1-\lambda}{\gamma}} + (1-\lambda)\,(\zeta_2^{\mathrm{s}})^{\lambda\gamma}\left(\frac{\zeta_2^{\mathrm{a}} - \lambda\zeta_2^{\mathrm{s}}}{1-\lambda}\right)^{1-\lambda} < 1 - \tau_1. \tag{A.24}$$

The following figure represents regions of the parameter space where each of the voting procedures dominates:

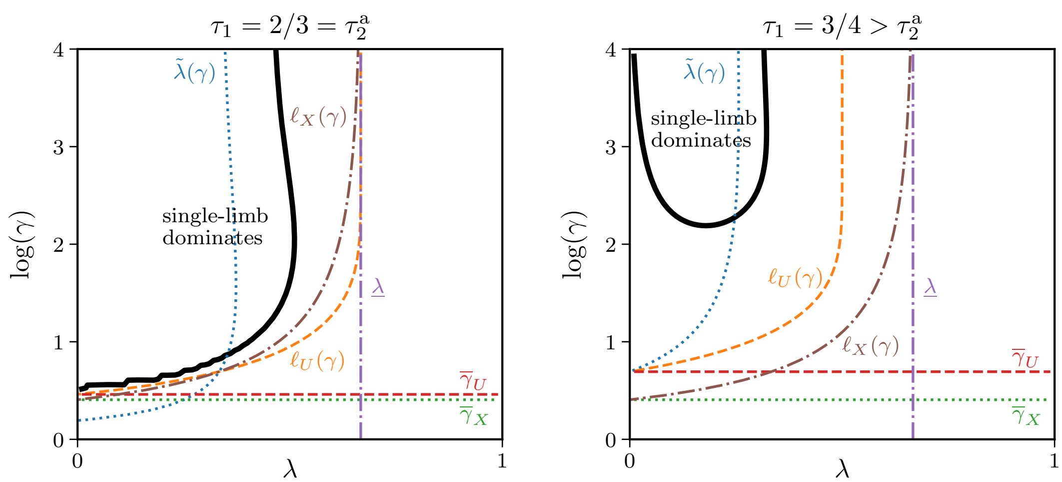

Figure A.1: Optimal Voting Rule as a function of Relative Size (λ) and Bond Heterogeneity (γ).

Notes: in each panel, the solid black line depicts the frontier delineating regions of the parameter space where single-limb or two-limb voting dominates, based on the *necessary and sufficient* condition (A.24). The other lines are related to *sufficient* conditions for one or the other method to dominate. Conditions $\gamma \leq \overline{\gamma}_X$, $\lambda \geq \underline{\lambda}$, or $\lambda \geq \ell_X(\lambda)$ guarantee that $F_H(\widehat{w}_H) \geq \tau_2^{\mathrm{s}}$, while conditions $\gamma \leq \overline{\gamma}_U$ or $\lambda \geq \ell_U(\lambda)$ guarantee that $F_H(u^*) \geq \tau_2^{\mathrm{s}}$. Part (ii) of Proposition 3 applies and single-limb dominates for λ close to $\tilde{\lambda}(\gamma)$ and γ sufficiently large. The notations involved in all these sufficient conditions are explicitly defined in the text of the appendix.

A.4 Derivations and Additional Results for Section 6

In the equilibrium described in Section 6.2, market clearing requires that $\widehat{r} = r_{\min} + \lambda_L R$, where $R \equiv r_{\max} - r_{\min}$ denotes the length of $\mathcal{R}$.

Reservation Value Distributions. The reservation values for the two bonds lie in

$$\begin{aligned} \mathcal{V}_L &= \left[\frac{c_L}{\widehat{r} + \delta_L + \chi}, \frac{c_L}{r_{\min} + \delta_L + \chi}\right] =: \left[\underline{w}_L, \overline{w}_L\right], \\ \mathcal{V}_S &= \left[\frac{c_S}{r_{\max} + \delta_S + \chi}, \frac{c_S}{\widehat{r} + \delta_S + \chi}\right] =: \left[\underline{w}_S, \overline{w}_S\right]. \end{aligned}$$

Using the notations $\{a_i, b_i\}_{i\in\{S,L\}}$ defined in the text by (23), one can check that $\underline{w}_i = b_i/a_i$, and $\overline{w}_i = b_i/(a_i - 1)$. For $w \in \mathcal{V}_L$, we have

$$\Pr\big(h_L(r) \le w \mid r \le \widehat{r}\big) = \frac{G(\widehat{r}) - G(c_L/w - (\delta_L + \chi))}{\lambda_L} = \frac{r_{\min} + \lambda_L R + \delta_L + \chi}{\lambda_L R} - \frac{c_L}{\lambda_L R}\frac{1}{w}.$$

Likewise, for $w \in \mathcal{V}_S$, we can write

$$\Pr\big(h_S(r) \le w \mid r \ge \widehat{r}\big) = \frac{1 - G(c_S/w - (\delta_S + \chi))}{\lambda_S} = \frac{R + \delta_S + \chi + r_{\min}}{\lambda_S R} - \frac{c_S}{\lambda_S R}\frac{1}{w}.$$

Hence, using the notations $\{a_i, b_i\}_{i\in\{S,L\}}$, the CDFs for the two bonds can be written as

$$F_i(w) = \begin{cases} 0 & \text{if} \quad w < \underline{w}_i, \\ a_i - b_i/w & \text{if} \quad w \in \mathcal{V}_i, \\ 1 & \text{if} \quad w > \overline{w}_i, \qquad i \in \{S, L\}. \end{cases}$$

Auxiliary Problem and Two-Limb Offer. The auxiliary problem is

$$\min_{w_S,\, w_L} \; \lambda_S w_S + \lambda_L w_L \quad \text{subject to} \quad \lambda_S F_S(w_S) + \lambda_L F_L(w_L) = \tau_2^{\mathrm{a}},$$

and we denote its solution by $\widehat{\mathbf{w}} = (\widehat{w}_S, \widehat{w}_L)$. In particular, when $\widehat{\mathbf{w}} \in \mathrm{int}(\mathcal{V}_S) \times \mathrm{int}(\mathcal{V}_L)$, the first-order optimality condition (14) implies that

$$\widehat{w}_S = \frac{\lambda_S b_S}{\lambda_S a_S + \lambda_L a_L - \lambda_L \frac{b_L}{\widehat{w}_L} - \tau_2^{\mathrm{a}}} \qquad \text{and} \qquad \widehat{w}_L = \frac{\lambda_S b_S \sqrt{\frac{b_L}{b_S}} + \lambda_L b_L}{\lambda_S a_S + \lambda_L a_L - \tau_2^{\mathrm{a}}}, \tag{A.25}$$

which, after substituting the expressions for $\{a_i, b_i\}_{i\in\{S,L\}}$, yields

$$\widehat{w}_S = \frac{c_S\left(\sqrt{\frac{c_L}{c_S}\frac{\lambda}{1-\lambda}}+1\right)}{(\lambda-\tau_2^{\mathrm{a}})R + r_{\max} + r_{\min} + \delta_S + \delta_L + 2\chi},$$

$$\widehat{w}_L = \frac{c_L\left(\sqrt{\frac{c_S}{c_L}\frac{1-\lambda}{\lambda}}+1\right)}{(\lambda-\tau_2^{\mathrm{a}})R + r_{\max} + r_{\min} + \delta_S + \delta_L + 2\chi}.$$

As in Section 2.2, the optimal two-limb offer coincides with the auxiliary solution $\widehat{\mathbf{w}}$ when the latter satisfies the series-by-series constraint for both bonds.

Single-Limb Offer. Let $F(w) = \lambda_S F_S(w) + \lambda_L F_L(w)$ denote the aggregate consent share associated with uniform offer w, with $F'(w) \geq 0$. The optimal single-limb offer is given by

$$u^* = \inf\{w \geq 0 \mid F(w) = \tau_1\}.$$

In particular, for parameter values such that

$$\lambda_S \in \left[\frac{t_S + \frac{c_S}{c_L}(R\tau_1 - t_L)}{R}, \frac{t_L + \frac{c_L}{c_S}(R\tau_1 - t_S)}{R}\right],$$

where $t_i = r_{\max} + \delta_i + \chi$, one can show that

$$u^* = \frac{\lambda_L b_L + \lambda_S b_S}{\lambda_L a_L + \lambda_S a_S - \tau_1} = \frac{c_L + c_S}{(\lambda-\tau_2^{\mathrm{s}})R + r_{\max} + r_{\min} + \delta_S + \delta_L + 2\chi} \in \mathcal{V}_S \cap \mathcal{V}_L.$$

B Extension with Stochastic Consent Shares

This appendix extends the analysis of the two-bond case presented in Sections 2 and 3 to consider a situation where the government faces some uncertainty on the consent shares that a given restructuring offer may attract. We first consider a general formulation of the government's problem in the presence of uncertainty, and then provide a parametric example.

Assumptions. We proceed under the following assumptions. First, the share of consent among the holders of each bond is a random variable, whose distribution depends on the recovery rate offered to that bond. For now, we leave this distribution and its dependence on the government's offer unspecified, and allow for the possibility that consent shares may be correlated across bonds. Second, we assume that, if one or more series cannot be restructured through the activation of CACs, the bonds of these series are left unrestructured (i.e., the minimum participation thresholds is no smaller than the CAC thresholds) and the government incurs a pecuniary cost Z per unit of face value of the unrestructured series.

Under single-limb aggregation, the presence of a unique constraint implies that either both bonds are restructured, or none of them is. Under the two-limb procedure, there is the possibility that the consent shares satisfy both the aggregate constraint and the series-by-series constraint for one bond, but not that for the other bond. In this case, we assume the presence of a redesignation clause, implying that the latter bond drops out of the restructuring pool and is left unrestructured, while the former is restructured through the triggering of CACs.

Notation. We denote by $\boldsymbol{\tau} \equiv (\tau_2^{\mathrm{a}}, \tau_2^{\mathrm{s}})$ the pair of thresholds under the two-limb procedure. To formulate the problem of the sovereign in the presence of uncertainty, we denote by $p_{\mathrm{a}}(\mathbf{w}, \tau)$ the probability that a restructuring offer $\mathbf{w} = (w_H, w_L)$ attracts an aggregate consent share above the generic aggregate threshold τ, with $p_{\mathrm{a}}(u, \tau) \equiv p_{\mathrm{a}}((u, u), \tau)$; we denote by $p_{HL}(\mathbf{w}, \boldsymbol{\tau})$ the probability that the offer $\mathbf{w}$ attracts consent shares that satisfy *all* constraints under the two-limb voting rule, i.e. both the series-by-series constraints and the aggregate constraint; and we denote by $p_i(\mathbf{w}, \boldsymbol{\tau})$ the probability that *only* bond i is restructured via redesignation under two-limb, i.e. the aggregate constraint and the series-by-series constraint for bond i are satisfied, but the series-by-series constraint for bond $j \neq i$ is not satisfied. The probability that both bonds are left unrestructured under the two-limb procedure is

$$p_0(\mathbf{w}, \boldsymbol{\tau}) \equiv 1 - p_{HL}(\mathbf{w}, \boldsymbol{\tau}) - \sum_i p_i(\mathbf{w}, \boldsymbol{\tau}).$$

Government's Problem. Let $\boldsymbol{\lambda} = (\lambda_H, \lambda_L)$ denote the relative sizes of the two bonds. The cost-minimization problem of the government under two-limb aggregation is given by

$$\mathbb{E}[C_2] = \min_{\mathbf{w}} \left\{ p_{HL}(\mathbf{w}, \boldsymbol{\tau}) \, (\boldsymbol{\lambda} \cdot \mathbf{w}) + \sum_i p_i(\mathbf{w}, \boldsymbol{\tau}) \Big(\lambda_i w_i + (1 - \lambda_i) Z \Big) + p_0(\mathbf{w}, \boldsymbol{\tau}) Z \right\}.$$

Instead, under the single-limb procedure, the government's problem is given by

$$\mathbb{E}[C_1] = \min_u \Big\{ p_{\mathrm{a}}(u, \tau_1) u + \big(1 - p_{\mathrm{a}}(u, \tau_1)\big) Z \Big\}.$$

Special Case. In one particular specification of the model with stochastic consent shares, an offer w_i to bond i attracts a consent share $F_i(w_i) - \nu_i$, where the noise terms (ν_H, ν_L) are distributed according to the multivariate standard normal distribution

$$\begin{bmatrix} \nu_H \\ \nu_L \end{bmatrix} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \qquad \text{where} \qquad \boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \boldsymbol{\Sigma} = \begin{bmatrix} \sigma^2 & \rho\,\sigma^2 \\ \rho\,\sigma^2 & \sigma^2 \end{bmatrix}.$$

It immediately follows that the expected consent shares are given by $\mathbb{E}[F_i(w_i) - \nu_i] = F_i(w_i)$. Under this specification, the aggregate consent share is $\sum_i \lambda_i (F_i(w_i) - \nu_i)$, implying that

$$p_{\mathrm{a}}(\mathbf{w}, \tau) = \Phi\left(\frac{\sum_i \lambda_i F_i(w_i) - \tau}{\sigma \sqrt{\lambda_H^2 + \lambda_L^2 + 2\rho\,\lambda_H\,\lambda_L}} \right). \tag{B.1}$$

Under two limb, all consent requirements are satisfied when the following conditions hold:

$$\begin{aligned} F_i(w_i) - \tau_2^{\mathrm{s}} &\geq \nu_i, \\ F_j(w_j) - \tau_2^{\mathrm{s}} &\geq \nu_j, \\ F_i(w_i) + \frac{\lambda_j F_j(w_j) - \tau_2^{\mathrm{a}} - \lambda_j \nu_j}{\lambda_i} &\geq \nu_i, \end{aligned}$$

for $i, j \in \{H, L\}$, $j \neq i$. Instead, the restructuring only includes bond i and leaves bond j unrestructured in case the second inequality is reversed. It follows that

$$\begin{aligned} p_{HL}(\mathbf{w}, \boldsymbol{\tau}) &= \Pr\left(\nu_i \leq \min\left\{ F_i(w_i) - \tau_2^{\mathrm{s}},\ F_i(w_i) + \frac{\lambda_j F_j(w_j) - \tau_2^{\mathrm{a}} - \lambda_j \nu_j}{\lambda_i} \right\} \ \wedge\ \nu_j \leq F_j(w_j) - \tau_2^{\mathrm{s}} \right), \\ p_i(\mathbf{w}, \boldsymbol{\tau}) &= \Pr\left(\nu_i \leq \min\left\{ F_i(w_i) - \tau_2^{\mathrm{s}},\ F_i(w_i) + \frac{\lambda_j F_j(w_j) - \tau_2^{\mathrm{a}} - \lambda_j \nu_j}{\lambda_i} \right\} \ \wedge\ \nu_j > F_j(w_j) - \tau_2^{\mathrm{s}} \right). \end{aligned}$$

Remark. We use the above additively separable specification to have simple expressions, even though it implies that consent shares can in principle lie outside the unit interval. This can be readily fixed by assuming that consent shares are instead given by

$$\begin{aligned} \widehat{F}_a(\mathbf{w}) &= \min\left\{ \max\left\{ \sum_i \lambda_i \big(F_i(w_i) - \nu_i\big),\, 0 \right\},\, 1 \right\}, \\ \widehat{F}_i(\mathbf{w}) &= \min\{\max\{F_i(w_i) - \nu_i,\, 0\},\, 1\}. \end{aligned}$$

This formulation delivers probabilities p_{a}, p_i, p_{HL} that are identical to those specified above, but at the cost of more involved expressions.

Illustration. We now illustrate the results of the model assuming that the expected consent share functions F_i are as per (17), with the same parameters (ϕ_L, ϕ_H, γ) as the ones set in Section 3.2. The CAC thresholds are $\tau_1 = \tau_2^{\mathrm{a}} = 2/3$ and $\tau_2^{\mathrm{s}} = 1/2$, and the cost of leaving a bond unrestructured is $Z = 7$. Figures B.1-B.3 illustrate comparative statics with respect to the relative size (λ) of the expensive bond H, the uncertainty about consent shares (σ), and their correlation (ρ), respectively. Unless specified otherwise, we set $\sigma = 0.05$ and $\rho = 0$.

In each figure, the top left panel depicts restructuring offers under two-limb voting (dashed red for bond H and dash-dotted blue for bond L) and the uniform offer under single-limb voting (dotted black). The top right panel depicts, for each aggregation method, the expected consent shares for each bond, along with the expected aggregate consent share (with the solid orange line corresponding to the expected aggregate consent under two-limb voting). The bottom right panel depicts as a dotted black line the probability $p_{\mathrm{a}}(u^*, \tau_1)$ that the restructuring goes through under single-limb, and as a solid orange line the probability $p_{HL}(\mathbf{w}, \boldsymbol{\tau})$ that the activation of CACs allows the restructuring of both bonds under two-limb voting, as well as the probabilities $p_i(\mathbf{w}, \boldsymbol{\tau})$ that only bond $i = H, L$ is being restructured. Finally, the bottom left panel depicts the expected restructuring costs $\mathbb{E}[C_2]$ and $\mathbb{E}[C_1]$ under the two-limb and single-limb procedures (solid orange and dotted black lines, respectively).

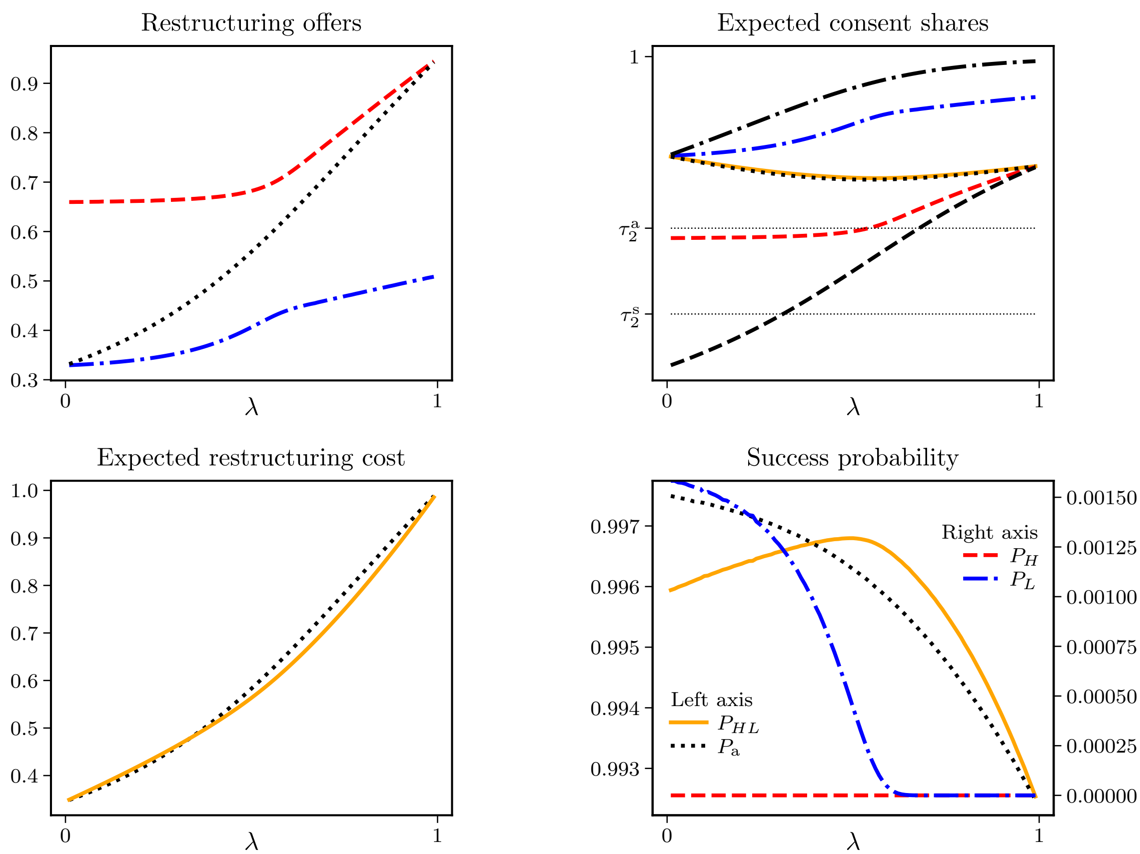

Figure B.1: Comparative Statics with Respect to λ.

Figure B.1 demonstrates that the findings illustrated in Figure 1 on the optimal restructuring approach generalize to a setting where the government faces uncertainty about consent shares. Optimal offers under either aggregation method are increasing in the share of the more demanding bond. In the presence of uncertainty, offers are made to ensure that, in expectation, consent shares are comfortably above the required thresholds, yet leaving the possibility that some bond(s) may be left unrestructured. For low values of λ, the optimal uniform offer under single limb is expected to attract from bond H a consent share below the series-by-series threshold τ_2^{s}, whereas the optimal offer to this bond under two limb is sufficiently generous to ensure that, in all likelihood, the series-by-series requirement will be met. For low values of λ, the unique advantage of single-limb voting thus makes it optimal.

Naturally, as illustrated in the top-left panel of Figure B.2, restructuring offers—as well as expected consent shares—are increasing in the degree of uncertainty. As $\sigma \downarrow 0$, offers can be tailored to just meet each of the consent requirements as an equality. When instead there is some amount of uncertainty, offers are made more generous to guarantee a wider comfort margin. Despite the optimal offers being more generous, an increase in uncertainty lowers the probability of a smooth restructuring. The bottom left panel of the figure reveals that an increase in uncertainty is favorable to the use of two-limb aggregation.

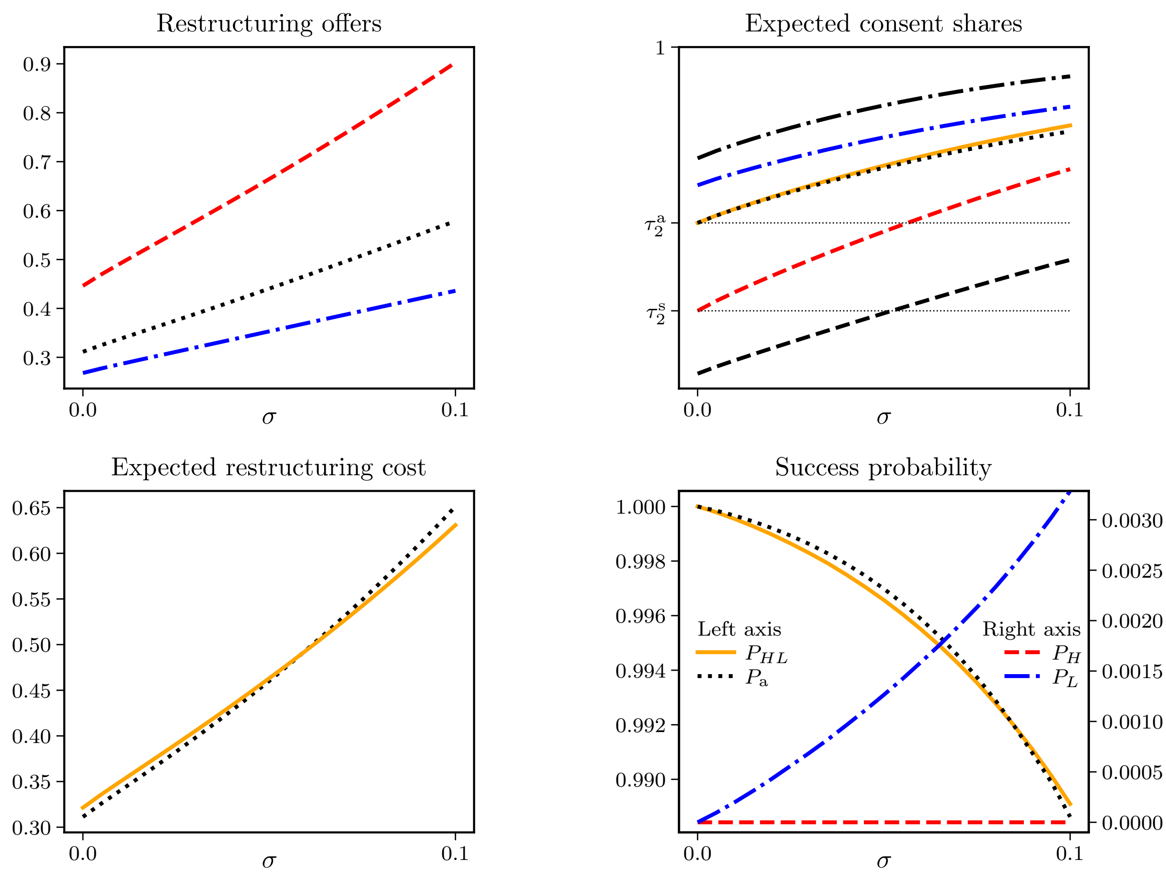

Figure B.2: Comparative Statics with Respect to σ, for $\lambda = 0.3$.

Figure B.3 illustrates the impact of the correlation of shocks to consent shares on the optimal restructuring approach. One may think of this correlation as being driven (at least partly) by the extent to which some creditors are common across the bonds. As can be seen from Equation (B.1), holding everything else constant, an increase in ρ lowers the probability of reaching a given aggregate threshold. Under single limb, despite a slight upward adjustment in the uniform offer, an increase in ρ is accompanied by an approximately linear drop in the likelihood of a smooth restructuring. Under two-limb voting, the drop is less pronounced and the probability p_{HL} of a smooth restructuring even reverts back up for high values of ρ due to the adjustment in the government's offers. Overall, an increase in the correlation of shocks to consent shares contributes to making the two-limb procedure more appealing.

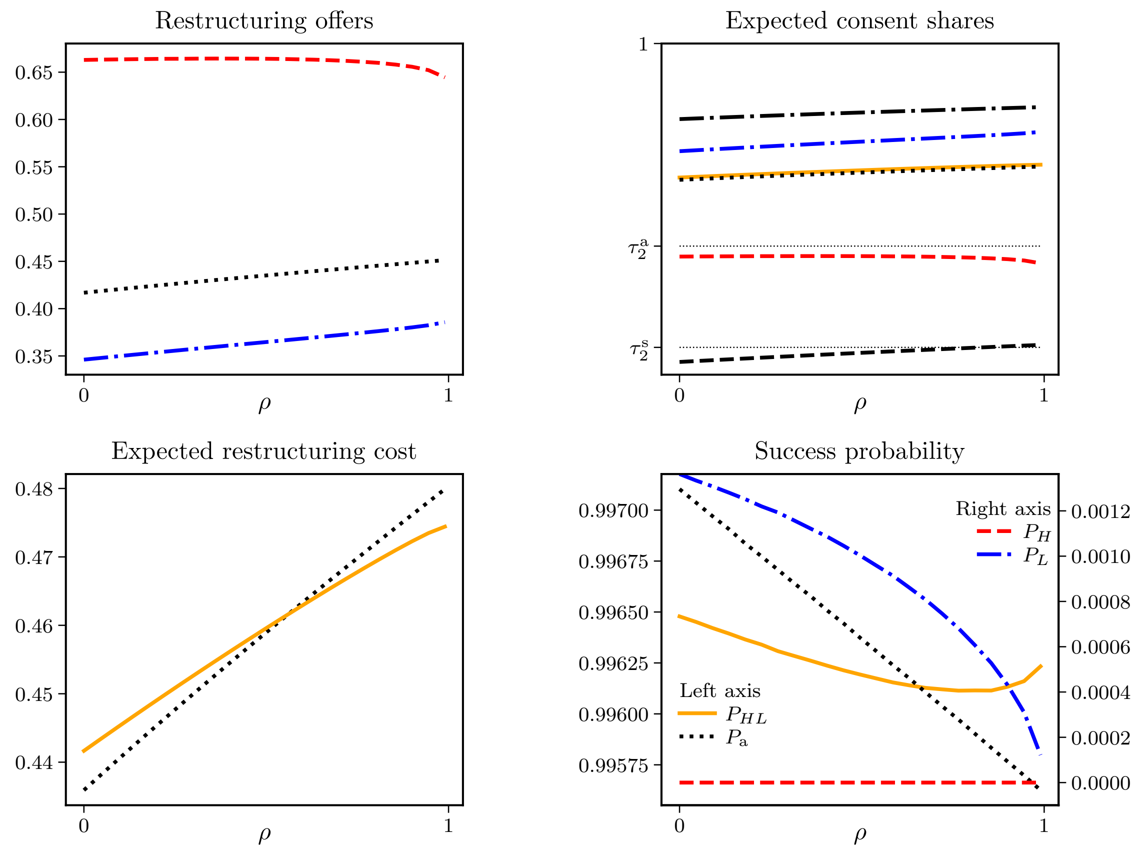

Figure B.3: Comparative Statics with Respect to ρ, for $\lambda = 0.25$.

Finally, Figure B.4 generalizes and extends the results from Sections 2 and 3 on the determinants of the optimal aggregation method. Consistent with Figure 2 (left panel, for $\tau_1 = \tau_2^{\mathrm{a}} = 2/3$), even in the presence of shocks, the single-limb procedure is found to be optimal only when there is substantial heterogeneity across bonds and the more demanding bond is relatively small. Moreover, consistent with the insights from Figures B.2 and B.3, an increase in uncertainty and in the correlation of shocks on consent shares are both favorable to the use of two-limb aggregation.

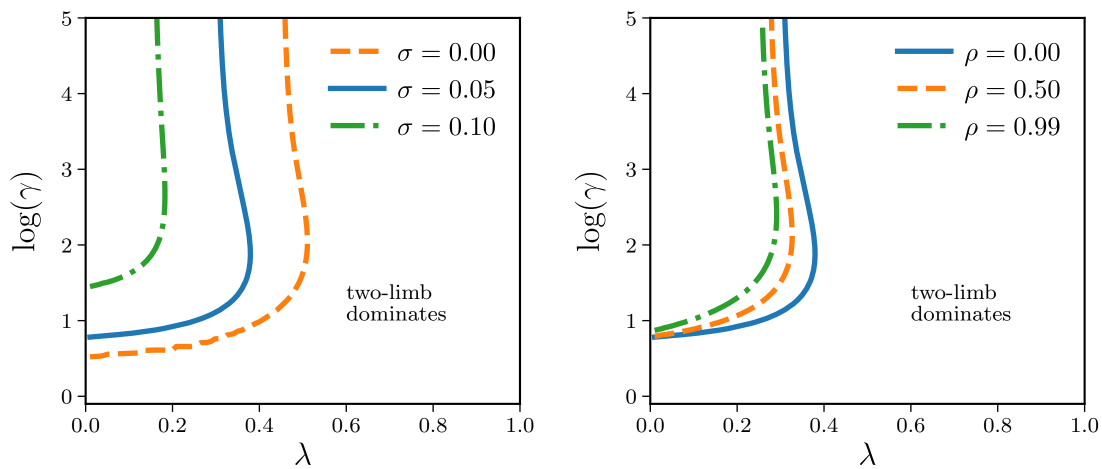

Figure B.4: Optimal Aggregation Method as a Function of Relative Size (λ) and Bond Heterogeneity (γ), for Different Levels of Volatility (σ) and Correlation (ρ) of Shocks to Consent Shares.