# Asset Purchases and Default-Inflation Risks when Investors Learn from Prices

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#### Motivation

APs typically a monetary policy tool when at the ZLB

- reduce long term rates
- restore appropriate function of monetary policy transmission mechanism

APs as a *fiscal* tool, to prevent sovereign debt crises and support govt debt service or not?



Lagarde: We are not here to close spreads, there are other tools and other actors to deal with these issues

3:10 PM · Mar 12, 2020 · Twitter Web App

"The problem with QE is that it works in practice, but it does not work in theory" Ben Bernanke, January 16th, 2014

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#### Contribution

- 1. Can APs compress credit spreads? How? When is that useful?
- 2. How do APs affect the information contained in market prices?

We answer these questions with a model that features

- fiscal-monetary interactions (Sargent and Wallace, 1981)
- sovereign default (Eaton and Gersovitz, 1981)
- noisy financial markets (Hellwig, Mukherji and Tsyvinski, 2006)

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#### What we find

#### Asset purchases

- expose the CB balance sheet (hence inflation) to default risk
- crowd out private investors
  - ⇒ relevant if beliefs are heterogeneous
  - reduce nominal and real sovereign yields
  - ▶ affect the informational content of market prices, asymmetrically
- net welfare effect > 0 under some conditions
- ⇒ information frictions as a rationale for why APs may work "in theory"

# Implications

• degree of belief heterogeneity key for AP elasticity of the interest rate

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#### Outline

- 1. Model setup
- 2. Homogeneous information
- 3. Heterogeneous information

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#### Model: Government

Two periods, t = 1, 2

First period t = 1

• fund stochastic spending by issuing nominal + defaultable debt

$$b = S$$
 where  $S \sim \textit{U}[0, 1]$ 

Second period t = 2

• raise taxes, can default ( $\delta \in \{0,1\}$ ) with haircut h and deadweight loss  $\theta$ 

$$b\frac{R(1-\delta h)}{\Pi} = \tau S \quad \rightarrow \quad \underbrace{\frac{R(1-\delta h)}{\Pi}}_{\psi(R,\Pi,\delta)} = \tau$$

•  $\delta$  decision minimises distortions from taxes ( $\zeta$ ) & default ( $\theta$ )

$$\mathcal{L} := (1 - \delta)\zeta(\psi(R, \Pi, 0)) + \delta\left[\zeta(\psi(R, \Pi, 1)) + \theta\right]$$

default iff  $\zeta(\psi(R,\Pi,0)) > \zeta(\psi(R,\Pi,1)) + \theta$  for today, default iff  $\theta < \hat{\theta}$ 

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# Model: Households

Continuum of risk-neutral agents  $i \in [0,1]$ 

First period t = 1

- receive information on APs, R and  $\theta$  (with noise)
- receive endowment  $e_1$ , save it in 3 assets

$$e_1 \geq b^i + m^i + s^i$$

Second period t = 2

$$c^{i} = b^{i} \frac{R(1 - \delta h)}{\Pi} + \frac{m^{i}}{\Pi} + \rho s^{i} - \tau S - \mathcal{L}$$

- pay taxes, consume
- tax & default distortions  $\mathcal{L}$  create deadweight losses

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# Model: Central Bank /1

First period t = 1: issue money, save via storage (real + risk-free) or bonds

$$s^{cb} + b^{cb} = m \rightarrow \frac{s^{cb}}{m} = 1 - \alpha$$

Second period t=2: reimburse money with returns from saving

$$\rho s^{cb} + \frac{b^{cb} R(1 - \delta h)}{\Pi} = \frac{m}{\Pi} \quad \rightarrow \quad (1 - \alpha)\rho + \alpha \frac{R(1 - \delta h)}{\Pi} = \frac{1}{\Pi}$$

Let share of money invested in bonds be  $\alpha := \frac{b}{a}$ 

- $\bullet$  return of money as  $\alpha$ -weighted average
- $\bullet$   $\alpha \rightarrow$  degree of fiscal dominance

CB balance sheet price level & bond returns

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# Model: Central Bank /2

Solving for the real return on money

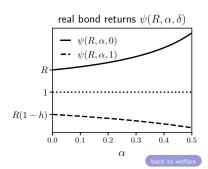
$$\frac{1}{\Pi} \underbrace{\begin{bmatrix} 1 - \alpha R (1 - \delta h) \end{bmatrix}}_{\begin{subarray}{l} \textbf{net return} \\ \textbf{nominal} \\ \textbf{liabilities} \end{subarray}}_{\begin{subarray}{l} \textbf{net return} \\ \textbf{real assets} \end{subarray}$$

Plug into real bond returns

$$\psi(R,\alpha,\delta) = \frac{R(1-\delta h)}{\Pi(R,\delta,\alpha)} = \rho \frac{1-\alpha}{\frac{1}{R(1-\delta h)} - \alpha}$$

Note that 
$$R \in \left[1, rac{1}{1-h}
ight]$$

- ullet Repayment:  $\delta=0$  and R>1
  - ▶ CB makes profits,  $\frac{1}{\Pi} > \rho$
  - $ightharpoonup \uparrow \alpha \Rightarrow \downarrow \Pi \Rightarrow \uparrow \psi$
- Default:  $\delta = 1$  and R(1 h) < 1
  - ▶ CB makes losses,  $\frac{1}{\Pi} < \rho$
  - $ightharpoonup \uparrow \alpha \Rightarrow \uparrow \Pi \Rightarrow \downarrow \psi$



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# Market clearing

#### Bonds market clearing

$$\int b^i di + b^{cb} = b$$

#### Goods market clearing

$$c = \rho[e_1 - S] - \mathcal{L}$$

#### Timing

- 1. First period
  - 1.1 asset purchases  $\alpha$  are unconditional, CB does not observe shocks
  - 1.2 shocks  $(\theta, S)$  realise
  - 1.3 agents receive information and make portfolio decisions
- 2. Second period
  - 2.1 government observes shocks perfectly, takes default decision
  - 2.2 payoffs realise & agents consume

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#### Roadmap

- 1. Perfect foresight
- 2. Uncertainty + homogeneous information
- 3. Uncertainty + heterogeneous information & learning from prices

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# 1) Perfect foresight

Everyone knows  $\delta = \mathbb{1}[\theta < \widehat{\theta}]$ 

Equilibrium interest rate

$$R(1-\delta h)=1$$

Inflation is anchored

$$\frac{1}{\Pi} = \rho \quad \perp \alpha$$

Real bond return = real money return =  $\rho$ 

⇒ Asset purchases are irrelevant

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# 2) Uncertainty + homogeneous information

Agents and CB share same uncertainty:  $Prob(\delta = 0) = p$ Equilibrium R solves no-arbitrage condition

$$p \psi(R, \alpha, 0) + (1 - p) \psi(R, \alpha, 1) = 1$$

Expected welfare loss

$$p \zeta \Big(\psi(R, \alpha, 0)\Big) + (1 - p) \zeta \Big(\psi(R, \alpha, 1)\Big)$$

Effect of asset purchases

- ↑ CB exposure to default risk
- ullet increase the variance of inflation  $(\uparrow \mathbb{V}(\Pi))$  and real bond returns  $(\uparrow \mathbb{V}(\psi))$
- but  $\mathbb{E}(\psi) \leftrightarrow$  by no-arbitrage
- $\Rightarrow$  With convex distortions,  $\alpha^* = 0$

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# 3) Uncertainty + heterogeneous information & learning from prices

For simplicity, assume that the fundamental

$$heta = egin{cases} heta^H ext{ (repay)} & ext{w.p.} & q \ heta^L ext{ (default)} & ext{w.p.} & 1-q \end{cases}$$

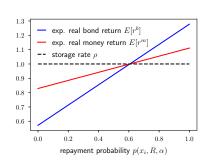
Agent i observes

- private signal  $x_i = \theta + \sigma_x \xi$  where  $\xi \sim N(0, 1)$
- equilibrium price R (endogenous public signal)
- $\Rightarrow$  subjective repayment probability  $p(x_i, R, \alpha)$

Assumption:  $b^i < 1$ 

Agent i's portfolio decisions:

$$\mathbb{E}[r^{b} \mid x_{i}, R, \alpha] \begin{cases} > 1 & b^{i} = 1; \quad m^{i} = e_{1} - 1 \\ = 1 & b^{i} + m^{i} + s^{i} = e_{1} \\ < 1 & e_{1} = s^{i} \end{cases}$$



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# Market Clearing

Agents follow monotone threshold strategies: hold bonds & money iff  $x_i \ge \widehat{x}(R, \alpha)$ 

$$\underbrace{P(x_i > \widehat{x}(R, \alpha))}_{\substack{\text{mass of } \\ \text{optimists}}} \begin{bmatrix} \underbrace{1}_{\substack{\text{direct}}} & \underbrace{(e_1 - 1)}_{\substack{\text{money}}} & \underbrace{\alpha}_{\substack{\text{AP}}} \end{bmatrix} = \underbrace{S}_{\substack{\text{random} \\ \text{bond}}}$$

Solving for the cutoff signal:

$$\widehat{\chi}(R,\alpha) = \overbrace{\theta - \sigma_x \Phi^{-1}\left(\frac{S}{d(\alpha)}\right)}^{:=Z(\theta,S,\alpha)} \quad \text{ where } d(\alpha) = 1 + \alpha(e_1 - 1) \geq 1$$

Effect of asset purchases:

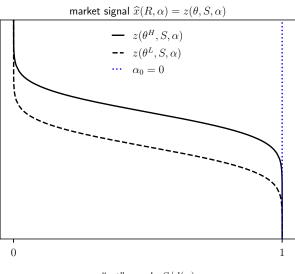
- ullet market signal truncation and full revelation of heta
- APs select a more optimistic marginal agent

• 
$$\widehat{S}'(\alpha) < 0$$

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# Market Signal

$$\widehat{x}(R,\alpha) = \theta - \sigma_x \Phi^{-1}\left(\frac{S}{d(\alpha)}\right)$$

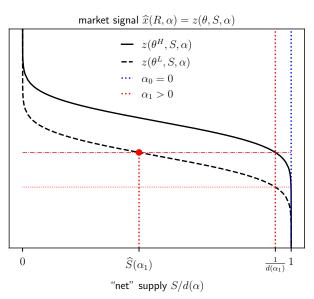


"net" supply  $S/d(\alpha)$ 

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# Market Signal

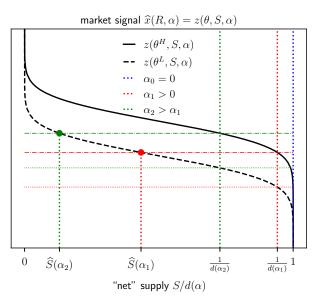
$$\widehat{x}(R,\alpha) = \theta - \sigma_x \Phi^{-1}\left(\frac{S}{d(\alpha)}\right)$$



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# Market Signal

$$\widehat{x}(R,\alpha) = \theta - \sigma_x \Phi^{-1}\left(\frac{S}{d(\alpha)}\right)$$



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$$z \mid \theta \sim N(\theta, \sigma_x^2)$$

over 
$$\mathcal{Z} = \left[\theta - \sigma_{\mathsf{x}}\Phi^{-1}(1/d(\alpha)), +\infty\right)$$

"Market" posterior beliefs over  $\theta$ 

$$\mathsf{P}(\theta^H|x_i,z,\alpha) = \begin{cases} \frac{\frac{q}{\sigma_{post}}\phi\left(\frac{\theta^H - \frac{x_i + z}{2}}{\sigma_{post}}\right)}{\frac{q}{\sigma_{post}}\phi\left(\frac{\theta^H - \frac{x_i + z}{2}}{\sigma_{post}}\right) + \frac{1 - q}{\sigma_{post}}\phi\left(\frac{\theta^L - \frac{x_i + z}{2}}{\sigma_{post}}\right)} & \text{for } z \geq \underline{z}(\theta^H,1,\alpha) \\ 0 & \text{for } z \in [\underline{z}(\theta^L,1,\alpha),\underline{z}(\theta^H,1,\alpha)) \end{cases}$$

where  $\sigma_{post}^2 = \sigma_x^2/2$ 

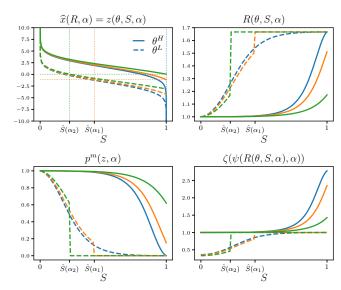
Finally, equilibrium interest rate

$$p^{m}(z,\alpha)\frac{1-\alpha}{\frac{1}{R}-\alpha}+\left(1-p^{m}(z,\alpha)\right)\frac{1-\alpha}{\frac{1}{R(1-\alpha)}-\alpha}=1$$

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#### Effect of asset purchases:

- ullet market signal truncation and full revelation of  $heta \qquad \Rightarrow \widehat{S}'(lpha) < 0$
- APs select a more optimistic marginal agent  $\Rightarrow rac{d\ z}{dlpha} > 0$



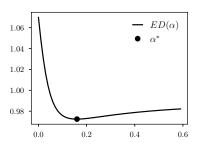
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#### Ex-ante welfare

Integrating over all  $(\theta, S)$  realisations

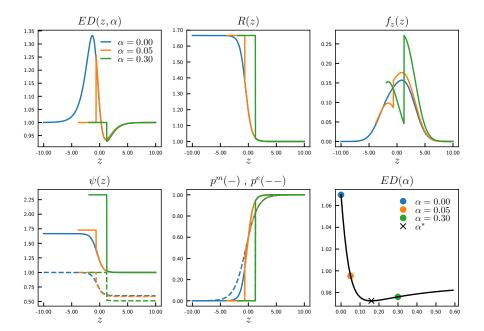
$$ED(\alpha) = q \int_0^1 \zeta \Big( \psi^r (z(\theta^H, s, \alpha), \alpha), \Big) dS$$
$$(1 - q) \left\{ [1 - \hat{S}(\alpha)] \zeta(1) + \int_0^{\hat{S}(\alpha)} \zeta \Big( \psi^d (z(\theta^L, s, \alpha), \alpha), \Big) dS \right\}$$



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# Thank You!

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# Government Budget Normalisation

$$\gamma y(\epsilon) = b$$
 
$$\frac{B_1}{P_2}R(1 - \delta h) = \hat{\tau}$$

which becomes

$$b\frac{P_1}{P_2}R(1-\delta h) = \tau y(\epsilon)$$
$$\gamma y(\epsilon)\frac{P_1}{P_2}R(1-\delta h) = \tau y(\epsilon)$$
$$\gamma \frac{P_1}{P_2}R(1-\delta h) = \tau$$

back

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#### Central Bank Balance Sheet

Central bank balance sheet at t = 1

| Assets                  | Liabilities |
|-------------------------|-------------|
| bonds b <sup>cb</sup>   | money m     |
| storage s <sup>cb</sup> |             |

Central bank balance sheet at t=2

| Assets                                   | Liabilities           |
|--|-----------------------|
| bonds $b^{cb} \frac{R(1-\delta h)}{\Pi}$ | money $\frac{m}{\Pi}$ |
| storage $ ho s^{cb}$                     |                       |

back

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#### Price level determination and real bond returns

Solving for the real return on money

$$\frac{1}{\Pi} = \rho \frac{1 - \alpha}{1 - \alpha R(1 - \delta h)}$$

Plug into real bond returns

$$\psi(R, \alpha, \delta) = \rho \frac{1 - \alpha}{\frac{1}{R(1 - \delta h)} - \alpha}$$

Since  $R \in \left[1, \frac{1}{1-h}\right]$ 

- ullet in repayment  $\delta=0$  and R>1
  - central bank makes profits
  - there is deflation:  $\frac{1}{\Pi} > \rho$
  - ▶ larger APs imply larger deflation and debt service:  $\uparrow \alpha \Rightarrow \uparrow \psi(0)$
- in default  $\delta = 1$  and R(1 h) < 1
  - central bank makes losses
  - there is inflation:  $\frac{1}{\Pi} < \rho$
  - ▶ larger APs imply larger inflation and lower debt service:  $\uparrow \alpha \Rightarrow \downarrow \psi(1)$

