Fiscal-Monetary Interactions and the FTPL: A Review

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Some course info

Course material: slides + class notes
Grade from final project: research proposal. A few pages long, with motivation, contribution and review of literature, modelling and/or empirical strategy.

Tentative syllabus (will evolve)

1. Classic references on the consolidated government budget constraint and price level determination
2. Some empirical evidence
3. Central bank balance sheets and fiscal-monetary interactions
4. Strategic models of fiscal-monetary interactions
5. Expectations, coordination, monetary policy and debt crises

So far

Fiscal and monetary policy are unavoidably intertwined, can only "ignore" one by making extreme assumptions and leaving it in the background

FTPL, "debt valuation equation"

Combination of M-F policy rules can lead to multiple/stable/unstable equilibria

A 2-period model

Quick review of concepts seen so far with a simple 2-period model similar to Leeper (1991)

Households' problem

$$\max \quad c_0 + v\left(m_0\right) + \beta c_1 \quad \text{where } m_0 = \frac{M_0}{P_0}$$
$$\text{s.t.} \quad c_t + \frac{M_t + B_t}{P_t} + \tau_t = y + \frac{M_{t-1} + B_{t-1}(1+i)}{P_t}$$

Government BC

$$\frac{M_t + B_t}{P_t} + \tau_t = g + \frac{M_{t-1} + B_{t-1}(1+i)}{P_t}$$

Market clearing

$$\begin{aligned} c_t + g &= y \\ B_t^{hh} &= B_t \\ M_t^{hh} &= M_t \end{aligned}$$

A 2-period model
Equilibrium conditions

Let $s_t := \tau_t - g$

$$M_1 = B_1 = 0 \qquad \text{TVCs}$$

$$1 = \beta(1+i)\frac{P_0}{P_1} \qquad \text{(EE)}$$

$$\frac{i}{1+i} = v'(m_0) \qquad \text{(money demand)}$$

$$\frac{L_{-1}}{P_0} = s_0 + \beta s_1 + m_0 \frac{i}{1+i} \qquad \text{(gPVBC)}$$

3 equations in 6 variables $(s_0, s_1, i, M_0, P_0, P_1) \rightarrow$ 3 policy variables are free to choose

A 2-period model

Quantity theory determines the price level

$$M_1 = B_1 = 0 \qquad \text{TVCs}$$

$$1 = \beta(1+i)\frac{P_0}{P_1} \qquad \text{(EE)}$$

$$\frac{i}{1+i} = v'(m_0) \qquad \text{(money demand)}$$

$$\frac{L_{-1}}{P_0} = s_0 + \beta s_1 + m_0 \frac{i}{1+i} \qquad \text{(gPVBC)}$$

Let CB choose (i, M_0). Then

- (money demand) $\rightarrow P_0$
- (EE) $\rightarrow P_1$
- (gPVBC) $\rightarrow s_0 + \beta s_1$

A 2-period model

Fiscal theory determines the price level

$$
\begin{aligned}
M_1 = B_1 &= 0 && \text{TVCs} \\
1 &= \beta(1+i)\frac{P_0}{P_1} && \text{(EE)} \\
\frac{i}{1+i} &= v'\left(m_0\right) && \text{(money demand)} \\
\frac{L_{-1}}{P_0} &= s_0 + \beta s_1 + m_0 \frac{i}{1+i} && \text{(gPVBC)}
\end{aligned}
$$

Let Treasury choose (s_0, s_1), CB choose i. Then

- (money demand) $\to m_0$
- (gPVBC) $\to P_0$ (and then M_0 via money demand)
- (EE) $\to P_1$

A 2-period model

Cashless environment

We can also do the same exercise without money and seigniorage: P_t is the "conversion rate" btw government paper and goods

$$B_1 = 0 \qquad \text{TVCs}$$

$$1 = \beta(1+i)\frac{P_0}{P_1} \qquad \text{(EE)}$$

$$\frac{L_{-1}}{P_0} = s_0 + \beta s_1 \qquad \text{(gPVBC)}$$

2 equations in 5 variables $(s_0, s_1, i, P_0, P_1) \rightarrow$ 3 policy variables are free to choose

Let Treasury choose (s_0, s_1), CB choose i. Then

- (gPVBC) $\rightarrow P_0$
- (EE) $\rightarrow P_1$

A 2-period model

Taking stock

Equilibrium variables depend on both F's and M's policies. What determines what depends on assumptions/rules/etc

So far, i determines (future) inflation, F or M determines P_0 (or current inflation)

We considered *actions*, not *rules*.
But we could make i contingent on L_{-1} or s_0, and then F would determine future inflation too

The gPVBC is an equilibrium condition: hhPVBC + market clearing + Euler equation

$$\frac{L_{-1}}{P_0} = s_0 + \beta s_1 + f\left(\frac{i}{1+i}\right)$$

$1/P_0$ is the value of gov't liabilities, it is what adjusts off equilibrium

The “debt valuation equation”

The gPVBC is an equilibrium condition: hhPVBC + market clearing + Euler equation

$$\frac{L_{-1}}{P_0} = s_0 + \beta s_1 + f\left(\frac{i}{1+i}\right)$$

- P_0 is the value of gov't liabilities, it is what adjusts off equilibrium
- How? NPV(taxes) = public debt = private wealth = private demand
- Think of surpluses as dividends, P_0 as stock price (“money as stock”, Cochrane (2005))

A dynastic model

A simple infinite-horizon model from Christiano and Fitzgerald (2000)

Straight to equilibrium conditions

$$1 + \rho = (1 + i_t)\frac{P_t}{P_{t+1}} \tag{EE}$$

$$\frac{M_t}{P_t} = A i_t^{-\alpha} \tag{MD}$$

$$\lim_{t\to\infty} \frac{B_t}{\Pi_{j=0}^{t}(1 + i_j)} = 0 \tag{TVC}$$

Government budget

$$\frac{B_{t+1}}{P_t}\frac{1}{1 + i_t} + \tau_t - g_t + \frac{M_t - M_{t-1}}{P_t} = \frac{B_t}{P_t}$$

A dynastic model

Fiscal and monetary policy

First policy assumption: CB sets $i_t = i$

- implies constant money demand: $m_t = Ai^{-\alpha}$
- in turn implies constant seigniorage:

$$\frac{M_t - M_{t-1}}{P_t} = m_t - m_{t-1}\frac{P_{t-1}}{P_t} = m\left(1 - \frac{1+\rho}{1+i}\right) = Ai^{-\alpha}\frac{i-\rho}{1+i} =: s^m$$

Second policy assumption: Treasury sets $\tau_t - g_t = s^f$

$\Rightarrow$ Total government "tax" revenues constant $s := s^m + s^f$

Law of motion for debt

$$\frac{B_{t+1}}{P_t}\frac{1}{1+i} + s = \frac{B_t}{P_t} \quad \Leftrightarrow \quad b_{t+1} = (1+\rho)(b_t - s)$$

A dynastic model

Price level determination

Use the (EE) to write the TVC as $\lim_{t\to\infty} \dfrac{1}{(1+\rho)^t}\dfrac{B_t}{P_t} = 0$ Iterating the gBC forward we get

$$\frac{B_0}{P_0} = \sum_{t=0}^{\infty} \frac{s}{(1+\rho)^t} + \lim_{T\to\infty} \frac{1}{(1+\rho)^T}\frac{B_T}{P_T} = s\frac{1+\rho}{\rho}$$

What determines what?

- i determines inflation via the Fisher equation
- s^f determines P_0 via the gPVBC. Real debt B_t/P_t is constant over time

Can also see this by looking at the difference equation for real debt $b_{t+1} = (1+\rho)(b_t - s)$, which has general solution $b_t = (1+\rho)^t\left(b_0 - b^*\right) + b^*$ for $b^* := s\frac{1+\rho}{\rho}$

A dynastic model

Different policies lead to indeterminacy

Suppose that total tax revenues follow

$$s_t = \begin{cases} s & b_t \leq \bar{b} \\ \frac{1+\rho-\gamma}{1+\rho} b_t - \frac{\xi}{1+\rho} & b_t > \bar{b} \end{cases} \quad \text{where} \quad \gamma \in [0,1),\ \bar{b} \in \left(s\frac{1+\rho}{\rho}, \frac{\xi}{1-\gamma} \right)$$

Then the law of motion for real debt becomes

- $b_{t+1} = (1+\rho)(b_t - s)$ when $b_t \leq \bar{b}$
- $b_{t+1} = \gamma b_t + \xi$ otherwise, in which case $b_t \to \xi/(1-\gamma)$

The price level is *not* determined!

- for all $b_0 \geq b^*$, real debt converges and the TVC holds
- this policy rule is reminiscent of Maastricht-Treaty rules
- similar in spirit to passive fiscal-passive monetary

Summing up

Dynastic and 2-period model share same conclusions

In dynastic model we consider policy rules, don't count variables, focus on convergence of real debt to satisfy the TVC

Debt valuation equation intuition is exactly the same

Aiyagari and Gertler (1985)

Simple setting that models F-M dominance with a single parameter, develops idea of *backing* at the basis of many modern papers

OLG setup, generations live for two periods

Pure exchange economy, endowment e^y when young, nothing when old

Three assets: money, bonds, equity with constant dividend (Lucas tree)

Household problem

$$\max_{c^y, c^o, M, B, A} \quad \mathbb{E}[(c^y)^\alpha \left(\frac{M}{P_t}\right)^\beta c^o]$$

$$\text{s.t.} \quad c^y = e^y - \tau_t^y - \frac{M}{P_t} - \frac{B}{(1+i_t)P_t} - Av_t$$

$$c^o = \frac{M+B}{P_{t+1}} + A(d + v_{t+1}) - \tau_{t+1}^o$$

Features: MIU, linear utility when old, taxes in both periods

Optimality conditions

No-arbitrage between bonds and equity

$$(1 + i_t)\mathbb{E}\frac{P_t}{P_{t+1}} = \mathbb{E}\frac{d + v_{t+1}}{v_t}$$

Money demand as a function of consumption, or the PV of wealth

$$\frac{M}{P_t} = \frac{\beta}{\alpha}\frac{i_t}{1 + i_t}c^y, \qquad \frac{M}{P_t} = \frac{1 + i_t}{i_t}\beta\eta[e^y - \tau_t^y - \tau_t^{o'}]$$

Bond and equity demand as a function of the PV of wealth

$$\frac{B}{P_t(1 + i_t)} + Av_t - \tau_t^{o'} = (1 - \beta/i_t)\eta[e^y - \tau_t^y - \tau_t^{o'}]$$

where

$$\tau^{o'} = \frac{\mathbb{E}[\tau_{t+1}^o]}{(1 + i_t)\mathbb{E}[P_t/P_{t+1}]}; \quad \eta = \frac{1}{1 + \alpha + \beta}$$

Government

Spending = random fraction $\tilde{g}_t$ (with mean $\bar{g}$) of total endowment e^y

gBC

$$\frac{B_{t-1}}{P_t} + \tilde{g}_t e^y = \tau_t^y + \tau_t^o + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t(1+i_t)}$$

Key assumption: PV of taxes (T_t) backs fraction $1-\delta$ of the real value of debt B_{t-1}/P_t
A policy that satisfies this is

$$\tau_t^y + \tau_t^o = (1-\delta)\left[\frac{B_{t-1}}{P_t} - \frac{B_t}{P_t(1+i_t)}\right]$$

Taxes in t pay for share $1-\delta$ of debt not rolled over

Present value of policies

Let $\tilde{\beta} := \dfrac{1}{(1+i_t)\mathbb{E}[P_t/P_{t+1}]}$

PV of taxes, spending, seigniorage revenues in recursive form

$$\begin{aligned} T_t &= \tau_t^y + \tau_{t+1}^o + \tilde{\beta}\, \mathbb{E}[T_{t+1}] \\ G_t &= \tilde{g}_t e^y + \tilde{\beta}\, \mathbb{E}[G_{t+1}] \\ \mathcal{M}_t &= \frac{M_t - M_{t-1}}{P_t} + \tilde{\beta}\, \mathbb{E}[\mathcal{M}_{t+1}] \end{aligned}$$

Then the usual gPVBC/debt valuation equation becomes

$$\frac{B_{t-1}}{P_t} = \mathcal{M}_t + T_t - G_t$$

and since $T_t = (1-\delta)B_{t-1}/P_t$ we have that $\mathcal{M}_t - G_t = \delta B_{t-1}/P_t$

Some observations

Plugging our tax rule in the period-by-period gBC

$$\delta \frac{B_{t-1}}{P_t} + \tilde{g}_t e^y = \frac{M_t - M_{t-1}}{P_t} + \delta \frac{B_t}{P_t(1+i_t)}$$

If $\delta = 0$ we have the "Ricardian" case, $\tilde{g}_t e^y = \frac{M_t - M_{t-1}}{P_t}$ and seigniorage only depends on spending since debt is fully backed by taxes

Note: Ricardian "usually" means that taxes *can* fully back debt *and* spending, so seigniorage needs not adjust to spending

Further assume that the policy keeps households' lifetime income independent of govt bonds

$$\tau_t^y = -(1-\delta)\frac{B_t}{(1+i_t)P_t}$$

$$\tau_{t+1}^o = (1-\delta)\frac{B_{t-1}}{P_t}$$

Equilibrium

Given initial M_0, B_0, it is a sequence of allocations and prices such that households maximise and markets clear

$$A = 1$$
$$B = B_t$$
$$M = M_t$$

We will now plug govt policies in the households' optimality conditions and study the implications of Ricardian vs non-Ricardian policies

Analysis

Look for a stationary equilibrium where $v_t = \theta \eta e^y$, with θ undetermined

Using asset demand functions and govt policy, the equilibrium interest rate and price level satisfy

$$P_t = \frac{1}{\tilde{v}_t} \frac{M_{t-1} + \delta B_{t-1}}{e^y}$$
$$i_t = \left(\delta \left(\frac{M_t + B_t}{M_t} - 1 \right) + 1 \right) \frac{\beta}{1-\theta}$$

where $\tilde{v}_t := (1 - \beta - \theta)\eta - \tilde{g}_t$

Using the no-arbitrage condition between bonds and equity we get $\theta = \dfrac{d}{d + e^y(\beta\eta - \bar{g})}$

Analysis

Equilibrium is now fully characterised. Consumption

$$
\begin{aligned}
c_t^y &= \alpha\eta e^y \\
c_t^o &= d + [(1+\beta)\eta - \tilde{g}_t]e^y
\end{aligned}
$$

and real returns (laws of motion for prices)

$$
\begin{aligned}
\frac{(1+i_t)P_t}{P_{t+1}} &= \frac{\tilde{v}_{t+1}}{\eta(1-\theta)} \\
\frac{d+v_{t+1}}{v_t} &= 1 + \frac{d}{\theta\eta e^y}
\end{aligned}
$$

$\Rightarrow$ none of this is a a function of govt policy!!! Model is frictionless, and policy keeps income insulated from debt stock

Nominal variables instead depend on policy and Ricardian regime

Ricardian vs non-Ricardian regimes

δ determines the extent to which seigniorage depends on debt

$$P_t = \frac{1}{\tilde{v}_t} \frac{M_{t-1} + \delta B_{t-1}}{e^y} \qquad \uparrow \text{ in } \delta$$

$$i_t = \left(\delta \left(\frac{M_t + B_t}{M_t} - 1 \right) + 1 \right) \frac{\beta}{1-\theta} \qquad \uparrow \text{ in } \delta$$

Remarks

- $\delta = 0$ is Ricardian regime: P_t depends on M_{t-1} alone (as in quantity theory), and i_t is independent of M_t/B_t because intertemporal taxes keep relative demand constant
- $\delta > 0$ is non-Ricardian: debt is partially backed by future money creation/seigniorage, which in turn implies future inflation (note that δ affects P_t/P_{t+1} via i_t)
- when $\delta < 1$, OMO affect price level and inflation: ΔB_t less than 1-to-1 with ΔM_t
- when instead $\delta = 1$, debt is fully backed by future money creation and only the total size of government liabilities matters. i_t still depends on M_t/B_t to affect relative demand

Further remarks

- Depending on δ, debt matters for price stability: bond issuance is inflationary when it is backed by future seigniorage

- Basic intuition carries through to most models of F-M interactions: debt must be backed by the PV of either fiscal or monetary revenues

- Govt policy can have real effects if we consider non-neutral tax policies or nominal frictions

References

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