On the Adoption Problem

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Semantic Groundedness: the truth value of sentences involving semantic notions can be ascertained on the basis of non-semantic facts (the notion can be made precise in different ways: e.g. Kripke (1975), Yablo (1982), Leitgeb (2004)).

Semantic Closure: semantical notions ought to be expressible in the object-language.

Prompted by truth-preservation, and granted that now 'true' and 'false' now mean *groundedly true* and *groundedly false*:

(CONSEQUENCE1) ψ is a CONSEQUENCE1 of φ iff whenever φ is true, ψ is also true.

Any inference of the form (φ, φ) — including (LIAR, LIAR) — will be licensed by CONSEQUENCE1, regardless of the semantic status of φ. More generally, CONSEQUENCE1 validates *Strong Kleene* logic: besides reflexivity, we do not have in general that

(NEG) if Δ is a CONSEQUENCE1 of Γ, φ, then $\Delta, \neg\varphi$ is a consequence of Γ.

By **Semantic Closure**, the notions involved in our *logical theory* should be expressible in the object-language. The following look plausible for a new connective $\twoheadrightarrow$ in the language:

($\twoheadrightarrow$1) if ψ is CONSEQUENCE1 of φ, then $\varphi \twoheadrightarrow \psi$ is true;

($\twoheadrightarrow$2) if φ and $\varphi \twoheadrightarrow \psi$ are true, then ψ is also true.

This creates absurdities, e.g. by considering a Curry-like sentence

$$\varkappa := (\text{`}\varkappa\text{' is true} \twoheadrightarrow \text{the earth is flat})$$

The paradoxical reasoning reveals that the inference $(\varkappa, \varkappa)$ played a fundamental role in the argument: i.e. we reasoned with 'semantically defective' sentences. This suggests the revision:

(CONSEQUENCE2) ψ is a CONSEQUENCE2 of φ iff either φ is false or ψ is true.

CONSEQUENCE2 can indeed be internalized as prescribed by **Semantic Closure**. It is non-paradoxical, as shown in Nicolai and Rossi (2017). What is crucial for our purposes is that its associated logic (often called TS from 'tolerant-strict'), with respect to Strong Kleene,

- gives up inferences of the form (Γ, Δ) for $\Gamma \cap \Delta \neq \varnothing$;
- features (NEG) unrestrictedly reformulated for CONSEQUENCE2.

The revision has the peculiar feature that instances of ADD and DROP are both present.

If our *logical theory* includes also structural properties of our inference patterns, CONSEQUENCE2 can be shown to be superior as the associated infinitary calculus enjoys cut-elimination, whereas this is not true for CONSEQUENCE1.

References

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