

# Self-Fulfilling Debt Crises and Government Policy

Carlo Galli

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Role for self-fulfilling beliefs in sovereign default models

- Motivated by emerging markets experience and Eurozone crisis
- Bond spreads high and volatile...
- ...but often disconnected to fundamentals and actual defaults
- EZ debt crisis: high spreads as bad equilibrium, motivation for OMT

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Link between spreads, gov't policy and fundamentals important

- Two-way empirical relationship between country spreads and business cycle  
[Neumeyer-Perri (2005), Uribe-Yue (2006)]
- Austerity policies *in response to* EZ crisis (Italy, Spain)
- Micro evidence of gov't spreads pass-through to investment, output  
[Arellano et al. (2017), Bocola (2016), Bottero et al. (2017)]

## This Paper

Standard sovereign default model, with endogenous output

- Circular feedback: spreads  $\Leftrightarrow$  govt debt  $\Leftrightarrow$  domestic policy (gov't investment)
- Non-contractible gov't policy
- Austerity induced by debt crises can generate belief-driven equilibria

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Debt crisis mechanism

- confidence crisis  $\Rightarrow$  higher spreads, costlier to borrow
- $\Rightarrow$  gov't raises less funds, cuts down on consumption *and* investment instead
- $\Rightarrow$  growth  $\downarrow$ , future default incentives  $\uparrow \Rightarrow$  pessimistic expectations verified

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Model properties

- multiplicity mechanism: dynamic, real effect of spreads
- standard debt auction timing [Aguiar-Gopinath (2006), Arellano (2008)]
- debt overhang dynamics:  $\uparrow$  debt may  $\downarrow$  investment incentives
- crisis periods (bad eqm) may feature large & finite spreads, lower debt (consistent with Aguiar et al. (2016))

# Outline

## 1. Stylised 2-period model

- highlight multiplicity mechanism
- characterize equilibria
- (appendix) analytical proof for deterministic case

## 2. Infinite-horizon numerical example

- show quantitative properties
- static vs dynamic multiplicity

## Government Problem

- Two periods,  $t = 0, 1$
- Agents: risk-averse SOE government, continuum of risk-neutral lenders.
- Government born with  $w(:= f(k_0) - b_0)$ , solves

$$V(w) = \max_{c_0, c_1^R, c_1^D} U(c_0) + \int \max_{R, D} \left\{ U(c_1^R), U(c_1^D) \right\} dG(\gamma)$$
$$\text{s.t. } c_0 = w + qb - k$$
$$c_1^R = f(k) - b$$
$$c_1^D = f(k)\gamma$$

- Govt cannot commit to either  $k$  or repay
- If default, production loss  $(1 - \gamma)$ , with  $\gamma \sim G(0, 1)$
- Repay iff default costs are high:  $1 - \gamma \geq \frac{b}{f(k)}$
- Discount bonds, perfectly patient lenders  $\Rightarrow$  risk-free debt price = 1



## Timing in $t = 0$

1. Government chooses debt issuance  $b$
2. Lenders pay price  $q$ , government raises  $qb$  resources
3. Consumption/Investment chosen **after** debt issuance, taking  $(q, b)$  as given
  - objective function for investment, given  $(w, q, b)$ 
$$W(k; w, q, b) = u(w + qb - k) + \int \max_{R, D} \{u[f(k) - b], u[f(k)\gamma]\} dG(\gamma)$$
  - $k^*(w, q, b)$  unique solution to  $\max_k W(k; w, q, b)$
  - consumption determined residually

## Lenders' Problem

Lenders are atomistic, perfectly competitive  $\Rightarrow$  make zero-profits in expectations

- must anticipate government's investment strategy  $k^*$

Set of zero-profit prices at which lenders are willing to buy  $b$

$$Q(w, b) = \left\{ q : q = \text{Prob} \left( (1 - \gamma) \geq \frac{b}{f[k^*(w, q, b)]} \right) \right\}$$

$$\Rightarrow \text{Calvo timing setup: repay if } y' - b'R \geq \gamma y' \quad \Rightarrow \quad (1 - \gamma) \geq \frac{b'R}{y'}$$

Pricing equations review

$Q(w, b)$  may be a correspondence for some values of  $(w, b)$

# Equilibrium

Collection of

- government policies  $b_i^*(w)$ ,  $k_i^*(w)$ , value functions  $V_i(w)$
- creditors debt price schedules  $q_i(w, b)$

such that

- $b_i^*(w)$  and  $k_i^*(w)$  solve the government's problem and achieve  $V_i(w)$ , conditional on  $q_i$
- given government policies, price functions  $q_i(w, b)$  satisfy lenders' zero-profit condition *for all*  $b$

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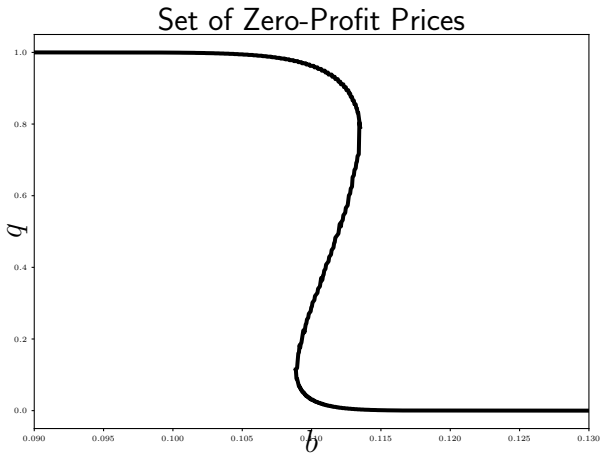
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Now:

1. show conditions for existence of **multiple debt price schedules**
2. show conditions & states for **multiple equilibria**

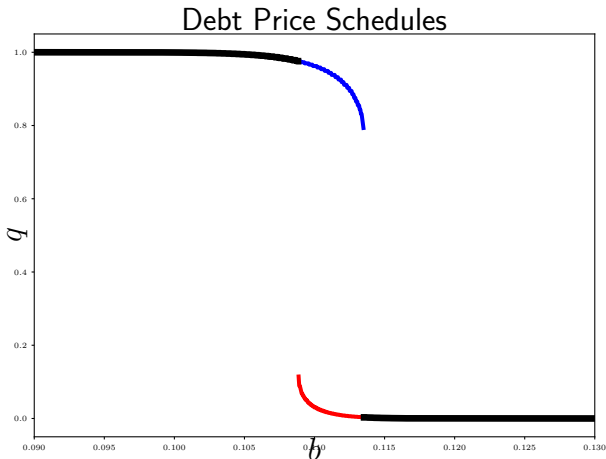
# When is $Q(w, b)$ a Correspondence?

Given state  $w$



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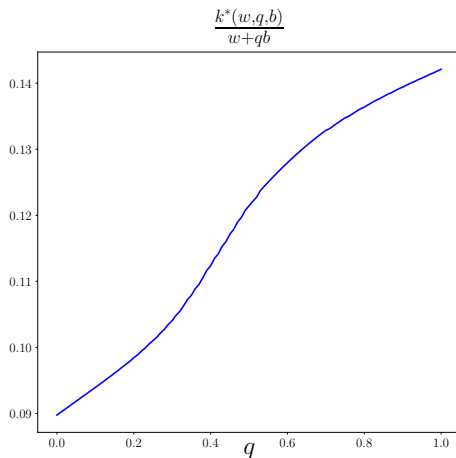
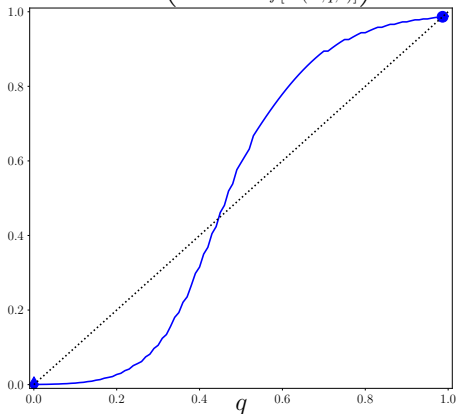


- ignore unstable part of schedule, split into single-valued fns  $q_i(w, b)$
- assume government observes  $i$  before issuing debt  
 $\approx$  observing secondary market conditions

# When is $Q(w, b)$ a Correspondence?

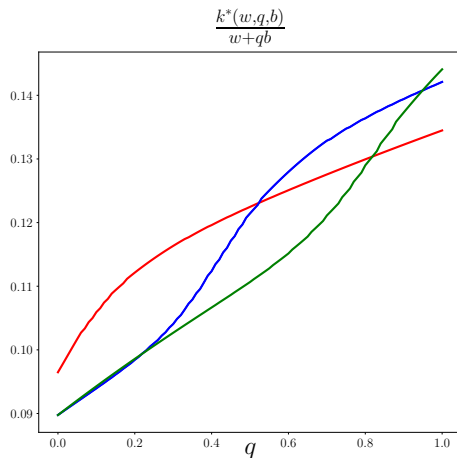
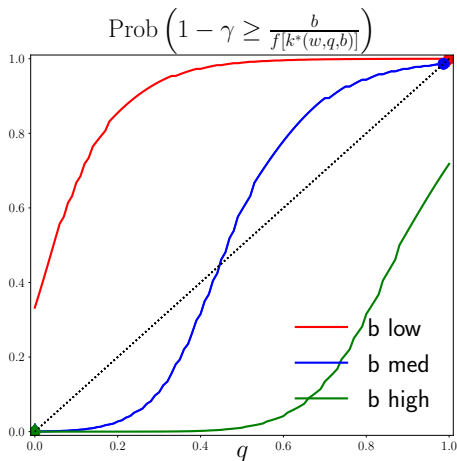
Fix  $w, b$

$$\text{Prob} \left( 1 - \gamma \geq \frac{b}{f[k^*(w, q, b)]} \right)$$



# When is $Q(w, b)$ a Correspondence?

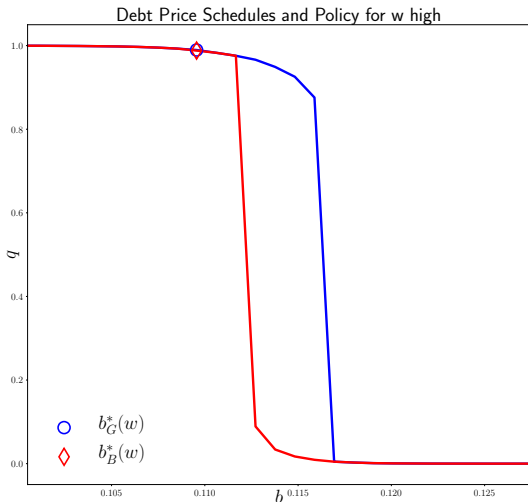
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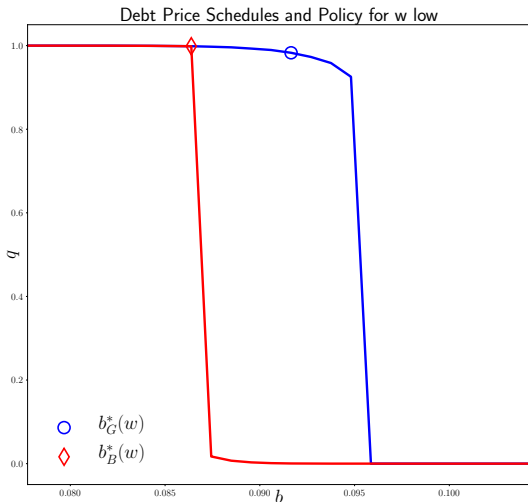
## Equilibrium Policy: High Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?



## Equilibrium Policy: Low Endowment

- There might exist multiple schedules...
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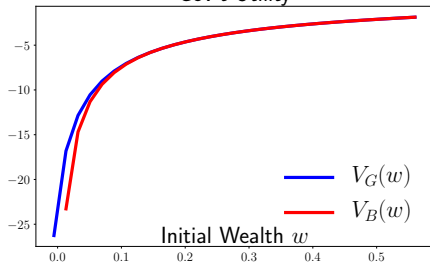
# Equilibrium Policy

When govt policy is risk-free

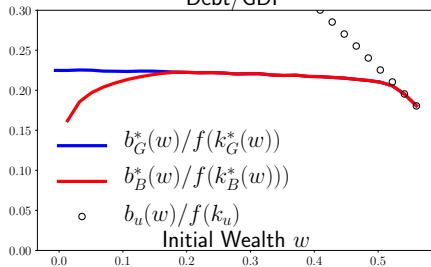
- $f'(k_u) = 1$  (MPK = return on savings/cost of borrowing)
- $b_u(w) = \frac{f(k_{rf}) + k_{rf} - w}{2}$  (when feasible, first-best)

# Multiple Equilibria

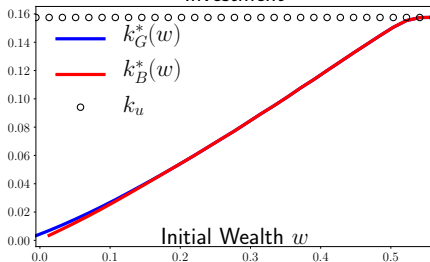
### Gov't Utility



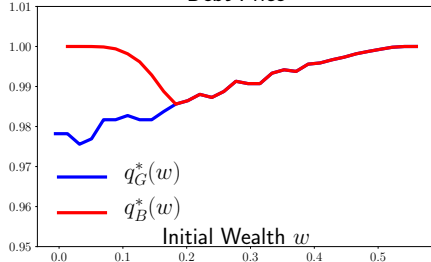
### Debt/GDP



### Investment



### Debt Price



# Summing Up

## Confidence crisis

- more expensive to borrow, tighter govt budget set
- cut borrowing, consumption & investment ( $\approx$  raise taxes)
- debt/GDP $\downarrow$ , but lower utility and depressed output

## Here “austerity” is bad but necessary

- fiscal tightening to avoid high (extreme here) borrowing costs
- not desirable, but only alternative during crisis

# Infinite Horizon Numerical Example

- 1 period = 1 quarter
- Default causes
  - random iid production loss  $\gamma$ , permanent
  - permanent exclusion from debt markets
- Qualitative predictions very similar to 2-period model

## Value Functions

Start-of-period value function:

$$V(k, b, \gamma) = \max_{R, D} \{ V^R[f(k) - b], V^D(k, \gamma) \}$$

Repay value function:

$$V^R[\underbrace{f(k) - b}_{:=w}] = \max_{k', b'} u[w + q(w, b')b' - k'] + \beta \sum_{\gamma} P(\gamma) V(k', b', \gamma)$$

Default value function:

$$V^D(k, \gamma) = \max_{k'} u[\gamma f(k) - k'] + \beta V^D(k', \gamma)$$

Debt price correspondence

$$Q(w, b') = \left\{ q : q = \frac{1}{R} \sum_{\gamma} P(\gamma) \mathbb{1} \left[ V^R[f(k^*) - b'] \geq V^D(k^*, \gamma) \right] \right\}$$

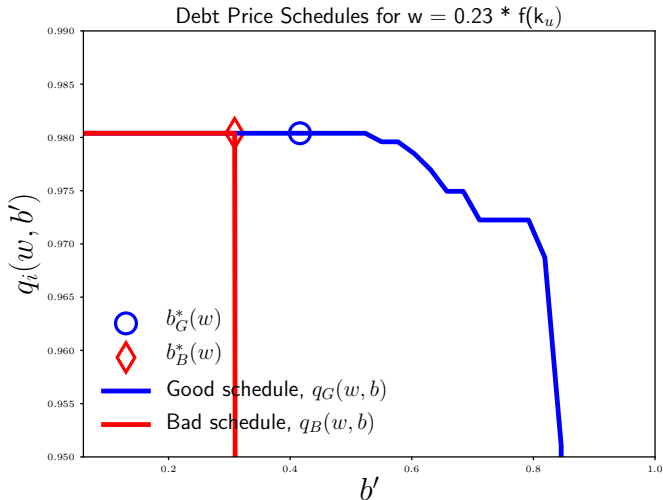
$$k^* := k^*(w, q, b')$$

- Debt price schedule is still a correspondence
- To coordinate lenders' beliefs, iid sunspot

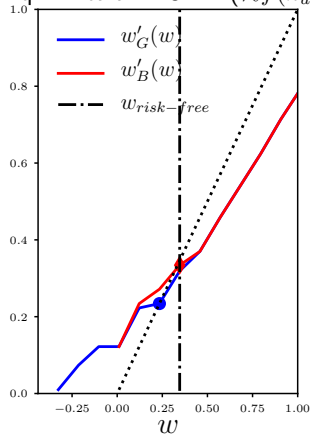
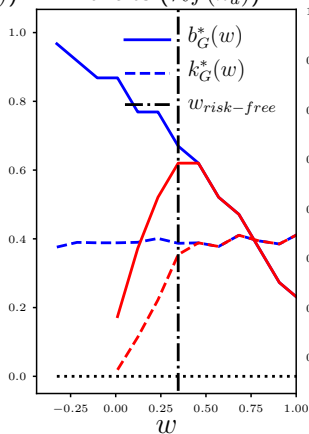
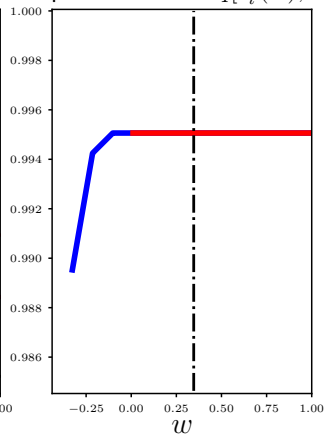
$$i = \begin{cases} G & \text{w.p. } \pi \rightarrow V_G[f(k) - b], Q_G(w, b') \\ B & \text{w.p. } 1 - \pi \rightarrow V_B[f(k) - b], Q_B(w, b') \end{cases}$$



# Debt Price Function Example

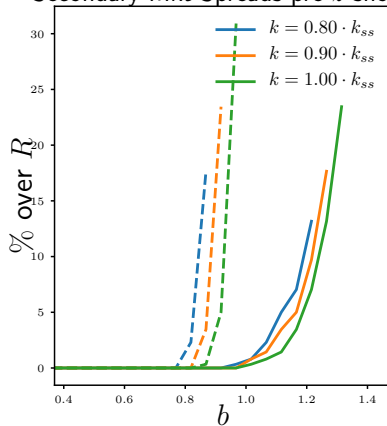


# Policies and Equilibrium Prices

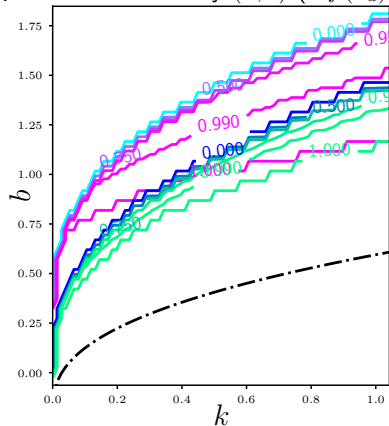
Eqm Wealth L.O.M. ( $\%f(k_u)$ )Policies ( $\%f(k_u)$ )Eqm Debt Prices  $q[b_i^*(w), w]$ 

# “Secondary Mkt” Spreads and Default Probabilities

## Secondary Mkt Spreads pre- $z$ shock



## Default Probs by $(k, b)$ ( $\%f(k_u)$ )



## What about “dynamic” multiplicity?

So far, “static” multiplicity: given  $E[V(k', b', \gamma)]$ , self-confirming beliefs over  $k'$  today

- Limitation? In bad eqm no risky borrowing, “endogenous austerity”

Dynamic, circular mechanism typical of sovereign default models:

- $V^R \rightarrow$  default cutoff  $\hat{b}(k, \gamma)$       via  $V^R[f(k) - \hat{b}] = V^D[\gamma f(k)]$
- cutoff  $\hat{b}(k, \gamma) \rightarrow$  price fn  $Q(w, b')$
- price fn  $Q(w, b') \rightarrow V^R$

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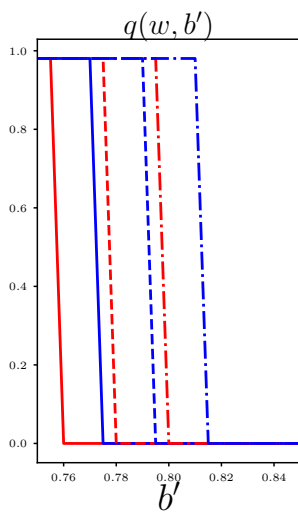
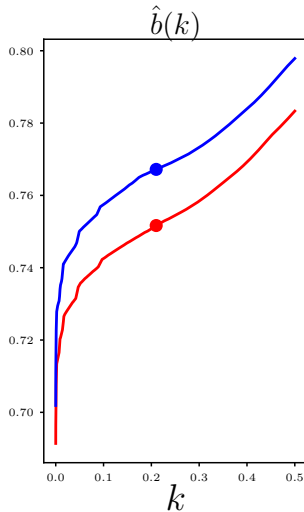
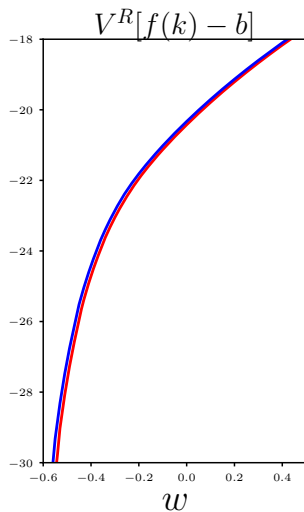
Is mechanism strong enough to generate multiple  $(V^R, \hat{b}, Q)$  triplets?

- No, in canonical Eaton-Gersovitz models (Auclert-Rognlie (2016))
- Answer seems different here with endogenous income

Different beliefs on continuation values

- parallel debt schedules
- more realistic equilibrium prices?

# Deterministic Example: Dynamic Multiplicity



# Conclusion

- Real, dynamic effect of spreads in standard sovereign default model
- Two-way feedback between spreads, policy and real activity
  - ⇒ beliefs interact with both debt *and* domestic policy

## Extensions

- add private sector and govt taxation → feedback spreads-govt-pvt sector
- dynamic multiplicity in infinite horizon

# Appendix



## Related Literature

Problems tackled separately in the literature:

- Quantitative literature (debt policy only [Aguiar-Gopinath(2006), Arellano(2008)], reform effort [Mueller et al.(2016), Marimon et al.(2017)], investment [Bai-Zhang(2012), Gordon-Guerron(2017)])
  - (fundamentals, policy)  $\rightarrow$  spreads
- Lending channel [Bocola (2016), Arellano et al. (2017), Ari (2017), Balke (2017), Bottero et al. (2017)]
  - spreads  $\rightarrow$  fundamentals (via banking sector)
- Austerity policies [Arellano-Bai (2016), Conesa-Kehoe-Ruhl (2017)]
  - spreads  $\rightarrow$  fundamentals (via tax policy)
- Self-fulfilling debt crises literature [Calvo (1988), Cole-Kehoe (2000), Lorenzoni-Werning (2014), Aguiar et al. (2016), Ayres et al. (2018)]
  - spreads  $\leftrightarrow$  debt policy (no fundamentals)

## Pricing Equations Review

### PR := Probability of Repayment

Eaton-Gersovitz tradition (Aguiar-Gopinath (2006), Arellano (2008))

- issue  $b'$ , get price  $q$ , repay tmr if  
 $y' - b' \geq h(y') \Rightarrow q = \text{PR}_{y'}[b']$ 
  - off-equilibrium, adjust  $c$

Government as price-taker (Lorenzoni-Werning (2014))

- given  $q$ , issue  $b'$ , repay if  $y' - b' \geq h(y') \Rightarrow q = \text{PR}_{y'}[b'(q)]$ 
  - off-equilibrium, adjust  $b'$

Calvo (1988) timing

- issue  $b'$  at interest rate  $1/q$ , repay if  
 $y' - b' \frac{1}{q} \geq h(y') \Rightarrow q = \text{PR}_{y'}[b' \frac{1}{q}]$

If output is endogenous:  $y' = \mathcal{H}(q, b', \cdot) \leftrightarrow$  This paper:

$$y' = f[k^*(w, q, b')]$$

- issue  $b'$ , get  $q$ , debt price  $q = \text{PR}_{y'}[b', \mathcal{H}(q, b', \cdot)]$ 
  - same timing/commitment of Eaton-Gersovitz framework
  - $\mathcal{H}$  can be many things

## First-Order Conditions

- Govt repays iff  $\gamma \leq \hat{\gamma} := 1 - \frac{b}{f(k)}$
- Define debt price schedule as  $q_i(w, b)$
- Capital FOC:

$$u'(c_0) = \beta f'(k) \left[ G(\hat{\gamma}) u'(c_R) + \int_{\hat{\gamma}} \gamma u'(c_D) dG(\gamma) \right]$$

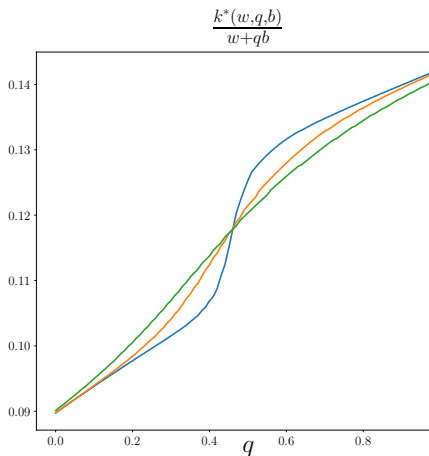
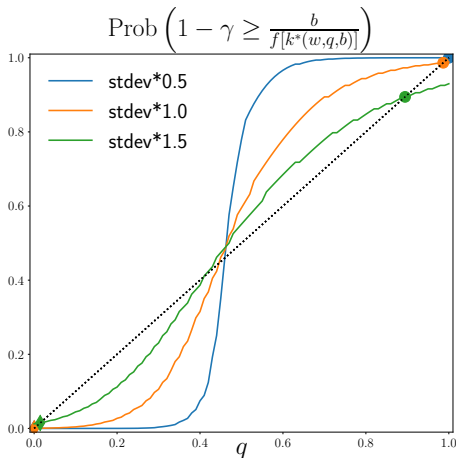
- Debt FOC:

$$u'(c_0) = \beta \frac{1}{q_i(w, b) + \frac{\partial q_i(w, b)}{\partial b} b} G(\hat{\gamma}) u'(c_R)$$

- When debt is risk-free:

$$q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b' = \frac{1}{R} \quad \text{and} \quad G(\hat{\gamma}) = 1 \quad \Rightarrow \quad \begin{cases} f'(k) = R \\ u'(c_0) = \beta R u'(c_R) \end{cases}$$

# When is $Q(w, b)$ a Correspondence?

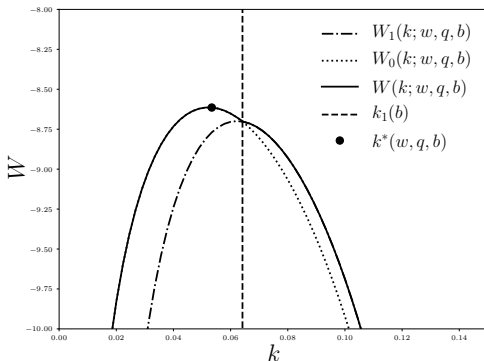


## Investment Objective Fn

- Examine the investment decision, keeping everything else ( $w, q, b$ ) fixed

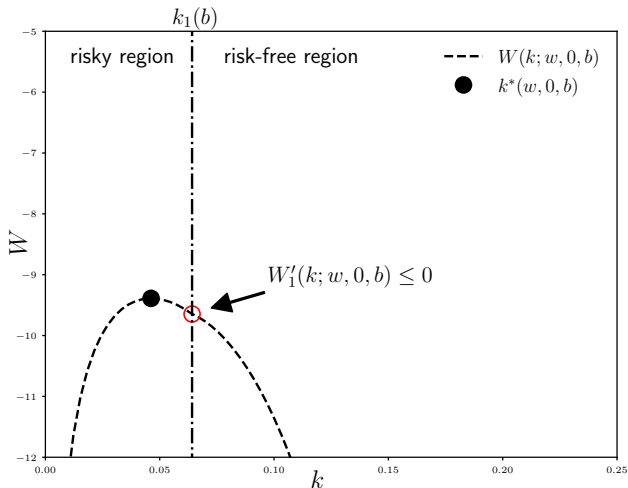
$$W(k; w, q, b) = u(w + qb - k) + \max \{u[f(k) - b], u[f(k)\gamma]\}$$

- $k_1(b) :=$  lowest  $k$  s.t. govt repays  $b$
- $W_p(k; w, q, b)$  is obj. fn. assuming govt will repay w.p.  $p$



# When is $Q(w, b)$ a Correspondence? Sufficient Conditions

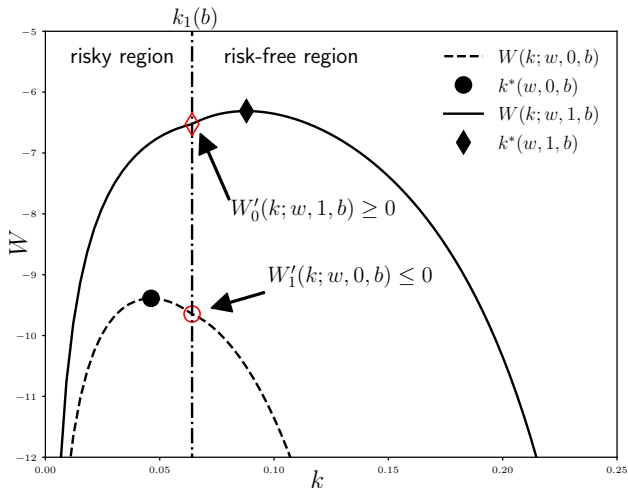
$$k^*(w, 0, b) < k_1(b) \Leftrightarrow 0 \in Q(w, b) \Leftrightarrow W'_1(k_1(b); w, 0, b) \leq 0$$



# When is $Q(w, b)$ a Correspondence? Sufficient Conditions

$$k^*(w, 0, b) < k_1(b) \Leftrightarrow 0 \in Q(w, b) \Leftrightarrow W'_1(k_1(b); w, 0, b) \leq 0$$

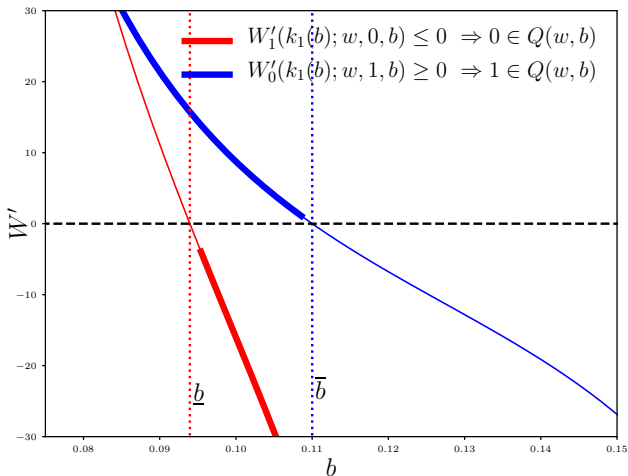
$$k^*(w, 1, b) \geq k_1(b) \Leftrightarrow 1 \in Q(w, b) \Leftrightarrow W'_0(k_1(b); w, 1, b) \geq 0$$



# When is $Q(w, b)$ a Correspondence? Characterization

- For each state  $w$ , characterize debt levels such that  $(0, 1) \in Q(w, b)$
- for all  $b \in [\underline{b}(w), \bar{b}(w)]$ , there are multiple zero-profit prices

ZP condition graph





## Sufficient Condition

### Proposition

*Given state  $w$ , if*

$$\frac{u'(w + \underline{b}(w) - k_1[\underline{b}(w)])}{u'(w - k_1[\underline{b}(w)])} \leq \gamma$$

*then*

$$\underline{b}(w) \leq \bar{b}(w) \quad \text{and} \quad (0, 1) \in Q(w, b) \quad \forall b \in [\underline{b}(w), \bar{b}(w)]$$

# Sufficient Condition

## Proposition

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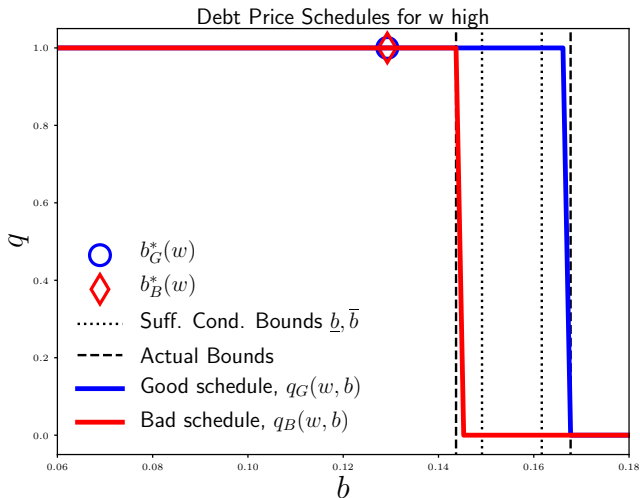
$$\underline{b}(w) \leq \bar{b}(w) \quad \text{and} \quad (0, 1) \in Q(w, b) \quad \forall b \in [\underline{b}(w), \bar{b}(w)]$$

In words, there exist multiple zero-profit prices if  $k^*$

- implies default when  $q = 0$ 
  - low auction revenues, high  $u'(c_0) \rightarrow MC(\text{risk-free } k) \gg MB(\text{risk-free } k)$
- is risk-free when  $q = 1$ 
  - high auction revenues, low  $u'(c_0) \rightarrow MC(\text{default } k) \ll MB(\text{default } k)$

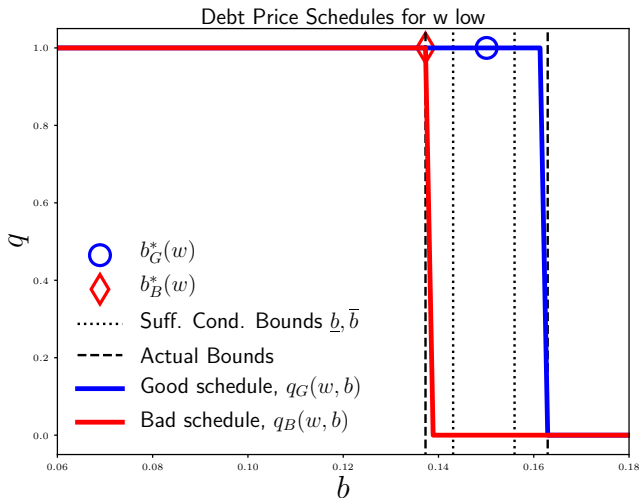
# Equilibrium Policy: High Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?



# Equilibrium Policy: Low Endowment

- There might exist multiple schedules...
- ...but does the government ever select them?



## Unconstrained Risk-Free Policy

When govt policy is risk-free

- $f'(k_u) = 1$  (MPK = return on savings/cost of borrowing)
- $b_u(w) = \frac{f(k_{rf}) + k_{rf} - w}{2}$  (when feasible, Pareto-efficient)

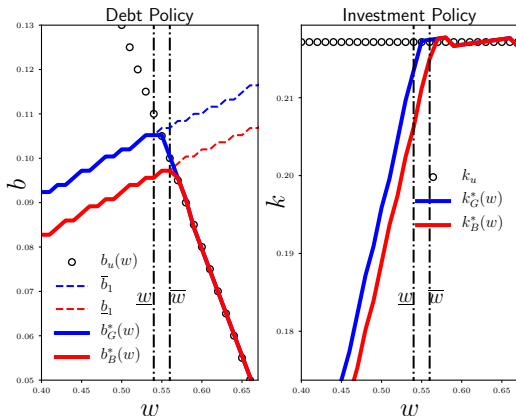
If  $1 \notin Q(w, b_u(w))$ , policy is not feasible!

- $\Rightarrow$  depends on schedule
- if so, govt is borrowing constrained
- borrowing constraint depends on schedule

# Multiple Equilibria

$[\underline{w}, \bar{w}]$  where  $k_u, b_u(w)$

- are feasible under  $q_G$
- are not feasible under  $q_B \Rightarrow$  constrained policy (borrow less, invest less)



# Optimality Conditions (Infinite Horizon Model)

Let  $\hat{\gamma} := \hat{\gamma}(k', b')$

Capital FOC

$$u'[w + q_i(w, b')b' - k'] = \beta f'(k) \left[ G(\hat{\gamma})u'[f(k) - b] + \int_{\hat{\gamma}} \gamma u'[\gamma' f(k) - k'] dG(\gamma) \right]$$

Debt FOC

$$u'[w + q_i(w, b')b' - k'] = \beta \frac{1}{q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b'} G(\hat{\gamma})u'[f(k) - b]$$

With risk-free debt

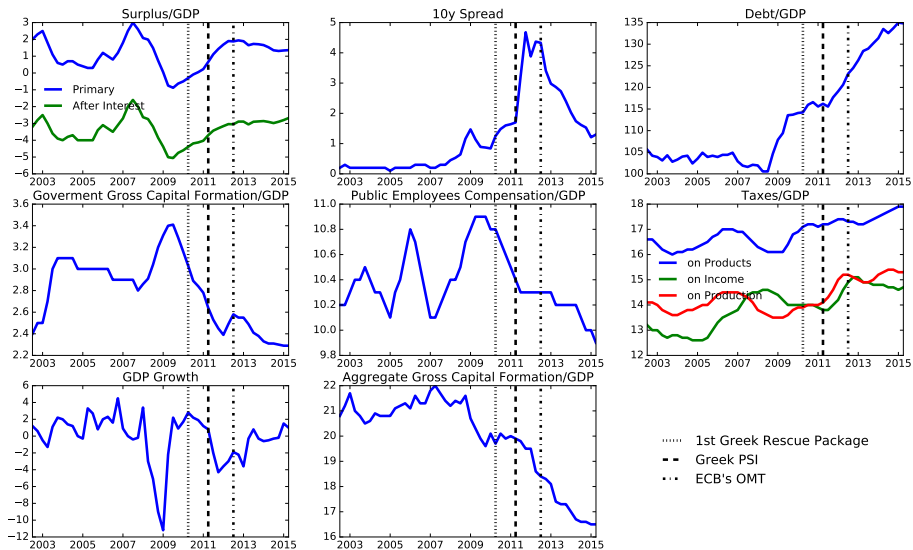
$$q_i(w, b') + \frac{\partial q_i(w, b')}{\partial b'} b' = \frac{1}{R} \quad \Rightarrow \quad f'(k_u) = R$$

## Parametrization

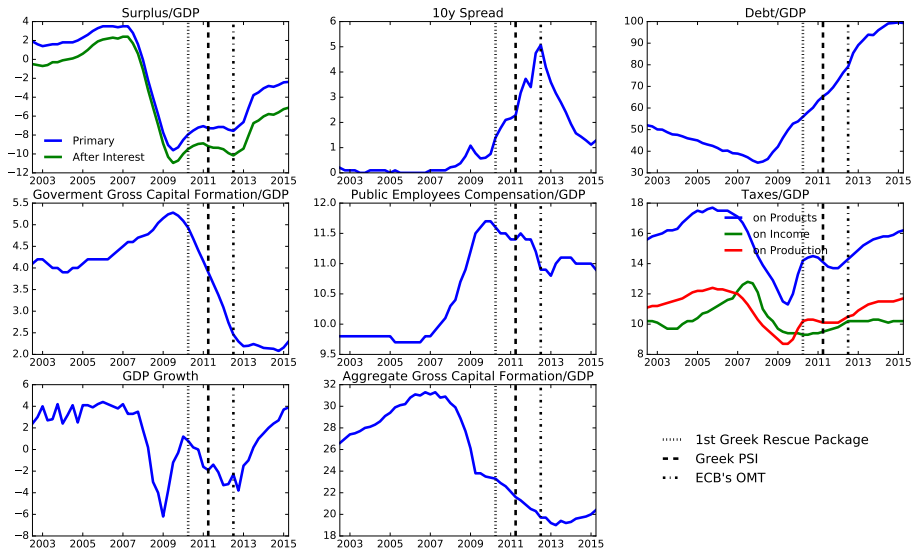
Parameter		Value
Capital share of output	$\alpha$	0.4
CRRA risk aversion parameter	$\sigma$	2
Government discount factor	$\beta$	0.89
Risk-free rate (annual)	$R$	2%
Sunspot probability	$\pi$	0.75
Default cost distribution	$\gamma$	$N(0.8, 0.05)$



## Italy



## Spain



## Some EZ Debt Crisis Quotes

Italian Government Press Release on “Salva Italia” measures, 4/12/2011

*“These urgent measures were necessary to face a serious financial crisis that has hit [...] sovereign bond markets, Italy included.”*

Italian PM Mario Monti, 29/12/2011

*“Our economic fundamentals do not justify such a high government bond spread.”*