# Is Inflation Default? The Role of Information in Debt Crises\*

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#### Abstract

We study the information sensitivity of government debt denominated in domestic vs. foreign currency: the former is subject to inflation risk and the latter to default. Default only affects sophisticated bond traders, whereas inflation concerns a larger and less informed group. Within a two-period Bayesian trading game, differential information manifests itself in the secondary market, and we display conditions under which debt prices are more resilient to bad news even in the primary market, where only sophisticated players operate. Our results can explain debt prices across countries following the 2008 financial crisis, and also provide a theory of "original sin."

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#### 1 Introduction

The sovereign borrowing experience of advanced economies in the aftermath of the financial crisis of 2008 has once again highlighted the important role of the currency in which debt is denominated. Countries which had control over their monetary policy, such as the United States, the United Kingdom, and Japan, were able to borrow at extremely low rates throughout the episode, even though they experienced very high deficit/GDP ratios (the UK) or debt/GDP ratios (Japan). In contrast, peripheral Eurozone countries were either unable to borrow from the market (Portugal, Ireland) or faced volatile interest rates when doing so (Italy, Spain).<sup>1</sup>

In previous crises, such as Latin America in the 1980s and Asia in 1998, currency mismatch was identified as a source of instability, and hence many authors have studied the role of the "original sin" or other causes of financial underdevelopment that led to the mismatch. In the presence of nominal rigidities, having an own currency may allow for a quick devaluation as a means to adjust to domestic shocks, preserving the country's economy and ability to repay its debt, but only if this debt is denominated in domestic currency.<sup>2</sup>

Compared to those crises, 2008 presents some important differences. First, financial underdevelopment of the debt market was not a cause of the Eurozone countries' difficulties, since they all had an ample and liquid market for government debt denominated in their home currency before joining the Euro. Second, it is not clear that the ability to devalue and thereby spare the economy from a deeper recession was a major factor in explaining the different behavior of interest rates: while it is true that the United Kingdom depreciated the Pound in the wake of the recession, the Yen appreciated substantially against the Euro, exacerbating the slump in Japan.

Our goal is to dig deeper in the source of frictions that may make the price of a country's debt less sensitive to adverse news on the government solvency. A premise of our analysis is

<sup>&</sup>lt;sup>1</sup>See e.g. Plender [43] and De Grauwe [26].

<sup>&</sup>lt;sup>2</sup>Krugman [38,39] sketches a theory whereby an asymmetry arises because defaults would lead to larger real haircuts for bondholders than inflation. While it is true that a default is a discrete event and inflation erodes the value of repayments over time, it is not a priori obvious that the cumulative losses would be different in the two scenarios. We consider a benchmark in which losses are the same. Our mechanism would of course remain at work even if inflation were less costly for creditors, as the two channels would complement each other.

that a domestic currency partially insulates a country from default risk, as the government may be able to lean on the central bank to act as a residual claimant on government debt securities. However, the resulting increase in the money supply would be bound to generate inflation, so that default risk would be replaced by inflation risk and we might expect interest rates to spike similarly under the two scenarios. Yet in practice inflation expectations, as well as the behavior of actual inflation, are very sluggish compared to the speed with which default crises, such as Greece's, unfold.

To reconcile these facts, we study an economy where private agents have dispersed and heterogeneous information about the government's ability to repay its debt. We contrast two situations: in the first one, contracts are denominated in an outside currency (the "Euro"), and the government is forced to outright default when its tax revenues fall short of debt promises, while in the second one a domestic currency is present (the "Yen"), and the government resorts to the printing press and eventual inflation to cover any shortfalls. To assess the implications of this difference, we analyze a two-period Bayesian trading game. In both the Euro and the Yen economy, government debt is subscribed in the first period by (sophisticated) bond traders, such as banks or relatively wealthy investors.<sup>3</sup> In deciding their actions, these traders take into account that the different nature of risk across the two cases (inflation vs. default) implies that different actors will be pivotal for future prices. Specifically, in the Euro economy, the default premium in the secondary market is driven by the beliefs of other players entering the bond market, who are likely to be as sophisticated as the original bond traders. In the Yen economy, the evolution of prices is driven by the beliefs about fiscal solvency of a (larger and) less sophisticated section of the population that uses Yen to trade but does not participate in bond markets.<sup>4</sup> Other than this difference, we impose as much symmetry as possible between the two economies: agents start with identical priors over government solvency, bond traders receive signals with equal precision across the two economies, and the fixed haircut upon default is matched to the loss in value due

<sup>&</sup>lt;sup>3</sup>The key distinction for us is not whether these traders are foreign or domestic, but rather that they are a restricted set of the population that is active in financial markets.

<sup>&</sup>lt;sup>4</sup>We do not model explicitly the frictions that prevent this population from accessing government bond markets. Any fixed cost of access would exclude poorer individuals and is enough to generate our story.

to inflation. All these assumptions allow us to concentrate on the consequences of heterogeneous information.

We analyze this problem in a dynamic version of a noisy rational expectations equilibrium in the tradition of Grossman [32] and Hellwig [35]. Admati [1] first studied learning spillovers in a static environment with multiple assets. The connection between spillovers across assets and over time has been emphasized by Brennan and Cao [16] in the context of a model that features a one-time private signal. Brennan and Cao [17] study trade among long-lived and heterogeneously informed agents who learn fundamentals gradually from each other. They use their model to characterize the sign of the flows and their covariance with price movements. They do not focus on the volatility of the price, which in their economy is dominated by the cumulative effect of learning.<sup>5</sup> Our results are particularly related to Allen, Morris, and Shin [5] (AMS), who studied an environment in which an asset goes through multiple rounds of trade among overlappinggenerations, as is our case. They emphasized the dampening effect of higher-order beliefs on price movements and conversely the greater emphasis that public signals take in that context. Their results represent a polar case of the analysis that we entertain along two dimensions. First, they only consider assets with a linear payoff (as is the case for Brennan and Cao [17]). As a premise to our application, we derive results that apply to nonlinear payoffs in general. This is useful to us because we are interested in debt contracts, which are inherently nonlinear. Second, AMS only consider the effect of adding rounds of trade. In our context, this can be viewed as an extreme version of the comparative statics of interest. Specifically, our model with two rounds of trade collapses to a model with a single round if we assume that second-period agents are perfectly informed about fundamentals, so that the price in the second period is equal to the terminal payoff of the asset. The AMS result applies thus to comparing an economy with infinite precision of signals in the second round to one where the precision is finite. We are interested in studying comparative statics about the precision of the information of second-period agents without going to this extreme. This is important because, as shown in our main proposition (Proposition 3), the comparative statics are not necessarily globally monotone.

<sup>&</sup>lt;sup>5</sup>Amador and Weill [6,7] also considered learning from aggregate prices, in the context of stylized macroeconomic models.

The structure of our model is closely related to Hellwig, Mukherji, and Tsyvinski [34] and Albagli, Hellwig, and Tsyvinski [4] (AHT), where a flexible and particularly tractable specification of noisy information aggregation in market prices is developed. Our paper considers a version of their model in which trade occurs repeatedly.

Due to the nature of the payoff that we consider in our application, we are also related to the literature that has used the global-games approach pioneered by Carlsson and van Damme [22] and Morris and Shin [40].<sup>6</sup> Our theorems are related to Iachan and Nenov [36], whose paper presents a systematic analysis of comparative statics results with respect to the precision of information in static global games; in contrast to their analysis, we are interested in the role of dynamics.<sup>7</sup>

When analyzing the effect of changes in second-period information on the first-period price of an asset, we can distinguish between effects arising from the first and second moment of beliefs of first-period agents about future prices. For linear payoffs, first moments are all that matters and results are simpler. As in AMS, our comparative statics are driven by the failure of prices to satisfy the equivalent of a law of iterated expectations. This failure arises both because first-period agents have some private information that is not fully aggregated in the price and possibly because the first-period price itself may not be perfectly observed by second-period traders. Both of these reasons lead traders in the first period to respond less aggressively to their incoming information about fundamentals. This effect is more pronounced, the lower is the quality of information available to agents in the second period. Hence, for linear payoffs we can unambiguously conclude that the price in both periods will be less responsive to incoming news, the less well informed agents are in the second period of trade. In the case of nonlinear payoffs, their concavity or convexity interacts with the second moment of beliefs entertained by the agents. When studying the effect of second-period information on first-period prices, a trade-

<sup>&</sup>lt;sup>6</sup>The role of signaling in global games has been studied by Angeletos, Hellwig, and Pavan [8], and the efficiency of information acquisition has been further analyzed by Angeletos and Pavan [10,11]. Dasgupta [25] and Angeletos, Hellwig and Pavan [9] studied learning in dynamic global games.

<sup>&</sup>lt;sup>7</sup>In a static context, the role of differential information has also been studied Corsetti et al. [24], who consider a global game with a single large player who may be more or less informed than a continuum of small players.

off emerges: better information implies that the second-period price tracks fundamentals more closely, but it also implies that traders in the second period will rely more on their own signals and less on the common information contained in the prior and the first-period price. Whether the second-period price becomes more or less predictable from the perspective of agents in the first period is thus ambiguous. After deriving results for general nonlinear payoffs, we apply them to our case of sovereign debt and default, where clear-cut comparisons are possible.

Our two main propositions compare the responsiveness of the first-period price to incoming news in the Euro economy and the Yen economy, which only differ by the precision of signals observed by agents in the second period. We prove that the price in the Euro economy always responds more to new information in both periods when either of the following is true:

- 1. The bond traders which participate in the secondary market of Euro bonds have at least as precise information as first-period traders;<sup>8</sup> or
- 2. The first-period price is observed in the second period with sufficient noise.

In sum, our results confirm that heterogeneity between a small sophisticated group of bond traders and a large, less informed population that drives the aggregate price level can explain why domestic-currency debt may be less information-sensitive than foreign-currency debt (or debt denominated in a common currency not directly controlled by the domestic central bank). This result can account for why a country which starts from a favorable prior condition may be able to better withstand the arrival of bad news. Conversely, our results also suggest that a country who is perceived as very likely to default may find it easier to borrow in foreign currency in the few instances in which its fundamentals are comparatively more favorable: sophisticated bond traders would find it easier to spot the presence of such conditions, while a pessimistic population may immediately fear (and trigger) hyperinflation. In this way, our paper provides an alternative explanation for the "original sin" and connects to the vast literature in international

<sup>&</sup>lt;sup>8</sup>Note that this proposition does not require comparing the precision of information of second-period vs. first-period agents in the *Yen economy*. In principle, this comparison could be ambiguous, because second-period agents are assumed to be less sophisticated in the Yen economy, but the passage of time might have revealed extra information about government solvency.

economics that has studied the role of currency mismatch, particularly in the years that follow the 1998 Asian crisis. A review of competing theories of the origins of the mismatch appears in Eichengreen and Hausmann [29]. Examples of theories of crises where foreign-currency debt plays an important role are Aghion, Bacchetta, and Banerjee [2] and Calvo, Izquierdo, and Talvi [21]. Particularly relevant for our analysis is Bordo and Meissner [15]: they show that currency mismatch and "original sin" are not necessarily harbingers of more frequent crises, provided fundamentals are managed correctly. This is reminiscent of our result, in which it is not necessarily the unconditional probability of eventual default or inflation that increases when debt is denominated in foreign currency: fragility manifests itself instead as a greater volatility of debt prices.

The interplay between secondary markets and sovereign spreads has received attention in other contexts. The participation of foreign vs. domestic investors in secondary markets and the resulting incentives for the government to default have been analyzed by Broner, Martin and Ventura [18]. In Arellano, Bai, and Lizarazo [13] movements in secondary-market prices after default cause spillovers among sovereign debtors, as risk-averse creditors pull back from risky lending in the aftermath of losses. Imperfect information in sovereign debt markets plays an important role in Yuan [47], where losses stemming from bad news in one market may lead liquidity-constrained informed creditors to pull resources invested in other sovereign bonds, leading to less informative prices and contagion across markets. Sandleris [44] studies an economy where a default reveals adverse information about the state of the economy, with negative consequences for private investment, and Gu and Stangebye [33] study variations in risk premia driven by endogenous time-varying information precision. None of the papers above studies the role of currency denomination and differential information in determining bond prices. Combining our mechanism with those of several of these other papers is likely to yield interesting interactions worthy of future exploration.

The rest of the paper is structured as follows. Section 2 introduces the basic setup. In Section 3 we analyze in detail a general version of the two-stage Bayesian trading game that is the building block of our model. This abstract setup allows for a clear exposition of the forces at

play. Section 4 deals with our application to sovereign debt. Specifically, Subsection 4.1 provides more technical intuition based on the general results of the previous section, while Subsection 4.2 presents and discusses our main conclusions emphasizing the macro implications for sovereign debt. In Section 5 we show that results are robust to an extension in which the default threshold is endogenous to the price at which government debt is issued, as in Calvo [20]. Conclusions and avenues for future research appear in Section 6. Appendix A provides a full description of a stylized macroeconomic model which maps into the Bayesian trading game that we study.

## 2 The Setup

We consider an economy that lasts for three periods, t = 1, 2, 3. The economy is populated by two overlapping generations of traders and a government. The government follows a mechanical strategy: it auctions one unit of debt in period 1 and repays it in period 3. We follow here Eaton and Gersovitz [28] and fix the face value of debt at redemption, which is normalized to one, letting  $q_1$  be the endogenous price at which debt will be issued.<sup>9</sup> Repayment in period 3 depends on the realization of a fiscal capacity shock  $\theta$ . Specifically, if  $\theta \geq \bar{\theta}$ , then the government has enough revenues to repay the debt in full. In contrast, when  $\theta < \bar{\theta}$ , a default occurs, in which case we assume a fixed haircut and the government only repays  $\delta \in (0,1)$ . All agents share a common prior about  $\theta$ , which is normal with mean  $\mu_0$  and variance  $1/\alpha_0$ .

Each generation of private agents is composed by a unit measure of informed traders and a random mass of noise traders. In their first period of life, informed agents have an endowment that they divide between storage (at a rate normalized to zero) and purchases of the asset. In the second period of their life, they liquidate any asset position and consume the proceeds of their investment. The first generation buys bonds from the government in the first period and resells them in the secondary market in the second period; the second generation of traders buys in the secondary market and keeps the asset until maturity in the final period. It follows that in both periods of trade the supply is inelastic, and all the action occurs on the buyers' side. Informed traders are risk neutral and choose their portfolio to maximize expected consumption.

<sup>&</sup>lt;sup>9</sup>The proceeds of the sale are consumed by the government in period 1.

Each informed trader i in period t receives a noisy private signal  $x_{i,t} = \theta + \xi_{i,t}$ , where  $\xi_{i,t}$  is distributed according to a normal distribution  $N(0, 1/\beta_t)$ , and we assume that a law of large numbers across agents applies as in Judd [37]. Based on this signal, informed traders submit price-contingent demand schedules. In submitting their demand, they take into account that the price  $q_t$  of the asset in period t is affected by all other traders' demand and is thus an endogenous, public source of information. To preserve tractability, we assume that asset holdings are limited to  $\{0,1\}$ . Since our model is dynamic, we must also specify how agents learn from the past: we will assume that second-period traders receive a noisy public signal  $\rho$  of the first-period price, with a distribution that we will specify later on. We are interested in the special cases where recall of the past price is either perfect or infinitely noisy, but adopting a general specification will allow us to draw broader results.

Noise traders generate a residual uncontingent demand  $\Phi(\epsilon_t/\sqrt{\psi_t})$  for the asset, where  $\Phi$  is the cumulative standard normal distribution function,  $\psi_t > 0$ , and  $\epsilon_t$  is itself distributed according to a standard normal distribution.<sup>11</sup> The mass of noise traders is independent of the fundamental and of informed traders' signals. As is standard in this class of models, the presence of noise traders ensures that equilibrium prices do not fully aggregate information, thereby revealing the fundamental.

We will contrast two economies, one in which government debt is denominated in local currency (the "Yen") and one in which it is denominated in a currency over which the government has no control (the "Euro"). The role of issuing debt in domestic currency is that it avoids explicit default, which is replaced by inflation instead. From the perspective of the dynamic trading game, we treat inflation and default symmetrically, that is, the haircut suffered by holders of government liabilities and the probability of the haircut are the same. Through this channel,

<sup>&</sup>lt;sup>10</sup>Given that we assume risk neutrality, the optimal demand schedule will take the form of a reservation price, below which strategic agents are willing to buy the asset. The lower bound of 0 is equivalent to a short-selling constraint. The upper bound of 1 is equivalent to an indivisibility assumption, which implies that traders cannot hold a non-integer position and do not have enough resources to buy two units. Consistently with the indivisibility assumption, we impose that their holdings must be either 0 or 1, but risk neutrality implies that the analysis is unchanged if traders are instead allowed any position in [0, 1].

<sup>&</sup>lt;sup>11</sup>When  $\psi_t = 1$ , this implies that the residual demand is uniform in [0,1].

bad news about fiscal solvency have the same negative effect on the price of government debt: in one case bad news imply high interest rates because of inflation risk, and in the other because of default risk.

The asymmetry between the inflation and default risk that we emphasize in this paper concerns differential information by secondary-market participants. The motivation for this asymmetry stems from a different interpretation of who is the relevant participant in the "secondary market" in period two. In the case of the Euro economy, the secondary-market price is dictated by the new generation of bond traders who will take over; short of an immediate fiscal adjustment, which we rule out, there is nothing that the government can do to dampen fluctuations in the price of its debt. In the Yen economy, the ability to print money to intervene in the market for government debt can temporarily substitute for varying demand by bond traders. The extent by which these interventions are stabilizing depends on the beliefs about eventual fiscal solvency of a larger section of the population that uses Yen to trade but does not participate in bond markets; it is likely that they are less well informed about government finances. In Appendix A we provide a microfounded model which features "workers" who use exclusively cash and "bond traders" who hold the government bonds, and we show formally how this can lead workers to be the pivotal agents in pricing inflation risk in the second period for the Yen economy, while bond traders are pivotal in pricing default risk for the Euro economy. From the perspective of the Bayesian trading game that we have described here, the only difference between the two economies is the precision of the private signal received by agents in the second period  $(\beta_2)$ : we will assume that this is lower for the Yen economy than it is for the Euro economy. 12

# 3 Strategies, Beliefs, and Equilibrium

The game that we described in the previous section is a dynamic version of the static game analyzed in AHT. It is also related to AMS, who study a dynamic game like ours but only consider linear payoffs and do not analyze the comparative statics which are relevant in our

<sup>&</sup>lt;sup>12</sup>In what follows, "traders" in the second period refers to the relevant players that determine the price in either economy, which can be either workers or bond traders depending on the case.

application. To better understand the economic forces at work in the model and compare our results to those previous papers, it is useful to derive the equilibrium for an asset whose payoff is a generic increasing function  $\pi(\theta)$ . We will then apply this intuition to our specific payoff in the next section.

In each period, the optimal decision by each informed agent takes the form of a demand schedule  $d_1(x_{i,1}, q_1)$  or  $d_2(x_{i,2}, q_2, \rho)$  that maps from signals into a desired asset position  $\{0, 1\}$ .

**Definition 1.** A Perfect Bayesian Equilibrium consists of bidding strategies  $d_1(x_{i,1}, q_1)$  and  $d_2(x_{i,2}, q_2, \rho)$  for informed traders in t = 1 and t = 2 respectively, price functions  $q_1(\theta, \epsilon_1), q_2(\theta, \epsilon_2, \rho)$  and posterior beliefs  $p_1(x_{i,1}, q_1), p_2(x_{i,2}, q_2, \rho)$  such that

- (i) demand schedules  $d_t$  are optimal given posterior beliefs  $p_t$ ,
- (ii) prices  $q_t$  clear the market for all  $(\theta, \epsilon_t, \rho)$ , and
- (iii) posterior beliefs  $p_t$  satisfy Bayes' Law for all market clearing prices  $q_t$ .

To characterize the equilibrium we work backwards, starting from period 2. The derivation of the second-period equilibrium follows AHT. The expected payoff of buying the risky asset for agent i in period 2 is  $\mathbb{E}(\pi(\theta)|x_{i,2}, q_2, \rho) - q_2$ . Proposition 5 in Appendix C proves that, whenever  $q_2$  and  $\rho$  do not fully reveal the value of  $\pi(\theta)$ , <sup>13</sup> posterior beliefs over  $\theta$  are strictly increasing in  $x_{i,2}$  in the sense of first-order stochastic dominance and agents' expected payoffs are a strictly increasing function of  $x_{i,2}$ . This implies that agents follow monotone strategies of the form

$$d_2(x_{i,2}, q_2, \rho) = \mathbb{1}[x_{i,2} \ge \hat{x}_2(q_2, \rho)], \tag{1}$$

where  $\mathbb{1}$  is the indicator function and  $\hat{x}_2(q_2, \rho)$  is a private signal threshold which is endogenous to the equilibrium.

Integrating strategic players' demand schedules over the signal distribution, the market clearing condition in period 2 is

$$\int d_2(x, q_2, \rho) d\Phi \left[ \sqrt{\beta_2} (x - \theta) \right] + \Phi(\epsilon_2 / \sqrt{\psi_2}) = 1.$$
 (2)

<sup>&</sup>lt;sup>13</sup>Equilibria in which prices reveal more than what is collectively known by the informed traders are ruled out by all the papers in this literature; as an example, a discussion of this point appears in Diamond and Verrecchia [27], page 227.

Using equation (1), the aggregate demand of strategic agents is  $\operatorname{Prob}[x_{i,2} \geq \hat{x}_2(q_2, \rho)|\theta]$ , and the market clearing condition becomes

$$z_2 := \theta + \frac{\epsilon_2}{\sqrt{\beta_2 \psi_2}} = \hat{x}_2(q_2, \rho).$$
 (3)

Henceforth we will focus on equilibria where  $z_2$  and  $q_2$  convey the same information, given  $\rho$ , and in which  $\rho$  does not fully reveal  $\theta$ .<sup>14</sup> In this case, conditioning beliefs on the endogenous price is equivalent to conditioning them on the exogenous signal  $z_2$ .

An agent whose private signal is at the threshold  $\hat{x}_2(q_2, \rho)$  must be indifferent in equilibrium between buying the risky asset or storing its endowment. Combining this with equation (3),  $q_2(z_2, \rho)$  must satisfy the indifference condition

$$q_2(z_2, \rho) = \mathbb{E}[\pi(\theta)|x_{i,2} = z_2, z_2, \rho]. \tag{4}$$

The analysis of equilibrium strategies in t=1 follows that of period two quite closely. Proposition 7 in Appendix C proves that the second-period price is strictly increasing in  $z_2$ , and that this is in turn sufficient to ensure that the beliefs of first-period traders are strictly increasing in their private signal  $x_{i,1}$  in the sense of first-order stochastic dominance, as long as the first-period price is not fully revealing. Hence, they too optimally follow monotone strategies described by a threshold signal of the form  $d(x_{i,1}, q_1) = \mathbb{1}[x_{i,1} \geq \hat{x}_1(q_1)]$ . Repeating the steps that led to (3), the market clearing condition in the first period can be rewritten as

$$z_1 := \theta + \frac{\epsilon_1}{\sqrt{\beta_1 \psi_1}} = \hat{x}_1(q_1) \tag{5}$$

and we have shown that  $z_1$  is an unbiased public signal of  $\theta$ , with precision  $\tau_{q_1} := \beta_1 \psi_1$ . As in period two, we focus on equilibria where  $q_1$  and  $z_1$  convey the same information.<sup>15</sup>

We assume that the price signal  $\rho$  observed by second-period agents is given by

$$\rho = z_1 + \sigma_\eta \eta_1,\tag{6}$$

<sup>&</sup>lt;sup>14</sup>Proposition 6 in Appendix C proves that, for debt payoffs and in the absence of recall (i.e. when  $\rho$  has infinite variance), a sufficient condition for  $z_2$  and  $q_2$  to convey the same information is that the equilibrium price is a continuous function of  $\theta$  and  $\epsilon_2$ .

<sup>&</sup>lt;sup>15</sup>For debt payoffs and in the case of no recall, Proposition 6 applies to period 1 as well, so that continuity of  $q_1$  as a function of  $\theta$  and  $\epsilon_1$  is sufficient.

with  $\sigma_{\eta} \geq 0$  and  $\eta_1 \sim N(0,1)$ .<sup>16</sup>  $\rho$  is therefore an unbiased public signal of  $\theta$ , with conditional variance  $1/\tau_{\rho} := \text{Var}(\rho|\theta) = 1/\tau_{q_1} + \sigma_{\eta}^2$ .  $\tau_{\rho}$  represents the precision of the information on  $\theta$  contained in  $\rho$  for t=2 agents.

The equilibrium price in the first period is therefore given by

$$q_1(z_1) = \mathbb{E}[q_2(z_2, \rho)|x_{i,1} = z_1, z_1]. \tag{7}$$

#### 3.1 Equilibrium in the Second Period

From (3) and (6) we can derive the beliefs about  $\theta$  of a strategic trader in period two:

$$\theta|x_{i,2}, z_2, \rho \sim N\left(\frac{\alpha_0 \mu_0 + \beta_2 x_{i,2} + \tau_{q_2} z_2 + \tau_{\rho} \rho}{\alpha_0 + \beta_2 + \tau_{q_2} + \tau_{\rho}}, \frac{1}{\alpha_0 + \beta_2 + \tau_{q_2} + \tau_{\rho}}\right), \tag{8}$$

where  $\tau_{q_2} := \beta_2 \psi_2$  represents the precision of information revealed by the market price in the second period. For the marginal trader, for whom  $x_{i,2} = z_2$ , we thus get

$$\theta|x_{i,2} = z_2, z_2, \rho \sim N\left(\frac{\alpha_0\mu_0 + (\beta_2 + \tau_{q_2})z_2 + \tau_{\rho}\rho}{\alpha_0 + \beta_2 + \tau_{q_2} + \tau_{\rho}}, \frac{1}{\alpha_0 + \beta_2 + \tau_{q_2} + \tau_{\rho}}\right). \tag{9}$$

Using the beliefs of the marginal agent in equation (4), the equilibrium price is given by

$$q_2(z_2, \rho) = \int \pi(\theta) d\Phi \left( \frac{\theta - (1 - w_\rho - w_{z_2})\mu_0 - w_{z_2}z_2 - w_\rho \rho}{\sigma_2} \right)$$
 (10)

where  $w_{\rho} := \frac{\tau_{\rho}}{\alpha_{0} + \beta_{2} + \tau_{q_{2}} + \tau_{\rho}}$  and  $w_{z_{2}} := \frac{\beta_{2} + \tau_{q_{2}}}{\alpha_{0} + \beta_{2} + \tau_{q_{2}} + \tau_{\rho}}$  are the Bayesian weights given by the second-period marginal trader to  $\rho$  and  $z_{2}$  respectively, and  $\sigma_{2} := (\alpha_{0} + \beta_{2} + \tau_{q_{2}} + \tau_{\rho})^{-1/2}$  is the standard deviation of her conditional beliefs. As is clear from equation (10),  $q_{2}$  exists and is unique for all  $z_{2} \in \mathbb{R}$ .

As the precision of the private information received by second-period traders increases ( $\beta_2$  increases), the information revealed by the second-period price becomes more precise as well ( $\tau_{q_2}$  increases). Both of these forces imply that the beliefs of the marginal trader become more

<sup>&</sup>lt;sup>16</sup>The parametric expression of the noise is assumed for tractability. Of course, in our main cases  $\sigma_{\eta} = 0$  or  $\sigma_{\eta} = \infty$ , in which case the specific distribution of  $\eta$  is irrelevant. It is worth noting that, since the price is in equilibrium a nonlinear function of  $z_1$ , the signal structure implies that the noise in the observation of the price is higher in regions of the fundamentals in which the price itself is more volatile. This is a plausible assumption.

concentrated, and more responsive to  $z_2$ . However, in what follows an important role will also be played by the predictability of the second-period price  $q_2$ , based on period-1 information. When  $q_2$  responds more to  $z_2$ , it is affected more by the fundamental  $\theta$ , but also by the noise  $\epsilon_2$ , over which period-1 agents have no information.

#### 3.2 Equilibrium in the First Period

Having derived  $q_2$  explicitly, we can now do the same for the equilibrium price in t = 1. First-period traders' posterior beliefs about  $\theta$  are given by

$$\theta|x_{i,1}, z_1 \sim N\left(\frac{\alpha_0 \mu_0 + \beta_1 x_{i,1} + \tau_{q_1} z_1}{\alpha_0 + \beta_1 + \tau_{q_1}}, \frac{1}{\alpha_0 + \beta_1 + \tau_{q_1}}\right). \tag{11}$$

First-period traders are not affected by  $\theta$  directly, but rather they use these beliefs to forecast  $q_2$ , which in turn is a function of  $z_2$  and  $\rho$ . The marginal trader's posterior beliefs on  $z_2$  and  $\rho$  are given by

$$z_{2}|(x_{i,1}=z_{1},z_{1}) \sim N\left((1-w_{1})\mu_{0}+w_{1}z_{1},\sigma_{2|1}^{2}:=\frac{1}{\alpha_{0}+\beta_{1}+\tau_{q_{1}}}+\frac{1}{\tau_{q_{2}}}\right)$$

$$\rho|(x_{i,1}=z_{1},z_{1}) \sim N\left(z_{1},\sigma_{n}^{2}\right)$$
(12)

where  $\sigma_{2|1}^2$  is the variance of *new* second-period information  $z_2$  conditional on first-period information  $(x_1, z_1 \text{ and prior})$ , and  $w_1 := \frac{\beta_1 + \tau_{q_1}}{\alpha_0 + \beta_1 + \tau_{q_1}}$  is the Bayesian weight given to  $z_1$  by the marginal trader in the first period. Imposing market clearing and the indifference condition of the marginal trader, Appendix C shows that the first-period price is given by

$$q_1(z_1) = \int \pi(\theta) \frac{1}{\sqrt{w_{z_2}^2 \sigma_{2|1}^2 + w_{\rho}^2 \sigma_{\eta}^2 + \sigma_2^2}} \phi \left( \frac{\theta - \mu_0 (1 - w_{\rho} - w_{z_2} w_1) - z_1 (w_{\rho} + w_{z_2} w_1)}{\sqrt{w_{z_2}^2 \sigma_{2|1}^2 + w_{\rho}^2 \sigma_{\eta}^2 + \sigma_2^2}} \right) d\theta, (13)$$

where  $\phi$  is the density function of a standard normal distribution. Equation (13) expresses the period-1 price as an expectation with respect to  $\theta$ , according to a distorted measure that accounts for the fact that period-1 agents care about forecasting  $q_2$  and not  $\theta$  directly. Both this distorted measure and the true beliefs about  $\theta$  conditional on the information of the marginal trader (given by equation (11)) are normal; let their means and variances be  $(\tilde{\mu}_1, \tilde{\sigma}_1^2)$  and  $(\mu_1, \sigma_1^2)$  respectively.

We can derive intuition about the key drivers of our results by comparing these moments, which, after some algebra, can be rewritten as

$$\mu_1 = \mu_0 + (z_1 - \mu_0) \frac{\beta_1 + \tau_{q_1}}{\alpha_0 + \beta_1 + \tau_{q_1}} \tag{14}$$

$$\tilde{\mu}_{1} = \mu_{0} + (z_{1} - \mu_{0}) \frac{\beta_{1} + \tau_{q_{1}}}{\alpha_{0} + \beta_{1} + \tau_{q_{1}}} \left[ 1 - \left( 1 - \frac{\tau_{\rho}}{\beta_{1} + \tau_{q_{1}}} \right) \frac{\alpha_{0}}{\alpha_{0} + \beta_{2} + \tau_{q_{2}} + \tau_{\rho}} \right]$$
(15)

and

$$\sigma_1^2 = \frac{1}{\alpha_0 + \beta_1 + \tau_{q_1}} \tag{16}$$

$$\tilde{\sigma}_{1}^{2} = \sigma_{1}^{2} \left\{ 1 + \frac{\tau_{\rho} \left( 1 - \frac{\tau_{\rho}}{\tau_{q_{1}}} \right) \frac{\alpha_{0} + \beta_{1} + \tau_{q_{1}}}{\alpha_{0} + \beta_{2} + \tau_{q_{2}} + \tau_{\rho}} + (\beta_{1} + \tau_{q_{1}} - \tau_{\rho}) + (\beta_{2} + \tau_{q_{2}}) \left[ \frac{1 + \frac{\alpha_{0} + \beta_{1} + \tau_{q_{1}}}{\tau_{q_{2}}}}{1 + \frac{\alpha_{0} + \tau_{\rho}}{\beta_{2} + \tau_{q_{2}}}} - 1 \right]}{\alpha_{0} + \beta_{2} + \tau_{\rho} + \tau_{q_{2}}} \right\}. (17)$$

For our purposes, an important observation is that  $\tilde{\mu}_1$  is an affine function of  $z_1$ , the information contained in the first-period price, which we will henceforth call the "market signal." When  $z_1$  is less than the prior mean  $\mu_0$  (either because of a bad realization of the fundamentals  $\theta$  or a bad realization of the noise-trader shock  $\epsilon_1$ ), we get  $\tilde{\mu}_1 > \mu_1$ , with the reverse occurring when  $z_1 > \mu_0$ . In contrast, the difference between  $\sigma_1^2$  and  $\tilde{\sigma}_1^2$  is entirely driven by the moments of the signal distributions and is independent of the realization of any shock.

We are interested in studying how the first-period price is affected by the quality of information in the second period. When  $\pi$  is differentiable, we can integrate (13) by parts and we obtain

$$\frac{\partial q_1}{\partial \beta_2} = \left[ \int \pi'(\theta) \frac{1}{\tilde{\sigma}_1} \phi \left( \frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) d\theta \right] \frac{\partial \tilde{\mu}_1}{\partial \beta_2} + \left[ \int \left( \frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) \pi'(\theta) \frac{1}{\tilde{\sigma}_1} \phi \left( \frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) d\theta \right] \frac{\partial \tilde{\sigma}_1}{\partial \beta_2}. \tag{18}$$

AMS study linear payoffs. In this case,  $\pi'$  is constant and the second term is zero: only distortions in the mean have an effect. Comparing equations (14) and (15), we notice that the difference is driven by two wedges:

• When  $\tau_{\rho} < \beta_1 + \tau_{q_1}$ , the information of the marginal trader in the first period is not fully passed on to the marginal trader in the second-period. As a consequence, the second-period price direct response to  $z_1$  is dampened, which in turn spills over to the incentives for the first-period agents to incorporate the incoming news.

• The extent by which dampening occurs depends on the quality of information in the second period. In the limiting case in which  $\beta_2 + \tau_{q_2} \to \infty$ , the second-period price tracks  $\theta$  perfectly anyway and the loss of period-1 information arising from  $\tau_{\rho} < \beta_1 + \tau_{q_1}$  is irrelevant. In contrast, the less precise the information is in the second period, the more the information loss contributes to a muted response of  $q_2$  to fundamentals with a corresponding effect in period 1.

Turning to the implications for the first-period price, we then obtain the following proposition:

**Proposition 1.** If  $\pi(\theta)$  is affine in  $\theta$ , the price  $q_1$  is unaffected by the second-period information if and only if  $\tau_{\rho} = \beta_1 + \tau_{q_1}$ . When  $\tau_{\rho} < \beta_1 + \tau_{q_1}$ , there exists a cutoff  $\bar{z}$  such that the price is increasing in  $\beta_2$  for  $z_1 > \bar{z}$  and decreasing for  $z_1 < \bar{z}$ .

**Proof.** If  $\pi(\theta) = \bar{\pi} + \pi_{\theta}\theta$ , we get that  $\partial q_1/\partial \beta_2 = \pi_{\theta}\partial \tilde{\mu}_1/\partial \beta_2$ , so the result follows immediately from equation (15). Note that, by the definition of  $\rho$  in equation (6),  $\tau_{\rho} \leq \tau_{q_1}$ , so the proposition covers all the possibilities.

Remark 1. Within our setup,  $\beta_1 = 0$  implies also that  $\tau_{q_1} = 0$ , so  $\tau_{\rho} = \beta_1 + \tau_{q_1}$  only happens in the degenerate case when period-1 agents rely uniquely on prior information. Nonetheless, the proposition would apply to more general environments where an exogenous public signal is present and  $\tau_{q_1}$  may be strictly positive even if no agent has any private information in the first period.

Proposition 1 proves a single-crossing property of the price  $q_1$ : conditional on good [bad] news (high  $z_1$ ), the *first period* price is increasing [decreasing] in the precision of the signal of second-period agents.<sup>17</sup> In what follows, we will informally discuss this single-crossing property as meaning that the price is more (or less) responsive to incoming news.<sup>18</sup>

This single-crossing only applies outside of the degenerate case in which  $\tau_{\rho} = \beta_1 + \tau_{q_1}$ .

 $<sup>^{18}</sup>$ An even stronger notion of responsiveness requires the derivative of the price with respect to  $z_1$  to be increasing in  $\beta_2$  at any given point. In the case of linear payoffs, this is also true. However, we focus our attention to the single-crossing property because it is the one relevant in our application to sovereign debt. In that case, the price function flattens out away from the default threshold, since debt either becomes risk free or is defaulted upon for sure. Additional precision makes it more likely that debt falls into these regions, in which the price is locally not as responsive, while the single-crossing property still applies.

When  $\pi'$  is not constant, distortions in the variance also have an effect on the price. In a static context, this effect has been analyzed by AHT. When  $\pi$  is twice differentiable, we can rewrite the second integral in equation (18) as

$$\int \left(\frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1}\right) \pi'(\theta) \frac{1}{\tilde{\sigma}_1} \phi\left(\frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1}\right) d\theta = \int \pi''(\theta) \phi\left(\frac{\theta - \tilde{\mu}_1}{\tilde{\sigma}_1}\right) d\theta. \tag{19}$$

Equation (19) highlights the role of concavity or convexity of the asset payoff function. For example, in the case of a convex payoff, any force that increases  $\tilde{\sigma}_1$  would increase the first-period price.

In a dynamic context, the factors that drive  $\tilde{\sigma}_1$  are richer than the simpler static case, as they reflect a two-way interaction between the information in the first and second period. We can highlight four main factors.

- All the distortions rely on the fact that the second-period price does not perfectly track the fundamentals. The term  $\alpha_0 + \beta_2 + \tau_\rho + \tau_{q_2}$  at the denominator of (17) represents the information available to the marginal trader in the second period: the more precise this information is, the smaller the wedge between  $\sigma_1^2$  and  $\tilde{\sigma}_1^2$ , since in the limit we converge to a situation in which  $q_2 = \pi(\theta)$  and dynamic trading is irrelevant.
- When second-period agents observe the first-period price imperfectly, the noise in their observation acts as a second disturbance in the determination of  $q_2$ . From the perspective of first-period traders, this extra noise increases the variance of their payoff. This effect is captured by the first term at the numerator of the wedge in equation (17). It vanishes when the first-period price is observed without noise ( $\tau_{\rho} = \tau_{q_1}$ ) or when it is not observed (equivalent to  $\tau_{\rho} = 0$ ), in which case second-period agents do not react to  $\rho$  (and hence to this new source of noise).
- Even when  $q_1$  is perfectly observed by traders in the second period, a wedge arises from the fact that they do not share the same private information as that of the marginal trader in the first period. This effect, represented by the second term at the numerator of the wedge in equation (17), is similar to what we already discussed in the case of the first moment.

• Finally, the last term in the same numerator combines the role of information frictions in the two periods and also reflects the fact that increased precision in the second period makes  $q_2$  less predictable in the first period. The ratio inside the square bracket is bigger than one both because second-period agents have less information about the first period  $(\beta_1 + \tau_{q_1} \geq \tau_{\rho})$  and because they have their own private information, which makes them more responsive to the market signal  $z_2$   $(\beta_2 + \tau_{q_2} \geq \tau_{q_2})$ .

The total effect of  $\beta_2$  on  $\tilde{\sigma}_1$  is ambiguous. The first and second economic force described above lower  $\tilde{\sigma}_1$  when  $\beta_2$  increases, while the last one goes in the opposite direction.<sup>19</sup> With some algebra, it can be shown that either  $\tilde{\sigma}_1$  is monotonically decreasing in  $\beta_2$  or it has one interior maximum.<sup>20</sup>

For a fully general payoff, the total effect of nonlinearities is thus a complicated combination of the way in which concavity and convexity move within the state space and of the non-monotonic behavior of the variance. To establish more definite results, we return to our specific payoff of interest.

## 4 The Role of Differential Information for Sovereign Debt

In our application, the payoff function is given by

$$\pi(\theta) = \begin{cases} \delta & \text{if } \theta < \bar{\theta} \\ 1 & \text{if } \theta \ge \bar{\theta}. \end{cases}$$
 (20)

For this payoff, equation (13) simplifies to

$$q_1(z_1) = \delta + (1 - \delta)\Phi\left[\frac{\mu_0 - \bar{\theta}}{\tilde{\sigma}_1} + K(z_1 - \mu_0)\right],$$
 (21)

with

$$K := \frac{w_{\rho} + w_{z_2} w_1}{\tilde{\sigma}_1}.\tag{22}$$

<sup>&</sup>lt;sup>19</sup>The third force is independent of  $\beta_2$ .

<sup>&</sup>lt;sup>20</sup>An interior maximum arises if  $\alpha_0 > \max\{0, \tau_{\rho}[2\psi_2(1-\tau_{\rho}/\tau_{q_1})-1]\}$ , see the online appendix for details.

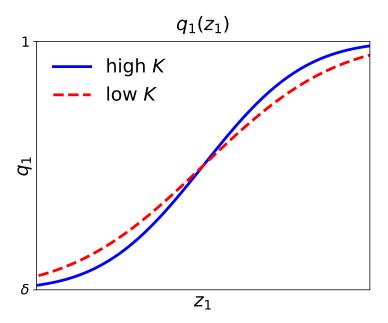


Figure 1: Illustration of  $q_1(z_1)$ . A higher K corresponds to a first-period price that is more sensitive to the realization of the market signal  $z_1$ .

Figure 1 illustrates this price as a function of the realization of the shock  $z_1$  for two different values of K: the higher K is, the more sensitive the price is. More precisely, a higher K leads to a lower price when first-period agents receive negative news about government solvency ( $z_1$  low), while the converse is true for high values of  $z_1$ .

The coefficient K illustrates some of the trade-offs that we have discussed in the general case. When second-period agents receive better information, they rely more on their signals and less on the information conveyed by the first-period price, that is,  $w_{\rho}$  decreases, while  $w_{z_2}$  increases. From the perspective of the first-period price, the second-period price tracks fundamentals better, but may track the period-1 information  $z_1$  less closely. The presence of  $\tilde{\sigma}_1$  at the denominator reflects the further effects from the nonlinearity in the payoff.

#### 4.1 Mean and Variance Effects in the Case of Debt

As in the case of a generic increasing payoff, the effect of  $\beta_2$  on  $q_1$  can be decomposed into one component primarily driven by the monotonicity of the payoff (the first term of the sum

in equation (18)) and a second component related to convexity/concavity (the second term). Since the debt payoff in equation (20) is not differentiable, equation (18) does not apply directly. However, after some algebra we obtain the following similar expression:

$$\frac{\partial q_1}{\partial \beta_2} = (1 - \delta) \frac{1}{\tilde{\sigma}_1} \phi \left( \frac{\bar{\theta} - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) \frac{\partial \tilde{\mu}_1}{\partial \beta_2} + (1 - \delta) \left( \frac{\bar{\theta} - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) \frac{1}{\tilde{\sigma}_1} \phi \left( \frac{\bar{\theta} - \tilde{\mu}_1}{\tilde{\sigma}_1} \right) \frac{\partial \tilde{\sigma}_1}{\partial \beta_2}$$
(23)

Intuitively, the connection between (18) and (23) stems from the fact that the debt payoff concentrates all the slope in a single point, being "convex" just to the left and "concave" just to the right.<sup>21</sup>

Consider first the effect of  $\beta_2$  on  $q_1$  arising from monotonicity, which is captured by changes in  $\tilde{\mu}_1$ . This effect has the same sign as  $z_1 - \mu_0$ : in the case of bad news  $(z_1 < \mu_0)$ , the price is decreasing in  $\beta_2$ , with the reverse occurring when  $z_1 > \mu_0$ . As second-period agents become better informed, any bad (good) news will be better detected in the second period and drive  $q_2$  further down (up), so first-period agents are also led to react more aggressively and trigger bigger price movements in  $q_1$  as well.

The role of convexity is more complex. For the step function that represents the debt payoff, the convexity element dominates at low values of  $z_1$ , while concavity dominates for high values. For low values of  $z_1$ , equation (15) shows that  $\tilde{\mu}_1$  is below the default threshold  $\bar{\theta}$ ; equation (23) then shows that increases in the distorted variance  $\tilde{\sigma}_1$  raise the price. The reverse occurs when  $z_1$  is high and  $\tilde{\mu}_1 > \bar{\theta}$ . The effect of  $\beta_2$  on  $\tilde{\sigma}_1$  itself is ambiguous and reflects the race between how closely  $q_2$  tracks fundamentals vs. how closely it tracks the first-period information  $z_1$ .

To establish our comparative-statics results, what is important is that, in the case of debt, both the effects on the mean and the variance are driven by the jump in the payoff at the threshold. This gives the two terms in equation (23) a common structure that collapses all of these effects in the coefficient K defined in equation (22) above and thus permits clear-cut comparisons.

<sup>&</sup>lt;sup>21</sup>A more formal statement to this end can be made by considering a family of approximating payoffs:  $\pi^a(\theta; \lambda) = \delta + (1 - \delta)\Phi\left(\frac{\theta - \bar{\theta}}{\lambda}\right)$ . This family is indeed convex for  $\theta < \bar{\theta}$  and concave otherwise. As  $\lambda \to 0$ , all the change is concentrated in a neighborhood of  $\bar{\theta}$  and (18) converges to (23).

#### 4.2 Main Results

From equation (21) we derive our main results, which provide conditions under which a government that faces a bad shock realization compared to its prior would benefit from a decrease in second-period agents' information precision. In terms of our "Euro" vs. "Yen" interpretation, these conditions ensure that bond prices are more resilient to bad shocks when second-period agents are unsophisticated households (the Yen economy) than in the Euro economy, where the relevant agents in the second period are well-informed bond traders.

**Proposition 2.** When the first-period price is observed with sufficient noise by second-period traders, the responsiveness of the price is strictly increasing in the precision of second-period information, even in the first period. Formally, there exists a cutoff level  $\hat{\tau}_{\rho} \in (0, \beta_1 \psi_1]$  such that, when  $\tau_{\rho} \leq \hat{\tau}_{\rho}$ , K is strictly increasing in  $\beta_2$ .<sup>22</sup>

#### **Proof.** See Appendix B.

When the first-period price is observed with sufficient noise, the proposition provides a global result justifying our motivating observation, that bond prices will react more to incoming news in the Euro economy than they will in the Yen economy.

In terms of the comparative statics of equation (23), Proposition 2 ensures that the mean effect always dominates the variance component.

Depending on the values of all other parameters, Proposition 2 may apply even when the first-period price is perfectly recalled in the second period. However, when this is not the case, we can still prove the following result:

**Proposition 3.** Assume that  $\psi_2 \geq \psi_1$  and  $\beta_2^A \geq \beta_1$ . Let  $\beta_2^B < \beta_2^A$ . Holding all other parameters fixed, K evaluated at  $\beta_2^A$  is greater than at  $\beta_2^B$ .

**Proof.** See Appendix B.

<sup>&</sup>lt;sup>22</sup>The two extreme cases of  $\tau_{\rho} = 0$  and  $\tau_{\rho} = \beta_1 \psi_1$  correspond to the cases in which the first-period price is either unobserved or perfectly observed by second-period agents, respectively.

#### t = 1 Price Responsiveness

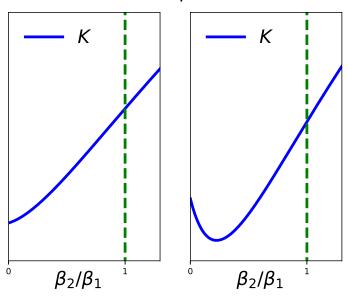


Figure 2: Coefficient K as a function of  $\beta_2/\beta_1$ , for  $\tau_{\rho} \leq \hat{\tau}_{\rho}$  (left) and  $\tau_{\rho} > \hat{\tau}_{\rho}$  (right).

Proposition 3 compares two values of the signal precision,  $\beta_2^A$  and  $\beta_2^B$ . According to our interpretation, greater values of  $\beta_2$  and  $\psi_2$  arise when debt is denominated in a currency over which the country has no control. In this case (represented by  $\beta_2^A$ ), inflation is not an option, debt is subject to the risk of outright default, and second-period agents correspond to a new cohort of well-informed bond traders. When the marginal agent is a bond trader in both periods, it is natural to assume that the information received (through their private signal and the market signal) by players in the second period is at least as good as that of the first period, which is reflected in the assumption that  $\beta_2^A \geq \beta_1$  and  $\psi_2 \geq \psi_1$ . In the second case (represented by  $\beta_2^B$ ), a country issues debt denominated in local currency which allows recourse to inflation rather than outright default. Here, second-period agents are workers setting their prices in the local currency. We do not take a stance on the quality of the workers' information relative to first-period traders: on the one hand, we assume that they are less sophisticated, but on the other hand the passage of time might have revealed more news about government finances. We

<sup>&</sup>lt;sup>23</sup>In addition, second-period agents also learn from the first-period price; this proposition applies regardless of the quality of this extra piece of information.

only need to compare the quality of the workers' information to that of second-period traders: here, less sophistication by the workers means that their precision  $\beta_2^B$  is strictly smaller than the traders'  $\beta_2^A$ . Under these assumptions, Proposition 3 shows that our main result holds: in Figure 1, the red line represents the price of debt when it is denominated in the domestic currency (the Yen economy), and the blue line is the price when it is issued in a foreign currency (the Euro economy).<sup>24</sup>

Proposition 3 shows that the price of debt is more resilient to bad shocks when issued in domestic currency. We view this result as particularly relevant for countries that start from a favorable prior: for them, there is limited upside from further confirming the creditors' belief that there is ample fiscal space, while there is substantial downside risk should they find out that fiscal constraints are tighter than they appeared. This is a good description of Eurozone countries in 2008, as well as other major developed economies, all of which paid very low interest rates before the onset of the crisis.

Our result also highlights a potentially opposite conclusion for a country that starts from an adverse prior. For such a country, issuing domestically-denominated debt may immediately lead workers to expect high inflation, and this pessimism will spill over to the traders who underwrite the debt, through the channels that we emphasize. When realized fiscal space is indeed limited, as will happen often if the prior is correct, there is not much that can be done to sustain the price of debt. However, in the event that fundamentals are more favorable, well-informed traders will be better placed to detect the situation, and debt will correspondingly fetch a higher price when issued in foreign currency. We view this as more relevant for countries such as those of Latin America and this may be another explanation for their past inclination to issue dollar-denominated debt.<sup>25</sup>

 $<sup>^{24}</sup>$ In the main text, we focus on the comparative statics with respect to  $\beta_2$ . In the online appendix, we show that K is globally strictly increasing in  $\psi_2$  as well, so that the analogous of Proposition 2 applies to that parameter independently of the degree of price recall. The comparative statics with respect to  $\psi_2$  are simpler than those with respect to  $\beta_2$ , because  $\psi_2$  only affects the precision of the market signal and does not interact with the private information accruing to individual investors.

<sup>&</sup>lt;sup>25</sup>This reason is complementary to the time-inconsistency forces emphasized by Calvo [19], Bohn [14], Aguiar et al. [3], Engel and Park [30], and Ottonello and Perez [41].

## 5 Endogenous Default Threshold

In the previous sections we assumed that the terminal payoff is independent of the price at which the security is issued in the first period. In particular, in the case of government debt, this means that the default cutoff is exogenous. We now consider an extension in which the debt default threshold is given by a function  $\bar{\theta}(q_1)$ . As an example, this happens if the debt auction follows the same structure as in Calvo [20]: the government requires a given revenue from the auction, which we normalize to unity, while its repayment obligations in the final period depend on the interest rate and are given by  $1/q_1$ . Since higher interest rates (lower  $q_1$ ) imply a higher promised repayment, in general the default threshold  $\bar{\theta}(q_1)$  will be a decreasing function.

For simplicity, we focus here on the case in which there is perfect recall of the first-period price:  $\tau_{\rho} = \beta_1 \psi_1$ . In this case, the construction of an equilibrium is very similar to what we did in Section 3. All the steps that lead to equation (21) remain the same, where  $\bar{\theta}$  is replaced by  $\bar{\theta}(q_1)$ . As of period 2,  $\bar{\theta}(q_1)$  is a given, so that existence and uniqueness given  $q_1$  are established as before. The main difference arises in equation (21), where now the endogenous threshold implies that  $q_1(z_1)$  is only implicitly characterized by a solution to the following equation:

$$q_1 = \delta + (1 - \delta)\Phi \left[ \frac{\mu_0 - \bar{\theta}(q_1)}{\tilde{\sigma}_1} + K(z_1 - \mu_0) \right], \tag{24}$$

where  $\tilde{\sigma}_1$  and K are given by the same expressions as in the case of an exogenous threshold, as defined in equations (17) and (22).

In Section 4, we could establish results about the sensitivity of the price to  $z_1$  by simply studying the properties of the coefficient K. Now, the analysis is complicated by the fact that  $q_1$  appears on the right-hand side through its effect on the default threshold, and we can no longer prove that the price functions drawn for two different values of  $\beta_2$  cross only once, as in Figure 1. In fact, the introduction of an endogenous default threshold creates a new source of complementarity and could even generate multiple equilibria if information is sufficiently precise (Hellwig, Mukherji and Tsyvinski [34], Angeletos and Werning [12]). However, even if single-crossing fails, or even if multiple equilibria arise, we can still prove that none of these complications affect the behavior following tail events, where multiplicity does not arise and

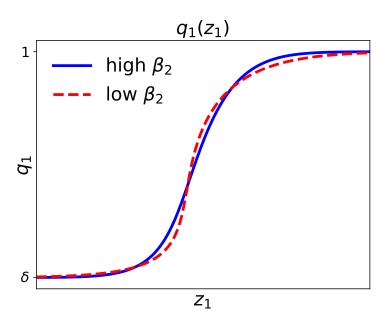


Figure 3:  $q_1(z_1)$  for a case in which multiple crossings occur with an endogenous default threshold. comparative statics remain the same as what we established in Section 4. Formally:

**Proposition 4.** Assume that  $\psi_2 \geq \psi_1$  and  $\beta_2^A \geq \beta_1$ . Let  $\beta_2^B < \beta_2^A$ . Then there exist two cutoff levels  $\hat{z}_1^L \leq \hat{z}_1^H \in \mathbb{R}$  such that when  $z_1 < \hat{z}_1^L$ ,  $q_1$  evaluated at  $\beta_2^A$  is smaller than at  $\beta_2^B$ , whereas the reverse occurs for  $z_1 > \hat{z}_1^H$ , holding all other parameters fixed.

**Proof.** See Appendix B. 
$$\Box$$

Figure 3 illustrates a case with multiple crossings.<sup>26</sup> The intuition behind Proposition 4 is that, for  $z_1$  large in absolute value, the dominant force determining how the price moves with  $\beta_2$  remains K, for which we already proved theorems in the previous section.

The price functions in the figure are obtained under the following parametrization:  $\bar{\theta}(q_1) := 1/q_1, \alpha = 15, \beta_1 = 10, \beta_2^{\text{high}} = 10, \beta_2^{\text{low}} = 0.001, \psi_1 = \psi_2 = 0.1, \tau_\rho = \beta_1 \psi_1, \mu = 1.22, \delta = 0.654$ . The extreme difference between  $\beta_2^{\text{high}}$  and  $\beta_2^{\text{low}}$  reflects the fact that, while conceptually possible, parameter values where multiple crossings occur are not as common, and it is especially difficult to find values where the multiple crossings can be seen easily in a graph.

### 6 Conclusion

Inflation risk and default risk affect the real value of maturing government debt in a similar way. However, the general price level is driven by the interaction among a much larger fraction of the population than the restricted group of people who actively participate in the government debt market. To the extent that information about government finances is unevenly distributed within the population, we have shown that this asymmetry has important implications for the resilience of debt prices in the face of adverse shocks.

In this paper, we emphasized one reason why inflation reacts sluggishly to fundamentals. Our results would also apply in different contexts where other frictions force a slower adjustment in the prices of goods relative to asset prices, such as sticky-price models.

Our analysis opens a new dimension for the study of optimal debt management, in addition to the traditional channels of fiscal hedging and time consistency. The next step in this direction is to further develop a full theory of the optimal denomination of debt. Such a theory would take into account the insurance aspect that we have studied here, together with the effects of different structures of debt on the ex ante expected borrowing costs.<sup>27</sup>

Finally, the information sensitivity of assets play a major role in the work of Gorton and Ordoñez [31]. While combining their forces and ours in a self-contained model is beyond the scope of our project, their theory and our work are complementary in accounting for sudden sovereign crises: as debt becomes more information-sensitive through the channels that we emphasize, Gorton and Ordoñez' forces would lead first-period agents to invest in even greater information acquisition, leading to further volatility and possibly market freezes.

<sup>&</sup>lt;sup>27</sup>As emphasized in AHT, in the context of the model that we adopt, the relationship between the expected price of a security and its fundamental expected value ex ante is driven by the concavity or convexity of the payoff as a function of the underlying fundamental. In our case, the payoff of the first-period traders takes the shape of a normal cumulative distribution function, with both a convex and a concave piece that play against each other, so that we cannot establish a definite ranking.

# Appendix A Microfoundations

In this Section we introduce a stylized macroeconomic model that underpins our assumption from the main text that the inflation expectations of a (larger and) unsophisticated group of agents drive the secondary-market price of debt for the Yen economy, while the default expectations of sophisticated bond traders determine the secondary-market price in the Euro economy. Other than this distinction, the model is such that the two economies are the same; the model maps exactly into the setup of our main text, where the only difference between the two economies is the precision of information of the agents that are pivotal in the second period.

We consider an economy that lasts for three periods. There is a single consumption good in each period. We consider two alternative scenarios: in the first one, the unit of account is exogenously fixed (the "Euro") and the price of the consumption good is normalized to 1. In the second case, the value of a unit of account (the "Yen") is endogenous.

The economy is populated by multiple generations of four types of agents: strategic workers, noise workers, strategic bond traders, and noise bond traders. In addition, a government is also present.

Workers are born in period 2 and die in period 3.<sup>28</sup> Strategic workers are endowed with one unit of the consumption good in period 2 and wish to consume in period 3; they are risk neutral and have access to a storage technology which has a yield normalized to zero. Negative storage is not allowed. Noise workers demand one unit of consumption in period 2, and can produce exclusively in period 3. To consume, they trade with strategic workers using nominal contracts, denominated in Euros or Yen, depending on the regime.<sup>29</sup> The relative mass of noise vs. strategic workers is  $\Phi(\epsilon_2^w/\sqrt{\psi_2^w})$ , where  $\Phi$  is the normal cumulative distribution function and  $\epsilon_2^w$  is i.i.d. with a standard normal distribution. Neither strategic workers nor noise workers have access to the bond market. Their asset position is limited to storage, trade credit with each other, and

<sup>&</sup>lt;sup>28</sup>We could add workers that live in periods 1 and 2, but these would not interact with bond traders, and so their presence would not have any effect on our results.

<sup>&</sup>lt;sup>29</sup>We do not model the reason why workers coordinate on nominal contracts. Euro contracts are equivalent to real contracts in our setup. Yen-denominated contracts favor strategic workers, as they can reap information rents at the expense of noise workers.

cash, which they may acquire from the bond traders.<sup>30</sup>

Under the Euro scenario, workers do not interact with bond traders, and their interaction with the government is limited to paying a lump-sum tax which is a negligible fraction of their endowment.

Bond traders live for two periods, and there will be overlapping generations of them. Their mass is negligible compared to workers; hence, when the two groups trade, the price is set by the workers. Bond traders are endowed with goods in the first period of their life,<sup>31</sup> which they want to consume in the second period. Strategic traders can store their endowment at a return normalized to 0. Alternatively, they can sell some of their endowment in exchange for a government bond, which in period 1 can be purchased from the primary market and in period 2 from the secondary market, soon to be described. To preserve tractability, we assume that holdings of government debt are limited to  $\{0,1\}$ . Noise traders do not get a choice; they absorb a fraction  $\Phi(\epsilon_t^b/\sqrt{\psi_t^b})$  of the government bonds supplied to the market, where  $\epsilon_t^b$  is i.i.d. with a standard normal distribution.

We next describe the government. We normalize its positions in per capita terms with respect to one cohort of strategic bond traders. In the first period, the government issues nominal bonds, backed by taxes that will be collected in period 3. Revenues from bond issuance are spent in a public good which does not affect the marginal utility of private consumption. When government bonds are denominated in Euros, they mature only in period 3, when the government has access to tax revenues. When instead the Yen is present, bonds are repaid in cash in period 2, and period-3 revenues are used to repurchase cash, as in Cochrane [23].<sup>32</sup> This arrangement corresponds to one of the important observations from which we started: that inflation is sluggish in advanced countries and workers often do not realize immediately that the government is resorting to the

<sup>&</sup>lt;sup>30</sup>The assumption that workers cannot buy government bonds could be justified by indivisibility constraints as in Wallace [46].

<sup>&</sup>lt;sup>31</sup>We assume that their endowment is always sufficient to buy one unit of government bonds.

<sup>&</sup>lt;sup>32</sup>The interpretation of this assumption is that the government can always smooth over any rollover risk by temporarily leaning on the central bank to purchase bonds. In the case of Japan, a more literal interpretation is actually true, since the Bank of Japan has monetized about half of the government net debt, that is, 80% of GDP.

printing press to cover its fiscal needs.<sup>33</sup> In period 1, the government auctions one unit of bonds with a promised repayment  $\hat{s}(q_1)$  in period 3, where  $q_1$  is the inverse of the (gross) nominal interest rate. Two examples of the function  $\hat{s}$  are the following:

- $\hat{s}(q_1) \equiv \hat{s} \equiv 1$ , corresponding to the Eaton-Gersovitz [28] timing, in which the government offers bonds making a fixed unit future repayment in period 3, and  $q_1$  represents the first-period discount; this is the case that we consider in most of the paper, up to Section 5.
- $\hat{s}(q_1) \equiv 1/q_1$ , corresponding to the Calvo [20] timing, in which the government offers bonds to raise a fixed amount of revenues (one) in period 1 and  $1/q_1 1$  represents promised interest payments in period 3.

The ability of the government to raise revenues without a default in period 3 is limited by a single random variable  $\theta$ . If  $\theta \geq \hat{s}(q_1)$ , revenues from current and future taxes are sufficient to repay the debt in full (under the Euro interpretation) or to maintain the price of goods pegged at parity with the Yen (when the government has its own currency). When instead  $\theta < \hat{s}(q_1)$ , tax revenues are insufficient to avoid explicit default or inflation. In this case, we assume that the government imposes an exogenous haircut and only repays  $\delta \hat{s}(q_1)$  units of the consumption good in period 3. When debt is denominated in Euros, this is implemented directly as a haircut upon default. When instead debt is denominated in Yen, the revenues  $\delta \hat{s}(q_1)$  are available to repurchase Yen, implying that the price level at which Yen are withdrawn becomes  $1/\delta$ .

The prior about  $\theta$  and the signals observed by the agents are described in Section 2. The key distinction is about the information of workers vs. bond traders in the second period: we assume that workers get a private signal with precision  $\beta_2^w$ , while bond traders get a signal with precision  $\beta_2^b > \beta_2^w$ .

<sup>&</sup>lt;sup>33</sup>We view this assumption as particularly appropriate for a government who has in the past established a reputation for stability. There are examples in history where this assumption would be violated. Sargent [45] discusses cases in which inflation responded quickly to fiscal news, and other, more recent cases in which doubts about the fiscal stance led to sluggish adjustments.

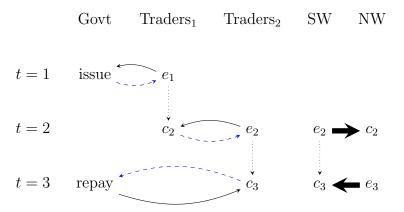


Figure 4: Markets and trading in the Euro scenario. Goods (solid black); Bonds (dashed blue); Storage (dotted black).  $e_t$  and  $c_t$  are endowment and consumption in period t, SW and NW stand for strategic and noise workers respectively.

#### A.1 Trading in the Euro Economy

The pattern of trade for the Euro economy is described in Figure 4, where flows of goods are represented by black arrows and flows of bonds by blue arrows. In the Euro economy, there is no uncertainty about the value of nominal contracts, which is fixed at 1. At these prices, strategic workers ("SW") are indifferent between storing their endowment or lending it at a rate zero to the noise workers ("NW"). Hence, they will absorb all of the demand  $\Phi(\epsilon_2^w/\sqrt{\psi_2^w}) \in (0,1)$  with no effect on their lending rate and no interaction with the bond market.

Next, we consider bond trading in the secondary market (period 2). Bond supply is fixed at one: both strategic and noise traders who purchased the bond in period 1 ("Traders<sub>1</sub>") must sell it to consume.

Strategic bond traders born in period 2 ("Traders<sub>2</sub>") must choose whether to store their entire endowment or purchase a government bond in the secondary market.<sup>34</sup> Defining  $q_2 := 1/(1+R_2)$ , where  $R_2$  is the nominal interest rate (yield to maturity) in the secondary market, the expected

<sup>&</sup>lt;sup>34</sup>They could also lend to noise workers at the same rate as storage; since their mass is negligible compared to workers, this would not affect the market-clearing condition for trade credit between periods 2 and 3.

net profit from buying the bond is

$$\hat{s}(q_1) \left[ \delta + (1 - \delta) \operatorname{Prob} \left( \theta \ge \hat{s}(q_1) | \mathcal{I}_{i,2}^b \right) - q_2 \right], \tag{25}$$

where  $\mathcal{I}_{i,t}^b$  is the information available to bond trader i in period t. We denote by  $D_t^b$  the demand for bonds by strategic bond traders in period t; this demand depends on the price  $q_t$ , but also on the details of available information, in particular the degree of price recall  $\tau_{\rho}$ . Second-period strategic bond traders must absorb a fraction  $1 - \Phi(\epsilon_2^b/\sqrt{\psi_2^b})$  of bonds in equilibrium, with the balance purchased by noise traders. Market clearing will then require

$$D_2^b = 1 - \Phi\left(\epsilon_2^b / \sqrt{\psi_2^b}\right). \tag{26}$$

Going back to period 1, strategic bond traders born at that time must choose whether to store their entire endowment or purchase a government bond in the primary market. The expected profit from buying a bond is

$$\hat{s}(q_1)\left\{\mathbb{E}[q_2|\mathcal{I}_{i,1}^b]-q_1\right\}.$$

Market clearing in the first period requires

$$D_1^b = 1 - \Phi\left(\epsilon_1^b / \sqrt{\psi_1^b}\right). \tag{27}$$

#### A.2 Trading in the Yen Economy

Figure 5 represents trading in the Yen economy. As in Figure 4, black arrows represent flows of goods and blue arrows represent flows of bonds; in addition, red arrows represent now the flows of cash. In this case, there is no uncertainty about the nominal repayment from government bonds, which happens in cash in period 2. However, the terminal value of cash in period 3 depends on tax revenues. Strategic workers must decide whether to store their endowment until period 3 or to sell their goods in period 2 for cash or trade credit, at a price  $P_2$ . Noise workers will demand goods in period 2 in exchange for trade credit, in a fixed amount  $\Phi(\epsilon_2^w/\sqrt{\psi_2^w}) \in (0,1)$ . Traders born in period 1 will also use their cash to buy goods in period 2; by assumption, their demand is negligible compared to that of the workers (the thick arrows between SW/Traders<sub>2</sub> and NW in Figure 5 are designed to remind the reader of this).

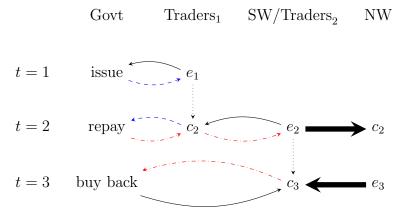


Figure 5: Markets in the Yen scenario. Goods (solid black); Bonds (dashed blue); Cash (dot-dashed red); Storage (dotted black).  $e_t$  and  $c_t$  are endowment and consumption in period t, SW and NW stand for strategic and noise workers respectively.

The payoff for a strategic worker of selling a unit of goods right away relative to storing it is

$$\mathbb{E}\left(\frac{1}{P_3}|\mathcal{I}_{i,2}^w\right) - \frac{1}{P_2},\tag{28}$$

where  $\mathcal{I}_{i,2}^w$  is the information available to the worker and  $P_3$  is the nominal price level in period 3, which is either 1 or  $1/\delta$ , depending on whether  $\theta \geq \hat{s}(q_1)$ . Hence, equation (28) becomes

$$\delta + (1 - \delta) \operatorname{Prob}\left(\theta \ge \hat{s}(q_1) | \mathcal{I}_{i,2}^w\right) - \frac{1}{P_2}.$$
 (29)

Letting  $D_2^w$  be the fraction of strategic workers selling the goods in period 2 (demanding cash or trade credit), market clearing in period 2 requires

$$D_2^w = \Phi(\epsilon_2^w / \sqrt{\psi_2^w}) = 1 - \Phi(-\epsilon_2^w / \sqrt{\psi_2^w}). \tag{30}$$

Since there is no secondary market for government bonds in period 2, noise traders are not active.<sup>35</sup> Strategic traders face the same choice as the workers: either store their endowment or sell it for cash or trade credit. Since their mass is negligible relative to that of the workers, their choice has no effect on market clearing and prices.

<sup>&</sup>lt;sup>35</sup>Recall that we assumed that the demand from noise traders is a fraction of the supply of bonds.

Going back to period 1, the problem of strategic bond traders in period 1 is similar to the Euro economy, except that their payoff is now a fixed amount of Yen with uncertain value rather than an uncertain amount of Euros. The expected profit from buying a bond is

$$\hat{s}(q_1) \left\{ \mathbb{E}\left[\frac{1}{P_2} | \mathcal{I}_{i,1}^b\right] - q_1 \right\},\,$$

and market clearing is still given by (27).

#### A.3 Comparing the Two Economies

The construction of an equilibrium in the two economies is very similar. The only difference between the two concerns the identity of the marginal agent in period 2. In the Euro scenario, this is a bond trader active in the secondary market, while in the case of Yen-denominated debt it is a worker selling her goods in exchange for nominal payments. This is seen comparing equations (25) and (26) for the Euro economy with equations (29) and (30) of the Yen economy.

The parameters of interest are thus the relative information that workers and second-period traders have about the government's ability to raise taxes in the final period. Our key assumption is that bond traders are more informed than workers, that is, they have a more precise signal  $(\beta_2^b > \beta_2^w)$  and face less market noise  $(\psi_2^b > \psi_2^w)$ .

Table 1 highlights the symmetry between the two scenarios, which we exploit to collapse the two cases into a single problem. Accordingly, we drop the superscripts referring to workers and traders, we define  $q_2 := 1/P_2$  in the case of the Yen, and we refer to "demand" by second-period strategic agents as their real demand for risky assets, which is their *supply* of goods: in the case of the Euro, traders acquire government bonds in the secondary market, whereas in the case of the Yen workers acquire cash or trade credit.

We thus proceed by analyzing a single problem. Defining  $\bar{\theta}(q_1) := \hat{s}(q_1)$ , this is precisely the economy of Section 2 when  $\hat{s}(q_1)$  is constant and normalized to 1, and that of Section 5 for the extension in which the default threshold depends on the first-period price of debt and  $q_1$  is the inverse of the gross interest rate at issuance. Our analysis proceeds from here by studying comparative statics with respect to  $\beta_2$ .<sup>36</sup>

<sup>&</sup>lt;sup>36</sup>We exploit the symmetry of the normal distribution in equation (30) and renormalize  $\epsilon_2^w = -\epsilon_2^w$  in the case

	Euro	Yen
Identity of marginal buyer	bond trader	worker
Goods given up at $t=2$	$\hat{s}(q_1)q_2$	1
Goods received at $t = 3$ :		
- w/o default/inflation	$\hat{s}(q_1)$	$P_2/P_3 = P_2$
- with default/inflation	$\delta \hat{s}(q_1)$	$P_2/P_3 = \delta P_2$
Return:		
- w/o default/inflation	$1/q_2$	$P_2$
- with default/inflation	$\delta/q_2$	$\delta P_2$

Table 1: Comparison of the payoffs of strategic agents in period 2.

## Appendix B Proofs

**Proof of Proposition 2.** First, with some algebra it is possible to show that the sign of  $\partial K/\partial \beta_2$  is equal to the sign of the following expression<sup>37</sup>

$$\beta_2 \psi_1 (1 + \psi_2) [\beta_1 (1 + \psi_1 + 2\psi_2) + 2\psi_2 (\beta_1 \psi_1 - \tau_\rho)]$$

$$+ (1 + \psi_1) \tau_\rho [\beta_1 \psi_1 (2\psi_2 - 1) - 2\psi_1 \tau_\rho] + \alpha \psi_1 [2\beta_1 (1 + \psi_1) \psi_2 - (1 + 2\psi_2) \tau_\rho]$$

which is linear in  $\beta_2$  with a positive slope coefficient. Let  $\hat{\beta}_2$  be the unique value of  $\beta_2$  for which the above expression is zero.

Second, note that  $\hat{\beta}_2$  is a quadratic function of  $\tau_{\rho}$ , with a positive coefficient on the quadratic term and a unique positive root which we denote with  $\tilde{\tau}_{\rho}$ . It follows that  $\hat{\beta}_2 \leq 0$  if and only if  $\tau_{\rho} \leq \tilde{\tau}_{\rho}$ . This condition is always verified when  $\beta_1 \leq \frac{\alpha(2\psi_2 - \psi_1)}{\psi_1(1+\psi_1)}$ , because in this case  $\tilde{\tau}_{\rho}$  is greater than its upper bound  $\beta_1\psi_1$ ; when instead  $\beta_1 > \frac{\alpha(2\psi_2 - \psi_1)}{\psi_1(1+\psi_1)}$ , we have that  $\tilde{\tau}_{\rho} \in (0, \beta_1\psi_1)$ . Together with the definition of  $\hat{\tau}_{\rho} := \min\{\tilde{\tau}_{\rho}, \beta_1\psi_1\}$ , this proves the Proposition.

**Proof of Proposition 3.** To prove the proposition it is sufficient to prove that  $K(\beta_2 = \beta_1) > K(\beta_2 \to 0)$ . This implies that, even when  $\tau_{\rho} \in (\hat{\tau}_{\rho}, \beta_1 \psi_1]$  and K has a positive interior minimum of the Yen economy.

<sup>&</sup>lt;sup>37</sup>Further details are provided in the online appendix.

in  $\beta_2$  (defined by  $\hat{\beta}_2$  in the previous proof), such minimum is smaller than  $\beta_1$  and that K is locally increasing at  $\beta_2 = \beta_1$ . Please refer to the online appendix for explicit algebra derivations.

**Proof of Proposition 4.** Define  $q_1^A(z_1)$  and  $q_1^B(z_1)$  to be the equilibrium prices in the first period when information precision in the second period is  $\beta_2^A$  and  $\beta_2^B$  respectively. Similarly, define  $K^A$  and  $K^B$  to be the values of K from equation (22) for the two precision values. Examine the argument of the cumulative distribution function on the right-hand side of (24). The second term is linear in  $z_1$  with coefficient K, while the first term is a bounded function of  $z_1$  since  $\bar{\theta}(q_1) \in (\bar{\theta}(1), \bar{\theta}(\delta))$ . It follows that, if (and only if)  $K^A > K^B$ , then there exists  $\hat{z}_1^L$  such that  $q_1^A(z_1) < q_1^B(z_1)$  for all  $z_1 < \hat{z}_1^L$ . The same argument applies for the upper threshold  $\hat{z}_1^H$ . The comparison of  $K^A$  and  $K^B$  was derived in Proposition 3.

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# Appendix C Appendix for Online Publication

**Proposition 5** (Belief Stochastic Dominance). In each period, agents' posterior beliefs over  $\theta$  are increasing in their private signal in the sense of first-order stochastic dominance. Whenever prices do not fully reveal the value of  $\theta$ , this monotonicity property is strict.

**Proof of Proposition 5.** Denote with  $F(\theta|x_{i,2}, q_2, \rho)$  the cumulative distribution function (cdf) of the posterior beliefs on  $\theta$  for a second-period agent with private signal  $x_{i,2}$ , after observing a signal  $\rho$  of the primary-market price and when the secondary-market price is  $q_2$ . Similarly, let  $h(x|\theta, q_2, \rho)$  be the probability density function of the second-period idiosyncratic signal conditional on  $(\theta, q_2, \rho)$ , and  $G(\theta|q_2, \rho)$  be the conditional cdf of  $\theta$  given  $q_2$  and  $\rho$ . By Bayes' rule,

$$F(\theta|x, q_2, \rho) = \frac{\int_{-\infty}^{\theta} h(x|y, q_2, \rho) dG(y|q_2, \rho)}{\int_{-\infty}^{+\infty} h(x|y, q_2, \rho) dG(y|q_2, \rho)}.$$
 (31)

To prove the proposition, we show that, if  $x_2 < \hat{x}_2$ , then  $\frac{F(\theta|x_2,q_2,\rho)}{F(\theta|\hat{x}_2,q_2,\rho)} > 1$  whenever the two cumulative distribution functions are strictly between 0 and 1.<sup>38</sup> First, note that the ratio converges to 1 as  $\theta \to +\infty$ . We obtain

$$\frac{F(\theta|x_2, q_2, \rho)}{F(\theta|\hat{x}_2, q_2, \rho)} = \frac{\int_{-\infty}^{\theta} h(x_2|y, q_2, \rho) dG(y|q_2, \rho)}{\int_{-\infty}^{\theta} h(\hat{x}_2|y, q_2, \rho) dG(y|q_2, \rho)} \cdot \frac{\int_{-\infty}^{+\infty} h(\hat{x}_2|y, q_2, \rho) dG(y|q_2, \rho)}{\int_{-\infty}^{+\infty} h(x_2|y, q_2, \rho) dG(y|q_2, \rho)}.$$

The second fraction on the right-hand side is independent of  $\theta$ .  $h(\cdot|\theta, q_2, \rho)$  is independent of  $(q_2, \rho)$  and normally distributed, so that  $\frac{h(x_2|y)}{h(\hat{x}_2|y)} > \frac{h(x_2|\theta)}{h(\hat{x}_2|\theta)}$  for all  $y < \theta$ . We next prove that  $W(\theta) := \frac{\int_{-\infty}^{\theta} h(x_2|y,q_2,\rho)dG(y|q_2,\rho)}{\int_{-\infty}^{\theta} h(\hat{x}_2|y,q_2,\rho)dG(y|q_2,\rho)}$  is decreasing in  $\theta$ , and strictly so in regions of positive probability. This completes the proof, since we know that  $\frac{F(\theta|x_2,q_2,\rho)}{F(\theta|\hat{x}_2,q_2,\rho)}$  converges to 1 in the limit. Let  $\theta_2 > \theta_1$  such that  $G(\theta_1|q_2,\rho) > 0$ ,<sup>39</sup> then

 $<sup>\</sup>overline{\phantom{a}}^{38}$ Since h is a normal density (with unbounded support), equation (31) implies that  $F(\cdot|x_2,q_2,\rho)$  and  $F(\cdot|\hat{x}_2,q_2,\rho)$  are absolutely continuous with respect to each other, for any values of  $x_2$  and  $\hat{x}_2$ ; hence, the sets on which they are 0 and 1 coincide.

<sup>&</sup>lt;sup>39</sup>If  $G(\theta_1|q_2, \rho) = 0$ , then  $F(\theta_1|x, q_2, \rho) = 0$  for all x.

$$\begin{split} W(\theta_2) - W(\theta_1) &= \frac{\int_{y \leq \theta_1} h(x_2|y) dG(y|q_2, \rho) + \int_{\theta_1}^{\theta_2} h(x_2|y) dG(y|q_2, \rho)}{\int_{y \leq \theta_1} h(\hat{x}_2|y) dG(y|q_2, \rho) + \int_{\theta_1}^{\theta_2} h(\hat{x}_2|y) dG(y|q_2, \rho)} - \frac{\int_{y \leq \theta_1} h(x_2|y) dG(y|q_2, \rho)}{\int_{y \leq \theta_1} h(\hat{x}_2|y) dG(y|q_2, \rho)} = \\ &\frac{\int_{\theta_1}^{\theta_2} h(x_2|y) dG(y|q_2, \rho) \int_{y \leq \theta_1} h(\hat{x}_2|y) dG(y|q_2, \rho) - \int_{y \leq \theta_1} h(x_2|y) dG(y|q_2, \rho) \int_{\theta_1}^{\theta_2} h(\hat{x}_2|y) dG(y|q_2, \rho)}{\int_{y \leq \theta_2} h(\hat{x}_2|y) dG(y|q_2, \rho) \int_{y \leq \theta_1} h(\hat{x}_2|y) dG(y|q_2, \rho)} \leq \\ &\frac{h(\hat{x}_2|\theta_1)}{h(x_2|\theta_1) \int_{y \leq \theta_2} h(\hat{x}_2|y) dG(y|q_2, \rho) \int_{y \leq \theta_1} h(\hat{x}_2|y) dG(y|q_2, \rho)} \cdot \\ &\left[\int_{\theta_1}^{\theta_2} h(x_2|y) dG(y|q_2, \rho) \int_{y \leq \theta_1} h(x_2|y) dG(y|q_2, \rho) - \int_{\theta_1}^{\theta_2} h(x_2|y) dG(y|q_2, \rho) \int_{y \leq \theta_1} h(x_2|y) dG(y|q_2, \rho) \right] = 0, \end{split}$$

where the inequality is strict if G has positive mass on  $(\theta_1, \theta_2]$ .

The posterior beliefs on  $\theta$  of a first-period trader with private signal  $x_{i,1}$  are given by  $F(\theta|x_{i,1},q_1)$ . Proving these are increasing in  $x_{i,1}$  in the sense of first-order stochastic dominance follows the same steps used above for second-period beliefs.

**Proposition 6** (Informational Equivalence of z and q in the case of debt payoff (Section 4) and no recall  $(\tau_{\rho} = 0)$ ). Let  $\pi(\theta)$  be the debt payoff in equation (20). Assume that in equilibrium the first-period price  $q_1$  is a continuous function of  $(\theta, \epsilon_1)$  and the second-period price  $q_2$  is a continuous function of  $(\theta, \epsilon_2)$ . Let  $\Sigma_1$  be the  $\sigma$ -algebra generated by the  $\pi$ -system  $\{q \in \mathbb{R} : q_1 \leq q\}$  and  $\hat{\Sigma}_1$  by  $\{z \in \mathbb{R} : z_1 \leq z\}$ , with  $z_1$  as defined in (5). Similarly, let  $\Sigma_2$  be the  $\sigma$ -algebra generated by the  $\pi$ -system  $\{q \in \mathbb{R} : q_2 \leq q\}$  and  $\hat{\Sigma}_2$  by  $\{z \in \mathbb{R} : z_2 \leq z\}$ , with  $z_2$  as defined in (3). Then  $\Sigma_1 = \hat{\Sigma}_1$  and  $\Sigma_2 = \hat{\Sigma}_2$ .

**Proof.** First, note that equation (3) follows directly from Proposition 5 and risk neutrality. Second, note that the function  $\hat{x}_2(q_2)$  is defined via the indifference condition

$$\delta + (1 - \delta)\operatorname{Prob}(\theta \ge \bar{\theta}|x_{i,2} = \hat{x}_2, q_2) = q_2. \tag{32}$$

Consider interior prices  $q_2 \in (\delta, 1)$ . Since conditional repayment probabilities are strictly increasing in the private signal  $\hat{x}_2$ , it follows that  $\hat{x}_2(q_2)$  exists and is unique.<sup>40</sup> Then the market

<sup>40</sup> Existence follows because, when  $q_2 \in (\delta, 1)$ , the price does not reveal fully whether  $\theta \geq \bar{\theta}$ . Bayes' rule then implies that the left-hand side converges to  $\delta$  as  $\hat{x}_2 \to -\infty$  and to 1 as  $\hat{x}_2 \to \infty$ .

clearing condition (3) is a single-valued mapping from the price  $q_2$  to the linear combination of shocks  $z_2 := \theta + \epsilon_2/\sqrt{\beta_2 \psi_2} = \hat{x}_2(q_2)$ .

Next, we use the property above to prove that corner prices cannot arise with positive probability in equilibria in which the price is continuous in  $(\theta, \epsilon_2)$ . Suppose by contradiction that a positive-probability set H can be found for which  $q_2$  is equal to  $\delta$ .<sup>41</sup> Since H has positive probability, we can find two pairs  $(\theta^A, \epsilon_2^A)$  and  $(\theta^B, \epsilon_2^B)$  that correspond to two different values of  $z_2$ :  $z_2^A$  and  $z_2^B$ . Next, consider the price as a function of  $\theta$  moving along the two lines  $\theta + \epsilon_2/\sqrt{\beta_2\psi_2} = z_2^A$  and  $\theta + \epsilon_2/\sqrt{\beta_2\psi_2} = z_2^B$ . As  $\theta$  increases along the lines, the price will eventually have to increase, since a price of  $\delta$  implies that H must lie below  $\bar{\theta}$  almost surely. Since  $q_2$  is continuous, there must be two points  $(\tilde{\theta}^A, \tilde{\epsilon}_2^A)$  and  $(\tilde{\theta}^B, \tilde{\epsilon}_2^B)$  on the two lines where the price is interior and the same. This contradicts what we have proved, since we showed that, whenever the price is interior,  $z_2 = \hat{x}_2(q_2)$ , with  $\hat{x}_2$  being single valued.

Having established that the price is almost surely interior, notice that  $z_2$  is continuous in  $(\theta, \epsilon_2)$  by construction and so is  $q_2$  by assumption. Hence, the mapping from  $q_2$  to  $z_2$  that exists from the arguments in the previous paragraphs must be continuous and thus measurable. This then implies that  $z_2$  is also  $\Sigma_2$ -measurable.

We next prove that  $q_2$  is  $\hat{\Sigma}_2$ -measurable. This proof follows the arguments of Pálvölgyi and Venter [42]. By contradiction, suppose that (on a set of positive measure) there are two vectors  $(\theta^C, \epsilon_2^C) \neq (\theta^D, \epsilon_2^D)$  that lie on the same straight line indexed by  $z_2$  but that correspond to different prices  $q_2^C$  and  $q_2^D$ , i.e. such that

$$\theta^{C} + \epsilon_{2}^{C} / \sqrt{\beta_{2} \psi_{2}} = z_{2}$$
, and  $q_{2}(\theta^{C}, \epsilon_{2}^{C}) = q_{2}^{C}$   
 $\theta^{D} + \epsilon_{2}^{D} / \sqrt{\beta_{2} \psi_{2}} = z_{2}$ , and  $q_{2}(\theta^{D}, \epsilon_{2}^{D}) = q_{2}^{D}$ 

Since  $q_2$  is continuous, the intermediate value theorem ensures that, for any curve that connects  $(\theta^C, \epsilon_2^C)$  to  $(\theta^D, \epsilon_2^D)$ , there must be at least one point  $(\theta, \epsilon_2)$  such that  $q_2(\theta, \epsilon_2) = \frac{q_2^C + q_2^D}{2}$ . First we apply the theorem to the curve represented by the straight line connecting  $(\theta^C, \epsilon_2^C)$  to  $(\theta^D, \epsilon_2^D)$ , and denote with  $(\hat{\theta}, \hat{\epsilon}_2)$  the point on such line such that  $q_2(\hat{\theta}, \hat{\epsilon}_2) = (q_2^C + q_2^D)/2$ . Along this

<sup>&</sup>lt;sup>41</sup>The same logic applies to the case in which  $q_2 = 1$ .

line  $z_2$  remains constant. Second, we apply the theorem to any other curve which intersects our straight line  $z_2$  only at  $(\theta^C, \epsilon_2^C)$  and  $(\theta^D, \epsilon_2^D)$ , again such that  $(\tilde{\theta}, \tilde{\epsilon}_2)$  lies on the curve and  $q_2(\tilde{\theta}, \tilde{\epsilon}_2) = (q_2^C + q_2^D)/2$ . It follows that we have found two different points,  $(\hat{\theta}, \hat{\epsilon}_2)$  and  $(\tilde{\theta}, \tilde{\epsilon}_2)$ , that correspond to the same price but are such that  $\hat{\theta} + \hat{\epsilon}_2/\sqrt{\beta_2\psi_2} \neq \tilde{\theta} + \tilde{\epsilon}_2/\sqrt{\beta_2\psi_2}$ . This contradicts the necessary market clearing condition (3).

The proof for the first period repeats the same steps as above.

It is possible to generalize the proposition to the generic increasing payoff function  $\pi(\theta)$  of Section 3, but the proof is considerably more involved, so here we choose to focus on the debt application.

**Proposition 7.** Assume that neither  $q_1$  nor  $q_2$  fully reveals the state of the economy. In any equilibrium in which  $q_2$  and  $z_2$  convey the same information given  $\rho$ ,  $q_2$  is a strictly increasing function of  $z_2$ . Furthermore, the expected resale price for a first-period trader is strictly increasing in her private signal.

**Proof.** Proposition 5 proves that  $\mathbb{E}[\pi(\theta)|x_{i,2}, z_2, \rho]$  is strictly increasing in the private signal. Repeating the same steps, we can also prove that it is increasing in the market signal  $z_2$ , since  $z_2$  also satisfies the monotone likelihood ratio property. Combining the two facts, the expected value perceived by the marginal trader,  $\mathbb{E}[\pi(\theta)|x_{i,2}=z_2,z_2,\rho]$ , is strictly increasing in  $z_2$ , which implies from equation (4) that  $q_2$  is also increasing in  $z_2$ .

The expected resale price for a first-period trader who received a private signal  $x_{i,1}$  is given by

$$\mathbb{E}[q_{2}|x_{i,1}, q_{1}] = \mathbb{E}[\mathbb{E}[\pi(\theta)|x_{i,2} = z_{2}, z_{2}, \rho]|x_{i,1}, q_{1}]$$

$$= \mathbb{E}[\mathbb{E}[\mathbb{E}[\pi(\theta)|x_{i,2} = z_{2}, z_{2}, \rho]|x_{i,1}, q_{1}, \theta]|x_{i,1}, q_{1}]$$

$$= \mathbb{E}[\mathbb{E}[\mathbb{E}[\pi(\theta)|x_{i,2} = z_{2}, z_{2}, \rho]|q_{1}, \theta]|x_{i,1}, q_{1}].$$
(33)

In the equation above, the last step follows from the fact that  $x_{i,1}$ ,  $\rho$ , and  $z_2$  are independent of each other conditional on  $\theta$ .  $\rho$  is a noisy public signal of the first-period price observed by second-period agents. As such, conditional on the actual first-period price  $q_1$ , it is independent of fundamentals and of the private signal  $x_{i,1}$ . Hence, the beliefs of the first-period trader about

 $\rho$  are independent of  $x_{i,1}$ . The distribution of  $z_2$  conditional on  $\theta$ ,  $\rho$  is equal to the distribution conditional on  $\theta$  alone and it is strictly increasing in  $\theta$  in the sense of first-order stochastic dominance. It follows that  $\mathbb{E}[\mathbb{E}[\pi(\theta)|x_{i,2}=z_2,z_2,\rho]|q_1,\theta]$  is strictly increasing in  $\theta$ . Repeating the steps of Proposition 5, the distribution of  $\theta$  conditional on  $x_{i,1},q_1$  is strictly increasing in  $x_{i,1}$  and therefore  $\mathbb{E}[q_2|x_{i,1},q_1]$  is strictly increasing in  $x_{i,1}$ .

#### C.1 Derivation of $q_1$

We start from equation (7):

$$q_1(z_1) = \mathbb{E}[q_2(z_2, \rho)|x_{i,1} = z_1, z_1].$$

We then use the expression for  $q_2$  from (10), and the distributions of  $z_2$  and  $\rho$  conditional on  $z_1$  given by (12):

$$q_{1}(z_{1}) = \int \int \int \pi(\theta) d\Phi \left( \frac{\theta - (1 - w_{\rho} - w_{z_{2}})\mu_{0} - w_{\rho}\rho - w_{z_{2}}z_{2}}{\sigma_{2}} \right) d\Phi \left( \frac{z_{2} - (1 - w_{1})\mu_{0} - w_{1}z_{1}}{\sigma_{2|1}} \right) d\Phi \left( \frac{\rho - z_{1}}{\sigma_{\eta}} \right).$$

Defining  $y := (z_2 - (1 - w_1)\mu_0 - w_1 z_1)/\sigma_{2|1}$  and changing the variables of integration we get

$$\begin{split} q_{1}(z_{1}) &= \\ &\int \pi(\theta) \int \int \frac{1}{\sigma_{2}} \phi \left( \frac{\theta - \mu_{0}(1 - w_{\rho} - w_{z_{2}}w_{1}) - z_{1}(w_{\rho} + w_{z_{2}}w_{1}) - \eta_{1}(w_{\rho}\sigma_{\eta}) - y(w_{z_{2}}\sigma_{2|1})}{\sigma_{2}} \right) d\Phi(y) d\Phi(\eta_{1}) d\theta \\ &= \int \pi(\theta) \int \frac{1}{\sqrt{w_{z_{2}}^{2}\sigma_{2|1}^{2} + \sigma_{2}^{2}}} \phi \left( \frac{\theta - \mu_{0}(1 - w_{\rho} - w_{z_{2}}w_{1}) - z_{1}(w_{\rho} + w_{z_{2}}w_{1}) - \eta_{1}(w_{\rho}\sigma_{\eta})}{\sqrt{w_{z_{2}}^{2}\sigma_{2|1}^{2} + \sigma_{2}^{2}}} \right) \phi(\eta_{1}) d\eta_{1} d\theta \\ &= \int \pi(\theta) \frac{1}{\sqrt{w_{z_{2}}^{2}\sigma_{2|1}^{2} + \sigma_{2}^{2} + w_{\rho}^{2}\sigma_{\eta}^{2}}} \phi \left( \frac{\theta - \mu_{0}(1 - w_{\rho} - w_{z_{2}}w_{1}) - z_{1}(w_{\rho} + w_{z_{2}}w_{1})}{\sqrt{w_{z_{2}}^{2}\sigma_{2|1}^{2} + \sigma_{2}^{2} + w_{\rho}^{2}\sigma_{\eta}^{2}}} \right) d\theta, \end{split}$$

This shows that  $q_1$  exists and is unique for all  $z_1 \in \mathbb{R}$  and yields the expression (13) in the main text.