The least of all evils

Carlo Nicolai



Bristol, May 3, 2019 (slides at carlonicolai.github.io)

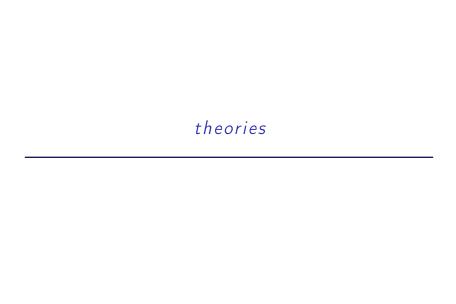
plan

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language

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- 「·¬ somehow yields a name for each sentence of the language.
- on occasion I will work with $\mathcal{L}_{\mathbb{N}}$, which yields a proper way of dealing with formal syntax

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$$(\neg I) \frac{\Gamma \Rightarrow A, \Delta}{\Gamma, \neg A \Rightarrow \Delta} \qquad (\neg r) \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg A, \Delta}$$

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Observation

This logic is the substructural 'dual' of FDE: $\Gamma \Rightarrow \Delta / \Gamma' \Rightarrow \Delta'$ is admissible iff $\Lambda \Gamma \rightarrow V \Delta \Rightarrow \Lambda \Gamma' \rightarrow V \Delta'$ is FDE-valid.

(CI)
$$\frac{\Gamma \Rightarrow \Delta, A \qquad B, \Gamma \Rightarrow \Delta}{\Gamma, C(\lceil A \rceil, \lceil B \rceil) \Rightarrow \Delta}$$
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... from which one can easily obtain (assuming $\Gamma \Rightarrow \Delta, \top$):

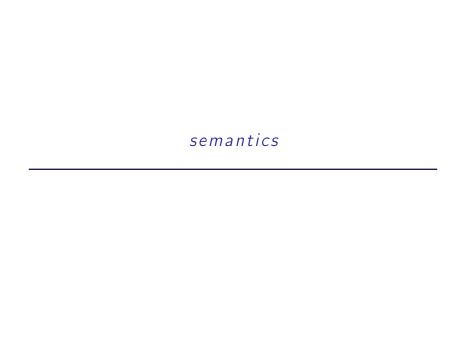
$$(\operatorname{Tr} I) \frac{A, \Gamma \Rightarrow \Delta}{\operatorname{Tr} \Gamma A^{\gamma}, \Gamma \Rightarrow \Delta} \qquad (\operatorname{Tr} r) \frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, \operatorname{Tr} \Gamma A^{\gamma}}$$

compositional truth: RKF

It is possible to formulate compositional extensions of LPC, e.g.:

$$\frac{\Gamma, C(\sigma, \delta) \Rightarrow \Delta \qquad \Gamma, C(\tau, \delta) \Rightarrow \Delta}{\Gamma, C(\sigma, \delta) \Rightarrow \Delta} \qquad \frac{\Gamma \Rightarrow C(\sigma, \delta), C(\tau, \delta), \Delta}{\Gamma \Rightarrow C(\sigma, \delta), \Delta}$$

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LPC – and its compositional extension – has a natural model (\mathbb{N}, X_{Φ}) where X_{Φ} is a fixed point of a monotone operator Φ such that:

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- with $(\mathbb{N}, X) \models^* \Gamma \Rightarrow \Delta$ iff all $\gamma \in \Gamma$ are true/both/neither only if there is a $\delta \in \Delta$ that is true/both, we have:

$$\#(\Gamma, A \Rightarrow B, \Delta) \in X \text{ iff } \#(\Gamma \Rightarrow C(\Gamma A^{\neg}, \Gamma B^{\neg}), \Delta) \in X.$$

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Observation

$$\left(\mathbb{N}, X_{\Phi \upharpoonright \operatorname{Tr}}\right) \vDash \mathsf{KF} \upharpoonright \mathsf{iff} \; \left(\mathbb{N}, X_{\Phi \upharpoonright \operatorname{Tr}}\right) \vDash \mathsf{PKF} \upharpoonright \mathsf{iff} \; \left(\mathbb{N}, X_{\Phi \upharpoonright \operatorname{Tr}}\right) \vDash^* \mathsf{RKF} \upharpoonright$$

LPC $^{\infty}$ is obtained from LPC in $\mathcal{L}_{\mathbb{N}}$ by omitting free variables and:

- adding axioms $\Gamma \Rightarrow r = s, \Delta$ and $\Gamma, r = s \Rightarrow \Delta$; where r, s are closed terms of $\mathcal{L}_{\mathbb{C}}^{\mathbb{N}}$ and, respectively, $r^{\mathbb{N}} = s^{\mathbb{N}}$ and $r^{\mathbb{N}} \neq s^{\mathbb{N}}$.
- Replacing (∀r) with:

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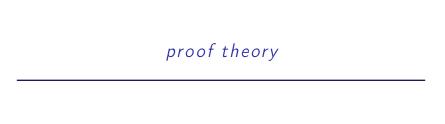
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Proposition

For \mathcal{I}_{Φ} the (Π_1^1 -complete) minimal fixed point of Φ , one has (by employing cut-elimination):

$$A\Rightarrow B\in\mathcal{I}_\Phi\quad\text{iff}\quad\vdash^\alpha_{\mathsf{LPC}^\infty}\Rightarrow\mathrm{C}(\ulcorner A\urcorner,\ulcorner B\urcorner)\text{ for some }\alpha<\omega_1^{\mathsf{ck}}$$



Cut-elimination for theories with non-logical rules such as (Cl) and (Cr) is a piece of cake for non-contractive systems (Cantini 2003, Grišin 1982): derivation trees don't 'grow' in the reduction, so one can simply count the applications of Tr - or C-rules.

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- ... there is one for LPC (and extensions thereof), and RKF\[\cdot\].

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• If \mathcal{D} ends with

$$\frac{\gamma_1^1, \dots, \gamma_n^1 \Rightarrow \delta_1^1, \dots, \delta_m^1, A \quad B, \gamma_1^2, \dots, \gamma_n^2 \Rightarrow \delta_1^2, \dots, \delta_m^2}{\gamma_1^3, \dots, \gamma_n^3, C(\lceil A \rceil, \lceil B \rceil) \Rightarrow \delta_1^3, \dots, \delta_m^3}$$

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$$\kappa(\mathbf{C}(\lceil A \rceil, \lceil B \rceil)) = \max(\kappa(A), \kappa(B)) + 1$$

$$\kappa(\gamma_i^3) = \max(\kappa(\gamma_i^1), \kappa(\gamma_i^2)) \quad 1 \le i \le n$$

$$\kappa(\delta_j^3) = \max(\kappa(\delta_j^1), \kappa(\delta_j^2)) \quad 1 \le j \le m$$

Lemma (Inversion)

All rules of LPC are κ -invertible: the C-complexity of the complex formulae is **no** greater than the one of their constituents, and strictly smaller for $\kappa > 0$.

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For instance: If
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, then there are $\mathcal{D}' \vdash_{\mathsf{LPC}} \Gamma^{10} \Rightarrow \Delta^{10}, \varphi$ and $\mathcal{D}'' \vdash_{\mathsf{LPC}} \psi, \Gamma^{11} \Rightarrow \Delta^{11}$ with $\kappa(\Gamma^{10}), \kappa(\Gamma^{11}) \leq \kappa(\Gamma^1)$, with $\kappa(\Delta^{10}), \kappa(\Delta^{11}) \leq \kappa(\Delta^1)$, and
$$\kappa(\varphi), \kappa(\psi) \leq \kappa(\mathsf{C}(\ulcorner \varphi \urcorner, \ulcorner \psi \urcorner)), \text{ if } \kappa(\mathsf{C}(\ulcorner \varphi \urcorner, \ulcorner \psi \urcorner)) = 0, \text{ or } \kappa(\varphi), \kappa(\psi) < \kappa(\mathsf{C}(\ulcorner \varphi \urcorner, \ulcorner \psi \urcorner)), \text{ if } \kappa(\mathsf{C}(\ulcorner \varphi \urcorner, \ulcorner \psi \urcorner)) > 0.$$

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Lemma

All **truth rules** of RKF \uparrow are κ -invertible – again, with a strict inequality if the active formula has Tr -complexity > 0.

full cut elimination

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Corollary

LPC $^{\infty}$ admits full cut elimination.

partial cut elimination

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Lemma (Tr-cut elimination)

Let \mathcal{D}_0 , \mathcal{D}_1 be cut-free derivations, in RKF \uparrow , of $\Gamma^1 \Rightarrow \Delta^1$, $\operatorname{Tr}(t)$ and $\operatorname{Tr}(t)$, $\Gamma^2 \Rightarrow \Delta^2$ respectively. Then we can find a derivation \mathcal{D} of $\Gamma \Rightarrow \Delta$ with $\kappa(\Gamma^k) \leq \kappa(\Gamma)$ and $\kappa(\Delta^k) \leq \tau(\Delta)$ and $k \in \{1,2\}$.

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Corollary $(I\Delta_0(superexp))$

RKF \restriction is **conservative** over PA: if $\Rightarrow \varphi$ is derivable in RKF \restriction and $\varphi \in \mathcal{L}_{\mathbb{N}}$, then there is a derivation of $\Rightarrow \varphi$ without Tr -cuts, i.e. a PA-derivation of $\Rightarrow \varphi$. A fortiori, RKF \restriction is consistent if PA is.

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- The version of CT↑ with restricted initial sequents a subtheory of RKF↑ – admits, by contrast, a smooth cut-elimination procedure.
- What I presented for truth can be extended to a compositional theory of consequence in the style of RKF.

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