

Asset Purchases and Heterogeneous Beliefs*

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Abstract

We study central bank purchases (APs) of government debt in financial markets where rational investors face position limits and observe private signals, bond prices, and central bank policy. We show that learning creates a wedge between bond prices and fundamentals, and that APs affect this wedge. As a result, APs have a sizable, non-monotonic effect on asset prices, and lead to central bank losses that imply implicit government financing. Our paper provides a microfoundation for the functioning and optimality of APs, grounded in the endogeneity of investors' beliefs to policy.

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1 Introduction

The widespread adoption of large-scale asset purchase (AP) programs represents arguably the most significant shift in monetary policy over the past two decades. While debate persists over their general equilibrium effects, robust empirical evidence confirms that APs raise asset prices, particularly in the markets directly targeted by the interventions. This price response is unsurprising, given the well-established notion of a downward-sloping aggregate demand for assets. However, the specific microfoundations of the aggregate price elasticity of asset demand are crucial for the evaluation of the risks associated with APs, such as their impact on the balance sheets of central banks and governments, and their potential disruptions to the price formation mechanism.

A downward-sloping aggregate demand curve exists insofar as investors have heterogeneous demand functions and limited capacity to absorb asset supply. This is evident in primary sovereign debt markets, where investors submit different price-contingent demand schedules. Such heterogeneity may either stem from structural features—such as preferences over risk, maturity, or liquidity—or from a disagreement in beliefs about the fundamental value of the asset.¹ The difference is deep: while individual structural characteristics are unaffected by APs, beliefs change with APs as investors learn from prices.

To the best of our knowledge, this is the first paper characterizing the effects of APs in financial markets where the downward-sloping demand for assets derives from rational belief heterogeneity. We show that the learning-from-prices channel through which APs influence investors’ beliefs has profound implications, even within the narrow scope of a financial market equilibrium.

First, the impact of APs on asset prices is non-monotonic in the size of the intervention. APs raise prices by *crowding out* the demand of more pessimistic investors, but may also trigger price drops because they make prices more informative in states where the asset has a relatively low payoff. To fix ideas, consider a scenario where a large sovereign debt purchase program is implemented during a period of high yield spreads. If the intervention fails to inflate prices, each investor interprets such failure as more compelling evidence that others are pessimistic about fundamentals, generating information that lets prices fall in equilibrium.

Second, APs generate a transfer of resources from the central bank to the government—the issuer of the asset—and private investors. This happens because learning from prices in the

¹See Cole et al. (2022, 2024) for recent evidence on the importance of information asymmetries and spillovers in sovereign debt markets.

presence of information frictions generates a wedge between the equilibrium price and the fundamental value of the asset. The sign of this wedge drives gains and losses for all financial market participants. We show that, as a by-product of the intervention, APs mostly occur when the asset is overpriced rather than underpriced, implying expected losses for the central bank and gains for the government. Under the alternative scenario where investor heterogeneity is structural, APs instead always generate gains for the central bank, proportional to the premium (such as term, liquidity, or other compensations for risk) paid by the government to investors.²

We model a financial market populated by a continuum of risk-neutral, rational investors who trade government bonds. Both the bond payoff and its supply are stochastic and imperfectly observed. Investors face constraints on their trading positions and form expectations about the bond payoff—which we frequently refer to as the “fundamental” of the economy—based on private signals and the publicly observable equilibrium bond price. Given the supply of bonds, the equilibrium price is determined by the beliefs of the marginal investor—defined as that who makes zero gains in expectation.

Learning from prices generates an externality, as investors take the stochastic properties of the price signal as given, while these are endogenous to their collective behavior. This externality introduces momentum in bond prices, which overreact to the very same information content they convey. The intuition is the following. When agents observe a relatively high bond price, they infer that demand is likely to be high due to strong fundamentals. This further raises demand and the bond price itself, in a loop of general equilibrium amplification that atomistic investors do not internalize. As a result, bonds are overpriced by the market with respect to their fundamental value when their price is high, and vice versa when bond prices are low. The sign of this externality is state-contingent: there exists a positive (negative) *wedge* between the price and the fundamental value of the asset if and only if the combined realization of the supply and payoff shocks is above (below) a threshold level.

Central bank interventions through asset purchase policies affect both equilibrium prices and the information they convey, via their impact on the wedge we just described. In the first part of our analysis, we consider quantity-targeting AP policies, whereby the central bank commits to purchasing a fixed quantity of bonds regardless of their price. Our two main results obtain.

First, APs raise the average bond price, but the effect is non-monotonic in the size of the intervention due to two opposing forces. On the one hand, APs *crowd out* the demand of relatively pessimistic investors. This has an effect on the average price that is positive and decreasing in

²We analytically develop this argument in Section 6.

the program size, because large interventions also crowd out relatively optimistic investors. On the other hand, APs create a region of the state space where low bond prices provide a strong signal that a low payoff is likely, which depresses the average price. The relative strength of this *revelation* channel is increasing in the size of APs. It follows that the marginal effect of APs on average bond prices is positive for small interventions (when crowding-out dominates) and negative for large ones (when the revelation effect prevails). This result hinges on the presence of the learning-from-prices externality and the associated wedge between bond prices and fundamentals. In fact, we prove that if this externality is muted, APs have *no impact* on average prices, regardless of their size, because crowding out and revelation effects would exactly offset each other. Finally, we explore the model’s predictions when it is set to match the dispersion of beliefs about long-term Treasury bond returns in the Survey of Professional Forecasters prior to the Large-Scale Asset Purchase announcement of March 18, 2009. The model predicts an average impact of APs on yields that can account for about half of the observed decline in real yields on long-term Treasuries following the announcement.

Second, we characterize the distribution of gains and losses in the zero-sum game among investors, central bank, and government that is implicit in the financial market we consider. On average, the central bank incurs losses from quantity-targeting APs. This happens because APs crowd out investors who are relatively pessimistic, leaving in the market those that are more optimistic. This implies that the central bank is more likely to buy an overpriced—rather than underpriced—asset. Investors, in contrast, earn positive average gains by exploiting their private information and entering the bond market only when they expect to make net gains. Since central bank losses are generally larger than investor gains, APs imply a net resource transfer from the central bank to the government, a form of monetary financing. On the other hand, in the absence of our learning-from-prices externality, the market price of government liabilities remains tied to its fundamentals so that APs are balance-sheet neutral for the central bank, no matter the size of the intervention.

In the second part of our analysis, we study price-targeting AP policies, where the central bank purchases bonds to achieve a specific price target, subject to a constraint on the intervention size. The main difference with the previous setting is informational: when the central bank is intervening and the equilibrium bond price equals the target, investors do not learn from the price but rather from the size of APs, since the former is fixed by construction while the latter is state-contingent. However, we show that the intervention size is uninformative, and the wedge deriving from the learning externality vanishes. Intuitively, this happens because the central bank is effectively the marginal buyer in the market, and since it has no private information,

equilibrium outcomes do not convey additional information to investors.

With price-targeting rules, APs are effective—in the sense that the average equilibrium price is weakly larger than the target—when such target is close to the average value of the fundamental. Moreover, when the target is exactly equal to such value, APs are costless because the central bank is buying at a “fair” price, at which the learning wedge is zero. When, instead, the price target is higher, average prices fall below target and APs are less effective. In addition, we show that price-targeting APs are generally more expensive, i.e., generate larger central bank losses, than the quantity-targeting alternative. This occurs because, when the price target is above the fundamental value of the asset, the entirety of central bank purchases takes place when bonds are overpriced.

In the final part of the paper, we extend our framework to a simple consumption-saving model to address the issue of the optimality of APs. We assume a household sector that saves via investors, who act as financial intermediaries. We show that the very same frictions that make APs non-neutral in our model also provide a microfoundation for the rationale behind central bank interventions, without resorting to additional frictions such as price rigidities or inequality motives. Specifically, the assumptions of limited arbitrage and dispersed information generate an inefficiency—conceptually different from that deriving from learning from prices—implying that households save too much, because investors’ returns in the financial market exceed the socially optimal return on savings. AP policies correct this inefficiency by reducing investors’ returns and stimulating current household consumption.

Related Literature. Our paper relates to the literature on rational expectations equilibria where investors learn from prices, and to the literature on central bank intervention in macroeconomics and finance. To the best of our knowledge, this is the first paper characterizing the workings of APs when rational investors have heterogeneous beliefs and learn from prices.

Angeletos and Werning (2006); Hellwig et al. (2006) studies the effect of the aggregation of information by prices for equilibrium multiplicity. More recently, Albagli et al. (2024) shows that the emergence of a wedge between prices and fundamental valuations is a general implication of learning from prices, which applies to all models of rational expectations equilibria (e.g., the vast literature using the CARA-Normal model), independently of the presence of position bounds or risk-neutrality. They abstract from policy interventions, and show that the relevance of the wedge relies on the asymmetry of the payoff distribution (see also Albagli et al. (2023) and Bassetto and Galli (2019)). Instead, we focus on the effect of APs and show results which are qualitatively independent of the shape of the payoff distribution.

The interaction between APs and information frictions has been explored in several papers. Iovino and Sergeyev (2023) focus on k-level thinking, a form of bounded rationality, as a source of non-neutrality of APs in an otherwise frictionless model. In the model of Fontanier (2023), APs reduce the incentives of structurally heterogeneous investors to acquire information, reducing the likelihood of a sovereign crisis as prices become less sensitive to fundamentals. Candian et al. (2023) study the information effect of foreign exchange interventions in an open economy where investors’ expectations over-react to the information content of exchange rates. Mussa (1981), Jeanne and Svensson (2007), Christensen and Rudebusch (2012) and Bhattarai et al. (2022) study the signaling role of APs, whereby central bank interventions serve as a way to commit to, or communicate, its current or future objectives. In all this work, investors either do not have rational expectations or they do not learn from prices, the key variable affected by APs.

Workhorse models of APs in finance – for example, the seminal work of Vayanos and Vila (2021), Hamilton and Wu (2012), Greenwood and Vayanos (2014), King (2013), King (2019), and Costain et al. (2024) – typically builds on settings with limits to arbitrage and policy-invariant heterogeneity in investors’ preferences for risk, maturity, or liquidity. The main goal of this literature is to model the “local” impact of APs on the yield curve for different maturities, as documented in a vast empirical literature.³ In our model, we abstract from different maturities and instead explore microfoundations for aggregate asset demand that rely on investors’ heterogeneity in beliefs which, in contrast to preferences, do change with AP policy.

The emphasis on the “local” nature of APs effect contrasts with the applied and quantitative macroeconomics literature, which studies the general equilibrium effects of APs (see Bhattarai and Neely (2016) and Kim et al. (2020) for a survey of the literature). This literature has emphasized various forms of market segmentation and structural heterogeneity as the relevant dimension of departure from the theoretical neutrality benchmark of Wallace (1981). Market segmentation is essential for APs to induce “portfolio rebalancing” effects, i.e., a relative price change across asset classes and maturities, which in turn have broader macroeconomic implications. These effects have been measured since the great recession of 2008-2009 in a flourishing empirical literature, see Gagnon et al. (2011) for the US, Joyce et al. (2012) and Breedon et al. (2012) for the UK, and more recently Koijen et al. (2021) and Altavilla et al. (2021) for the

³A non-exhaustive list would include D’Amico and King (2013), Krishnamurthy and Vissing-Jorgensen (2011), D’Amico and King (2013), and McLaren et al. (2014) for the Fed Large-Scale Asset Purchases; Eser and Schwaab (2016) for the ECB Securities Markets Program (SMP); Altavilla et al. (2016) for the ECB Outright Monetary Transactions (OMT); Krishnamurthy et al. (2017), Koijen et al. (2017), Arrata et al. (2020) and Bernardini and Conti (2023) for various ECB AP programs; and Lucca and Wright (2024) for the yield-targeting policy of the Reserve Bank of Australia.

Eurozone, among others. These effects have been microfounded and studied with quantitative models by seminal papers like Cúrdia and Woodford (2011) and Gertler and Karadi (2015), followed by more recent work.⁴ Further work by Cahill et al. (2013), Li and Wei (2013), Gilchrist et al. (2015) and Rogers et al. (2018) has identified sizable “global” portfolio rebalancing effects that pervade financial markets beyond those that are targeted directly by the program. Recent work by Ray et al. (2024) combines a fully fledged macroeconomic framework with the insights of the finance literature on preferred-habitat investors. Finally, another strand of the literature has emphasized the role of APs in alleviating a lack of risk sharing or insurance on the side of firms or households, in the context of incomplete markets economies with structurally heterogeneous agents. See for example Gornemann et al. (2016), Auclert (2019), Luetticke (2018), Ravn and Sterk (2021), Kaplan et al. (2018), Debortoli and Galí (2017), Hagedorn et al. (2019) and Cui and Sterk (2021).

2 Model

2.1 The Financial Market

We consider a one period economy. The financial market consists of a continuum of investors facing a random supply of government bonds, and a given asset purchase rule by the central bank. Investors share the same preferences, but differ in their beliefs about bond payoffs.⁵ Central bank interventions affect prices and investors’ expected gains, bearing financial consequences for the government and central bank.

Government. At the beginning of the period, the government issues a quantity \tilde{S} of real bonds that is uniformly distributed between 0 and 1; that is:

$$\tilde{S} \sim \text{Uniform}[0, 1]. \tag{1}$$

The bonds are sold at the market-clearing price of Q , and at the end of the period pay a stochastic real payoff $\tilde{\theta}$, which potentially captures uncertainty relative to inflation or default. We assume

⁴See Chen et al. (2012), Del Negro et al. (2017), Wen (2014), Campbell et al. (2012), Harrison (2017) and Sims and Wu (2021).

⁵We compare our channel to a mechanism relying on heterogeneous term premia in Section 6.

$\tilde{\theta}$ is distributed according to the exogenous lottery

$$\tilde{\theta} = \begin{cases} \tilde{\theta}_H & \text{with probability } q, \\ \tilde{\theta}_L & \text{with probability } 1 - q, \end{cases} \quad (2)$$

where q and $1 - q$ respectively denote the probability of a high and low payoff. Despite its simplicity, this binary distribution allows for the asymmetry of bonds' payoffs (they are left-skewed if and only if $q < 1/2$).

Investors. There is a continuum of measure one of bond investors that are risk neutral and discount the end-of-period bond payoff by a common rate r . They do not observe the realization of the aggregate shocks $(\tilde{\theta}, \tilde{S})$, but receive noisy information about it. Specifically, we assume that each investor receives a noisy private signal on the payoff present value $\theta := \tilde{\theta}/(1 + r)$ that is given by

$$x_i = \theta + \sigma_x \xi_i,$$

where $\xi_i \sim N(0, 1)$ is independently and identically distributed across agents. We denote the marginal distribution of private signal x_i with \mathcal{N} .⁶ Investors also receive two pieces of public information: they observe the quantity of central bank APs b_{cb} , and the bond price Q . We thus denote the information set of each investor $i \in [0, 1]$ with $\Omega_i := \{x_i, b_{cb}, Q\}$.⁷

Each investor i chooses her bond position b_i , subject to position bounds $[0, 1]$, to maximize the expected present value of net trading gains, which are given by

$$\max_{b_i \in [0, 1]} \mathbb{E}[\pi_{inv, i} \mid \Omega_i], \quad (3)$$

where $\pi_{inv, i} := b_i(\theta - Q)$. As we will show later, the solution of the investor problem is a price-contingent demand function $b_i = b(\Omega_i)$ taking the form of a limit order, i.e., the investor buys up to the portfolio upper bound if the price is below her reservation value, and nothing otherwise.

In our model, the lack of individual arbitrage is due to the presence of exogenous position bounds. This is a necessary assumption for APs to have an impact on prices. Other models of APs combine limits to investors' arbitrage with structural heterogeneity in some dimension, such as preferred-habitat preferences (Vayanos and Vila (2021)) or risk-bearing capacity (Gertler

⁶The analytical expression for \mathcal{N} is postponed to Appendix B.3.

⁷Without any loss of generality, we abstract from any other information that could be common knowledge among investors. For example, θ could be interpreted as the degree of severity of a crisis, in which case (2) would represent the residual uncertainty conditional on the commonly known event of being in a crisis.

and Karadi (2015)), to cite some notable examples. Here instead we pair limited arbitrage with beliefs heterogeneity, while keeping investors structurally homogeneous.⁸

It is worth noting that, although we will refer to \tilde{S} as the gross supply of bonds, \tilde{S} could also be interpreted as an aggregate demand shock, accommodating the framework of Ray et al. (2024). For example, one could assume a fixed unitary supply of bonds, and a stochastic, non-fundamental demand of $1 - \tilde{S}$ by noise traders. Broadly speaking, \tilde{S} can be interpreted as any aggregate shock orthogonal to bonds' payoffs.

Central Bank. We are interested in studying the impact of central bank open market operations in financial markets. We consider two different policy rules: *quantity-* and *price-targeting* asset purchases.

Quantity-targeting is the simplest rule, and consists in the central bank buying bonds up to a fixed quantity target, independent of the market price, and subject to its demand being no larger than the supply.

Definition 1 (Quantity-targeting APs). *Under quantity-targeting APs, the central bank buys up to an announced quantity target b at market price Q according to*

$$b_{cb} = \begin{cases} b & \text{if } \tilde{S} \geq b \\ \tilde{S} & \text{if } \tilde{S} < b. \end{cases} \quad (4)$$

When $\tilde{S} \geq b$, some supply is absorbed by investors, and the equilibrium bond price Q is determined by the market. When instead $\tilde{S} < b$, we assume that the central bank buys all the supply at the price Q_{pas} and the market remains “passive”. We set $Q_{pas} = \theta$ to abstract from institutional transfers between the central bank and the government.⁹

Price-targeting policies instead are such that the central bank buys bonds only at or below a specific price target.

Definition 2 (Price-targeting APs). *Under price-targeting APs, the central bank submits, simultaneously to investors, a limit order to buy up to a quantity \bar{b}_n of bonds if the price is below*

⁸In Appendix B.1 we discuss the conditions under which central bank APs are neutral with respect to prices and allocations, as in Wallace (1981).

⁹To be clear, Q_{pas} is an exogenous parameter of the analysis that can be modified without any loss of generality or change in our main results. Nevertheless, we show in Appendix B.2 that the assumption $Q_{pas} = \theta$ can be microfounded with a trembling-hand limit argument. Specifically, we show that the assumption of an infinitesimal disturbance in the market clearing mechanism implies that the equilibrium bond price is determined by market forces even in states where $\tilde{S} < b$, and is equal to $Q = \theta$ in such states.

a target Q_n , and nothing otherwise, that is

$$b_{\text{cb}} \begin{cases} = \bar{b}_n & \text{if } Q < Q_n, \\ \in [0, \bar{b}_n] & \text{if } Q = Q_n, \\ = 0 & \text{if } Q > Q_n. \end{cases} \quad (5)$$

with $Q_n \in [\theta_L, \theta_H]$ being the announced price target.

The central bank has no information about θ and, in fact, does not need to observe the state (θ, \tilde{S}) to implement its price-targeting policy. It only needs to be able to condition its demand on the bond price Q . As we will see, the quantity b_{cb} that the central bank effectively purchases to achieve the target will be an equilibrium outcome and, as such, provide information to investors.

2.2 Market Clearing and Equilibrium

The bond market clearing condition is given by

$$\int_0^1 b_i \, di + b_{\text{cb}} = \tilde{S}. \quad (6)$$

In a market equilibrium, the demand of investors equals the supply by the government net of central bank demand b_{cb} . We are now ready to give a formal definition of an equilibrium.

Definition 3. (*Equilibrium*) Given an AP policy rule as per (4) or (5), a homogeneous discount rate r and distributions (1)-(2), a Perfect Bayesian Equilibrium consists of demand schedules $b(\Omega_i)$, a price function $Q(\theta, \tilde{S}, b_{\text{cb}})$, and posterior beliefs $\mathbb{E}[\theta \mid \Omega_i]$ such that

- (i) the demand schedules solve investors' problem (3) given their posterior beliefs;
- (ii) the price function $Q(\theta, \tilde{S}, b_{\text{cb}})$ clears the bond market, satisfying (6);
- (iii) posterior beliefs satisfy Bayes' law for all market clearing prices.

3 Quantity-Targeting Asset Purchases

In this section, we characterize the equilibrium in the financial market when the central bank follows the quantity-targeting rule (4).

3.1 Investors' Strategies and Price Signals

We start by working out how the demand by investors and the central bank determines the equilibrium price and the information it contains.

Monotone threshold strategies. The posterior beliefs of investor i on the likelihood of a high payoff are given by $p_i := \text{Prob}(\theta = \theta_H | \Omega_i)$, and are weakly increasing in the private signal x_i in the sense of first-order stochastic dominance. In particular, given that x_i is the only piece of information that is heterogeneous across investors, it is true that

$$x_i \geq x_j \Leftrightarrow p_i \geq p_j \Leftrightarrow \mathbb{E}[\theta | \Omega_i] \geq \mathbb{E}[\theta | \Omega_j] \quad \forall i, j,$$

where $\mathbb{E}[\theta | \Omega_i] = (\theta_H - \theta_L)p_i + \theta_L$. This implies that each investor's demand follows a monotone threshold strategy of the form

$$b_i(x_m) = \begin{cases} 1 & \text{if } x_i \geq x_m \\ 0 & \text{if } x_i < x_m, \end{cases} \quad (7)$$

where $b_i(x_m)$ is the portfolio choice of agent i when the private signal threshold is x_m , which is endogenous to the equilibrium. By construction, x_m coincides with the private signal received by the marginal agent $i = m$, who is indifferent between buying bonds or not.¹⁰

Price signal mapping. We assume that a law of large numbers across investors applies as in Judd (1985): for a given value of the fundamental, the mass of investors buying bonds is given by the share of agents with a private signal larger than x_m , that is

$$\int_0^1 b_i \, di = \text{Prob}(x_i \geq x_m | \theta) = \Phi\left(\frac{\theta - x_m}{\sigma_x}\right),$$

where Φ denotes the standard normal cumulative distribution function. It follows that, conditional on x_m , aggregate bond demand is a function of θ , since investors' beliefs are centered around it. We can thus rewrite the bond market clearing condition (6) as

$$\Phi\left(\frac{\theta - x_m}{\sigma_x}\right) = \tilde{S} - b_{cb}. \quad (8)$$

¹⁰Note that investors' optimal strategy (7) effectively amounts to the submission of a limit order, since there exists a one-to-one mapping between the private signal threshold x_m and the equilibrium price Q .

The left-hand side represents the mass of investors who buy bonds. The right-hand side represents net public bond supply, i.e., gross government supply net of central bank purchases. For brevity, we define $S := \tilde{S} - b_{cb}$ and refer to it as *net supply*. For the market to clear given (θ, S) , the identity (or private signal) x_m of the marginal agent must adjust, which suggests that x_m and Q are related. Henceforth, we focus on equilibria where Q and x_m convey the same information.¹¹ Rearranging terms, we can express x_m as

$$x_m(\theta, S) = \theta + \sigma_x \Phi^{-1}(1 - S). \quad (9)$$

Equation (9) states that the marginal agent's private signal x_m must be equal, in equilibrium, to a function that is linear in the fundamental shock θ , and decreasing and nonlinear in net supply S . In other words, x_m is an endogenous public signal of θ that aggregates private information imperfectly, because of the noise introduced by unobservable net supply S . We thus refer to x_m as the price signal (or market signal).

The *crowding-out* and *revelation* effects of APs. Figure 1 illustrates the mapping from fundamental and gross supply shocks (θ, \tilde{S}) and AP policy b to the equilibrium marginal investor (or price) signal x_m , as per equation (9). The figure plots the realization of x_m on the y-axis, as a function of the net supply shock S on the x-axis, for the two values of θ .¹²

The left panel illustrates the case with no APs: in this case, gross and net supply coincide ($\tilde{S} = S$). When $S \rightarrow 1$, the entire population of investors is needed to clear the market, so the most pessimistic investor, i.e., the one with the lowest private signal ($x_i \rightarrow -\infty$), is marginal. When instead $S \rightarrow 0$, there is an infinitesimal amount of supply, and only the most optimistic investor ($x_i \rightarrow +\infty$) buys bonds and is marginal. For any realization of net supply away from these limits, investors face an inference problem, since any observed price signal can be the result of either high demand (due to a high payoff) and high supply, or low demand and low supply.

The right panel shows that positive APs ($b_{cb} = b > 0$) have two main effects. First, there is a *crowding-out* effect: for a given realization of the shocks (θ, \tilde{S}) , APs reduce net supply so that a relatively more optimistic investor is marginal, i.e., a smaller mass of buyers is needed to clear the market. In particular, some investors on the left tail of the private signal distribution

¹¹This is equivalent to focusing on equilibria with continuous price functions. Pálvölgyi and Venter (2015) show that another class of equilibria with discontinuous and non-monotone price functions exists in noisy rational expectations economies.

¹²We use a common parametrization for all the figures in the text. We use a value of $b = 0.1$ for the case with APs, which is purely chosen for illustrative purposes. We set the other parameters to $q = 0.53, \theta_H = 2.5, \theta_L = -2.5, \sigma_x = 7$, which are chosen to match some features of the data, as we explain in detail in Section 3.5.

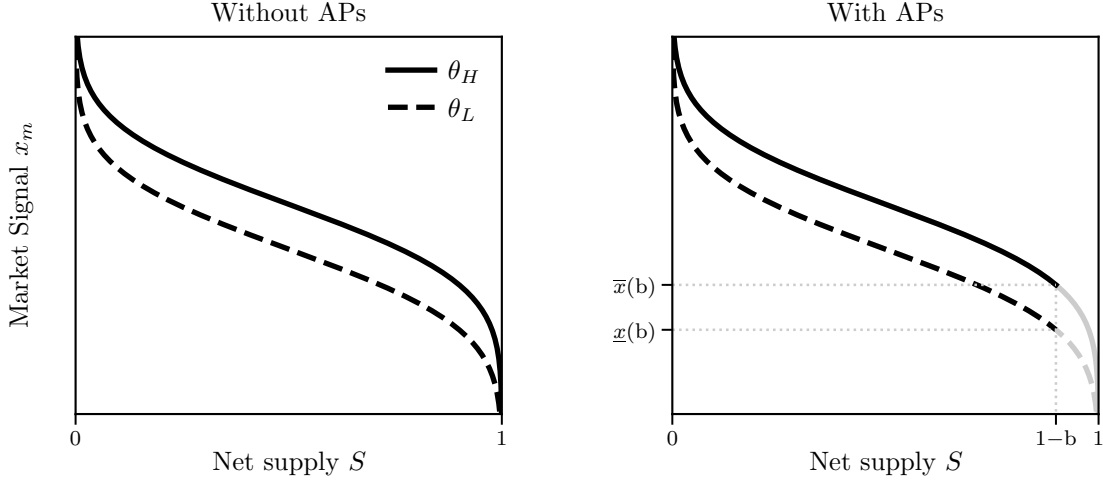


Figure 1: The price signal as a function of net supply S and fundamental θ .

will always be crowded out by APs and never be marginal, since the maximal net supply, $1 - b$, becomes smaller than one. It follows that the support of the price signal distribution is truncated from below and becomes $[\underline{x}(b), +\infty)$, where $\underline{x}(b) := x_m(\theta_L, 1 - b)$.

Second, there is a *revelation* effect specific to low payoff states: the truncation of the support of S induced by APs gives rise to an interval of price signals $[\underline{x}(b), \bar{x}(b))$, with $\bar{x}(b) := x_m(\theta_H, 1 - b)$, which only occur when $\theta = \theta_L$. Perfect revelation is an artifact of our simple assumption of a binary payoff, but the mechanism is more general. When the central bank is intervening, investors understand that a smaller mass of buyers is needed to clear the market, so the market signal will, in general, be higher. Observing a low market signal in this setting then implies a larger likelihood of low payoffs than in the absence of APs.

Price signal distribution. We denote by \mathcal{M}_b the distribution of the price signal x_m conditional on $\tilde{S} \geq b$, i.e., when the equilibrium bond price is determined by the market. The p.d.f. is given by $\frac{1}{1-b} f_{\mathcal{M}_b}(x_m)$, where

$$f_{\mathcal{M}_b}(x_m) = \begin{cases} \frac{1}{\sigma_x} \left[(1-q) \phi\left(\frac{x_m - \theta_L}{\sigma_x}\right) + q \phi\left(\frac{x_m - \theta_H}{\sigma_x}\right) \right] & \text{for } x_m \in [\bar{x}(b), +\infty) \\ \frac{1}{\sigma_x} (1-q) \phi\left(\frac{x_m - \theta_L}{\sigma_x}\right) & \text{for } x_m \in [\underline{x}(b), \bar{x}(b)), \end{cases} \quad (10)$$

and ϕ denotes the standard normal probability density function. Note that the market signal density is normalized by the probability $P(\tilde{S} \geq b) = 1 - b$ that the market is “active”. The remaining set of states has probability $P(\tilde{S} < b) = b$, and is such that the central bank buys all

the supply, and investors do not trade bonds nor learn from the price.

3.2 From Price Signal to Investors' Beliefs

Having characterized how investors learn from prices, we now derive their posterior beliefs and how the equilibrium price is determined. Before digging into it, it is useful to consider, as an intermediate step, the posterior beliefs of an agent who only observes public information, i.e., the price signal, and receives no private information. We will refer to these as the “public” posterior beliefs.

Public posterior beliefs. The posterior probability of $\theta = \theta_H$ conditional only on the public price signal x_m is given by standard Bayesian updating

$$\hat{p}(x_m) := P(\theta_H | x_m \sim \mathcal{M}_b) = \begin{cases} \frac{q \phi\left(\frac{\theta_H - x_m}{\sigma_x}\right)}{q \phi\left(\frac{\theta_H - x_m}{\sigma_x}\right) + (1-q) \phi\left(\frac{\theta_L - x_m}{\sigma_x}\right)} & \text{if } x_m \in [\bar{x}(b), +\infty), \\ 0 & \text{if } x_m \in [\underline{x}(b), \bar{x}(b)), \end{cases} \quad (11)$$

and reflects the fact that values of x_m in the fully revealing region are only consistent with θ_L .¹³ Importantly, these posterior beliefs satisfy the law of iterated expectations, i.e., $\mathbb{E}[\hat{p}(x_m)] = q$ since they are conditional on public information only.¹⁴ It is worth noting that APs do not directly affect the posterior repayment probability conditional on x_m , but they do determine the fully- and non-revealing partitions of the support of the price signal, as well as its density function.

Investors' posterior beliefs. We can now characterize the posterior beliefs of investor i conditional on their information set $\Omega_i = \{x_i \sim \mathcal{N}, x_m \sim \mathcal{M}_b\}$, that is, on both their private

¹³To lighten notation, we henceforth omit b as an argument when defining objects that are functions of the price signal x_m , since dependence on b is implicit in its effect on the distribution of x_m .

¹⁴See Proposition 2 below for a different version of this statement, and Appendix A.2 for the proof.

signal and the public price signal.¹⁵ Standard Bayesian updating yields

$$p(x_i, x_m) := P(\theta_H | x_i \sim \mathcal{N}, x_m \sim \mathcal{M}_b) = \begin{cases} \frac{q \phi\left(\frac{\theta_H - \frac{x_i + x_m}{2}}{\sigma_x / \sqrt{2}}\right)}{q \phi\left(\frac{\theta_H - \frac{x_i + x_m}{2}}{\sigma_x / \sqrt{2}}\right) + (1-q) \phi\left(\frac{\theta_L - \frac{x_i + x_m}{2}}{\sigma_x / \sqrt{2}}\right)} & \text{if } x_m \in [\bar{x}(b), +\infty), \\ 0 & \text{if } x_m \in [\underline{x}(b), \bar{x}(b)). \end{cases} \quad (12)$$

Note that the investor posterior p is identical to the public posterior \hat{p} —and equal to zero—in the fully revealing region, and is different otherwise. The difference derives from the fact that investors have two sources of information, while the public only has one, so the scale parameter of the former is smaller than that of the latter by a factor of $1/\sqrt{2}$.¹⁶

3.3 Market Prices and Fundamental Valuations

To characterize the equilibrium bond price, we consider the marginal investor whose private signal is at the threshold ($x_i = x_m$), and is thus indifferent with respect to her bond position. First, we define the posterior repayment probability of the marginal agent as

$$p(x_m) := p(x_i, x_m)|_{x_i=x_m}. \quad (13)$$

It is easy to see from (11) and (12) that $p(x_m) \neq \hat{p}(x_m)$, even though both depend solely on x_m .

From the marginal investor's indifference condition $Q = \mathbb{E}[\theta | x_i \sim \mathcal{N}, x_m \sim \mathcal{M}_b]|_{x_i=x_m}$ we derive the equilibrium bond price

$$Q(p(x_m)) = p(x_m) \theta_H + (1 - p(x_m)) \theta_L, \quad (14)$$

which is a simple linear function of the marginal investor's posterior beliefs. This concludes our derivation of how the state (θ, \tilde{S}) maps into the marginal investor's signal via market clearing (9), and how the latter maps into the equilibrium bond price via the marginal investor's indifference condition (14).

¹⁵Henceforth, we always characterize conditioning sets by indicating from which distribution— \mathcal{N} or \mathcal{M}_b —each realization is drawn. This is useful to avoid confusion and remind ourselves through which channels APs affect posterior beliefs and learning from prices.

¹⁶The fact that both the private and price signals are given the same Bayesian weight follows from the assumption that gross supply is uniformly distributed, which is made to preserve the normality of posterior beliefs in the presence of APs.

Using (14), we construct an alternative object, which we define as *fundamental valuation*, and is given by the expected bond payoff once we plug in the public posterior beliefs we defined in (11):

$$Q(\widehat{p}(x_m)) := \mathbb{E}[\theta | x_m \sim \mathcal{M}_b] = \widehat{p}(x_m) \theta_H + (1 - \widehat{p}(x_m)) \theta_L. \quad (15)$$

That is, $Q(\widehat{p}(x_m))$ is the expected value of the fundamental θ , conditional on x_m being drawn from \mathcal{M}_b . The following proposition characterizes the difference between equilibrium bond price and fundamental valuation.

Proposition 1 (Single-crossing). *For any $x_m \in [\bar{x}(b), \infty)$, we have that*

$$Q(p(x_m)) > Q(\widehat{p}(x_m)) \quad \text{if and only if} \quad x_m > x^\dagger := \frac{\theta_H + \theta_L}{2}.$$

For any $x_m \in [\underline{x}(b), \bar{x}(b))$, we have $Q(p(x_m)) = Q(\widehat{p}(x_m)) = \theta_L$.

Proof. Postponed to Appendix A.1. ■

The proposition establishes that there exists a single crossing between market price and fundamental valuation occurring at x^\dagger , the unique value of the price signal that makes posterior equal to prior probabilities, $p(x^\dagger) = \widehat{p}(x^\dagger) = q$.

In Figure 2, we plot the market price $Q(p(x_m))$ in solid lines, and the fundamental valuation $Q(\widehat{p}(x_m))$ in dotted lines, as functions of the price signal x_m , for the same two scenarios considered in Figure 1. The figure shows that, when $x_m > x^\dagger$, the market “overprices” bonds because the equilibrium price is larger than the valuation, and “underprices” them otherwise.¹⁷ The effect of quantity-targeting APs can be seen by comparing the baseline case of no central bank intervention ($b = 0$) in the left panel, to the case with intervention ($b > 0$) on the right panel. First, by the *crowding-out* effect, APs truncate the left tail of the support of market prices (i.e., $\underline{x}(b)$ increases from $-\infty$ to a finite number), excluding from the market the most pessimistic investors—those with a private signal $x_i \in (-\infty, \underline{x}(b))$ —for any realization of the fundamental and supply shocks. Second, by the *revelation* effect, APs imply that some price signal realizations in the left tail of the support of x_m —the interval $[\underline{x}(b), \bar{x}(b))$ —become fully revealing of the low payoff state θ_L . These channels affect both market prices and fundamental valuations in the same way.

¹⁷We use the fundamental valuation as the benchmark against which we assess “mispricing”, because it satisfies the law of iterated expectations and thus represents the actuarially fair value of the asset. Proposition 2 a few pages below will formalize this statement.

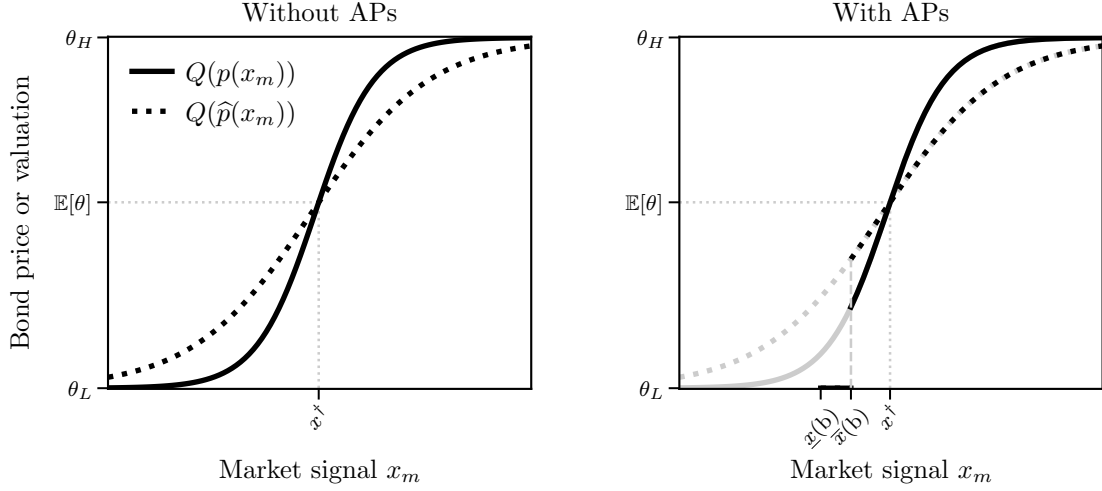


Figure 2: The equilibrium bond price $Q(p(x_m))$ and fundamental valuation $Q(\hat{p}(x_m))$ as a function of the price signal x_m .

A learning-from-prices externality. The difference between the fundamental valuation and the market price can be understood as the effect of learning from prices. This has been characterized in the form of a wedge by Albagli et al. (2023, 2024) for a general class of noisy rational expectations equilibria in financial markets with learning from prices. As they show, the wedge is also present in models where agents are risk-averse or have no position bounds.

The wedge could be understood as the effect of a “pecuniary” externality, in that atomistic investors take as given the stochastic properties of the equilibrium price from which they learn, while these properties actually depend on investors’ aggregate behavior. To gain a better understanding of the driver of this wedge, consider an increase in θ or a decrease in \tilde{S} . By the market clearing condition (9), x_m must increase, and so does the equilibrium price.¹⁸ Since investors learn from prices, a change in the price signal implies that agents update upwards their posterior beliefs about θ_H . This in turn increases bond demand for any price, and results in a further increase in the equilibrium price, in a loop of general equilibrium amplification that atomistic investors do not internalize.

To eliminate this externality and make the equilibrium price coincide with the fundamental valuation, i.e., the risk neutral expected payoff, investors would need to coordinate in weighting the private and the price signal according to half their equilibrium Bayesian weight (thus getting rid of the $\sqrt{2}$ term in the expression for posterior beliefs (12)).

¹⁸Depending on the shock, x_m increases for different reasons: either because lower supply selects a different, more optimistic agent, or because the same marginal agent receives a more optimistic signal when the fundamental improves.

Whereas Albagli et al. (2023, 2024) study the implication of the wedge for the pricing of assets with asymmetric payoffs, we consider the impact of APs on asset prices through their effect on the distribution of the wedge. A key difference is that our results do not rely on the asymmetry of payoffs.

3.4 Average Prices and Valuations

Let us turn attention to the average bond price and fundamental valuation, which are given by the following unconditional expectations:

$$\begin{aligned}\mathcal{Q}_{\text{qt}}(b) &:= \frac{1}{1-b} \int_{\underline{x}(b)}^{+\infty} Q(p(x_m)) f_{\mathcal{M}_b}(x_m) dx_m \\ \widehat{\mathcal{Q}}_{\text{qt}}(b) &:= \frac{1}{1-b} \int_{\underline{x}(b)}^{+\infty} Q(\widehat{p}(x_m)) f_{\mathcal{M}_b}(x_m) dx_m.\end{aligned}$$

The contrasting effect of APs on these unconditional averages can be grasped intuitively by looking back at Figure 2 and its discussion: APs increase bond prices and valuations because of the crowding out of pessimistic investors, and decrease them through the creation of a revealing region. The following proposition characterizes the overall reaction of the average price and valuation to the size of APs.

Proposition 2 (Average price and valuation). *The average valuation is equal to the prior mean of the fundamental and is independent of APs, that is,*

$$\widehat{\mathcal{Q}}_{\text{qt}}(b) = \mathbb{E}[\theta] \quad \text{for all } b \in [0, 1]. \quad (16)$$

The average price is a function of APs and is given by

$$\mathcal{Q}_{\text{qt}}(b) = \mathbb{E}[\theta] + \Delta(b) \quad \text{for all } b \in [0, 1], \quad (17)$$

where the average wedge

$$\Delta(b) := \frac{\theta_H - \theta_L}{1-b} \int_{\underline{x}(b)}^{+\infty} (p(x_m) - \widehat{p}(x_m)) f_{\mathcal{M}_b}(x_m) dx_m, \quad (18)$$

has the following properties: it is a single-peaked function of the AP size b , it is positive for b large enough, $\Delta(0) \geq 0$ if and only if $q \leq 1/2$, and $\lim_{b \rightarrow 1} \Delta(b) = 0$.

Proof. See Appendix [A.2](#). ■

The first important result of the proposition is that APs have no effect on the *fundamental valuation*, because the crowding-out and revelation effects exactly offset each other. It follows that the unconditional expected valuation equals the average payoff, regardless of central bank intervention, as a consequence of the law of iterated expectations.

The second and main result of the proposition is that the average price $\mathcal{Q}_{qt}(b)$ differs from the fundamental valuation, and is an inverse U-shaped function of APs. Equation (17) provides an intuitive representation for the difference between the average price and the average payoff $\mathbb{E}[\theta]$, which must come from the average wedge, denoted by $\Delta(b)$. The existence of this wedge and the comparison between $\hat{\mathcal{Q}}_{qt}$ and \mathcal{Q}_{qt} clearly show that APs are non-neutral, with respect to the average price, *because of* the learning-from-prices externality. The proposition states two key properties of the average wedge. First, its value in the absence of APs has the same sign as the skewness of the payoff distribution.¹⁹ Second, the average wedge is a single-peaked function of the size of the AP policy. In other words, the marginal effect of APs on the average bond price is positive provided that the size of the intervention is sufficiently small, and negative otherwise. This result is illustrated by the left panel of Figure 3, which plots with a solid line the average wedge $\Delta(b)$ as a function of the size of the AP program. The intuition behind this result can be understood by referring back to Figure 2, where the conditional value of the wedge is the difference $Q(p(x_m)) - Q(\hat{p}(x_m))$. The right panel shows that small interventions tend to crowd out relatively more pessimistic investors, removing states where bonds are underpriced by the market. In contrast, sufficiently large interventions crowd out less pessimistic investors, eliminating states where bonds are overpriced. As a result, APs raise the average wedge (i.e., the bond price) in the former case, and lower it in the latter.

3.5 Magnitudes and Interpretation

The analytical proposition above outlines our main result on the impact of APs on asset prices, but does not assess whether the implied price elasticity is empirically significant. To provide a first pass on this question, we set our stylized model to target key aspects of the U.S. Federal Reserve’s Large-Scale Asset Purchases announcement on March 18, 2009 (the first including Treasuries, LSAP1 henceforth). We show that the model-predicted magnitudes are empirically plausible—even though the framework is not designed for quantitative accuracy. This exercise

¹⁹This is consistent with the main result of Albagli et al. (2024).

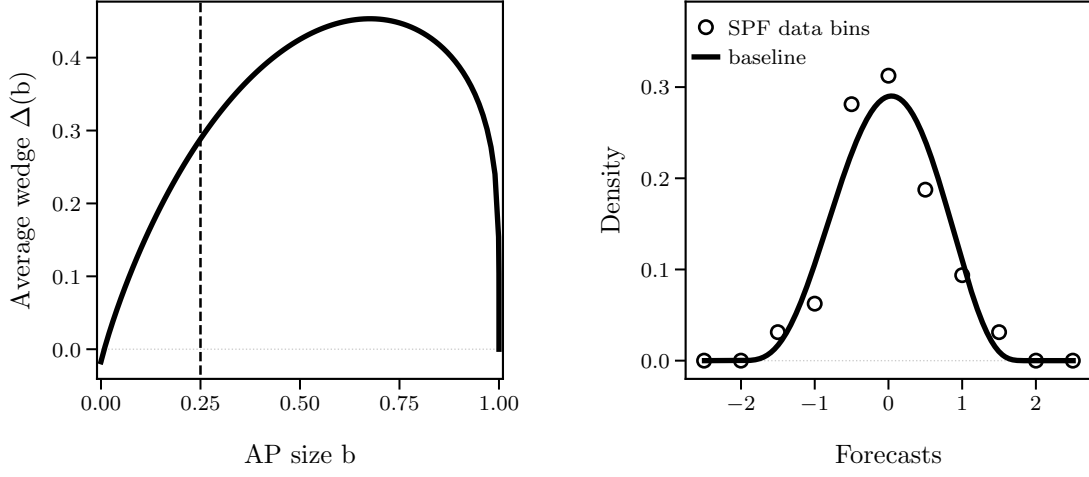


Figure 3: The left panel plots the average wedge as a function of the AP size; the right panel plots the distribution of investors’ forecasts in the SPF for Q1 2009 (circles) and in our model (solid line). The parameter values are: $\theta_H = 2.5, \theta_L = -2.5, q = 0.53, \sigma_x = 7.5$.

also provides a concrete interpretation of our model primitives in terms of data.

We calibrate model parameters to match the Q1-2009 distribution of inflation forecasts from the Survey of Professional Forecasters (SPF) for average 10-year inflation. This maps into disagreement about 10-year real Treasury yields, as the nominal yield is publicly observed. The binned empirical distribution of SPF forecasts is represented by circles in the right panel of Figure 3. It captures a structurally slow-moving dimension of forecast disagreement.²⁰

In terms of model counterparts, we interpret θ as the realized real gross payoff on 10-year U.S. Treasury bonds, discounted by the risk-free rate r , and Q as their price. Thus, $\mathbb{E}[\theta - Q(p(x_m)) \mid x_i]$ represents the negative of the expected yield premium conditional on observing x_i , averaged across all possible x_m states.²¹ We therefore interpret the analog of the SPF distribution of average beliefs as the distribution of

$$\mathbb{E}[\theta - Q(p(x_m)) \mid x_i]_{|b=0} = (\theta_H - \theta_L) \int (p(x_i, x_m) - p(x_m)) f_{\mathcal{M}_0|\mathcal{N}}(x_m \mid x_i) dx_m, \quad (19)$$

across all possible x_i , where $f_{\mathcal{M}_0|\mathcal{N}}$ is the conditional distribution of price signals given private signals, under no APs (see Appendix B.4). This model-implied distribution is shown as the solid

²⁰In Q1-2009, the distribution had a standard deviation of 0.6%, the same as in Q1-2010, both lying one standard deviation (0.15) above the historical mean for the period (0.44) 1992-2024.

²¹Market clearing implies that this premium is expected to be zero for the marginal agent: $Q(p(x_m)) = \mathbb{E}[\theta \mid x_i, x_m]_{|x_i=x_m}$. Further details are collected in Appendix C.

line in the right panel of Figure 3.

According to an ex-ante perspective, we look at the average wedge $\Delta(b) = -(\mathbb{E}[\theta] - \mathcal{Q}_{qt}(b))$ as capturing the *realized* yield premium averaged over all possible aggregate states x_m , and look at its change before ($b = 0$) and after ($b > 0$) an AP intervention. In practice, \mathcal{Q}_{qt} represents the price investors would agree upon ex ante—before aggregate and idiosyncratic shocks are realized. This is consistent with our assumption that APs convey no new information about fundamentals, or equivalently, that any such information is commonly known.

The left panel of Figure 3 shows how the average price \mathcal{Q}_{qt} —specifically, its wedge component—responds to interventions of size b . The model predicts that intermediate-scale APs can lower yields by up to 47 basis points. The dashed vertical line marks the LSAP1 intervention size as a fraction of outstanding Treasury bonds.²² For this intervention size, the model predicts an increase in \mathcal{Q}_{qt} , that is, a drop in yields of approximately 31 basis points ($\Delta(0.25) - \Delta(0) \approx 31$), about half of the 59 basis point decline in real yields observed around the LSAP1 announcement, as documented by for example by Krishnamurthy and Vissing-Jorgensen (2011).

3.6 Asset Purchases and the Distribution of Gains

The existence of a wedge between prices and fundamentals due to learning from prices has implications for how APs affect the distribution of gains and losses between the government, the central bank, and private investors.

The expected gains of an investor conditional on price signal x_m are given by

$$\mathbb{E}[\pi_{inv} | x_m] = \sum_{\theta \in \{\theta_H, \theta_L\}} \Phi\left(\frac{\theta - x_m}{\sigma_x}\right) (\theta - Q(p(x_m))) P(\theta | x_m \sim \mathcal{M}_b), \quad (20)$$

where $\theta - Q(p(x_m))$ represents the unitary gain or loss, and $\Phi((\theta - x_m)/\sigma_x)$ represents the probability of being a buyer (i.e., of receiving a private signal above the price signal).²³ Note that we consider a representative investor because all investors are identical *ex ante*, i.e., before receiving private information.

²²This is calculated as the announced “\$300 billion of long-term Treasury securities” over the total amount of marketable, non-indexed bonds and notes with over 5 years of residual maturity as of February 2009. Further details are collected in Appendix C.

²³By the law of large numbers, $\Phi((\theta - x_m)/\sigma_x)$ also represents the mass of investors buying the bond conditional on θ and x_m .

Central bank expected gains conditional on x_m are

$$\begin{aligned}\mathbb{E}[\pi_{cb} | x_m] &= b \sum_{\theta \in \{\theta_H, \theta_L\}} \left(\theta - Q(p(x_m)) \right) P(\theta | x_m \sim \mathcal{M}_b) \\ &= b \left[Q(\hat{p}(x_m)) - Q(p(x_m)) \right],\end{aligned}\tag{21}$$

Where the second line follows directly from the definition of the fundamental valuation in (15).

The expected gains of the government obtain residually, because of the zero-sum nature of the financial market: $\mathbb{E}[\pi_{gov} | x_m] + \mathbb{E}[\pi_{inv} | x_m] + \mathbb{E}[\pi_{cb} | x_m] = 0$, for any state $x_m \in [\underline{x}(b), +\infty)$.²⁴²⁵ The following proposition outlines some properties of these conditional gains.

Proposition 3 (Conditional gains and losses). *The expected gains of investors, central bank, and government conditional on the market signal x_m and the AP quantity b have the following properties:*

- $\mathbb{E}[\pi_{inv} | x_m] > 0$;
- for $b \in (0, 1)$, $\mathbb{E}[\pi_{cb} | x_m] < 0$ if and only if $x_m > x^\dagger$;
- $\mathbb{E}[\pi_{gov} | x_m] > 0$, if and only if $x_m > x_g(b)$ where $x_g(b) > x^\dagger$ and $x'_g(b) < 0$;

for all $x_m \in [\underline{x}(b), +\infty)$ and $b \in [0, 1)$.

Proof. Postponed to Appendix A.3. ■

The three panels in Figure 4 plot these conditional gains as a function of the market signal x_m for our baseline calibration. Gray and black solid lines respectively denote the case without and with APs. Thin dashed lines refer to the no-wedge case, where transactions occur at the fundamental valuation rather than the market price, which is a useful benchmark to illustrate how results depend on the learning-from-prices externality.

Consider first the case without APs. The left panel shows that investors' expected gains are positive in all x_m states, regardless of the presence of a wedge between prices and fundamentals. Investors buy bonds only when they expect a payoff higher than their outside option of staying

²⁴This interval of price signals corresponds to states where $\tilde{S} \geq b$. When instead $\tilde{S} < b$ and the market is not active, we have that $\mathbb{E}[\pi_{inv}] = 0$ and $E[\pi_{gov}] = -\mathbb{E}[\pi_{cb}] = (\theta - Q_{pas})b/2 = 0$, since we assume $Q_{pas} = \theta$.

²⁵The concept of trading gain or loss may be more natural for investors and the central bank than for the government. When we refer to the latter, losses can be thought of as the “excess” cost of debt issuance on top of its fundamental value.

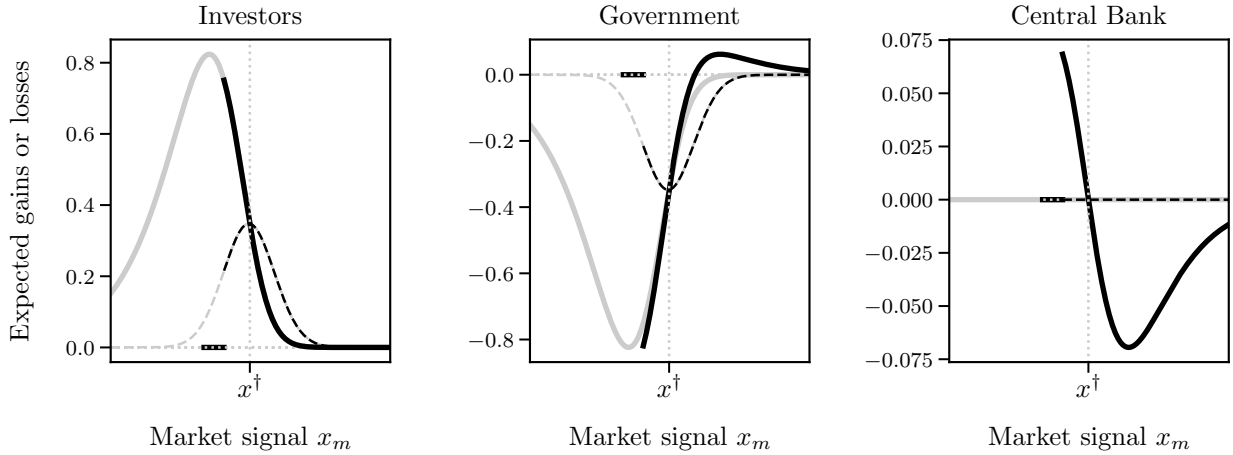


Figure 4: Expected gains conditional on x_m . Gray and black solid lines respectively denote the case without APs ($b = 0$) and with APs ($b = 0.1$).

put (which delivers zero gains with certainty). This is effectively a call option whose payoff is positively correlated with the fundamental: conditional on x_m , each investor is more likely to buy when fundamentals are good, and supply is in turn high. These two factors together imply that investors' gains are strictly positive even in the absence of the wedge due to learning, as shown by the dashed line in the left panel. In particular, positive gains are the result of limits to individual arbitrage that are due to the presence of position bounds and dispersed beliefs. The learning-from-prices externality introduces an additional force that generates skewness in the gains distribution: bonds are underpriced when $x_m < x^\dagger$, which implies that investors' gains are larger than in the no-wedge case, and vice versa when $x_m > x^\dagger$.

Consider now the case with APs, denoted by black solid lines. In contrast to investors, the central bank buys unconditionally, making losses when $x_m > x^\dagger$ and gains otherwise, as the right panel shows. The central panel shows that central bank losses result in net gains for the government. In this sense, APs have the potential to engender implicit monetary financing, i.e., a transfer of resources from the central bank to the government. This result emerges because of the existence of the wedge due to the learning from price externality. The effect of central bank APs on investors' and central bank gains is negative: the crowding-out effect truncates the left tail of the gains distribution, and the revelation effect makes gains drop to zero in the adjacent, fully revealing region. In the counterfactual where the externality is muted—which is plotted as a dashed line in all the three panels—APs are balance-sheet neutral, generating neither gains nor losses for the central bank, while government losses mirror investors' gains. We state the effect of APs on *average*—or unconditional—gains in the following proposition.

Proposition 4 (Average gains and losses). *Average gains have the following properties:*

- *Average investor gains are positive and decreasing in APs;*
- *Average central bank gains are negative for any AP quantity if $q \leq 1/2$, and for b large enough if $q > 1/2$.*
- *Average government gains are positive if APs are large enough, i.e., if b is such that $\bar{x}(b) > x_g(b)$.*
- *The average gains and losses of all players converge to zero as $b \rightarrow 1$.*

Proof. Postponed to Appendix A.4. ■

The proposition is illustrated in Figure 5: we plot the average gains of investors (dash-dotted line), central bank (dotted line), and government (dashed line), as a function of the quantity target of the AP policy. As in the previous figure, thin shaded lines denote these quantities in the absence of the wedge due to learning from prices.

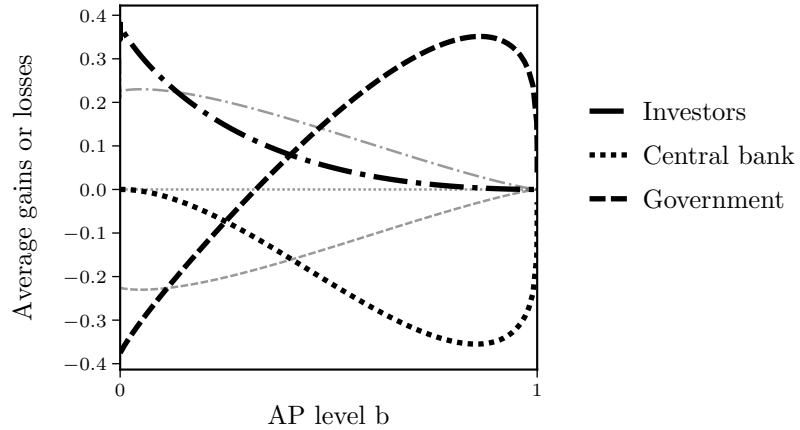


Figure 5: Average gains by investors, the central bank, and the government, as a function of the size of the AP program b . Thin shaded lines represent average gains for each player in the absence of the wedge between bond prices and valuations.

As APs increase in size, investors' gains fall and central bank losses rise, all to the benefit of the government. In particular, as the proposition states, sufficiently large APs *imply* monetary financing for the government. Net gains for the government emerge because a transfer of resources occurs: i) directly from the central bank that buys government debt at an average loss, ii) indirectly from investors, by reducing the scope for buying underpriced assets. As b approaches

one, all gains and losses converge to zero, because in most states the central bank is buying the whole market at the perfect information price, and there are no net transfers to the government.

Again, these results are tied to the existence of the wedge, but are generic to the degree of payoff skewness q . In the no-wedge case, denoted by thin shaded lines, the intervention is balance-sheet neutral for the central bank, while investors' gains are still decreasing in APs.

4 Price-Targeting Asset Purchases

In this section, we explore the workings of price-targeting AP policies according to (5), where the central bank submits a limit order to buy bonds up to a quantity \bar{b}_n if the price is below a target Q_n . Most of our derivations for the case of quantity-targeting policies carry on to this section, so we organize the presentation that follows around the key differences.

4.1 State Space Partitions

We have seen above that a quantity-targeting policy can be understood as partitioning the state space of shocks (θ, \tilde{S}) into three distinct regions: one of full revelation, one where the market operates under uncertainty, and one where the market remains passive. Price-targeting policies perform a similar partition of the state space into three regions: one of full revelation, one where the market operates under uncertainty, and one where the market price equals the central bank target. In both cases, the last region is one where the equilibrium price formation mechanism is disrupted. With quantity-targeting this disruption arises as the central bank buys all the supply. On the other hand, with price targeting, the equilibrium price is artificially held at its target value as a result of the central bank intervention. We proceed by characterizing such regions.

To start, we recover from equation (14) the mapping from the equilibrium price Q to the signal of the marginal investor

$$x(Q, \sigma) = x^\dagger + \frac{\sigma^2}{\theta_H - \theta_L} \log \left(\frac{1-q}{q} \frac{Q - \theta_L}{\theta_H - Q} \right) \quad (22)$$

where $\sigma \in \{\sigma_x, \sigma_x/\sqrt{2}\}$ depending on whether the relevant beliefs are those of the investors or of the public. We define two values of interest:

- $\tilde{x}_n := x(Q_n, \sigma_x/\sqrt{2})$, which defines the price signal at which the equilibrium bond price is equal to the target in the absence of APs, i.e., $Q_n = \mathbb{E}[\theta \mid \tilde{x}_n \sim \mathcal{N}, \tilde{x}_n \sim \mathcal{M}_0]$;

- $x_n := x(Q_n, \sigma_x)$, which defines the price signal at which the *fundamental valuation* is equal to the price target in the absence of APs, i.e., $Q_n = \mathbb{E}[\theta | x_n \sim \mathcal{M}_0]$.

We will focus on price targets such that $Q_n \geq \mathbb{E}[\theta]$, which implies that $x_n \geq \tilde{x}_n \geq x^\dagger$.²⁶ The pair (\tilde{x}_n, x_n) fully characterizes the regions in which price-targeting APs partition the state space.

No-intervention region. This region is characterized by all combinations of fundamental and supply shocks that deliver a price signal $x_m \geq \tilde{x}_n$ in the absence of APs, which implies an equilibrium price larger than the target ($Q \geq Q_n$), and indeed no central bank intervention ($b_{cb} = 0$) as a result. It occurs with probability

$$P(Q \geq Q_n) = q \Phi \left(\frac{\theta_H - \tilde{x}_n}{\sigma_x} \right) + (1 - q) \Phi \left(\frac{\theta_L - \tilde{x}_n}{\sigma_x} \right).$$

In these states, the equilibrium bond price is determined by the market in the way described in the previous section, i.e., according to (14), with the posterior beliefs in (13), such that investors learn from the price and the scale parameter in their beliefs is $\sigma_x/\sqrt{2}$.

Price target region. This region is characterized by the equilibrium price being equal to the policy target ($Q = Q_n$) and an AP quantity $b_{cb} \in [\underline{b}_n, \bar{b}_n]$, where $(\underline{b}_n, \bar{b}_n)$ are bounds that we derive below. It occurs with probability

$$P(Q = Q_n) = 1 - \Phi \left(\frac{\theta_H - \tilde{x}_n}{\sigma_x} \right). \quad (23)$$

In this region, investors learn from both the equilibrium price Q_n and the AP program size b_{cb} —which is endogenous to the equilibrium—that is needed to keep the price at the target. The following proposition establishes this mapping.

Proposition 5 (Price-targeting APs). *Let*

$$b_n(\theta, \tilde{S}) = \tilde{S} - \Phi \left(\frac{\theta - x_n}{\sigma_x} \right) \quad (24)$$

be the AP size that corresponds to price signal x_n and state (θ, \tilde{S}) . When the equilibrium price

²⁶Restricting our analysis to the cases where $Q_n \geq \mathbb{E}[\theta]$ ensures that the target can always be met without the need for central bank asset sales. While asset sales—a form of quantitative tightening—can be understood as the counterpart to reducing the scale of purchases, its analysis necessitates an explicit assumption about the central bank's initial bond holdings. For clarity and simplicity, we have chosen not to extend our discussion to this case.

is at the target Q_n , the size of central bank APs is given by

$$b_{cb} = \begin{cases} b_n(\theta, \tilde{S}) & \text{if } b_n(\theta, \tilde{S}) \in [\underline{b}_n, \bar{b}_n] \\ 0 & \text{otherwise,} \end{cases} \quad (25)$$

where $\underline{b}_n := b_n\left(\theta_H, \Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)\right)$ and $\bar{b}_n := b_n(\theta_H, 1)$. We can state the following properties:

- conditional on $Q = Q_n$, b_{cb} is distributed as a Uniform $[\underline{b}_n, \bar{b}_n]$, independently of θ ;
- for all $b_{cb} \in [\underline{b}_n, \bar{b}_n]$, the market signals $(Q = Q_n, b_{cb})$ are uninformative, i.e.
 $\mathbb{E}[\theta | Q = Q_n, b_{cb}] = \mathbb{E}[\theta]$.
- x_n is such that $Q_n = \mathbb{E}[\theta | x_n \sim \mathcal{N}]$,

Proof. Postponed to Appendix A.5. ■

The proposition establishes several properties of price-targeting APs. Let us just sketch the main intuition here. Equation (24) derives the size of the central bank intervention needed to make net supply consistent with a given x_n . It shows that, conditional on θ , the distribution of $b_n(\theta, \tilde{S})$ is uniform because it follows that of gross supply. For APs to not be fully revealing, any realization of b_n must be compatible with all values of the fundamental θ . The set of AP quantities that satisfy this property is given by the interval $[\underline{b}_n, \bar{b}_n]$. It follows that the likelihood of observing any $b_{cb} \in [\underline{b}_n, \bar{b}_n]$ is constant and independent of the fundamental θ . Standard Bayesian updating then implies that

$$\mathbb{E}[\theta | x_i \sim \mathcal{N}, Q = Q_n, b_{cb}] = \mathbb{E}[\theta | x_i \sim \mathcal{N}] \quad \text{for all } b_{cb} \in [\underline{b}_n, \bar{b}_n],$$

that is, the expected bond payoff for investor i , conditional on observing the public signals $Q = Q_n$ and any b_{cb} inside $[\underline{b}_n, \bar{b}_n]$, is uninformative about θ . The indifference condition $Q_n = \mathbb{E}[\theta | x_n \sim \mathcal{N}]$ finally pins down the marginal investor's (or price) signal $x_n = x(Q_n, \sigma_x)$ that is consistent with the equilibrium price being equal to target *and* investors learning nothing from public information. The intuition behind this uninformativeness result is that, when the price target is achieved, the central bank becomes the marginal investor in the market. Since the likelihood of its purchases is independent of θ , investors do not learn anything from observing them.

It is worth discussing the fact that the lower bound \underline{b}_n is generally greater than zero, which means a discontinuity exists at the margin when APs are implemented. To see why, consider

a situation where the equilibrium price in the absence of APs is just equal to the target, i.e., the market signal is equal to \tilde{x}_n and we are at the lower bound of the no-intervention region. Consider now a marginal increase in gross supply, which makes the equilibrium price drop below target and triggers central bank intervention. Such intervention implies a discontinuous drop in the precision of investors' beliefs, since the equilibrium outcome (the bond price and central bank purchases) becomes uninformative. This makes investors' demand shrink, which must be compensated by further purchases by the central bank. Using (24), we get the interval of APs larger than the minimal quantity we just discussed and compatible with a given value of θ , as a function of gross supply. We can show that the interval corresponding to θ_L is wider than that for θ_H , so the latter defines the range of AP quantities that achieve the price target, while its complement is fully revealing of the fundamental θ_L and consists of the region we discuss next.²⁷

Fully revealing region. This region is characterized by a price signal $x_m < \tilde{x}_n$, an equilibrium price equal to the lower bound ($Q = \theta_L$), and no central bank intervention ($b_{cb} = 0$). It occurs with probability

$$P(Q = \theta_L) := (1 - q) \left[\Phi \left(\frac{\theta_H - \tilde{x}_n}{\sigma_x} \right) - \Phi \left(\frac{\theta_L - \tilde{x}_n}{\sigma_x} \right) \right]$$

As discussed in the previous paragraph, the presence of bounds to b_{cb} generates cases where investors have common knowledge that $\theta = \theta_L$. This happens when gross supply takes values such that an intervention $b_{cb} \notin [b_n, \bar{b}_n]$ would be needed to sustain the price target. We slightly modify rule (5) and assume that, in these instances, the central bank refrains from intervening, and the market signal fully reveals the state. This assumption is innocuous, since these interventions would happen at the perfect information price $Q = \theta_L$ and the central bank would make zero gains.

An illustration. Figure 6 illustrates the equilibrium under a given price target level. The left panel plots the equilibrium price as a function of the price signal with black solid lines, the mass point where the price is equal to the target with a circle marker, and the equilibrium price and fundamental valuation in the absence of APs with thick and thin gray lines, respectively. The center and right panels respectively plot the price signal and central bank purchases as a function of gross bond supply (on the x-axis) and the value of the fundamental (solid and dashed lines

²⁷We focus on the case of a “natural” upper bound $\bar{b}_n = b_n(\theta_H, 1)$, for simplicity. One could, of course, exogenous set a lower value for it.

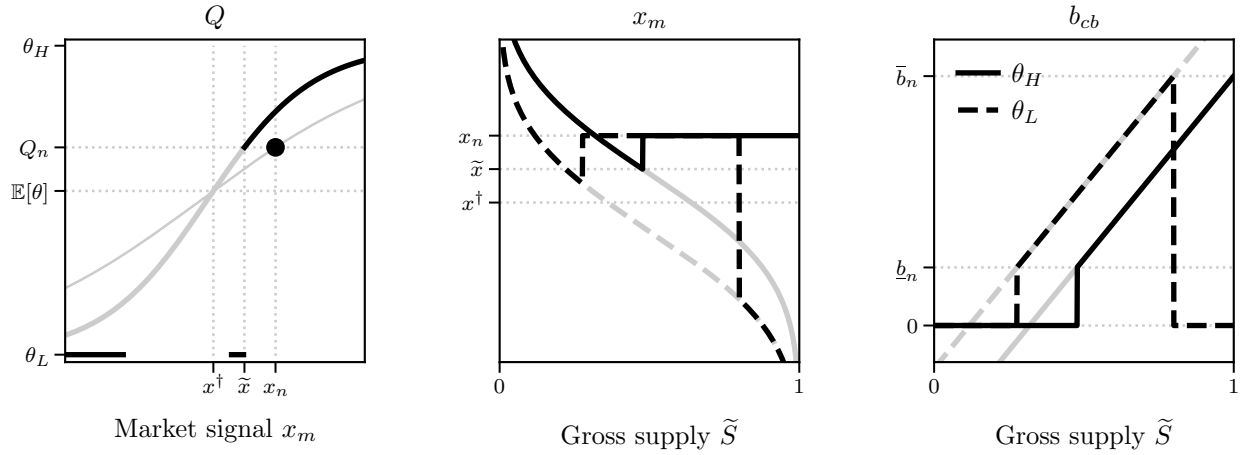


Figure 6: Illustration of the workings of price-targeting APs. The price target for this example is set at $Q_n = 1.71$, where for reference $(\mathbb{E}[\theta], \theta_H) = (1, 3.35)$.

represent θ_H and θ_L , respectively). The gray lines represent the price signal in the absence of APs in the center panel,²⁸ and $b_n(\theta, \tilde{S})$, which is the “unconstrained” version of APs as per (24), in the right panel. The no-intervention region is characterized by an equilibrium price above both the target and the fundamental valuation for all price signals above \tilde{x}_n , and APs equal to zero. When the equilibrium price equals the target, it lies on the fundamental valuation line because the public price signal is uninformative. In this region, the central bank expands APs in response to changes in gross supply to sustain the target price as a market outcome, b_{cb} is inside the $[\underline{b}_n, \bar{b}_n]$ interval, and the price signal is equal to x_n . The extent of the full revelation region is best understood by looking at the center and right panels. Full revelation never takes place when $\theta = \theta_H$, and occurs in two instances When $\theta = \theta_L$: first, when gross supply is large enough that the target price could only be sustained with APs larger than \bar{b}_n ; second, for all states where the marginal agent’s signal without APs drops below \tilde{x}_n , and the target price could only be sustained with APs smaller than \underline{b}_n .

4.2 The Distribution of Gains and Losses

We now characterize gains and losses under this class of AP policies. In the case of the central bank, we only need to consider states where APs are positive, i.e., when $Q = Q_n$, in which case ex-post gains or losses are given by $\pi_{cb}(\theta) = \theta - Q_n$. Since the distribution of APs is uniformly distributed and uncorrelated with the fundamental θ (see Proposition 5), average central bank

²⁸Note that these curves coincide with the functions depicted in Figures 2 and 1.

gains are a simple product of the per-unit gain or loss $\mathbb{E}[\theta] - Q_n$, the average purchased size $\frac{\bar{b}_n + b_n}{2}$, and the unconditional probability of hitting the target

$$\mathbb{E}[\pi_{cb}] = \mathbb{E}[b_{cb}(\theta - Q_n)] = P(Q = Q_n) \frac{\bar{b}_n + b_n}{2} (\mathbb{E}[\theta] - Q_n). \quad (26)$$

If $Q_n = \mathbb{E}[\theta]$, average gains are zero. This happens because the central bank is buying bonds at their risk-neutral valuation, and the probability it does so is uncorrelated with the fundamental. If instead $Q_n > \mathbb{E}[\theta]$, the central bank makes losses on average because it is intervening *exclusively* in states where the wedge between bond prices and valuations is positive, i.e., bonds are overpriced.

Investors' average gains are given by

$$\begin{aligned} \mathbb{E}[\pi_{inv}] = & \int_{\tilde{x}_n}^{\infty} \sum_{j \in \{H, L\}} \Phi\left(\frac{\theta_j - x}{\sigma_x}\right) (\theta_j - Q(p(x))) P(\theta_j | x \sim \mathcal{M}_0) f_{\mathcal{M}_0}(x) dx \\ & + P(Q = Q_n) \sum_{j \in \{H, L\}} \Phi\left(\frac{\theta_j - x_n}{\sigma_x}\right) (\theta_j - Q_n) q_j, \end{aligned} \quad (27)$$

where $q_H = q$, $q_L = 1 - q$, and the first and second lines account for states in the no intervention and price target regions, respectively. Two similarities with the case of quantity-targeting APs are worth noting. First, the probability that investors enter the market is positively correlated with bond payoffs, which implies investors' gains are always non-negative. Second, the first line of (27) coincides with average gains under quantity-targeting if $\bar{x}(b) = \tilde{x}_n$.

As before, the sum of the gains by the government, investors, and the central bank must equal zero. The following proposition characterizes how price-targeting APs shape the unconditional distribution of gains in the market.

Proposition 6 (Average properties of price-targeting APs). *Price-targeting APs have the following properties:*

- i) *The average size of APs is strictly increasing in Q_n ;*
- ii) *Central bank average gains are weakly negative and decreasing in Q_n ;*
- iii) *Investors' average gains are positive, decreasing in Q_n , and $\lim_{Q_n \rightarrow \theta_H} \mathbb{E}[\pi_{inv}] = 0$;*
- iv) *Government gains are positive if Q_n is sufficiently large;*

v) The average bond price as a function of the price target Q_n is given by

$$\mathcal{Q}_{\text{pt}}(Q_n) = \left(1 - P(Q = Q_n)\right) \mathcal{Q}_{\text{qt}}(\tilde{b}_n) + P(Q = Q_n)Q_n \quad (28)$$

where $\tilde{b}_n = \Phi((\tilde{x}_n - \theta_H)/\sigma_x)$ is the level of quantity-targeting APs that delivers $\bar{x}(b) = \tilde{x}_n$. The average price $\mathcal{Q}_{\text{pt}}(Q_n)$ is such that $\mathcal{Q}_{\text{pt}}(\mathbb{E}[\theta]) > \mathbb{E}[\theta]$, $\mathcal{Q}_{\text{pt}}(\theta_H) = \theta_H$, and $\mathcal{Q}_{\text{pt}}(Q_n) \leq Q_n$ for Q_n large enough.

Proof. See Appendix A.6. ■

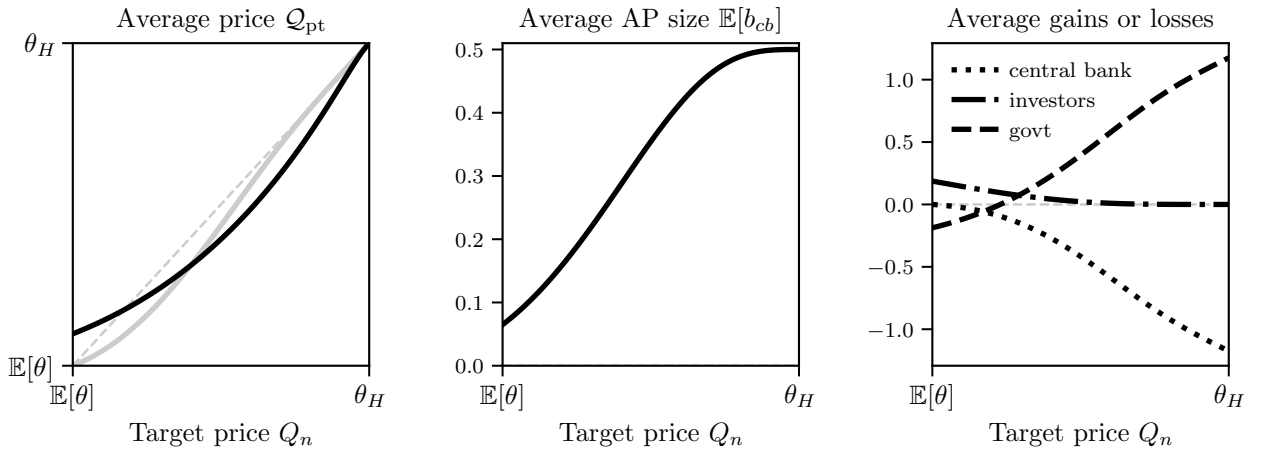


Figure 7: Average bond price, asset purchases, and gains with price-targeting APs.

Statement *v)* of the proposition characterizes the average price under price-targeting as a weighted average of the target, and the average price under a quantity-targeting program that delivers the same partitions of the state space. This is useful since we already derived the property of \mathcal{Q}_{qt} . Figure 7 illustrates the proposition. The left, center and right panel respectively plot the average equilibrium bond price, the average size of the price-targeting AP program, and average gains, all as a function of the central bank price target Q_n . The shape of the average equilibrium price \mathcal{Q}_{pt} as a function of Q_n is the result of two contrasting forces, which can be seen from equation (28). First, in the absence of the learning-from-prices wedge we know that $\mathcal{Q}_{\text{qt}}(\tilde{b}_n) = \mathbb{E}[\theta]$, which implies that the average price (represented by a gray solid line in the left panel) is never larger than the target ($\mathcal{Q}_{\text{pt}}(Q_n) \leq Q_n$), and is equal to it at $Q_n = \{\mathbb{E}[\theta], \theta_H\}$.²⁹

²⁹Note that, when APs target the highest possible price ($Q_n = \theta_H$), the central bank must intervene and buy the whole bond supply at all times ($P(Q = Q_n) = 1$), which implies its average size equals average gross supply ($\mathbb{E}[b_{cb}] = \mathbb{E}[\tilde{S}] = 1/2$).

Second, the learning-from-prices wedge is strictly positive in all states in which the central bank does not intervene, i.e. $\Delta(\tilde{b}_n) > 0$, and its average value is decreasing in Q_n and converges to zero as $Q_n \rightarrow \theta_H$, because so does the measure $P(Q > Q_n)$ of the no-intervention region. The combined effects of these two forces implies the single-crossing result in the last statement of the proposition.

Statement *ii)* of the proposition establishes that central bank gains are zero at a target of $\mathbb{E}[\theta]$, and negative above it. The former part of this statement implies that price-targeting APs can inflate average bond prices to some extent, at no balance sheet cost. The latter part states that the higher the price target, the larger the required central bank intervention, and the larger the associated losses. As equation (26) shows, losses are inevitable because the central bank is, by construction, buying bonds only when they are overpriced by the market.

Statements *iii)* and *iv)* of the proposition say that investor and government gains behave as in the case of quantity targeting APs. The former converge to zero as $Q_n \rightarrow \theta_H$, because so does the extent of investors' participation in the market. This implies that the government ultimately benefits from the intervention of the central bank, because it gets large positive resource transfers from it. In summary, the key difference from the quantity-targeting case is that the central bank must always pay the marginal investor's valuation to intervene, leading to significant losses when attempting to purchase the entire market.

5 Welfare Rationale for Asset Purchases

In this section, we extend our framework to address the natural question of what frictions justify central bank intervention through large-scale asset purchases. These frictions are often modelled using nominal rigidities, a zero lower bound on interest rates, or a financially constrained banking sector—none of which are essential for our mechanism. We demonstrate that the same frictions underpinning our approach—namely, limited arbitrage and dispersed information—are sufficient, on their own, to generate inefficiencies in a simple consumption-saving model where the investors are financial intermediaries. Specifically, investors make inefficiently high average gains in the financial market. This inefficiency stems from dispersed information and limited arbitrage, which is distinct from the externality in learning from prices, i.e., is present even without any wedge in prices. AP policies can effectively fight this inefficiency and lower investor gains by affecting the equilibrium price wedge that arises from the learning externality.

Environment. Time is discrete and there are two periods, $t = 0, 1$. We consider an economy populated by a government, a central bank, and two continua of measure one of households and investors. For simplicity, we assume there is no time discounting. In the first period, the government issues bonds to finance spending; households receive an endowment that they either consume or lend to investors; investors observe noisy signals and allocate funds to government bonds or central bank reserves; and the central bank issues reserves to investors to purchase bonds or save through storage. In the second period, the government levies lump-sum taxes to finance spending, debt service, and make transfers to the central bank; investors repay their loans and transfer their trading gains to households; households consume and pay taxes. The analysis that follows applies to both quantity- and price-targeting asset purchase policies by the central bank. We now consider the problem of each class of agents in detail.

Households. A continuum of homogeneous households enjoy utility from consumption in two periods. In the first period, household j solves the following consumption-saving problem:

$$\begin{aligned} & \max_{c_{j,0}, c_{j,1}, \{s_{j,i}\}_{i \in [0,1]}} u(c_{j,0}) + u(c_{j,1}) \\ \text{s.t. } & c_{j,0} = y - \int_0^1 s_{j,i} \, di \quad \text{and} \quad c_{j,1} = \int_0^1 \mathcal{R}_i s_{j,i} \, di + D - \tau. \end{aligned} \tag{29}$$

where y is the initial endowment of the consumption good,³⁰ $s_{j,i}$ is the amount of savings that the household j lends to investor $i \in [0, 1]$, \mathcal{R}_i is the fixed interest rate on such loan contract, D are dividends paid out by investors, and τ is the amount of lump-sum taxes paid by the household to the government. Lending contracts are signed before any shock realizes. Since investors are ex-ante identical and perfectly compete for savings, in equilibrium they all borrow at the same interest rate \mathcal{R} , such that they make zero gains in expectation. It follows that, in equilibrium, $s_{j,i} = s$ since households are identical and indifferent with respect to which investor they lend to. Finally, we assume that households hold an equal ownership share of each investor, so the dividends they receive are independent of their lending choice, and are thus taken as given. Note that dividends are generally non-zero because investors as a whole always make gains or losses ex post.

Government and central bank. The government issues an amount of debt whose face value is stochastic and given by \tilde{S} , according to equation (1). The total amount of government spending

³⁰We shall assume that the endowment is sufficiently large to allow liquidity to the financial sector in any state of the world.

in the two periods must sum up to G units of the consumption good, so the fraction that is consumed in $t = 0$ depends on the revenues from debt issuance according to the budget constraint $g_0 = \tilde{S}Q$, and the rest relies on tax revenues net of central bank transfers and debt service, according to $G - g_0 = \tau - \tilde{S}\theta - \tau_{cb}$.

In the first period, the central bank issues reserves to purchase bonds or save in a safe storage technology k_{cb} , so its budget constraint is given by $a_{cb} = Qb_{cb} + k_{cb}$. Reserves are assumed to offer a zero net interest rate. In the second period, the central bank uses the proceeds from APs, storage, and government transfers to reimburse reserves, and its budget constraint is given by $\theta b_{cb} + k_{cb} + \tau_{cb} = a_{cb}$.

The present-value budget constraint of the government and the central bank are respectively given by

$$\tau = \tau_{cb} + G + \pi_{gov}, \quad (30)$$

and

$$-\tau_{cb} = \pi_{cb}, \quad (31)$$

where $\pi_{gov} := \tilde{S}(\theta - Q)$ and $\pi_{cb} := b_{cb}(\theta - Q)$, matching the payoff functions we use in Section 4.2. Joining these two conditions, we get the present-value budget constraint of the public sector $\tau - G = (\tilde{S} - b_{cb})(\theta - Q)$, which states that public sector surplus must equal the net losses from bond issuance. In the absence of information frictions $Q = \theta$, the government runs a balanced budget, and debt is only needed because of a mismatch between the timing of taxes and government spending.

Investors. Before receiving any information, investors borrow funds $s_i := \int s_{j,i} dj$ from households to maximize *ex ante* expected dividends. After learning their private signal, each investor i allocates their funds between central bank reserves a_i and government debt b_i , according to $s_i = a_i + Qb_i$, taking bond price Q and loan interest rate \mathcal{R} as given. Their objective is to maximize expected dividends conditional on their information set

$$\begin{aligned} \max_{s_i, b_i \in [0,1], d_i} \quad & \mathbb{E}[d_i \mid \Omega_i] \\ \text{s.t.} \quad & d_i = b_i(\theta - Q) - s_i(\mathcal{R} - 1) \end{aligned} \quad (32)$$

where bond position bounds and the information set Ω_i are as in Section 2.1, so that investors' bond demand follows (7).

Market clearing. The market clearing condition for bonds is as in (6); that for central bank reserves is $\int_0^1 a_i di = a_{cb}$; that for households' savings is $\int_0^1 s_{j,i} dj = s_i$; and those for goods in the two periods are $c_0 + g_0 + k_{cb} = y$ and $c_1 + g_1 = k_{cb}$, respectively. The equilibrium interest rate on investors' loan contracts is given by their *ex ante* zero-profit condition

$$\mathcal{R} = 1 + \frac{1}{s} \mathbb{E} [\pi_{inv}], \quad (33)$$

where $\mathbb{E} [\pi_{inv}]$ denotes investors' average gains in the bond market, and is the same object we characterized in Proposition 4.³¹ Finally, dividends $D := \int d_i di$ are such that ex post gains are rebated to—and losses recapitalized by—households, ensuring that all investors can meet their loan obligation.

Competitive equilibrium. A competitive equilibrium in this extended model consists of two blocks. First, consumption and loan allocations for households and investors, government tax and spending policies, central bank transfer policies, and an interest rate \mathcal{R} such that: the allocations solve the household problem (29) and the ex ante investor problem; government and central bank policies satisfy the budget constraints (30) and (31); the interest rate satisfies equations (33); and the markets for goods, loans, and reserves clear. Second, a generic AP policy, a bond price function Q , and investors' portfolio allocations and posterior beliefs that form a Perfect Bayesian Equilibrium of the financial market according to Definition 3.

Efficiency. The Euler equation for the representative household is given by

$$u'(c_0) = \mathcal{R} u'(c_1) \quad (34)$$

which equates the marginal cost and expected benefit of lending to investors at rate \mathcal{R} , taking as given dividends D and lump-sum taxes τ . Imposing symmetry and plugging in market clearing conditions and budget constraints, equation (34) becomes

$$u'(y - s) = \mathcal{R} u'(s - G). \quad (35)$$

In contrast, the planner problem is given by $\max_s u(y - s) + u(s - G)$, which yields the first

³¹That is, average investor profits obtain by integrating (20) with respect to the market signal distribution \mathcal{M}_b . The analytical derivation can be found in Appendix A.4.

best condition

$$u'(y - s) = u'(s - G). \quad (36)$$

The key observation here is that the planner ignores the average gains of investors in the financial market because—in contrast to households—it internalizes the effect of aggregate savings on taxes and investors’ gains. The following proposition states our main result on the welfare effects of APs.

Proposition 7. *Household welfare is increasing in the central bank quantity-target b_{cb} or price-target Q_n insofar as*

$$\mathcal{R} > 1 \quad \Leftrightarrow \quad \mathbb{E}[\pi_{inv}] > 0. \quad (37)$$

Proof. The equivalence in condition (37) follows from (33). Comparing equation (36) with (35), one can see that the competitive equilibrium is not efficient when average investor gains are positive. Propositions 4 and 6 then prove that such gains are decreasing in the AP quantity- or price-target, respectively. ■

The key takeaway from the proposition is that the very same assumptions—limited arbitrage and dispersed information—that make APs non-neutral in our model also provide a microfoundation for the rationale behind central bank interventions, without resorting to additional frictions such as price rigidities or inequality motives. In particular, policy has a role because households fail to internalize that investor gains in the financial market lead to higher lump-sum taxes, thus saving too much relative to the social optimum. Proposition 7 states that APs reduce these expected returns in the financial market and thus increase welfare.

In this setting, the optimal AP quantity or price target would be the largest possible, because the central bank bears no cost from performing APs or transferring resources directly to the treasury. Introducing inflation and limits to the transfer policy between central bank and treasury would be one way of thinking about the potential inflationary costs of APs and central bank balance sheet expansion. We leave this as an interesting direction for future research.

6 Heterogeneous Term Premia: a Simple Comparison

This section compares our mechanism to more traditional models, where the elasticity of aggregate asset demand depends on the distribution of required premia across the investor population. To explore this, we consider a modification of our model where investors demand have heterogeneous preferences for risk. Our findings reveal a stark contrast in predictions: this different form

of heterogeneity implies that asset prices increase monotonically with the size of APs, while the expected impact on the central bank's balance sheet is strictly positive.

Let us first show how our framework extends to include heterogeneity in term premia. As before, bonds pay according to the lottery specified in (2), and each investor i is risk neutral, can buy up to one unit of the asset, and has an expected payoff given by $\mathbb{E}[\tilde{\theta} | Q, x_i]$. We assume that investor i discounts the payoff at a discount rate given by an idiosyncratic premium λ_i , and thus buys bonds if $\frac{\mathbb{E}[\tilde{\theta} | Q, x_i]}{1 + \lambda_i} > Q$.³² That is, investor i buys bonds if her subjective premium is smaller than the equilibrium one. The case investigated in the main text obtains in the case of a homogeneous term premium ($\lambda_i = \lambda$) through a simple rescaling of $\tilde{\theta}$. Here we explore the case of homogeneous expectations ($\mathbb{E}[\tilde{\theta} | Q, x_i] = \mathbb{E}[\tilde{\theta}]$ for all i) and heterogeneous term premia.

The distribution of required premia across investors is assumed to be exogenous, with a cumulative distribution function \tilde{F} on the support $[\underline{\lambda}, \bar{\lambda}]$. We denote by F the c.d.f. of reservation prices $\mathbb{E}[\tilde{\theta}]/(1 + \lambda)$ that is induced by \tilde{F} . The marginal agent m is the one whose reservation price is equal to the equilibrium one, so we write her subjective premium as a function of the equilibrium price: $\lambda_m(Q) = \mathbb{E}[\tilde{\theta}]/Q - 1$.

The bond market is assumed to work as in the previous sections: the supply of bonds is given by \tilde{S} and distributed as in (1), and the central bank implements APs according to the quantity-targeting rule (4).³³ Market clearing in the bond market requires

$$1 - F(Q) = \max\{\tilde{S} - b, 0\}$$

where aggregate demand $1 - F(Q) = \tilde{F}(\lambda_m(Q))$ denotes the mass of investors with a reservation price larger than Q , or equivalently a subjective premium smaller than $\lambda_m(Q)$. The equilibrium price is given by

$$Q(\tilde{S}, b) = \begin{cases} F^{-1}(1 - \tilde{S} + b) & \text{if } \tilde{S} \geq b \\ Q_{\text{pas}} & \text{if } \tilde{S} < b \end{cases} \quad (38)$$

where we set $Q_{\text{pas}} = \mathbb{E}[\tilde{\theta}]$ to rule out direct transfers between the central bank and government, as we do in our baseline model. Equation (38) shows that the equilibrium bond price is independent of the realization of $\tilde{\theta}$. It follows that the equilibrium value of the marginal agent's subjective premium is given by $\lambda_m(Q) = \tilde{F}^{-1}(\tilde{S} - b)$.

Assuming that subjective premia are uniformly distributed, we obtain simple results stated

³²This can be easily extended to also include a common component r (e.g., the short-term risk-free rate).

³³We consider quantity-targeting APs for simplicity, but our results here would also go through with price-targeting policies.

in the following proposition, including an equilibrium price—or yield—expression conceptually close to that typically assumed in the finance literature.

Proposition 8 (APs with structural heterogeneity). *Let $\tilde{F} = \text{Uniform}[\underline{\lambda}, \bar{\lambda}]$. Approximating to a first order, bond demand is given by*

$$\tilde{F}(\lambda_m(Q)) = \frac{\lambda_m(Q) - \underline{\lambda}}{\bar{\lambda} - \underline{\lambda}} \approx \beta - \alpha \log(Q) \quad (39)$$

where $\alpha := 1/(\bar{\lambda} - \underline{\lambda})$ and $\beta := \alpha [\log(\mathbb{E}[\tilde{\theta}]) - \underline{\lambda}]$. The equilibrium yield premium is given by

$$\lambda_m(\tilde{S}, b) = \underline{\lambda} + (\tilde{S} - b)(\bar{\lambda} - \underline{\lambda}).$$

A quantity-targeting intervention of size b leads to a drop in the average equilibrium yield

$$\mathbb{E}_b[\lambda_m] - \mathbb{E}_0[\lambda_m] = -b \frac{\bar{\lambda} - \underline{\lambda}}{2},$$

and to central bank average gains

$$\mathbb{E}[\pi_{cb}] = \frac{b}{1-b} \mathbb{E}[\tilde{\theta}] \int_b^1 \frac{\lambda_m(\tilde{S}, b)}{1 + \lambda_m(\tilde{S}, b)} d\tilde{S} > 0.$$

Proof. Postponed to Appendix A.7. ■

The proposition characterizes a simple example where aggregate asset demand is downward-sloping and microfounded by heterogeneity in risk premia. This is useful to highlight some key differences with our model of belief heterogeneity. First, the effect of APs on yields is strictly positive and is a monotonic function of the intervention size. Second, the effect on the central bank balance sheet is strictly positive. These conclusions go beyond the specific functional form explored in this example. APs have monotonic effects because they do not affect the fundamental characteristics of investors, so their only effect is to crowd out those less willing to enter the market. Since investors require positive risk premia, the central bank always intervenes at a price that is below the fundamental value of the asset, generating positive average gains. These results imply that APs have no downsides, and any reason to tame their size, if any, is external to their functioning. In our model, in contrast, investors respond to central bank intervention by adjusting their beliefs. This has two important implications: first, APs can backfire and depress asset prices by making them more informative in low payoff states; second, central banks may

incur losses if they purchase assets when they are overpriced by the market.

7 Conclusion

This paper studies central bank purchases of government debt when investors hold heterogeneous beliefs and learn from prices. Learning from prices introduces an externality that creates a wedge between bond prices and fundamentals. We show that the effect of APs crucially depends on how they interact with the magnitude and sign of this wedge. First, APs have a non-monotonic effect on bond prices which depends on the intervention size: they raise prices by crowding out pessimistic investors, but lower them by making prices more informative in states where bond payoffs are low. Second, APs generate implicit transfers from the central bank to the government, because the central bank tends to purchase bonds when they are overpriced relative to their fundamental value. Our findings contrast sharply with settings where investor heterogeneity is structural and thus invariant to policy.

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Appendix

A Proofs

A.1 Proof of Proposition 1 (Single crossing)

For notational convenience, let

$$\gamma := \frac{\theta_H - \theta_L}{\sigma_x^2}; \quad \tilde{q} := \frac{1 - q}{q}. \quad (40)$$

Using equation (11) for $\hat{p}(x_m)$ and (12) and (13) for $p(x_m)$, we can rewrite posterior beliefs as

$$\begin{aligned} p(x_m) &= \frac{1}{1 + \tilde{q} \exp\{2\gamma(x^\dagger - x_m)\}} \\ \hat{p}(x_m) &= \frac{1}{1 + \tilde{q} \exp\{\gamma(x^\dagger - x_m)\}}. \end{aligned} \quad (41)$$

It is straightforward to show that $p(x_m) \geq \hat{p}(x_m)$ if and only if $x_m \geq x^\dagger$.

A.2 Proof of Proposition 2 (Average price and valuation)

A.2.1 Derivation of Equations (16)-(18)

First, we derive (16). The average fundamental valuation is

$$\begin{aligned} \mathbb{E}[Q(\hat{p}(x_m))] &= \int_{\underline{x}(b)}^{+\infty} Q(\hat{p}(x_m)) \frac{f_{\mathcal{M}_b}(x_m)}{1 - b} dx_m \\ &= \theta_L + \frac{(\theta_H - \theta_L)}{1 - b} \int_{\underline{x}(b)}^{+\infty} \hat{p}(x_m) f_{\mathcal{M}_b}(x_m) dx_m \\ &= \theta_L + \frac{(\theta_H - \theta_L)}{1 - b} \int_{\underline{x}(b)}^{+\infty} \frac{q \phi\left(\frac{\theta_H - x_m}{\sigma_x}\right)}{\sum_{j \in \{L, H\}} q_j \phi\left(\frac{\theta_j - x_m}{\sigma_x}\right)} \frac{1}{\sigma_x} \sum_{j \in \{L, H\}} q_j \phi\left(\frac{\theta_j - x_m}{\sigma_x}\right) dx_m \\ &= \theta_L + q \frac{(\theta_H - \theta_L)}{1 - b} \int_{\underline{x}(b)}^{+\infty} \phi\left(\frac{\theta_H - x_m}{\sigma_x}\right) \frac{1}{\sigma_x} dx_m \\ &= \theta_L + q(\theta_H - \theta_L) = \mathbb{E}[\theta] \end{aligned}$$

where in the summations we use $q_L = 1 - q$, $q_H = q$.

Second, we derive (17) and (18). We write the average price as a function of the average fundamental valuation

$$\begin{aligned}\mathbb{E}[Q(p(x_m))] &= \mathbb{E}[Q(\widehat{p}(x_m))] + \mathbb{E}[Q(p(x_m)) - Q(\widehat{p}(x_m))] = \\ &= \mathbb{E}[\theta] + \frac{(\theta_H - \theta_L)}{1 - b} \int_{\bar{x}(b)}^{+\infty} (p(x_m) - \widehat{p}(x_m)) f_{\mathcal{M}_b}(x_m) dx_m \\ &= \mathbb{E}[\theta] + \Delta(b).\end{aligned}$$

A.2.2 Average Wedge Without APs

We now turn to prove that

$$\Delta(0) = \int_{-\infty}^{+\infty} (p(x_m) - \widehat{p}(x_m)) f_{\mathcal{M}_0}(x_m) dx_m \geq 0 \quad (42)$$

if and only if $q \leq 1/2$, and holds with equality at $q = 1/2$. We start by rearranging the expression as follows

$$\begin{aligned}\int_{-\infty}^{+\infty} (p(x_m) - \widehat{p}(x_m)) f_{\mathcal{M}_0}(x_m) dx_m &\geq 0 \\ \int_{x^\dagger}^{+\infty} (p(x_m) - \widehat{p}(x_m)) f_{\mathcal{M}_0}(x_m) dx_m &\geq \int_{-\infty}^{x^\dagger} (\widehat{p}(x_m) - p(x_m)) f_{\mathcal{M}_0}(x_m) dx_m \\ \int_0^{+\infty} (p(x^\dagger + h) - \widehat{p}(x^\dagger + h)) f_{\mathcal{M}_0}(x^\dagger + h) dh &\geq \int_{-\infty}^0 (\widehat{p}(x^\dagger + h) - p(x^\dagger + h)) f_{\mathcal{M}_0}(x^\dagger + h) dh \\ \int_0^{+\infty} (p(x^\dagger + h) - \widehat{p}(x^\dagger + h)) f_{\mathcal{M}_0}(x^\dagger + h) dh &\geq \int_0^{+\infty} (\widehat{p}(x^\dagger - h) - p(x^\dagger - h)) f_{\mathcal{M}_0}(x^\dagger - h) dh.\end{aligned}$$

First, we show that $f_{\mathcal{M}_0}(x^\dagger + h) \geq f_{\mathcal{M}_0}(x^\dagger - h)$ for all $h \in (0, +\infty)$ if and only if $q \geq 1/2$

$$\begin{aligned}f_{\mathcal{M}}(x^\dagger + h) &\geq f_{\mathcal{M}}(x^\dagger - h) \\ q\phi\left(\frac{x^\dagger - h}{\sigma_x}\right) + (1 - q)\phi\left(\frac{-x^\dagger - h}{\sigma_x}\right) &\geq q\phi\left(\frac{x^\dagger + h}{\sigma_x}\right) + (1 - q)\phi\left(\frac{-x^\dagger + h}{\sigma_x}\right) \\ q\phi\left(\frac{x^\dagger - h}{\sigma_x}\right) + (1 - q)\phi\left(\frac{x^\dagger + h}{\sigma_x}\right) &\geq q\phi\left(\frac{x^\dagger + h}{\sigma_x}\right) + (1 - q)\phi\left(\frac{x^\dagger - h}{\sigma_x}\right) \\ (1 - 2q)\left[\phi\left(\frac{x^\dagger + h}{\sigma_x}\right) - \phi\left(\frac{x^\dagger - h}{\sigma_x}\right)\right] &\geq 0,\end{aligned}$$

since the term inside square brackets in the last inequality is strictly negative for all $h > 0$. At

$q = 1/2$, the above condition holds with equality because $p(x^\dagger + h) - \widehat{p}(x^\dagger + h) = \widehat{p}(x^\dagger - h) - p(x^\dagger - h)$ and $f_{\mathcal{M}_0}(x^\dagger + h) = f_{\mathcal{M}_0}(x^\dagger - h)$ for all $h \in (0, +\infty)$.

Second, we show that

$$p(x^\dagger + h) - \widehat{p}(x^\dagger + h) \geq \widehat{p}(x^\dagger - h) - p(x^\dagger - h)$$

if and only if $q \geq 1/2$. We use (41), rescaling $h = h/\gamma$ to simplify the algebra, to rewrite the condition as

$$\begin{aligned} \frac{1}{1 + \widetilde{q}e^{-2h}} - \frac{1}{1 + \widetilde{q}e^{-h}} &\geq \frac{1}{1 + \widetilde{q}e^h} - \frac{1}{1 + \widetilde{q}e^{2h}} \\ \frac{\widetilde{q}(e^{-h} - e^{-2h})}{1 + \widetilde{q}(e^{-2h} + e^{-h}) + \widetilde{q}^2e^{-3h}} &\geq \frac{\widetilde{q}(e^{2h} - e^h)}{1 + \widetilde{q}(e^{2h} + e^h) + \widetilde{q}^2e^{3h}} \\ (e^{-h} - e^{-2h}) + \widetilde{q}(e^h + 1 - 1 - e^{-h}) + \widetilde{q}^2(e^{2h} - e^h) &\geq (e^{2h} - e^h) + \widetilde{q}(1 + e^h - e^{-h} - 1) + \widetilde{q}^2(e^{-h} - e^{-2h}) \\ (1 - \widetilde{q}^2)(e^{-h} + e^h - e^{-2h} - e^{2h}) &\geq 0. \end{aligned}$$

Since the second parenthesis is negative for any $h > 0$, the above condition is satisfied for $q \leq 1/2$, and holds with equality at $q = 1/2$. This concludes the proof of (42).

A.2.3 Properties of the Average Wedge

To rewrite the average wedge in a more convenient way, we perform the following change of variable

$$x_m = \theta_H + \sigma_x \Phi^{-1}(y),$$

which we previously defined as $\bar{x}(y)$. This implies that $dx_m = \sigma_x \frac{1}{\phi\left(\frac{x_m - \theta_H}{\sigma_x}\right)} dy$, so the expression for the average price becomes

$$\mathcal{Q}_{\text{qt}}(b) = \mathbb{E}[\theta] + q(\theta_H - \theta_L) \left(\frac{1}{1 - b} \int_b^1 \frac{p(\bar{x}(y))}{\widehat{p}(\bar{x}(y))} dy - 1 \right)$$

and we define the following convenient linear transformation of the average wedge

$$\widetilde{\Delta}(b) := \frac{\Delta(b)}{q(\theta_H - \theta_L)} + 1 = \frac{1}{1 - b} \int_b^1 \delta(\bar{x}(y)) dy,$$

where $\delta(x) := \frac{p(x)}{\widehat{p}(x)}$ denotes the *conditional wedge ratio* function. We now derive some properties of the function $\delta(\cdot)$, which will be useful to prove the properties of the average wedge $\Delta(\cdot)$ that

are stated in the Proposition.

Lemma 1. *The conditional wedge ratio $\delta(x_m) := \frac{p(x_m)}{\widehat{p}(x_m)}$*

(i) is single-peaked, has maximizer $x^ := x^\dagger + \frac{\sigma_x^2}{\theta_H - \theta_L} \log\left(\frac{1+\sqrt{q}}{\sqrt{q}}\right)$, and maximum $\delta(x^*)$ which depends solely on parameter q .*

(ii) is increasing (decreasing) in x_m to the left (right) of x^ , and is such that $\lim_{x \rightarrow -\infty} \delta(x) = 0$ and $\lim_{x \rightarrow +\infty} \delta(x) = 1$.*

Proof. The derivative of the wedge is given by

$$\delta'(x) = \frac{\gamma \tilde{q} e^{\gamma(x^\dagger - x)}}{(1 + \tilde{q} e^{2\gamma(x^\dagger - x)})^2} \left[\tilde{q} e^{2\gamma(x^\dagger - x)} + 2 e^{\gamma(x^\dagger - x)} - 1 \right]$$

where γ, \tilde{q} are defined in (40). It follows that $\delta'(x) \geq 0$ if and only if $x \leq x^* := x^\dagger + \frac{1}{\gamma} \log\left(\frac{\tilde{q}}{\sqrt{1+\tilde{q}}-1}\right) = x^\dagger + \frac{1}{\gamma} \log\left(\frac{1+\sqrt{q}}{\sqrt{q}}\right)$. Since $\log\left(\frac{1+\sqrt{q}}{\sqrt{q}}\right) > 0$, x^* is larger than x^\dagger and is increasing in σ_x (i.e., decreasing in γ). It follows that $\delta'(x^*) = 0$, $\delta'(x) > 0$ for $x < x^*$, and $\delta'(x) < 0$ otherwise. Moreover, the maximum $\delta(x^*) = \frac{2(1+\sqrt{q})+q}{1+\sqrt{q}+q}$ is such that $\delta(x^*) > \delta(x^\dagger) = 1$, it is only a function of the prior distribution parameter q , and it is decreasing in it. The two statements for $\lim_{x_m \rightarrow \pm\infty} \delta(x_m)$ can be proved using L'Hôpital's rule.

This completes the proof that the wedge function $\delta(x)$ is single-peaked, is increasing in x iff $x < x^*$, is smaller than one if and only if $x < x^\dagger$, its maximum is only function of q , and its maximizer is increasing in σ_x . ■

We now return to the proof of the proposition, where we will use the results of the lemma. To help the reader with a visual representation, Figure 8 plots the average and conditional wedge functions on the left and right panel, respectively.

Limit of $\Delta(\mathbf{b})$ for $\mathbf{b} \rightarrow 1$. Using L'Hôpital's rule, we get

$$\lim_{\mathbf{b} \rightarrow 1} \tilde{\Delta}(\mathbf{b}) = \frac{\lim_{\mathbf{b} \rightarrow 1} -\delta(\bar{x}(\mathbf{b}))}{-1} = 1,$$

since $\lim_{\mathbf{b} \rightarrow 1} \delta(\bar{x}(\mathbf{b})) = \lim_{x \rightarrow +\infty} \delta(x) = 1$ as per Lemma 1. It follows that $\lim_{\mathbf{b} \rightarrow 1} \Delta(\mathbf{b}) = 0$.

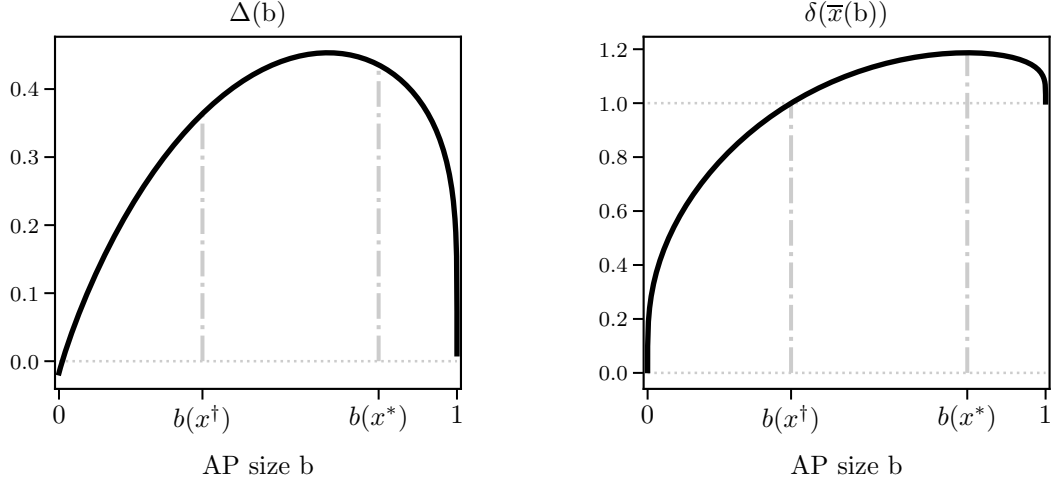


Figure 8: The average wedge (left panel) and the conditional wedge ratio (right panel) as a function of APs b .

Single-peakedness of $\Delta(b)$. The derivative of the (transformed) average wedge with respect to APs is given by

$$\tilde{\Delta}'(b) = \frac{1}{1-b} \left(\tilde{\Delta}(b) - \delta(\bar{x}(b)) \right). \quad (43)$$

To show that it is an inverse U-shaped function of APs, we again use the properties of $\delta(x)$ described in Lemma 1. For convenience, let $b(x)$ denote the AP quantity that delivers a non-revealing region with lower bound x , that is

$$b(x) = \{b : \bar{x}(b) = x\} = \Phi \left(\frac{x - \theta_H}{\sigma_x} \right) \quad (44)$$

First, $\tilde{\Delta}'(b) > 0$ for all $b \in [0, b(x^\dagger)]$, because $\delta(\bar{x}(b))$ is the lowest value the wedge ratio δ can take in the interval $[\bar{x}(b), +\infty]$, so $\tilde{\Delta}(b) > \delta(\bar{x}(b))$. This also implies that $\tilde{\Delta}(b) > \delta(\bar{x}(b))$ in this range of AP quantities, and specifically $\tilde{\Delta}(b(x^\dagger)) > 1$.

Second, $\tilde{\Delta}'(b) < 0$ for all $b \in [b(x^*), 1]$, because $\delta(\bar{x}(b))$ is the largest value the wedge ratio can take in the interval $[\bar{x}(b), +\infty]$, so $\tilde{\Delta}(b) < \delta(\bar{x}(b))$.

Third, we prove that $\tilde{\Delta}(b)$ has a unique stationary point b^* such that $\tilde{\Delta}'(b^*) = 0$, and $b^* \in [b(x^\dagger), b(x^*)]$. To do so, we compute the second derivative

$$\tilde{\Delta}''(b) = \frac{1}{(1-b)^2} [2\tilde{\Delta}'(b) - \delta'(\bar{x}(b))\bar{x}'(b)].$$

Since $\bar{x}'(b) > 0$ for all b , and $\delta'(x) > 0$ for $x < x^*$, it follows that $\tilde{\Delta}''(b) < 0$ when $\tilde{\Delta}'(b) = 0$, so there can only be one value of b where that is the case, because there are no locally convex stationary points.

Fourth, since $\Delta(b)$ is single-peaked and converges to 0 from above, it must be that it is positive for all $b \geq \tilde{b}(q)$, where $\tilde{b}(q) = 0$ for all $q \leq 1/2$, and $\tilde{b}(q) \in (0, b(x^\dagger))$ otherwise.

This concludes the proof that $\Delta(b)$ is a single-peaked function of the AP size b , it is positive for b large enough, $\Delta(0) \geq 0$ if and only if $q \leq 1/2$, and $\lim_{b \rightarrow 1} \Delta(b) = 0$.

A.3 Proof of Proposition 3 (Conditional gains and losses)

Note that the proposition focuses on the non-revealing region $x_m \in [\underline{x}(b), +\infty)$, because otherwise the fundamental is fully revealed, $Q(p(x_m)) = \theta$, and gains and losses are zero.

Investors. Since $\mathbb{E}[\theta - Q(p(x_m)) | x_i, x_m] \geq 0$ if and only if $x_i \geq x_m$, investor gains conditional on x_m are positive ($E[\pi_{\text{inv}} | x_m] > 0$) for any x_m in the non-revealing region.

Central bank. From (21) it is straightforward to see that central bank conditional gains are negative if and only if the fundamental valuation is smaller than the equilibrium price, and bonds are overpriced by the market. By Proposition 1, this is true for all $x_m > x^\dagger$.

Government. Government gains can be rewritten as

$$\begin{aligned} \mathbb{E}[\pi_{\text{gov}} | x_m] &= \sum_{\theta \in \{\theta_H, \theta_L\}} \Phi\left(\frac{\theta - x_m}{\sigma_x}\right) \left(Q(p(x_m)) - \theta\right) P(\theta | x_m \sim \mathcal{M}_b) + \\ &+ \sum_{\theta \in \{\theta_H, \theta_L\}} b \left(Q(p(x_m)) - \theta\right) P(\theta | x_m \sim \mathcal{M}_b). \end{aligned}$$

The first line represents losses on the bonds sold to investors, and is strictly negative for all (x_m, b) because it is the flip side of investors' gains. The second line represents gains or losses on the bonds sold to the central bank, and is strictly positive whenever $b > 0$ and $x_m > x^\dagger$, since it is the flip side of the central bank losses: when bonds are overpriced, the seller gains and the buyer loses.

We now characterize the states where the government makes net gains or losses. Let $\mathcal{B}(\theta, x_m) := \Phi\left(\frac{\theta - x_m}{\sigma_x}\right)$ denote the mass of investors who are buying bonds. We derive the following condition,

with each step explained below:

$$\begin{aligned}
\mathbb{E}[\pi_{\text{gov}} | x_m] &\geq 0 \\
b(\theta_H - \theta_L)(p(x_m) - \widehat{p}(x_m)) &\geq (\theta_H - \theta_L) \left(\mathcal{B}(\theta_H, x_m) \widehat{p}(x_m) (1 - p(x_m)) - \mathcal{B}(\theta_L, x_m) p(x_m) (1 - \widehat{p}(x_m)) \right) \\
b(\delta(x_m) - 1) &\geq \mathcal{B}(\theta_H, x_m) (1 - p(x_m)) - \mathcal{B}(\theta_L, x_m) \delta(x_m) (1 - \widehat{p}(x_m)) \\
b \left(e^{-\gamma(x_m - x^\dagger)} - e^{-2\gamma(x_m - x^\dagger)} \right) &\geq \mathcal{B}(\theta_H, x_m) e^{-2\gamma(x_m - x^\dagger)} \left(1 - \frac{\mathcal{B}(\theta_L, x_m)}{\mathcal{B}(\theta_H, x_m)} e^{\gamma(x_m - x^\dagger)} \right) \\
b \left(e^{\gamma(x_m - x^\dagger)} - 1 \right) &\geq \mathcal{B}(\theta_H, x_m) \left(1 - \frac{\mathcal{B}(\theta_L, x_m)}{\mathcal{B}(\theta_H, x_m)} e^{\gamma(x_m)} \right).
\end{aligned}$$

The second step follows from the definitions of $Q(p(x_m))$ in (14) and of $\widehat{p}(x_m)$ in (11), the third and fourth from the definition of $\delta(x_m)$ in Lemma 1 and the notation defined in (40), and the last from simple algebra. Finally, we get

$$e^{\gamma(x_m - x^\dagger)} \geq \frac{b + \mathcal{B}(\theta_H, x_m)}{b + \mathcal{B}(\theta_L, x_m)}. \quad (45)$$

We define x_g as the value for which (45) holds with equality, and prove that the inequality is strict for all $x_m > x_g$ by showing that the right-hand side of the condition is decreasing in x_m whenever $x_m > x_g$.

$$\begin{aligned}
\frac{\partial}{\partial x} \frac{b + \mathcal{B}(\theta_H, x)}{b + \mathcal{B}(\theta_L, x)} &< 0 \\
\phi \left(\frac{\theta_L - x}{\sigma_x} \right) [b + \mathcal{B}(\theta_H, x)] &< \phi \left(\frac{\theta_H - x}{\sigma_x} \right) [b + \mathcal{B}(\theta_L, x)] \\
e^{\frac{\gamma}{2}(x - x^\dagger)} &< \frac{b + \mathcal{B}(\theta_H, x)}{b + \mathcal{B}(x)} \leq e^{\gamma(x - x^\dagger)}
\end{aligned}$$

where the last inequality follows from (45).

A.4 Proof of Proposition 4 (Average gains and losses)

Average (or unconditional) gains for $j \in \{\text{inv}, \text{cb}, \text{gov}\}$ are given by

$$\mathbb{E}[\pi_j] = b \mathbb{E}[\pi_j | \widetilde{S} < b] + (1 - b) \mathbb{E}[\pi_j | \widetilde{S} \geq b] = \int_{\bar{x}(b)}^{\infty} \mathbb{E}[\pi_j | x_m] f_{\mathcal{M}_b}(x_m) dx_m, \quad (46)$$

where we use the fact that $\mathbb{E}[\pi_j | \tilde{S} < b] = 0$ for government and central bank because of the assumption that $Q_{\text{pas}} = \theta$, and for investors because they do not participate in the bond market. The set of states where $\tilde{S} \geq b$ corresponds to $x_m \in [\underline{x}(b), +\infty)$. The integration interval in the last equation is the non-revealing region $[\bar{x}(b), +\infty)$, because gains and losses in the fully revealing region $[\underline{x}(b), \bar{x}(b))$ are zero for all players. From (46) it is straightforward to see that average gains converge to zero as $b \rightarrow 1$.

Investors. Proposition 3 shows that $\mathbb{E}[\pi_{\text{inv}} | x_m] > 0$ for all $x_m \in [\bar{x}(b), +\infty)$, which implies that investors' average gains are strictly positive. To show they are decreasing in b , we take the derivative

$$\frac{\partial \mathbb{E}[\pi_{\text{inv}}]}{\partial b} = -\bar{x}'(b) \mathbb{E}[\pi_{\text{inv}} | \bar{x}(b)] f_{\mathcal{M}_b}(\bar{x}(b)) < 0$$

since $\bar{x}'(b) = \sigma_x / \phi(\Phi^{-1}(b))$ and $\mathbb{E}[\pi_{\text{inv}} | x_m] > 0$ for any $x_m \in [\bar{x}(b), +\infty)$.

Central bank. Applying (46) to the central bank, and combining it with (21) and Proposition 2, we get that central bank average gains are given by

$$\mathbb{E}[\pi_{\text{cb}}] = -b(1-b)\Delta(b).$$

We can thus use the properties of the average wedge stated in Proposition 2 and derived in Appendix A.2. Specifically, following directly from the properties of the average wedge, central bank gains are negative for $b \geq \tilde{b}(q)$, where $\tilde{b}(q) = 0$ for all $q \leq 1/2$, and $\tilde{b}(q) \in (0, b(x^\dagger))$ otherwise.

Government. Proposition 3 states that government gains conditional on x_m are positive for all $x_m > x_g(b)$. It follows that a sufficient condition for average government gains to be positive is that they are positive at all market signals x_m in the non-revealing region, which happens for all b such that $[\bar{x}(b), +\infty) \subseteq [x_g(b), +\infty)$.

A.5 Proof of Proposition 5 (Price-targeting APs)

For convenience and to lighten the notation, define

$$\tilde{B}_i := \Phi\left(\frac{\theta_i - \tilde{x}_n}{\sigma_x}\right) \quad \text{and} \quad B_i := \Phi\left(\frac{\theta_i - x_n}{\sigma_x}\right) \quad (47)$$

as the mass of investors buying bonds when the fundamental is θ_i for $i \in \{H, L\}$ and the marginal agent's signal is \tilde{x}_n as defined in the paragraph of equation (22). We consider states where the central bank intervenes, which requires

$$x_m \in (-\infty, \tilde{x}_n] \quad \Leftrightarrow \quad \tilde{S} \in [\tilde{B}_i, 1].$$

For now, suppose that the AP policy must deliver a marginal agent's signal equal to some value x_n to achieve the price target, which results in the central bank purchasing a quantity $b_n(\theta, \tilde{S})$ as defined in (24). It follows that $b_n | \theta$ is uniformly distributed over the interval $[b_n(\theta, \tilde{B}_i), b_n(\theta, 1)]$.

If investors were to observe an AP quantity that is only compatible with one value of the fundamental, then APs would be fully revealing. This implies that the interval of non-revealing purchases must be the intersection of the supports of $b_n | \theta_H$ and $b_n | \theta_L$. This coincides with the support of θ_H . The fact that $b_n(\theta_H, 1) < b_n(\theta_L, 1)$ is straightforward to show. To see why $b_n(\theta_H, \tilde{B}_H) > b_n(\theta_L, \tilde{B}_L)$, we plug in the definitions of \tilde{x}_n and x_n , and after a bit of algebra we get to the expression $\int_{L_n/\sqrt{2}}^{L_n} \phi(A+y)dy < \int_{L_n/\sqrt{2}}^{L_n} \phi(A-y)dy$, where $L_n := \frac{\sigma_x}{\theta_H - \theta_L} \log\left(\frac{1-q}{q} \frac{Q_n - \theta_L}{\theta_H - Q_n}\right) > 0$ and $A := \frac{\theta_H - \theta_L}{2\sigma_x} > 0$, which is satisfied for all Q_n because $\phi(A+y) < \phi(A-y)$ for any y , due to the symmetry of the standard normal density.

We have thus shown that the price-targeting, non-revealing AP policy $b_n | \theta \sim \text{Uniform}[\underline{b}_n, \bar{b}_n]$. Crucially, this distribution is independent of θ , so it is also the marginal distribution of b_n , whose c.d.f. and p.d.f. are given by

$$P(b_n < y | Q_n) = \frac{y - \underline{b}_n}{\bar{b}_n - \underline{b}_n} \quad \text{and} \quad P(b_n = y | Q_n) = \frac{1}{\bar{b}_n - \underline{b}_n} \quad \text{for all } y \in [\underline{b}_n, \bar{b}_n].$$

We now have all the elements to show that, when $b_{cb} \in [\underline{b}_n, \bar{b}_n]$, observing $(Q = Q_n, b_{cb})$ does not convey any new information. Using Bayes' law

$$P(\theta_H | Q_n, b_{cb}) = \frac{P(Q_n, b_{cb} | \theta_H)P(\theta_H)}{\sum_{j \in \{H, L\}} P(Q_n, b_{cb} | \theta_j)P(\theta_j)} = \frac{P(\theta_H)}{\sum_{j \in \{H, L\}} P(\theta_j)} = q$$

where $P(\theta_H) = q, P(\theta_L) = 1 - q$, and the second equality uses the fact we derived above that the distribution of APs is independent of the fundamental.

As a result, the expected bond payoff for investor i conditional on observing $(Q = Q_n, b_{cb})$ is given by

$$\mathbb{E}[\theta | x_i \sim \mathcal{N}, Q_n, b_n > 0] = \mathbb{E}[\theta | x_i \sim \mathcal{N}],$$

so that $Q_n = \mathbb{E}[\theta \mid x_n \sim \mathcal{N}]$ pins down the price (or marginal agent's) signal $x_n = x(Q_n, \sigma_x)$ that needs to obtain for the price to equal the target when the central bank is intervening in the bond market.

A.6 Proof of Prop. 6 (Average properties of price-targeting APs)

A.6.1 Proof of statement (i)

We continue to use the notation introduced in (47) for brevity. The average size of APs is

$$\mathbb{E}[b_{cb}] = P(Q_n) \mathbb{E}[b_{cb} \mid Q_n] = (1 - \tilde{B}_H) \frac{1}{2} \left[1 - 2B_H + \tilde{B}_H \right]$$

where

$$\begin{aligned} \mathbb{E}[b_{cb} \mid Q_n] &= \frac{\bar{b}_n + \underline{b}_n}{2} = \frac{1}{2} \left[1 - 2B_H + \tilde{B}_H \right] \\ P(Q_n) &= 1 - \tilde{B}_H. \end{aligned}$$

To show that the average size is increasing in Q_n , let $\tilde{B}'_H := \frac{\partial \tilde{B}_H}{\partial \tilde{x}_n} \frac{\partial \tilde{x}_n}{\partial Q_n}$ and $B'_H := \frac{\partial B_H}{\partial x_n} \frac{\partial x_n}{\partial Q_n}$ denote derivatives with respect to the price target, both of which can be shown to be negative. We have

$$\begin{aligned} \frac{\partial \mathbb{E}[b_{cb}]}{\partial Q_n} &= \frac{1}{2} \left[-\tilde{B}'_H (1 - 2B_H + \tilde{B}_H) + (1 - \tilde{B}_H) (-2B'_H + \tilde{B}'_H) \right] \\ &= -B'_H (1 - \tilde{B}_H) - \tilde{B}'_H (\tilde{B}_H - B_H). \end{aligned}$$

which is positive since $\tilde{B}_H > B_H$ when $Q_n > \mathbb{E}[\theta]$.

A.6.2 Proof of statement (ii)

Since average APs are positive, the sign of central bank gains follows directly from the sign of $\mathbb{E}[\theta] - Q_n$, and is thus either zero or negative when $Q_n > \mathbb{E}[\theta]$. Deriving with respect to Q_n we get

$$\frac{\partial \mathbb{E}[\pi_{cb}]}{\partial Q_n} = \frac{\partial \mathbb{E}[b_{cb}]}{\partial Q_n} (\mathbb{E}[\theta] - Q_n) - \mathbb{E}[b_{cb}]$$

which is strictly negative, since average APs are increasing in Q_n as per statement (i).

A.6.3 Proof of statement (iii)

This requires taking the derivative of equation (27) with respect to the price target Q_n . Before doing so, we rewrite (27) replacing $P(Q = Q_n)$ with $\Phi\left(\frac{\tilde{x}_n - \theta_H}{\sigma_x}\right)$ and $P(\theta_j | x \sim \mathcal{M}_0) f_{\mathcal{M}_0}(x)$ with $\frac{1}{\sigma_x} \phi\left(\frac{\theta_j - x}{\sigma_x}\right) q_j$, where $j \in \{H, L\}$ and $q_H = q, q_L = 1 - q$. Finally, since x_n and Q_n are one-to-one and $\tilde{x}_n = (x_n + x^\dagger)/2$, we can take derivatives with respect to x_n rather than Q_n , since we are only concerned with the sign of derivatives. We compute

$$\frac{\partial}{\partial x_n} \left[\int_{\tilde{x}_n}^{\infty} \sum_j \Phi\left(\frac{\theta_j - x}{\sigma_x}\right) (\theta_j - Q(p(x))) \frac{1}{\sigma_x} \phi\left(\frac{\theta_j - x}{\sigma_x}\right) q_j dx + \Phi\left(\frac{\tilde{x}_n - \theta_H}{\sigma_x}\right) \sum_j \Phi\left(\frac{\theta_j - x_n}{\sigma_x}\right) (\theta_j - Q_n) q_j \right]$$

which yields the following derivative (that we explain below)

$$\begin{aligned} &= -\frac{1}{2} \sum_j \Phi\left(\frac{\theta_j - \tilde{x}_n}{\sigma_x}\right) (\theta_j - Q_n) \frac{1}{\sigma_x} \phi\left(\frac{\theta_j - \tilde{x}_n}{\sigma_x}\right) q_j + \\ &\quad + \frac{1}{2} \frac{1}{\sigma_x} \phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) \sum_j \Phi\left(\frac{\theta_j - x_n}{\sigma_x}\right) (\theta_j - Q_n) q_j + \\ &\quad - \Phi\left(\frac{\tilde{x}_n - \theta_H}{\sigma_x}\right) \sum_j \frac{1}{\sigma_x} \phi\left(\frac{\theta_j - x_n}{\sigma_x}\right) (\theta_j - Q_n) q_j \\ &\quad - \Phi\left(\frac{\tilde{x}_n - \theta_H}{\sigma_x}\right) \sum_j \Phi\left(\frac{\theta_j - x_n}{\sigma_x}\right) \frac{\partial Q_n}{\partial x_n} q_j. \end{aligned} \tag{48}$$

In words, a higher price target has the following effects, each corresponding to one line of the derivative in (48): first, it shrinks the set of states in which there are no interventions (first line); second, it expands the set of states in which there are interventions and $Q = Q_n$ (second line); third, it lowers the amount of bonds absorbed by investors in such states (third line); fourth, it increases the price and shrinks the gains margin $\theta - Q$ in such states (fourth line).

To show that this derivative is negative for all $Q_n \geq \mathbb{E}[\theta]$, we proceed as follows. The fourth line is strictly negative since $\frac{\partial Q_n}{\partial x_n} > 0$. The third line is equal to zero: to see that, we first rearrange the statement as

$$\frac{\phi\left(\frac{\theta_L - x_n}{\sigma_x}\right)}{\phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)} \frac{1 - q}{q} \frac{Q_n - \theta_L}{\theta_H - Q_n} = 1,$$

and then define $T := \frac{1-q}{q} \frac{Q_n - \theta_L}{\theta_H - Q_n}$ and note that

$$\frac{\phi\left(\frac{\theta_L - x_n}{\sigma_x}\right)}{\phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)} = \exp\left\{\frac{(x^\dagger - x_n)(\theta_H - \theta_L)}{\sigma_x^2}\right\} = T^{-1}, \quad (49)$$

according to (22), which verifies that the third line of (48) is indeed zero.

To complete the proof, it is thus sufficient to compare the first two lines of the derivative in (48). Collecting some terms, simplifying, and opening up the summation over θ we get

$$\begin{aligned} & q(\theta_H - Q_n)\phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)\left[\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)\right] \geq \\ & \geq (1-q)(Q_n - \theta_L)\left[\phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right)\Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right) - \phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)\Phi\left(\frac{\theta_L - x_n}{\sigma_x}\right)\right]. \end{aligned}$$

Dividing through by $q(\theta_H - Q_n)\phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)$ we get

$$\left[\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)\right] \geq T \left[\frac{\phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right)}{\phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)}\Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_L - x_n}{\sigma_x}\right)\right]$$

where T is defined in the previous paragraph, and we know $T \geq 1$ when $Q_n \geq \mathbb{E}[\theta]$. Derivations similar to (49) yield

$$\frac{\phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right)}{\phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)} = e^{\frac{(x^\dagger - \tilde{x}_n)(\theta_H - \theta_L)}{2\sigma_x^2}} = T^{-1/2}.$$

We thus have

$$\left[\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)\right] \geq \sqrt{T}\Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right) - T\Phi\left(\frac{\theta_L - x_n}{\sigma_x}\right). \quad (50)$$

Given that with $Q_n > \mathbb{E}[\theta]$ we have $T > 1$, a sufficient condition for (50) to hold is

$$\left[\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_H - x_n}{\sigma_x}\right)\right] \geq \sqrt{T}\left(\Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_L - x_n}{\sigma_x}\right)\right). \quad (51)$$

The proof of this is tedious and proceeds as follows. To lighten the notation, let $\Delta := \frac{\theta_H - \theta_L}{2\sigma_x}$ and $t := \frac{1}{2} \frac{\sigma_x}{\theta_H - \theta_L} \log(T) = \frac{\tilde{x}_n - x^\dagger}{\sigma_x} = \frac{x_n - x^\dagger}{2\sigma_x}$, where we note $\Delta > 0$ and $t > 0$, and recall $x^\dagger = \frac{\theta_H + \theta_L}{2}$.

The sufficient condition (51) can be rewritten as

$$\begin{aligned}
\Phi(\Delta - t) - \Phi(\Delta - 2t) &\geq e^{2\Delta t} [\Phi(-\Delta - t) - \Phi(-\Delta - 2t)] \\
\int_{\Delta-2t}^{\Delta-t} \phi(x) dx &\geq e^{2\Delta t} \int_{\Delta+t}^{\Delta+2t} \phi(x) dx \\
t \int_1^2 \phi(\Delta - ty) dy &\geq e^{2\Delta t} t \int_1^2 \phi(\Delta + ty) dy \\
\int_1^2 e^{-\frac{1}{2}(\Delta-ty)^2} dy &\geq \int_1^2 e^{-\frac{1}{2}(\Delta+ty)^2} e^{2\Delta t} dy \\
\int_1^2 e^{-\frac{1}{2}(\Delta+ty)^2} e^{2\Delta ty} dy &\geq \int_1^2 e^{-\frac{1}{2}(\Delta+ty)^2} e^{2\Delta t} dy \\
\int_1^2 e^{-\frac{1}{2}(\Delta+ty)^2} (e^{2\Delta ty} - e^{2\Delta t}) dy &\geq 0
\end{aligned}$$

which concludes the proof since $2\Delta ty > 2\Delta t$ for $y \in [1, 2]$.

A.6.4 Proof of statement (iv)

This follows from the fact that government gains are the sum of central bank losses and investors' gains. Since the former are increasing in the price target, and the latter are decreasing in it and eventually converge to zero, there exists a price target $Q_n \in (\mathbb{E}[\theta], \theta_H)$ such that government gains are positive if the target is at least as high.

A.6.5 Proof of statement (v)

We compute the average price in three different regions of the state space.

First, with probability $P(Q > Q_n) = q\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) + (1 - q)\Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right)$, the price signal is above \tilde{x}_n , there are no APs and $Q > Q_n$, and the average price can be written as:

$$\frac{\int_{\tilde{x}_n}^{\infty} Q(p(x_m)) dx_m}{P(Q > Q_n)}.$$

With some algebra, the numerator of this expression can be rewritten as

$$\begin{aligned}
\int_{\tilde{x}_n}^{\infty} Q(p(x_m)) dx_m &= \int_{\tilde{x}_n}^{\infty} Q(\hat{p}(x_m)) dx_m + \int_{\tilde{x}_n}^{\infty} (Q(p(x_m)) - Q(\hat{p}(x_m))) dx_m = \\
&= P(Q > Q_n) \theta_L + (\theta_H - \theta_L) \left[\int_{\tilde{x}_n}^{\infty} \hat{p}(x_m) f_{\mathcal{M}_b}(x_m) dx_m + \int_{\tilde{x}_n}^{\infty} (p(x_m) - \hat{p}(x_m)) f_{\mathcal{M}_b}(x_m) dx_m \right] \\
&= P(Q > Q_n) \theta_L + (\theta_H - \theta_L) \Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) \left[q + \Delta(\tilde{b}_n) \right]
\end{aligned}$$

where the last step uses (18) to rewrite the third term of the summation as $\Delta(\tilde{b}_n)(1 - \tilde{b}_n)$ with $\tilde{b}_n = \Phi((\tilde{x}_n - \theta_H)/\sigma_x)$.

Second, with probability $P(Q = Q_n) = 1 - \Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right)$ there are APs and $Q = Q_n$; the average price in this case is Q_n .

Third, with the remaining probability $P(Q = \theta_L) = (1 - q) \left[\Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) - \Phi\left(\frac{\theta_L - \tilde{x}_n}{\sigma_x}\right) \right]$ there are no APs and there is full revelation that $\theta = \theta_L$; the average price in this case is θ_L .

Finally, we write the average price as

$$\begin{aligned}
\mathcal{Q}_{\text{pt}} &= \int_{\tilde{x}_n}^{\infty} Q(p(x_m)) dx_m + P(Q = Q_n) Q_n + P(Q = \theta_L) \theta_L = \\
&= \Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) \left(\theta_L + (\theta_H - \theta_L) \left(q + \Delta(\tilde{b}_n) \right) \right) + P(Q = Q_n) Q_n \\
&= (1 - P(Q = Q_n)) \left(\mathbb{E}[\theta] + (\theta_H - \theta_L) \Delta(\tilde{b}_n) \right) + P(Q = Q_n) Q_n \\
&= (1 - P(Q = Q_n)) \mathcal{Q}_{\text{qt}}(\tilde{b}_n) + P(Q = Q_n) Q_n
\end{aligned}$$

where we used $P(Q > Q_n) + P(Q = \theta_L) = \Phi\left(\frac{\theta_H - \tilde{x}_n}{\sigma_x}\right) = 1 - P(Q = Q_n)$.

We now move on to prove how the average price compares with the price target. First, consider $Q_n = \mathbb{E}[\theta]$. Note that $\mathcal{Q}_{\text{pt}}(\mathbb{E}[\theta]) > \mathbb{E}[\theta]$ if and only if $\Delta(\tilde{b}_n) > 0$. In this case $\tilde{b}_n = \Phi\left(\frac{-(\theta_H - \theta_L)}{2\sigma_x}\right) = b(x^\dagger)$, and we know from Appendix A.2 that $\Delta(b(x^\dagger)) > 0$.³⁴

Second, when $Q_n = \theta_H$, we have that $P(Q = Q_n) = 1$ and thus $\mathcal{Q}_{\text{pt}} = \theta_H$. In this case, the central bank must crowd out all investors, thus paying the valuation of the most optimistic one.

Third, with some algebra we can show that $\mathcal{Q}_{\text{pt}}(Q_n) < Q_n$ if and only if $\tilde{\Delta}(\tilde{b}_n) < \frac{p_n}{q}$, where $p_n = (Q_n - \theta_L)/(\theta_H - \theta_L)$. We have shown in Appendix A.2.3 that $\tilde{\Delta}'(b) < 0$ for all $b \geq b^*$,

³⁴The function $b(\cdot)$ is defined in (44).

which implies $\tilde{\Delta}(\tilde{b}) < \delta(\bar{x}(\tilde{b}))$. It follows that, for all price targets such that $\tilde{b}_n > b^*$,

$$\tilde{\Delta}(\tilde{b}_n) < \delta(\bar{x}(\tilde{b}_n)) = \delta(\tilde{x}_n) = \frac{p_n}{\hat{p}(\tilde{x}_n)} < \frac{p_n}{q}$$

which must hold since $\tilde{b}_n > b^*$ implies $\tilde{x}_n > x^\dagger$ and in turn $\hat{p}(\tilde{x}_n) > \hat{p}(x^\dagger) = q$.

This concludes the proof of all statements of Proposition 6.

A.7 Proof of Proposition 8 (APs with structural heterogeneity)

To derive (39), we take the equilibrium expression for the subjective premium of the marginal agent $1 + \lambda_m(Q) = \mathbb{E}[\tilde{\theta}]/Q$, take logs and use $\log(1 + x) \approx x$ to get

$$\lambda_m(Q) \approx \log(\mathbb{E}[\tilde{\theta}]) - \log(Q).$$

Plugging this into $\tilde{F}(\lambda_m(Q))$ yields expression (39).

The average effects of APs on yields conditional on $\tilde{S} \geq b$ is

$$\begin{aligned} \mathbb{E}_b[\lambda_m] - \mathbb{E}_0[\lambda_m] &= \frac{1}{1-b} \int_b^1 \lambda_m(\tilde{S}, b) d\tilde{S} - \int_0^1 \lambda_m(\tilde{S}, 0) d\tilde{S} \\ &= \frac{1}{1-b} \int_0^{1-b} \lambda_m(S, 0) dS - \int_0^{1-b} \lambda_m(\tilde{S}, 0) d\tilde{S} - \int_{1-b}^1 \lambda_m(\tilde{S}, 0) d\tilde{S} \\ &= \frac{b}{1-b} \int_0^{1-b} \lambda_m(\tilde{S}, 0) d\tilde{S} - \int_{1-b}^1 \lambda_m(\tilde{S}, 0) d\tilde{S} \\ &= \frac{b}{1-b} ((1-b)\underline{\lambda} + (1-b)^2(\bar{\lambda} - \underline{\lambda})/2) - (b\underline{\lambda} + b(2-b)(\bar{\lambda} - \underline{\lambda})/2) \\ &= (b\underline{\lambda} + b(1-b)(\bar{\lambda} - \underline{\lambda})/2 - (b\underline{\lambda} + b(2-b)(\bar{\lambda} - \underline{\lambda})/2)) \\ &= (b(1-b) - b(2-b))(\bar{\lambda} - \underline{\lambda})/2 \\ &= -b(\bar{\lambda} - \underline{\lambda})/2 \end{aligned}$$

The average gains of the central bank are

$$\begin{aligned}
\mathbb{E}[\pi_{cb}] &= \frac{b}{1-b} \int_b^1 \left(\mathbb{E}[\tilde{\theta}] - Q(\tilde{S}, b) \right) d\tilde{S} \\
&= \frac{b}{1-b} \int_b^1 \left(\mathbb{E}[\tilde{\theta}] - \mathbb{E}[\tilde{\theta}]/(1 + \lambda_m) \right) d\tilde{S} \\
&= \frac{b}{1-b} \mathbb{E}[\tilde{\theta}] \int_b^1 \frac{\lambda_m(\tilde{S}, b)}{1 + \lambda_m(\tilde{S}, b)} d\tilde{S} \\
&= \frac{b}{1-b} \mathbb{E}[\tilde{\theta}] \int_b^1 \frac{\underline{\lambda} + (\bar{\lambda} - \underline{\lambda})(\tilde{S} - b)}{1 + \underline{\lambda} + (\bar{\lambda} - \underline{\lambda})(\tilde{S} - b)} d\tilde{S} \\
&= \frac{b}{1-b} \mathbb{E}[\tilde{\theta}] \left[\tilde{S} - \frac{1}{\bar{\lambda} - \underline{\lambda}} \log \left(1 + \underline{\lambda} + (\bar{\lambda} - \underline{\lambda})(\tilde{S} - b) \right) \right]_b^1 = \\
&= \frac{b}{1-b} \mathbb{E}[\tilde{\theta}] \left[(1-b) - \frac{1}{\bar{\lambda} - \underline{\lambda}} \log \left(1 + \frac{(\bar{\lambda} - \underline{\lambda})(1-b)}{1 + \underline{\lambda}} \right) \right] = \\
&= b \mathbb{E}[\tilde{\theta}] \left[1 - \frac{1}{(1-b)(\bar{\lambda} - \underline{\lambda})} \log \left(1 + \frac{(\bar{\lambda} - \underline{\lambda})(1-b)}{1 + \underline{\lambda}} \right) \right]
\end{aligned}$$

which is strictly positive for any $b \in (0, 1)$.

B Analytical details

B.1 Price Neutrality

We discuss here the conditions under which we obtain that central bank APs are neutral with respect to prices and allocations, as in Wallace (1981). To do so, we take the stylized framework introduced in Section 5 and simplify it further, assuming that the household is the investor, the utility function has curvature, and position bounds are absent.

The household-investor has a generic increasing (and possibly concave) utility function $u(\cdot)$, and an information set $\Omega_i = \{x_i, Q, b_{cb}\}$, which includes an exogenous private signal, the equilibrium price Q , and the AP quantity b_{cb} . As in Section 5, they have an initial endowment y , can buy a risk-free asset with a unitary gross rate of return, and a risky bond b that has a price of Q and a stochastic payoff of θ . The bond is issued by the government in random supply \tilde{S} , and the central bank purchases a quantity b_{cb} . The consolidated public sector levies lump-sum taxes $\tau = (\tilde{S} - b)(\theta - Q) + G$ to pay for exogenous spending and net losses or gains from bond issuance.

The portfolio allocation of the investor is

$$\max_{c_i, b_i} \mathbb{E} [u(c_i) \mid \Omega_i],$$

subject to

$$c_i = b_i(\theta - Q) + y - \tau.$$

Taking the first-order condition with respect to b_i and plugging in the expression for taxes, we get the optimality condition

$$\mathbb{E} \left[u' \left((b_i + b_{cb} - \tilde{S})(\theta - Q) + w \right) (\theta - Q) \mid \Omega_i \right] = 0,$$

which pins down individual bond demand as a function of beliefs, expected net bond supply, and the equilibrium price.

It is easy to see that, everything else equal, the investor only cares about their known net risk exposure $b_i + b_{cb}$, so any changes in central bank APs will be met by a one-for-one change in bond demand. This is true for each and every investor in the market. Formally, we can use the implicit function theorem to show that $\frac{\partial b_i}{\partial b_{cb}} = -1$, that is, central bank APs perfectly crowd out each investor's demand in exactly the same way.

Let $b_i^*(b_{cb}, Q)$ denote individual bond demand as a function of APs and the bond price. The reasoning above implies that

$$b_i^*(0, Q) = b_i^*(b_{cb}, Q) + b_{cb} \quad \text{for all } b_{cb}, Q.$$

It follows that the total quantity demanded by investors and the central bank is invariant to b_{cb} and is given by $\int b_i^*(b_{cb}, Q) di + b_{cb} = \int b_i^*(0, Q) di$. As a consequence, the price Q that clears the market (i.e., such that $\int b_i^*(0, Q) di = \tilde{S}$) is also invariant to APs. In this setting, APs are thus neutral because they crowd investors out in the same way. That is, all investors participate in the bond market, and APs have a homogeneous effect on their demand (the intensive margin).

A direct implication of this reasoning is that any departure of the model such that aggregate demand does vary with APs delivers the results that central bank intervention is non-neutral. The assumption of heterogeneous beliefs and position bounds is one such departure, because it implies that the bounds bind for some investors, who therefore do not respond one-for-one to APs. Our framework features an extreme version of this mechanism, because risk neutrality implies that *all* investors are against their position constraint. As a result, APs crowd out a

specific part of the investor distribution, namely the most pessimistic agents among those that would buy bonds absent APs. That is, APs affect the market participation of some investors (the extensive margin), while leaving the demand of all other investors unaffected.

B.2 Microfoundation of Q_{pas} when $\tilde{S} < b$

We provide explicit microfoundations for the argument in Footnote 9.

We assume there exists a small disturbance in the bond market, such that there is always an infinitesimal residual quantity of bonds traded by investors, even when the central bank quantity target exceeds supply. Formally, net supply is

$$S = \epsilon \quad \text{if} \quad \tilde{S} < b,$$

where ϵ is a random variable uniformly distributed in $[0, \bar{\epsilon}]$, with $\bar{\epsilon}$ arbitrarily small.

First, notice that investors correctly recognize states where $\tilde{S} < b$, because they observe central bank purchases below the quantity target ($b_{\text{cb}} < b$). In these states, x_m has the same distribution that it would have when the central bank has a target $b = 1 - \bar{\epsilon}$ in the main text, as discussed in the paragraph around equation (10).

Therefore, in analogy to \mathcal{M}_b , we denote the distribution of x_m conditional on $S = \epsilon$ by $\mathcal{M}_{1-\bar{\epsilon}}$, whose p.d.f is $f_{\mathcal{M}_{1-\bar{\epsilon}}}(x_m)/\bar{\epsilon}$ and is such that $\lim_{\epsilon \rightarrow 0} f_{\mathcal{M}_{1-\bar{\epsilon}}}(x_m) = \lim_{b \rightarrow 1} f_{\mathcal{M}_b}(x_m)$. This means that, no matter how small is $[0, \bar{\epsilon}]$, the support of x_m will always consist of a fully revealing region $[\underline{x}(1 - \bar{\epsilon}), \bar{x}(1 - \bar{\epsilon})]$ and a non-revealing region $[\bar{x}(1 - \bar{\epsilon}), +\infty)$. Using the last statement of Proposition 2 and the fact that $\lim_{\bar{\epsilon} \rightarrow 0} \{\underline{x}(1 - \bar{\epsilon}), \bar{x}(1 - \bar{\epsilon})\} = \lim_{b \rightarrow 1} \{\underline{x}(b), \bar{x}(b)\}$, we finally have that $\lim_{\bar{\epsilon} \rightarrow 0} \mathbb{E}[Q] = \lim_{b \rightarrow 1} \mathbb{E}[Q] = \mathbb{E}[\theta]$, which is equal to $\mathbb{E}[Q_{\text{pas}}]$ under our assumption that $Q_{\text{pas}} = \theta$.

B.3 Distribution of x_i

The marginal p.d.f. of x_i is

$$f_{\mathcal{N}}(x) = \sum_{j \in \{L, H\}} \frac{1}{\sigma_x} q_j \phi\left(\frac{\theta_j - x}{\sigma_x}\right) \quad (52)$$

where $q_H = q$ and $q_L = 1 - q$.

B.4 Distribution of x_m conditional on x_i

The p.d.f. of the market signal x_m conditional on x_i is given by

$$\begin{aligned} f_{\mathcal{M}_b|\mathcal{N}}(x_m | x_i) &= \sum_{j \in \{H, L\}} f_{\mathcal{M}_b|\mathcal{N}, \theta}(x_m | x_i, \theta_j) f_{\theta|\mathcal{N}}(\theta | x_i) \\ &= \sum_{j \in \{H, L\}} f_{\mathcal{M}_b|\theta}(x_m | \theta_j) f_{\theta|\mathcal{N}}(\theta | x_i) \\ &= \sum_{j \in \{H, L\}} f_{\mathcal{M}_b|\theta}(x_m | \theta_j) \frac{f_{\mathcal{N}|\theta}(x_i | \theta_j) q_j}{f_{\mathcal{N}}(x_i)} \end{aligned}$$

where the second line follows from the conditional independence of x_m and x_i given θ , the third line from Bayes' law, and the distributions $\mathcal{M}_b, \mathcal{N}$ are given in (10) and (52), from which the distributions $\mathcal{M}_b | \theta, \mathcal{N} | \theta$ can be backed out.

C Calibration

We provide further details about Section 3.5. First, we present the generic bond pricing framework that we use to compare our model with the data and interpret our results. Second, we provide further details on the data we use for our calibration exercise.

C.1 Interpretation

A generic bond pricing model. We consider a textbook asset pricing model, such as that in Chapter 1 of Cochrane (2005). Take a nominal bond that, at the end of the period, delivers a stochastic real payoff denoted by $\tilde{\theta}$. This random payoff reflects various sources of risk, including inflation or default during the holding period, as well as resale price fluctuations in the case of a long-term bond. The equilibrium bond price conditional on a generic aggregate state x_m satisfies $Q(x_m) = \mathbb{E}[\zeta \tilde{\theta} | x_m]$, where ζ is a generic stochastic discount factor such that the real risk-free rate is given by $1 + r = 1/\mathbb{E}[\zeta | x_m]$. Using the definition of covariance and recalling that $\theta := \tilde{\theta}/(1 + r)$, we can rewrite the bond pricing equation as

$$\lambda(x_m) = \mathbb{E}[\theta | x_m] - Q(x_m) \tag{53}$$

where $\lambda(x_m) := \text{Cov}(\zeta, \theta | x_m)$ is a generic risk adjustment, which includes various premia such as those for term, liquidity, default, and inflation risk.

Our model. Consider now our model, where equation (17) implies that the average wedge

$$-\Delta(b) = \mathbb{E}[\theta] - \mathcal{Q}$$

is the natural (negative) counterpart of the average equilibrium bond premium in the simple generic bond pricing framework of (53); in particular $\Delta(b) = -\mathbb{E}[\lambda]$.

Belief heterogeneity. To account for the distribution of individual beliefs in our model, we compute the individual expectation of the distribution of the learning wedge across states

$$\mathbb{E}[\theta - Q(p(x_m)) | x_i] = \int_{-\infty}^{+\infty} [\theta - Q(p(x_m))] f_{\mathcal{M}_0 | \mathcal{N}}(x_m | x_i) dx_i \quad (54)$$

where note the following holds: $\mathbb{E}[\theta | x_i] = \mathbb{E}[\mathbb{E}[\theta | x_m] | x_i]$. This represents the expected gain for an investor observing private signal x_i , but before observing the state x_m . By (53), $\mathbb{E}[\theta - Q(p(x_m)) | x_i] = \mathbb{E}[\lambda | x_i]$ in a generic bond pricing framework.

Remarks on ex-ante view. Finally, to address the dynamic nature of the data, we adopt an ex-ante perspective—that is, before any shock realizes. This approach also avoids taking a stand on what identifies x_m in the data. In this case, investors would agree the value of the asset is given by $\mathbb{E}[\lambda] = \int \mathbb{E}[\lambda | x_i] f_{\mathcal{N}}(x_i) dx_i$.

C.2 Calibration Exercise and Data Details

Here we explain how we map elements of our model to observables for calibration. We focus on the 18th of March 2009, the date of the first announcement of Large-Scale Asset Purchases that would be directed to long-term treasuries. To compute expected real bond returns at the individual forecaster level, we take the nominal yield to maturity of the 10-year constant-maturity US Treasury bond on March 17th, 2009, and subtract expected average inflation over the maturity of the bond (i.e., 10 years) which we take from the 2009 Q1 release of the Survey of Professional Forecasters (SPF) run by the Philadelphia Fed. An alternative approach to derive the distribution of real return forecasts would be to take bond yield forecasts and subtract

inflation forecasts. We choose not to follow this approach because, as is well known, in the SPF the survey question about long-term forecasts of Treasury yields was ambiguously phrased—and the wording was indeed changed from 2014:Q1.

LSAP1 data. As a reference for the empirical effect of the LSAP1 announcements on real yields, we take the two-day change in the real yield of 10-year TIPS from Table 4 of Krishnamurthy and Vissing-Jorgensen (2011), which is -59 basis points. An alternative approach would be to take the two-day change in the yield of nominal 10-year bonds from Table 1, which is -41 basis points, and then subtract the 22 basis point increase in 10-year inflation swaps, which delivers a comparable total change of -63 basis points.

As a reference for the size of APs in the model, we take the size of the LSAP1 announcement relative to the face value of marketable, non-indexed U.S. Treasury bonds and notes with residual maturity larger than 5 years as of February 2009, amounting to \$585.821 billions in Treasury notes and \$609.353 billions in Treasury bonds, as stated in the monthly statement of the public debt of the United States on the 28 of February 2009. The Fed announcement of purchasing up to “300 billion of longer-term Treasury securities over the next six months” is thus equivalent to about 25% of the existing marketable stock of debt (or about 20% if we include TIPS).

Notably, the absolute values of θ do not affect the average wedge—only the 500 basis point range matters—reinforcing that our model captures price elasticities rather than price levels.