Econometrica Supplementary Material

SUPPLEMENT TO "OPTIMAL TRANSPORT NETWORKS IN SPATIAL EQUILIBRIUM"

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PABLO D. FAJGELBAUM
UCLA and NBER

EDOUARD SCHAAL
CREI, Universitat Pompeu Fabra, Barcelona GSE, and CEPR

APPENDIX A: APPENDIX TO SECTION 3 (MODEL)

A.1. *Planner's Problem*

IN THIS SECTION, WE PRESENT the first-order conditions to the planner's problem. We refer to these conditions in some of the characterizations in the text and in the proofs below. We present the problem adopting a formulation of the transport technology that nests the approach with own-good congestion in which the transport cost is denominated in units of the good being shipped, as well as the approach with congestion across goods in which the transport cost is denominated in units of the bundle of traded goods (discussed in Section 3.7). In the formulations below, the parameter $\chi \in \{0, 1\}$ takes a value of 0 in the case with own-good congestion and a value of 1 with cross-good congestion. The case $\chi = 0$ corresponds to the equations presented in the body of the paper.

Immobile Labor

The Lagrangian of the problem in Definition 1 is

$$
\begin{aligned}
\mathcal{L} = {} & \sum_j \omega_j L_j U(c_j, h_j) - \sum_j P_j^D \Big[c_j L_j + \chi \sum_{k \in \mathcal{N}(j)} \tau_{jk}(Q_{jk}, I_{jk}) Q_{jk} - D_j\big(D_j^1, \dots, D_j^N\big) \Big] \\
& - \sum_j P_j^H (h_j L_j - H_j) \\
& - \sum_j \sum_n P_j^n \Big[D_j^n + \sum_{k \in \mathcal{N}(j)} \big(Q_{jk}^n + (1-\chi) \tau_{jk}^n \big(Q_{jk}^n, I_{jk}\big) Q_{jk}^n\big) \\
& - F_j^n\big(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\big) - \sum_{i \in \mathcal{N}(j)} Q_{ij}^n \Big] \\
& - \sum_j W_j \Big[\sum_n L_j^n - L_j \Big] - \sum_j \sum_l R_j^m \Big[\sum_n V_j^{mn} - V_j^m \Big] \\
& - P_K \Bigg(\sum_j \sum_{k \in \mathcal{N}(j)} \delta_{jk}^I I_{jk} - K \Bigg) + \sum_{j,k} \zeta_{jk}^{\underline{I}} (I_{jk} - \underline{I}_{jk}) + \sum_{j,k} \zeta_{jk}^{\overline{I}} (\overline{I}_{jk} - I_{jk})
\end{aligned}
$$

Pablo D. Fajgelbaum: pfajgelbaum@econ.ucla.edu
Edouard Schaal: eschaal@crei.cat

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$$+\sum_{j,k,n}\zeta^{Q}_{jkn}Q^{n}_{jk}+\sum_{j,n}\zeta^{L}_{jn}L^{n}_{j}+\sum_{j,n,l}\zeta^{V}_{jnl}V^{ln}_{j}+\sum_{j,n,l}\zeta^{X}_{jnl}X^{ln}_{j}+\sum_{j,n}\zeta^{D}_{jn}D^{n}_{j}$$
$$+\sum_{j}\zeta^{c}_{j}c_{j}+\sum_{j}\zeta^{h}_{j}h_{j},$$

where $Q_{jk}=\sum_{n=1}^{N}m^{n}Q^{n}_{jk}$ and where P^{D}_{j}, P^{H}_{j}, P^{N}_{j}, W_j, R^{l}_{j}, P_K, ζ^{I}_{jk}, ζ^{Q}_{jkn}, ζ^{L}_{jn}, ζ^{V}_{jnl}, ζ^{C}_{jn}, ζ^{c}_{j}, ζ^{h}_{j} are the multipliers of all constraints implied by (i)–(v) in Definition 1. The first-order conditions with respect to consumption and production are

$$[c_j]\qquad \omega_j L_j U_C(c_j,h_j)+\zeta^{c}_{j}=P^{D}_{j}L_j,$$
$$[h_j]\qquad \omega_j L_j U_H(c_j,h_j)+\zeta^{h}_{j}=P^{H}_{j}L_j,$$
$$\left[D^{n}_{j}\right]\qquad P^{D}_{j}\frac{\partial D_j}{\partial D^{n}_{j}}+\zeta^{D}_{jn}=P^{n}_{j},$$
$$\left[L^{n}_{j}\right]\qquad P^{n}_{j}\frac{\partial Y^{n}_{j}}{\partial L^{n}_{j}}+\zeta^{L}_{jn}=W_j,$$
$$\left[V^{n}_{j}\right]\qquad P^{n}_{j}\frac{\partial Y^{n}_{j}}{\partial V^{ln}_{j}}+\zeta^{V}_{jnl}=R^{l}_{j},$$
$$\left[X^{n}_{j}\right]\qquad P^{n}_{j}\frac{\partial Y^{n}_{j}}{\partial X^{ln}_{j}}+\zeta^{X}_{jnl}=P^{l}_{j}.$$

The first-order condition with respect to flows is

$$\begin{aligned}\left[Q^{n}_{jk}\right]\qquad & -\chi P^{D}_{j}\left(\tau^{n}_{jk}(Q_{jk},I_{jk})+\frac{\partial\tau^{n}_{jk}(Q_{jk},I_{jk})}{\partial Q_{jk}}Q_{jk}\right)\\ & -(1-\chi)P^{n}_{j}\left(\tau^{n}_{jk}\left(Q^{n}_{jk},I_{jk}\right)+\frac{\partial\tau^{n}_{jk}\left(Q^{n}_{jk},I_{jk}\right)}{\partial Q^{n}_{jk}}Q^{n}_{jk}\right)+P^{n}_{k}-P^{n}_{j}+\zeta^{Q}_{jkn}\\ & =0,\end{aligned}\tag{A.1}$$

which, along with the complementary slackness condition for Q^{n}_{jk}, implies (8) in the main text.

Finally, the first-order condition with respect to the network investment is

$$\begin{aligned}[I_{jk}]\qquad & \chi\sum_{n=1}^{N}P^{D}_{j}Q^{n}_{jk}\left(-\frac{\partial\tau^{n}_{jk}(Q_{jk},I_{jk})}{\partial I_{jk}}\right)+(1-\chi)\sum_{n=1}^{N}P^{n}_{j}Q^{n}_{jk}\left(-\frac{\partial\tau^{n}_{jk}\left(Q^{n}_{jk},I_{jk}\right)}{\partial I_{jk}}\right)\\ & +\left(\zeta^{\underline{I}}_{jk}-\zeta^{\overline{I}}_{jk}\right)=P_K\delta^{I}_{jk},\end{aligned}\tag{A.2}$$

which, along with the complementary slackness condition for I^{n}_{jk}, implies (9) in the text.

Mobile Labor

The Lagrangian of the problem in Definition 2 is

$$
\begin{aligned}
\mathcal{L} = {} & u - \sum_j \tilde{\omega}_j L_j\big(u - U(c_j, h_j)\big) - W^L\left(\sum_j L_j - L\right) \\
& - \sum_j P_j^D\left[c_j L_j + \chi \sum_{n=1}^N \sum_{k\in\mathcal{N}(j)} \tau_{jk}^n\left(\sum_{n=1}^N m^n Q_{jk}^n, I_{jk}\right) Q_{jk}^n - D_j\big(D_j^1, \dots, D_j^N\big)\right] \\
& - \sum_j P_j^H (h_j L_j - H_j) \\
& - \sum_j \sum_n P_j^n \Bigg[D_j^n + \sum_{k\in\mathcal{N}(j)} \big(Q_{jk}^n + (1-\chi)\tau_{jk}^n\big(Q_{jk}^n, I_{jk}\big) Q_{jk}^n\big) \\
& - F_j^n\big(L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\big) - \sum_{i\in\mathcal{N}(j)} Q_{ij}^n\Bigg] \\
& - \sum_j W_j\left[\sum_n L_j^n - L_j\right] - \sum_j \sum_l R_j^l\left[\sum_n V_j^n - V_j\right] \\
& - P_K\left(\sum_j \sum_{k\in\mathcal{N}(j)} \delta_{jk}^I I_{jk} - K\right) + \sum_{j,k} \zeta_{jk}^{\underline{I}}(I_{jk} - \underline{I}_{jk}) + \sum_{j,k} \zeta_{jk}^{\overline{I}}(\overline{I}_{jk} - I_{jk}) \\
& + \sum_{j,k,n} \zeta_{jkn}^Q Q_{jk}^n + \sum_{j,n} \zeta_{jn}^L L_j^n + \sum_{j,n,l} \zeta_{jnl}^V V_j^{ln} + \sum_{j,n,l} \zeta_{jnl}^X X_j^{ln} + \sum_{j,n} \zeta_{jn}^D D_j^n \\
& + \sum_j \zeta_j^c c_j + \sum_j \zeta_j^h h_j,
\end{aligned}
$$

where, in addition to the previous notation for the multipliers, in the first line we have defined $\tilde{\omega}_j$ and W^L as the multipliers of constraints (vi) and (vii) in Definition 2.

The first-order conditions with respect to consumption of traded services $[C_j^n]$, factor allocation within locations $[L_j^n]$, $[V_j^n]$, and $[X_j^n]$, optimal transport $[Q_{jk}^n]$, and optimal investment $[I_{jk}]$ are the same as in the problem without labor mobility. The first-order conditions with respect to u and L_j are

$$
\begin{aligned}
&[u] \qquad 1 = \sum_j L_j \tilde{\omega}_j, \\
&[L_j] \qquad P_j^D c_j + P_j^H h_j - \tilde{\omega}_j\big[U(c_j, h_j) - u\big] = W_j - W^L,
\end{aligned}
$$

where, from monotonicity of $U(c_j, h_j)$, it follows that

$$
U(c_j, h_j) = \begin{cases} u & \text{if } L_j > 0, \\ 0 & \text{if } L_j = 0. \end{cases}
$$

In addition, the first-order conditions with respect to consumption of traded and non-traded services, $[c_j]$ and $[h_j]$, are the same as in the problem without labor mobility replacing the planner's weights ω_j with the multipliers of the mobility constraint $\tilde{\omega}_j$. Combining $[L_j]$ with $[c_j]$ and $[h_j]$ gives the multiplier on the labor-mobility constraint. For populated locations:

$$\tilde{\omega}_j = \frac{W_j - W^L}{U_C(c_j, h_j)c_j + U_H(c_j, h_j)h_j}.$$

A.2. *Symmetry in Infrastructure Investments*

For the applications in Section 5, we impose symmetry in infrastructure levels as an additional restriction in the planner's problem, that is, $I_{jk} = I_{kj}$. This section provides the first-order condition for I_{jk} in that case. The first-order condition with respect to I_{jk} is

$$\begin{aligned}[I_{jk}] \qquad & -\chi \sum_{n=1}^{N} \left(P_j^D Q_{jk}^n \frac{\partial \tau_{jk}^n(Q_{jk}, I_{jk})}{\partial I_{jk}} + P_k^D Q_{kj}^n \frac{\partial \tau_{kj}^n(Q_{kj}, I_{jk})}{\partial I_{jk}} \right) \\ & - (1-\chi) \sum_{n=1}^{N} \left(P_j^n Q_{jk}^n \frac{\partial \tau_{jk}^n(Q_{jk}^n, I_{jk})}{\partial I_{jk}} + P_k^n Q_{kj}^n \frac{\partial \tau_{kj}^n(Q_{jk}^n, I_{jk})}{\partial I_{jk}} \right) \\ & + \left(\zeta_{jk}^{\underline{I}} - \zeta_{jk}^{\overline{I}}\right) = P_K\left(\delta_{jk}^I + \delta_{kj}^I\right). \end{aligned} \tag{A.3}$$

Assuming symmetry leaves all the remaining first-order conditions presented in Section A.1 unchanged. Under the log-linear specification (10) of the transport technology, the optimal infrastructure investment, conditional on $I_{jk} \in (\zeta_{jk}^{\underline{I}}, \zeta_{jk}^{\overline{I}})$, is

$$\begin{aligned} I_{jk}^* = \Bigg[& \frac{\gamma}{P_K\left(\delta_{jk}^I + \delta_{kj}^I\right)} \Bigg(\chi\left(\delta_{jk}^\tau P_j^D Q_{jk}^{1+\beta} + \delta_{kj}^\tau P_k^D Q_{kj}^{1+\beta}\right) \\ & + \sum_{n=1}^{N} (1-\chi)\left(\delta_{jk}^\tau P_j^n \left(Q_{jk}^n\right)^{1+\beta} + \delta_{kj}^\tau P_k^n \left(Q_{jk}^n\right)^{1+\beta}\right) \Bigg) \Bigg]^{\frac{1}{1+\gamma}}. \end{aligned} \tag{A.4}$$

As discussed in Section 5.2, to build $\delta_{jk}^{I,\mathrm{FOC}}$ we use (A.4) under the symmetry assumption $\delta_{jk}^I = \delta_{kj}^I$. Setting $I_{jk}^* = I_{jk}^{\mathrm{obs}}$, $\delta_{jk}^{I,\mathrm{FOC}}$ can be backed out as function of calibrated parameters, the observed network I_{jk}^{OBS}, and the equilibrium prices generated by the calibrated model. Note that, to generate these prices, we use the model calibrated given the network I_{jk}^{OBS}, as discussed in Section 5.2.

A.3. *Proofs of the Propositions*

PROPOSITION 1—Convexity of the Planner's Problem: (i) *Given the network* $\{I_{jk}\}$, *the joint optimal transport and allocation problem in the fixed* (*respectively mobile*) *labor case is a convex* (*respectively quasiconvex*) *optimization problem if* $Q\tau_{jk}(Q, I_{jk})$ *is convex in* Q *for all* j *and* $k \in \mathcal{N}(j)$; *and* (ii) *if, in addition,* $Q\tau_{jk}(Q, I)$ *is convex in both* Q *and* I *for all* j *and* $k \in \mathcal{N}(j)$, *then the full planner's problem including the network design problem from Definition* 1 (*resp. Definition* 2) *is a convex* (*respectively quasiconvex*) *optimization problem. In either the joint transport and allocation problem, or the full planner's problem, strong duality holds when labor is fixed.*

PROOF: Consider the planner's problem from Definition 1. We can write it as

$$\max_{\{C_j,\{D_j^n,\{Q_{jk}^n,I_{jk}\}_{k\in\mathcal{N}(j)}\}\}\forall j} f=\sum_j \omega_j L_j U\left(\frac{C_j}{L_j},\frac{H_j}{L_j}\right)$$

subject to: (i) availability of traded commodities,

$$g_j^1 = C_j+\chi\sum_{k\in\mathcal{N}(j)}\tau_{jk}(Q_{jk},I_{jk})Q_{jk}-D_j\big(D_j^1,\ldots,D_j^N\big)\le 0 \quad \text{for all } j;$$

(ii) the balanced-flows constraint,

$$g_{jn}^2 \equiv D_j^n+\sum_{k\in\mathcal{N}(j)}Q_{jk}^n\big[1+(1-\chi)\tau_{jk}\big(Q_{jk}^n,I_{jk}\big)\big]-F_j^n\big(L_j^n,\mathbf{V}_j^n,\mathbf{X}_j^n\big)-\sum_{i\in\mathcal{N}(j)}Q_{ij}^n\le 0 \quad \text{for all } j,n;$$

(iii) the network-building constraint,

$$\sum_j\sum_{k\in\mathcal{N}(j)}\delta_{jk}^I I_{jk}\le K;$$

and conditions (iv)–(v) in the text. Since constraints (iii)–(v) are linear, we need f to be concave and g_j^1 and g_{jn}^2 to be convex. Since U is jointly concave in both its arguments, f is concave. $D_j(\{D_j^n\})$ is concave, hence g_j^1 is convex. If $Q\tau_{jk}(Q,I)$ is convex, then g_{jn}^2 is the sum of linear and convex functions, hence it is convex. To show that this problem admits strong duality, a constraint qualification is required. Note first that constraints g_j^1 and g_{jn}^2 must hold with equality at an optimum and therefore can be substituted into the objective function. The remaining constraints (iii)–(v) are all linear and thus satisfy the Arrow–Hurwicz–Uzawa qualification constraint (Takayama (1985), Theorem 1.D.4). Hence, the global optimum must satisfy the KKT conditions and the duality gap is 0.[42]

Consider now the planner's problem with labor mobility from Definition 2. Because U is homothetic, we can express it as $U=G(U_0(c,h))$, where G is an increasing continuous function and U_0 is homogeneous of degree 1. Therefore, imposing the change of variables $\tilde{u}=G^{-1}(u)$, the planner's problem can be restated as maximizing $\tilde{u}$ subject to the convex constraints (i)–(v) and $L_j\tilde{u}\le U_0(C_j,H_j)$. To make the latter constraint convex, let us denote $U_j=L_j\tilde{u}$ and replace $\tilde{u}$ in the objective function by $\min_{j|L_j>0}\{\frac{U_j}{L_j}\}$,[43] so that the problem becomes

$$\max_{C_j,\{D_j^n,L_j^n,\mathbf{V}_j^n,\{Q_{jk}^n,I_{jk}^n\}_{k\in\mathcal{N}(j)}\},U_j,L_j}\ \min_{j|L_j>0}\left\{\frac{U_j}{L_j}\right\}$$

subject to the convex restrictions (i)–(v) above as well as

$$U_j\le U_0(C_j,H_j)\quad \text{for all } j.$$

[42]Despite having substituted constraints g_j^1 and g_{jn}^2 into the objective function, the multipliers for these constraints, P_j^D and P_j^n, can be recovered from the above KKT conditions such that $\omega_j U_C(c_j,h_j)=P_j^D$ and $P_j^D\partial C_j^T/\partial C_j^n=P_j^n$.

[43]Since the objective function is strictly increasing in $\tilde{u}$ and because $\tilde{u}$ only shows up in the constraints $L_j\tilde{u}\le U_0(C_j,H_j)$ for all j, it is necessarily the case that $\tilde{u}=\min_{j|L_j>0}U_j/L_j$.

The objective function is quasiconcave because U_j/L_j is quasiconcave and the minimum of quasiconcave functions is quasiconcave. In addition, all the restrictions are convex. The work of Arrow and Enthoven (1961) then implies that the Karush–Kuhn–Tucker conditions are sufficient if the gradient of the objective function is different from zero at the candidate for an optimum, and here the gradient never vanishes. *Q.E.D.*

PROPOSITION 2—Optimal Network in Log-Linear Case: *When the transport technology is given by* (10), *the full planner's problem is a convex* (*resp. quasiconvex*) *optimization problem if* $\beta \geq \gamma$. *The optimal infrastructure is given by* (12).

PROOF: First, note that if $\beta \geq \gamma$, then $Q\tau(Q, I) \propto Q^{1+\beta} I^{-\gamma}$ is convex in $Q \in \mathbb{R}_+$ and $I \in \mathbb{R}_+$. To see this, note that the determinant of the Hessian of $Q^{1+\beta} I^{-\gamma}$ is $(1+\beta)\gamma(\beta - \gamma) Q^{2\beta} I^{-2(\gamma+1)}$, which is positive for $Q \in \mathbb{R}_+$ and $I \in \mathbb{R}_+$ if $\beta \geq \gamma \geq 0$. Next, from the first-order condition for optimal infrastructure (9), if the solution to the planning problem implies $I_{jk} = \underline{I}_{jk}$ so that there is no investment, then

$$
\begin{aligned}
P_K &\geq -\frac{1}{\delta^I_{jk}} \sum_n P^n_j Q^n_{jk} \frac{\partial \tau^n_{jk}}{\partial I_{jk}} \Big|_{I_{jk} = \underline{I}_{jk}} \\
&\geq \gamma \frac{\delta^\tau_{jk}}{\delta^I_{jk}} \frac{\sum_n P^n_j \left(Q^n_{jk}\right)^{1+\beta}}{\underline{I}^{\gamma+1}_{jk}} \\
&\geq \frac{\gamma(1+\beta)^{-\frac{1+\beta}{\beta}}}{\delta^I_{jk} \left(\delta^\tau_{jk}\right)^{\frac{1}{\beta}}} \frac{\sum_{n: P^n_k > P^n_j} P^n_j \left(\frac{P^n_k}{P^n_j} - 1\right)^{\frac{1+\beta}{\beta}}}{\underline{I}^{\frac{\beta-\gamma}{\beta}}_{jk}},
\end{aligned}
$$

where the second line follows from (10) and the third line follows from (11). The last inequality is equivalent to $\underline{I}_{jk} \geq I^*_{jk}$ for I^*_{jk} defined in (14). Therefore, if $\underline{I}_{jk} < I^*_{jk}$, then $I_{jk} > \underline{I}_{jk}$ and $I_{jk} = I^*_{jk}$. Moreover, if there is any n such that $P^n_k \neq P^n_j$, then $I^*_{jk} > 0$. *Q.E.D.*

PROPOSITION 3—Tree Property: *Assume that* $\lim_{c \to 0^+} U_C(c, h) = \infty$. *In the absence of a pre-existing network* (*i.e.*, $\underline{I}_{jk} = 0$), *if the transport technology is given by* (10) *and satisfies* $\gamma > \beta$, *and if there is a unique commodity produced in a single location*, *then the optimal transport network is a tree*.

PROOF: See Section E of the Supplementary Material. *Q.E.D.*

DEFINITION 1: The decentralized equilibrium without labor mobility consists of quantities c_j, h_j, D_j, D^n_j, L^n_j, $\mathbf{V}^n_j$, $\mathbf{X}^n_j$, $\{Q^n_{jk}\}_{k \in \mathcal{N}(j)}$, goods prices $\{p^n_j\}_n$, p^D_j, p^H_j and factor prices w_j, $\{r^m_j\}_m$ in each location j such that:

(i)(a) consumers optimize:

$$
\{c_j, h_j\} = \arg\max_{\hat{c}_j, \hat{h}_j} U(\hat{c}_j, \hat{h}_j),
$$

$$
p^D_j \hat{c}_j + p^H_j \hat{h}_j = e_j \equiv w_j + t_j,
$$

where e_j are expenditures per worker in j and where p_j^D is the price index associated with $D_j(D_j^1, \dots, D_j^N)$ at prices $\{p_j^n\}_n$ and t_j is a transfer per worker located in j. The set of transfers satisfy

$$\sum_j t_j L_j = \Pi,$$

where Π adds up the aggregate returns to the portfolio of fixed factors and the government tax revenue,

$$\Pi = \sum_j p_j^H H_j + \sum_j \sum_m r_j^m V_j^m + \sum_j \sum_{k \in \mathcal{N}(j)} \sum_n t_{jk}^n p_k^n Q_{jk}^n;$$

(i)(b) firms optimize:

$$\{L_j^n, \mathbf{V}_j^n, \mathbf{X}_j^n\} = \underset{\hat{L}_j^n, \hat{\mathbf{V}}_j^n, \hat{\mathbf{X}}_j^n}{\operatorname{argmax}} \, p_j^n F_j^n(\hat{L}_j^n, \hat{\mathbf{V}}_j^n, \hat{\mathbf{X}}_j^n) - w_j \hat{L}_j^n - \sum_m r_j^m \hat{V}_j^{mn};$$

(i)(c) the transport companies optimize:

$$\pi_{od}^n = \max_{r=(j_0,\dots,j_\rho)\in\mathcal{R}_{od}} p_d^n - p_o^n - \sum_{k=0}^{\rho-1}\left(\chi P_{j_k}^D m^n \tau_{j_k j_{k+1}} + (1-\chi) P_{j_k}^n \tau_{j_k j_{k+1}}^n\right) - \sum_{k=0}^{\rho-1} p_{j_{k+1}} t_{j_k j_{k+1}}^n,$$

for all $(o, d) \in \mathcal{J}^2$, where $\mathcal{R}_{od} = \{(j_0, \dots, j_\rho) \in \mathcal{J}^{\rho+1}, \rho \in \mathbb{N} \mid j_0 = o, j_\rho = d, j_{k+1} \in \mathcal{N}(j_k)$ for all $0 \le k < \rho\}$ is the set of routes from o to d, and there is free entry to delivering products from every source to every destination: $\pi_{od}^n \le 0$ for all $(o, d) \in \mathcal{J}^2$, $=$ if good n is shipped from o to d.

(i)(d) producers of final commodities optimize:

$$\{D_j^n\} = \underset{\hat{D}_j^n}{\operatorname{argmax}} \, D_j(\{\hat{D}_j^n\}) - \sum_j p_j^n \hat{D}_j^n;$$

as well as the market-clearing and non-negativity constraints (i), (ii), (iv), and (v) from Definition 1.

If, in addition, labor is mobile, then the decentralized equilibrium also consists of utility u and employment $\{L_j\}$ such that

$$u = U_j(c_j, h_j)$$

whenever $L_j > 0$, and the labor market-clearing condition (vii) from Definition 2 holds.

PROPOSITION 4—First and Second Welfare Theorems: *If the tax on shipments of product n from j to k, denominated in the same unit as transport costs, is $t_{jk}^n = \chi m^n \varepsilon_{Q,jk}^\tau \tau_{jk}(Q_{jk}, I_{jk}) + (1-\chi)\varepsilon_{Q,jkn}^\tau \tau_{jk}^n(Q_{jk}^n, I_{jk})$, where $\varepsilon_{Q,jk}^n = \partial \log \tau_{jk}^n / \partial \log Q_{jk}^n$ $(\chi = 0)$ and $\varepsilon_{Q,jk} = \partial \log \tau_{jk} / \partial \log Q_{jk}$ $(\chi = 1)$, then*: (i) *if labor is immobile, the competitive allocation coincides with the planner's problem under specific planner's weights ω_j and, conversely, the planner's allocation can be implemented by a market allocation with specific transfers t_j; and* (ii) *if labor is mobile, the competitive allocation coincides with the planner's problem if and only if all workers own an equal share of fixed factors and tax revenue, that is, $t_j = \frac{\Pi}{L}$. In either case, the price of good n in location j, p_j^n, equals the multiplier on the balanced-flows constraint in the planner's allocation, P_j^n.*

PROOF: *Equivalence of the First-order Conditions.* Condition (i)(c) from the definition of the market allocation implies that the free entry condition of shippers holds for every pair of neighbors; that is, for every $j \in \mathcal{J}$ and $k \in \mathcal{N}(j)$,

$$p_k^n \leq p_j^n + (1-\chi) p_j^n \big(\tau_{jk}^n + t_{jk}^n\big) + \chi P_j^D \big(m^n \tau_{jk} + t_{jk}^n\big), \quad = \text{if } Q_{jk}^n > 0. \tag{A.5}$$

This condition is consistent with the first-order condition (8) from the planner's problem if and only if the tax scheme is defined as in the proposition. We must further show that, under this tax scheme, a route is the solution to (i)(c) if and only if it is used in the solution to the planner's problem, which we establish at the end of this proof.

Without labor mobility, the rest of the allocation corresponds to a standard neoclassical economy with convex technologies and preferences where the welfare theorems hold. Specifically, the first-order conditions from the consumer and firm optimization problems (i)(a) and (i)(b) yield

$$\begin{aligned}
&[\hat{c}_j] && \left(\frac{1}{\lambda_j}\right) U_C(c_j, h_j) = p_j^D, \\
&[\hat{h}_j] && \left(\frac{1}{\lambda_j}\right) U_H(c_j, h_j) = p_j^H, \\
&\big[\hat{D}_j^n\big] && p_j^D \frac{\partial D_j}{\partial D_j^n} = p_j^n, \\
&\big[\hat{L}_j^n\big] && \frac{\partial Y_j^n}{\partial L_j^n} P_j^n \leq w_j, \quad = \text{if } L_{jn} > 0, \\
&\big[\hat{V}_j^{mn}\big] && \frac{\partial Y_j^n}{\partial V_j^{mn}} P_j^n \leq r_j^m, \quad = \text{if } V_j^{mn} > 0.
\end{aligned}$$

Since the market-clearing constraints are the same in the market's and the planner's allocation, the planner's allocation coincides with the market if the planner's weights are such that the planner's FOC for c_j coincides with the market. This is the case if the weight ω_j from the planner's problem equals the inverse of the multiplier on the budget constraint from the consumer's optimization problem (i)(a) in the market allocation. To find that weight, using that U is homothetic, we can write $U = G(U_0(c, h))$, where U_0 is homogeneous of degree 1. Then, the planner's allocation coincides with the market's under weights

$$\omega_j = \frac{e_j}{G'\big(U_0(c_j, h_j)\big) U_0(c_j, h_j)},$$

where e_j is the expenditure per worker and c_j, h_j are the consumption per worker of the traded and non-traded good in the market allocation. If U is homogeneous of degree 1, then $\omega_j = P_j^U$, where P_j^U is the price index associated with $U(c_j, h_j)$ at the market equilibrium prices p_j^D, p_j^H. In the opposite direction, given arbitrary weights ω_j, the market allocation implements the planner's under the transfers $t_j = P_j^D c_j + P_j^H h_j - W_j$ constructed using the quantities $\{c_j, h_j\}$ from the planner's allocation and the multipliers $\{P_j^D, P_j^H\}$ and W_j corresponding to the constraints (i) and (iv) of the planner's problem, respectively.

For the case with labor mobility, note that, for populated locations, the planner's first-order condition with respect to L_j implies

$$P_j^D c_j + P_j^H h_j = W_j - W^L.$$

Therefore, the market allocation and the planner's solution coincide if and only if, in the market allocation, expenditure per worker in location j takes the form $e_j = w_j + Constant$ for all j. The only transfer scheme delivering the same transfer per capita is $t_j = \frac{\Pi}{L}$.

Equivalence of Least-Cost Routes. We want to establish that any route used in the planner's problem is a solution to (i)(c) in Definition 1 under the proposed tax scheme. Fix good n. We introduce the following notation: for a given route $r = (j_0, \dots, j_\rho) \in \mathcal{R}_{od}$, we denote by f_r^n the matrix of flows

$$f_r^n(j,k) = \begin{cases} 1 & \text{if } \exists l, 0 \le l \le \rho - 1 \text{ and } j = j_l, k = j_{l+1}, \\ 0 & \text{otherwise.} \end{cases}$$

Consider an optimal route from o to d, $r^* = (j_0^*, \dots, j_{\rho^*}^*) \in \mathcal{R}_{od}$, that is, such that $Q^n_{j_k^* j_{k+1}^*} > 0$ at the optimum of the planner's problem ($\zeta^Q_{j_k^* j_{k+1}^*} = 0$). We now consider redirecting a marginal amount of goods $\varepsilon > 0$ from r^* to some other route $r = (j_0, \dots, j_\rho) \in \mathcal{R}_{od}$. In other words, denoting $\mathbf{Q}^n = (Q^n_{jk})_{j \in \mathcal{J}, k \in \mathcal{N}(j)}$, we consider the perturbation $\mathbf{Q}^n + \varepsilon f_r^n - \varepsilon f_{r^*}^n$. The first-order effect of the deviation around the optimum must reduce the Lagrangian:

$$\mathcal{L}\big(\mathbf{Q}^n + \varepsilon f_r^n - \varepsilon f_{r^*}^n\big) - \mathcal{L}\big(\mathbf{Q}^n\big) \le 0.$$

To translate this condition into a minimum-cost route problem, we decompose the first-order impact on the Lagrangian,

$$\mathcal{L}\big(\mathbf{Q}^n + \varepsilon f_r^n - \varepsilon f_{r^*}^n\big) - \mathcal{L}\big(\mathbf{Q}^n\big) = \nabla_Q \mathcal{L}\big(\mathbf{Q}^n\big) \cdot \big(f_r^n - f_{r^*}^n\big)\varepsilon + o(\varepsilon),$$

and evaluate each term separately. The first deviation term,

$$\begin{aligned} \nabla_Q \mathcal{L}\big(\mathbf{Q}^n\big) \cdot f_r^n = \sum_{l=0}^{\rho-1} \Bigg[& -P^n_{j_l} + P^n_{j_{l+1}} + \zeta^Q_{j_l j_{l+1} n} - \chi P^D_{j_l} \left(m^n \tau^n_{j_l j_{l+1}} + Q_{j_l j_{l+1}} m^n \frac{\partial \tau_{j_l j_{l+1}}}{\partial Q_{j_l j_{l+1}}} \right) \\ & - (1-\chi) P^n_{j_l} \left(\tau_{j_l j_{l+1}} + Q^n_{j_l j_{l+1}} \frac{\partial \tau^n_{j_l j_{l+1}}}{\partial Q^n_{j_l j_{l+1}}} \right) \Bigg], \end{aligned}$$

simplifies to

$$\begin{aligned} \nabla_Q \mathcal{L}\big(\mathbf{Q}^n\big) \cdot f_r^n = \Bigg[& P^n_d - P^n_o + \sum_{l=0}^{\rho-1} \zeta^Q_{j_l j_{l+1} n} - \sum_{l=0}^{\rho-1} \big(\chi P^D_{j_l} m^n \tau_{j_l j_{l+1}} + (1-\chi) P^n_{j_l} \tau^n_{j_l j_{l+1}} \big) \\ & - \sum_{l=0}^{\rho-1} \left(\chi P^D_{j_l} Q_{j_l j_{l+1}} m^n \frac{\partial \tau_{j_l j_{l+1}}}{\partial Q_{j_l j_{l+1}}} + (1-\chi) P^n_{j_l} Q^n_{j_l j_{l+1}} \frac{\partial \tau^n_{j_l j_{l+1}}}{\partial Q^n_{j_l j_{l+1}}} \right) \Bigg]. \end{aligned}$$

Using the definition of the optimal tax, we have that

$$t^n_{jk} = \chi Q_{jk} m^n \frac{\partial \tau_{jk}}{\partial Q_{jk}} + (1-\chi) Q^n_{jk} \frac{\partial \tau^n_{jk}}{\partial Q^n_{jk}}.$$

Substituting into the previous deviation term, we obtain

$$\nabla_Q \mathcal{L}(\mathbf{Q}^n)\cdot f_r^n = \Bigg[P_d^n - P_o^n - \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D m^n \tau_{j_l j_{l+1}} + (1-\chi)P_{j_l}^n \tau_{j_l j_{l+1}}^n\big) - \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D + (1-\chi)P_{j_l}^n\big)t_{j_l j_{l+1}}^n + \sum_{l=0}^{\rho-1}\zeta_{j_l j_{l+1} n}^Q\Bigg].$$

By assumption, the total deviation $\mathbf{Q}^n + \varepsilon f_r^n - \varepsilon f_{r^*}^n$ has a negative impact on the Lagrangian for the feasible deviation $\varepsilon > 0$, so that $\nabla_Q \mathcal{L}(\mathbf{Q}^n)\cdot (f_r^n - f_{r^*}^n) \le 0$. Using that $\zeta_{j_l^* j_{l+1}^* n}^Q = 0$, we get

$$\begin{aligned}
&P_o^n + \sum_{l=0}^{\rho^*-1}\big(\chi P_{j_l^*}^D m^n \tau_{j_l^* j_{l+1}^*} + (1-\chi)P_{j_l^*}^n \tau_{j_l^* j_{l+1}^*}^n\big) + \sum_{l=0}^{\rho^*-1}\big(\chi P_{j_l^*}^D + (1-\chi)P_{j_l^*}^n\big)t_{j_l^* j_{l+1}^*}^n \\
&\quad\le P_o^n + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D m^n \tau_{j_l j_{l+1}} + (1-\chi)P_{j_l}^n \tau_{j_l j_{l+1}}^n\big) + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D + (1-\chi)P_{j_l}^n\big)t_{j_l j_{l+1}}^n - \sum_{l=0}^{\rho-1}\zeta_{j_l j_{l+1} n}^Q \\
&\quad\le P_o^n + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D m^n \tau_{j_l j_{l+1}} + (1-\chi)P_{j_l}^n \tau_{j_l j_{l+1}}^n\big) + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D + (1-\chi)P_{j_l}^n\big)t_{j_l j_{l+1}}^n.
\end{aligned}$$

Hence, the optimal route r^* is solution to the least-cost route problem

$$\min_{r=(j_0,\dots,j_\rho)\in\mathcal{R}_{od}} P_o^n + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D m^n \tau_{j_l j_{l+1}} + (1-\chi)P_{j_l}^n \tau_{j_l j_{l+1}}^n\big) + \sum_{l=0}^{\rho-1}\big(\chi P_{j_l}^D + (1-\chi)P_{j_l}^n\big)t_{j_l j_{l+1}}^n,$$

where we recognize condition (i)(c) of Definition 1.

Finally, the minimum-cost route problem in the case of own-good congestion ($\chi = 0$) is equivalent to

$$\min_{r=(j_0,\dots,j_\rho)\in\mathcal{R}_{od}} P_o^n \prod_{l=0}^{\rho-1}\big(1+\tau_{j_l j_{l+1}}^n\big) + \sum_{l=0}^{\rho-1} P_{j_{l+1}}^n t_{j_l j_{l+1}}^n \prod_{k=l}^{\rho-1}\big(1+\tau_{j_k j_{k+1}}^n\big),$$

since $P_{j_{l+1}}^n = (1+\tau_{j_l j_{l+1}}^n)P_{j_l}^n + P_{j_{l+1}}^n t_{j_l j_{l+1}}^n$ along any used path. *Q.E.D.*

PROPOSITION 5: *If the global convexity condition of Proposition 1 is satisfied and the toll is consistent with the optimal Pigouvian tax ($\theta_{jk}^n = \chi P_j^D m^n \varepsilon_{Q,jkn}^\tau \tau_{jk} + (1-\chi)P_j^n \varepsilon_{Q,jkn}^\tau \tau_{jk}^n$), then the decentralized infrastructure choice implements the optimal network investment.*

PROOF: To ease notation, we focus on the case with own-good congestion ($\chi = 0$), but the case with cross-good congestion ($\chi = 1$) can be derived following similar steps. Consider the problem of a regulated monopoly on link jk allowed to charge a per-unit toll θ_{jk}^n on good n. The monopolist can purchase asphalt at price p_K. We assume that the government forbids entry on unused links or sets a price too low for entry, allowing us to focus on links used at the social optimum ($Q_{jk}^n > 0$ for some n, $I_{jk} > 0$). Free-entry of shipping companies on link jk yields

$$p_k^n \le p_j^n\big(1+\tau_{jk}^n(Q_{jk}^n, I_{jk}^n)\big) + \theta_{jk}^n, \quad = \text{if } Q_{jk}^n > 0.$$

Under the assumption that τ_{jk}^n is strictly increasing in Q_{jk}^n, the demand for transport at any level of infrastructure I_{jk} given prices is

$$Q_{jk}^n(I_{jk};p) = \mathrm{inv}_Q \tau_{jk}\left(\frac{p_k^n - p_j^n - \theta_{jk}^n}{p_j^n}, I_{jk}\right) \quad \text{for } p_k^n \geq p_j^n + \theta_{jk}^n, \tag{A.6}$$

where $\mathrm{inv}_Q\tau(Q,I)$ denotes the inverse of function τ with respect to Q. The monopoly solves the profit maximization problem

$$\max_{I_{jk}} \sum_n \theta_{jk}^n Q_{jk}^n(I_{jk}) - p_K \delta_{jk}^I I_{jk}$$

subject to (A.6). It can be shown that convexity of $\tau(Q,I)$ in Q and I is sufficient for the problem to be concave. The first-order condition over infrastructure is

$$\sum_n \frac{\partial}{\partial I_{jk}}\left[\mathrm{inv}_Q \tau\left(\frac{p_k^n - p_j^n - \theta_{jk}^n}{p_j^n}, I_{jk}\right)\right]\theta_{jk}^n = p_K \delta_{jk}^I.$$

From the implicit function theorem, we have that $\frac{\partial}{\partial I_{jk}}[\mathrm{inv}_Q\tau_{jk}(\frac{p_k^n - p_j^n - \theta_{jk}^n}{p_j^n}, I_{jk})] = -\frac{\partial \tau_{jk}}{\partial I_{jk}} / \frac{\partial \tau_{jk}}{\partial Q_{jk}^n}$. In turn, implementing the efficient flows Q_{jk}^n requires the toll θ_{jk}^n corresponding to the Pigouvian congestion tax from Proposition 4, that is,

$$\theta_{jk}^n = p_j^n t_{jk}^n = p_j^n \varepsilon_{Q,jkn}^{\tau} \tau_{jk}\big(Q_{jk}^n, I_{jk}\big) = p_j^n Q_{jk}^n \frac{\partial \tau_{jk}^n}{\partial Q_{jk}^n}.$$

Combining the last two expressions, the monopolist's first-order condition thus becomes the same as (13), except for the price of asphalt p_K. Imposing market clearing in K, the set of conditions in the decentralized allocation coincide with the planner, implying that p_K equals the Lagrange multiplier P_K and other prices equal their corresponding multipliers, $p_j^n = P_j^n$ and $p_j^D = P_j^D$. Moreover, under the global convexity condition from Proposition 1, first-order conditions of the builders are sufficient, which demonstrates the result. *Q.E.D.*

APPENDIX B: APPENDIX TO SECTION 4 (CALIBRATION AND COUNTERFACTUALS)

Construction of $\mathcal{P}(j,k)$. The definition of the weights $\omega_{jk}(s)$ assigned to the construction of I_{jk}^{obs} involves the cheapest path $\mathcal{P}(j,k)$ for all $j \in \mathcal{J}$ and $k \in \mathcal{N}(j)$ in every country. To find $\mathcal{P}(j,k)$, we first convert the shapefile with all the road segments from EuroGeographics into a weighted graph, where each edge corresponds to a segment s on the road network. We define $\mathcal{P}(j,k)$ as the shortest path between j and k under the segment-specific weights $\text{length}_s * \text{lanes}_s^{-\chi_{\text{lane}}} * \chi_{\text{use}}^{1-\text{nat}_s} * \chi_{\text{paved}}^{1-\text{paved}_s} * \chi_{\text{median}}^{1-\text{median}_s}$, where length_s is the length of the segment, lanes_s is the number of lanes, nat_s equals 1 if the segment belongs to a national road, paved_s equals 1 if the segment is paved, and median_s equals 1 if the segment has a median. When information on number of lanes is missing, we assign a number of lanes equal to the minimum of 1 observed in the data. When the information is missing, we define the road use as non-national. We parameterize χ_{lane}, χ_{use}, χ_{paved}, and χ_{median} based on the extent by which adding a lane, using a national road, using paved

TABLE A.I

SUMMARY STATISTICS OF ACTUAL AND DISCRETIZED ROAD NETWORK BY COUNTRY[a]

		Actual Road Network			Discretization		
		Length (Km.)	Number of Segments	Average Lanes per Km.	Number of Cells	Length (Km.)	Average Infrastructure Index
Country	Code	(1)	(2)	(3)	(4)	(5)	(6)
Austria	AT	17,229	6156	2.36	46	9777	1.53
Belgium	BE	19,683	10,472	2.49	21	3535	2.12
Switzerland	CHLI	14,451	11,043	2.26	24	5307	1.71
Cyprus	CY	2818	947	2.29	20	2077	0.69
Czech Republic	CZ	28,651	10,183	2.20	48	10,593	1.01
Germany	DE	115,173	66,423	2.42	54	27,227	2.61
Denmark	DK	15,358	6297	2.20	30	5418	0.99
Spain	ES	101,990	18,048	2.39	61	30,686	1.80
Finland	FI	70,394	9221	2.04	70	34,057	0.28
France	FR	128,787	38,668	2.05	74	38,754	2.12
Georgia	GE	28,682	9009	1.95	39	8420	0.34
Hungary	HU	32,728	9235	2.10	50	11,572	0.95
Ireland	IE	24,952	4144	2.10	47	10,671	0.63
Italy	IT	77,586	44,155	2.32	36	14,742	2.18
Lithuania	LT	10,682	1586	2.39	43	9408	0.59
Luxembourg	LU	1778	863	2.31	8	574	0.73
Latvia	LV	11,495	2102	2.03	47	9857	0.33
Moldova	MD	8527	1457	2.21	20	4359	0.38
Macedonia	MK	5578	908	2.15	13	2205	0.27
Northern Ireland	ND	7087	1888	2.18	12	1780	0.67
Netherlands	NL	14,331	8383	2.68	20	3884	2.55
Portugal	PT	15,034	4933	2.10	43	11,116	1.49
Slovenia	SI	7801	2441	2.19	12	1525	1.25
Slovakia	SK	11,406	2601	2.18	30	5880	1.04

[a]Columns (1) to (3) report statistics from EuroGlobalMap, and Columns (4) to (6) report statistics from the discretization of road networks described in Section 5.1.

road, or using a road with a median reduces road user costs. Table 4 of Combes and Lafourcade (2005) reports that, in France, the reference cost per km. in a national road with at least 4 lanes is 25% higher than in other national roads. In our road network data for France, the average number of lanes in national roads with at least 4 lanes is 4.43, and the average number of lanes in national roads with less than 4 lanes is 1.9. From this, we infer that adding 2.5 lanes on top of 2 lanes, a 125% increase in the number of lanes, reduces costs by 25%, implying an elasticity of user costs with respect to number of lanes of $\chi_{\text{lane}} = \frac{25\%}{125\%} = 0.2$ in absolute value. In addition, Table 4 in Combes and Lafourcade (2005) reports that the total reference cost is about 7% higher on "secondary roads" relative to "other national roads," from which we infer $\chi_{\text{use}} = 1.07$. According to Figueroa, Fotsch, Hubbard, and Haddock (2013), road user costs are 35% higher on gravel relative to paved roads, implying $\chi_{\text{paved}} = 1.35$, and according to Tay and Churchill (2007), adding a median increases speed by 5%, implying $\chi_{\text{median}} = 1.05$.

Calibration of β and γ. To parameterize β and γ we assume that: (i) trade costs are a linear function of shipping time, $\tau_{jk} = a^{\tau}_{jk}\frac{dist_{jk}}{S_{jk}}$; (ii) shipping speed is a log-linear function

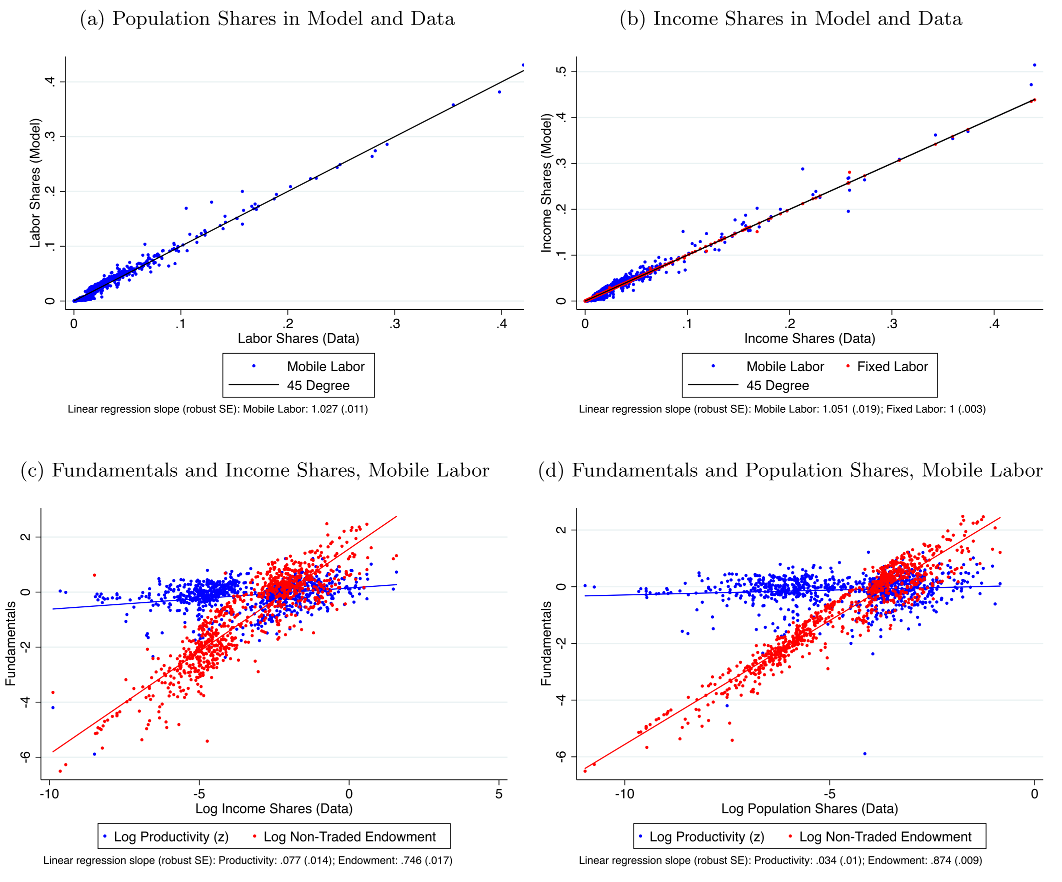


FIGURE A.1.—Calibration of population and income shares, all locations and countries. *Notes:* The figures pool the 868 cells across the 24 countries in the convex case of the parameters for the calibration with 10 differentiated goods. Similar relationships hold for the non-convex case. In the panels (c) to (e), log-productivity and log-endowment of the non-traded good per capita are demeaned within each country.

of the number of vehicles and road lane kilometers,

$$S_{jk} = a^S_{jk} I^{\gamma}_{jk} V^{-\beta}_{jk}; \tag{A.7}$$

and (iii) the total number of vehicles is a linear function of the quantity of goods that is shipped, $V_{jk} = a^D_{jk} Q_{jk}$. These assumptions are consistent with the functional form (10) for $\tau_{jk}(Q, I)$. To recover the parameters γ and β, one would ideally like to estimate the relationship between speed, roads, and vehicles in (A.7) across links. This relationship was estimated by Couture, Duranton, and Turner (2018) across cities.[44] Equation (2) in

[44] Couture, Duranton, and Turner (2018) estimated the parameters from data on movements of vehicles, regardless of the trip purpose (i.e., it may include transport of passengers or goods). We assume that the speed of vehicles transporting goods responds to traffic and to highway lanes similarly to vehicles transporting passengers, and that the total number of vehicles V_{jk} is a linear function of the number vehicles transporting goods. Data from the 2015 E-Road traffic census in Europe shows a correlation of 0.81 between the average daily traffic of all vehicles and of vehicles used to transport goods across measuring posts in European highways. On average across measuring stations, 16% of all vehicles are used for transport.

TABLE A.II

WELFARE GAINS FROM OPTIMAL REALLOCATION OR EXPANSION OF CURRENT NETWORKS, FIXED LABOR

	Non-Convex			Convex		
ICC	Expansion (GEO)	Expansion (FOC)	Misallocation	Expansion (GEO)	Expansion (FOC)	Misallocation
Austria	3.5%	1.0%	3.1%	2.3%	0.3%	2.3%
Belgium	1.0%	0.4%	0.8%	0.6%	0.1%	0.5%
Cyprus	1.5%	0.6%	1.3%	1.0%	0.2%	1.0%
Czech Republic	1.6%	0.7%	1.3%	1.0%	0.2%	1.0%
Denmark	8.3%	1.0%	7.8%	4.3%	0.4%	4.3%
Finland	5.3%	1.2%	4.7%	2.6%	0.4%	2.3%
France	4.0%	1.0%	3.4%	2.7%	0.4%	2.5%
Georgia	3.4%	1.3%	3.1%	2.2%	0.3%	2.2%
Germany	2.7%	1.2%	2.3%	1.8%	0.3%	1.7%
Hungary	3.1%	0.9%	2.7%	1.7%	0.2%	1.7%
Ireland	2.8%	0.9%	2.4%	1.8%	0.3%	1.7%
Italy	2.5%	1.2%	1.9%	1.6%	0.4%	1.3%
Latvia	4.0%	0.9%	3.6%	2.5%	0.3%	2.7%
Lithuania	3.4%	1.0%	3.0%	2.4%	0.3%	2.3%
Luxembourg	0.2%	0.2%	0.1%	0.2%	0.1%	0.1%
Macedonia	1.0%	0.6%	0.9%	0.5%	0.1%	0.5%
Moldova	1.8%	0.6%	1.5%	1.0%	0.2%	0.9%
Netherlands	1.6%	0.5%	1.4%	1.2%	0.2%	1.1%
Northern Ireland	1.2%	0.6%	1.0%	0.8%	0.1%	0.8%
Portugal	2.6%	0.9%	2.1%	1.4%	0.3%	1.2%
Slovakia	2.7%	1.3%	2.3%	1.9%	0.3%	1.9%
Slovenia	1.8%	0.6%	1.6%	1.4%	0.2%	1.4%
Spain	5.6%	2.3%	4.7%	3.7%	0.5%	3.5%
Switzerland	1.8%	0.7%	1.5%	1.0%	0.2%	0.9%
Average	2.8%	0.9%	2.4%	1.7%	0.3%	1.7%

their paper assumes a log-linear relationship between speed, roads, and vehicle travel time, defined as vehicle-kilometers (i.e., vehicles times distance) over speed. To measure roads, they used the log of lane-kilometers of interstate highways in the United States, which corresponds to our measure of I_{jk}. Assuming that their estimates would hold at the level of a connection between populated areas in our data, in our notation equation (2) of their paper can be written[45]

$$\ln S_{jk} = \alpha^{\mathrm{CDT}} \ln I_{jk} - \theta^{\mathrm{CDT}} \ln\left(\frac{V_{jk} * dist_{jk}}{S_{jk}}\right) + \varepsilon_{jk}, \tag{A.8}$$

where we use α^{CDT} and θ^{CDT} to refer to α and β in their paper. These parameters translate to ours as follows: $\alpha^{\mathrm{CDT}} \equiv \frac{\gamma}{1+\beta}$ and $\theta^{\mathrm{CDT}} \equiv \frac{\beta}{1+\beta}$. When $\alpha^{\mathrm{CDT}} < \theta^{\mathrm{CDT}}$, there are decreasing returns to scale in the provision of vehicle kilometers traveled. Couture, Duranton, and Turner (2018) found decreasing returns to scale ($\alpha^{\mathrm{CDT}} < \theta^{\mathrm{CDT}} \rightarrow \gamma < \beta$) across all their specification (Tables 5 and 6 of their paper) using a variety of OLS and IV approaches.

[45]In their notation, vehicle travel time (number of vehicles times time of travel) is $VTT_i = \frac{VKT_i}{S_i}$, where VKT is vehicle kilometers (number of vehicles times distance) and S_i is speed (distance over time of travel). In our notation, therefore, $VKT_i = V_{jk} \times dist_{jk}$.

TABLE A.III

WELFARE GAINS FROM OPTIMAL REALLOCATION OR EXPANSION OF CURRENT NETWORKS, MOBILE LABOR

	Non-Convex			Convex		
ICC	Expansion (GEO)	Expansion (FOC)	Misallocation	Expansion (GEO)	Expansion (FOC)	Misallocation
Austria	3.4%	1.9%	2.8%	2.4%	0.4%	2.5%
Belgium	1.0%	0.5%	0.8%	0.6%	0.1%	0.5%
Czech Republic	1.6%	1.3%	1.3%	1.1%	0.2%	1.1%
Denmark	9.6%	1.8%	8.9%	4.5%	0.4%	4.5%
France	4.3%	2.2%	3.4%	3.1%	0.5%	2.9%
Germany	2.4%	2.1%	2.3%	2.0%	0.3%	1.8%
Hungary	3.1%	1.1%	2.7%	1.8%	0.3%	1.8%
Ireland	2.8%	1.5%	2.3%	1.8%	0.3%	1.7%
Finland	6.4%	1.6%	5.7%	3.1%	0.4%	2.8%
Italy	2.6%	2.4%	2.0%	1.6%	0.4%	1.3%
Latvia	4.0%	1.1%	3.6%	2.5%	0.3%	2.6%
Lithuania	3.8%	1.1%	3.3%	2.6%	0.3%	2.6%
Moldova	1.8%	0.6%	1.6%	1.1%	0.2%	1.0%
Luxembourg	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%
Macedonia	0.9%	0.6%	0.8%	0.5%	0.1%	0.4%
Northern Ireland	1.1%	0.6%	0.9%	0.8%	0.1%	0.8%
Netherlands	1.6%	0.5%	1.3%	1.4%	0.2%	1.3%
Slovakia	3.1%	1.9%	2.7%	2.2%	0.3%	2.2%
Portugal	2.4%	1.3%	2.1%	1.3%	0.3%	1.1%
Slovenia	1.7%	0.7%	1.5%	1.3%	0.1%	1.4%
Switzerland	1.7%	0.9%	1.4%	1.1%	0.2%	1.0%
Spain	5.2%	3.2%	4.6%	4.0%	0.6%	3.8%
Georgia	3.6%	1.6%	3.2%	2.3%	0.3%	2.4%
Cyprus	1.5%	0.5%	1.3%	1.0%	0.2%	0.9%
Average	2.9%	1.3%	2.5%	1.8%	0.3%	1.8%

TABLE A.IV

OPTIMAL INFRASTRUCTURE INVESTMENT, POPULATION GROWTH, AND LOCAL CHARACTERISTICS FOR DIFFERENT NUMBER OF SECTORS[a]

	(1) Investment	(2) Investment	(3) Investment	(4) Pop. Growth	(5) Pop. Growth	(6) Pop. Growth
Population	0.114	0.125	0.125	−0.000	−0.000	−0.000
Tradable Income per Capita	0.146	0.071	0.083	0.013	0.008	0.011
Infrastructure	−0.214	−0.235	−0.236	−0.001	−0.001	−0.001
Consumption per Capita				−0.064	−0.060	−0.065
Infrastructure Growth				0.000	0.002	0.002
Differentiated Producer				0.009	0.010	0.010
Observations	868	868	868	868	868	868
Adjusted R-squared	0.22	0.24	0.25	0.56	0.54	0.57

[a]Each column corresponds to a different regression pooling all locations in the optimal expansion counterfactual across the 24 countries in the convex case with mobile labor and $\delta = \delta^{I,\text{GEO}}$. The dependent variable is investment in columns (1)–(3) and population growth in columns (4)–(6). All regressions include country fixed effects. Standard errors are clustered at the country level. Dependent variables: population growth is defined as $\Delta \ln L_j$, where Δx denotes the difference between variable x in the counterfactual and in the calibrated allocation. Investment growth is defined as the difference over the average, $\Delta I_j/(\frac{1}{2}(I_j + \Delta I_j))$, where total infrastructure at the node level defined as $I_j = \sum_{k \in \mathcal{N}(j)} I_{jk}$. Independent variables correspond to the log of the level of each variable in the calibrated model. Population and income per capita are the two outcomes matched by the calibration. Consumption per capita corresponds to traded goods, c_j in the calibrated model. Differentiated producer is a dummy for whether the location is a producer of differentiated goods in the calibration.

TABLE A.V

AVERAGE WELFARE GAINS FOR DIFFERENT NUMBER AND ALLOCATION OF DIFFERENTIATED GOODS[a]

	Allocation of Goods							
	Benchmark				Within NUTS			
	Number of Sectors				Number of Sectors			
	$N = 10$		$N = 15$		$N = 10$		$N = 15$	
	Labor		Labor		Labor		Labor	
	Fixed	Mobile	Fixed	Mobile	Fixed	Mobile	Fixed	Mobile
Optimal Reallocation								
$\delta = \delta^{I,\mathrm{GEO}}$	1.7%	1.8%	1.7%	1.8%	1.8%	1.9%	2.2%	2.2%
Optimal Expansion								
$\delta = \delta^{I,\mathrm{GEO}}$	1.7%	1.8%	1.8%	1.8%	1.9%	2.0%	2.3%	2.3%
$\delta = \delta^{I,\mathrm{FOC}}$	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.4%	0.4%

[a]Each element of the table shows the average welfare gain in the corresponding counterfactual across the 24 countries for the convex case.

TABLE A.VI

AVERAGE WELFARE GAINS WITH AND WITHOUT CONGESTION ACROSS GOODS[a]

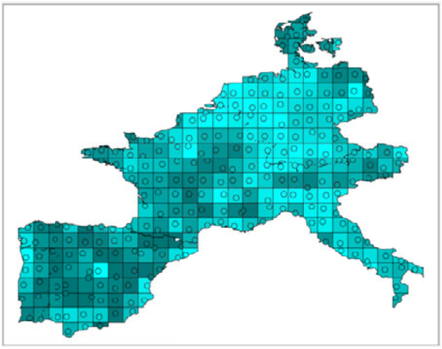
	Congestion			
	Across Goods		Own Good (Iceberg)	
	Labor		Labor	
	Fixed	Mobile	Fixed	Mobile
Optimal Reallocation				
$\delta = \delta^{I,\mathrm{GEO}}$	1.7%	1.8%	1.5%	1.4%
Optimal Expansion				
$\delta = \delta^{I,\mathrm{GEO}}$	1.7%	1.8%	1.6%	1.5%
$\delta = \delta^{I,\mathrm{FOC}}$	0.3%	0.3%	0.2%	0.2%

[a]Each element of the table shows the average welfare gain in the corresponding counterfactual across the 24 countries for the convex case with $N = 10$ goods.

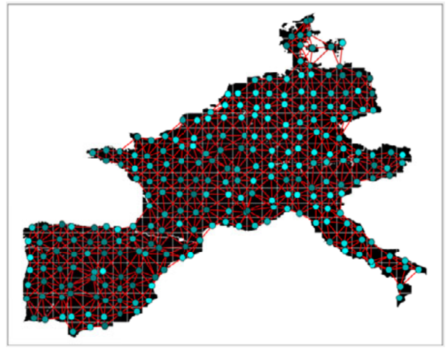
Their preferred estimate (column (6) of Table 5) yields $\alpha^{\mathrm{CDT}} = 0.09$ and $\theta^{\mathrm{CDT}} = 0.13$, implying $\gamma = 0.10$ and $\beta = 0.13$.

Construction of Ruggedness Measure. We use elevation data from the ETOPO1 Global Relief Model. The ETOPO1 data set corresponds to a 1 arc-minute degree grid. We construct ruggedness for each cell as the average ruggedness across the 900 arc-minute cells from the ETOPO1 data set contained in each 0.5 arc-degree cell in our discretized maps. We use the standard ruggedness index by Riley, DeGloria, and Elliot (1999). Letting $\mathcal{J}^{\mathrm{etopo}}(j)$ be the set of cells in ETOPO1 contained in each cell $j \in \mathcal{J}$ of our discretization and $\mathcal{N}^{\mathrm{etopo}}(i)$ be the 8 neighboring cells to each cell in ETOPO1, this index is defined as: $rugged_j = (\sum_{i \in \mathcal{J}^{\mathrm{etopo}}(j)} \sum_{k \in \mathcal{N}^{\mathrm{etopo}}(i)} (elev_i - elev_k)^2)^{1/2}$; that is, it is the standard deviation of the difference in elevation across neighboring cells. Then, we define $rugged_{jk}$ in (21) as $rugged_{jk} = \frac{1}{2}(rugged_j + rugged_k)$.

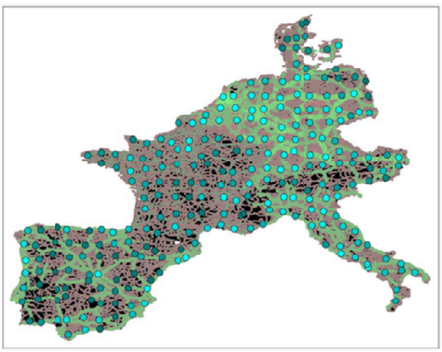
(a) Population on the Discretized Map



(b) Nodes and Edges in the Baseline Graph



(c) Nodes in the Actual Road Network



(d) Actual Road Network on the Baseline Graph

FIGURE A.2.—Discretization of the European road network. *Notes:* Panel (a) shows total population from GPW aggregated into 1 arc-degree (approximately 100 km) cells. Panel (b) shows the nodes $\mathcal{J}$ corresponding to the population centroids of each cell in panel (a), reallocated to their closest point on the actual road network, and the edges $\mathcal{E}$ corresponding to all the vertical and diagonal links between cells. Panel (c) shows the centroids and the actual road network. Light gray segments correspond to national roads, dark gray segments are all other roads, and the width of each segment is proportional to the number of lanes. Panel (d) shows the same centroids and the edges as the baseline graph in panel (b), where each edge is weighted proportionally to the average number lanes on the cheapest path between each pair of nodes on the road network. The brightness varies according to the fraction of the shortest path traveled on a national road.

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