Progress Report

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1 Prediction of Foreground Jobs

1.1 Statistics of Data

The data provided by Microsoft Azure Data Center contains maximum and average of CPU in 5 minutes windows. The length of traces in this analysis are limited to 8000 data points, which is roughly 30 days, and the number of traces used in this section is 3000.

1.1.1 Characteristics of Traces

We examined the empirical distribution of Coefficient of Variation, Entropy, Variance for both maximum and average traces across all 3000 traces to see the dispersion of maximum values aggregated in different windows.

As suggested by Fig 1, the variance of maxes increases as window size increases from 5 minutes to 5 hours and decreases from 6 hours to 24 hours.

Fig 2 indicates by using Coefficient of variation as a measurement of dispersion, as window size increases, the dispersion of maxes increases.

Fig 3, the entropy demonstrates similar behaviour as Fig 1, the entropy of maxes increases as window size increases from 5 minutes to 5 hours and decreases from 6 hours to 24 hours.

1.1.2 Correlations of Predictors

To find the best predictors for the maximum CPU of next window for a given size, we examined autocorrelations of maximum data, cross-correlation of maximum data and average data, and correlations of average of maximum in the past windows in different lags.

As an example, only window size of 12 observations (1 h) will be presented.

As shown in Fig 4, Fig 5 and Fig 6, amongst all the different window sizes of the past predictors, window size of 1h is shown to be one of the best choices for predicting maxes of next window size of 1h for all three different choices of statistics.

Among the three different choices of statistics, the Pearson correlation between Maximum of next one hour window and maximum of previous one hour window is strongest and the correlation using Averages of past windows and Average of Maxes of past windows is the weakest.

1.2 Scoring System

To schedule a background job at next window without interfering with the foreground job, we use prediction interval of next window to find a safe margin for the point estimate of maximum of next window, and then we schedule foreground job size as $100 - u(\alpha)$ with $u(\alpha)$ being the prediction upper bound of level α.

Two scores are designed to measure the performance of scheduled size $100 - u(\alpha)$ as a function of $u(\alpha)$ and *actual* which is the actual observation of maximum at next window:

$$\textbf{survival} = \begin{cases} 0 & u(\alpha) \leq actual \\ 1 & u(\alpha) \geq actual \\ \text{NA} & u(\alpha) \geq 100 \end{cases} \tag{1}$$

$$\textbf{utilization} = \begin{cases} \frac{100-u(\alpha)}{100-actual} & actual = 1 \\ 0 & actual = 0 \\ \text{NA} & actual = 100 \end{cases} \tag{2}$$

The survival score represents whether the scheduled background job will interfere with the foreground job, and utilization score represents how much of available resource the scheduling actually uses.

Outcomes of Scoring: For a single prediction, the survival score has three outcomes as described above; An *undefined(NA)* result is produced whenever the prediction upper bound equals or exceeds 100 percent meaning the scheduler refuses to assign jobs of any size at this particular interval. Similarly, an *undefined(NA)* is produced whenever the actual maximum hits 100 percent meaning there is no space to utilize regardless of the choice of the scheduler. To evaluate the outcome of an entire trace, the outcomes of disjoint windowed and consecutive predictions are aggregated using weighted sum. The *undefined(NA)* values are naturally considered to have weights of 0, as both circumstances should not be penalized or encouraged by our scoring system.

Formulation of Problem: Our algorithm is designed to solve is a constrained optimization problem: For predictions on fixed number of windows, denoted as *predictions*, and given target of survival rate, denoted as *target*:

$$\begin{aligned}\mathbf{max}\frac{\sum(utilization)}{predictions}\\ \mathbf{s.t.}\frac{\sum(survival)}{predictions} \geq goal\end{aligned}$$

Trade-off Between Survival Rate and Utilization: Survival rate quantifies how conservative our next prediction is while utilization rate penalizes the wideness of the safety margin of the prediction interval.

A lot of the designs in the mechanism of the algorithm relies on the trade-off between survival score and utilization score. A natural parameter that demonstrates such behaviour is the level of prediction interval, α or the cut-off probability referring to $0.5 * (1 - \alpha)$ in the following context.

1.3 Cut-off Probability

As described in Trade-Off Between Survival Rate and Utilization of Scoring System section, different cut-off probability values appear on different positions of trade-off curve between survival rate and utilization rate. Also, due to Limit of Fixed Ranged Observations described in Adjustment Policy, a smaller value of cut-off probability is needed to reach target survival rate than $0.5 * (1 - target)$, but such setting only works for less volatile traces with prediction interval upper bound smaller than 100 - granularity to leave minimal resource for scheduled job.

Fig 7 and Table 1 shows the trade-off between Survival Rate and Utilization Rate with AR1 model and offline training policy. Similar conclusion can be drawn with different models and different training policies. With smaller cut off probability, the survival rate is higher at the expense of lower utilization rate for most traces.

Trade-off Curve: A trade-off curve illustrates the trade-off between Survival Rate and Utilization Rate under a certain parameter setting. It can be represented by a line connecting a finite number of points with each point representing a different cut off probability on a 2D plane with Survival Rate and Utilization Rate on the axes. Comparisons against trade-off curves under different parameter settings may identify which parameter setting is a better option under a certain (or possibly all) region(s). In our case, a better Survival Rate and a better Utilization Rate means a superior option, therefore, whenever a trade-off curve is at the top right of another curve, it shows that the parameter for the parameter setting corresponding to the former curve is better.

Training Parameter: As the volatility of some traces changes over time, having a constant cut-off probability is not necessarily the optimal solution to the constrained optimization problem, especially cut-off probability has a interaction effect with adjustment policy on the performance on each batch.

One simple solution to dynamically train cut-off probability is to try all the discretized values in a prespecified range to solve the constrained optimization problem.

If cut-off probability is discretized, an iterative optimization algorithm EM algorithm can be used to simultaneously train both the parameters in adjustment policy and cut-off probability.

Since cut-off probability is a continuous parameter, a more complicated solution is to use Bayesian Optimization to calculate survival score and utilization score for each parameter setting and build a hyperplane.

1.4 Models

1.4.1 AR1

An AR1 model is a simple model based on the ECDF of Partial Autocorrelation plots which indicates that using one lag from the past window is significantly good predictor for most traces. A trained AR1 model contains intercept, slope, and variance of the residuals as trained parameters, therefore it needs relatively small training size to train these parameters.

Stationary Condition: By using ADF tests for unit roots, a huge proportion of the maximum traces shows insignificant statistical evidence to contain a unit root. However, it is very common to have irregular fluctuations or jumps that results in impossible survival at those windows, even though the ADF test shows insignificant test statistics, those traces appear to have low overall survival score and is not appropriate to directly fit AR1 model without preprocessing.

External Regressors: Since the average data over the same window as maximum data is also computed and recorded in the dataset, it may be helpful to include the average data of past window as an regressor to predict the maximum of current window according to the cross-correlation plot, Fig 5.

By comparing AR1 and AR1X in Fig 8 and Table 2, using AR1 with average trace as an external regressor results in a better trade-off curve than using pure AR1 model for most settings of cut off probabilities. Considering average value of past window results in more conservative prediction interval and therefore higher survival rate and lower utilization rate. However, the advantage for AR1X model becomes weaker as cut off probability becomes more and more extreme, such as 0.001.

Model adequacy Check: To check whether AR1 is a good fit statistically. Ljung-Box tests for testing white noise series are used on residuals whenever an AR1 model is fitted.

Experiments show that for most of the traces the Ljung-Box test does not conclude statistically significant test statistics to reject that the residuals are a white nosie process. However, some spikes, jumps or irregular behaviours are common for most traces.

Residual Distributions: Using Maximum Likelihood Estimation for parameter estimations in AR1 model and constructing prediction interval using normal quantiles both rely on the assumption that the error term is normally distributed. Histogram of residuals is compared to the normal distribution with estimated parameters.

Experiments show that the distribution of residuals for most traces are skewed, asymmetric with heavy tails, this suggests skewed normal seems to be a better fit. However, using skewed normal results in similar performance in survival scores and utilization scores. The parameter estimation of skewed normal innovations is provided by "Estimation of autoregressive models with epsilon-skew-normal innovations": https://www.sciencedirect.com/science/article/pii/S0047259X09000360.

Using bootstrapping methods to construct prediction interval and Conditional Sum of Squares estimation method can avoid the normality assumption of the residuals. Experiments how that Conditional Sum of Squares estimation may result in different estimated values than Maximum Likelihood estimation method, but the difference is minimal with respect to the variance of the estimates.

Fig 9 and Table 3 show that using empirical distribution yielded by bootstrapping as highest survival score and lowest utilization score using the same cut off probability, which indicates empirical distribution has prediction upper bound higher than using normal and skewed normal distribution approximation. For most traces, using skewed normal approximation has lower survival score and higher utilization score which indicates most approximated skewed normal are right skewed, and therefore yields less conservative prediction intervals. Even though using any of normal, skew-normal or empirical distribution for residuals yields similar trade-off curves, it is noticeable that using skew-normal is highly sensitive to the choice of cut off probabilities, whereas using empirical distribution is more robust for different choices of cut off probabilities.

Outlier Analysis: For most traces, outliers are very common throughout the whole trace. An occurrence of outlier normally results in failur of survival score at the window and over conservative prediction at the succeeding window when using AR1 model. Outliers also result in overestimate of variance and instability of other parameters of AR1 model. The density of occurrences of outliers varies in different segment of one trace and also varies from trace to trace.

A simple solution to deal with outliers is to use τ-trimmed sample to estimate variance and other parameters. However, experiments show that is critical to select τ since removing extreme values results in monotonically smaller estimate of variance which directly affects the width of prediction interval. It is hard to find an α that is suitable for an entire trace, but it can be trained using stepwise optimization methods when training.

Another statistical approach to deal with outliers involves in automatic detection of outliers using test statistics assuming the type of outliers, estimation of effect of outliers and remove the effect of outliers. An automatic procedure identifies the outliers as one of Additional Outliers(AO), Innovational Outliers(IO), Temporary Change(TC) or Level shift(LS). However, experiments show that for most traces, using above automatic procedure results in lower survival rate and lower utilization rate for most traces. The automatic procedure of outlier detection is provided by Joint Estimation of Model Parameters and Outlier Effects in Time Series: http://www.jstor.org/stable/2290724.

Fig 10 and Table 10 show that by considering outlier effect, the estimation of variance of residuals decreases as more observations are considered as outliers, resulting less conservative prediction intervals than not considering outliers. The survival score decreases and utilization score increases as the result. Another observation is that considering the outliers as LS yields similar scoring outcomes as not considering outliers at all. This shows that LS type of outliers are not as common as AO and IO outliers in most traces. By not considering any type of outlier, AR1 model yields the best trade-off curve.

1.4.2 ARI11

Since for a significant proportion of traces, the maximum traces are non-stationary due to change of behaviours, a significant number of outliers, temporal trend or seasonality. Those traces need to be preprocessed by taking difference by neighbouring observations to become stationary. The AR1 model is fitted to model the change from one window to the succeeding window. The prediction interval is calculated by first forecasting the prediction interval for the change from current window to the next window and then adding the maximum of current window to the interval.

Comparing Fig 8 and Table 2 show that ARI11 model has some improvement on performance over AR1 model for most traces under same cut off probability. Using ARI11 model also yields more conservative prediction intervals, and therefore higher survival rate and lower utilization. However, using ARI11 model with average trace as an external regressor yields much worse trade-off curve, ARI11X may result in overly conservative prediction interval where not much gain in survival rate but significant loss in utilization rate occurs when cut off probability takes lower values.

1.4.3 General ARIMA

A great proportion of the model has at least one significant change of behaviours in maxes. Also, a small proportion of the traces requires more than lag one in at least one of its regime. Therefore, a general ARIMA model is fitted at each retraining step in this method. The appropriate AR, MA order of the model is selected by minimizing AICc(Akaike Information Criterion corrected) with maximum order of 3.

The automatic order detection adds a small complexity to the simulation. However, Fig 8 and Table 2 show that for offline training, Auto-detect ARIMA model does not yield better trade-off curve than AR1 model. But by including average trace as an external regressor brings more significant improve on Auto-detect ARIMA model than AR1 model.

Overhead of ARIMA-like Models:

Memory Consumption: Since AR1 model, ARI11 model are simple models with only three parameters to be estimated and kept track of (intercept, slope and variance of residuals), using them are very light weighted in terms of memory consumption.

Training and Updating Models: There are two ways of training parameters for ARIMA-like models, one is using Maximum Likelihood under the assumption of observations follows conditional normal distribution, or by minimizing Conditional Sum of Squares(also called Conditional Least Squares). For simple AR1 model, it is easy to derive closed form of the parameters in terms of training data. Therefore, by using vector operations, the computational cost for training is very small. For general ARIMA models, it is somewhat more costly to train parameters since optimization algorithms such as gradient descent may be used. In terms of updating models, it is possible to use weighted sum of past trained parameter and new observations to update parameters. Hence, updating complicated ARIMA models or retraining simple AR1 model would be computationally cheap.

Doing Predictions: Under normality assumption or skew-normal assumption of innovations, doing point predictions and then constructing prediction interval is simple and constant cost. However, using bootstrapping strategies to from empirical distribution to construct prediction interval could be computationally costly depending on the number of paths sampled.

1.4.4 Markov

A Markov model requires a state number parameter to be trained. To predict the maximum of next window, either the maximum of the current window or the average of current window can be used as the regressor. The transition matrix is trained by mapping the desired type of regressor of the one window to its actual observation at the succeeding window. Experiments show that the training size of choice has great impact on the performance on a whole trace.

Partitioning Methods: There are two ways in the experiments that maximum or average observations are partitioned into states; Fixed partitioning from range 0 to 100 and quantile partitioning according to the quantile of the training observations.

Fig 11 and Table 5 indicate that using quantile partitioning method results in better trade-off curve than using fixed partitioning method, since using quantile partitioning method results in much fewer untraiend or undertrained states.

Limit of training size: A Markov model essentially trains the transition matrix which contains the square of the state number. The training size needs to be sufficiently large to be fully saturated to all the states. In the experiments using Markov models, we are limited by the length of the traces being one month, therefore it is hard to train Markov models with large state number greater than 16 for large window sizes.

A simple solution to the limit of training size is to aggregate by overlapping windows instead of disjoint windows to yield more data points for training. Taking maximum in aggregation by overlapping windows yields repetitive observations in training size.

Untrained States: It is common for some traces that some of the states are untrained in fixed partitioning method, that is, no observations in training set fall into those states. This may be the result of limit of training size or the states are too extreme for observations to fall into.

A simple solution to avoid testing observation to fall into the untrained states is to use unconditional distribution for the untrained states. Experiments show that using unconditional distribution for untrained states has better performance than using uniform distribution for untrained states.

Untrained states are not considered to be a serious problem for a well trained Markov model, if enough observations are used in training step, it is extremely rare to have observations in testing batch to fall into one of the untrained states.

Use of External Regressor: Aside of training the traditional past window max to current window max transition matrix, we also trained past window average to current window average transition matrix and made predictions.

Fig 11 and Table 5 shows that by using fixed partitioning method, MarkovX(avg-to-max) model yields much higher survival rate and lower utilization rate. This is because past average values may transit to wider range of states than past maximum values, which results in more conservative decisions for a given cut off probability. Another observation from Table 5 is that when reducing cut off probability to more extreme values, using Markov-like models eventually fails to trade utilization rate for survival rate unlike ARIMA-like models. This is because of the limit in granularity of states when using Markov-like models, and it shows lesser flexibility when Markov-like models cannot reach a pre-specified survival rate no matter how extreme the cut off probability is.

Another observation is that using MarkovX(avg-to-max) models results in better trade-off curve than using Markov(max-to-max) models, also MarkovX model has higher survival rate but lower utilization rate comparing to Markov models. This is because transitions from one state in average data results in more uniform range of maximum data, therefore MarkovX models tend to make more conservative decisions.

Granularity of States: For Markov model with larger number of state number, each state covers a smaller range of observation. If state number is small, then α with small differences may be inseparable, that is, different value of α results in exact same prediction interval upper bounds. Because of the granularity of states, the prediction interval upper bounds are overestimated resulting in more conservative decisions when scheduling.

Fig 12 and Table 6 indicate that using smaller number of states, survival rate is slightly higher and utilization rate is slightly lower. It is also noticeable that with smaller state number, the scheduler tend to refuse to schedule more frequently. With smaller cut off probability, using 8 states results in better trade-off curve.

Overhead of Markov-like Models:

Memory Consumption: The Markov models need to store one transition matrix, which contains the number of parameters equals to square of the number of states. This is significantly more than ARIMA-like models. However, experiments show that using larger number of states not necessarily increase the performance of the model.

Training and Updating Models: To train the transition matrix, one needs to loop through all the training data, identifying the starting state and the ending state and then add them to the corresponding entry of the transition matrix. Hence, the computational cost of training Markov-like models is proportional to the training size. As for updating the transition matrix, there is no good way of updating the matrix with new observations and forgetting the oldest observations. But updating the matrix without forgetting would be simple and proportional to the number of new observations.

Doing Predictions: Doing predictions is simple for Markov-like models, especially for one step prediction. The complexity is $O(n)$ where n is the number of states. For multiple steps prediction, the complexity becomes $O(n^3)$, since matrix multiplication is needed.

1.4.5 VAR

VAR models maximum and average of the same window simultaneously which is different than AR1 model with external regressor.

Higher Order of VAR models: Adding one additional order to AR order or MA order results in adding four additional parameters to the overall model. Experiments show that adding additional MA order is not necessary, inaccurate estimation of variance-covariance matrix of the residuals results in unstable prediction intervals.

Comparing all the Multivariate series models, Fig 13 and Table 7 indicate that VAR1 model has better trade-off curve comparing with higher orders of VAR models, such as VAR2 and VAR3. However, AR1X model appears to have better trade-off curve comparing to VAR1 model, which suggests using a simpler structure of only one time series is a better choice than more complicated dual time series structure.

Overhead of VAR Models:

Memory Consumption: For a two-series VAR model described above, only two 2 by 2 matrices storing the parameters, and one 2 by 2 matrix storing variance-covariance matrix is needed. Therefore, memory overhead for VAR model is constant.

Training and Updating Models: Training a simple VAR1 model is similar to training an AR1 model, only with matrix operations. Maximum Likelihood Estimation has complexity of $O(m^2)$ where m is the number of training observations, since a matrix multiplication of m by 2 matrix, 2 by 2 matrix and 2 by m matrix is required. Ordinary Least Squares Estimation yields the same result with same complexity. However, with more complicated VAR models, the complexity also scales with the number of lags.

Doing Predictions: Doing predictions for VAR models requires constant cost of 2 by 2 matrix multiplications.

1.4.6 More Compicated Models

Feed Forward Neural Network: Neural Network with lag 1 up to lag p and a single hidden layer contains more parameters than AR(p) model, providing stronger ability in longer term predictions. The parameters are estimated by taking the average of multiple repetitions of randomly picked starting points and optimized using forward and backward propagation. The prediction interval is constructed by bootstrapping.

Under the setting of the number of neurons being half of the number of input nodes (including external regressors, if given) plus 1, Fig 14 and Table 8 show that using Neural Network with pure maximum traces is almost consistently better than using both maximum and average traces. Also, a neural network with only lag 1 of maximum traces, containing the least number of parameters, has better trade-off curve than other neural networks with more input nodes and neurons. The lag marked as "auto" in the figure and table refers to automatically selected lags by minimizing AIC(Akaike Information Criterion).

Overhead of NN Models:

Memory Consumption: For a Neural Network model with only lag 1 maximum traces as input and no hidden layers, it is exactly the same as AR1 model. With only one hidden layer and only one neuron like the model described above, the number of parameters in total is 4. For general Neural Network with lag 1 up to lag p as input, the total number of parameters will be $p + 1$ for input layer, and $floor(p/2) + 1$ for hidden layer.

Training and Updating Models: Training a simple VAR1 model is similar to training an AR1 model, only with matrix operations. Maximum Likelihood Estimation has complexity of $O(m^2)$ where m is the number of training observations, since a matrix multiplication of m by 2 matrix, 2 by 2 matrix and 2 by m matrix is required. Ordinary Least Squares Estimation yields the same result with same complexity. However, with more complicated VAR models, the complexity also scales with the number of lags.

Doing Predictions: Doing predictions for NN models requires bootstrapping which is time consuming, and is proportional to the number of sample paths in a similar fashion as ARIMA models with empirical distribution as residual distribution estimations.

1.5 Aggregation VS Extrapolation

To predict the maximum of next window of size 12, we can train the models using window size of 1, 2, up to 12, the number of steps to predict forward would be 12, 6, down to 1 accordingly.

The prediction interval expands proportionally to autocorrelations between residuals as more steps are extrapolated. The results from experiments shows that the performance of extrapolation highly depends on the model of choice, for example, the prediction intervals for 12 steps extrapolations using AR1 model are much wider than 1 step predictions for most traces, but the Markov model does not appear to have too much difference for most traces.

The number of steps of extrapolation is shown to be a factor of **trade-off between survival rate and utilization**, more steps in extrapolation for most models results in more conservative prediction intervals. In reality, it is hard to determine the optimal extrapolation steps in training procedures, this is normally a parameter that needs to be predefined depending on the window size to be predicted and the feasible number of aggregated training size. In the below experiments, we always use aggregation and one step prediction for conventional purpose.

Fig 15 and Table 9 show the performance of predicting the next hour window (12 observations) with control variable of extrapolation steps. With fewer steps to extrapolate, most traces have higher survival rate but lower utilization rate. One-step predictions has better trade-off curve comparing to other multi-step predictions with smaller windows.

1.6 Offline Training VS Online Training

Offline Training refers to training the model once and use the model for predictions for the entire trace, and **Online Training** refers to training the model dynamically either at a fixed amount of time (in this section is referred as **Fixed Point Training**) or when the scheduler detects bad performances using current model (in this section is referred as **Dynamic Training**). Online training is more expensive than offline training, the computational cost depends on the model of choice.

The intuition for developing online training strategy over simple offline training is because of the change of behaviours for some traces that we observed. The change of behaviours includes temporary or permanent level shifts, change of volatility, or change of occurrences in outliers, and they all require new models to be refitted at some frequency.

Fig 16 and Table 10 show that with parameter setting of using AR1 model with 840 data points for training and aggregation over window size of 12 to predict the next hour window (12 observations), even with simple strategy as Fixed Point Training results in better performance than offline training policy for most traces. This represents that change of behaviours are common for most traces and one model would not suffice for the whole trace.

Dynamic training is a more complicated online training policy is designed to better reach the goal for survival rate for most traces: The testing set is divided into batches, after evaluating the performance on the current batch is well performed (described in Section 1.6), if not, a retrain signal is sent and a new model is retrained using most recent data before evaluating the next testing batch.

The whole point of dynamic training is to avoid excessive training when not needed and therefore reduce the overhead of training comparing to fixed point training. Also, dynamic training trades utilization rate to survival rate to better achieve the target on survival rate.

Fig 16 and Table 10 show that with naive parameter setting of the Dynamic Training, the performance of most traces are very similar, yet slightly worse comparing to performance using simple Fixed Point Training.

Batch Size: Batch size is a hyperparameter used for both Fixed Point Training and Dynamic Training. It controls the frequency of updating trained models. Fig 17 and Table 11 shows that for Dynamic online training policy, AR1 model and fixed training size, more frequent retraining results in better trade-off curve, but more expensive overhead. Similar conclusion can be drawn with fixed point training policy.

Performance of Various Models: From Fig 18 and Table 12 show that all models results in better trade-off curve with online training policy than offline training policy. In this experiment, the model chosed for Markov model is with 8 states, and the lag of input time series of Neural Network models is 1, as suggested to be the best choices from previous sections where parameter navigations are done for those models. From all of the models selected, AR1, AR1 with external regressor and VAR1 model have the best trade-off curve, making them best candidate models. Even though Neural Network model has similar trade-off curve as these models, it is considered to be worse since the model cannot continue trade-off between utilization and survival rate with lower cut off probabilities as demosntrated by the cluster of points at the lower right of the yellow curve. Similarly, for MarkovX with fixed partitioning method and fixed partitioning method, the model fails to trade utilization rate for more survival rate at lower cut off probabilities.

1.7 Parallel Models Policy

The purpose of keep tracking of multiple models is similar to online training. Online training policy can be combined with **parallel models policy**, that is, the model simultaneously trains multiple models but only one of the model is the active model, that is, the performance of active model is actually recorded to evaluate the overall performance for the whole trace. Depending on the performance of active model and other inactive models, the scheduler may decide to change the status of one of the inactive models to active model, or remove the one of the inactive models and retrain a model when combined with Dynamic Training Scheme.

This training policy defines comparison operations between results of simulations, evaluated on a small size of data sets, called **batches**:

- **well performed(A)**: A simulation A result is well performed if the survival rate of the result satisfies the constraint, i.e., $\frac{\sum(survival)}{predictions} \geq goal$ for a given goal.

- **out performs(A, B)**: A simulation A result out performs another simulation result B if both A and B are well performed, and utilization rate of A is higher than that of B, otherwise if A is well performed and B is not well performed, otherwise if both A and B are not well performed, the survival rate of A is higher than that of B.

At a specific time, if the active model is not well performed on the previous batch, the scheduler sends a swap signal and check whether the other inactive models are well performed on the previous batch. If some of the models are well performed, the scheduler picks the best model using out performs comparison operation to be the active model for the next batch. If no models are well performed, the scheduler sends a retrain signal and replace the worst model using out performs comparison operation by newly trained model, and the newly trained model is automatically the active model for the next batch.

Fig 19 and Table 13 show that using Dynamic Online Training Policy with AR1 model, with more model numbers, the survival rate increases to better achieve the goal set, same as the purpose of dynamic training policy, in expense of utilization rate. However, the increase of model number shows worse trade-off curve.

Fig 20 and Fig 21 show the difference in offline training and dynamic online training when combing with parallel model policy. Both of them keep track of three different AR1 models with different trained parameters at the same time. Different background colours represents that different AR1 models are used, and a change in colour represents either a new AR1 model is trained or a different model is being selected. One model is at least used for three consecutive predictions because of the setting of **Batch size** being three, which is explained in detail in the previous section.

1.8 Granularity

Since the allocation of CPU resource is achieved by assigning CPU cores, granularity is evaluated by diving 100 percent by total number of cores of the machine representing the percentage of the total CPU resource assigned to each cores. Each machine can only assign resource as multiples of granularity. Granularity of 0 is a special case where granularity is not considered, and CPU resource can be any real number between 0 and 100.

Fig 22 does not show significant differences in survival rate and utilization rate affected by the size of granularity with AR1 model and offline training policy. Similar conclusion can be drawn with different models and different training policies. However, it is observed that for some traces, smaller granularity results in lower survival rate but higher utilization for majority choices of models due to slightly larger prediction interval.

1.9 Adjustment Policy

Limit of Fixed Ranged Observations: Ideally, the construction of prediction interval does not have limit on the range of the observations. It is common that the prediction interval upper bound exceeds 100 for volatile traces. However it is not realistic to schedule a job size of negative CPU size, 0 or smaller than one CPU core. In these cases, the scheduler does not count them as predictions made, i.e, the survival score of this prediction is not 1 nor 0.

Due to the fixed range of observations, survival rate of the performance is smaller than the level of prediction interval.

Reduction of Consecutive Failures: The purpose of adjustment policy is to reduce consecutive 0s in survival scores. Adjustment policy dynamically turns on and off the adjustment switch. When adjustment switch is ON, the predictions made are not counted. However, adjustment policy as part of the scheduler decision while being conservative and prevent the scheduling of jobs in consecutive windows may forfeit the actual available CPU resources, that is, unless the actual maximum is above 100 - granularity, the utilization when adjustment switch is ON is always 0.

Adjustment policy is controlled by two integer parameters, the first parameter corresponds to the number of consecutive 0s to activate the adjustment switch, and the second parameter corresponds to the number of consecutive 1s to deactivate the adjustment switch.

For adjustment policy of parameter 1,1, a survival segment of 0,1,0 would have a total survival rate of 0 of 2 predictions after adjustment, which is lower than 1/3 of 3 predictions before adjustment. In some cases, not carefully chosen adjustment parameters would result in lower overall survival rates.

Training Parameters: As described in the section of Limit of Fixed Ranged Observations, it is impossible to have overall survival rate reaching the predefined level of prediction interval. By dynamically training the two parameters of adjustment policy and the initial status of the switch, it is possible to avoid lower survival rate in the testing batch after adjustment.

A simple strategy to train such two parameters is to try all the values in a pre-specified range and find the value that maximizes the training utilization rate while being able to reach the predefined target survival rate.

Fig 23 and Table 15 shows that with offline training policy, even the simplest parameter setting of 1,1, the trade-off curve benefits significantly from the adjustment policy, meaning consecutive failures in predictions are common with offline training of AR1 model. Fig 24 and Table 16 show that with fixed point online training policy, the configuration have to be chosen carefully to get a better trade-off curve. But in the region where the survival rate is high (and cut off probability is low), a wider choice of react speed would be plausible. It can be seen that a setting of 1,1 for react speed results in the best trade-off curve.

1.10 Predicting Average of Next Window

Comparing to Maxes of most traces, the Average data is much less volatile, and more stationary, which makes them easy to predict.

Fig 25 and Table 17 shows that with simple parameter setting using AR1 model and offline training, the performance to predict average at next window has very high performance on utilization rate. Also, although the average traces are a lot more stationary than max traces, sudden jumps and irregular fluctuation may still appear in some of the traces, therefore differenced average traces are more suitable for fitting an AR1 model, the performance of ARI11 model is better than other models in both survival score and utilization score.

With slightly more complicated online strategy, Fig 25 and Table 17 shows that the performance using online fixed point training results in higher utilization and survival rate than using offline training. Even though most average traces are very smooth comparing to

Comparing to predicting the maximum value of next windows, predictioning average of next windows with same model typically results in both higher utilization rate and higher survival rate. The reason is that average traces, comparing to maximum traces, are less volatile and contains fewer outliers and irregular behaviours. Therefore, the former results in much more narrow prediction interval and hence higher utilization, and the latter results in higher survival rate.

One may also notice that the trade-off curve for both models using offline training has an upward direction when cut off probabilities decreases, which doesn't really suggests a trade off between utilization rate and survival rate. The reason for such phenomenon is that the trade-off curve is normally a concave down quadratic curve if a wide enogh range of cut off probabilities are supplied. When cut off proabilities are high enough, the model schedules larger jobs, and if the larger jobs cannot survive, a utilization of 0 is recorded, therefore when cut off probability increases, smaller jobs that can survive are scheduled, resulting a higher utilization rate and survival rate than with the higher cut off probabilities.

1.11 Comparing Against Autopilot

By using generated dataset of granularity of $1s$, we are able to compare the performance between Autopilot recommender and our algorithms. The Autopilot recommender is introduced in source: https://dl.acm.org/doi/pdf/10.1145/3342195.3387524.

An obvious observation using Autopilot is that the simulation requires much more memory to keep track of hundreds of histograms, therefore the total number of parameters are number of histograms times number of breaks in each histogram.

1.11.1 jth-Quantile

It is obvious that the parameter "j", analogous to "cut off probability", is an important factor for trade-offs between survival rate and utilization. Naturally, due to the design of our experiments, both AR1 method and Autopilot method cannot reach desired goal, which is, $1 - \alpha$ for arbitrary cut off probability α.

1.11.2 Generate more granular traces

In the data set, for each trace, the only available information for utilization is the average and max over five minutes period. We want to reconstruct the traces into more granular traces, namely, period of 300 observations.

Let $X(t)$ denote the utilization value of a trace at each second t. Let $\{t_0 = 0, t_1, ..., t_n\}$ $(t_i - t_{i-1} = 300$ for any i), denote the sample points where we observed the average utilization values $Y(t_i) = \int_{t_{i-1}}^{t_i} \frac{X(t)}{300} dt$. Consequently, we can compute the anti-derivative of $X(t)$, called $F(t)$, at sample points $\{t_0 = 0, t_1, ..., t_n\}$, by $F(t_i) = \sum_{j=1}^{i} 300 * Y(t_j) = \int_0^{t_i} X(t) dt$.

Now, we provide two methods to interpolate samples points for function F:

- **Version 1** Using a cubic interpolation spline and sampling noise to match exact max observations and approximately exact average observations.
- **Version 2** Using bandlimited interpolation based on Shannon's sampling theorem and window-wise scaling to match exact max observations.
- **Version 3** Using a cubic interpolation spline and constrained sampling noise to match exact max and average observations.

Either way, we are able to obtain the values of F at a finer granularity(i.e: at time grids $\{k_0 = 0, k_1, ..., k_m\}$

where $k_i - k_{i-1} = 1$). Approximating the derivative of F by taking consecutive difference, we can get an approximate for $X(t)$ at one minute granularity(i.e: $X(k_i) = F(k_i) - F(k_{i-1})$).

Fig 26 and Table 18 show that Version 1 and Version 2 have very similar trade-off curve and performance. Although Version 3 has poor performance with high cut off probabilities, when cut off probabilities are high, it has very similar trade-off curve with Version 1 and Version 2. In a actual situation, only cut off proabilities in the lower right region would be practical. The reason for the poor perforamnce on upper left region for Version 3 is because the constraints on maixmum and averages normally would result in a few artifical observations that are outliers. The reconstruction frequency for such reconstruction determines the frequency of the appearances of those outliers, which in our case is at least one per 300 observations. As cut off probabilities becomes smaller and the prediction interval becomes wider, the effect of those outliers eventually becomes diminished.

1.11.3 Choice of half-life

The half life parameter controls the rate of decay for the weights applied on the previous histograms. This parameter represents the length of time for the weight to decrease by $\frac{1}{2}$. The higher this parameter is, the model puts more weight on previous histograms and therefore is more long-memory.

Cut off weight: Ideally, the decay of weight can goes to infinitly small, which requires the scheduler to keep track of infinitly many of past histograms. In a more practical setup, a cut off weight is used to determine after which value the weight will be regarded as zero, which means the scheduler only needs to keep track of finitely many of past histograms. The values of cut off weight and half life determie the memory required for the Autopilot model.

1.11.4 Number of breaks

Since the prediction upperbound is based on the quantile of weighted histograms of past observations, the breaks controls the way of partitioning the histograms. Ideally, for different historgams of different ranges, shapes and variations, we can come up with a reasonable way of partitioning it, making sure there is enough observations in each bin to guarentee the histogram contains precise information of the empirical distribution. However, the model needs throusands of histograms of previous windows, each containing 300 observations of 5-minute windows, it is not feasible to manually examine each histogram to find out the best partitioning nor keep track of the breaks as parameters for thousands of histograms.

Therefore, we use the setting of breaks for all histograms to minizmie the number of parameters the scheduler needs to keep track of. A simple setting is to make sure the bin size is exactly the same in the range of 0 and 100, then we only need to specify the number of bins(or breaks).

1.11.5 Different Window Size

Since using "j-quantile" only requires to tune two important parameters, j and half-life, one should not use the same half life for predicting different window sizes into the future. For example,

1.12 Training Size

1.12.1 AR1

1.12.2 Markov

1.12.3 VAR

1.12.4 Neural Network

1.12.5 Autopilot

1.13 Appendix to Section 1

1.13.1 Statistics of Data

Ref to Section 1.1, include Fig 1, Fig 2, Fig 3, Fig 4, Fig 5, Fig 6.

1.13.2 Cut-off Probability

Ref to Section 1.2, include Fig 7, Table 1.

Table 1: Configuration and Result of AR1 Model with Different Cut Off Probability of Offline Training

cut off prob	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
0.005	0.9277	47487	0.3142	59249
0.01	0.9168	48124	0.3463	59249
0.03	0.8965	50300	0.4024	59249
0.05	0.8841	52904	0.4309	59249
0.07	0.8706	53640	0.4518	59249
0.10	0.8548	54444	0.4729	59249

Figure 1: ECDF of Varaince of Maxes

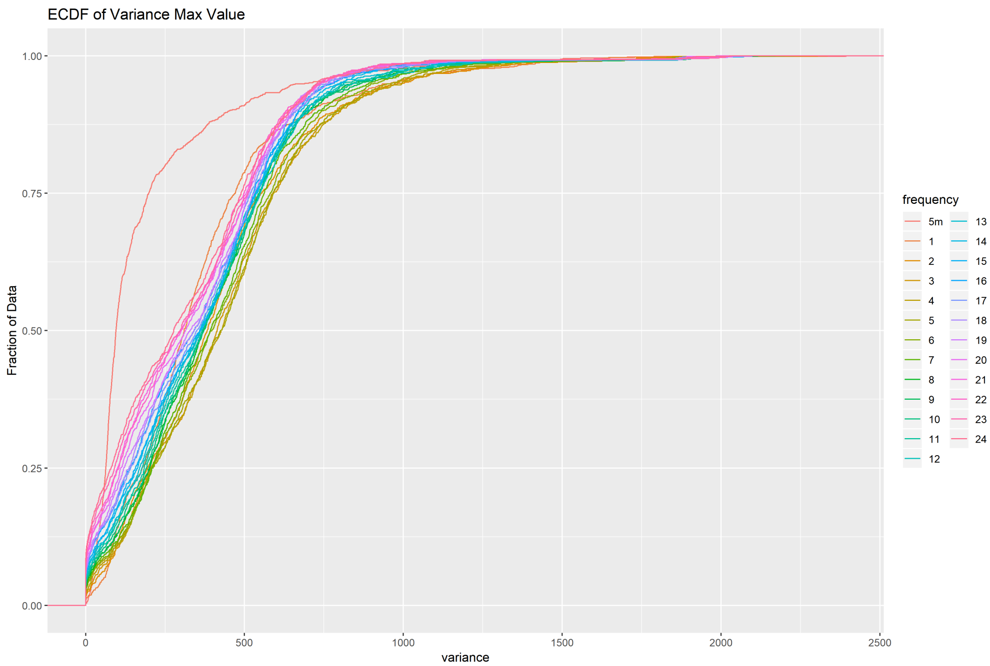


Figure 2: ECDF of CoV of Maxes

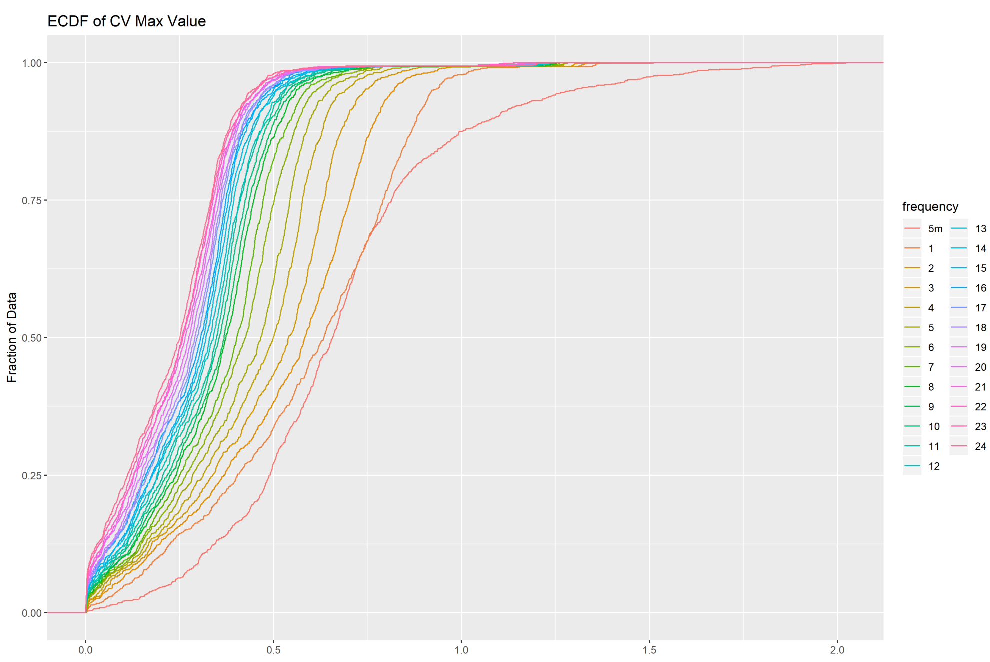
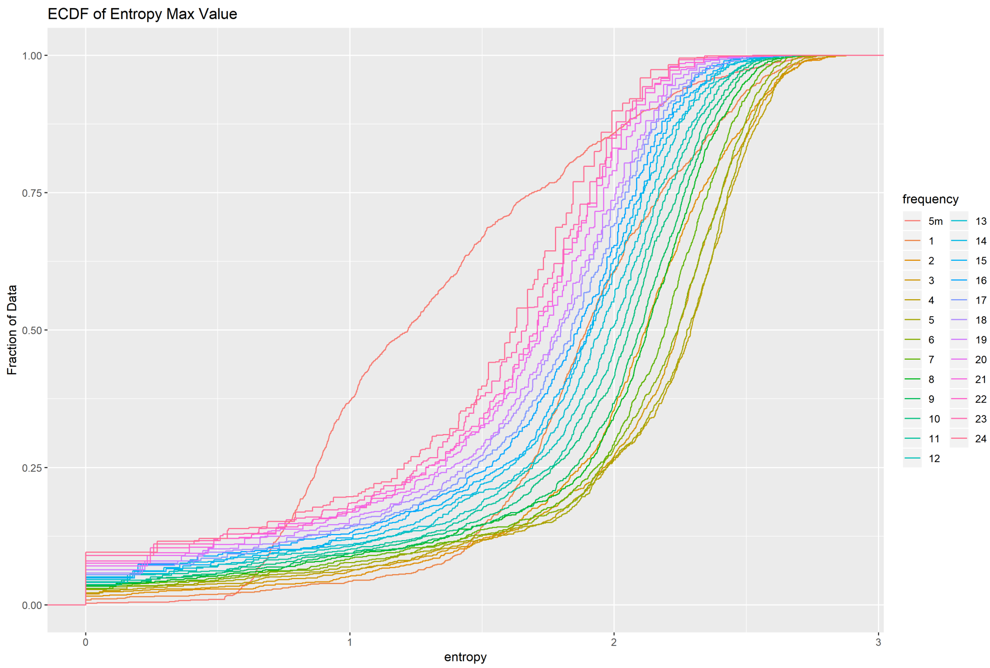


Figure 3: ECDF of Entropy of Maxes



1.13.3 Models

Ref to Section 1.3, include Fig 8, Table 2, Fig 9, Table 3, Fig 10, Table 4, Fig 11, Table 5, Fig 12, Table 6, Fig 13, Table 7, Fig 14, Table 8.

Table 2: Configuration and Result of ARI11,ARI11X Models with Offline Training

name	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
AR1	0.001	0.9452	42981	0.2468	59249
AR1	0.003	0.9352	46484	0.2915	59249
AR1	0.005	0.9277	47487	0.3142	59249
AR1	0.010	0.9168	48124	0.3463	59249
AR1	0.030	0.8965	50300	0.4024	59249
AR1	0.050	0.8841	52904	0.4309	59249
AR1X	0.001	0.9479	41306	0.2335	59249
AR1X	0.003	0.9407	43602	0.2746	59249
AR1X	0.005	0.9364	44299	0.2964	59249
AR1X	0.010	0.9293	45094	0.3270	59249
AR1X	0.030	0.9142	47570	0.3821	59249
AR1X	0.050	0.9034	50374	0.4111	59249

ARI11	0.001	0.9622	37735	0.2112	59249
ARI11	0.003	0.9541	42205	0.2576	59249
ARI11	0.005	0.9490	44002	0.2824	59249
ARI11	0.010	0.9410	46307	0.3205	59249
ARI11	0.030	0.9219	48903	0.3888	59249
ARI11	0.050	0.9064	50471	0.4240	59249
ARI11X	0.001	0.9399	38470	0.2069	59249
ARI11X	0.003	0.9339	42132	0.2524	59249
ARI11X	0.005	0.9303	43794	0.2768	59249
ARI11X	0.010	0.9238	45778	0.3140	59249
ARI11X	0.030	0.9065	48501	0.3808	59249
ARI11X	0.050	0.8910	49852	0.4131	59249
AUTO	0.001	0.9423	41566	0.2503	59249
AUTO	0.003	0.9353	46182	0.2934	59249
AUTO	0.005	0.9276	46954	0.3157	59249
AUTO	0.010	0.9152	47691	0.3464	59249
AUTO	0.030	0.8952	50206	0.4017	59249
AUTO	0.050	0.8820	52677	0.4309	59249
AUTOX	0.001	0.9505	41296	0.2474	59249
AUTOX	0.003	0.9439	43562	0.2880	59249
AUTOX	0.005	0.9394	44203	0.3089	59249
AUTOX	0.010	0.9328	45502	0.3395	59249
AUTOX	0.030	0.9180	48421	0.3950	59249
AUTOX	0.050	0.9082	51563	0.4254	59249

Table 4: Configuration and Result of AR1 Model with Different Outlier Detection Strategies

outlier type	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
None	0.001	0.9452	42981	0.2468	59249
None	0.003	0.9352	46484	0.2915	59249
None	0.005	0.9277	47487	0.3142	59249
None	0.010	0.9168	48124	0.3463	59249
None	0.030	0.8965	50300	0.4024	59249
None	0.050	0.8841	52904	0.4309	59249

AO	0.001	0.8719	48195	0.3885	59249
AO	0.003	0.8634	49576	0.4151	59249
AO	0.005	0.8569	49836	0.4272	59249
AO	0.010	0.8465	50288	0.4435	59249
AO	0.030	0.8269	52421	0.4704	59249
AO	0.050	0.8145	50288	0.4408	59249
IO	0.001	0.8796	47127	0.3638	59249
IO	0.003	0.8709	48489	0.3927	59249
IO	0.005	0.8648	48810	0.4065	59249
IO	0.010	0.8562	49834	0.4256	59249
IO	0.030	0.8382	52346	0.4591	59249
IO	0.050	0.8273	54894	0.4763	59249
LS	0.001	0.9337	42735	0.2589	59249
LS	0.003	0.9239	46469	0.3001	59249
LS	0.005	0.9164	47456	0.3206	59249
LS	0.010	0.9054	48360	0.3495	59249
LS	0.030	0.8852	50802	0.4011	59249
LS	0.050	0.8725	53029	0.4279	59249
All	0.001	0.8438	50238	0.4382	59249
All	0.003	0.8280	50563	0.4521	59249
All	0.005	0.8197	50671	0.4583	59249
All	0.010	0.8071	50820	0.4663	59249
All	0.030	0.7871	52401	0.4807	59249
All	0.050	0.7785	54721	0.4889	59249

Table 5: Configuration and Result of Markov Models with Different Partitioning Methods

model name	**partitioning type**	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
MARKOV	fixed	0.001	0.9046	29990	0.2349	59249
MARKOV	fixed	0.003	0.9045	30082	0.2360	59249
MARKOV	fixed	0.005	0.9045	30195	0.2381	59249
MARKOV	fixed	0.010	0.9052	31233	0.2439	59249
MARKOV	fixed	0.030	0.9014	37994	0.3025	59249
MARKOV	fixed	0.050	0.8930	41058	0.3497	59249

MARKOVX	fixed	0.001	0.9339	16849	0.1115	59249
MARKOVX	fixed	0.003	0.9339	16849	0.1115	59249
MARKOVX	fixed	0.005	0.9361	17443	0.1131	59249
MARKOVX	fixed	0.010	0.9361	17443	0.1131	59249
MARKOVX	fixed	0.030	0.9233	32287	0.2347	59249
MARKOVX	fixed	0.050	0.9126	36591	0.2906	59249
MARKOV	quantile	0.001	0.9598	56899	0.2096	59249
MARKOV	quantile	0.003	0.9598	56899	0.2096	59249
MARKOV	quantile	0.005	0.9598	56899	0.2096	59249
MARKOV	quantile	0.010	0.9573	56899	0.2168	59249
MARKOV	quantile	0.030	0.9493	56918	0.2532	59249
MARKOV	quantile	0.050	0.9436	56918	0.2705	59249
MARKOVX	quantile	0.001	0.9663	56511	0.191	59249
MARKOVX	quantile	0.003	0.9663	56511	0.191	59249
MARKOVX	quantile	0.005	0.9663	56511	0.191	59249
MARKOVX	quantile	0.010	0.9645	56650	0.1982	59249
MARKOVX	quantile	0.030	0.9617	56650	0.2109	59249
MARKOVX	quantile	0.050	0.9558	56650	0.2311	59249

Table 6: Configuration and Result of Markov Model with Different Number of States

state num	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
8	0.001	0.9225	36075	0.2870	59249
8	0.003	0.9193	40043	0.3256	59249
8	0.005	0.9101	41985	0.3689	59249
8	0.010	0.8792	45459	0.4231	59249
8	0.030	0.8139	49858	0.4873	59249
8	0.050	0.7913	51268	0.4983	59249
10	0.001	0.9075	39391	0.4306	59249
10	0.003	0.9032	42650	0.4306	59249
10	0.005	0.8939	44164	0.4306	59249
10	0.010	0.8683	46253	0.4306	59249
10	0.030	0.8029	50804	0.4306	59249
10	0.050	0.7800	51898	0.4306	59249

16	0.001	0.8975	42678	0.4282	59249
16	0.003	0.8956	45218	0.4282	59249
16	0.005	0.8862	45828	0.4282	59249
16	0.010	0.8612	47846	0.4282	59249
16	0.030	0.7882	51331	0.4282	59249
16	0.050	0.7582	52399	0.4282	59249
20	0.001	0.8760	45960	0.3840	59249
20	0.003	0.8743	46770	0.3962	59249
20	0.005	0.8628	47254	0.4178	59249
20	0.010	0.8475	48876	0.4505	59249
20	0.030	0.7806	52390	0.4884	59249
20	0.050	0.7457	52809	0.4886	59249

Table 8: Configuration and Result of NN Model with Different lags

model name	**p**	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
NN	1	0.001	0.9228	46859	0.3162	59249
NN	1	0.003	0.9235	46794	0.3176	59249
NN	1	0.005	0.9220	46972	0.3177	59249
NN	1	0.010	0.9143	48416	0.3478	59249
NN	1	0.030	0.8911	50528	0.4046	59249
NN	1	0.050	0.8774	52016	0.4328	59249
NN	2	0.001	0.9009	47134	0.3502	59249
NN	2	0.003	0.9015	47459	0.3520	59249
NN	2	0.005	0.9031	47391	0.3531	59249
NN	2	0.010	0.8913	48640	0.3747	59249
NN	2	0.030	0.8697	50890	0.4246	59249
NN	2	0.050	0.8558	52225	0.4460	59249
NN	3	0.001	0.8905	49009	0.3757	59249
NN	3	0.003	0.8892	48680	0.3736	59249
NN	3	0.005	0.8911	48482	0.3722	59249
NN	3	0.010	0.8742	49901	0.3960	59249
NN	3	0.030	0.8526	52100	0.4342	59249
NN	3	0.050	0.8413	53147	0.4574	59249

NN	4	0.001	0.8811	48268	0.3809	59249
NN	4	0.003	0.8749	48861	0.3820	59249
NN	4	0.005	0.8803	48171	0.3791	59249
NN	4	0.010	0.8713	50536	0.4020	59249
NN	4	0.030	0.8384	52674	0.4338	59249
NN	4	0.050	0.8301	53968	0.4541	59249
NN	auto	0.001	0.9090	47824	0.3406	59249
NN	auto	0.003	0.9065	47792	0.3399	59249
NN	auto	0.005	0.9105	47883	0.3448	59249
NN	auto	0.010	0.9010	49105	0.3705	59249
NN	auto	0.030	0.8773	51293	0.4175	59249
NN	auto	0.050	0.8632	52966	0.4430	59249
NNX	1	0.001	0.8761	46059	0.3255	59249
NNX	1	0.003	0.8760	46211	0.3285	59249
NNX	1	0.005	0.8774	46360	0.3316	59249
NNX	1	0.001	0.8619	47880	0.3457	59249
NNX	1	0.030	0.8466	49617	0.3917	59249
NNX	1	0.050	0.8398	50895	0.4136	59249
NNX	2	0.001	0.8888	45568	0.3438	59249
NNX	2	0.003	0.8896	45476	0.3449	59249
NNX	2	0.005	0.8860	46557	0.3472	59249
NNX	2	0.010	0.8840	47449	0.3657	59249
NNX	2	0.030	0.8649	50322	0.4080	59249
NNX	2	0.050	0.8547	52460	0.4306	59249
NNX	3	0.001	0.8855	45692	0.3487	59249
NNX	3	0.005	0.8871	46554	0.3583	59249
NNX	3	0.010	0.8865	48592	0.3851	59249
NNX	3	0.030	0.8551	50239	0.4043	59249
NNX	3	0.050	0.8456	51600	0.4267	59249
NNX	4	0.001	0.8564	48596	0.3862	59249
NNX	4	0.003	0.8639	48671	0.3873	59249
NNX	4	0.005	0.8570	49700	0.3894	59249
NNX	4	0.010	0.8488	49995	0.3943	59249
NNX	4	0.030	0.8173	51421	0.4222	59249

NNX	4	0.050	0.8023	52340	0.4274	59249
NNX	auto	0.001	0.8702	47413	0.3410	59249
NNX	auto	0.003	0.8677	47460	0.3436	59249
NNX	auto	0.005	0.8670	46978	0.3421	59249
NNX	auto	0.010	0.8652	48739	0.3672	59249
NNX	auto	0.030	0.8458	51894	0.4091	59249
NNX	auto	0.050	0.8303	52173	0.4251	59249

1.13.4 Aggregation VS Extrapolation

Ref to Section 1.4, include Fig 15 and Table 9.

Table 9: Configuration and Result of AR1 Model with Different Extrapolation Steps

window size	extrap step	cut off prob	score 1	score 1 weight	score 2	score 2 weight
12	1	0.001	0.9452	42981	0.2468	59249
12	1	0.003	0.9352	46484	0.2915	59249
12	1	0.005	0.9277	47487	0.3142	59249
12	1	0.010	0.9168	48124	0.3463	59249
12	1	0.030	0.8965	50300	0.4024	59249
12	1	0.050	0.8841	52904	0.4309	59249
6	2	0.001	0.9223	42894	0.2751	59249
6	2	0.003	0.9099	44826	0.3138	59249
6	2	0.005	0.9040	46383	0.3344	59249
6	2	0.010	0.8923	47700	0.3642	59249
6	2	0.030	0.8654	52518	0.4250	59249
6	2	0.050	0.8442	53601	0.4565	59249
4	3	0.001	0.8965	44584	0.3237	59249
4	3	0.003	0.8837	46569	0.3621	59249
4	3	0.005	0.8789	49140	0.3824	59249
4	3	0.010	0.8641	51370	0.4156	59249
4	3	0.030	0.8251	53782	0.4746	59249
4	3	0.050	0.8031	55304	0.5060	59249
3	4	0.001	0.8813	46600	0.3645	59249
3	4	0.003	0.8679	50354	0.4048	59249

3	4	0.005	0.8577	52049	0.4260	59249
3	4	0.010	0.8354	52424	0.4569	59249
3	4	0.030	0.7970	54986	0.5129	59249
3	4	0.050	0.7692	56591	0.5423	59249
2	6	0.001	0.8571	50336	0.4202	59249
2	6	0.003	0.8327	51827	0.4574	59249
2	6	0.005	0.8209	52433	0.4765	59249
2	6	0.010	0.7993	53626	0.5045	59249
2	6	0.030	0.7450	56532	0.5560	59249
2	6	0.050	0.6987	57175	0.5808	59249
1	12	0.001	0.7859	52731	0.5124	59249
1	12	0.003	0.7484	53427	0.5441	59249
1	12	0.005	0.7304	54588	0.5600	59249
1	12	0.010	0.6953	56316	0.5833	59249
1	12	0.030	0.6100	56991	0.6230	59249
1	12	0.050	0.5618	57010	0.6412	59249

1.13.5 Offline Training VS Online Training

Ref to Section 1.5, include Fig 16, Table 10, Fig 17, Table 11, Fig 18, and Table 12.

Table 12: Configuration and Result of Different Models and Different Training Policy

model	**training policy**	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
AR1	fixed	0.001	0.9698	42245	0.2849	59249
AR1	fixed	0.003	0.9627	45239	0.3297	59249
AR1	fixed	0.005	0.9583	46435	0.3530	59249
AR1	fixed	0.010	0.9504	47518	0.3873	59249
AR1	fixed	0.030	0.9319	50406	0.4498	59249
AR1	fixed	0.050	0.9181	52841	0.4848	59249
AR1	offline	0.001	0.9452	42981	0.2468	59249
AR1	offline	0.003	0.9352	46484	0.2915	59249
AR1	offline	0.005	0.9277	47487	0.3142	59249
AR1	offline	0.010	0.9168	48124	0.3463	59249
AR1	offline	0.030	0.8965	50300	0.4024	59249

AR1	offline	0.050	0.8841	52904	0.4309	59249
AR1X	fixed	0.001	0.9680	42594	0.2916	59249
AR1X	fixed	0.003	0.9607	45379	0.3354	59249
AR1X	fixed	0.005	0.9567	46476	0.3586	59249
AR1X	fixed	0.010	0.9488	47679	0.3922	59249
AR1X	fixed	0.030	0.9307	50578	0.4538	59249
AR1X	fixed	0.050	0.9165	52692	0.4875	59249
AR1X	offline	0.001	0.9479	41306	0.2335	59249
AR1X	offline	0.003	0.9407	43602	0.2746	59249
AR1X	offline	0.005	0.9364	44299	0.2964	59249
AR1X	offline	0.010	0.9293	45094	0.3270	59249
AR1X	offline	0.030	0.9142	47570	0.3821	59249
AR1X	offline	0.050	0.9034	50374	0.4111	59249
Markov fixed	fixed	0.001	0.9531	29904	0.2260	59249
Markov fixed	fixed	0.003	0.9532	30312	0.2295	59249
Markov fixed	fixed	0.005	0.9531	30917	0.2358	59249
Markov fixed	fixed	0.010	0.9520	32086	0.2512	59249
Markov fixed	fixed	0.030	0.9458	38918	0.3309	59249
Markov fixed	fixed	0.050	0.9371	42711	0.3925	59249
Markov fixed	offline	0.001	0.9046	29990	0.2349	59249
Markov fixed	offline	0.003	0.9045	30082	0.2360	59249
Markov fixed	offline	0.005	0.9045	30195	0.2381	59249
Markov fixed	offline	0.010	0.9052	31233	0.2439	59249
Markov fixed	offline	0.030	0.9014	37994	0.3025	59249
Markov fixed	offline	0.050	0.893	41058	0.3497	59249
MarkovX fixed	fixed	0.001	0.9789	21213	0.1438	59249
MarkovX fixed	fixed	0.003	0.9786	21296	0.1447	59249
MarkovX fixed	fixed	0.005	0.9785	21356	0.1454	59249
MarkovX fixed	fixed	0.010	0.9779	21520	0.1471	59249
MarkovX fixed	fixed	0.030	0.9619	34533	0.2702	59249
MarkovX fixed	fixed	0.050	0.9504	38975	0.3284	59249
MarkovX fixed	offline	0.001	0.9339	16849	0.1115	59249
MarkovX fixed	offline	0.003	0.9339	16849	0.1115	59249
MarkovX fixed	offline	0.005	0.9361	17443	0.1131	59249

MarkovX fixed	offline	0.010	0.9361	17443	0.1131	59249
MarkovX fixed	offline	0.030	0.9233	32287	0.2347	59249
MarkovX fixed	offline	0.050	0.9126	36591	0.2906	59249
MarkovX quantile	fixed	0.001	0.9795	55226	0.2233	59249
MarkovX quantile	fixed	0.003	0.9795	55241	0.2234	59249
MarkovX quantile	fixed	0.005	0.9794	55267	0.2236	59249
MarkovX quantile	fixed	0.010	0.9770	55437	0.2312	59249
MarkovX quantile	fixed	0.030	0.9725	55645	0.2475	59249
MarkovX quantile	fixed	0.050	0.9664	55860	0.2670	59249
MarkovX quantile	offline	0.001	0.9663	56511	0.1910	59249
MarkovX quantile	offline	0.003	0.9663	56511	0.1910	59249
MarkovX quantile	offline	0.005	0.9663	56511	0.1910	59249
MarkovX quantile	offline	0.010	0.9645	56650	0.1982	59249
MarkovX quantile	offline	0.030	0.9617	56650	0.2109	59249
MarkovX quantile	offline	0.050	0.9558	56650	0.2311	59249
NN	fixed	0.001	0.9531	46145	0.3657	59249
NN	fixed	0.003	0.9539	46128	0.3654	59249
NN	fixed	0.005	0.9542	46200	0.3676	59249
NN	fixed	0.010	0.9464	47654	0.3981	59249
NN	fixed	0.030	0.9259	50788	0.4618	59249
NN	fixed	0.050	0.9121	52772	0.4952	59249
NN	offline	0.001	0.9226	46819	0.3171	59249
NN	offline	0.003	0.9229	46805	0.3168	59249
NN	offline	0.005	0.9227	46923	0.3180	59249
NN	offline	0.010	0.9142	48401	0.3476	59249
NN	offline	0.030	0.8909	50527	0.4046	59249
NN	offline	0.050	0.8763	52059	0.4316	59249
VAR1	fixed	0.001	0.9618	44987	0.3284	59249
VAR1	fixed	0.003	0.9528	47098	0.3763	59249
VAR1	fixed	0.005	0.9464	47952	0.4005	59249
VAR1	fixed	0.010	0.9362	49559	0.4372	59249
VAR1	fixed	0.030	0.9067	53679	0.5056	59249
VAR1	fixed	0.050	0.8830	55244	0.5403	59249
VAR1	offline	0.001	0.9381	44419	0.2766	59249

VAR1	offline	0.003	0.9270	46114	0.3204	59249
VAR1	offline	0.005	0.9206	46533	0.3425	59249
VAR1	offline	0.010	0.9102	47837	0.3750	59249
VAR1	offline	0.030	0.8829	51860	0.4320	59249
VAR1	offline	0.050	0.8654	53962	0.4650	59249

1.13.6 Parallel Models Policy

Ref to Section 1.6, include Fig 19, Table 13, Fig 20, Fig 21.

Table 13: Configuration and Result of AR1 Model with Different Model Number and Dynamic Training

model num	cut off prob	score 1	score 1 weight	score 2	score 2 weight
1	0.001	0.9704	42449	0.2792	59249
1	0.003	0.9632	45137	0.3231	59249
1	0.005	0.9587	46419	0.3494	59249
1	0.010	0.9508	47422	0.3844	59249
1	0.030	0.9325	50363	0.4475	59249
1	0.050	0.9189	52866	0.4818	59249
2	0.001	0.9706	42352	0.2759	59249
2	0.003	0.9638	44989	0.3181	59249
2	0.005	0.9594	46071	0.3424	59249
2	0.010	0.9527	47453	0.3716	59249
2	0.030	0.9391	49963	0.4215	59249
2	0.050	0.9277	52260	0.4506	59249
3	0.001	0.9724	39050	0.2477	59249
3	0.003	0.9659	42414	0.2898	59249
3	0.005	0.9615	44330	0.3177	59249
3	0.010	0.9548	45913	0.3531	59249
3	0.030	0.9421	48813	0.4010	59249
3	0.050	0.9302	51368	0.4396	59249
4	0.001	0.9732	38110	0.2380	59249
4	0.003	0.9659	42477	0.2903	59249
4	0.005	0.9615	44285	0.3169	59249
4	0.010	0.9551	45760	0.3527	59249

4	0.030	0.9418	48358	0.4011	59249
4	0.050	0.9321	51371	0.4332	59249

1.13.7 Granularity

Ref to Section 1.7, include Fig 22, Table 14.

1.13.8 Adjustment Policy

Ref to Section 1.8, include Fig 23, Table 15.

Table 15: Configuration and Result of AR1 Model with Different React Speed of Adjustment Policy of Offline Training

react speed	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
0,0	0.9452	42981	0.2468	59249	
0,0	0.9352	46484	0.2915	59249	
0,0	0.9277	47487	0.3142	59249	
0,0	0.9168	48124	0.3463	59249	
0,0	0.8965	50300	0.4024	59249	
0,0	0.8841	52904	0.4309	59249	
1,1	0.9662	40631	0.2352	59249	
1,1	0.9599	43476	0.2763	59249	
1,1	0.9556	44064	0.2973	59249	
1,1	0.9486	44126	0.3258	59249	
1,1	0.9371	45102	0.3748	59249	
1,1	0.9278	46786	0.3977	59249	
1,2	0.9703	39262	0.2261	59249	
1,2	0.9647	41736	0.2644	59249	
1,2	0.9607	42116	0.2838	59249	
1,2	0.9544	41871	0.3092	59249	
1,2	0.9436	42277	0.3518	59249	
1,2	0.9358	43423	0.3701	59249	
1,3	0.9738	38100	0.2187	59249	
1,3	0.9685	40269	0.2546	59249	
1,3	0.9646	40475	0.2724	59249	

1,3	0.9589	39976	0.2952	59249
1,3	0.9485	39907	0.3324	59249
1,3	0.9414	40655	0.3468	59249
1,3	0.9599	42001	0.2436	59249
1,3	0.9520	45218	0.2870	59249
1,3	0.9472	46016	0.3094	59249
1,3	0.9392	46387	0.3407	59249
1,3	0.9263	47934	0.3949	59249
1,3	0.9152	50157	0.4215	59249
2,2	0.9634	41534	0.2405	59249
2,2	0.9571	44556	0.2830	59249
2,2	0.9527	45297	0.3052	59249
2,2	0.9448	45564	0.3355	59249
2,2	0.9322	46940	0.3881	59249
2,2	0.9225	48861	0.4125	59249
2,3	0.9671	41031	0.2375	59249
2,3	0.9604	43986	0.2793	59249
2,3	0.9563	44653	0.3011	59249
2,3	0.9487	44804	0.3302	59249
2,3	0.9359	46030	0.3811	59249
2,3	0.9272	47755	0.4039	59249

Table 16: Configuration and Result of AR1 Model with Different React Speed of Adjustment Policy of Fixed Point Training

react speed	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
0,0	0.001	0.9698	42245	0.2849	59249
0,0	0.003	0.9627	45239	0.3297	59249
0,0	0.005	0.9583	46435	0.3530	59249
0,0	0.010	0.9504	47518	0.3873	59249
0,0	0.030	0.9319	50406	0.4498	59249
0,0	0.050	0.9181	52841	0.4848	59249
1,1	0.001	0.9734	40973	0.2773	59249
1,1	0.003	0.9674	43557	0.3193	59249

1,1	0.005	0.9637	44503	0.3407	59249
1,1	0.010	0.9568	45166	0.3714	59249
1,1	0.030	0.9414	46979	0.4255	59249
1,1	0.050	0.9289	48523	0.4536	59249
1,2	0.001	0.9737	39888	0.2704	59249
1,2	0.003	0.9677	42141	0.3095	59249
1,2	0.005	0.9644	42896	0.3291	59249
1,2	0.010	0.9578	43223	0.3564	59249
1,2	0.030	0.9424	44234	0.4020	59249
1,2	0.050	0.9303	45085	0.4235	59249
1,3	0.001	0.9741	38844	0.2640	59249
1,3	0.003	0.9684	40789	0.3003	59249
1,3	0.005	0.9650	41381	0.3182	59249
1,3	0.010	0.9586	41407	0.3422	59249
1,3	0.030	0.9435	41700	0.3800	59249
1,3	0.050	0.9319	41960	0.3958	59249
2,1	0.001	0.9710	42061	0.2841	59249
2,1	0.003	0.9644	44974	0.3284	59249
2,1	0.005	0.9601	46115	0.3515	59249
2,1	0.010	0.9526	47112	0.3851	59249
2,1	0.030	0.9350	49732	0.4458	59249
2,1	0.050	0.9222	51969	0.4798	59249
2,2	0.001	0.9711	41919	0.2833	59249
2,2	0.003	0.9644	44780	0.3272	59249
2,2	0.005	0.9603	45864	0.3498	59249
2,2	0.010	0.9528	46782	0.3827	59249
2,2	0.030	0.9354	49175	0.4413	59249
2,2	0.050	0.9226	51282	0.4740	59249
2,3	0.001	0.9711	41775	0.2825	59249
2,3	0.003	0.9644	44577	0.3259	59249
2,3	0.005	0.9604	45609	0.3481	59249
2,3	0.010	0.9530	46446	0.3803	59249
2,3	0.030	0.9355	48610	0.4366	59249
2,3	0.050	0.9228	50560	0.4678	59249

Figure 4: ECDF of Pearson Correlation between Maxes and (Past) Maxes

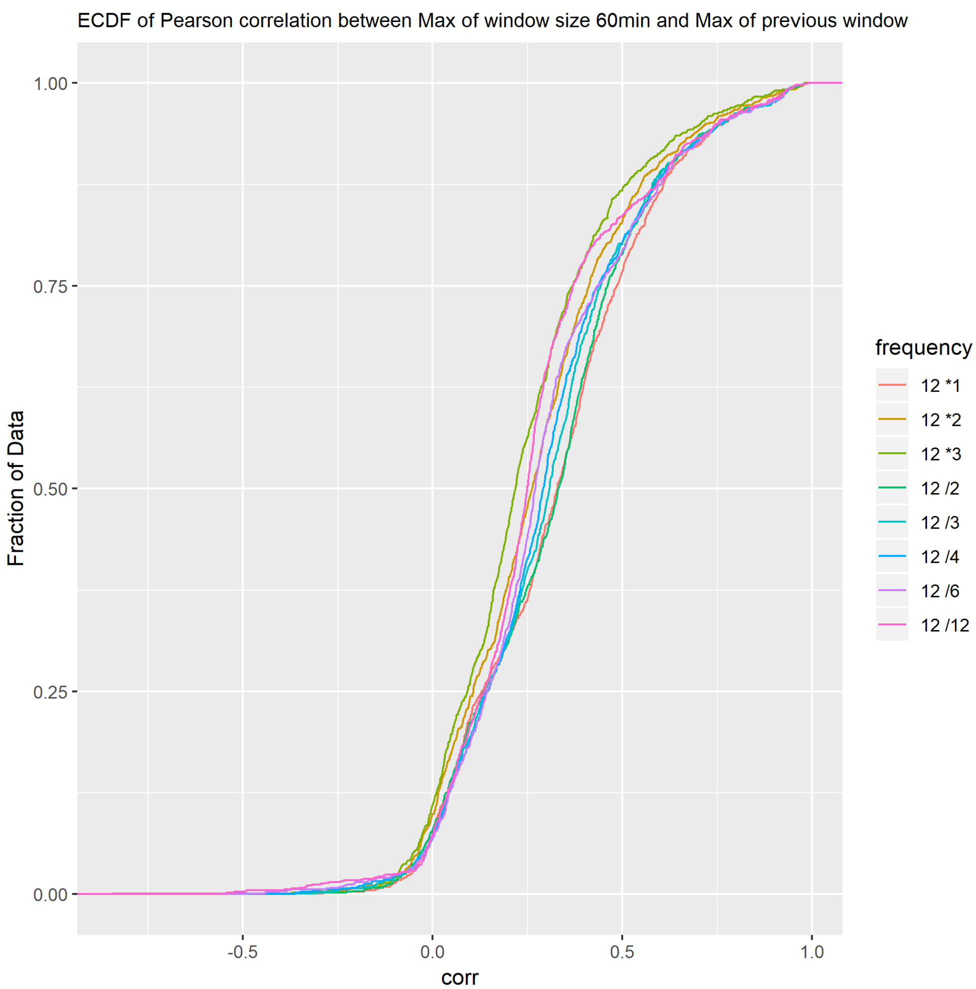


Figure 5: ECDF of Pearson Correlation between Maxes and (Past) Avgs

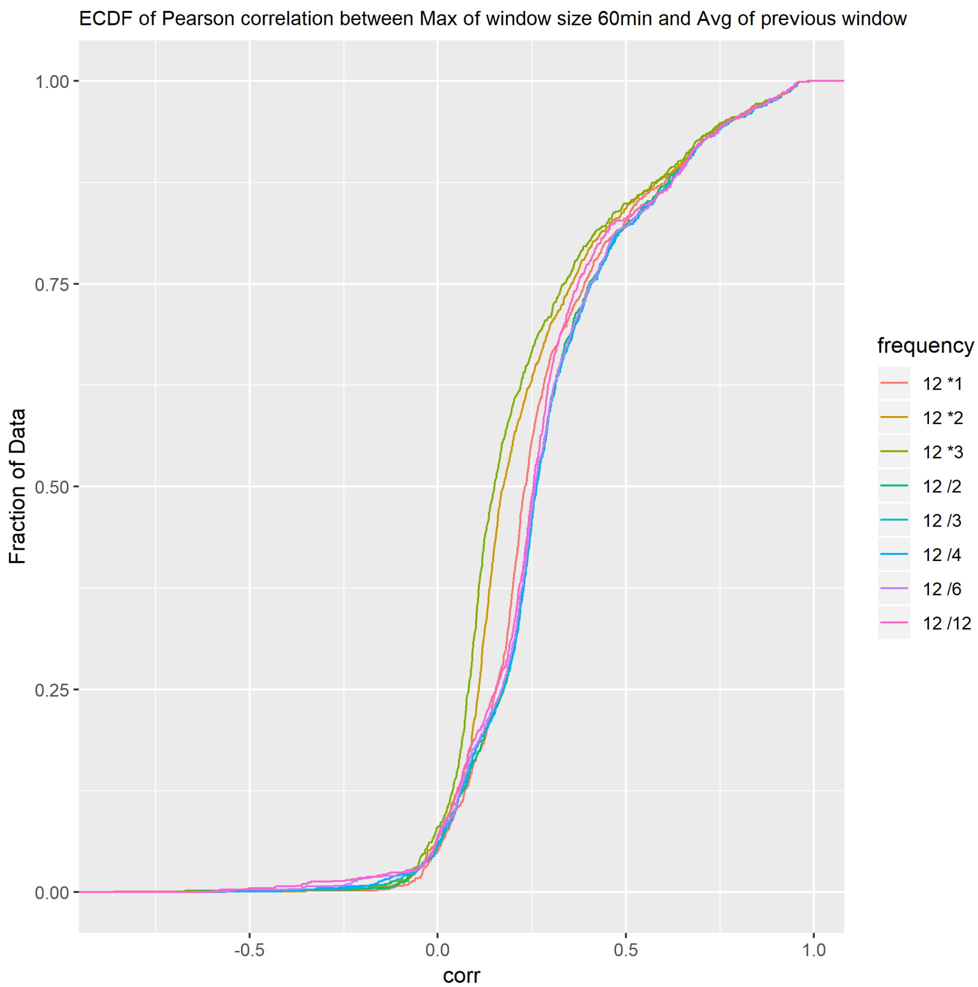


Figure 6: ECDF of Pearson Correlation between Maxes and (Past) Average of Maxes

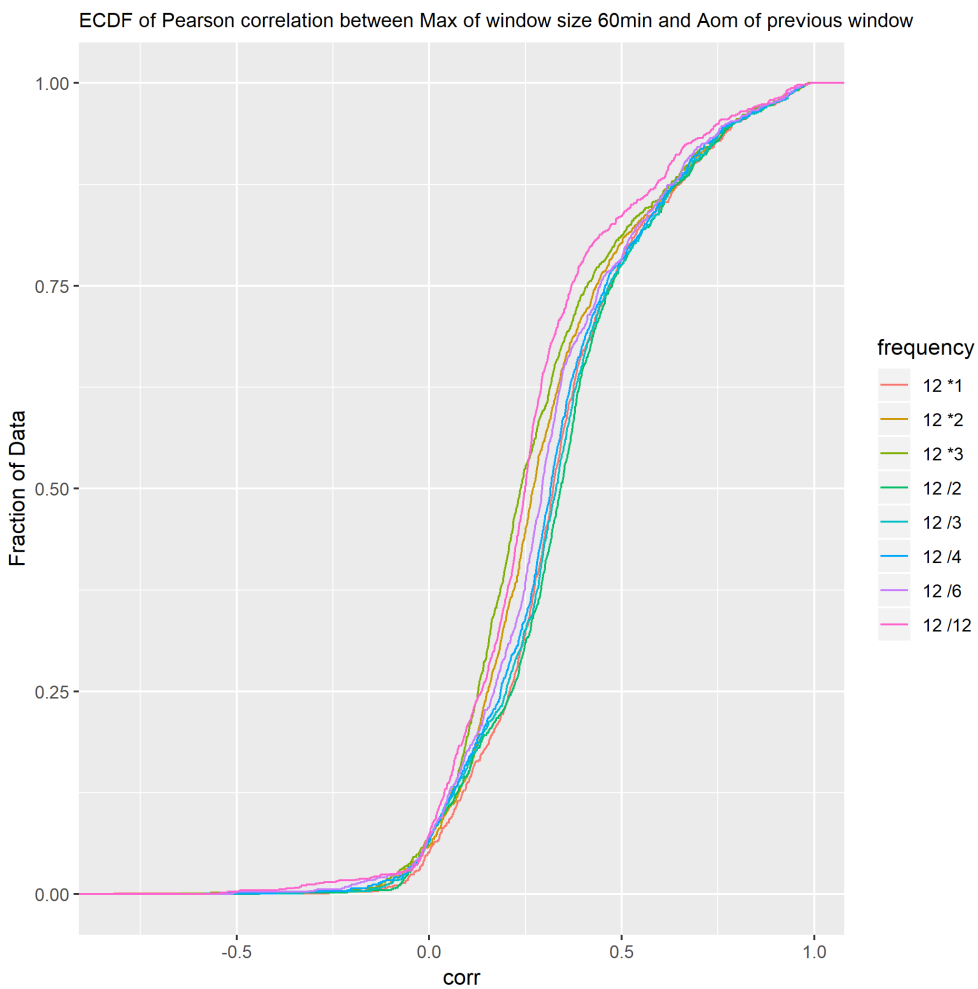


Figure 7: Trade-off Curve for AR1 Model With Offline Training

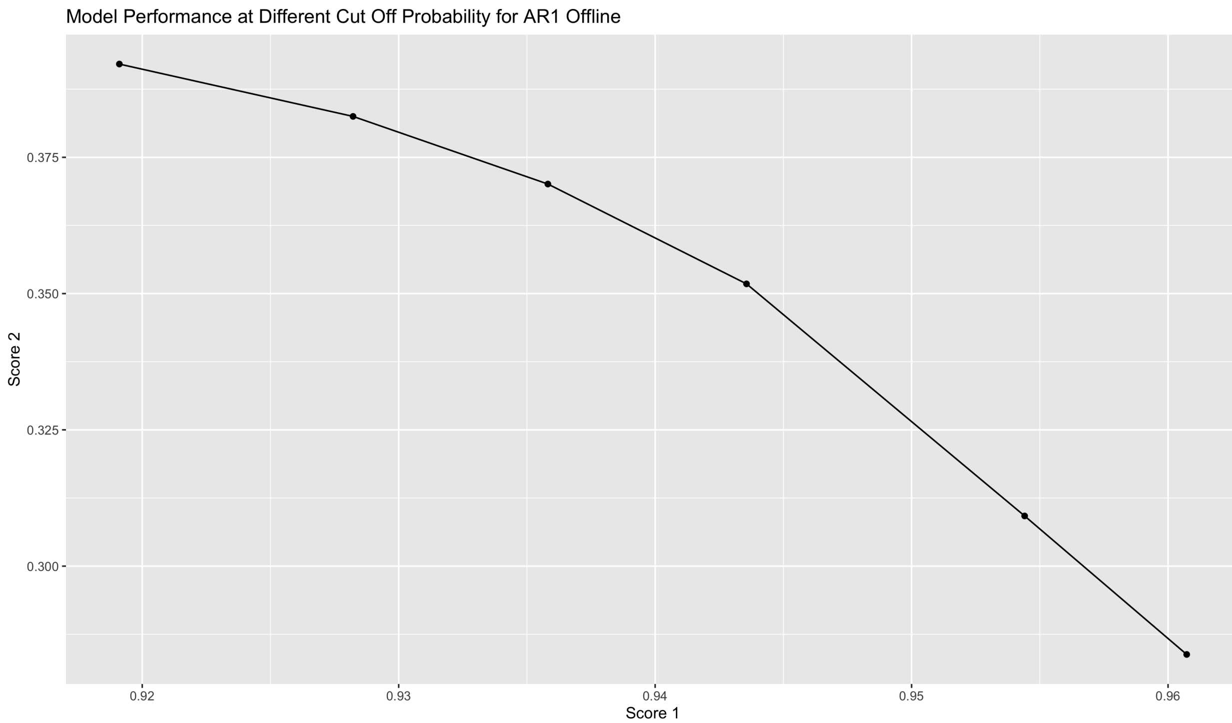


Figure 8: Trade-off Curves for ARIMA-like Models with Offline Training

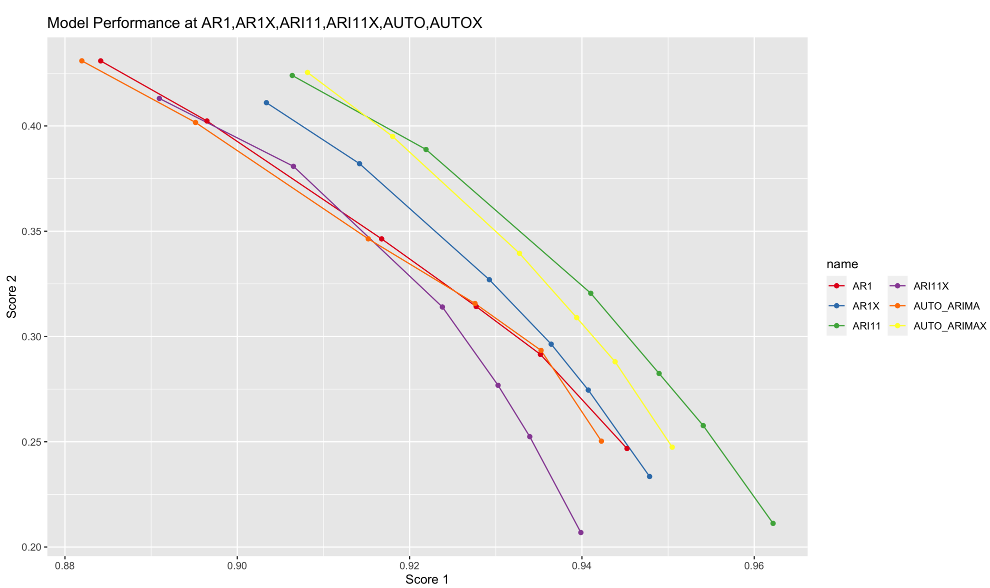


Figure 9: Trade-off Curves for AR1 with Different Residual Distribution Estimation

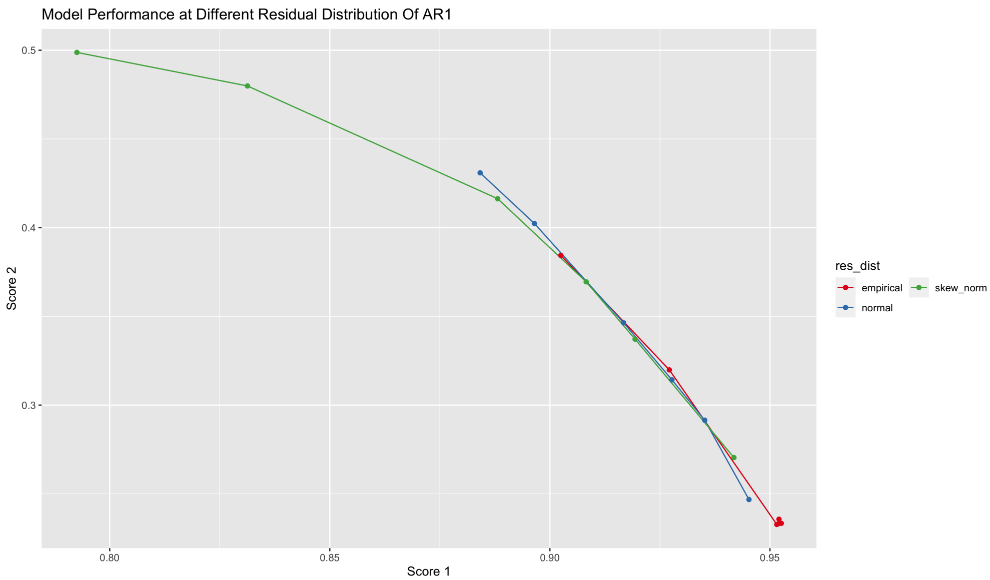


Figure 10: Trade-off Curves for AR1 with Different Outlier Detection Strategies

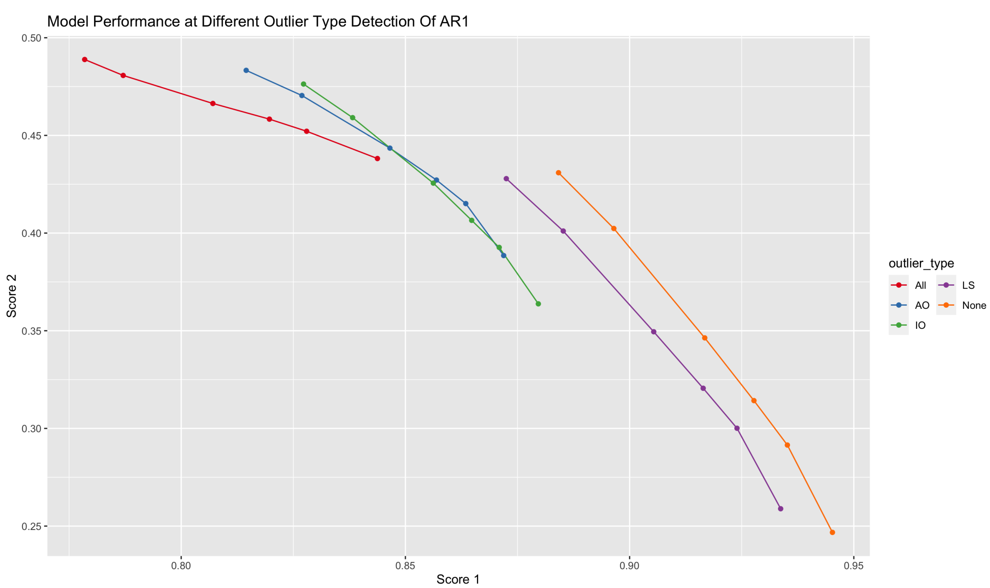


Table 3: Configuration and Result of AR1 Model with Different Residual Distribution Estimation

residual distribution	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
Normal	0.001	0.9452	42981	0.2468	59249
Normal	0.003	0.9352	46484	0.2915	59249
Normal	0.005	0.9277	47487	0.3142	59249
Normal	0.010	0.9168	48124	0.3463	59249
Normal	0.030	0.8965	50300	0.4024	59249
Normal	0.050	0.8841	52904	0.4309	59249
Skew-normal	0.001	0.9418	46008	0.2704	59249
Skew-normal	0.003	0.9194	47926	0.3371	59249
Skew-normal	0.005	0.9082	49578	0.3695	59249
Skew-normal	0.010	0.8881	51728	0.4163	59249
Skew-normal	0.030	0.8313	53154	0.4799	59249
Skew-normal	0.050	0.7925	54091	0.4988	59249
Empirical	0.001	0.9516	50437	0.2327	59249
Empirical	0.003	0.9526	50398	0.2334	59249
Empirical	0.005	0.9521	50434	0.2333	59249
Empirical	0.010	0.9520	50717	0.2357	59249
Empirical	0.030	0.9271	53533	0.3199	59249
Empirical	0.050	0.9025	55347	0.3844	59249

Figure 11: Trade-off Curves for Different Markov Models

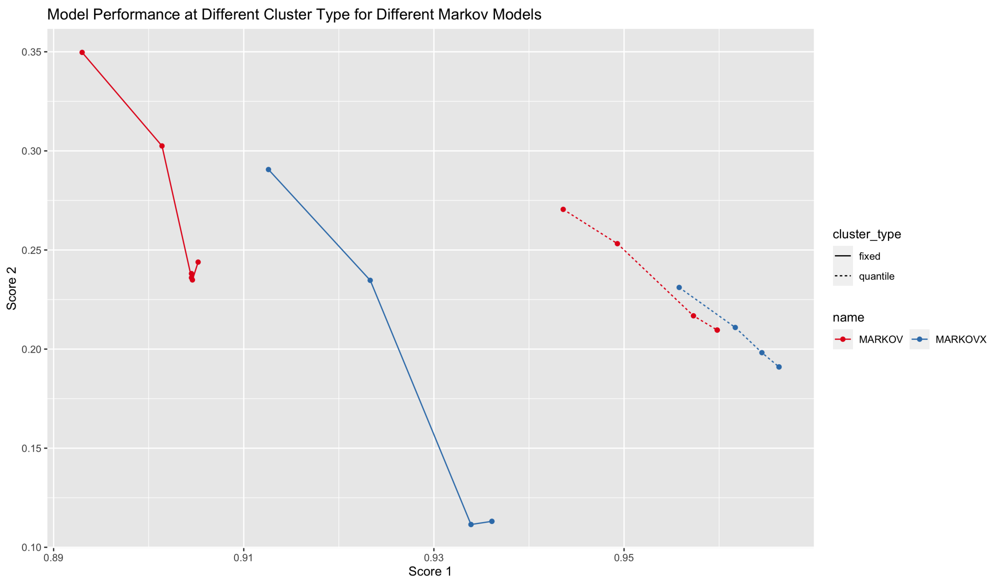


Figure 12: Trade-off Curves for Different States for Markov Models

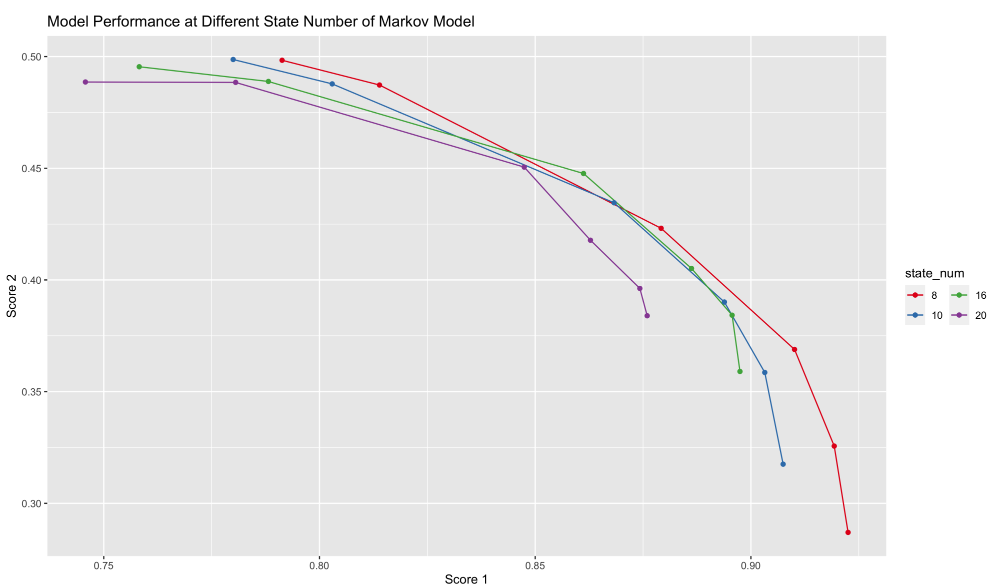


Figure 13: Trade-off Curves for Different Multivariate Models

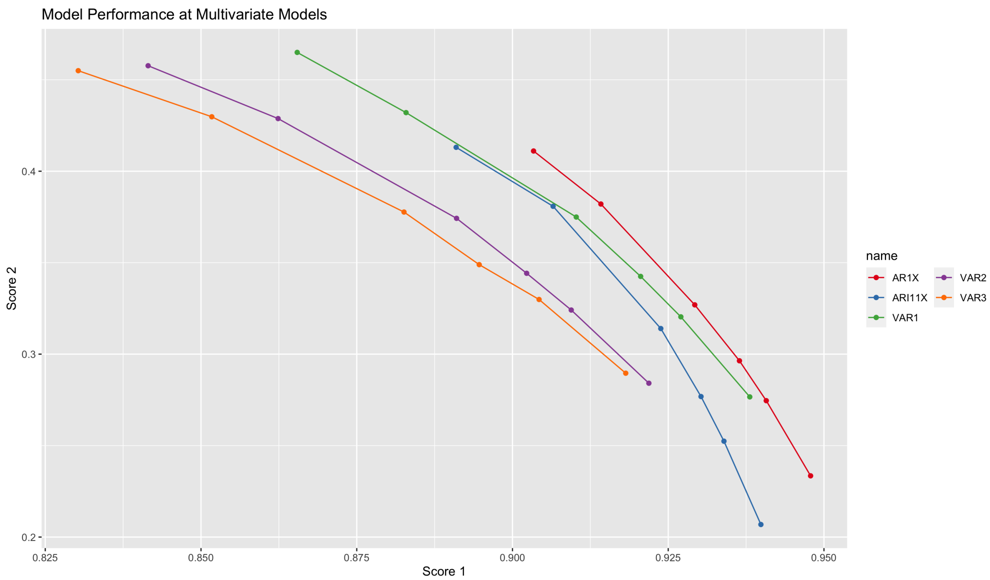


Figure 14: Trade-off Curves for Different Neural Network Models

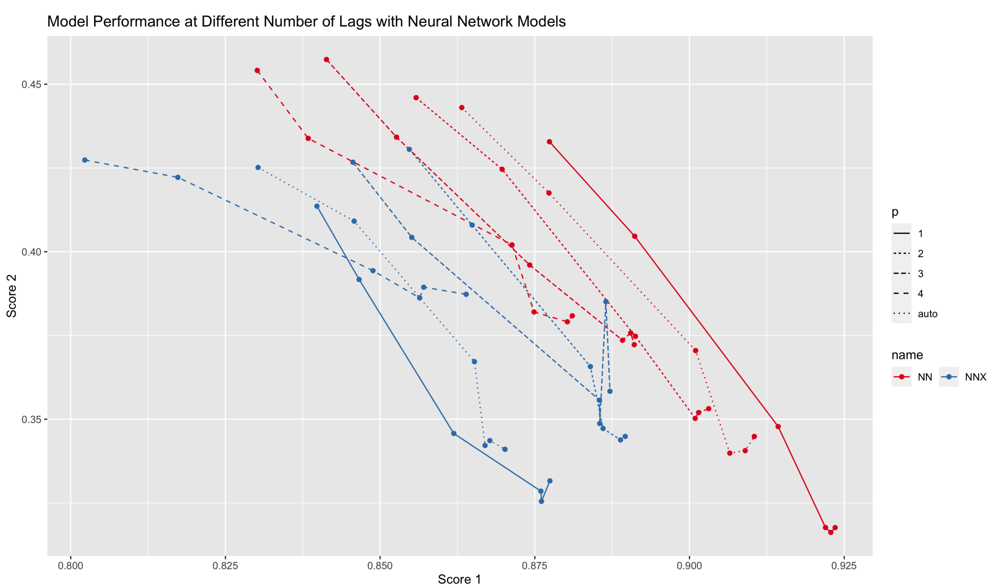


Table 7: Configuration and Result of AR1 Model with Different Outlier Detections

model name	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
AR1X	0.001	0.9479	41306	0.2335	59249
AR1X	0.003	0.9407	43602	0.2746	59249
AR1X	0.005	0.9364	44299	0.2964	59249
AR1X	0.010	0.9293	45094	0.3270	59249
AR1X	0.030	0.9142	47570	0.3821	59249
AR1X	0.050	0.9034	50374	0.4111	59249
ARI11X	0.001	0.9399	38470	0.2069	59249
ARI11X	0.003	0.9339	42132	0.2524	59249
ARI11X	0.005	0.9303	43794	0.2768	59249
ARI11X	0.010	0.9238	45778	0.3140	59249
ARI11X	0.030	0.9065	48501	0.3808	59249
ARI11X	0.050	0.8910	49852	0.4131	59249
VAR1	0.001	0.9381	44419	0.2766	59249
VAR1	0.003	0.9270	46114	0.3204	59249
VAR1	0.005	0.9206	46533	0.3425	59249
VAR1	0.010	0.9102	47837	0.3750	59249
VAR1	0.030	0.8829	51860	0.4320	59249
VAR1	0.050	0.8654	53962	0.4650	59249
VAR2	0.001	0.9219	44733	0.2841	59249
VAR2	0.003	0.9094	46551	0.3241	59249
VAR2	0.005	0.9023	47142	0.3442	59249
VAR2	0.010	0.8910	48459	0.3742	59249
VAR2	0.030	0.8624	52483	0.4288	59249
VAR2	0.050	0.8415	54281	0.4577	59249
VAR3	0.001	0.9181	44719	0.2895	59249
VAR3	0.003	0.9043	46798	0.3299	59249
VAR3	0.005	0.8947	47713	0.3489	59249
VAR3	0.010	0.8826	48861	0.3777	59249
VAR3	0.030	0.8517	52978	0.4298	59249
VAR3	0.050	0.8303	53852	0.4550	59249

Figure 15: Trade-off Curves for Different Extrapolation Steps

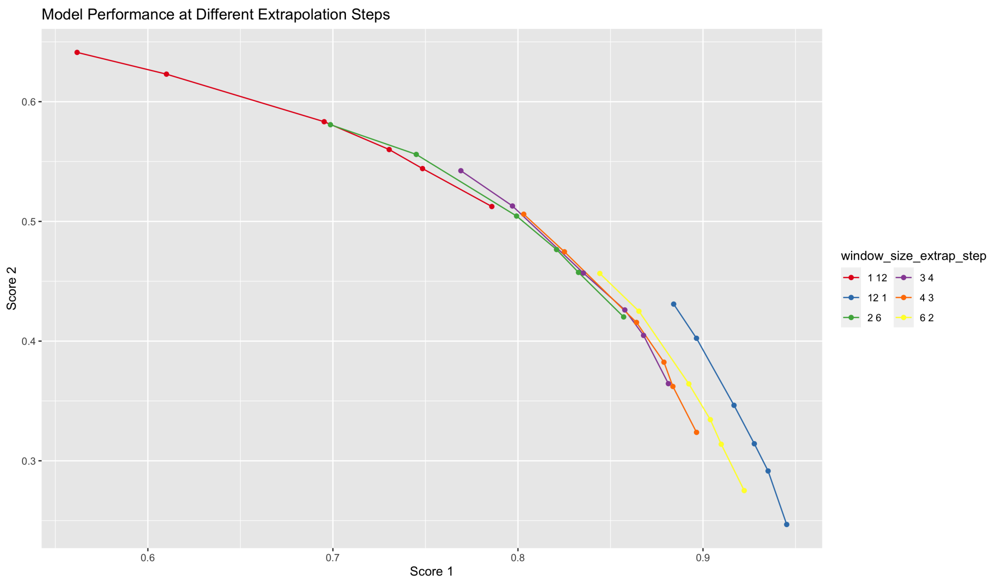


Figure 16: Trade-off Curves for Different Training Policies

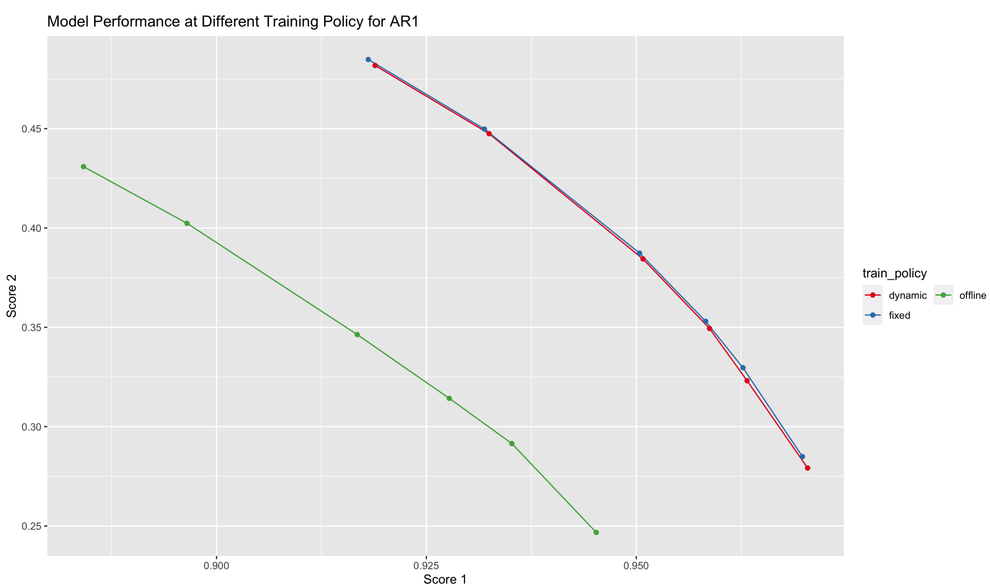


Table 10: Configuration and Result of AR1 Model with Different Training Policies

training policy	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
offline	0.001	0.9452	42981	0.2468	59249
offline	0.003	0.9352	46484	0.2915	59249
offline	0.005	0.9277	47487	0.3142	59249
offline	0.010	0.9168	48124	0.3463	59249
offline	0.030	0.8965	50300	0.4024	59249
offline	0.050	0.8841	52904	0.4309	59249
fixed	0.001	0.9698	42245	0.2849	59249
fixed	0.003	0.9627	45239	0.3297	59249
fixed	0.005	0.9583	46435	0.3530	59249
fixed	0.010	0.9504	47518	0.3873	59249
fixed	0.030	0.9319	50406	0.4498	59249
fixed	0.050	0.9181	52841	0.4848	59249
dynamic	0.001	0.9704	42449	0.2792	59249
dynamic	0.003	0.9632	45137	0.3231	59249
dynamic	0.005	0.9587	46419	0.3494	59249
dynamic	0.010	0.9508	47422	0.3844	59249
dynamic	0.030	0.9325	50363	0.4475	59249
dynamic	0.050	0.9189	52866	0.4818	59249

Figure 17: Model Performance for Different Batch Size of Dynamic Training

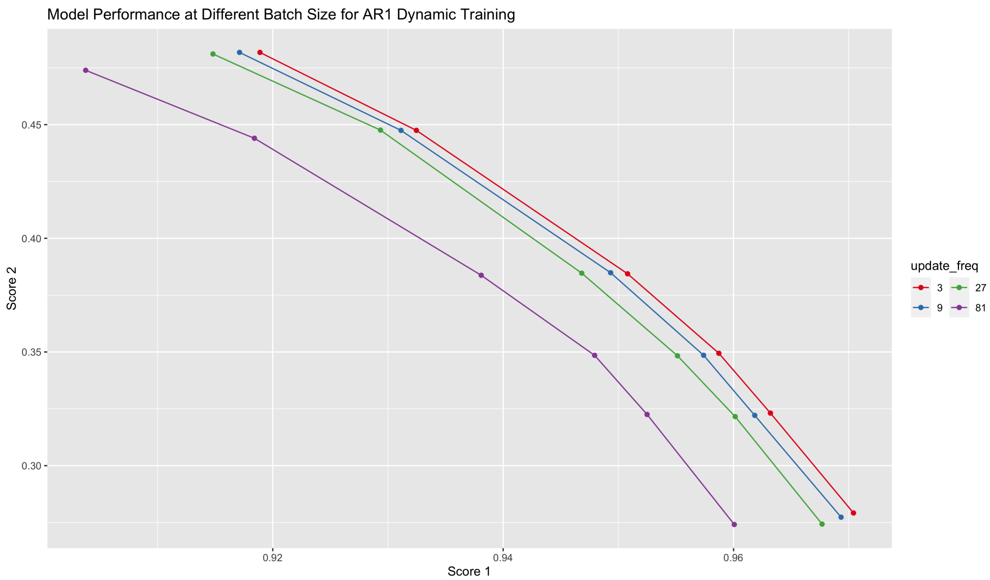


Figure 18: Different Models with Different Training Policy

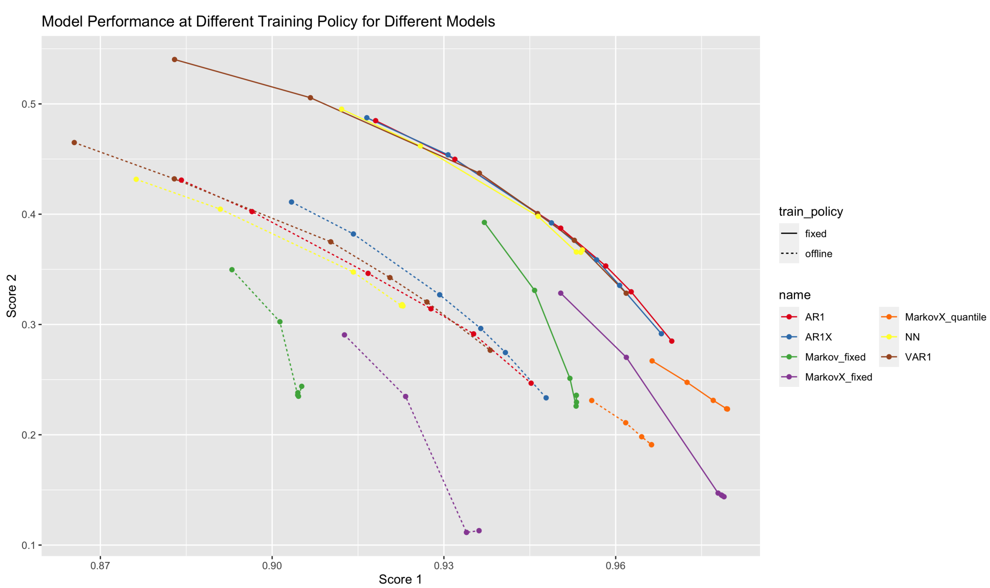


Table 11: Configuration and Result of AR1 Model with Different Batch Size of Dynamic Training

batch size	cut off prob	score 1	score 1 weight	score 2	score 2 weight
3	0.001	0.9704	42449	0.2792	59249
3	0.003	0.9632	45137	0.3231	59249
3	0.005	0.9587	46419	0.3494	59249
3	0.010	0.9508	47422	0.3844	59249
3	0.030	0.9325	50363	0.4475	59249
3	0.050	0.9189	52866	0.4818	59249
9	0.001	0.9693	42427	0.2773	59249
9	0.003	0.9618	45075	0.3221	59249
9	0.005	0.9574	46478	0.3486	59249
9	0.010	0.9493	47562	0.3849	59249
9	0.030	0.9311	50533	0.4475	59249
9	0.050	0.9171	52966	0.4818	59249
27	0.001	0.9677	42087	0.2743	59249
27	0.003	0.9601	45188	0.3216	59249
27	0.005	0.9551	46716	0.3483	59249
27	0.010	0.9468	47764	0.3847	59249
27	0.030	0.9294	50702	0.4476	59249
27	0.050	0.9148	53010	0.4811	5924
81	0.001	0.9601	40290	0.2741	56561
81	0.003	0.9525	43764	0.3225	56561
81	0.005	0.9479	45007	0.3485	56561
81	0.010	0.9381	45813	0.3837	56561
81	0.030	0.9184	48803	0.4440	56561
81	0.050	0.9038	50572	0.4739	56561

Table 14: Configuration and Result of AR1 Model with Different Granularity and Offline Training

granularity	score 1	score 1 weight	score 2	score 2 weight
0.00000	0.9168	48124	0.3463	59249
0.78125	0.9183	48070	0.3577	57170
1.56250	0.9197	47992	0.3638	56005
3.12500	0.9224	47725	0.3667	55163

Figure 19: Trade-off Curves for Different Model Number with Dynamic Training

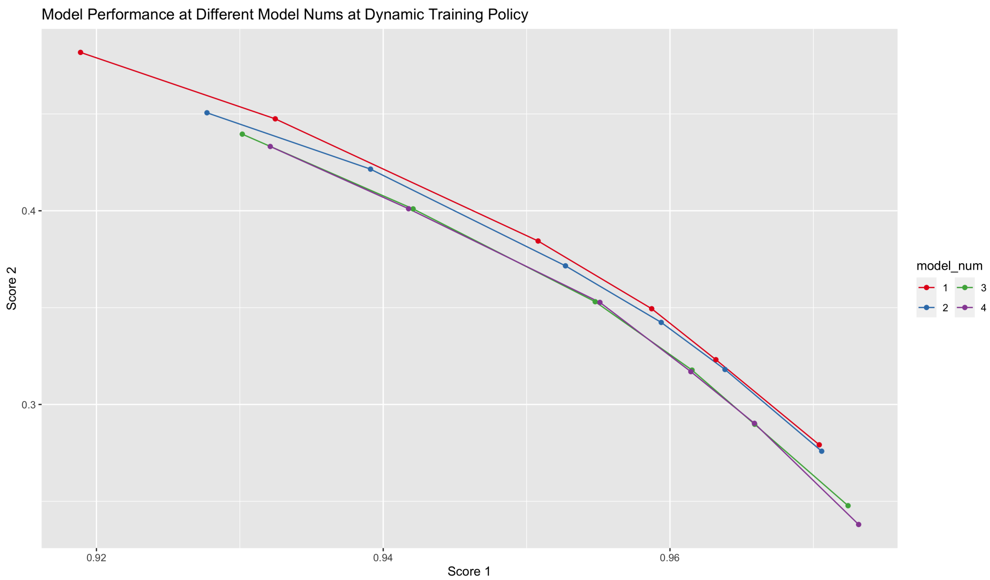


Figure 20: An Example Trace with Three Parallel AR1 Models and Offline Training

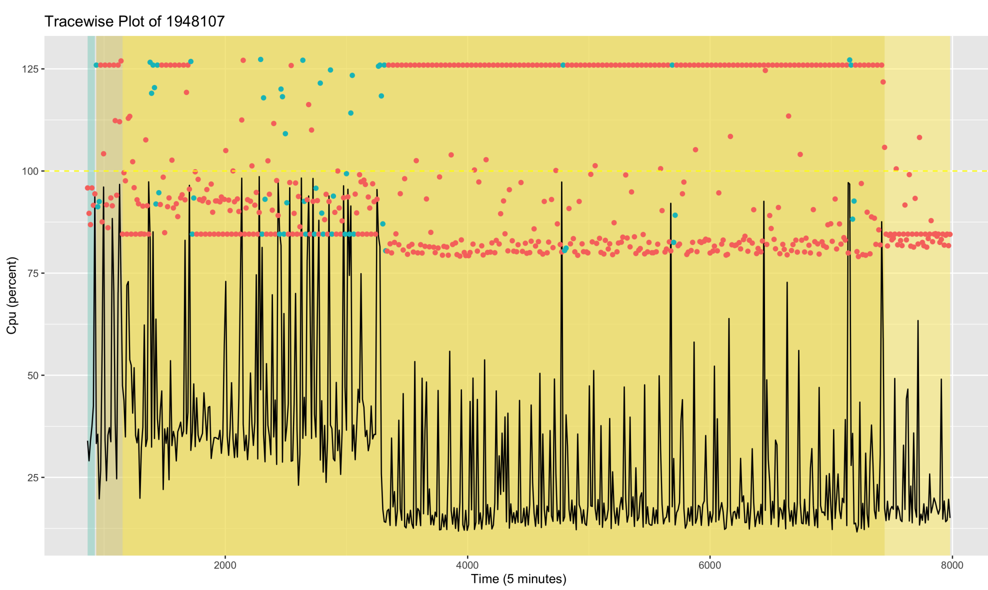


Figure 21: An Example Trace with Three Parallel AR1 Models and Dynamic Training

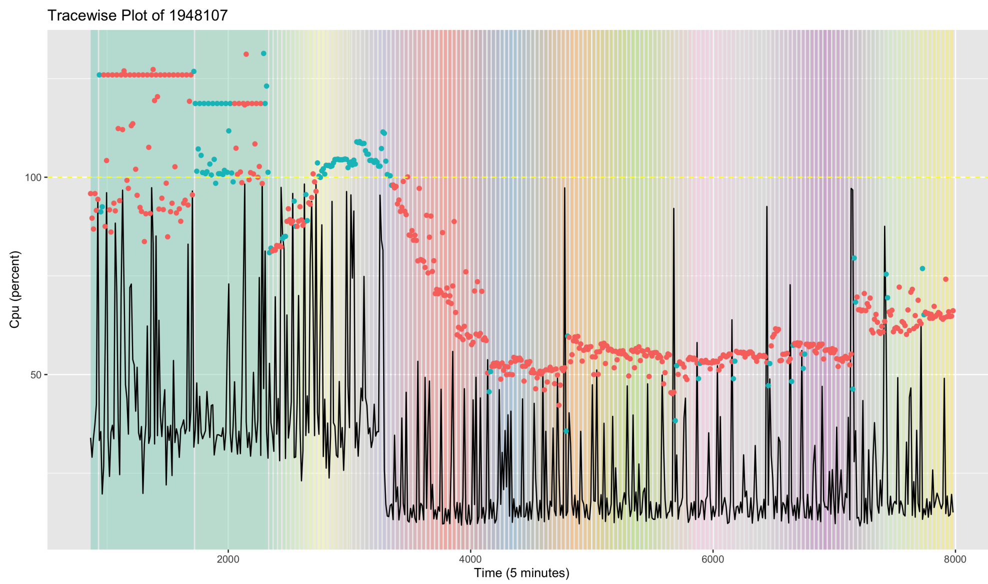


Figure 22: ECDF of Scores for Different Granularity of Offline Training

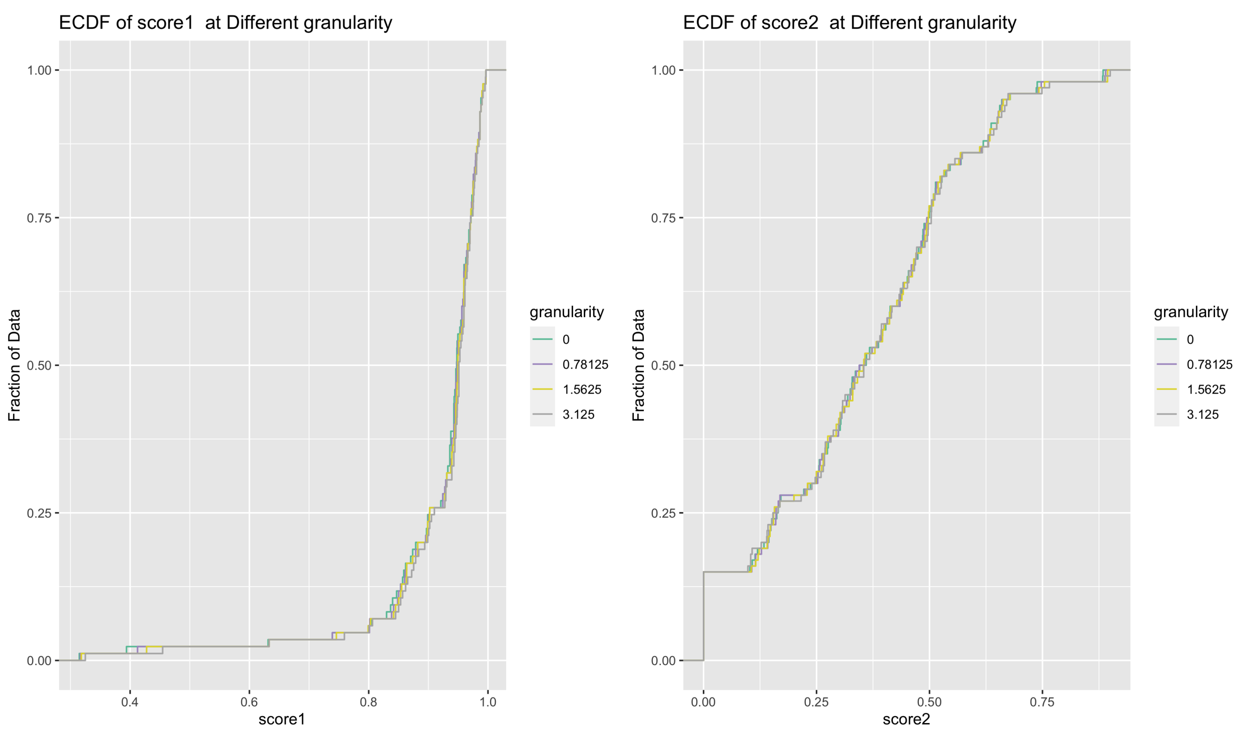
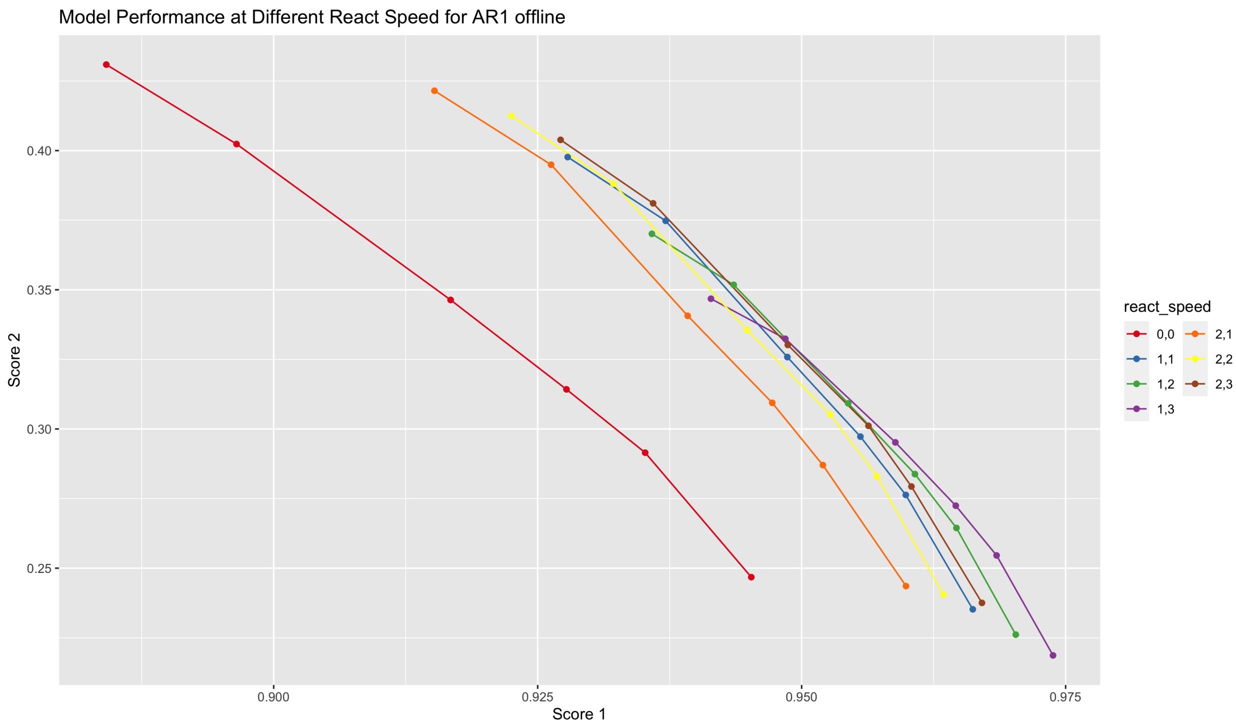


Figure 23: Trade-off Curves for Different React Speed of Adjustment Policy of Offline Training



1.13.9 Predicting Average of Next Window

Ref to Section 1.10, include Fig 25, Table 25.

Table 17: Configuration and Result of Different Models Predicting Averages of Next Window with Offline Training

model	training policy	cut off prob	score 1	score 1 weight	score 2	score 2 weight
AR1	fixed	0.0002	0.9828	59305	0.9117	59400
AR1	fixed	0.0004	0.9808	59320	0.9133	59400
AR1	fixed	0.0006	0.9798	59327	0.9146	59400
AR1	fixed	0.0008	0.9789	59332	0.9153	59400
AR1	fixed	0.0010	0.9782	59342	0.9158	59400
AR1	fixed	0.0030	0.9735	59366	0.9179	59400
AR1	fixed	0.0050	0.9702	59385	0.9179	59400
AR1	fixed	0.0100	0.9646	59398	0.9173	59400
AR1	fixed	0.0200	0.9560	59399	0.9139	59400
AR1	fixed	0.0300	0.9490	59399	0.9104	59400

AR1	fixed	0.0400	0.9433	59399	0.9071	59400
AR1	fixed	0.0500	0.9378	59399	0.9036	59400
AR1	offline	0.0002	0.8537	59399	0.8006	59400
AR1	offline	0.0004	0.8491	59399	0.7984	59400
AR1	offline	0.0006	0.8461	59399	0.7969	59400
AR1	offline	0.0008	0.8440	59399	0.7959	59400
AR1	offline	0.0010	0.8419	59399	0.7947	59400
AR1	offline	0.0030	0.8314	59399	0.7887	59400
AR1	offline	0.0050	0.8251	59399	0.7845	59400
AR1	offline	0.0100	0.8141	59399	0.7768	59400
AR1	offline	0.0200	0.7996	59399	0.7657	59400
AR1	offline	0.0300	0.7900	59399	0.7583	59400
AR1	offline	0.0400	0.7815	59399	0.7513	59400
AR1	offline	0.0500	0.7750	59399	0.7461	59400
ARI11	fixed	0.0002	0.9875	59188	0.9088	59400
ARI11	fixed	0.0004	0.9857	59236	0.9110	59400
ARI11	fixed	0.0006	0.9847	59255	0.9124	59400
ARI11	fixed	0.0008	0.9837	59264	0.9132	59400
ARI11	fixed	0.0010	0.9831	59271	0.9139	59400
ARI11	fixed	0.0030	0.9792	59332	0.9174	59400
ARI11	fixed	0.0050	0.9766	59360	0.9186	59400
ARI11	fixed	0.0100	0.9718	59386	0.9192	59400
ARI11	fixed	0.0200	0.9646	59395	0.9179	59400
ARI11	fixed	0.0300	0.9586	59395	0.9157	59400
ARI11	fixed	0.0400	0.9530	59395	0.9129	59400
ARI11	fixed	0.0500	0.9481	59396	0.9103	59400
ARI11	offline	0.0002	0.9052	59388	0.8464	59400
ARI11	offline	0.0004	0.9020	59391	0.8460	59400
ARI11	offline	0.0006	0.8995	59394	0.8453	59400
ARI11	offline	0.0008	0.8977	59394	0.8447	59400
ARI11	offline	0.0010	0.8964	59395	0.8443	59400
ARI11	offline	0.0030	0.8854	59395	0.8385	59400
ARI11	offline	0.0050	0.8806	59398	0.8361	59400
ARI11	offline	0.0100	0.8716	59399	0.8308	59400

ARI11	offline	0.0200	0.8614	59399	0.8246	59400
ARI11	offline	0.0300	0.8533	59399	0.8189	59400
ARI11	offline	0.0400	0.8462	59399	0.8137	59400
ARI11	offline	0.0500	0.8394	59399	0.8084	59400

1.13.10 Comparison Against Autopilot

Table 18: Configuration and Result of Different j-quantile for Predicting Maximum of Next 5min Window Using Autopilot

reconstruct method	**j**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
Version 1	0.001	0.9969	414249	0.2298	716235
Version 1	0.010	0.9834	614705	0.5077	716235
Version 1	0.050	0.9385	678510	0.7093	716235
Version 1	0.070	0.9185	682062	0.7367	716235
Version 1	0.080	0.9105	683368	0.7412	716235
Version 1	0.100	0.8948	686723	0.7450	716235
Version 2	0.001	0.9949	519172	0.3020	714840
Version 2	0.010	0.9786	642181	0.5350	714840
Version 2	0.050	0.9405	680123	0.7025	714840
Version 2	0.070	0.9283	682283	0.7243	714840
Version 2	0.080	0.9215	684080	0.7316	714840
Version 2	0.100	0.9092	688740	0.7411	714840
Version 3	0.0001	0.9896	556478	0.4110	716235
Version 3	0.0003	0.9793	611494	0.5097	716235
Version 3	0.0005	0.9724	623478	0.5465	716235
Version 3	0.0010	0.9597	637230	0.5888	716235
Version 3	0.0100	0.8603	694175	0.6774	716235
Version 3	0.0500	0.7884	709186	0.6857	716235
Version 3	0.0700	0.7519	711860	0.6691	716235
Version 3	0.0800	0.7366	713910	0.6600	716235
Version 3	0.1000	0.7104	716169	0.6429	716235

Figure 24: Trade-off Curves for Different React Speed of Adjustment Policy of Offline Training

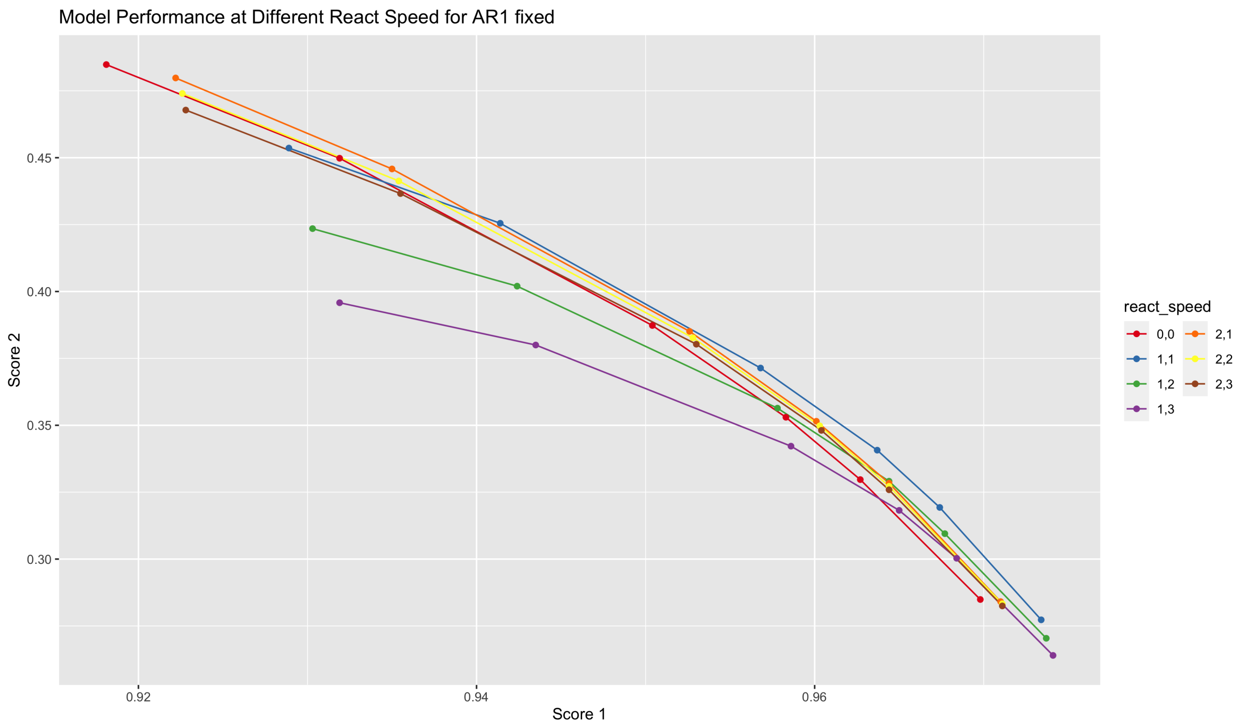


Table 19: Configuration and Result of Different half-life for Predicting Maximum of Next 5min Window Using Autopilot

half-life	cut off prob	score 1	score 1 weight	score 2	score2 weight
1	0.001	0.9297	665495	0.6843	716235
1	0.003	0.9292	666128	0.6850	716235
1	0.005	0.9284	666998	0.6858	716235
1	0.007	0.9275	667945	0.6871	716235
1	0.008	0.9224	670520	0.6968	716235
1	0.009	0.9222	670818	0.6971	716235
1	0.010	0.9219	671071	0.6973	716235
1	0.020	0.9142	675662	0.7082	716235
1	0.030	0.9107	677272	0.7105	716235
1	0.050	0.8960	682473	0.7181	716235
3	0.001	0.9724	631807	0.5967	716235
3	0.003	0.9691	639489	0.6128	716235

3	0.005	0.9661	644261	0.6236	716235
3	0.007	0.9622	648268	0.6327	716235
3	0.008	0.9611	649218	0.6340	716235
3	0.009	0.9541	650933	0.6342	716235
3	0.01	0.9532	651747	0.6357	716235
3	0.02	0.9424	660513	0.6612	716235
3	0.03	0.9351	666856	0.6814	716235
3	0.05	0.9239	674907	0.7062	716235
6	0.001	0.9822	604936	0.534	716235
6	0.003	0.9791	619677	0.5633	716235
6	0.005	0.9753	628036	0.5807	716235
6	0.007	0.9733	633727	0.5939	716235
6	0.008	0.9722	635977	0.5989	716235
6	0.009	0.9713	638131	0.6040	716235
6	0.010	0.9704	640185	0.6089	716235
6	0.020	0.9593	654099	0.6406	716235
6	0.030	0.9479	663104	0.6651	716235
6	0.050	0.9323	676348	0.7099	716235
12	0.001	0.9873	570639	0.4632	716235
12	0.003	0.9846	594920	0.5071	716235
12	0.005	0.9825	608231	0.5322	716235
12	0.007	0.9806	618105	0.5518	716235
12	0.008	0.9797	622374	0.5608	716235
12	0.009	0.9787	625993	0.5684	716235
12	0.01	0.9778	629084	0.5746	716235
12	0.02	0.9677	652495	0.6288	716235
12	0.03	0.9554	666568	0.6674	716235
12	0.05	0.9354	677635	0.7148	716235
36	0.001	0.9934	499442	0.344	716235
36	0.003	0.9903	548546	0.4111	716235
36	0.005	0.988	578375	0.4582	716235
36	0.007	0.9858	600206	0.4965	716235
36	0.008	0.9847	608979	0.5136	716235
36	0.009	0.9834	617026	0.5299	716235

36	0.010	0.9821	623970	0.5447	716235
36	0.020	0.9712	656286	0.6253	716235
36	0.030	0.9603	668408	0.6644	716235
36	0.050	0.9379	677320	0.7147	716235
72	0.001	0.9956	453026	0.2787	716235
72	0.003	0.9929	524223	0.364	716235
72	0.005	0.9898	569209	0.4281	716235
72	0.007	0.9869	594523	0.4744	716235
72	0.008	0.9856	604047	0.4938	716235
72	0.009	0.9843	612429	0.5113	716235
72	0.010	0.983	619511	0.5267	716235
72	0.020	0.9716	656032	0.6184	716235
72	0.030	0.9611	668939	0.6612	716235
72	0.050	0.9384	677264	0.7117	716235
144	0.001	0.9969	414249	0.2298	716235
144	0.010	0.9834	614705	0.5077	716235
144	0.050	0.9385	678510	0.7093	716235

Table 20: Configuration and Result of Different Number of Breaks for Predicting Maximum of Next 5min Window Using Autopilot

number of breaks	**cut off prob**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**

Table 21: Configuration and Result of Different Window Sizes for Predicting Maximum of Next Window Using Autopilot

window size	**half life**	**cut off prob**	**score 1**	**score1 weight**	**score 2**	**score 2 weight**

Table 22: Configurations and Results of Different models for Predicting Maximum of Next Windows

model	**cut off prob**	**window**	**score 1**	**score 1 weight**	**score 2**	**score 2 weight**
AR1 online(fixed)	0.01	10m	0.9643	334769	0.5805	357435
AR1 online(fixed)	0.01	15m	0.9609	217810	0.5295	238038
AR1 online(fixed)	0.01	30m	0.9542	101981	0.4499	118942
AR1 online(fixed)	0.01	01h	0.9504	047518	0.3873	059249

AR1 online(fixed)	0.01	03h	0.9287	012797	0.2439	019661
AR1 online(fixed)	0.03	10m	0.9467	339404	0.6279	357435
AR1 online(fixed)	0.03	15m	0.9463	223975	0.5867	238038
AR1 online(fixed)	0.03	30m	0.9372	108094	0.5123	118942
AR1 online(fixed)	0.03	01h	0.9319	050406	0.4498	059249
AR1 online(fixed)	0.03	03h	0.9101	014947	0.3134	019661
Autopilot	0.01	10m	0.9555	309666	0.5366	357899
Autopilot	0.01	15m	0.9433	205504	0.5280	238514
Autopilot	0.01	30m	0.9081	111959	0.5142	119133
Autopilot	0.01	01h	0.8492	051011	0.5014	059443
Autopilot	0.01	03h	0.6809	017156	0.4245	019662
Autopilot	0.03	10m	0.9031	334336	0.6493	357899
Autopilot	0.03	15m	0.8754	223237	0.6379	238514
Autopilot	0.03	30m	0.7987	111959	0.6001	119133
Autopilot	0.03	01h	0.6744	056028	0.5312	059443
Autopilot	0.03	03h	0.4237	018608	0.3370	019662

2 Prediction of Background Jobs

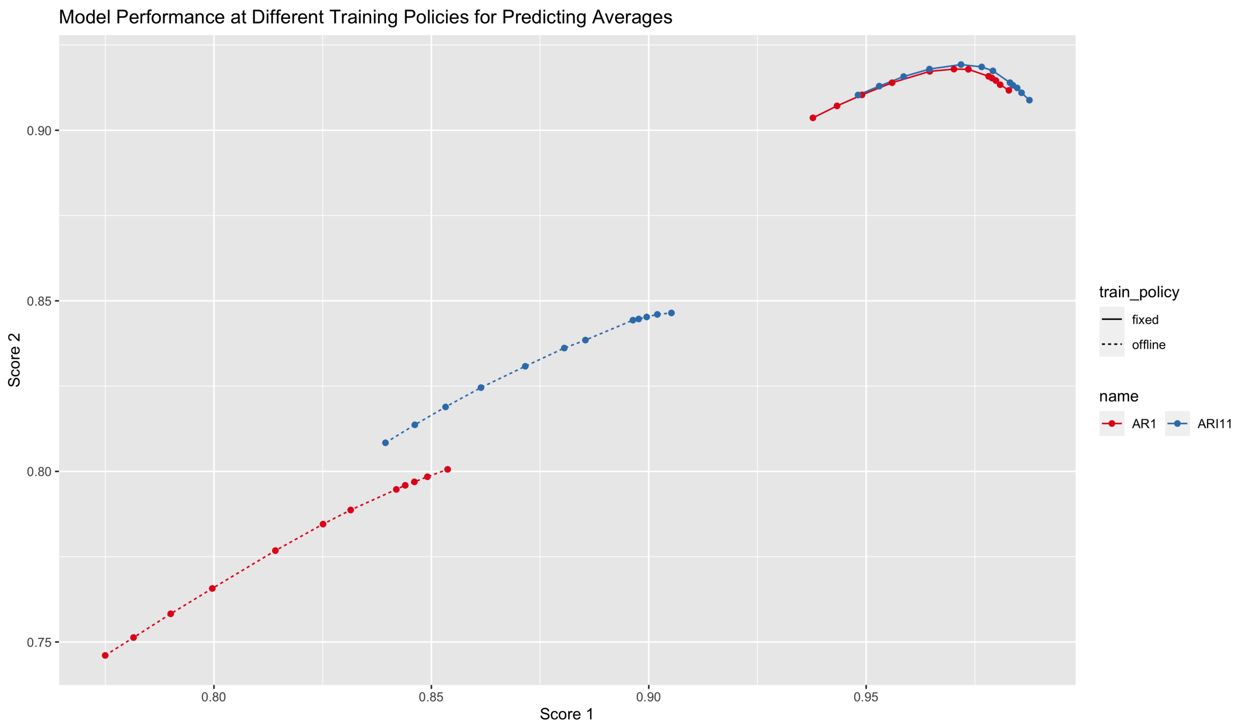
2.1 Main Motivation for the problem

In the foreground part, the time series models do the prediction of the amount of available resource that can be scheduled in the next period of time, with the constraint for the overall survival rate. However, we do not know for sure how long a specific job will last in the machine. Therefore, we are aiming to predict the distribution of job's duration times, as a discrete probability vector: $(p_1, p_2, ..., p_m)$, which can be used later with the models for the foreground part, to decide which job should be assigned to which machine.

The methodology in this part can be decomposed into the following parts:

- Discretization of the job duration times.
- Clustering of jobs
- Building probability vector within each cluster
- Diagnosis and Updating of the probability vectors

Figure 25



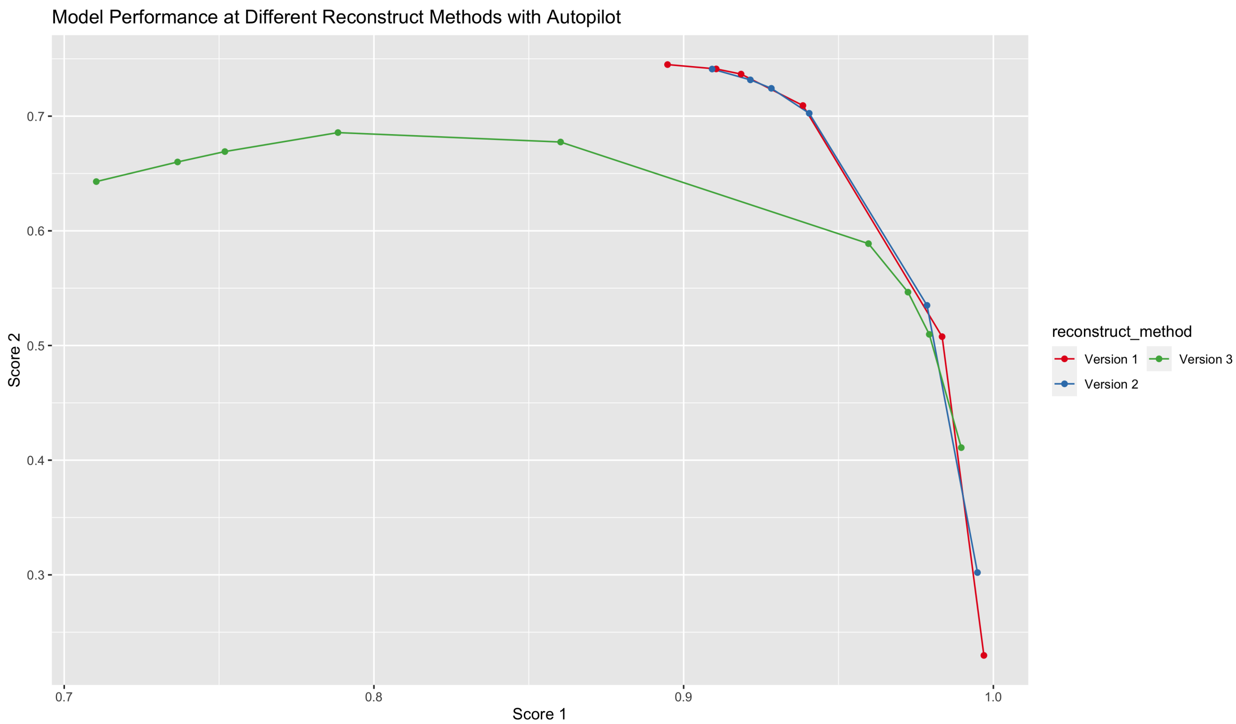
2.2 Discretization

The reason that we need to do the discretization for job's duration time is that for each unique value of job's duration time, we need to correspond with a separate model in the foreground part. Since in practice we can only have finitely many models in each part, we need to discretize the duration times into finitely many distinct bins.

The first step for do a visualization of the unconditional job duration times, and decide the number and placement of our bins.

Based on this histogram Fig 28, a reasonable bin vector will be:
$(0, 1, 2, 6, 10, 14, 18, 22, 26, 30, 50, 80, 205)$, which covers the full range of job duration in this case, and places bins with smaller widths for region with greater density. Therefore, the discretized values of job duration will be
$(1, 2, 6, 10, 14, 18, 22, 26, 30, 50, 80, 205)$ depending on which bin that job duration is in.

Figure 26: Trade-off Curves for Different Reconstructed Methods



2.3 Clustering of jobs

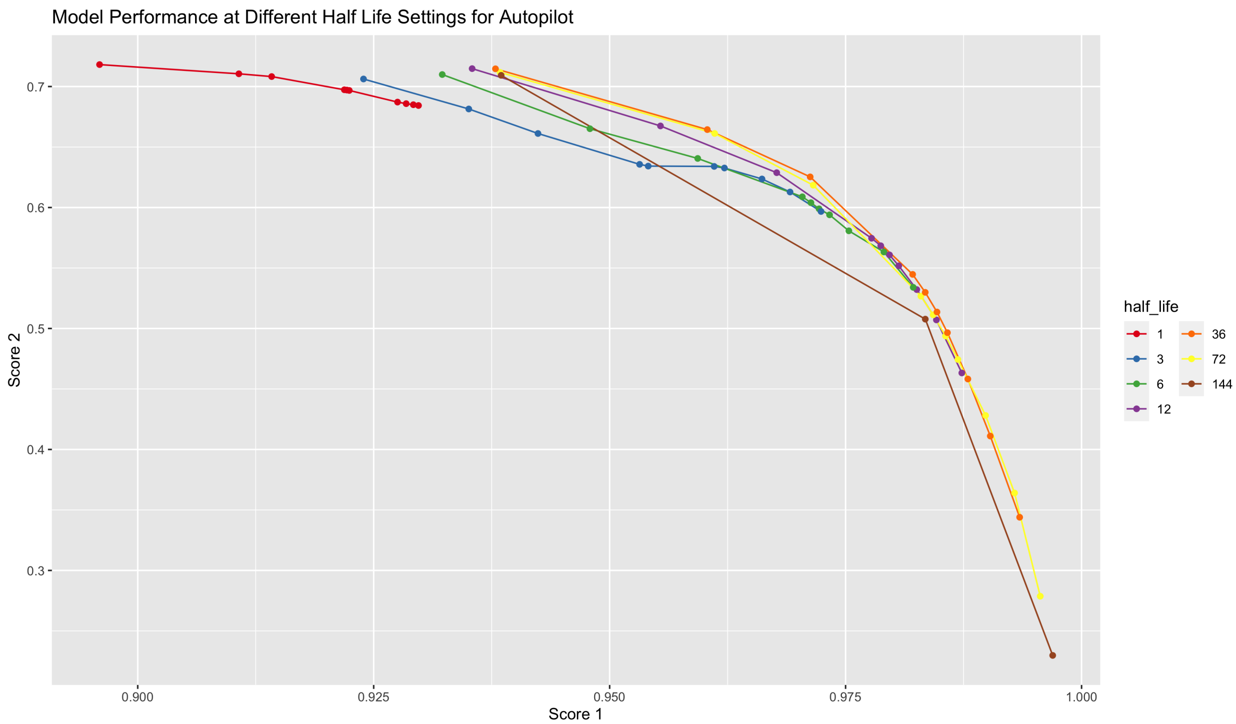
To make the probability vector a more accurate approximation for job's duration's distribution, jobs are assigned to different clusters. The clustering algorithms use job's features: priority, scheduling class, requested CPU, requested RAM and requested local disk space. There are two options available for the clustering algorithm.

2.3.1 Tree Method

Using the training set, a **survival tree** or a **regression tree** that aims to predict the job's duration is fitted, and each terminal node will be regarded as a cluster for jobs. The survival tree treats each observation of the job's duration as a realization from a specific exponential distribution, with each terminal node in the tree represents a different type of exponential distribution, and seeks to maximize a likelihood criterion. The regression tree just treats each duration as a usual continuous numerical variable and seeks to minimize the sum of squares. The number of clusters will be the number of terminal nodes in the tree, so it does not need to be specified beforehand.

minsize: Minsize used when the tree method is selected for the clustering. It specifies the minimum number

Figure 27: Trade-off Curves for Different Half-life Settings for Autopilot



of observations in each terminal node of the tree. To make sure later the probability vector in each cluster is estimated accurately, it is necessary to make sure that there are enough observations in each cluster.

2.3.2 Gaussian Mixture Model Method

Different from the tree method above, the GMM method does the clustering without looking at the response variable duration time. This method simply treats the features of each job are multivariate normally distributed, and models the whole data-set as a mixture of normal distributions. To use this algorithm, it is necessary to know the number of clusters in the data set. So without this information, the algorithm will automatically try for a bunch of numbers and select the one with the optimal BIC value.

upper limit BIC: Upper Limit BIC used when the GMM method is selected. It specifies the maximum number of cluster that the algorithm will try when it is selecting the number of clusters in the model.

2.4 Building probability vector

After obtaining a model for the clustering, and a clustered training set, a probability vector can be estimated in each cluster by looking at the empirical distribution(histogram). For example, if we discretized the job duration time into m discrete values, and found out that there are k clusters after the clustering, then we are looking for a list of k vectors, each with m components, i.e.: $\{(p_{i1}, p_{i2}, ..., p_{im}) : i \in (1, ..., k)\}$. The value of p_{ij} can be interpreted as the probability that a job in cluster i, has duration time t_j (the j-th distinct duration time after discretization).

2.5 Diagnosis and Updating

To test the accuracy of probability vectors we obtained from training set, we use the probability vectors obtained from the testing set, by measuring the average squared error weighted by the number of observations in each labeled testing set. (For this measure to make sense, we need the number of observations in the unconditional testing set to be large enough, but not necessary for each labeled testing set, as they are already weighted). For example, denote the number of observations in the (unconditional) testing set as T, and the number of observations in cluster i in the testing set as T_i, and let $\{(q_{i1}, q_{i2}, ..., q_{im}) : i \in (1, ..., k)\}$ be the list of probability vectors obtained from the testing set, then our measure of discrepancy will be:

$$\sum_{i=1}^{k} \frac{T_i}{T} \sum_{j=1}^{m} \frac{(q_{ij} - p_{ij})^2}{m} \tag{3}$$

When this value is too large, it might be that users have now changed their usual behavior, and we need to gradually adopt their new behavior by updating our probability vector. It could be done by randomly dropping observations from the unconditional training set, and replace these observations by the testing set we used and recompute the probability vector.

2.6 Appendix

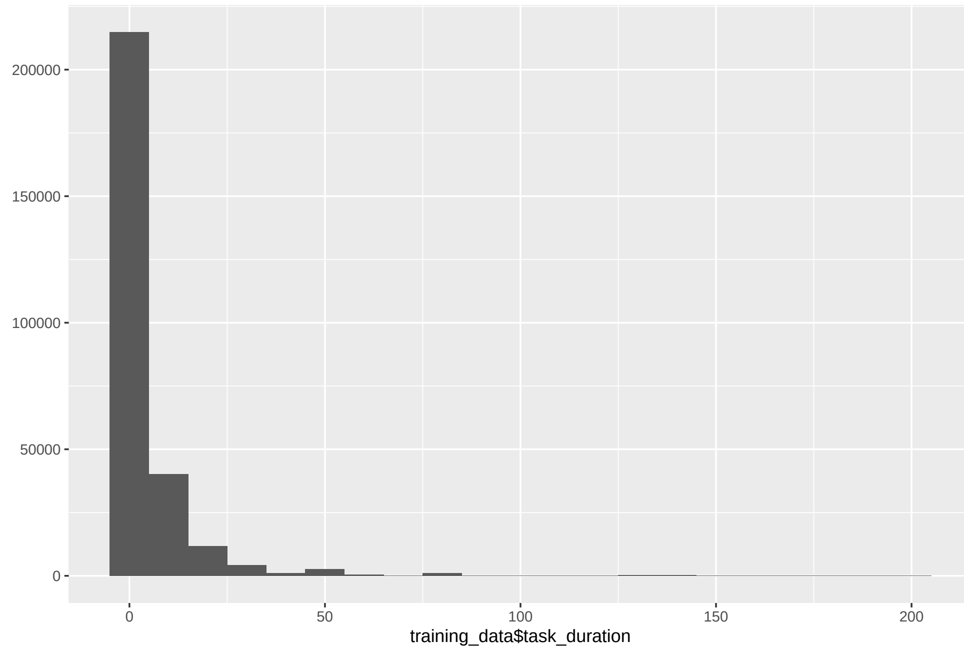
2.6.1 Discretization

2.6.2 Clustering of jobs

Table 23: Different Background Models With Actual Job Length Information

Background Model	**alpha**	**Use Actual Job Length**	**finished utilization**	**survival rate**	**load**	**unfinishe**

Figure 28: Unconditional Histogram of Job Duration



3 Combining Predictions of Two Parts

3.1 Overview of Current method

Our current method of combining foreground jobs model and background jobs model can be summarized as following:

Step 1: Given a job, we use the clustering model from the background part to identify which cluster it is in. Let its requested CPU be R.

Step 2: Using the probability vector of this cluster from background part, we know the probability of this job finishing at different times. Let's suppose that the probability vector is $P = (p_1, ..., p_m)$

Step 3: There will be also m models running on the foreground part, and each of them will have a aggregation time correspond to a component of the discretized job duration times. Each of these models will give a one-step forward prediction for the available resource on the machine. Call this vector of one step predictions as $(r_1, ..., r_m)$.

Step 4: Notice that r_i can be interpreted as the amount of available resource given that time T equals t_i. So the expected available resource for this job is computed as $U = \sum_{i=1}^{m} p_i r_i$.

Step 5: For the j-th machine, we can compute the probability that the i-th job can be finished, by summing over the components in the probability vector such that $r_j \geq R_i$. If this sum is greater than the specified threshold (a default value is $\sqrt{99}$ percent), we call this machine is plausible for the job.

Step 6: For the j-th machine, compute the cost D_j as:$U_j - R$ if $R \leq U_j$, and ∞ otherwise. Then among all the plausible machines, we will choose to allocate a job to the machine with minimum cost.

Order of Scheduling Jobs: Since scheduling multiple jobs on multiple machines with some capacities is a multi-dimensional knapsack problem which is an NP hard problem. A simple approximation algorithm we proposed is to order the jobs by their requested CPU and consecutively scheduled (following Step 4, Step 5 and Step 6).

Killing Jobs: Sometimes outliers or irregular fluctuations may occur and the foreground CPU resource may interfere with background scheduled jobs. In this case, we need to at least free up the CPU resource scheduled subtracted by CPU resource actually available, denoted by C.

In order to find the suitable jobs to kill to minimize the number of jobs being killed while also minimizing CPU utilization lost, a two-step dynamic programming algorithm is implemented: First, order the current scheduled jobs by their requested CPU in descending order, then by consecutively removing the jobs in order, we can find the minimum number of jobs that needs to be killed in order to accommodate the spike in foreground CPU resource. Second, we consecutively select the minimum number of jobs to kill with minimum resource occupied of C at this time of while minimizing the loss of CPU resource as the result of killing such selected jobs, that is, the sum of product of CPU resource requested and the amount of time the job has been running. A dynamic programming algorithm in the second step would significantly reduce the run time.

It is also noticeable that the running time of the above is approximately $O(n^k)$ where n is the number of jobs active at any timestamp, k is an integer greater than 0 as the number of jobs that needs to be killed. Another alternative is simply order the jobs by their requested CPU in descending order and select jobs from top of the list to kill. This approach significantly reduce the running time and does not have any impact on survival rate since the number of jobs being killed is exactly the same as dynamic programming algorithm described above, but the utilization rate will be lower since the whole purpose of the dynamic programming algorithm is to find smallest job to kill.

Randomized Simulation: There are two parts of the simulations that are random; the arrival time of the jobs are uniformly assigned across all simulation time length and the machine selected. In order to generate more stable results, each simulation is repeated at least 10 times and averaged as the final performance.

3.2 Simulation Study

The purpose of these simulations is to deal with the case of how to allocate each job into the optimal machine (we want it to survive, but not to waste so much resource). We used the google data set for jobs, and the Microsoft data set for foreground predictions.

The foreground setting of simulation is to use simple AR1 model with offline training strategy, with training size of 3000 data points and the length of simulation is 100 unit time (500 minutes). The background setting of simulation is to use ANOVA tree model on clustering and 5000 jobs as training set.

3.2.1 Arrival Rate

Arrival rate refers to the number of jobs arrivals per unit of time. If all the jobs can be ideally scheduled onto the machines, the arrival rate is considered to be **sparse**, and if the jobs are competing for resources and a significant proportion of jobs cannot be scheduled eventually, then the arrival rate is considered to be **dense**.

Load of Jobs: The load of jobs is an indicator of how dense the arrival rate is for fixed choices of machines. It is simply the sum of CPU resource requested by all jobs times their running time divided by the resource available across all machines summed at all simulation time. When the load is close to 1, a small proportion of the jobs may eventually fail to be scheduled, and when the load is much greater than 1, the jobs are competing for resources and all machines can be presumed to be fully occupied at all time.

Table 24 shows that the performance of survival rate is not affected by the arrival rate or the load, the utilization becomes better as arrival rate increases because more choices of jobs becomes available at higher rates, and the algorithm may have better selections.

Another method of changing the arrival rate is to increase the simulation length, Table 25 shows that fixing the number of background jobs to be 5000 as simulation length increases, the utilization decreases but there is not significant change in survival rate.

3.2.2 Cut off Probability

In the above experiments, we have used cut off probability

3.2.3 Models for Background Jobs

As described in the Tree Methods and GMM Methods in the previous section, we aim to compare the performance of simulation results using the different models for clustering background jobs. Evidently different tree methods yields similar tree structure and Table 26 suggests that the two tree methods have similar performance in both survival rate and utilization rate.

3.2.4 Models for Foreground Jobs

Since using different models yields different prediction upper bounds, which are used to determine the available resource at each time window.

3.3 Appendix

3.3.1 Simulation Study

Table 24: Combined Simulation of AR1 Model and ANOVA tree with Different Number of Jobs

job num	finished utilization	survival rate	load	unfinished rate	unscheduled rate
1000	0.1185	0.9966	0.8077	0.0070	0.4038
2000	0.1808	0.9956	1.5285	0.0024	0.5262
3000	0.1877	0.9937	1.8106	0.0017	0.6019
5000	0.2950	0.9965	2.7880	0.0015	0.6405

Table 25: Combined Simulation of AR1 Model and ANOVA tree with Different Length of Simulation

sim length	finished utilization	survival rate	load	unfinished rate	unscheduled rate
100	0.2950	0.9965	2.7880	0.0015	0.6405
200	0.2490	0.9953	1.3927	0.0016	0.5003
500	0.2465	0.9923	0.5649	0.0009	0.1608
1000	0.1645	0.9926	0.2828	0.0005	0.1435
1500	0.0938	0.9965	0.1857	0.0003	0.1897

Table 26: Combined Simulation of AR1 Model and Different Background Models

background model	alpha	finished utilization	survival rate	load	unfinished rate	unscheduled rate
anova tree	0.005	0.2950	0.9965	2.7880	0.0015	0.6405
anova tree	0.010	0.2950	0.9965	2.7880	0.0015	0.6405
anova tree	0.020	0.2950	0.9965	2.7880	0.0015	0.6405
anova tree	0.050	0.2950	0.9965	2.7880	0.0015	0.6405
survival tree	0.005	0.2993	0.9956	2.5562	0.0011	0.5568
survival tree	0.010	0.2993	0.9956	2.5562	0.0011	0.5568
survival tree	0.020	0.2993	0.9956	2.5562	0.0011	0.5568

survival tree	0.050	0.2993	0.9956	2.5562	0.0011	0.5568	

Table 27: Combined Simulation of AR1 and Autopilot with Different Length of Simulation

model	**sim length**	**alpha**	**finished utilization**	**survival rate**	**load**	**unfinished rate**	**unscheduled**
Autopilot	1000	0.005	0.1932	0.9947	0.2858	0.0006	0.1594
Autopilot	1000	0.01	0.2055	0.9927	0.2858	0.0006	0.0928
Autopilot	1000	0.02	0.2137	0.9889	0.2858	0.0006	0.0778
Autopilot	1000	0.05	0.1898	0.9743	0.2858	0.0006	0.0307
Autopilot	1500	0.005	0.1390	0.9945	0.1681	0.0007	0.0227
Autopilot	1500	0.01	0.1358	0.9914	0.1681	0.0005	0.0221
Autopilot	1500	0.02	0.1274	0.9886	0.1681	0.0006	0.0221
Autopilot	1500	0.05	0.1238	0.9852	0.1681	0.0005	0.0221
Autopilot	2000	0.005	0.1187	0.9948	0.1391	0.0003	0.0382
Autopilot	2000	0.01	0.1168	0.9914	0.1391	0.0003	0.0148
Autopilot	2000	0.02	0.1026	0.9816	0.1391	0.0003	0.0148
Autopilot	2000	0.05	0.0916	0.9733	0.1391	0.0002	0.0148
AR1 online	1000	0.01	0.1424	0.9973	0.2858	0.0005	0.1409
AR1 online	1000	0.02	0.1712	0.9948	0.2858	0.0006	0.0884
AR1 online	1000	0.05	0.2013	0.9928	0.2858	0.0006	0.0490
AR1 online	1000	0.06	0.2028	0.9913	0.2858	0.0007	0.0474
AR1 online	1000	0.07	0.2078	0.9905	0.2858	0.0007	0.0415
AR1 online	1000	0.08	0.2106	0.9916	0.2858	0.0006	0.0433
AR1 online	1000	0.09	0.2091	0.9908	0.2858	0.0006	0.0425
AR1 online	1500	0.005	0.1174	0.9975	0.1681	0.0005	0.0483
AR1 online	1500	0.01	0.1301	0.9971	0.1681	0.0006	0.0338
AR1 online	1500	0.02	0.1358	0.9957	0.1681	0.0006	0.0277
AR1 online	1500	0.05	0.1397	0.9942	0.1681	0.0006	0.0230
AR1 online	1500	0.06	0.1413	0.9942	0.1681	0.0006	0.0227
AR1 online	1500	0.07	0.1404	0.9938	0.1681	0.0006	0.0225
AR1 online	1500	0.08	0.1408	0.9933	0.1681	0.0006	0.0223
AR1 online	1500	0.09	0.1387	0.9927	0.1681	0.0006	0.0222
AR1 online	2000	0.005	0.0860	0.9968	0.1391	0.0003	0.0901
AR1 online	2000	0.01	0.0966	0.9971	0.1391	0.0003	0.0607

AR1 online	2000	0.02	0.1075	0.9973	0.1391	0.0003	0.0436
AR1 online	2000	0.05	0.1167	0.9958	0.1391	0.0003	0.0188
AR1 online	2000	0.06	0.1186	0.9952	0.1391	0.0003	0.0166
AR1 online	2000	0.07	0.1186	0.9955	0.1391	0.0003	0.0161
AR1 online	2000	0.08	0.1191	0.9950	0.1391	0.0003	0.0161
AR1 online	2000	0.09	0.1193	0.9947	0.1391	0.0003	0.0160

Table 28: Combined Simulation of Different Foreground Models with ANOVA model

model	**cut off prob**	**finished utilization**	**survival rate**	**load**	**unfinished rate**	**unscheduled rate**
AR1 online	0.005	0.1877	0.9970	0.5776	0.0007	0.3005
AR1 online	0.010	0.2173	0.9974	0.5776	0.0007	0.2588
AR1 online	0.020	0.2519	0.9955	0.5776	0.0009	0.1800
AR1 online	0.050	0.3005	0.9893	0.5776	0.0010	0.1272
AR1 online	0.060	0.3135	0.9894	0.5776	0.0015	0.1292
AR1 online	0.070	0.3204	0.9880	0.5776	0.0011	0.1277
AR1 online	0.080	0.3195	0.9882	0.5776	0.0010	0.0986
AR1 online	0.090	0.3197	0.9878	0.5776	0.0011	0.1163
VAR1 online	0.005	0.2313	0.9912	0.5776	0.0009	0.2258
VAR1 online	0.010	0.2726	0.9944	0.5776	0.0010	0.1568
VAR1 online	0.020	0.3052	0.9931	0.5776	0.0010	0.1281
VAR1 online	0.050	0.3423	0.9910	0.5776	0.0011	0.0960
VAR1 online	0.060	0.3402	0.9900	0.5776	0.0011	0.0881
VAR1 online	0.070	0.3486	0.9899	0.5776	0.0011	0.0847
VAR1 online	0.080	0.3405	0.9873	0.5776	0.0011	0.0778
VAR1 online	0.090	0.3380	0.9871	0.5776	0.0011	0.0774
Autopilot	0.005	0.2998	0.9938	0.5776	0.0000	0.0000
Autopilot	0.010	0.3301	0.9905	0.5776	0.0011	0.2254
Autopilot	0.020	0.3410	0.9873	0.5776	0.0011	0.1368
Autopilot	0.050	0.3330	0.9736	0.5776	0.0011	0.0980

Figure 29: Histogram of percent performance