Transportation Networks

Esteban Rossi-Hansberg

University of Chicago

Optimal Transport Networks in Spatial Equilibrium

Fajgelbaum and Schaal, 2020, *Econometrica*

- Large investments in infrastructure
  - 20% of World Bank spending, 6% of government spending around the world
  - How should these investments be allocated in a transport network?
- Challenges
  - Investments in one segment affect returns in others
  - Reallocation of economic activity and trading routes
  - Dimensionality

This Paper

- **Develop a framework to study optimal transport networks in general equilibrium**

1. **Solve global optimization over the space of networks**
   - given any primitive fundamentals
   - in a neoclassical trade framework (with labor mobility)
2. **Apply to road networks in 24 European countries**
   - how large are the gains from expansion and the losses from misallocation of current networks?
   - what are the regional effects?

Key Features

- **Neoclassical Trade Model on a Graph**
  - Infrastructure impacts shipping cost in each link
- **Sub-Problems:**
  - how to ship goods through the network? ("Optimal Flows")
  - how to build infrastructure? ("Optimal Network")
- **Optimal flows/routes**
  - Numerically very tractable
  - Especially using dual approach (convex optimization in space of prices)
- **Full problem (Flows+Network+GE) inherits numerical tractability**
  - Infrastructure investment expressed as function of equilibrium prices
  - Requires general problem to be convex (congestion in transport)

Preferences and Technologies

- $\mathcal{J} = \{1, ..., J\}$ locations
  - N traded goods aggregated into c_j
  - 1 non-traded good $h_j$in fixed supply (can make it variable)
  - L_j workers located in j (fixed or mobile)
- Homothetic and concave utility in j,

$$U\left(c_j, h_j\right)$$

where

$$c_j L_j = D_j\left(D_j^1, \ldots, D_j^N\right)$$

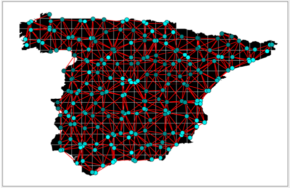
  - $D_j\left(\cdot\right)$ homogeneous of degree 1 and concave (e.g., CES)
- Output of sector n in location j is:

$$Y_j^n = F_j^n\left(L_j^n, V_j^n, X_j^n\right)$$

  - $F_j^n\left(\cdot\right)$ is neoclassical
  - $V_j^n, X_j^n =$ other primary factors and intermediate inputs
- Special cases
  - Ricardian model, Armington, Specific-factors, Heckscher-Ohlin, Endowment economy, Rosen-Roback...

Underlying Graph

- The locations are arranged on an *undirected* graph
  - $\mathcal{J} = \{1, ..., J\}$ nodes
  - $\mathcal{E}$ edges
- Each location j has a set $\mathcal{N}(j)$ of "neighbors" (directly connected)
  - Shipments flow through neighbors
  - "Neighbors" may be geographically distant
    - ⋆ Fully connected case $\mathcal{N}(j) = \mathcal{J}$ is nested
- E.g.: Spain, $\sim$ 50 km$\times$50 km square network, $\#\mathcal{N}(j) = 8$



Transport Technology

- Per-unit cost $\tau^n_{jk}\left(Q^n_{jk}, I_{jk}\right)$ denominated in units of good itself (iceberg)
  - Q^n_{jk} = quantity of commodity n from j to $k \in \mathcal{N}(j)$
  - I_{jk} = index of road quality/capacity (number of lanes, highway,...)
- **Decreasing returns to transport:** $\frac{\partial \tau^n_{jk}}{\partial Q^n_{jk}} \geq 0$
  - "Congestion" in short, may account for travel times, road damage, fixed factors in transport technologies...
  - Alternatively, cross-good congestion: $\tau^n_{jk}\left(\sum_n Q^n_{jk}, I_{jk}\right)$ denominated in units of the bundle of traded goods
- **Positive returns to infrastructure:** $\frac{\partial \tau^n_{jk}}{\partial I_{jk}} < 0$
  - Infrastructure investment lower trade costs (# of lanes, whether road is paved, etc.)
  - **The transport network chosen by planner is defined by** $\left\{I_{jk}\right\}_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$

Network Technology

- **Constraint on flows:**

$$\underbrace{D_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(1 + \tau_{jk}^n\right) Q_{jk}^n}_{\text{Consumption +Intermediate Use+ Exports}} \leq \underbrace{Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n}_{\text{Production + Imports}}$$

- Building infrastructure I_{jk} takes up $\delta_{jk}^I I_{jk}$ units of a scarce resource in fixed supply K ("asphalt")
  - Building cost δ_{jk}^I may vary across links
  - e.g. due to ruggedness, distance...

- Aggregate resource constraint:

$$\sum_j \sum_{k \in \mathcal{N}(j)} \delta_{jk}^I I_{jk} \leq K$$

Planner's Problem Given Network

Definition

The planner's problem given the infrastructure network is

$$W_0\left(\{I_{jk}\}\right) = \max_{\mathbf{C}_j, \mathbf{L}_j, \mathbf{V}_j^n, \mathbf{X}_j^n, L_j} \max_{Q_{jk}^n} u$$

subject to (i) availability of traded and non-traded goods,

$$c_j L_j \leq D_j\left(\mathbf{D}_j\right) \text{ and } h_j L_j \leq H_j \text{ for all } j;$$

(ii) the balanced-flows constraint,

$$D_j^n + \sum_{n'} X_j^{nn'} + \sum_{k \in \mathcal{N}(j)} \left(1 + \tau_{jk}^n\left(Q_{jk}^n, I_{jk}\right)\right) Q_{jk}^n = Y_j^n + \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \text{ for all } j,\ n;$$

(iii) free labor mobility,

$$L_j u \leq L_j U\left(c_j, h_j\right) \text{ for all } j;$$

(iv) local and aggregate labor-market clearing; and
(v) factor market clearing and non-negativity constraints.

Optimal Flows Problem

- Multipliers P_j^n of balanced-flow constraints = price of n in j in decentralization
- No-arbitrage condition:

$$P_k^n \leq P_j^n \left(1 + \tau_{jk}^n + \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n} Q_{jk}^n\right), = \text{if } Q_{jk}^n > 0$$

  - Key property: Q_{jk}^n invertible from P_j^n, P_k^n
- Dual solution coincides with primal
  - Dual: convex optimization with linear constraints in smaller space (of prices):

$$\inf_P \mathcal{L}\left(C\left(P\right), L\left(P\right), Q\left(P\right); P\right)$$

    - Use FOCs and substitute for C, Q,..., as function of P, then minimize over Lagrange multipliers
    - Convex minimization problem in fewer variables with just non-negativity constraints (just P)
  - Efficient algorithms are guaranteed to converge to global optimum (Bertsekas, 1998)

Optimization over Transport Network

Definition

The full planner's problem with labor mobility is

$$W = \max_{\{I_{jk}\}} W_0\left(\{I_{jk}\}\right)$$

subject to: (a) the building constraint, $\sum_j \sum_{k \in \mathcal{N}(j)} \delta^I_{jk} I_{jk} = K$*; and (b) the bounds* $\underline{I}_{jk} \leq I_{jk} \leq \overline{I}_{jk}$

- At the global optimum, the optimal network satisfies

$$\underbrace{P_K \delta^I_{jk}}_{\text{Building Cost}} \geq \underbrace{\sum_n P^n_j Q^n_{jk} \times \left(-\frac{\partial \tau^n_{jk}}{\partial I_{jk}}\right)}_{\text{Gain from Infrastructure}}, = \text{ if } I_{jk} > \underline{I}_{jk}$$

  - Reduces optimization to space of prices $\rightarrow$ Full problem inherits tractability of optimal flows

Proposition

If the function $Q\tau_{jk}(Q, I)$ *is convex in* Q *and* I*, the full planner's problem with mobile labor (immobile labor) is a quasiconvex (convex) optimization problem*

Log-Linear Transport Technology

- Log-linear transport technology:

$$\tau_{jk}\left(Q, I\right) = \delta_{jk}^{\tau} \frac{Q^{\beta}}{I^{\gamma}}$$

  - Global convexity if $\beta > \gamma$

- Optimal network

$$I_{jk}^{*} \propto \left[\frac{1}{\delta_{jk}^{I} \left(\delta_{jk}^{\tau}\right)^{\frac{1}{\beta}}} \left(\sum_{n: P_k^n > P_j^n} P_j^n \left(\frac{P_k^n}{P_j^n} - 1 \right)^{\frac{1+\beta}{\beta}} \right) \right]^{\frac{\beta}{\beta - \gamma}}$$

Decentralization

- Atomistic traders into shipping each good n from o to d solve:

$$\pi_{od}^{n} = \max_{r=(j_0,\ldots,j_\rho)\in\mathcal{R}_{od}} p_d^n - \underbrace{p_o^n}_{\text{Sourcing Costs}} - \underbrace{\sum_{k=0}^{\rho-1} p_{j_k}^n \tau_{j_k j_{k+1}}^n}_{\text{Transport costs}} - \underbrace{\sum_{k=0}^{\rho-1} p_{j_k}^n t_{j_k j_{k+1}}^n}_{\text{Taxes}}$$

Proposition

(Welfare Theorems) If the tax on shipments of product n from j to k is $t_{jk}^n = \frac{\partial \log \tau_{jk}^n}{\partial \log Q_{jk}^n}\tau_{jk}^n$*,then the competitive allocation coincides with the planner's problem.*

- Builders choose infrastructure, receive a per-unit toll θ_{jk}^n, and take prices as given:

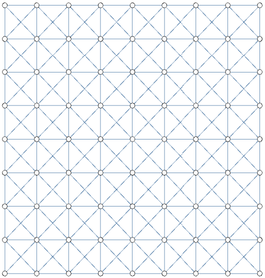
$$\max_{I_{jk}} \sum_n \theta_{jk}^n \underbrace{Q_{jk}^n\left(I_{jk}; p_k^n, p_j^n\right)}_{\text{consistent with ZP}} - p_K \delta_{jk}^I I_{jk}$$

Proposition

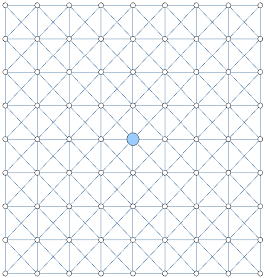
(Network Decentralization) If the toll θ_{jk}^n *is consistent with the optimal Pigouvian tax, then the decentralized infrastructure implements the optimal network investment.*

One Good in a Square

One Traded Good, Endowment Economy, Output 10x Larger at Center, Uniform Fixed Population



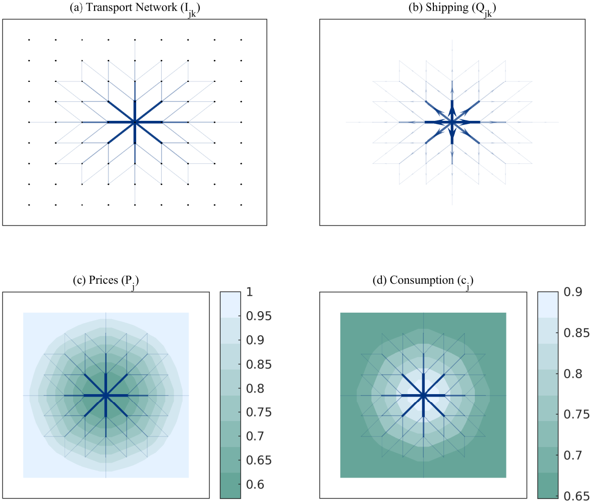
(a) Population



(b) Productivity

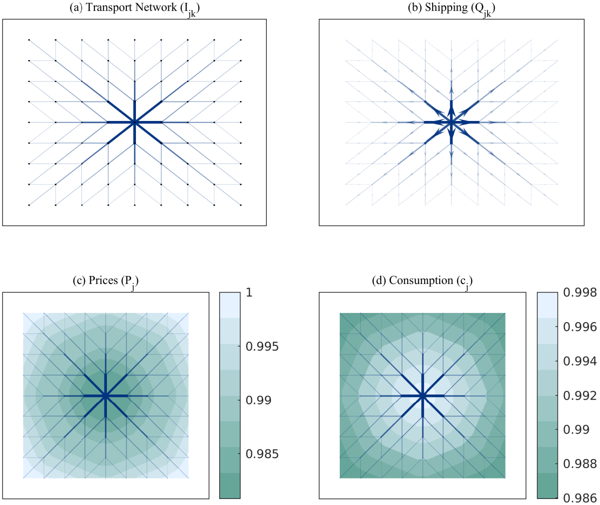
One Good in a Square

Optimal Network, $K = 1$



One Good in a Square

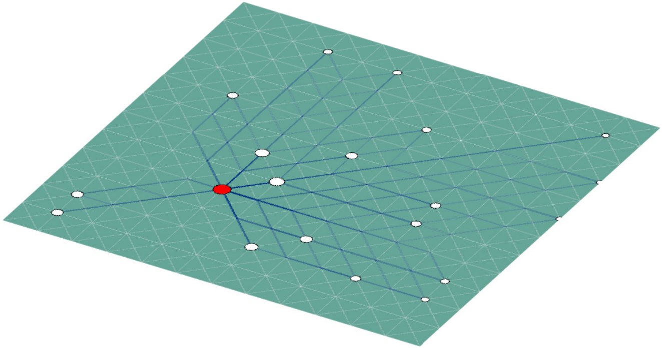
Optimal Network, $K = 100$



Role of Building Costs

20 random cities across uniform geography, convex case

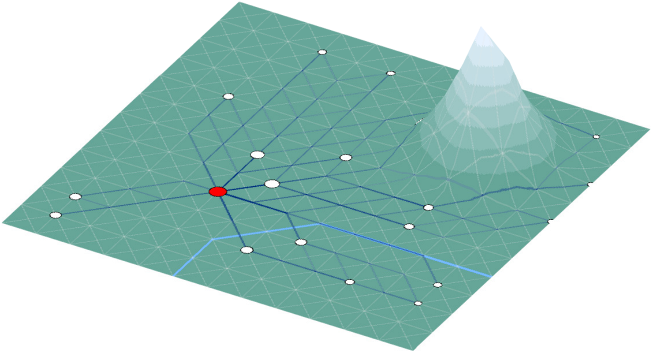
Building Cost: $\delta^I_{jk} = \delta_0 \text{Distance}^{\delta_1}_{jk}$



Role of Building Costs

Adding a mountain, a river, bridges, and water transport

Building Cost: $\delta^{I}_{jk} = \delta_0 \text{Distance}_{jk}^{\delta_1} \left(1 + |\Delta\text{Elevation}|_{jk}\right)^{\delta_2} \delta_3^{\text{CrossingRiver}_{jk}} \delta_4^{\text{AlongRiver}_{jk}}$



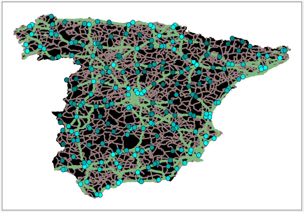
Application

- Compute optimal road network and efficiency gains across European countries
- In 24 European countries we observe
  - Road networks with features of each segment: number of lanes and national/local road (EuroGeographics)
  - Value Added (G-Econ 4.0)
  - Population (GPW)

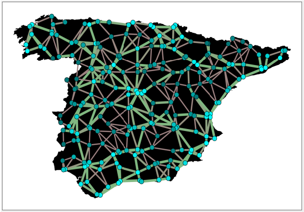
Underlying Graph and Observed Infrastructure: Spain

- Construct the graph $(\mathcal{J}, \mathcal{E})$ with quality of actual road network for each country
  - $\mathcal{J}$: population centroids of 0.5 x 0.5 degree ($\sim$50 km) cells
  - $\mathcal{E}$: all links among contiguous cells (8 neighbors per node)
  - I_{jk}^{obs}: observed infrastructure between all connected $jk \in \mathcal{E}$

Actual Road Network



Discretized Road Network



Calibration

- **Production technologies**: $Y_j^n = z_j^n L_j^n$
- **Preferences**: $U(c,h) = c^{\alpha} h^{1-\alpha}$
  - $N \in \{5, 10, 15\}$ tradeable sectors with CES demand ($\sigma = 5$)
- **Fundamentals**: $\{z_j, H_j\}$ such that $\left\{GDP_j^{obs}, L_j^{obs}\right\}$ is the model's outcome given $\left\{I_{jk}^{obs}\right\}$
- **Building costs** δ_{jk}^I
  - Use estimates from Collier et al. (2016) $\rightarrow$ Set $\delta_{jk}^{I,GEO}$ as function of distance and ruggedness

$$\ln\left(\frac{\delta_{jk}^{I,GEO}}{dist_{jk}}\right) = \ln\left(\delta_0^I\right) - 0.11 \times \left(dist_{jk} > 50km\right) + 0.12 \times \ln\left(rugged_{jk}\right)$$

  - Alternative: assume that observed road networks are optimal $\rightarrow$ Back out $\delta_{jk}^{I,FOC}$ from FOC's using I_{jk}^{obs}

Transport Costs

- **Transport technology with cross-goods congestion**:

$$\tau_{jk}^{n} = \delta_{jk}^{\tau} \frac{\left(\sum_n Q_{jk}^{n}\right)^{\beta}}{I_{jk}^{\gamma}}$$

  - Choose scale to match intra-regional share of intra-national trade in Spain

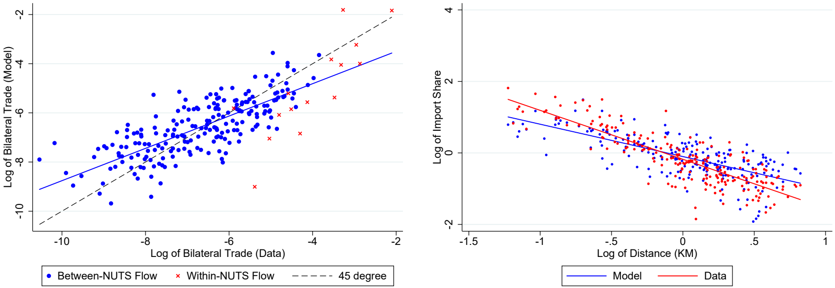
- **Consistent with assuming:**
  - trade costs (τ) are a linear function of shipping time
  - speed (S) is a log-linear function of the vehicles (V) and road lane kilometers (I)
  - vehicles (V) is a linear function of the shipments

- **Implies**

$$\ln S_{jk} = \left(\frac{\gamma}{1+\beta}\right) \ln I_{jk} - \left(\frac{\beta}{1+\beta}\right) \ln \left(\frac{V_{jk} * dist_{jk}}{S_{jk}}\right) + \varepsilon_{jk}$$

  - Use estimates from Couture, Duranton, and Turner (2018)
  - Estimates imply $\gamma = 0.10$ and $\beta = 0.13$

Model Implied Trade Flows across Spanish NUTS



Optimal Expansion and Reallocation

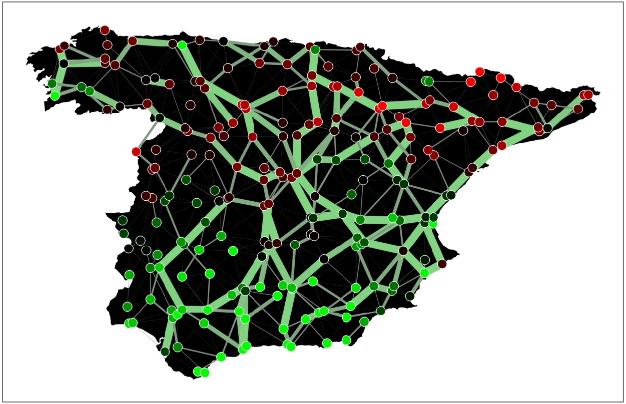
- Optimal expansion
  - Increase K by 50% relative to calibration in every country
  - Build on top of existing network ($I_{jk} \geqslant I_{jk}^{obs}$)
  - Using both $\delta_{jk}^{I,FOC}$ and $\delta_{jk}^{I,GEO}$
- Optimal reallocation
  - K is equal to calibrated model
  - Build anywhere ($I_{jk} \geqslant 0$)
  - Using $\delta_{jk}^{I,GEO}$

Aggregate Effects (Cross-Country Average)

Returns to Scale:	Benchmark		Non-Convex	
Labor:	Fixed	Mobile	Fixed	Mobile
Optimal Reallocation				
$\delta = \delta^{I,GEO}$	1.7%	1.8%	2.4%	2.5%
Optimal 50% Expansion				
$\delta = \delta^{I,GEO}$	1.7%	1.8%	2.8%	2.9%
$\delta = \delta^{I,FOC}$	0.3%	0.3%	0.9%	1.3%

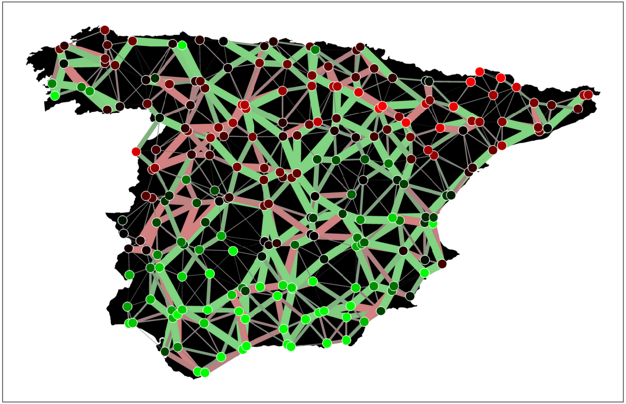
Regional Effects (Spain)

Optimal Expansion



Regional Effects (Spain)

Optimal Reallocation



Optimal Placement and Regional Effects

Panel A: Dependent Variable: Infrastructure Growth

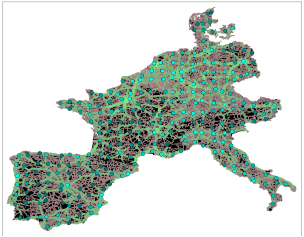
	(1) Reallocation	(2) Expansion (GEO)	(3) Expansion (FOC)
Population	0.343***	0.125***	0.002
Tradeable Income per Capita	0.151	0.071	0.007*
Infrastructure	-0.418***	-0.235***	-0.010
Observations	868	868	868
Adjusted R-squared	0.29	0.24	0.04

Panel B: Dependent Variable: Population Growth

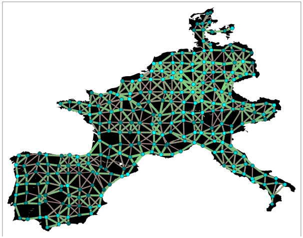
	(1) Reallocation	(2) Expansion (GEO)	(3) Expansion (FOC)
Population	-0.001	-0.000	-0.000
Tradeable Income per Capita	0.008	0.008	0.001
Consumption per Capita	-0.061***	-0.060***	-0.008***
Infrastructure	-0.001	-0.001	-0.000
Infrastructure Growth	0.002**	0.002*	-0.001
Differentiated Producer	0.010***	0.010***	0.002***
Observations	868	868	868
Adjusted R-squared	0.53	0.54	0.80

European Countries

Actual Road Network

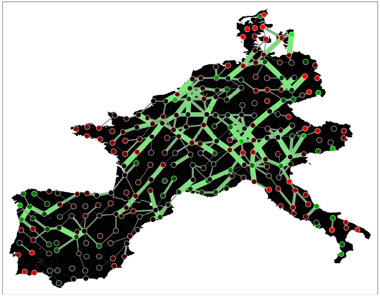


Discretized Road Network

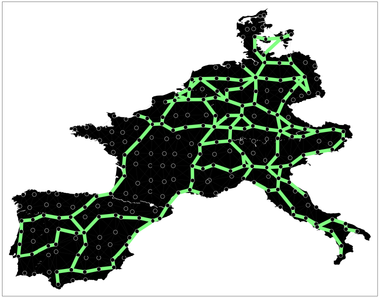


Optimal vs. Planned Investments

Optimal Expansion



TEN-T Network



Conclusion

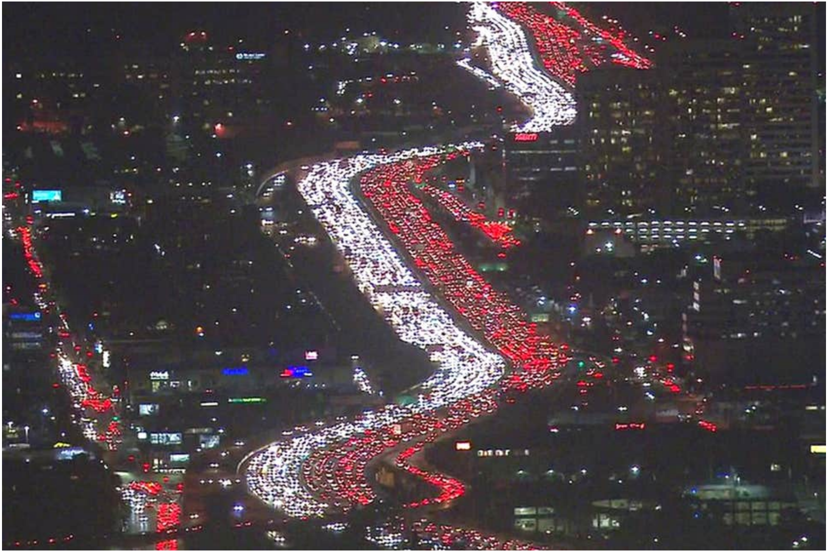
- We develop and implement a framework to study optimal transport networks
  1. Neoclassical model (with labor mobility) on a graph
  2. Optimal Transport with congestion
  3. Optimal Network
- Application to road networks in Europe
- Other potential applications
  - Political economy / competing planners
  - Model-based instruments for empirical work on impact of infrastructure
  - Optimal investments in developing countries
  - Optimal transport of workers
  - Absent forces: agglomeration, dynamics

The Welfare Effects of Transportation Infrastructure Improvements

Allen and Arkolakis, 2021, *Review of Economic Studies* (forthcoming)

- Recent "quantitative" revolution in spatial economics
  - Spearheaded by flexible theory
    Eaton Kortum '02, Allen Arkolakis '14, Ahlfeldt Redding Sturm Wolf '15
  - Fueled with swaths of spatial data
- Key benefit: evaluation of major infrastructure projects
  - Trains (Donaldson '18), subways (Severen '19), BRT (Tsivanidis '19)
- The "elephant in the room": Roads

Motivation



This Paper: Quantitative Spatial Modeling with Traffic

- Two quantitative spatial frameworks:
  - Economic geography: Agents choose source & route of each good.
  - Urban: Agents choose where to live, work, & commuting route.
- New technique to solve the fixed point problem:
  - Economic activity & route choice $\rightarrow$ traffic, transportation costs
  - Transportation costs $\rightarrow$ spatial distribution of economic activity
- Retains benefits of quantitative modeling:
  - Analytically tractable
  - "Exact hat" link to data to perform counterfactuals
- ...But with new twists:
  - A gravity equation for traffic.
  - Counterfactuals use (easily observed) traffic data.
- Estimate the ROI for improving each link in:
  - The U.S. highway network.
  - The Seattle road network.

Related literature

- Technique applies to burgeoning quantitative spatial literature
  - Eaton Kortum '02, Allen and Arkolakis '14, Redding '16, Redding & Rossi-Hansberg '17
- Quantitative evaluations of transportation infrastructure
  - Donaldson '12, Allen and Arkolakis '14, Ahlfeldt et. al. '15, Donaldson and Hornbeck '16, Alder '16, Severen '19, Tsivanidis '19, Heblich Redding Sturm '20
- Empirical evidence on importance of congestion
  - Duranton and Turner '11, Anderson '14
- Optimal transportation policy computationally
  - Alder '16, Fajgelbaum and Schaal '20

Standard components

- City comprises $i \in \{1, ..., N\} \equiv \mathcal{N}$ locations, $\bar{L}$ agents.
- Agents choose where to live & work, yielding *commuting gravity*:

$$L_{ij} = \left(\frac{\overbrace{u_i}^{\text{amenity at home}} \times \overbrace{A_j}^{\text{productivity at work}}}{\underbrace{\tau_{ij}}_{\text{commuting cost}}} \right)^{\theta} \times \left(\frac{\overbrace{\bar{L}}^{\text{aggregate population}}}{\underbrace{W^{\theta}}_{\text{expected welfare}}} \right)$$

- Productivities and amenities in each location can be written as:

$$A_i = \underbrace{\bar{A}_i}_{\text{first nature}} \times \underbrace{\left(L_i^F\right)^{\alpha}}_{\text{second nature}}$$

$$u_i = \underbrace{\bar{u}_i}_{\text{first nature}} \times \underbrace{\left(L_i^R\right)^{\beta}}_{\text{second nature}}$$

- Given elasticities $\{\alpha, \beta, \theta\}$, geography $\{\bar{A}_i, \bar{u}_i\}$, and costs τ_{ij}, equilibrium is $\{L_i^F, L_i^R\}$ such that:

$$L_i^R = \sum_{j \in \mathcal{N}} L_{ij}, \; L_j^F = \sum_{i \in \mathcal{N}} L_{ij}$$

New component: Endogenous commuting costs

- Commuting costs τ_{ij} are *endogenous*, depend on:
  - Agents' routing problem: What is the optimal path through the infrastructure network (taking traffic as given)?
  - Traffic congestion: How do agents' route choice, choice of where to live and work affect use of each link in the infrastructure network?
- Feedback loop: traffic congestion affects route choice & choice of where to live and work.

Infrastructure network

- N locations arrayed on a weighted network.
- Let $t_{kl} \geq 1$ be the iceberg commuting cost incurred by traveling from k to l on the infrastructure network, where:

$$t_{kl} = \bar{t}_{kl} \times (\Xi_{kl})^{\lambda} \tag{1}$$

  where:
  - $\bar{t}_{kl} \geq 1$ is the (first nature) quality of the infrastructure connection.
  - If $\bar{t}_{kl} < \infty$, we say that k and l are a *link*.
  - Ξ_{kl} is the traffic on link k to l.
  - λ is strength of traffic congestion ($\lambda = 0$ in a standard model).

The routing choice problem

- A *route* from i to j of length K is a sequence of locations beginning with i and ending with j:
$$r = \{i, r_1, r_2, ...r_{K-1}, j\}$$
- Let $\Re_{ij}$ be the set of all possible routes from i to j. Then a route $r \in \Re_{ij}$ incurs an iceberg cost of:
$$\tau_{ij,r} = \prod_{l=1}^{K} t_{r_{l-1},r_l}$$
- Assume agents choose where to live, where to work, & route to maximize:
$$V_{ij,r}(\nu) = \left(A_j u_i / \prod_{l=1}^{K} t_{r_{l-1},r_l} \right) \times \varepsilon_{ij,r}(\nu).$$

with Fréchet distributed idiosyncratic shock $\varepsilon_{ij,r}(\nu)$.

Endogenous commuting costs

- Solving the maximization problem and summing across all possible routes from i to j yields commuting gravity equation from above:

$$L_{ij} = \left(\frac{u_i \times A_j}{\tau_{ij}}\right)^{\theta} \times \left(\frac{\bar{L}}{W^{\theta}}\right)$$

where:

$$\tau_{ij} \equiv \left(\sum_{r \in \Re_{ij}} \left(\prod_{l=1}^{K} t_{r_{l-1}, r_l}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$$

is the *endogenous* commuting cost.

An analytical solution

- Define the *weighted adjacency matrix* $\mathbf{A} \equiv \left[a_{ij} \equiv t_{ij}^{-\theta}\right]$.
  - Define $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ and $b_{ij} \equiv [\mathbf{B}]_{ij}$.
- If $\rho(\mathbf{A}) < 1$ then:

$$\tau_{ij} = c b_{ij}^{-\frac{1}{\theta}} \tag{2}$$

  - Mapping from infrastructure network to commuting costs (!)
- Notes:
  - As $\theta \to \infty$, τ_{ij} converge to commuting cost for least cost route (generalization of Dijkstra algorithm)
    - In practice, $\sim$0.99 correlation.
  - Same operational complexity as Dijkstra ($O(N \ln N)$)
    - In practice, $\sim$1,000x faster.

From routing to traffic

- Equation (2) yields the commuting cost taking traffic congestion as given. But what is the equilibrium traffic?
- First step: calculate the intensity with which a particular link is used on the way from i to j:

$$\pi_{ij}^{kl} = \left(\frac{\tau_{ij}}{\tau_{ik} \times t_{kl} \times \tau_{lj}} \right)^{\theta}$$

  - Intuition: More out of the way links are used less.
- Second step: Sum over all origins and destinations to get traffic:

$$\Xi_{kl} = \sum_{i,j \in \mathcal{N}} L_{ij} \pi_{ij}^{kl}$$

A gravity equation for traffic

- Standard *commuting gravity equation*:

$$L_{ij} = \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i} \times \frac{L_j^F}{FMA_j},$$

where

  - Residential market access: $RMA_i = \sum_j \tau_{ij}^{-\theta} \times \frac{L_j^F}{FMA_j}$
  - Firm market access: $FMA_j = \sum_i \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i}$.

- New *traffic gravity equation*:

$$\Xi_{kl} = t_{kl}^{-\theta} \times FMA_k \times RMA_l \tag{3}$$

  - *Intuition*: Greater FMA_k, more traffic flowing in. Greater RMA_l, more traffic flowing out.

The effect of traffic congestion

- To summarize:
  - Traffic gravity (equation 3) shows how market access affects equilibrium traffic flows...
  - ... Through congestion (equation 1), traffic flows affects travel costs, which through routing choice (equation 2) affects commuting costs...
  - ... And commuting costs affect market access through (standard) equilibrium channels.
- A massive fixed point problem!
  - ...but it turns out to not be too bad at all.

Equilibrium

- Eqm. conditions $L_i^R = \sum_j L_{ij}$, $L_i^F = \sum_j L_{ji}$ in a standard model are:

$$\left(l_i^R\right)^{1-\theta\beta} = \chi \sum_j \tau_{ij}^{-\theta} \bar{u}_i^\theta \bar{A}_j^\theta \left(l_j^F\right)^{\theta\alpha}$$

$$\left(l_i^F\right)^{1-\theta\alpha} = \chi \sum_j \tau_{ji}^{-\theta} \bar{u}_j^\theta \bar{A}_i^\theta \left(l_j^R\right)^{\theta\beta}$$

- With traffic congestion, they become:

$$\left(l_i^R\right)^{1-\theta\beta} \left(l_i^F\right)^{\frac{\theta\lambda(1-\alpha\theta)}{1+\theta\lambda}} = \chi \bar{A}_i^\theta \bar{u}_i^\theta \left(l_i^F\right)^{\frac{\theta(\alpha+\lambda)}{1+\theta\lambda}} + \chi^{\frac{\theta\lambda}{1+\theta\lambda}} \sum_j \left(\bar{t}_{ij} \bar{L}^\lambda\right)^{-\frac{\theta}{1+\theta\lambda}} \bar{A}_i^{\theta \frac{\theta\lambda}{1+\theta\lambda}} \bar{u}_i^\theta \bar{u}_j^{-\frac{\theta}{1+\theta\lambda}} \left(l_j^R\right)^{\frac{1-\beta\theta}{1+\theta\lambda}}$$

$$\left(l_i^R\right)^{\frac{\theta\lambda(1-\beta\theta)}{1+\theta\lambda}} \left(l_i^F\right)^{1-\theta\alpha} = \chi \bar{A}_i^\theta \bar{u}_i^\theta \left(l_i^R\right)^{\frac{\theta(\beta+\lambda)}{1+\theta\lambda}} + \chi^{\frac{\theta\lambda}{1+\theta\lambda}} \sum_j \left(\bar{t}_{ji} \bar{L}^\lambda\right)^{-\frac{\theta}{1+\theta\lambda}} \bar{A}_i^\theta u_i^{\theta \frac{\theta\lambda}{1+\theta\lambda}} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda}} \left(l_j^F\right)^{\frac{1-\alpha\theta}{1+\theta\lambda}}$$

where $\chi \equiv \frac{\bar{L}^{\alpha+\beta}}{W}$, $l_i^R \equiv L_i^R / \bar{L}$ and $l_i^F \equiv L_i^F / \bar{L}$ are labor shares.

- Same number of equations & unknowns, new structure!

Existence and Uniqueness

Proposition

For any strictly positive local geography $\{\bar{A}_i > 0, \bar{u}_i > 0\}_{i \in \mathcal{N}}$*, aggregate labor endowment* $\bar{L} > 0$*, strongly connected infrastructure network* $\bar{\mathbf{T}} \equiv [\bar{t}_{kl}]$*, and model parameters* $\{\alpha \in \mathbb{R}, \beta \in \mathbb{R}, \theta > 0, \lambda \geq 0\}$*, then:*

1. *There exists a strictly positive equilibrium.*
2. *For any* $\alpha \in [-1, 1]$ *and* $\beta \in [-1, 1]$*, the equilibrium is unique if:*

$$\alpha \leq \frac{1}{2}\left(\frac{1}{\theta} - \lambda\right) \text{ and } \beta \leq \frac{1}{2}\left(\frac{1}{\theta} - \lambda\right)$$

- Intuition: Unlike traditional congestion externalities (α and β), stronger traffic congestion (λ) can induce *greater* economic concentration by reducing commuting between locations.

Counterfactuals

- Standard model: write system in changes using observed data:

$$\left(\hat{I}_i^R\right)^{1-\theta\beta} = \hat{\chi}\sum_j \left(\frac{L_{ij}}{L_i^R}\right)\hat{\tau}_{ij}^{-\theta}\left(\hat{I}_j^F\right)^{\theta\alpha}$$

$$\left(\hat{I}_i^F\right)^{1-\theta\alpha} = \hat{\chi}\sum_j \left(\frac{L_{ji}}{L_i^F}\right)\hat{\tau}_{ji}^{-\theta}\left(\hat{I}_j^R\right)^{\theta\beta}$$

- With traffic congestion:

$$\left(\widehat{I_i^R}\right)^{-\theta\beta+1}\left(\widehat{I_i^F}\right)^{\frac{\theta\lambda(1-\alpha\theta)}{1+\theta\lambda}} = \widehat{\chi}\left(\frac{L_i^F}{L_i^F+\sum_j \Xi_{ij}}\right)\left(\widehat{I_i^F}\right)^{\frac{\theta(\alpha+\lambda)}{1+\theta\lambda}} + \widehat{\chi}^{\frac{\theta\lambda}{1+\theta\lambda}}\sum_j\left(\frac{\Xi_{ij}}{L_i^F+\sum_j \Xi_{ij}}\right)\widehat{\bar{t}}_{ij}^{-\frac{\theta}{1+\theta\lambda}}\left(\widehat{I_j^R}\right)^{\frac{1-\beta\theta}{1+\theta\lambda}}$$

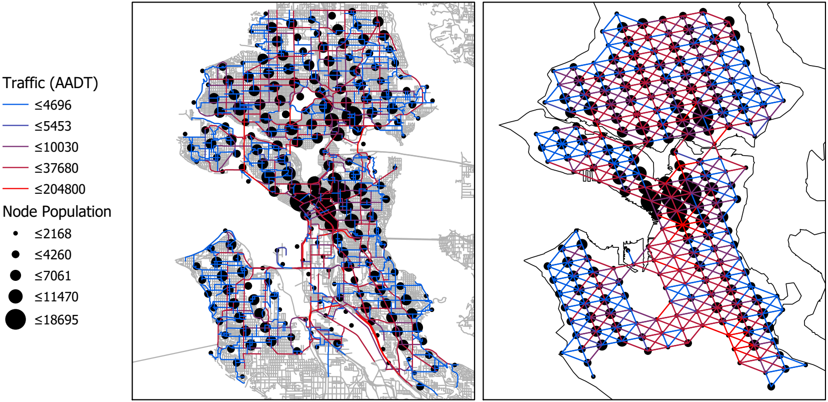
$$\left(\widehat{I_i^R}\right)^{\frac{\theta\lambda(1-\beta\theta)}{1+\theta\lambda}}\left(\widehat{I_i^F}\right)^{-\theta\alpha+1} = \widehat{\chi}\left(\frac{L_i^R}{L_i^R+\sum_j \Xi_{ji}}\right)\left(\widehat{I_i^R}\right)^{\frac{\theta(\beta+\lambda)}{1+\theta\lambda}} + \widehat{\chi}^{\frac{\theta\lambda}{1+\theta\lambda}}\sum_j\left(\frac{\Xi_{ji}}{L_i^R+\sum_j \Xi_{ji}}\right)\widehat{\bar{t}}_{ji}^{-\frac{\theta}{1+\theta\lambda}}\left(\widehat{I_j^F}\right)^{\frac{1-\alpha\theta}{1+\theta\lambda}}$$

- Same close marriage between theory and data, but now using traffic data!

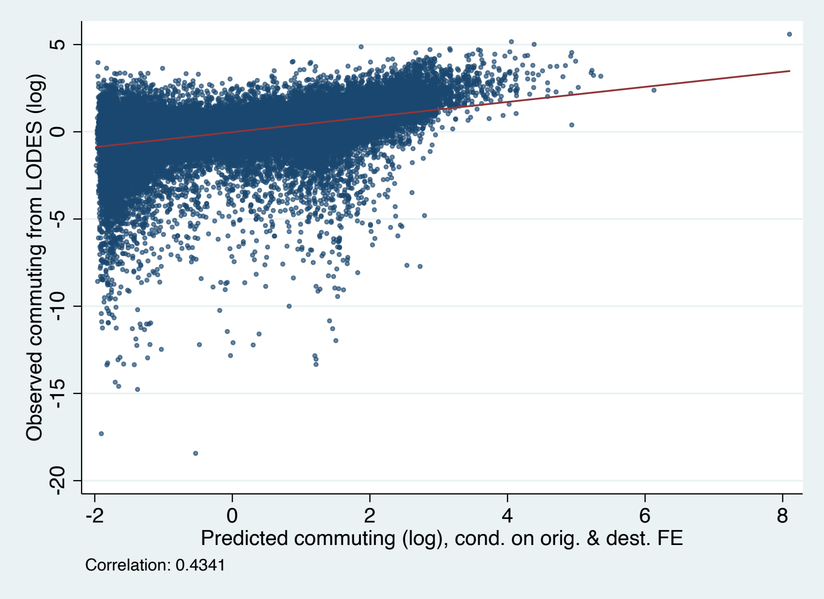
Why Seattle?

- The traffic in Seattle is bad.
  - Second highest commute times in the U.S.
  - No major public transportation system.
- The data in Seattle is good.
  - For roughly 1,500 miles of roads, we observe traffic, length, location, number of lanes, speed limit (HPMS)
  - Residential, workplace populations in each census block group (LODES).
  - Note: HPMS & LODES available throughout U.S.
- The geography is interesting.
  - Water & bridges result in natural bottlenecks in road network.

The Seattle Road Network



A first pass: predicting commuting from traffic



Estimation overview

- To evaluate welfare impacts, only need to know four elasticities:
  1. Preference heterogeneity θ. $\leftarrow$ *We set* $\theta = 6.83$ *(Ahlfeldt et al. '15).*
  2. Productivity spillover α. $\leftarrow$ *We set* $\alpha = -0.12$ *(Ahlfeldt et al. '15).*
  3. Amenity spillover β. $\leftarrow$ *We set* $\beta = -0.1$ *(Ahlfeldt et al. '15).*
  4. Traffic congestion parameter λ. $\leftarrow$ *We estimate this.*

Estimation of Traffic Congestion

- Assume:
  1. Link costs increasing in travel time:

$$\ln t_{kl} = \delta_0 \ln time_{kl}$$

  2. Speed is decreasing in traffic (AADT) per lane-mile:

$$speed_{kl} = -\delta_1 \ln \left(\frac{\Xi_{kl}}{lanes_{kl}} \right) + \delta_{kl},$$

- Then:
  1. Consistent with theory, we have:

$$\ln t_{kl} = \underbrace{\delta_0 \ln distance_{kl} - \delta_0 \delta_1 lanes_{kl} - \delta_0 \delta_{kl}}_{\equiv \ln \bar{t}_{kl}} + \underbrace{\delta_0 \delta_1}_{\equiv \lambda} \ln \Xi_{kl}$$

     - ⋆ Note: Constructing more lane-miles reduces $\bar{t}_{kl}$.

  2. Simple estimating equation:

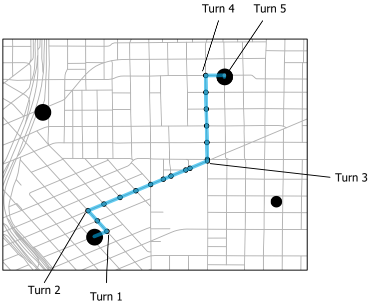
$$speed_{kl} = -\delta_1 \ln \left(\frac{\Xi_{kl}}{lanes_{kl}} \right) + \underbrace{\mathbf{X}_{kl} \beta + \varepsilon_{kl}}_{\equiv \delta_{kl}}$$

Estimation of Traffic Congestion (ctd.)

- Estimating equation from last slide:

$$speed_{kl} = -\delta_1 \ln\left(\frac{\Xi_{kl}}{lanes_{kl}}\right) + \underbrace{\mathbf{X}_{kl}\beta + \varepsilon_{kl}}_{\equiv \delta_{kl}}$$

- Need an IV for traffic uncorrelated with free flow rate of speed.
  - *Solution*: Number of turns (conditional on number of intersections).



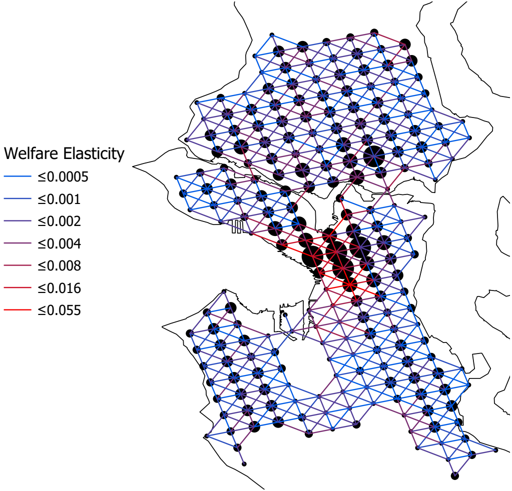
  - *Intuition*: Intersections uniformly costly, turns annoying.

Table: ESTIMATING THE STRENGTH OF TRAFFIC CONGESTION

	(1)	(2)	(3)	(4)	(5)
Travel Time Optimized	OLS	IV: 1st stage	IV	IV: 1st stage	IV
AADT per Lane	-0.048***		0.118**		0.488*
	(0.007)		(0.048)		(0.278)
Turns along Route		-0.252***		-0.091**	
		(0.049)		(0.039)	
F-statistic	41.546	26.347	6.191	5.336	3.084
R-squared	0.766	0.721	-0.450	0.875	-2.757
Observations	1338	1338	1338	1338	1338
Start-location FE	Yes	Yes	Yes	Yes	Yes
End-location FE	Yes	Yes	Yes	Yes	Yes
No. of Intersections	No	Yes	Yes	Yes	Yes
Bilateral Route Quality	No	No	No	Yes	Yes

- Implies $\lambda = 0.07$, elasticity of traffic to additional lanes of 0.33.

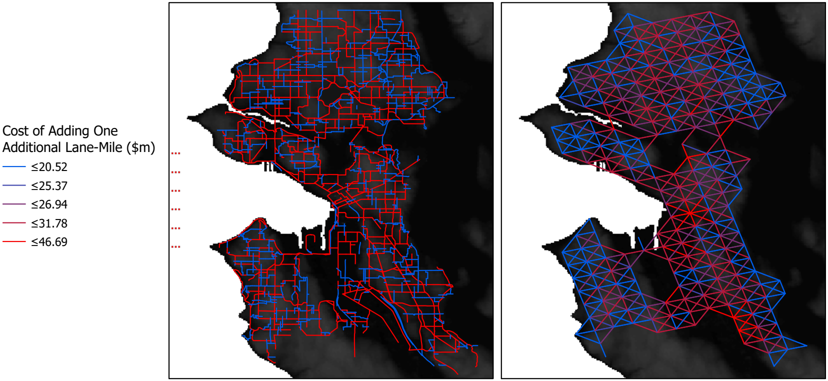
Welfare elasticities $\left(\frac{\partial \ln W}{\partial \ln \bar{t}_{kl}}\right)$ of improving each link



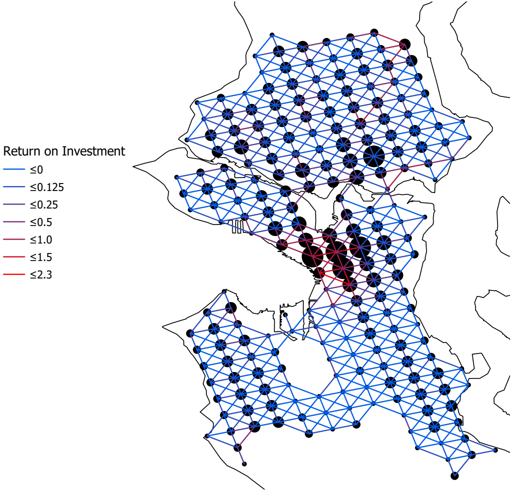
Calculating the Returns on Investment

- Calculate the annual return on investment for an additional lane-mile on every segment.
- *Benefits*:
$$\frac{\partial \ln W}{\partial \ln lanes_{kl}} = \delta_0 \delta_1 \times \frac{\partial \ln W}{\partial \ln \bar{t}_{kl}}$$
- *Costs*: Latest estimates from Federal Highway Administration (FHWA) by road type & location. Assume 10 year linear depreciation.

Estimated Annualized cost of an Additional Lane-mile



Return on Investment of Infrastructure Investment



- Huge heterogeneity in ROI: Mean: 17%, Median: 8%, SD: 37%.

Conclusion

- To bolster the quantitative revolution, introduce new spatial framework with traffic congestion:
  - Same analytical tractability, close marriage between theory and data.
  - New implications for welfare impacts of road construction.
- Future work could leverage wide-spread availability of traffic data to better design infrastructure networks in locations where commuting data is scarce (e.g. in developing countries).