State learning tutorial

Now that we are more familiar with Strawberry Fields, let's have a go using the TensorFlow backend to do some quantum circuit optimization. This functionality is provided via the Tensorflow simulator backend. By leveraging Tensorflow, we have access to a number of additional funtionalities, including GPU integration, automatic gradient computation, built-in optimization algorithms, and other machine learning tools.

Background information

Using the TensorFlow backend

Using the TensorFlow backend is similar to using the Fock and Gaussian backend in the previous tutorials, however there are a couple of slight subtleties that need to be accounted for.

We import Strawberry Fields as before, and create the quantum engine:

```
In [1]: import strawberryfields as sf
        from strawberryfields.ops import *

        eng, q = sf.Engine(2)
```

When a circuit contains only numerical parameters, the Tensorflow simulator backend works the same as the other backends. However, with the Tensorflow backend, we have the additional option to use Tensorflow objects (e.g., `tf.Variable`, `tf.constant`, `tf.placeholder`, or `tf.Tensor`) for the parameters of quantum states, gates, and measurements.

In [2]:
```
import tensorflow as tf

alpha = tf.Variable(0.5)
theta_bs = tf.constant(0.0)
phi_bs = tf.sigmoid(0.0) # this will be a tf.Tensor object
phi = tf.placeholder(tf.float32)

with eng:
    # States
    Coherent(alpha)             | q[0]

    # Gates
    BSgate(theta_bs, phi_bs)    | (q[0], q[1])

    # Measurements
    MeasureHomodyne(phi)        | q[0]
```

To run a Strawberry Fields simulation with the Tensorflow backend, we need to specify `'tf'` as the backend argument when calling `eng.run()`. However, directly evaluating a circuit which contains Tensorflow objects using `eng.run()` will produce errors. The reason for this is that `eng.run()` tries, by default, to numerically evaluate any measurement result. But Tensorflow requires several extra ingredients to do this:

- Numerical computations must be carried out using a `tf.Session`.
- All `tf.Variable` objects must be initialized within this `tf.Session` (the initial values are supplied when creating the variables).
- Numerical values must be provided for any `tf.placeholder` objects using a feed dictionary (`feed_dict`).

The solution is to perform the simulation *symbolically*, using the keyword argument `eval=False`. The final state and the register q will both instead contain unevaluted Tensors. These Tensors can be evaluated numerically by running the `tf.Session` and supplying the desired values for any placeholders:

In [3]:
```
# eval=False tells it to perform the simulation not numerically, so the tensor (
tensorflow) returned is not evaluated numerically, it's symbolicall
state = eng.run('tf', cutoff_dim=7, eval=False)
```

Example

In [4]:
```
eng, q = sf.Engine(2)

with eng:
    Dgate(alpha)         | q[0]
    MeasureHomodyne(phi) | q[0]

state = eng.run('tf', cutoff_dim=7, eval=False)
```

Measurement values and state properties/methods will now return unevaluated, symbolic tensors:

In [5]:
```
state_density_matrix = state.dm()
```

In [6]:
```
print(state.dm())
```

```
Tensor("einsum_1/transpose_2:0", shape=(7, 7, 7, 7), dtype=complex64)
```

In [7]:
```
homodyne_meas = q[0].val
```

```
In [8]: print(q[0].val)
```

```
Tensor("Measure_homodyne_1/Meas_result:0", shape=(), dtype=float64)
```

We can evaluate these tensors by calling Tensorflow's `sess.run` method:

```
In [9]: sess = tf.Session()
        sess.run(tf.global_variables_initializer())
        dm_x, meas_x = sess.run([state_density_matrix, homodyne_meas], feed_dict={phi: 0
        .0})
```

```
In [10]: print(meas_x)
```

```
2.389923899238992
```

Note
When being used as a numerical simulator (similar to the other backends), the Tensorflow backend creates temporary sessions in order to evaluate measurement results numerically.

Processing data

The parameters for Blackbird states, gates, and measurements may be more complex than just raw data or machine learning weights. These can themselves be the outputs from some learnable function like a neural network:

```
In [11]: input_ = tf.placeholder(tf.float32, shape=(2,1))
         weights = tf.Variable([[0.1,0.1]])
         bias = tf.Variable(0.0)

         NN = tf.sigmoid(tf.matmul(weights, input_) + bias)
         NNDgate = Dgate(NN)
```

We can also use the `sf.convert()` decorator to allow arbitrary processing of measurement results with the Tensorflow backend:

```
In [12]: @sf.convert
         def sigmoid(x):
             return tf.sigmoid(x)

         with eng:
             MeasureX              | q[0]
             Dgate(sigmoid(q[0])) | q[1]
```

Working with batches

It is common in machine learning to process data in batches. Strawberry Fields supports both unbatched and batched data when using the Tensorflow backend. Unbatched operation is the default behaviour (shown above). To enable batched operation, you should provide an extra batch_size argument when calling `eng.run()`, e.g.,

```
In [13]: # run simulation in batched-processing mode
         batch_size = 3
         eng, q = sf.Engine(2)

         with eng:
             Dgate(tf.Variable([0.1] * batch_size)) | q[0]

         state = eng.run('tf', cutoff_dim=7, eval=False, batch_size=batch_size)
```

Parameters supplied to a circuit in batch-mode operation can either be scalars or vectors (of length batch_size). Scalars are **automatically broadcast** over the batch dimension.

```
In [14]: alpha = tf.Variable([0.5] * batch_size)
         theta = tf.constant(0.0)
         phi = tf.Variable([0.1, 0.33, 0.5])
```

Measurement results will be returned as Tensors with shape `(batch_size,)`. We can picture batch-mode operation as simulating multiple circuit configurations at the same time. Combined with appropriate parallelized hardware like GPUs, this can result in significant speedups compared to serial evaluation.

```
In [15]: state

Out[15]: <FockStateTF: num_modes=2, cutoff=7, pure=True, batched=True, hbar=2>
```

```
In [16]: state.ket().shape

Out[16]: TensorShape([Dimension(3), Dimension(7), Dimension(7)])
```

Example: variational quantum circuit optimization

A key element of machine learning is optimization. We can use Tensorflow’s automatic differentiation tools to optimize the parameters of variational quantum circuits. In this approach, we fix a circuit architecture where the states, gates, and/or measurements may have learnable parameters associated with them. We then define a loss function based on the output state of this circuit.

> **Exercise**
> Complete the Strawberry Fields program below, to optimize a Dgate to produce an output with the largest overlap with the Fock state $n = 1$.

In [17]:
```
#!/usr/bin/env python3
import strawberryfields as sf
from strawberryfields.ops import *
import tensorflow as tf

# create a one mode engine
eng, q = sf.Engine(1)

# The displacement parameter is a TF variable - the one we want to optimize
alpha = tf.Variable(0.1)

# Apply a Dgate with this parameter to the qumode
# and run the engine using the Tensorflow backend
# in symbolic mode

with eng:
    Dgate(alpha) | q[0]

state = eng.run('tf', cutoff_dim=6, eval=False)

# calculate the probability of the
# resulting state being in Fock state |1>
prob = state.fock_prob([1])


# The loss function:
# Loss is the probability for the Fock state n=1.
# Note that we add a negative sign since
# we wish to *maximize* the probability overlap
loss = -prob

# Set up optimization
optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1)
minimize_op = optimizer.minimize(loss)

# Create Tensorflow Session and initialize variables
sess = tf.Session()
sess.run(tf.global_variables_initializer())

# Carry out optimization
for step in range(50):
    prob_val, alpha_val, _ = sess.run([prob, alpha, minimize_op])
    print("Value at step {}: {}".format(step, prob_val, alpha_val))
```

```
Value at step 0: 0.00990049634128809
Value at step 1: 0.014101705513894558
Value at step 2: 0.019992981106042862
Value at step 3: 0.0281626395881176
Value at step 4: 0.039317384362220764
Value at step 5: 0.054226942360401154
Value at step 6: 0.07359194755554199
Value at step 7: 0.09781551361083984
Value at step 8: 0.12670716643333435
Value at step 9: 0.15923479199409485
Value at step 10: 0.19351372122764587
Value at step 11: 0.22716644406318665
Value at step 12: 0.2579536437988281
Value at step 13: 0.2843506336212158
Value at step 14: 0.30577120184898376
Value at step 15: 0.32241666316986084
Value at step 16: 0.3349469006061554
Value at step 17: 0.3441750109195709
Value at step 18: 0.35087576508522034
Value at step 19: 0.3556998074054718
Value at step 20: 0.3591567277908325
Value at step 21: 0.3616286516189575
Value at step 22: 0.36339545249938965
Value at step 23: 0.36465880274772644
Value at step 24: 0.365563303232193
Value at step 25: 0.36621156334877014
Value at step 26: 0.36667701601982117
Value at step 27: 0.36701157689094543
Value at step 28: 0.3672524690628052
Value at step 29: 0.3674260973930359
Value at step 30: 0.367551326751709
Value at step 31: 0.36764174699783325
Value at step 32: 0.3677072525024414
Value at step 33: 0.36775457859039307
Value at step 34: 0.3677889406681061
Value at step 35: 0.36781373620033264
Value at step 36: 0.3678317368030548
Value at step 37: 0.367844820022583
Value at step 38: 0.3678542971611023
Value at step 39: 0.36786115169525146
Value at step 40: 0.3678661584854126
Value at step 41: 0.3678697645664215
Value at step 42: 0.3678724467754364
Value at step 43: 0.3678743243217468
Value at step 44: 0.36787569522857666
Value at step 45: 0.3678767681121826
Value at step 46: 0.3678774833679199
Value at step 47: 0.3678780794143677
Value at step 48: 0.36787837743759155
Value at step 49: 0.36787864565849304
```

> **Exercise**
> Plot the Wigner function for the single photon state, and compare it to the optimized state above.

First, you need to numerically evaluate the density matrix of the state.

```
In [18]: eng, q = sf.Engine(1)

         with eng:
             # prepare the initial states
             Coherent(1+0j) | q[0]

         state = eng.run('fock', cutoff_dim=100, eval=False)

         rho = state.dm()
         print(rho.shape)
         prob = state.fock_prob([1])
         print(prob)
```

```
(100, 100)
0.36787944117144233
```

The following Wigner function is provided for convenience.

In [19]:
```
# Wigner function for plotting purposes
def wigner(rho):
    import copy
    # Domain parameter for Wigner function plots
    l = 5.0
    cutoff = rho.shape[0]

    # Creates 2D grid for Wigner function plots
    x = np.linspace(-l, l, 100)
    p = np.linspace(-l, l, 100)

    Q, P = np.meshgrid(x, p)
    A = (Q + P * 1.0j) / (2 * np.sqrt(2 / 2))

    Wlist = np.array([np.zeros(np.shape(A), dtype=complex) for k in range(cutoff
)])

    # Wigner function for |0><0|
    Wlist[0] = np.exp(-2.0 * np.abs(A) ** 2) / np.pi

    # W = rho(0,0)W(|0><0|)
    W = np.real(rho[0, 0]) * np.real(Wlist[0])

    for n in range(1, cutoff):
        Wlist[n] = (2.0 * A * Wlist[n - 1]) / np.sqrt(n)
        W += 2 * np.real(rho[0, n] * Wlist[n])

    for m in range(1, cutoff):
        temp = copy.copy(Wlist[m])
        # Wlist[m] = Wigner function for |m><m|
        Wlist[m] = (2 * np.conj(A) * temp - np.sqrt(m)
                    * Wlist[m - 1]) / np.sqrt(m)

        # W += rho(m,m)W(|m><m|)
        W += np.real(rho[m, m] * Wlist[m])

        for n in range(m + 1, cutoff):
            temp2 = (2 * A * Wlist[n - 1] - np.sqrt(m) * temp) / np.sqrt(n)
            temp = copy.copy(Wlist[n])
            # Wlist[n] = Wigner function for |m><n|
            Wlist[n] = temp2

            # W += rho(m,n)W(|m><n|) + rho(n,m)W(|n><m|)
            W += 2 * np.real(rho[m, n] * Wlist[n])

    return Q, P, W / 2
```

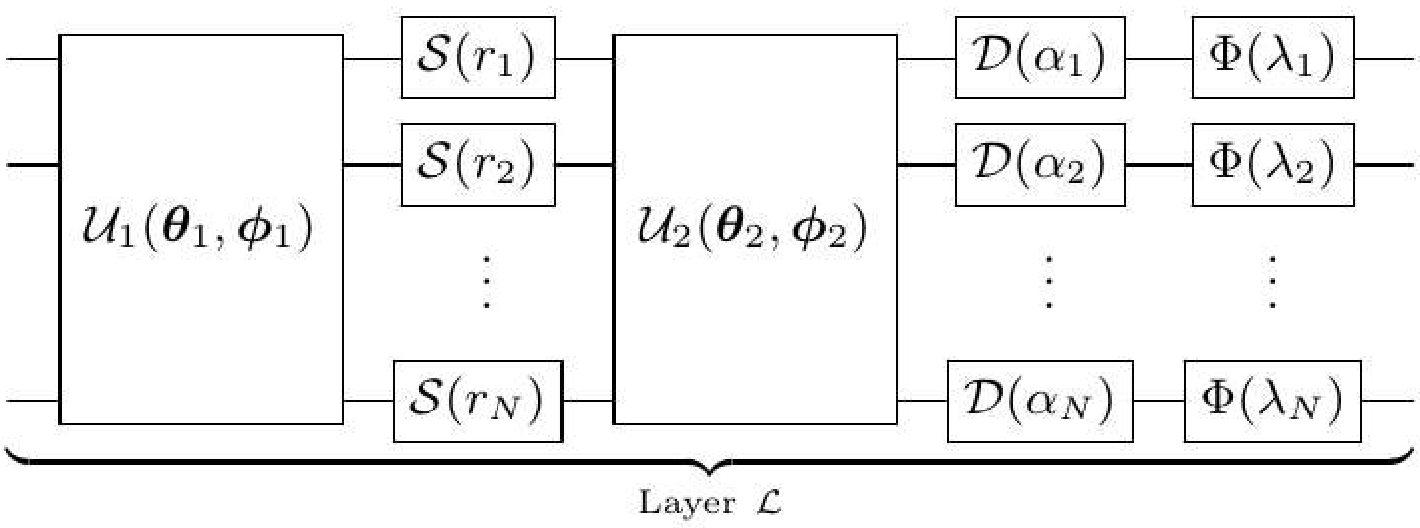
In [20]:
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

fig = plt.figure()
ax = fig.add_subplot(111, projection="3d")
X, P, W = wigner(rho)
ax.plot_surface(X, P, W, cmap="RdYlGn", lw=0.5, rstride=1, cstride=1)
```

```
Out[20]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7fc335a3ab38>
```

Layers and state learning

For arbitrary state preparation using optimization, we need to make use of a quantum circuit with a layer structure that is **universal** - that is, by 'stacking' the layers, we can guarantee that we can produce *any* CV state. Therefore, the architecture we choose must consist of layers with each layer containing parameterized Gaussian *and* non-Gaussian gates. **The non-Gaussian gates provide both the nonlinearity and the universality of the model.**



> **Reference**
> Killoran, N., Bromley, T. R., Arrazola, J. M., Schuld, M., Quesada, N., & Lloyd, S. (2018). "Continuous-variable quantum neural networks." [arXiv:1806.06871.](https://arxiv.org/abs/1806.06871)

First, we must define the **hyperparameters** of our layer structure:

- `cutoff`: the Fock space truncation we will use in the optimization. The TensorFlow backend will perform operations in this truncated Fock space when performing the optimization.
- `depth`: The number of layers in our quantum circuit. As a general rule, increasing the number of layers (and thus, the number of parameters we are optimizing over) increases the optimizers chance of finding a reasonable local minimum in the optimization landscape.
- `reps`: the number of steps in optimization routine performing gradient descent

Some other optional hyperparameters include:

- `penalty_strength`: this allows us to *penalize* deviations from a trace of 1 in the cost function. This helps ensure that the optimization process never results in operations that transform the state beyond the truncated Fock space.
- The standard deviation of initial parameters. Note that we make a distinction between the standard deviation of *passive* parameters (those that preserve photon number when changed, such as phase parameters), and *active* parameters (those that introduce or remove energy from the system when changed).

In [21]:
```
# Cutoff dimension
cutoff = 6 # 100

# Number of layers
depth = 8

# Number of steps in optimization routine performing gradient descent
reps = 1000

# Penalty coefficient to ensure the state is normalized
penalty_strength = 100

# Standard deviation of initial parameters
passive_sd = 0.1
active_sd = 0.001
```

The layer parameters

We use TensorFlow to create the variables corresponding to the gate parameters. Note that each variable has shape `[depth]`, with each individual element representing the gate parameter in layer i.

In [22]:
```
# parameters for one layer

# squeeze gate
sq_r = tf.Variable(tf.random_normal(shape=[depth], stddev=active_sd))
sq_phi = tf.Variable(tf.random_normal(shape=[depth], stddev=passive_sd))

# displacement gate
d_r = tf.Variable(tf.random_normal(shape=[depth], stddev=active_sd))
d_phi = tf.Variable(tf.random_normal(shape=[depth], stddev=passive_sd))

# rotation gates
r1 = tf.Variable(tf.random_normal(shape=[depth], stddev=passive_sd))
r2 = tf.Variable(tf.random_normal(shape=[depth], stddev=passive_sd))

# kerr gate
kappa = tf.Variable(tf.random_normal(shape=[depth], stddev=active_sd))
```

The layer function

Now, we can create a function to define the ith layer, acting on qumode `q`. This allows us to simply call this function in a loop later on when we build our circuit.

In [23]:
```
# layer architecture
def layer(i, q):
    Rgate(r1[i]) | q
    Sgate(sq_r[i], sq_phi[i]) | q
    Rgate(r2[i]) | q
    Dgate(d_r[i], d_phi[i]) | q
    Kgate(kappa[i]) | q
```

Constructing the circuit

Now that we have defined our gate parameters and our layer structure, we can construct our variational quantum circuit. Note that, as before, we use the TensorFlow backend with `eval=False`.

```
In [24]: # Start SF engine
         engine, q = sf.Engine(1)

         # Apply circuit of layers with corresponding depth
         with engine:
             for k in range(depth):
                 layer(k, q[0])

         # Run engine
         state = engine.run('tf', cutoff_dim=cutoff, eval=False)
         ket = state.ket()
         # dm = state.dm()
```

Performing the optimization

With the TensorFlow backend calculating the resulting state of the circuit symbolically, we can use TensorFlow to optimize the gate parameters to minimize the cost function we specify. With state learning, the cost function we often wish to minimize is the **fidelity of the output state $|\psi\rangle$ with some target state $|\psi_t\rangle$**. This is defined as the overlap between the two states:

$$\text{fidelity} = |\langle\psi \mid \psi_t\rangle|^2$$

> **Exercise**
> Define a target state you wish to learn using the variational quantum circuit below, and run the optimization. A good starting example is the single photon state $|1\rangle$.

```
In [25]: # Fock basis
         target_state = np.zeros([cutoff])
         target_state[1] = 1
         target_state

Out[25]: array([0., 1., 0., 0., 0., 0.])
```

Using this target state, we calculate the fidelity with the state exiting the variational circuit. Note that we must use TensorFlow functions to manipulate this data, as were are working with symbolic variables!

```
In [26]: fidelity = tf.abs(tf.reduce_sum(tf.conj(ket) * target_state)) ** 2
```

We can use this to construct our cost function. We can also include the penalty on the trace, to ensure the trace of the system remains close to 1.

```
In [27]: # Objective function to minimize
         cost = (1-fidelity) + penalty_strength*tf.abs(1-state.trace())
         tf.summary.scalar('cost', cost)

Out[27]: <tf.Tensor 'cost:0' shape=() dtype=string>
```

Now that the cost function is defined, we can define and run the optimization. Below, we choose the Adam optimizer that is built into TensorFlow.

```
In [28]: # Using Adam algorithm for optimization
         optimiser = tf.train.AdamOptimizer()
         min_cost = optimiser.minimize(cost)

         # Begin Tensorflow session
         session = tf.Session()
         session.run(tf.global_variables_initializer())
```

We then loop over all repetitions, storing the best predicted fidelity value.

```
In [29]: fid_progress = []
         best_fid = 0

         # Run optimization
         for i in range(reps):

             # one repitition of the optimization
         #     _, cost_val, fid_val, ket_val, dm_val = session.run([min_cost, cost, fidel
         ity, ket, dm])
             _, cost_val, fid_val, ket_val = session.run([min_cost, cost, fidelity, ket])

             # Stores fidelity at each step
             fid_progress.append(fid_val)

             if fid_val > best_fid:
                 # store the new best fidelity and best state
                 best_fid = fid_val
                 best_state = ket_val
         #         best_dm = dm_val

             # Prints progress at every 10 reps
             if i % 10 == 0:
                 print(i, fid_val)
```

```
0 2.9181033e-08
10 0.005481824
20 0.027850525
30 0.0744978
40 0.14822125
50 0.24235348
60 0.3436347
70 0.4413434
80 0.52769023
90 0.6052944
100 0.6735166
110 0.72180563
120 0.7501099
130 0.768204
140 0.7836428
150 0.8040208
160 0.8297338
170 0.85698134
180 0.88309246
190 0.90707046
200 0.92677295
210 0.9426541
220 0.95581067
230 0.96630925
240 0.97420555
250 0.97996145
260 0.9842429
270 0.9875092
280 0.99002254
290 0.99197525
300 0.9935064
310 0.99470925
320 0.9956582
330 0.99641454
340 0.9970221
350 0.9975141
360 0.99791706
370 0.99824876
380 0.9985252
390 0.99875665
400 0.99895066
410 0.9991146
420 0.9992515
430 0.99936604
440 0.9994623
450 0.9995418
460 0.99960834
470 0.99966455
480 0.99971175
490 0.9997512
500 0.99978495
510 0.999812
520 0.9998356
530 0.9998547
540 0.9998704
550 0.99988365
560 0.9998944
570 0.9999033
580 0.9999112
590 0.99991786
600 0.9999236
610 0.999928
620 0.99993205
630 0.99993527
640 0.9999392
650 0.9999416
```

Exercise
Plot the Wigner function of the learnt state and the target state.

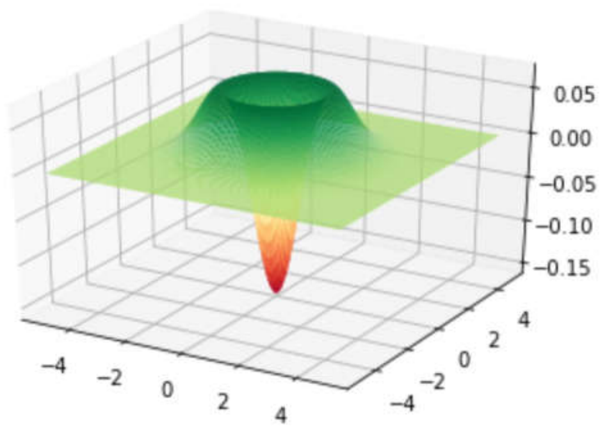
```
In [30]: # desired state

         rho = np.outer(target_state, target_state.conj())
         print(rho)

         fig = plt.figure()
         ax = fig.add_subplot(111, projection="3d")
         X, P, W = wigner(rho)
         ax.plot_surface(X, P, W, cmap="RdYlGn", lw=0.5, rstride=1, cstride=1)
```

```
[[0. 0. 0. 0. 0. 0.]
 [0. 1. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]
 [0. 0. 0. 0. 0. 0.]]
```

```
Out[30]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7fc323ea9ac8>
```



The following cell computes the density matrix $\rho = |\psi\rangle \langle\psi|$ of a pure state vector. This can be used in the Wigner function defined in the previous section.

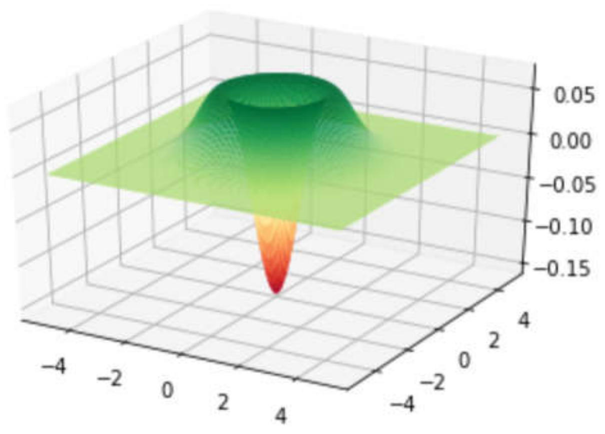
In [31]:
```
# learned state

rho = np.outer(best_state, best_state.conj())
print(rho)
print(rho.shape)
```

```
[[ 9.5106213e-07+0.0000000e+00j  1.1713692e-04+9.6814934e-04j
  -7.7388108e-07+4.5322793e-07j  1.3412501e-06-1.3571201e-06j
   3.1778040e-06-3.1490690e-06j  4.3360382e-07-6.8333929e-07j]
 [ 1.1713692e-04-9.6814934e-04j  9.9997067e-01+0.0000000e+00j
   3.6605631e-04+8.4360648e-04j -1.2163087e-03-1.5324964e-03j
  -2.8142547e-03-3.6227512e-03j -6.4221205e-04-5.2555720e-04j]
 [-7.7388108e-07-4.5322793e-07j  3.6605631e-04-8.4360648e-04j
   8.4569399e-07+0.0000000e+00j -1.7381124e-06+4.6511963e-07j
  -4.0864720e-06+1.0480235e-06j -6.7846906e-07+3.4940098e-07j]
 [ 1.3412501e-06+1.3571201e-06j -1.2163087e-03+1.5324964e-03j
  -1.7381124e-06-4.6511963e-07j  3.8280641e-06+0.0000000e+00j
   8.9751193e-06+9.3551307e-08j  1.5865891e-06-3.4495798e-07j]
 [ 3.1778040e-06+3.1490690e-06j -2.8142547e-03+3.6227512e-03j
  -4.0864720e-06-1.0480235e-06j  8.9751193e-06-9.3551307e-08j
   2.1044973e-05+0.0000000e+00j  3.7114198e-06-8.4754731e-07j]
 [ 4.3360382e-07+6.8333929e-07j -6.4221205e-04+5.2555720e-04j
  -6.7846906e-07-3.4940098e-07j  1.5865891e-06+3.4495798e-07j
   3.7114198e-06+8.4754731e-07j  6.8866683e-07+0.0000000e+00j]]
(6, 6)
```

In [32]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection="3d")
X, P, W = wigner(rho)
ax.plot_surface(X, P, W, cmap="RdYlGn", lw=0.5, rstride=1, cstride=1)
```

Out[32]:
```
<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7fc323006208>
```


0.05
0.00
−0.05
−0.10
−0.15
−4
−2
0
2
4
−4
−2
0
2
4