Linear programming

By example

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Introduction

In this presentation we layout several linear programming problems (20 in total), we analize them and finally we resolve them. We present for each problem the objetive function, the restrictions, the full model and the solution given by a linear problem solver Software.

Example 01

Example 01 - Problem

Farmer Jones must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at $4 a bushel, and all corn can be sold at $3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year. Formulate an LP whose solution will tell Farmer Jones how to maximize the total revenue from wheat and corn.

Example 01 - Analisis

Each bushel of wheat can be sold at \$4 and each one of corn at \$3. Also it says that for each acre of wheat it will produce 25 bushels (The profit by acre of wheat will be 25 bushels, at \$4 each bushel) and for each acre of corn it will be produced 10 bushels (The profit by acre of corn will be 10 bushels, at \$3 each bushel). With this information we could stablish the function that refers to maximize the profits of planting corn and wheat as:

$$Z = 100AT + 30AM$$

$$AT \longrightarrow \text{weat acres}$$

$$AM \longrightarrow \text{corn acres}$$

Example 01 - Function

Maximize the profits of planting corn and wheat:

$$Z = 100AT + 30AM$$

Example 01 - Restrictions

About the restrictions, the first one would be about the working hours; 1 acre of wheat requires 10 hours of labor per week and 1 acre of corn requires 4 hours of labor per week; and there are 40 hours available, this restricted us that the sum of 10AT and 4AM must less or equal than 40 (labour hours available). The second restriction refers to the availability of land. There are 7 acres of land available so AT + AM must be less or equal than 7. Also there is a restriction from Government which that at least 30 bushels of corn be produced during the current year, so AM must be greater or equal that 3, because each acre produce 10 bushels so $3*10 = 30$ that is the minimun required. Finally we have the trivial restrictions that all variables must be greater or equal than 0.

Example 01 - Model

Maximize:

$$Z = 100AT + 30AM$$

Subject to:

$$\begin{aligned} 10AT + 4AM &\leq 40 \\ AT + AM &\leq 7 \\ AM &\geq 3 \\ AT, AM &\geq 0 \end{aligned}$$

Example 01 - Solution

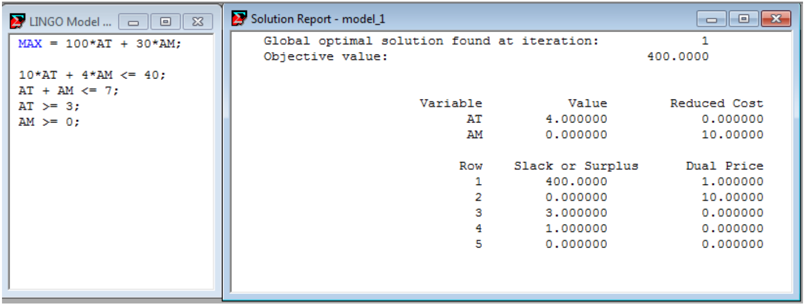

Figure: Example 01 model and solution in Lingo Software.

Example 01 - Final solution

Global optimal solution found at iteration: 1

Objective value: 400.0000

Variable	Value	Reduced Cost
AT	4.000000	0.000000
AM	0.000000	10.00000

Row	Slack or Surplus	Dual Price
1	400.0000	1.000000
2	0.000000	10.00000
3	3.000000	0.000000
4	1.000000	0.000000
5	0.000000	0.000000

Example 01 - Final analisis

The optimal solution given by LINGO says that Farmer Jones have to plant **4** acres of wheat and **0** acres of corn, this will generate a profit of **$400**. If Farmer Jones plants corn, each acre planted will reduce the profit in $10.

Example 02

Example 02 - Problem

Truckco manufactures two types of trucks: 1 and 2. Each truck must go through the painting shop and assembly shop. If the painting shop were completely devoted to painting type 1 trucks, 800 per day could be painted, whereas if the painting shop were completely devoted to painting type 2 trucks, 700 per day could be painted. If the assembly shop were completely devoted to assembling truck 1 engines, 1500 per day could be assembled, and if the assembly shop were completely devoted to assembling truck 2 engines, 1200 per day could be assembled. Each type 1 truck contributes $300 to profit; each type 2 truck contributes $500. Formulate an LP that will maximize Truckco's profit.

Example 02 - Analisis

We must maximize the profit of the company. TruckCo manufactures two types of trucks: type 1 and type 2. Each truck type 1 can be sold at $300 and each truck type 2 at $500.

$x_1 \longrightarrow$ type 1 trucks

$x_2 \longrightarrow$ type 2 trucks

Example 02 - Function

Maximize TruckCo's profits:

$$Z = 300x_1 + 500x_2$$

Example 02 - Restrictions

The first restriction is abour the amount of trucks that could be painted by day. If the painting shop were completely devoted to painting type 1 trucks, 800 per day could be painted, whereas if the painting shop were completely devoted to painting type 2 trucks, 700 per day could be painted. So just painting 1 truck type 1 could be expressed as $\frac{1}{800}x_1$ and just painting 1 truck type 2 could be expressed as $\frac{1}{700}x_2$. The same happens with assembly, just assembling 1 truck type 1 could be represented as $\frac{1}{1500}x_1$ and just assembling 1 truck type 2 could be expressed as $\frac{1}{1200}x_2$. These 2 restrictions refers just to 1 truck, so both are less or equal than 1. The last restriction is the trivial one, that all variables must be greater or equal than 0.

Example 02 - Model

Maximize:

$$Z = 300x_1 + 500x_2$$

Subject to:

$$\begin{aligned} \frac{1}{800}x_1 + \frac{1}{700}x_2 &\leq 1 \\ \frac{1}{1500}x_1 + \frac{1}{1200}x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Example 02 - Solution

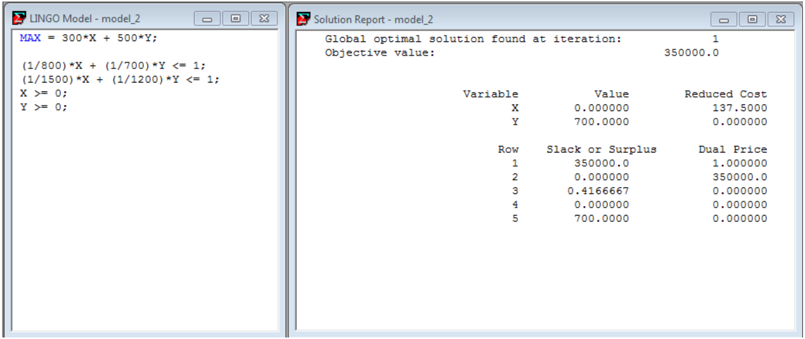

Figure: Example 02 model and solution in Lingo Software.

Example 02 - Final solution

Global optimal solution found at iteration: 1

Objective value: 350000.0

Variable	Value	Reduced Cost
X	0.000000	137.5000
Y	700.0000	0.000000

Row	Slack or Surplus	Dual Price
1	350000.0	1.000000
2	0.000000	350000.0
3	0.4166667	0.000000
4	0.000000	0.000000
5	700.0000	0.000000

Example 02 - Final analisis

The optimal solution given by LINGO says that TruckCo has to manufacture **700** type 2 trucks and **0** type 1 trucks, this will generate a profit of **$350000**. If they manufacture type 1 trucks, each truck manufactured will reduce the profit in $137,5.

Example 03

Example 03 - Problem

Leary Chemical manufactures three chemicals: A, B and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs \$4 and yields 3 units of A, 1 of B, and 1 of C. Running process 2 for an hour costs \$1 and produces 1 unit of A and 1 of B. To meet customer demands, at least 10 units of A, 5 of B, and 3 of C must be produced daily. Graphically determine a daily production plan that minimizes the cost meeting Leary Chemical's daily demands.

Example 03 - Analisis

We have to minimize the cost while meeting Leary Chemical's daily demands. We know the duration hours and the cost per hour of each process. Running process 1 for an hour costs \$4 and running process 2 for an hour costs \$1. So we need to minimize 4*HoursUsedByProcess1 + 1*HoursUsedByProcess2.

$x_1 \longrightarrow$ hours for process 1

$x_2 \longrightarrow$ hours for process 2

Minimize the cost meeting Leary Chemical's daily demands:

$$Z = 4x_1 + x_2$$

Example 03 - Restrictions

The only restrictions are the minimum amounts of units of each chemicals that must be produced. These are at least 10 units of chemical A, 5 units of chemical B and 3 units of chemical C, this is that for A: $3x_1 + x_2$ must be greater or equal than 10, for B $x_1 + x_2$ must be greater or equal than 5, and for C x_1 must be greater or equal than 3. Also the trivial restriction that all variables must be greater or equal than 0.

Example 03 - Model

Minimize:

$$Z = 4x_1 + x_2$$

Subject to:

$$\begin{aligned} 3x_1 + x_2 &\geq 10 \\ x_1 + x_2 &\geq 5 \\ x_1 &\geq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Example 03 - Solution

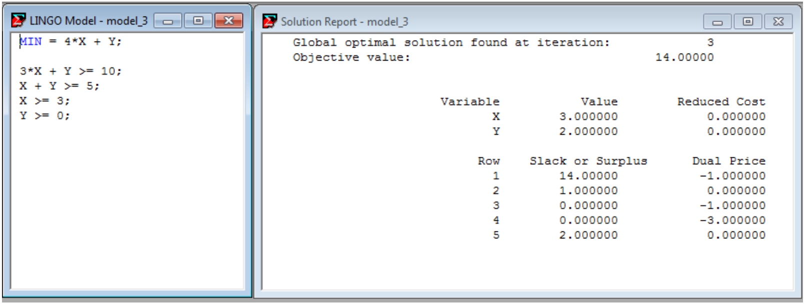

Figure: Example 03 model and solution in Lingo Software.

Example 03 - Final solution

Global optimal solution found at iteration: 3

Objective value: 14.00000

Variable	Value	Reduced Cost
X	3.000000	0.000000
Y	2.000000	0000000

Row	Slack or Surplus	Dual Price
1	14.00000	-1.000000
2	1.000000	0.000000
3	0.000000	-1.000000
4	0.000000	-3.000000
5	2.000000	0.000000

Example 03 - Final analisis

The optimal solution given by LINGO says that Leary Chemicals two production processes might use **3** hours (Process 1) and **2** hours (Process 2). It will generate a cost of **$14** on daily demands.

Example 04

Example 04 - Problem

Farmer Jane owns 45 acres of land. She is going to plant each with wheat or corn. Each acre planted with wheat yields \$200 profit; each with corn yields \$300 profit. The labor and fertilizer used for each acre are given in the following table. One hundred workers and 120 tons of fertilizer are available. Use LP to determine how Jane can maximize profits from her land.

Resource	**Wheat**	**Corn**
Labor	3 workers	2 workers
Fertilizer	2 tons	4 tons

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Example 04 - Analisis

We must maximize profits from Jane land. We know each acre of wheat yields \$200 and each acre of corn yields \$300. So we need to maximize 200*EachAcreOfWheat + 300*EachAcreOfCorn.

$AT \longrightarrow$ weat acres

$AM \longrightarrow$ corn acres

Example 04 - Function

Maximize Jane's profits:

$$Z = 200AT + 300AM$$

Example 04 - Restrictions

The first restriction is about the acres available in Jane's Farm. She owns 45 acres of land, so $AT + AM \leq 45$. Also there is restricted the amount of workers available, there are 100 workers and for wheat production there are needed 3 workers and for corn production there are needed 2 workers; so these could be expressed as $3AT + 2AM \leq 100$. Also wheat needs 2 tons of fertilizer, corn needs 4 tons, and there are just 120 tons of fertilizer available , so $2AT + 4AM \leq 120$. Then we have the last restriction that all variables must be greater or equal than 0.

Example 04 - Model

Maximize:

$$Z = 200AT + 300AM$$

Subject to:

$$\begin{aligned} AT + AM &\leq 45 \\ 3AT + 2AM &\leq 100 \\ 2AT + 4AM &\leq 120 \\ AT, AM &\geq 0 \end{aligned}$$

Example 04 - Solution

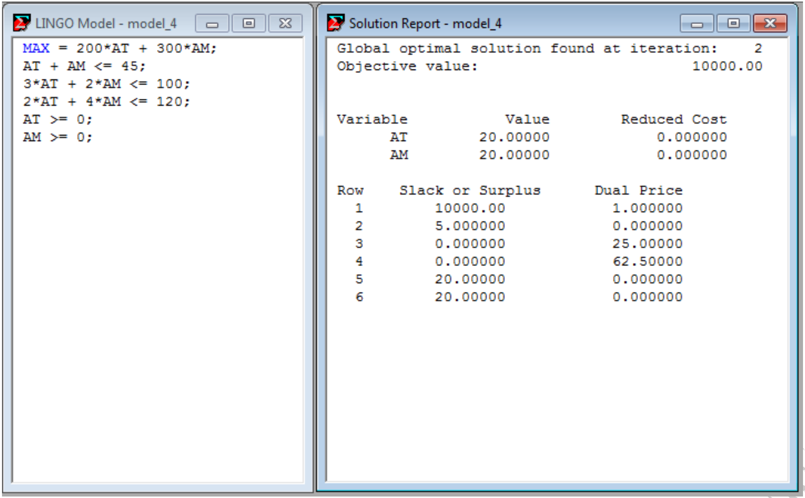


Figure: Example 04 model and solution in Lingo Software.

Example 04 - Final solution

Global optimal solution found at iteration: 2
Objective value: 10000.00

Variable	Value	Reduced Cost
AT	20.00000	0.000000
AM	20.00000	0.000000

Row	Slack or Surplus	Dual Price
1	10000.00	1.000000
2	5.000000	0.000000
3	0.000000	25.00000
4	0.000000	62.50000
5	20.00000	0.000000
6	20.00000	0.000000

Example 04 - Final analisis

The optimal solution given by LINGO says that Farmer Jane must plant **20** acres of wheat and 20 acres of corn to maximize profits from her land. This will generate a profit of **$10000**.

Example 05

Example 05 - Problem (1)

There are three factories on the Momiss River (1, 2 and 3). Each emits two types of pollutants (1 and 2) into the river. If the waste from each factory is processed, the pollution in the river can be reduced. It costs $15 to process a ton of factory 1 waste, and each ton processed reduces the amount of pollutant 1 by 0.10 ton and the amount of pollutant 2 by 0.45 ton. It costs $10 to process a ton of factory 2 waste, and each ton processed will reduce the amount of pollutant 1 by 0.20 ton and the amount of pollutant 2 by 0.25 ton. It costs $20 to process a ton of factory 3 waste, and each ton processed will reduce the amount of pollutant 1 by 0.40 ton and the amount of pollutant 2 by 0.30 ton.

Example 05 - Problem (2)

The state wants to reduce the amount of pollutant 1 in the river by at least 30 tons and the amount of pollutant 2 in the river by at least 40 tons. Formulate an LP that will minimize the cost of reducing pollution by the desired amounts.

Example 05 - Analisis (1)

The previous problem statement can be represented with the following table:

Factory	Cost per ton	Contaminant 1	Contaminant 2
Factory 1	15	0.10	0.45
Factory 2	10	0.20	0.25
Factory 3	20	0.40	0.30

Example 05 - Analisis (2)

We know that 1 ton of waste processed at Factory 1 costs \$15, also it costs \$10 processing 1 ton of waste in Factory 2 and 1 ton of waste processed at Factory 3 costs \$20. We need to minimize the cost of reducing pollution. So we could express this like minimize 15*TonsOfWasteFactory1 + 10*TonsOfWasteFactory2 + 20*TonsOfWasteFactory3.

Example 05 - Variables

$x \longrightarrow$ tons of waste processed by factory 1

$y \longrightarrow$ tons of waste processed by factory 2

$z \longrightarrow$ tons of waste processed by factory 3

Example 05 - Function

Minimize the cost of reducing pollution by the desired amounts:

$$Z = 15x + 10y + 20z$$

Example 05 - Restrictions

To what restrictions refers the first ones would be given by the fact that the contaminants needs to be reduced by al least 30 tons for contaminant 1 and at least 30 tons for contaminant 2. That is, we take the amount of contaminant per ton for each factory and we multiply it with the variables: $0.1x + 0.2y + 0.4z \geq 30$; in the same way for the contaminant 2 we have: $0.45x + 0.25y + 0.3z \geq 40$ At last we have a trivial restrictions that x, y, z must be greater or equal to 0 because tons can't be negative.

Example 05 - Model

Minimize:

$$Z = 15x + 10y + 20z$$

Subject to:

$$\begin{aligned} 0.1x + 0.2y + 0.4z &\geq 30 \\ 0.45x + 0.25y + 0.3z &\geq 40 \\ x, y, z &\geq 0 \end{aligned}$$

Example 05 - Solution

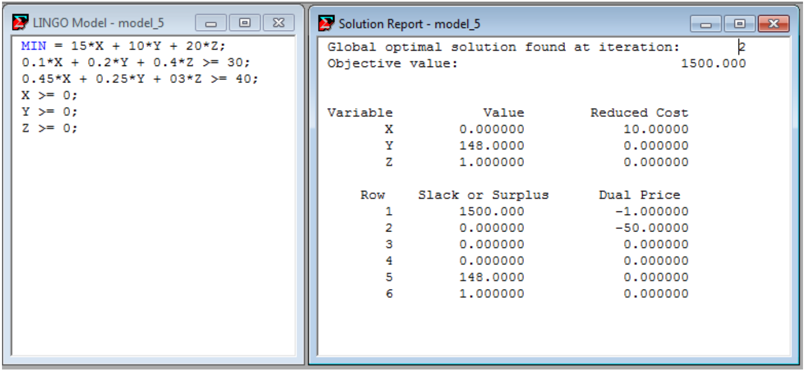


Figure: Example 05 model and solution in Lingo Software.

Example 05 - Final solution

Global optimal solution found at iteration: 2

Objective value: 1500.000

Variable	Value	Reduced Cost
X	0.000000	10.00000
Y	148.0000	0.000000
Z	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	1500.000	-1.000000
2	0.000000	-50.00000
3	0.000000	0.000000
4	0.000000	0.000000
5	148.0000	0.000000
6	1.000000	0.000000

Example 05 - Final analisis

The optimal solution given by LINGO says that Factory 1 must process **0** tons of waste, Factory 2 must process **148** tons of waste and Factory 3 must process **1** ton of waste. The cost of reducing pollution by these amounts will be of **$1500**. Each ton processed by Factory 1 will increase the cost in $10.

Example 06

Example 06 - Problem (1)

U.S Labs manufactures mechanical heart valves from the heart valves of pigs. Different heart operations require valves of different sizes. U.S. Labs purchases pig valves from three different suppliers. The cost and size mix of the valves purchased from each supplier are given in the following table.

Supplier	Cost per valve	% large	% medium	% small
Sup. 1	\$5	40	40	20
Sup. 2	\$4	30	35	35
Sup. 3	\$3	20	20	60

Example 06 - Problem (2)

Each month, U.S. Labs places one order with each supplier. At least 500 large, 300 medium, and 300 small valves must be purchased each month.
Because of limited availability of pig vales, at most 500 valves per month can be purchased from each supplier. Formulate an LP that can be used to minimize the cost of acquiring the needed valves.

Example 06 - Analisis

U.S Labs purchases the valves from 3 different suppliers. Each of these suppliers sells the valves at different prices. Supplier 1 sells the valves at a cost per value of \$5, Supplier 2 sells them at \$4 and the last one, Supplier 3 sells the valves at \$3. U.S labs needs to minimize the cost of acquiring the needed valves, so it could be expressed as minimize \$5*ValvesFromSupplier1 + \$4*ValvesFromSupplier2 + \$3*ValvesFromSupplier3

Example 06 - Variables

$x_1 \longrightarrow$ valves ordered from supplier 1

$x_2 \longrightarrow$ valves ordered from supplier 2

$x_3 \longrightarrow$ valves ordered from supplier 3

Example 06 - Function

Minimize cost of acquiring the needed valves:

$$Z = 5x_1 + 4x_2 + 3x_3$$

Example 06 - Restrictions

The first restriction would be the amount or percentual distribution of each type of valve que each supplier needs; that is each percentage is divided by 100 and is multiplied by the x that corresponds to the total amount of valves.
We know the we need to buy at least 500 large valves, with 40% being from supplier 1, 30% from supplier 2 and 20% for supplier 3; that is: $0.4x_1 + 0.3x_2 + 0.2x_3 \geq 500$. In a similar way we have some restrictions for medium size and small size valves, which give $0.4x_1 + 0.35x_2 + 0.2x_3 \geq 300$ and $0.2x_1 + 0.35x_2 + 0.6x_3 \geq 300$.
The last restrictions is that we can't buy more than 500 total valves from each supplier, and obviously we can't buy negative valves. Both restriction can be modeled like: $x_1, x_2, x_3 \leq 500$ and $x_1, x_2, x_3 \geq 0$.

Example 06 - Model

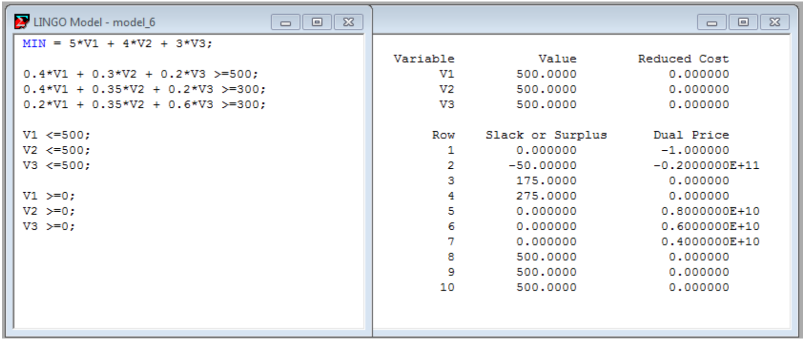
Minimize:

$$Z = 5x_1 + 4x_2 + 3x_3$$

Subject to:

$$\begin{aligned} 0.4x_1 + 0.3x_2 + 0.2x_3 &\geq 500 \\ 0.4x_1 + 0.35x_2 + 0.2x_3 &\geq 300 \\ 0.2x_1 + 0.35x_2 + 0.6x_3 &\geq 300 \\ x_1, x_2, x_3 &\leq 500 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Example 06 - Solution



LINGO Model - model_6

```
MIN = 5*V1 + 4*V2 + 3*V3;

0.4*V1 + 0.3*V2 + 0.2*V3 >=500;
0.4*V1 + 0.35*V2 + 0.2*V3 >=300;
0.2*V1 + 0.35*V2 + 0.6*V3 >=300;

V1 <=500;
V2 <=500;
V3 <=500;

V1 >=0;
V2 >=0;
V3 >=0;
```

Variable	Value	Reduced Cost
V1	500.0000	0.000000
V2	500.0000	0.000000
V3	500.0000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	-50.00000	-0.2000000E+11
3	175.0000	0.000000
4	275.0000	0.000000
5	0.000000	0.8000000E+10
6	0.000000	0.6000000E+10
7	0.000000	0.4000000E+10
8	500.0000	0.000000
9	500.0000	0.000000
10	500.0000	0.000000

Figure: Example 06 model and solution in Lingo Software.

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Example 06 - Final solution

Global optimal solution found at iteration:
Objective value:

Variable	Value	Reduced Cost
V1	500.000	0.00000
V2	500.000	0.00000
V3	500.000	0.00000

Row	Slack or Surplus	Dual Price
1	0.00000	-1.000000
2	-50.0000	-0.2000000E+11
3	175.000	0.000000
4	275.000	0.000000
5	0.00000	0.8000000E+10
6	0.00000	0.6000000E+10
7	0.00000	0.4000000E+10
8	500.000	0.000000
9	500.000	0.000000
10	500.000	0.000000

Example 06 - Final analisis

LINGO detects an “Unbounded Solution” in the model. When the “Unbounded Solution” termination occurs, it implies the formulation admits the unrealistic result that an infinite amount of profit can be made. A more realistic conclusion is that an important constraint has been omitted or the formulation contains a critical typographical error.

Example 07

Example 07 - Problem

Highland's TV-Radio Store must determine how many TVs and radios to keep in stock. A TV requires 10 sq ft of floorspace, whereas a radio requires 4 sq ft; 200 sq ft of floorspace is available. A TV will earn Highland \$60 in profits, and a radio will earn \$20. The store stocks only TVs and radios. Marketing requirements dictate that at least 60% of all appliances in stock be radios. Finally, a TV ties up \$200 in capital, and a radio, \$50. Highland wants to have at most \$3000 worth of capital tied up at any time. Formulate an LP that can be used to maximize Highland's profit.

Example 07 - Analisis (1)

The previous problem statement can be represented with the following table:

Item	Required space	Profit	Capital
TV	10	\$60	\$200
Radio	4	\$20	\$50

Example 07 - Analisis (2)

We need to maximize Highland's TV-Radio Store profits. We know that each TV sold yields \$60 and each radio sold yields \$20. So the profit to maximize could be represented as \$60*TVsInStock + \$20RadiosInStock

Example 07 - Variables

$TV \longrightarrow$ amount of TV's in stock

$R \longrightarrow$ amount of radios in stock

Example 07 - Function

Maximize Highland's TV-Radio Store profit:

$$Z = 60\,TV + 20\,R$$

Example 07 - Restrictions

The first restriction would be about the space available in the store and the space both TVs and radios requires. There is 200ft^2 available and a TV requires 10ft^2 and a radio requires 4ft^2; from there we have the restriction $10TV + 4R \leq 200$. Another restriction is about the requires that from marketing, which would give the equation: $\frac{R}{TV+R} \geq 0.6$. Futhermore, the are presented by the fact that Highlands want to invest un maximum of \$3000 in capital and both TVs and radios requires \$200 and \$50 respectively; that gives the restriction: $200TV + 50R \leq 3000$. Finally we need not to forget the trivial restriction that both TVs and radios needs to be positive as we can buy negative things.

Example 07 - Model

Maximize:

$$Z = 60TV + 20R$$

Subject to:

$$10TV + 4R \leq 200$$
$$200TV + 50R \leq 3000$$
$$\frac{R}{TV + R} \geq 0.6$$
$$TV, R \geq 0$$

Example 07 - Solution

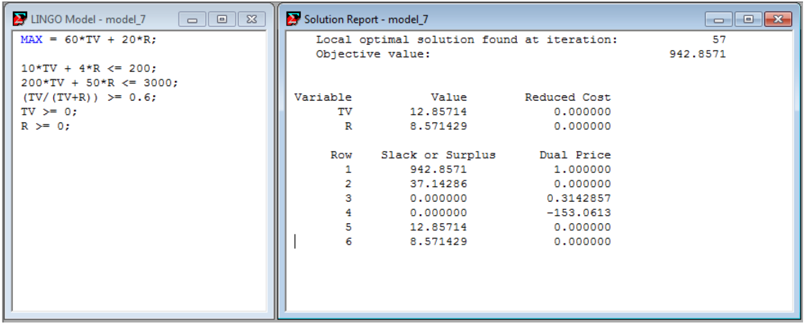

Figure: Example 07 model and solution in Lingo Software.

Example 07 - Final solution

Global optimal solution found at iteration: 57
Objective value: 942.8571

Variable	Value	Reduced Cost
TV	12.85714	0.000000
R	8.571429	0.000000

Row	Slack or Surplus	Dual Price
1	942.8571	1.000000
2	37.14286	0.000000
3	0.000000	0.3142857
4	0.000000	-153.0613
5	12.85714	0.000000
6	8,571429	0.000000

Example 07 - Final analisis

The optimal solution given by LINGO says that Highland's TV-Radio Store must keep **12.85** TVs in stock and **8.57** Radios. It will generate a profit of **$942.8571**.

Example 08

Example 08 - Problem (1)

Coalco produces coal in three mines and ships it to four customers. The cost per ton of producing coal, the ash and sulfur content (per ton) of the coal, and the production capacity (in tons) for each mine are given in the following table.

Production	**Mine 1**	**Mine 2**	**Mine 3**
Cost	\$50	\$55	\$62
Capacity	120	100	140
Ash	.08 ton	.06 ton	.04 ton
Sulfur	.05 ton	.04 ton	.04 ton

Example 08 - Problem (2)

The numbers of tons of coal demanded by each customer are given in the following table.

Customer	**Demand**
Customer 1	80
Customer 2	70
Customer 3	60
Customer 4	90

Example 08 - Problem (3)

The cost (in dollars) of shipping a ton of coal from a mine to each customer is given in the following table.

Mine	**Cust. 1**	**Cust. 2**	**Cust. 3**	**Cust. 4**
Mine 1	4	6	8	12
Mine 2	9	6	7	11
Mine 3	8	12	3	5

It is required that the total amount of coal shipped contains at most 5% ash and at most 4% sulfur. Formulate an LP that minimizes the cost of meeting customer demands.

Example 08 - Analisis

Coalco needs to minimize the cost of meeting customer demands. The coal is taking from 3 mines and it's demanded by 4 customers. Each customer demand a different amount of coal and shipping it to each customer has a cost of \$8, \$10, \$11 and \$14. The coal shipped must contains at most 5% ash and at most 4% sulfur. So putting these data together with the production cost we could express all as minimizing the sum of (cost of shipping from a mine to each customer + Production Cost) for each mine and customer.

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Example 08 - Variables

$$X_{i,j}$$

$X \longrightarrow$ tons of coal

$i \longrightarrow$ mine number from where the coal will be extrated [1;3]

$j \longrightarrow$ number of customer [1;4]

Example 08 - Function

Minimize the cost of meeting customer coal demands:

$$\begin{aligned} Z =& 200X_{11} + 300X_{12} + 400X_{13} + 600X_{14} + \\ & 495X_{21} + 330X_{22} + 385X_{23} + 605X_{24} + \\ & 496X_{31} + 744X_{32} + 186X_{33} + 310X_{34} \end{aligned}$$

Example 08 - Restrictions

The first restrictions are quite straightforward, the first 4 restrictions are (see model in the next slide) restrict the demand from clients (1, 2, 3 and 4). The following 3 equations restrict the capacity of the mine (1, 2 and 3). The final two restrictions are the more complicated ones, the first one restricts the percentage of ash in the coal, and the next one restrict the percentage of sulfur in the coal.

Example 08 - Model (1)

Minimize:

$$
\begin{aligned}
Z = & 54X_{11} + 56X_{12} + 58X_{13} + 62X_{14} + \\
& 64X_{21} + 61X_{22} + 62X_{23} + 66X_{24} + \\
& 70X_{31} + 74X_{32} + 65X_{33} + 68X_{34}
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
X_{11} + X_{21} + X_{31} &= 80 \\
X_{12} + X_{22} + X_{32} &= 70 \\
X_{13} + X_{23} + X_{33} &= 60 \\
X_{14} + X_{24} + X_{34} &= 90 \\
X_{11} + X_{12} + X_{13} + X_{14} &\leq 120 \\
X_{21} + X_{22} + X_{23} + X_{24} &\leq 100 \\
X_{31} + X_{32} + X_{33} + X_{34} &\leq 140 \\
X_{ij} &\geq 0
\end{aligned}
$$

Example 08 - Model (2)

And...

$$\frac{0.8(X_{11} + X_{12} + X_{13} + X_{14}) + 0.6(X_{21} + X_{22} + X_{23} + X_{24}) + 0.4(X_{31} + X_{32} + X_{33} + X_{34})}{X_{11} + X_{12} + X_{13} + X_{14} + X_{21} + X_{22} + X_{23} + X_{24} + X_{31} + X_{32} + X_{33} + X_{34}} \leq 0.05$$

$$\frac{0.5(X_{11} + X_{12} + X_{13} + X_{14}) + 0.4(X_{21} + X_{22} + X_{23} + X_{24}) + 0.3(X_{31} + X_{32} + X_{33} + X_{34})}{X_{11} + X_{12} + X_{13} + X_{14} + X_{21} + X_{22} + X_{23} + X_{24} + X_{31} + X_{32} + X_{33} + X_{34}} \leq 0.04$$

LINGO Model - model_8

```
MIN = 54*x11 + 56*x12 + 58*x13 + 62*x14 + 64*x21 + 61*x22 + 62*x23 + 66*x24 + 70*x31 + 74*x32 + 65*x33 + 68*x34;

x11 + x21 + x31 = 80;
x12 + x22 + x32 = 70;
x13 + x23 + x33 = 60;
x14 + x24 + x34 = 90;

x11 + x12 + x13 + x14 <= 120;
x21 + x22 + x23 + x24 <= 100;
x31 + x32 + x33 + x34 <= 140;

(0.8*(x11 + x12 + x13 + x14) + 0.6*(x21 + x22 + x23 + x24)+ 0.4*(x31 + x32 + x33 + x34))
/(x11 + x12 + x13 + x14 + x21 + x22 + x23 + x24 + x31 + x32 + x33 + x34) <= 0.05;

(0.5*(x11 + x12 + x13 + x14) + 0.4*(x21 + x22 + x23 + x24)+ 0.3*(x31 + x32 + x33 + x34))
/(x11 + x12 + x13 + x14 + x21 + x22 + x23 + x24 + x31 + x32 + x33 + x34) <= 0.04;

x11 >= 0; x12 >= 0; x13 >= 0; x14 >= 0; x21 >= 0; x22 >= 0; x23 >= 0; x24 >= 0; x31 >= 0; x32 >= 0; x33 >= 0; x34 >= 0;
```

Figure: Example 08 model and solution in Lingo Software.

Example 08 - Solution (2)

Solution Report - model_8

Variable	Value	Reduced Cost
X11	60.00000	0.000000
X12	0.000000	0.000000
X13	0.000000	0.000000
X14	0.000000	0.000000
X21	9.999999	0.000000
X22	70.00000	0.000000
X23	20.00000	0.000000
X24	0.000000	0.000000
X31	10.00000	0.000000
X32	0.000000	0.000000
X33	40.00000	0.000000
X34	90.00000	0.000000

Figure: Example 08 model and solution in Lingo Software.

Example 08 - Solution (3)

Solution Report - model_8

Row	Slack or Surplus	Dual Price
1	18810.00	-1.000000
2	-0.3722023E-07	-0.1266667E-02
3	-0.2381191E-07	-0.1266667E-02
4	-0.3231968E-07	-0.1266667E-02
5	-0.1268634E-06	-0.1266667E-02
6	60.00000	0.000000
7	0.000000	0.9999999E-03
8	0.000000	0.2000000E-02
9	-0.4966667	1.000000
10	-0.3333333	1.000000
11	60.00000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	9.999999	0.000000
16	70.00000	0.000000
17	20.00000	0.000000
18	0.000000	0.000000
19	10.00000	0.000000
20	0.000000	0.000000
21	40.00000	0.000000
22	90.00000	0.000000

Figure: Example 08 model and solution in Lingo Software.

Example 08 - Final solution (1)

Global optimal solution found at iteration:
Objective value:

Variable	Value	Reduced Cost
X11	60.00000	0.000000
X12	0.000000	0.000000
X13	0.000000	0.000000
X14	0.000000	0.000000
X21	9.999999	0.000000
X22	70.00000	0.000000
X23	20.00000	0.000000
X24	0.000000	0.000000
X31	10.00000	0.000000
X32	0.000000	0.000000
X33	40.00000	0.000000
X34	90.00000	0.000000

TEC

Example 08 - Final solution (2)

	Slack or Surplus	Dual Price
Row	18810.00	-1.000000
1	-0.3722023E-07	-0.1266667E-02
2	-0.2381191E-07	-0.1266667E-02
3	-0.3231968E-07	-0.1266667E-02
4	-0.1268634E-06	-0.1266667E-02
5	60.00000	0.000000
6	0.000000	0.9999999E-03
7	0.000000	0.2000000E-02
8	-0.4966667	1.000000
9	-0.3333333	1.000000
10	60.00000	0.000000
11		

Example 08 - Final solution (3)

Variable		Dual Price
Row	Slack or Surplus	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	9.999999	0.000000
16	70.00000	0.000000
17	20.00000	0.000000
18	0.000000	0.000000
19	10.00000	0.000000
20	0.000000	0.000000
21	40.00000	0.000000
22	90.00000	

Example 08 - Final analisis

LINGO detects an "Unbounded Solution" in the model. When the "Unbounded Solution" termination occurs, it implies the formulation admits the unrealistic result that an infinite amount of profit can be made. A more realistic conclusion is that an important constraint has been omitted or the formulation contains a critical typographical error.

Example 09

Example 09 - Problem (1)

Furnco manufactures tables and chairs. A table requires 40 board ft of wood, and a chair requires 30 board ft of wood. Wood may be purchased at a cost of $1 per board ft, and 40000 board ft of wood are available for purchase. It takes 2 hours of skilled labor to manufacture an unfinished table or an unfinished chair. Three more hours of skilled labor will turn an unfinished table into a finished table, and 2 more hours of skilled labor will turn an unfinished char into a finished chair.

Example 09 - Problem (2)

A total of 6000 hours of skilled labor are available (and have already been paid for). All furniture produced can be sold at the following unit prices: unfinished table, \$70; finished table, \$140; unfinished chair, \$60; finished chair, \$110. Formulate an LP that will maximize the contribution to profit from manufacturing tables and chairs.

Example 09 - Analisis

The solution asked is about maximize the contribution to profit from manufacturing tables and chairs. Each unfinished table can be sold at \$70 (70*UT), each finished table at \$140 (140*FT), an unfinished chair can be sold at \$60 (60*UC) and a finished chair at \$110 (110*FC). But we need to deduct from these profits the cost of buying the wood require to manufacture the chairs and tables. Each board ft of wood may be purchased at \$1, so a table requires 40 board ft that means \$40*(FT + UT); a chair requires 30 board ft of wood so: \$30*(FC + UC). Now putting all together we have that the function to maximize is MAX Z= 70UT + 140FT + 60UC + 110FC-40(FT+UT)-30(FC+UC).

Example 09 - Variables

$UT \longrightarrow$ unfinished table

$FT \longrightarrow$ finished table

$UC \longrightarrow$ unfinished chair

$FC \longrightarrow$ finished char

Example 09 - Function

Maximize the contribution to Furnco profit from manufacturing tables and chairs:

$$Z = 70UT + 140FT + 60UC + 110FC - 40(FT + UT) - 30(FC + UC)$$

Example 09 - Restrictions

There is a restriction about the total of wood available. A table requires 40 board ft of wood to be manufactured, a chair requires 30 board ft and there are only 40000 board ft of wood available, that means that $40(FT + UT) + 30(FC + UC) \leq 40000$. Also we could see a time restriction on the hours of skilled labor available, there are 6000 hours available; manufacturing an unfinished table or an unfinished chairs takes 2 hours and 3 more hours of skilled labor will turn an unfinished table into a finished table (5 in total), and 2 more hours of skilled labor will turn an unfinished char into a finished chair (4 in total). That means that the sum of these hours must be less or equal than the 6000:
$5FT + 2UT + 4FC + 2UC \leq 6000$.

Example 09 - Model

Maximize:

$$
\begin{aligned}
Z =& 70UT + 140FT + 60UC + 110FC - \\
& 40(FT + UT) - 30(FC + UC)
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
40(FT + UT) + 30(FC + UC) &\leq 40000 \\
5FT + 2UT + 4FC + 2UC &\leq 6000
\end{aligned}
$$

Example 09 - Solution (1)

LINGO Model - model_9

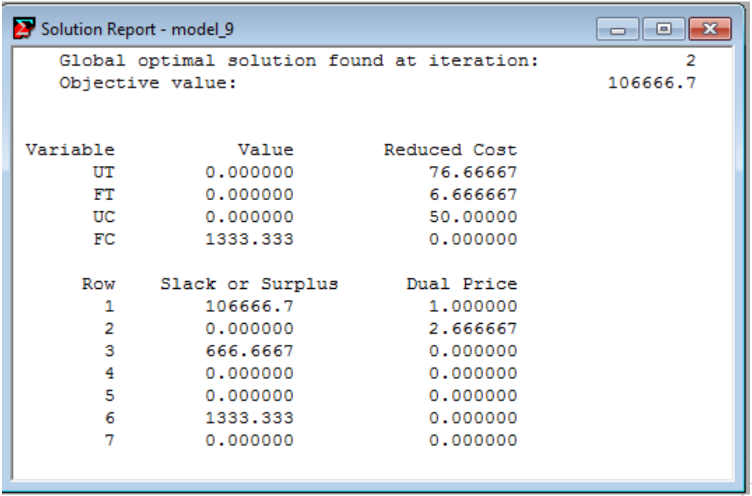
```
MAX = 70*UT + 140*FT + 60*UC + 110*FC - 40*(FT + UT) - 30*(FC + UC);

40*(FT + UT) +30*(FC + UC) <= 40000;
5*FT + 2*UT + 4*FC + 2*UC <= 6000;

FT >= 0;
UT >= 0;
FC >= 0;
UC >= 0;
```

Figure: Example 09 model and solution in Lingo Software.

Example 09 - Solution (2)



Solution Report - model_9

Global optimal solution found at iteration: 2
Objective value: 106666.7

Variable	Value	Reduced Cost
UT	0.000000	76.66667
FT	0.000000	6.666667
UC	0.000000	50.00000
FC	1333.333	0.000000

Row	Slack or Surplus	Dual Price
1	106666.7	1.000000
2	0.000000	2.666667
3	666.6667	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	1333.333	0.000000
7	0.000000	0.000000

Figure: Example 09 model and solution in Lingo Software.

Example 09 - Final solution

Global optimal solution found at iteration: 2
Objective value: 106666.7

Variable	Value	Reduced Cost
UT	0.000000	76.66667
FT	0.000000	6.666667
UC	0.000000	50.00000
FC	1333.333	0.000000
————	————	————
Row	Slack or Surplus	Dual Price
1	106666.7	1.000000
2	0.000000	2.666667
3	666.6667	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	1333.333	0.000000
7	0.000000	0.000000

Example 09 - Final analisis

The optimal solution given by LINGO says that Furnco must manufacture **0** unfinished tables, **0** finished tables, **0** unfinished chairs and **1333.333** finished chairs. It will generate the optimal profit of **$106666.7**. Each unfinished table manufactured will reduce the profit by $76.66667, each finished table by $6.66667 and each unfinished table by $50.

Example 10

Example 10 - Problem (1)

A company produces six products in the following fashion. Each unit of raw material purchased yields four units of product 1, two units of product 2, and one unit of product 3. Up to 1200 units of product 1 can be sold, and up to 300 units of product 2 can be sold. Each unit of product 1 can be sold or processed further. Each unit of product 1 that is processed yields a unit of product 4. Demand for products 3 and 4 is unlimited. Each unit of product 2 can be sold or processed further. Each unit of product 2 that is processed further yields 0.8 unit of product 5 and 0.3 unit of product 6. Up to 1000 units of product 5 can be sold, and up to 800 units of product 6 can be sold. Up to 3000 units of raw material can be purchased at $6 per unit.

Example 10 - Problem (2)

Leftover units of products 5 and 6 must be destroyed. It costs \$4 to destroy each leftover unit of product 5 and \$3 to destroy each leftover unit of product 6. Ignoring raw material purchase costs, the per-unit sales price and production costs for each product are shown in the following table. Formulate an LP whose solution will yield a profit-maximizing production schedule.

Product	**Sales price**	**Production cost**
Product 1	\$7	\$4
Product 2	\$6	\$4
Product 3	\$4	\$2
Product 4	\$3	\$1
Product 5	\$20	\$5
Product 6	\$35	\$5

TEC

Example 10 - Analisis

A company produces six products in the following fashion. Each unit of raw material purchased yields four units of product 1, two units of product 2, and one unit of product 3. We need to maximize the profit so we need to get the sum of (Production Costs - Unit Sale Price) * Product for each product produced, minus the cost of destroying product 5 and product 6 and the cost of raw material.

Example 10 - Variables

$$x_i$$

$x \longrightarrow$ amount of product to produce
$i \longrightarrow$ number of product to produce [1;6]

$$d_i$$

$d \longrightarrow$ amount of product to destroy
$i \longrightarrow$ number of product to destroy [1;6]

$$m$$

$m \longrightarrow$ amount of material to buy

Example 10 - Function

Get a profit-maximizing production schedule:

$$\begin{aligned} Z = &3x_1 + 2x_2 + 2x_3 + \\ &2x_4 + 15x_5 + 30x_6 - \\ &4d_1 - 3d_2 - 6m \end{aligned}$$

Example 10 - Restrictions

This problem has many restrictions, so we will enumerate them accordingly to their position in the next slide (model):

1. Restricts the amount of material that can be bought.
2. Restricts the amount of products that M can generate.
3. Another equation that restricts the amount of products that M can generate.
4. And yet another one.
5. Restricts the amount of type 1 products to create.
6. Restricts the amount of type 2 products to create.
7. Restricts the amount of type 5 products to create.
8. Restricts the amount of type 6 products to create.
9. The final and trivial restrictions.

Example 10 - Model

Maximize:

$$Z = 3x_1 + 2x_2 + 2x_3 + 2x_4 + 15x_5 + 30x_6 - 4d_1 - 3d_2 - 6m$$

Subject to:

$$\begin{aligned}
m &\geq 3000 \\
4m - x_1 - x_4 &= 0 \\
2m - x_2 - x_5 - x_6 - d_1 - d_2 &= 0 \\
m - x_3 &= 0 \\
x_1 &\leq 1200 \\
x_2 &\leq 300 \\
x_5 &\leq 1000 \\
x_6 &\leq 800 \\
d_1, d_2 &\geq 0
\end{aligned}$$

Example 10 - Solution (1)

LINGO Model - model_10

```
MAX = 3*x1 + 2*x2 + 2*x3 + 2*x4 + 15*x5 + 30*x6 -4*d1 - 3*d2 -6*m;

m >= 3000;
4*m - x1 - x4 = 0;
2*m - x2 - x5 - x6 - d1 - d2 = 0;
m - x3 = 0;
x1 <= 1200;
x2 <= 300;
x5 <= 1000;
x6 <= 800;

x1 >= 0;
x2 >= 0;
x3 >= 0;
x4 >= 0;
x5 >= 0;
x6 >= 0;

d1 >= 0;
d2 >= 0;
m >= 0;
```

Figure: Example 10 model and solution in Lingo Software.

Example 10 - Solution (2)

Solution Report - model_10

Global optimal solution found at iteration: 0

Objective value: 41100.00

Variable	Value	Reduced Cost
X1	1200.000	0.000000
X2	300.0000	0.000000
X3	3000.000	0.000000
X4	10800.00	0.000000
X5	1000.000	0.000000
X6	800.0000	0.000000
D1	0.000000	1.000000
D2	3900.000	0.000000
M	3000.000	0.000000

Figure: Example 10 model and solution in Lingo Software.

Example 10 - Solution (3)

Solution Report - model_10

Row	Slack or Surplus	Dual Price
1	41100.00	1.000000
2	0.000000	-2.000000
3	0.000000	-2.000000
4	0.000000	3.000000
5	0.000000	-2.000000
6	0.000000	1.000000
7	0.000000	5.000000
8	0.000000	18.00000
9	0.000000	33.00000
10	1200.000	0.000000
11	300.0000	0.000000
12	3000.000	0.000000
13	10800.00	0.000000
14	1000.000	0.000000
15	800.0000	0.000000
16	0.000000	0.000000
17	3900.000	0.000000
18	3000.000	0.000000

Figure: Example 10 model and solution in Lingo Software.

Example 10 - Final solution (1)

Global optimal solution found at iteration: 0
Objective value: 41100.00

Variable	Value	Reduced Cost
X1	1200.000	0.000000
X2	300.0000	0.000000
X3	3000.000	0.000000
X4	10800.00	0.000000
X5	1000.000	0.000000
X6	800.0000	0.000000
D1	0.000000	1.000000
D2	3900.000	0.000000
M	3000.000	0.000000

Example 10 - Final solution (2)

Row	Slack or Surplus	Dual Price
1	41100.00	1.000000
2	0.000000	-2.000000
3	0.000000	-2.000000
4	0.000000	3.000000
5	0.000000	-2.000000
6	0.000000	1.000000
7	0.000000	5.000000
8	0.000000	18.00000
9	0.000000	33.00000

TEC

Example 10 - Final solution (3)

Row	Slack or Surplus	Dual Price
10	1200.000	0.000000
11	300.0000	0.000000
12	3000.000	0.000000
13	10800.00	0.000000
14	1000.000	0.000000
15	800.0000	0.000000
16	0.000000	0.000000
17	3900.000	0.000000
18	3000.000	0.000000

Example 10 - Final analisis

The optimal solution given by LINGO says that the company must produce **1200** type 1 products, **300** type 2 products, **3000** type 3 products, **10800** type 4 products, **1000** type 5 products and **800** type 6 products. Also it has to destroy **3900** type 2 products and **0** type 1 products; if this happens, profits will be reduced by $1. Also the company must buy **3000** units of raw material. The optimal profits will be of **$41100**.

Example 11

Example 11 - Problem (1)

Gandhi Clothing Company produces shirts and pants. Each shirt requires 2 sq yd of cloth, each pair of pants, 3. During the next two months, the following demands for shirts and pants must be met (on time): month 1-10 shirts, 15 pairs of pants; month 2-12 shirts, 14 pairs of pants. During each month, the following resources are available: month 1-90 sq yd of cloth; month 2 -60 sq yd. (Cloth that is available during month 1 may, if unused during month 1, be used during month2.)

Example 11 - Problem (2)

During each month, it costs \$4 to make an article of clothing with regular-time labor and \$8 with overtime labor. During each month, a total of at most 25 articles of clothing may be produced with regular-time labor, and an unlimited number of articles of clothing may be produced with overtime labor. At the end of each month, a holding cost of \$3 per article of clothing is assessed. Formulate an LP that can be used to meet demands for the next two months (on time) at minimum cost. Assume that at the beginning of month 1, 1 shirt and 2 pairs of pants are available.

Example 11 - Analisis

Gandhi Clothing Company need to meet demands por the next two months (on time) at minimum cost. So we could express that as the sum of (Cost of each article + the holding cost)*Amount of shirts/pants

Example 11 - Variables

C_i Ce_i P_i Pe_i

$C \longrightarrow$ amount of shirts to make at normal working time
$i \longrightarrow$ number of the month

$Ce \longrightarrow$ amount of shirts to make at extra working time
$i \longrightarrow$ number of the month

$P \longrightarrow$ amount of pants to make at normal working time
$i \longrightarrow$ number of the month

$Pe \longrightarrow$ amount of pants to make at extra working time
$i \longrightarrow$ number of the month

Example 11 - Function

Gandhi Clothing need to meet demands for the next two months (on time) at minimum cost. So there is needed to minimize:

$$Z = 7C_1 + 7P_1 + 11Ce_1 + 11Pe_1 + 7C_2 + 7P_2 + 11Ce_2 + 11Pe_2$$

Example 11 - Restrictions

This problem has many restrictions, so we will enumerate them accordingly to their position in the next slide (model):

1. Restricts the amount of clothing to produce in the first month.
2. Restricts the amount of of shits to produce in the first month.
3. Restricts the amount of pants to produce in the first month.
4. Restricts the amount of cloth available in the first month.
5. Restricts the amount of clothing to produce in the second month.
6. Restricts the amount of of shits to produce in the second month.
7. Restricts the amount of pants to produce in the second month.
8. Restricts the amount of cloth available in the second month plus what was leftover the first month.
9. Trivial restrictions.

TEC

Example 11 - Model

Minimize:

$$Z = 7C_1 + 7P_1 + 11Ce_1 + 11Pe_1 + 7C_2 + 7P_2 + 11Ce_2 + 11Pe_2$$

Subject to:

$$\begin{aligned}
4C_1 + 4P_1 &\leq 25 \\
C_1 + Ce_1 &= 9 \\
P_1 + Pe_1 &= 13 \\
2C_1 + 2Ce_1 + 3P_1 + 3Pe_1 &\leq 90 \\
4C_2 + 4P_2 &\leq 25 \\
C_2 + Ce_2 &= 12 \\
P_2 + Pe_2 &= 14 \\
2C_1 + 2Ce_1 + 3P_1 + 3Pe_1 + 2C_2 + 2Ce_2 + 3P_2 + 3Pe_2 &\leq 150 \\
All \quad variables &\geq 0
\end{aligned}$$

Example 11 - Solution (1)

LINGO Model - model_11

```
MIN = 7*c1 + 7*p1 + 11*ce1 + 11*pe1 + 7*c2 + 7*p2 + 11*ce2 + 11*pe2;

4*c1 + 4*p1 <= 25;
c1 + ce1 = 9;
p1 + pe1 = 13;
2*c1 + 2*ce1 + 3*p1 + 3*pe1 <= 90;

4*c2 + 4*p2 <= 25;
c2 + ce2 = 12;
p2 + pe2 = 14;
2*c2 + 2*ce2 + 3*p2 + 3*pe2 + 2*c1 + 2*ce1 + 3*p1 + 3*pe1 <= 60 + 90;
c1 >= 0;
c2 >= 0;
ce1 >= 0;
ce2 >= 0;
p1 >= 0;
p2 >= 0;
pe1 >= 0;
pe2 >= 0;
```

Figure: Example 11 model and solution in Lingo Software.

Example 11 - Solution (2)

Solution Report - model_11

```
Global optimal solution found at iteration:          0
Objective value:                              478.0000
```

Variable	Value	Reduced Cost
C1	0.000000	0.000000
P1	6.250000	0.000000
CE1	9.000000	0.000000
PE1	6.750000	0.000000
C2	0.000000	0.000000
P2	6.250000	0.000000
CE2	12.00000	0.000000
PE2	7.750000	0.000000

Figure: Example 11 model and solution in Lingo Software.

Example 11 - Solution (3)

Solution Report - model_11

Row	Slack or Surplus	Dual Price
1	478.0000	-1.000000
2	0.000000	1.000000
3	0.000000	-11.00000
4	0.000000	-11.00000
5	33.00000	0.000000
6	0.000000	1.000000
7	0.000000	-11.00000
8	0.000000	-11.00000
9	27.00000	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	9.000000	0.000000
13	12.00000	0.000000
14	6.250000	0.000000
15	6.250000	0.000000
16	6.750000	0.000000
17	7.750000	0.000000

Figure: Example 11 model and solution in Lingo Software.

Example 11 - Final solution (1)

Global optimal solution found at iteration: 0
Objective value: 478.0000

Variable	Value	Reduced Cost
C1	0.000000	0.000000
P1	6.250000	0.000000
CE1	9.000000	0.000000
PE1	6.750000	0.000000
C2	0.000000	0.000000
P2	6.250000	0.000000
CE2	12.00000	0.000000
PE2	7.750000	0.000000

Example 11 - Final solution (2)

Variable Row	Slack or Surplus	Dual Price
1	478.0000	-1.000000
2	0.000000	1.000000
3	0.000000	-11.00000
4	0.000000	-11.00000
5	33.00000	0.000000
6	0.000000	1.000000
7	0.000000	-11.00000
8	0.000000	-11.00000
9	27.00000	0.000000

Example 11 - Final solution (3)

Row	Slack or Surplus	Dual Price
10	0.000000	0.000000
11	0.000000	0.000000
12	9.000000	0.000000
13	12.00000	0.000000
14	6.250000	0.000000
15	6.250000	0.000000
16	6.750000	0.000000
17	7.750000	0.000000

Example 11 - Final analisis

The optimal cost of meeting demands will be of **$478** according to the solution given by LINGO. It says that Gandhi Cloth Company must make **0 shirts** and **6.25** pants at normal working time, **9** shirts and **6.75** pants at extra working time on the first month. Also the company must make **0** shirts and **6.35** pants at normal working time, **12** shirts and **7.75** pants at extra working time on the second month.

TEC

Example 12

Example 12 - Problem (1)

An insurance company believes that it will require the following numbers of personal computers during the next six months: January, 9; February, 5; March, 7; April, 9; May, 10; June, 5. Computers can be rented for a period of one, two or three months at the following unit rates: one-month rate, \$200; two-month rate, \$350; three-month rate \$450.

Example 12 - Problem (2)

Formulate an LP that can be used to minimize the cost of renting the required computers. You may assume that if a machine is rented for a period of time extending beyond June, the cost of the rental should be prorated. For example if a computer is rented for three months at the beginning of May, then a rental fee of $(2/3)(450) = \$300$, not \$450, should be assessed in the objective function.

Example 12 - Analisis

An insurance company requires to rent som personal computers, so the target is to minimize the cost of renting the required computers, that means to minimize the sum of Cost * Amount of Computer Required per month and period of renting

Example 12 - Variables

$$X_{i,j}$$

$X \longrightarrow$ amount of computers rented

$i \longrightarrow$ number of the month

$j \longrightarrow$ period of months for which the computer is rented

Example 12 - Function

Minimize the cost of renting the required computers by the insurance company:

$$\begin{aligned} Z = {} & 200X_{11} + 350X_{12} + 450X_{13} + 200X_{21} + 350X_{22} + 450X_{23} + \\ & 200X_{31} + 350X_{32} + 450X_{33} + 200X_{41} + 350X_{42} + 450X_{43} + \\ & 200X_{51} + 350X_{52} + 450X_{53} + 200X_{61} + 175X_{62} + 150X_{63} \end{aligned}$$

Example 12 - Restrictions

This problem has many restrictions, so we will enumerate them accordingly to their position in the next slide (model):

1. Restricts the amount of computers to lend in january.
2. Restricts the amount of computers to lend in february.
3. Restricts the amount of computers to lend in march.
4. Restricts the amount of computers to lend in april.
5. Restricts the amount of computers to lend in may.
6. Restricts the amount of computers to lend in june.
7. Trivial restrictions.

Example 12 - Model

Minimize:

$$\begin{aligned} Z = & 200X_{11} + 350X_{12} + 450X_{13} + 200X_{21} + 350X_{22} + 450X_{23} + \\ & 200X_{31} + 350X_{32} + 450X_{33} + 200X_{41} + 350X_{42} + 450X_{43} + \\ & 200X_{51} + 350X_{52} + 450X_{53} + 200X_{61} + 175X_{62} + 150X_{63} \end{aligned}$$

Subject to:

$$\begin{aligned} X_{11} + X_{12} + X_{13} &= 9 \\ X_{21} + X_{22} + X_{23} + X_{12} + X_{13} &= 5 \\ X_{31} + X_{32} + X_{33} + X_{22} + X_{23} + X_{13} &= 7 \\ X_{41} + X_{42} + X_{43} + X_{32} + X_{33} + X_{23} &= 9 \\ X_{51} + X_{52} + X_{53} + X_{42} + X_{43} + X_{33} &= 10 \\ X_{61} + X_{62} + X_{63} + X_{52} + X_{53} + X_{43} &= 5 \end{aligned}$$

Example 12 - Solution (1)

LINGO Model - model_12

```
MIN = 200*x11 + 350*x12 + 450*x13 +
      200*x21 + 350*x22 + 450*x23 +
      200*x31 + 350*x32 + 450*x33 +
      200*x41 + 350*x42 + 450*x43 +
      200*x51 + 350*x52 + 450*x53 +
      200*x61 + 175*x62 + 150*x63;


x11 + x12 + x13 = 9;
x21 + x22 + x23 + x12 + x13 =5;
x31 + x32 + x33 + x22 + x23 + x13 =7;
x41 + x42 + x43 + x32 + x33 + x23 = 9;
x51 + x52 + x53 + x42 + x43 + x33 = 10;
x61 + x62 + x63 + x52 + x53 + x43 = 5;
```

Figure: Example 12 model and solution in Lingo Software.

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Example 12 - Solution (2)

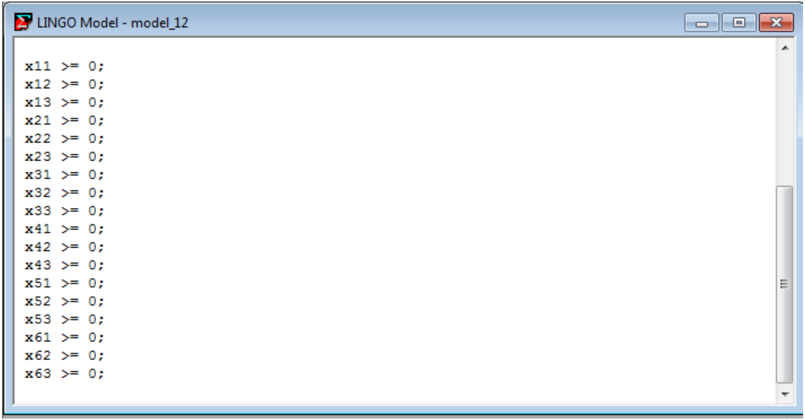

Figure: Example 12 model and solution in Lingo Software.

Example 12 - Solution (3)

Solution Report - model_12

Global optimal solution found at iteration: 3
Objective value: 7100.000

Variable	Value	Reduced Cost
X11	4.000000	0.000000
X12	2.000000	0.000000
X13	3.000000	0.000000
X21	0.000000	50.00000
X22	0.000000	100.0000
X23	0.000000	50.00000
X31	0.000000	100.0000
X32	0.000000	100.0000
X33	4.000000	0.000000
X41	0.000000	50.00000
X42	0.000000	0.000000
X43	5.000000	0.000000
X51	1.000000	0.000000
X52	0.000000	50.00000
X53	0.000000	150.0000
X61	0.000000	100.0000
X62	0.000000	75.00000
X63	0.000000	50.00000

Figure: Example 12 model and solution in Lingo Software.

Example 12 - Solution (4)

Solution Report - model_12

Row	Slack or Surplus	Dual Price
1	7100.000	-1.000000
2	0.000000	-200.0000
3	0.000000	-150.0000
4	0.000000	-100.0000
5	0.000000	-150.0000
6	0.000000	-200.0000
7	0.000000	-100.0000
8	4.000000	0.000000
9	2.000000	0.000000
10	3.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	4.000000	0.000000
17	0.000000	0.000000
18	0.000000	0.000000
19	5.000000	0.000000
20	1.000000	0.000000
21	0.000000	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	0.000000	0.000000

Figure: Example 12 model and solution in Lingo Software.

Example 12 - Final solution (1)

Global optimal solution found at iteration: 3
Objective value: 7100.000

Variable	Value	Reduced Cost
X11	4.000000	0.000000
X12	2.000000	0.000000
X13	3.000000	0.000000
X21	0.000000	50.00000
X22	0.000000	100.0000
X23	0.000000	50.00000
X31	0.000000	100.0000
X32	0.000000	100.0000
X33	4.000000	0.000000

Example 12 - Final solution (2)

Variable	Value	Reduced Cost
X41	0.000000	50.00000
X42	0.000000	0.000000
X43	5.000000	0.000000
X51	1.000000	0.000000
X52	0.000000	50.00000
X53	0.000000	150.0000
X61	0.000000	100.0000
X62	0.000000	75.00000
X63	0.000000	50.00000

Example 12 - Final solution (3)

Row	Slack or Surplus	Dual Price
1	7100.000	-1.000000
2	0.000000	-200.0000
3	0.000000	-150.0000
4	0.000000	-100.0000
5	0.000000	-150.0000
6	0.000000	-200.0000
7	0.000000	-100.0000
8	4.000000	0.000000
9	2.000000	0.000000
10	3.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000

Example 12 - Final solution (4)

Row	Slack or Surplus	Dual Price
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	4.000000	0.000000
17	0.000000	0.000000
18	0.000000	0.000000
19	5.000000	0.000000
20	1.000000	0.000000
21	0.000000	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	0.000000	0.000000

Example 12 - Final analisis

The optimal solution according to LINGO is renting the following amount of computers:

- January: 4 for a period of 1 month, 2 for a period of 2 months and 3 for a period of 3 months.
- February: 0 computers.
- March: 4 for a period of 3 months.
- April: 5 for a period of 3 months.
- May: 1 for a period of 1 month.
- June: 0 computers.

The optimal cost is **$7100**. By renting computers in february it will increase the cost in $50, $100, or $50 depending of the renting period. Same happens in June and it will increase the cost in $100, $75 or $50.

Example 13

Example 13 - Problem (1)

Sunco processes oil into aviation fuel and heating oil. It costs \$40 to purchase each 1000 barrels of oil, which is then distilled and yields 500 barrels of aviation fuel and 500 barrels of heating oil. Output from the distillation may be sold directly or processed in the catalytic cracker. If sold after distillation without further processing, aviation fuel sells for \$60 per 1000 barrels, and heating oil sells for \$40 per 1000 barrels.

Example 13 - Problem (2)

It takes 1 hour to process 1000 barrels of aviation fuel in the catalytic cracker, and these 1000 barrels can be sold or \$130. It takes 45 minutes to process 1000 barrels of heating oil in the cracker, and these 1000 barrels can be sold for \$90. Each day, at most 20000 barrels of oil can be purchased, and 8 hours of cracker time are available. Formulate an LP to maximize Sunco's profits.

Example 13 - Analisis

Sunco needs to maximize the profits of processing oil into aviation fuel and heating oil. So basically it's about the sum of the price * Product minus the cost of purchasing the oil. that means something like 40 * Heating Oil + 90 * Heating Oil Processed + 130 * Fuel Processed + 60 * Fuel - 40 * Oil

Example 13 - Variables

$O \longrightarrow$ thousand barrels of oil purchased

$HO \longrightarrow$ thousand barrels of heating oil

$HOP \longrightarrow$ thousand barrels of heating oil processed at the catalytic cracker

$F \longrightarrow$ thousand barrels of aviation fuel

$FP \longrightarrow$ thousand barrels of aviation fuel processed at the catalytic cracker

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Example 13 - Function

Maximize Sunco's profits by processing aviation fuel and heating oil:

$$Z = -40O + 40HO + 90HOP + 60F + 130FP$$

Example 13 - Restrictions

A maximum of 20000 barrels of oil can be bought daily, which means $O \leq 20000$. Every thousand barrels of oil after being distilled generate 500 barrels of airplane fuel and 500 heating oil, which means the each constitutes a 50% of product from the distillation, obtaining that $F + FP = 0.5O$ and $HO + HOP = 0.5O$. There is also a time restriction in minutes, for it takes an hour to process the airplane fuel (60 minutes) and 45 minutes the heating oil and Sunco has 8 hours available to do this process, that is $60FP + 45HOP \leq 480$. Last but not least the trivial restrictions that all the variables must be higher or equal to 0.

Example 13 - Model

Maximize:

$$Z = -40O + 40HO + 90HOP + 60F + 130FP$$

Subject to:

$$\begin{aligned} O &\leq 20000 \\ F + FP &= 0.5O \\ HO + HOP &= 0.5O \\ 60FP + 45HOP &\leq 480 \\ O &\geq 0 \\ HO &\geq 0 \\ HOP &\geq 0 \\ FP &\geq 0 \\ F &\geq 0 \end{aligned}$$

LINGO Model - model_12

```
MAX = 40*HO + 90*HOP + 130*FP + 60*F - 40*O;

O <= 20;
F + FP = 0.5*O;
HO + HOP = 0.5*O;
60*FP + 45*HOP <= 480;
O >= 0;
HO >= 0;
HOP >= 0;
FP >= 0;
F >= 0;
```

Solution Report - model_12

```
Global optimal solution found at iteration:         0
Objective value:                              760.0000
```

Variable	Value	Reduced Cost
HO	10.00000	0.000000
HOP	0.000000	2.500000
FP	8.000000	0.000000
F	2.000000	0.000000
O	20.00000	0.000000

Row	Slack or Surplus	Dual Price
1	760.0000	1.000000
2	0.000000	10.00000
3	0.000000	60.00000
4	0.000000	40.00000
5	0.000000	1.166667
6	20.00000	0.000000
7	10.00000	0.000000
8	0.000000	0.000000
9	8.000000	0.000000
10	2.000000	0.000000

Figure: Example 13 model and solution in Lingo Software.

Example 13 - Final solution

Global optimal solution found at iteration: 0
Objective value: 760.0000

Variable	Value	Reduced Cost
HO	10.00000	0.000000
HOP	0.000000	2.500000
FP	8.000000	0.000000
F	2.000000	0.000000
O	20.00000	0.000000

Row	Slack or Surplus	Dual Price
1	760.0000	1.000000
2	0.000000	10.00000
3	0.000000	60.00000
4	0.000000	40.00000
5	0.000000	1.166667
6	20.00000	0.000000
7	10.00000	0.000000
8	0.000000	0.000000
9	8.000000	0.000000
10	2.000000	0.000000

Example 13 - Final analisis

According to the solution given by LINGO, Sunco must process **10000** barrels of heating oil, **0** barrels of heating oil must be process at the catalytic cracker, **8000** barrels of jet fuel must be process at the catalytic cracker and **2000** of fuel needs to be processed. Also the company has to purchase **20000** barrels of oil. With these amounts, Sunco has profits of **$760.0000**. If they proccess barrels of heating oil theses profits will decrease by $2.5 per barrel.

Example 14

Example 14 - Problem

Steelco manufactures two types of steel at three different steel mills. During a given month, each steel mill has 200 hours of blast furnace time available. Because of differences in the furnaces at each mill, the time and cost to produce a ton of steel differs for each mill. The time and cost for each mill are shown in the following table.

Mill	Cost	Time (min)	Cost	Time (min)
Mill 1	\$10	20	\$11	22
Mill 1	\$12	24	\$9	18
Mill 1	\$14	28	\$10	30

Each month, Steelco must manufacture at least 500 tons of steel 1 and 600 tons of steel 2. Formulate an LP to minimize the cost of manufacturing the desired steel.

Example 14 - Analisis

We must minimize the cost of the desired steel so with the given costs, we must do something like minimize the sum of Cost * Steel per Mill for each Mill.

Example 14 - Variables

$$X_{i,j}$$

$X \longrightarrow$ amount of steel j manufactured at mill i

$i \longrightarrow$ mill number [1;3]

$j \longrightarrow$ steel number [1;2]

Example 14 - Function

Minimize the cost of Steelco of manufacturing the two types of steel:

$$Z = 10X_{11} + 11X_{12} + 12X_{21} + 9X_{22} + 14X_{31} + 10X_{32}$$

Example 14 - Restrictions (1)

In the restrictions every month Steelco must manufacture at least 500 tons of steel 1 and 600 of 2, which states that: $X_{11} + X_{21} + X_{31} \geq 500$ and $X_{12} + X_{22} + X_{32} \geq 600$. Also there is a restriction of baking time available. The oven of each mill is available for 200 hours that in minutes would be: $200 * 60 = 12000$ minutes per mill; and according to the data from the board about duration of steel baking on each mill would be that in the mill 1 the steel 1 needs 20 minutes and the steel 2, 22 minutes; in the mill 2 the steel 1 needs 24 minutes and the steel 2 needs 18 minute; finally in the mill 3 the steel 1 needs 28 minutes and the steel 2 30 minutes.

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Example 14 - Restrictions (2)

The last statements about time can be divided in 3 restrictions (One per mill) as follows:

$$20X_{11} + 22X_{12} \leq 12000$$
$$24X_{21} + 18X_{22} \leq 12000$$
$$28X_{31} + 30X_{32} \leq 12000$$
$$X_{11} + X_{21} + X_{31} \geq 500$$

Last there is the trivial restriction that all variables must be higher or equal to 0 for it can't be negative steel.

Example 14 - Model

Minimize:

$$Z = 10X_{11} + 11X_{12} + 12X_{21} + 9X_{22} + 14X_{31} + 10X_{32}$$

Subject to:

$$\begin{aligned}
20X_{11} + 22X_{12} &\leq 12000 \\
24X_{21} + 18X_{22} &\leq 12000 \\
28X_{31} + 30X_{32} &\leq 12000 \\
X_{11} + X_{21} + X_{31} &\geq 500 \\
X_{12} + X_{22} + X_{32} &\geq 600 \\
X_{11}, X_{12}, X_{21}, X_{22}, X_{31}, X_{32} &\geq 0
\end{aligned}$$

Example 14 - Solution (1)

LINGO Model - model_13

```
MIN = 10*x11 + 11*x12 + 12*x21 + 9*x22 + 14*x31 + 10*x32;

20*x11 + 22*x12 <= 12000;
24*x21 + 18*x22 <= 12000;
28*x31 + 0*x32 <= 12000;

x11 + x21 + x31 >= 500;
x12 + x22 + x32 >= 600;

x11 >= 0;
x12 >= 0;
x21 >= 0;
x22 >= 0;
x31 >= 0;
```

Figure: Example 14 model and solution in Lingo Software.

Example 14 - Solution (2)

Solution Report - model_13

Global optimal solution found at iteration: 3

Objective value: 10400.00

Variable	Value	Reduced Cost
X11	500.0000	0.000000
X12	0.000000	2.000000
X21	0.000000	2.000000
X22	600.0000	0.000000
X31	0.000000	4.000000
X32	0.000000	1.000000

Row	Slack or Surplus	Dual Price
1	10400.00	-1.000000
2	2000.000	0.000000
3	1200.000	0.000000
4	12000.00	0.000000
5	0.000000	-10.00000
6	0.000000	-9.000000
7	500.0000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	600.0000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000

Figure: Example 14 model and solution in Lingo Software.

Example 14 - Final solution (1)

Global optimal solution found at iteration: 3
Objective value: 10400

Variable	Value	Reduced Cost
X11	500.0000	0.000000
X12	0.000000	2.000000
X21	0.000000	2.000000
X22	600.0000	0.000000
X31	0.000000	4.000000
X32	0.000000	1.000000

Example 14 - Final solution (2)

Row	Slack or Surplus	Dual Price
1	10400.00	-1.000000
2	2000.000	0.000000
3	1200.000	0.000000
4	12000.00	0.000000
5	0.000000	-10.0000
6	0.000000	-9.000000
7	500.0000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	600.0000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000

Example 14 - Final analisis

According to LINGO the optimal cost of manufacturing the different types of steel is **$10400**. It must be manufactured **500** tons of steel 1 at mill 1 and **600** tons of steel 2 at mill 2. If some steel 2 is manufactured at mill 1 it will increase costs by $2 per ton, same happens manufacturing steel 1 at mill 2. And finally, no (**0** tons) steel must be manufactured at mill 3 or it will increase cost by $4 (Steel 1) and $1 (Steel 2) per ton.

Example 15

Example 15 - Problem (1)

Kiriakis Electronics produces three products. Each product must be processed on each of three types of machines. When a machine is in use, it must be manned by a worker. The time (in hours) required to process each product on each machine and the profit associated with each product are shown in following table.

Machine	**Product 1**	**Product 2**	**Product 3**
Machine 1	2	3	4
Machine 2	3	5	6
Machine 3	4	7	9
Profit	\$6	\$8	\$10

Example 15 - Problem (2)

At present, five type 1 machines, three type 2 machines, and four type 3 machines are available. The company has ten workers available and must determine how many workers to assign to each machine. The plant is open 40 hours per week, and each worker works 35 hours per week. Formulate an LP that will enable Kiriakis to assign workers to machines in a way that maximizes weekly profits.

Note: a worker need not spend the entire work week manning a single machine.

Example 15 - Analisis

Kiriakis wants to maximize weekly profits and each type of product yields different profits.Product 1 yields \$6, product 2 yields \$8 and product 3 yields \$10 . So it's about maximize the sum of Product Profit * Product_i, where i refers to type of product.

Example 15 - Variables

$X_1 \longrightarrow$ amount of type 1 products to produce

$X_2 \longrightarrow$ amount of type 2 products to produce

$X_3 \longrightarrow$ amount of type 3 products to produce

Example 15 - Function

Enable Kiriakis Electronics to assign workers to machines in a way that maximizes weekly profits:

$$Z = 6X_1 + 8X_2 + 10X_3$$

Example 15 - Restrictions (1)

The first restriction is given by the hours that are required for each product in the machine 1. Knowing that there are 5 machines of this type available and that the factory is open 40 hours, therefore with the hours given in the board of the machine 1 we have that 2 $2X_1 + 3X_2 + 4X_3 \leq 200$. Similarly a restriction of the same nature is obtained for the other remaining two machines: for machine 2 we have $3X_1 + 5X_2 + 6X_3 \leq 120$ for there are 3 machines available. For the machine 3 we have $4X_1 + 7X_2 + 9X_3 \leq 160$ for there are 4 machines available.

Example 15 - Restrictions (2)

There is also the restriction of each worker working a total of 35 hours per week for a total of 10 workers and when each product is in a machine it must be handled by a worker, because of this all the hours required by product are added, so for product 1 9 hours are required; for product 2, 15 hours and for product 3 19 hours from what we obtain: $9X_1 + 15X_2 + 19X_3 \leq 350$. Finally there is the trivial restriction that no negative products can exits.

Example 15 - Model

Maximize:

$$Z = 6X_1 + 8X_2 + 10X_3$$

Subject to:

$$\begin{aligned} 2X_1 + 3X_2 + 4X_3 &\leq 200 \\ 3X_1 + 5X_2 + 6X_3 &\leq 120 \\ 4X_1 + 7X_2 + 9X_3 &\leq 160 \\ 9X_1 + 15X_2 + 19X_3 &\leq 350 \\ X_1, X_2, X_3 &\geq 0 \end{aligned}$$

Example 15 - Solution

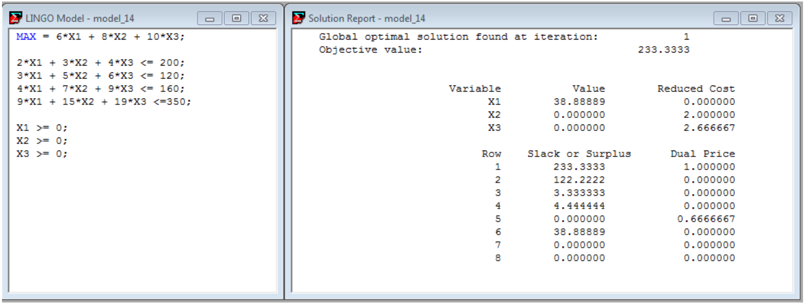

Figure: Example 15 model and solution in Lingo Software.

Example 15 - Final solution

Global optimal solution found at iteration: 1
Objective value: 233.3333

Variable	Value	Reduced Cost
X1	38.88889	0.000000
X2	0.000000	2.000000
X3	0.000000	2.666667

Row	Slack or Surplus	Dual Price
1	233.3333	1.000000
2	122.2222	0.000000
3	3.333333	0.000000
4	4.444444	0.000000
5	0.000000	0.6666667
6	38.88889	0.000000
7	0.000000	0.000000
8	0.000000	0.000000

Example 15 - Final analisis

LINGO give the optimal solution that says that Kiriakis must produce **38.88889** type 1 products and **0** type 2 and type 3 products, otherwise the profits will decrease by \$2 per type 2 product or \$2.67 per type 3 product. The optimal profit of Kiriakis is **\$233.3333**.

Example 16

Example 16 - Problem (1)

Oliver Winery produces four award-winning wines in Bloomington, Indiana. The profit contribution, labor hours, and tank usage (in hours) per gallon for each type of wine are given in the following table.

Wine	**Profit**	**Labor**	**Tank**
Wine 1	\$6	.2 hr	.5 hr
Wine 2	\$12	.3 hr	.5 hr
Wine 3	\$20	.3 hr	1 hr
Wine 4	\$30	.5 hr	1.5 hr

Example 16 - Problem (2)

By law at most 100000 gallons of wine can be produced each year. A maximum of 12000 labor hours and 32000 tank hours are available annually. Each gallon of wine 1 spends an average of (1/2) year in inventory; wine 2, an average of 1 year; wine 3, an average of 2 years; wine 4, an average of 3.333 years. The winery's warehouse can handle an average inventory level of 50000 gallons. Determine how much of each type of wine should be produced annually to maximize OliverWinery's profit.

Example 16 - Analisis

Oliver Winery wants to maximize profits produced by the wines. Each wine has a profit associated. So what we need to do is to maximize the sum of the ProfitPerWine * AmountOfWine .

Example 16 - Variables

$w_1 \longrightarrow$ type 1 wine to produce

$w_2 \longrightarrow$ type 2 wine to produce

$w_3 \longrightarrow$ type 3 wine to produce

$w_4 \longrightarrow$ type 4 wine to produce

Example 16 - Function

Maximize Oliver Winery's profit by determining how much of each type of wine should be produced anually:

$$Z = 6W_1 + 12W_2 + 20W_3 + 30W_4$$

Example 16 - Restrictions

$$
\begin{aligned}
W_1 + W_2 + W_3 + W_4 &\leq 100000 \\
0.2W_1 + 0.3W_2 + 0.3W_3 + 0.5W_4 &\leq 12000 \\
0.5W_1 + 0.5W_2 + W_3 + 1.5W_4 &\leq 32000 \\
\frac{1}{3}W_1 + W_2 + 2W_3 + \frac{1}{3}W_4 &\leq 50000 \\
W_1, W_2, W_3, W_4 &\geq 0
\end{aligned}
$$

Example 16 - Model

Maximize:

$$Z = 6W_1 + 12W_2 + 20W_3 + 30W_4$$

Subject to:

$$\begin{aligned}
W_1 + W_2 + W_3 + W_4 &\leq 100000 \\
0.2W_1 + 0.3W_2 + 0.3W_3 + 0.5W_4 &\leq 12000 \\
0.5W_1 + 0.5W_2 + W_3 + 1.5W_4 &\leq 32000 \\
\frac{1}{3}W_1 + W_2 + 2W_3 + \frac{1}{3}W_4 &\leq 50000 \\
W_1, W_2, W_3, W_4 &\geq 0
\end{aligned}$$

Example 16 - Solution

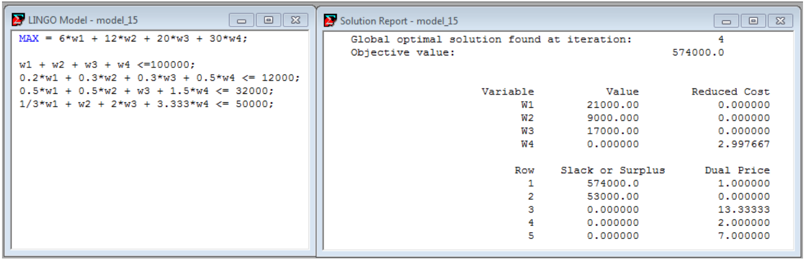

Figure: Example 16 model and solution in Lingo Software.

Example 16 - Final solution

Global optimal solution found at iteration: 4
Objective value: 574000.0

Variable	Value	Reduced Cost
W1	21000.00	0.000000
W2	9000.000	0.000000
W3	17000.00	0.000000
W4	0.000000	2.997667

Row	Slack or Surplus	Dual Price
1	574000.0	1.000000
2	53000.00	0.000000
3	0.000000	13.33333
4	0.000000	2.000000
5	0.000000	7.000000

Example 16 - Final analisis

The solution give by LINGO advice Oliver Winery to produce **21000** gallons of type 1 wine, **9000** gallons of type 2 wine and **17000** gallons of type 3 wine. With these 3 wines, Oliver Winery will generate profits of **$574000**. If the winery produce type 4 wine, it will decrease the profits by $2.997667 per each gallon produced.

Example 17

Example 17 - Problem (1)

Gotham City National Bank is open Monday-Friday from 9 A.M to 5 P.M. From past experience, the bank knows that it needs the number of tellers shown in the following table.

Time Period	Tellers Required
9-10	4
10-11	3
11-Noon	4
Noon-1	6
1-2	5
2-3	6
3-4	8
4-5	8

Example 17 - Problem (2)

The bank hires two types of tellers. Full-time tellers work 9-5 five days a week, except for 1 hour off for lunch. (The bank determines when a full-time employee takes lunch hour, but each teller must go between noon and 1 P.M or between 1 P.M and 2 P.M.).

Full-time employees are paid (including fringe benefits) $8 per hour (this includes payment for lunch hour). The bank may also hire part-time tellers. Each part-time teller must work exactly 3 consecutive hours each day.

A part-time teller is paid $5 per hour (and receives nofringe benefits). To maintain adequate quality of service, the bank has decided that at most five part-time tellers can be hired. Formulate an LP to meet the teller requirements at minimum cost.

Example 17 - Analisis

The bank need to minimize the costs of hiring bank tellers by hiring the minimal amount. We need to minimize the cost of hiring each teller * the amount of tellers hired.

Example 17 - Variables

$$X_i$$

$C \longrightarrow$ tellers grouped by shifts
$i \longrightarrow$ shifts

Example 17 - Function

Minimize the cost of hiring the minimum amount of bank tellers:

$$Z = 64(X_1 + X_2) + 15(X_3 + X_4 + X_5 + X_6 + X_7 + X_8)$$

Example 17 - Restrictions

Las restricciones son básicamente la cantidad de cajeros por turno. Esto lo podemos extraer de la misma tabla sacando las intersecciones de variables restringido por la cantidad es decir: en la primera columna tenemos a X1, X2 y X3 y una cantidad de 4; de ahí deducimos que $X_1 + X_2 + X_3 = 4$. Análogamente aplicamos la misma técnica para las demás columnas lo que nos da las restricciones que se muestran en el modelo (siguiente slide). Finalmente tenemos la restricciones triviales de que todas las variables deben ser mayor o iguales a 0.

Example 17 - Model

Minimize:

$$Z = 64(X_1 + X_2) + 15(X_3 + X_4 + X_5 + X_6 + X_7 + X_8)$$

Subject to:

$$\begin{aligned}
X_3 + X_4 + X_5 + X_6 + X_7 + X_8 &\leq 5 \\
X_1 + X_2 + X_3 &= 4 \\
X_1 + X_2 + X_3 + X_4 &= 3 \\
X_1 + X_2 + X_3 + X_4 + X_5 &= 4 \\
X_2 + X_4 + X_5 + X_6 &= 6 \\
X_1 + X_5 + X_6 + X_7 &= 5 \\
X_1 + X_2 + X_6 + X_7 + X_8 &= 6 \\
X_1 + X_2 + X_7 + X_8 &= 8 \\
X_1 + X_2 + X_6 + X_8 &= 8 \\
X_i &\geq 0
\end{aligned}$$

Example 17 - Solution (1)

LINGO Model - model_17

```
MIN = 64*(X1 + X2) + 15*(X3 + X4 + X5 + X6 + X7 + X8);

X3 + X4 + X5 + X6 + X7 + X8 <= 5;
X1 + X2 + X3 = 4;
X1 + X2 + X3 + X4 = 3;
X1 + X2 + X3 + X4 + X5 = 4;
X2 + X4 + X5 + X6 = 6;
X1 + X5 + X6 + X7 = 5;
X1 + X2 + X6 + X7 + X8 = 6;
X1 + X2 + X7 + X8 = 8;
X1 + X2 + X6 + X8 = 8;

X1 >= 0;
X2 >= 0;
X3 >= 0;
X4 >= 0;
X5 >= 0;
X6 >= 0;
X7 >= 0;
X8 >= 0;
```

Figure: Example 17 model and solution in Lingo Software.

Example 17 - Solution (2)

Solution Report - model_17

Variable	Value	Reduced Cost
X1	0.000000	0.000000
X2	4.000000	0.000000
X3	0.000000	0.3200000E+12
X4	-1.000000	0.6400000E+11
X5	1.000000	0.000000
X6	1.000000	0.000000
X7	3.000000	0.000000
X8	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.1280000E+12
3	0.000000	0.1280000E+12
4	0.000000	0.6400000E+11
5	0.000000	0.000000
6	-1.000000	-0.6400000E+11
7	0.000000	-0.6400000E+11
8	-3.000000	0.6400000E+11
9	0.000000	-0.1280000E+12
10	-2.000000	-0.6400000E+11
11	0.000000	0.000000
12	4.000000	0.000000
13	0.000000	0.000000
14	-1.000000	-0.6400000E+11

Figure: Example 17 model and solution in Lingo Software.

Example 17 - Solution (3)

Solution Report - model_17

X6	1.000000	0.000000
X7	3.000000	0.000000
X8	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.1280000E+12
3	0.000000	0.1280000E+12
4	0.000000	0.6400000E+11
5	0.000000	0.000000
6	-1.000000	-0.6400000E+11
7	0.000000	-0.6400000E+11
8	-3.000000	0.6400000E+11
9	0.000000	-0.1280000E+12
10	-2.000000	-0.6400000E+11
11	0.000000	0.000000
12	4.000000	0.000000
13	0.000000	0.000000
14	-1.000000	-0.6400000E+11
15	1.000000	0.000000
16	1.000000	0.000000
17	3.000000	0.000000
18	1.000000	0.000000

Figure: Example 17 model and solution in Lingo Software.

Example 17 - Final solution (1)

Global optimal solution found at iteration:

Objective value:

Variable	Value	Reduced Cost
X1	0.000000	0.000000
X2	4.000000	0.000000
X3	0.000000	0.3200000E+12
X4	-1.000000	0.6400000E+11
X5	1.000000	0.000000
X6	1.000000	0.000000
X7	3.000000	0.000000
X8	1.000000	0.000000

Example 17 - Final solution (2)

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
2	0.000000	0.1280000E+12
3	0.000000	0.1280000E+12
4	0.000000	0.6400000E+11
5	0.000000	0.000000
6	-1.000000	-0.6400000E+11
7	0.000000	-0.6400000E+11
8	-3.000000	0.6400000E+11
9	0.000000	-0.1280000E+12

Example 17 - Final solution (3)

Row	Slack or Surplus	Dual Price
10	-2.000000	-0.6400000E+11
11	0.000000	0.000000
12	4.000000	0.000000
13	0.000000	0.000000
14	-1.000000	-0.6400000E+11
15	1.000000	0.000000
16	1.000000	0.000000
17	3.000000	0.000000
18	1.000000	0.000000

Example 17 - Final analisis

LINGO detects an “Unbounded Solution” in the model. When the “Unbounded Solution” termination occurs, it implies the formulation admits the unrealistic result that an infinite amount of profit can be made. A more realistic conclusion is that an important constraint has been omitted or the formulation contains a critical typographical error.

Example 18

Example 18 - Problem (1)

A paper recycling plant processes box board, tissue paper, newsprint, and book paper into pulp that can be used to produce three grades of recycled paper (grades 1, 2, and 3). The prices per ton and the pulp contents of the four inputs are shown in the following table.

Input	**Cost**	**Pulp Content**
Box board	\$5	15%
Tissue paper	\$6	20%
Newsprint	\$8	30%
Book paper	\$10	40%

Example 18 - Problem (2)

Two methods, de-inking and asphalt dispersion, can be used to process the four inputs into pulp. It costs \$20 to de-ink a ton of any input. The process of de-inking removes 10 percent of the input's pulp, leaving 90% of the original pulp. It costs \$15 to apply asphalt dispersion to a ton of material. The asphalt dispersion process removes 20% of the input's pulp. At most 3000 tons of input can be run through the asphalt dispersion process or the de-inking process.

Example 18 - Problem (3)

Grade 1 paper can only be produced with newsprint or book paper pulp; grade 2 paper only with book paper, tissue paper, or box board pulp. To meet its current demands, the company needs 500 tons of pulp for grade 1 paper, 500 tons of pulp for grade 2 paper, and 600 tons of pulp for grade 3 paper. Formulate an LP to minimize the cost of meeting demands for pulp.

Example 18 - Analisis

The paper recylcing plant need to minimize the cost of meeting demands for pulp. It is needed some raw material and some processes like de-inking and asphalt dispersion, so to minimize the cost we have to sum the cost of the raw material purchase + the cost of sending materials (inputs) through the diferent processes (de-inking and asphalt dispersion)

Example 18 - Variables

Using the indexes:

$i \longrightarrow$ input type [1;4] (being 1 box board, 2 tissue paper, 3 newsprint, and 4 book paper)
$j \longrightarrow$ paper grade [1;3]

We can define the variables:

$M_i \longrightarrow$ tonnes of raw material i purchased
$D_i \longrightarrow$ tonnes of raw material i processed using de-inking
$DA_i \longrightarrow$ tonnes of raw material i processed using asphalt dispersion
$P_i \longrightarrow$ tonnes of type i pulp produced
$PG_{ij} \longrightarrow$ tonnes of pulp type i for paper grade j

Example 18 - Function

Minimize the cost of meeting demands for pulp of the paper recycling plant:

$$
\begin{aligned}
Z = & 5M_1 + 6M_2 + 8M_3 + 10M_4 + \\
& 20D_1 + 20D_2 + 20D_3 + 20D_4 + \\
& 15DA_1 + 15DA_2 + 15DA_3 + 15DA_4
\end{aligned}
$$

Example 18 - Restrictions

$$
\begin{aligned}
D_1 + DA_1 &\leq M_1\\
D_2 + DA_2 &\leq M_2\\
D_3 + DA_3 &\leq M_3\\
D_4 + DA_4 &\leq M_4\\
(0.15 * 0.90)D_1 + (0.15 * 0.80)DA_1 &= P_1\\
(0.20 * 0.90)D_2 + (0.15 * 0.80)DA_2 &= P_2\\
(0.30 * 0.90)D_3 + (0.15 * 0.80)DA_3 &= P_3\\
(0.40 * 0.90)D_4 + (0.15 * 0.80)DA_4 &= P_4\\
M_i, P_i, D_i, DA_i &\geq 0\\
PG_{ij} &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
D_1 + D_2 + D_3 + D_4 &\leq 3000\\
DA_1 + DA_2 + DA_3 + DA_4 &\leq 3000\\
PG_{22} + PG_{13} &\leq P_1\\
PG_{32} + PG_{23} &\leq P_2\\
PG_{32} + PG_{33} &\leq P_3\\
PG_{42} + PG_{43} &\leq P_4\\
PG_{31} + PG_{41} &\geq 500\\
PG_{12} + PG_{22} + PG_{42} &\geq 500\\
PG_{13} + PG_{23} + PG_{33} &\geq 500
\end{aligned}
$$

Example 18 - Model

Minimize:

$$
\begin{aligned}
Z = & 5M_1 + 6M_2 + 8M_3 + 10M_4 + \\
& 20D_1 + 20D_2 + 20D_3 + 20D_4 + \\
& 15DA_1 + 15DA_2 + 15DA_3 + 15DA_4
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
D_1 + DA_1 &\leq M_1 \\
D_2 + DA_2 &\leq M_2 \\
D_3 + DA_3 &\leq M_3 \\
D_4 + DA_4 &\leq M_4 \\
(0.15 * 0.90)D_1 + (0.15 * 0.80)DA_1 &= P_1 \\
(0.20 * 0.90)D_2 + (0.15 * 0.80)DA_2 &= P_2 \\
(0.30 * 0.90)D_3 + (0.15 * 0.80)DA_3 &= P_3 \\
(0.40 * 0.90)D_4 + (0.15 * 0.80)DA_4 &= P_4 \\
M_i, P_i, D_i, DA_i &\geq 0 \\
PG_{ij} &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
D_1 + D_2 + D_3 + D_4 &\leq 3000 \\
DA_1 + DA_2 + DA_3 + DA_4 &\leq 3000 \\
PG_{22} + PG_{13} &\leq P_1 \\
PG_{32} + PG_{23} &\leq P_2 \\
PG_{32} + PG_{33} &\leq P_3 \\
PG_{42} + PG_{43} &\leq P_4 \\
PG_{31} + PG_{41} &\geq 500 \\
PG_{12} + PG_{22} + PG_{42} &\geq 500 \\
PG_{13} + PG_{23} + PG_{33} &\geq 500
\end{aligned}
$$

Example 18 - Solution (1)

LINGO Model - model_18

```
MIN = 5*M1 + 6*M2 + 8*M3 + 10*M4 + 20*D1 + 20*D2 + 20*D3 + 20*D4 + 15*DA1 + 15*A2 + 15*A3 + 15*A4;

D1 + A1 <= M1;
D2 + A2 <= M2;
D3 + A3 <= M3;
D4 + A4 <= M4;

(0.15)*(0.90)*D1 + (0.15)*(0.80)*A1 = P1;
(0.20)*(0.90)*D2 + (0.15)*(0.80)*A2 = P2;
(0.30)*(0.90)*D3 + (0.15)*(0.80)*A3 = P3;
(0.40)*(0.90)*D4 + (0.15)*(0.80)*A4 = P4;

D1 + D2 + D3 +D4 <= 3000;
A1 + A2 + A3 + A4 <= 3000;

PG22 + PG13 <= P1;
PG32 + PG23 <= P2;
PG32 + PG33 <= P3;
PG42 + PG43 <= P4;

PG31 + PG41 >= 500;
PG12 + PG22 + PG42 >= 500;
PG13 + PG23 + PG33 >= 500;
```

Figure: Example 18 model and solution in Lingo Software.

Example 18 - Solution (2)

LINGO Model - model_18

```
D1 >= 0;
D2 >= 0;
D3 >= 0;
D4 >= 0;

DA1 >= 0;
DA2 >= 0;
DA3 >= 0;
DA4 >= 0;

M1 >= 0;
M2 >= 0;
M3 >= 0;
M4 >= 0;

PG22 >= 0;
PG13 >= 0;
PG32 >= 0;
PG23 >= 0;
PG32 >= 0;
PG33 >= 0;
PG42 >= 0;
PG43 >= 0;
```

Figure: Example 18 model and solution in Lingo Software.

Example 18 - Solution (3)

Solution Report - model_18

```
Global optimal solution found at iteration:          6
Objective value:                              29518.52
```

Variable	Value	Reduced Cost
M1	3000.000	0.000000
M2	0.000000	0.000000
M3	518.5185	0.000000
M4	0.000000	0.000000
D1	0.000000	11.00000
D2	0.000000	7.333333
D3	518.5185	0.000000
D4	0.000000	30.00000
DA1	0.000000	15.00000
A2	0.000000	16.00000
A3	0.000000	18.00000
A4	0.000000	32.44444
A1	3000.000	0.000000
P1	360.0000	0.000000
P2	0.000000	0.000000
P3	140.0000	0.000000
P4	0.000000	0.000000
PG22	0.000000	103.7037
PG13	360.0000	0.000000
PG32	0.000000	207.4074
PG23	0.000000	0.000000

Figure: Example 18 model and solution in Lingo Software.

Example 18 - Solution (4)

Solution Report - model_18

PG33	140.0000	0.000000
PG42	0.000000	0.000000
PG43	0.000000	0.000000
PG31	0.000000	0.000000
PG41	500.0000	0.000000
PG12	500.0000	0.000000
DA2	0.000000	0.000000
DA3	0.000000	0.000000
DA4	0.000000	0.000000

Row	Slack or Surplus	Dual Price
1	29518.52	-1.000000
2	0.000000	5.000000
3	0.000000	6.000000
4	0.000000	8.000000
5	0.000000	10.00000
6	0.000000	-103.7037
7	0.000000	-103.7037
8	0.000000	-103.7037
9	0.000000	0.000000
10	2481.481	0.000000
11	0.000000	7.444444
12	0.000000	103.7037
13	0.000000	103.7037
14	0.000000	103.7037
15	0.000000	0.000000

Figure: Example 18 model and solution in Lingo Software.

Example 18 - Solution (5)

Solution Report - model_18

15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000
18	0.000000	-103.7037
19	0.000000	0.000000
20	0.000000	0.000000
21	518.5185	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	0.000000	0.000000
26	0.000000	0.000000
27	3000.000	0.000000
28	0.000000	0.000000
29	518.5185	0.000000
30	0.000000	0.000000
31	0.000000	0.000000
32	360.0000	0.000000
33	0.000000	0.000000
34	0.000000	0.000000
35	0.000000	0.000000
36	140.0000	0.000000
37	0.000000	0.000000
38	0.000000	0.000000

Figure: Example 18 model and solution in Lingo Software.

Example 18 - Final solution (1)

Global optimal solution found at iteration: 6

Objective value: 29518.52

Variable	Value	Reduced Cost
M1	3000.000	0.000000
M2	0.000000	0.000000
M3	518.5185	0.000000
M4	0.000000	0.000000
D1	0.000000	11.00000
D2	0.000000	7.333333
D3	518.5185	0.000000
D4	0.000000	30.00000
DA1	0.000000	15.00000
A2	0.000000	16.00000
A3	0.000000	18.00000
A4	0.000000	32.44444
A1	3000.000	0.000000

Example 18 - Final solution (2)

Variable	Value	Reduced Cost
P1	360.0000	0.000000
P2	0.000000	0.000000
P3	140.0000	0.000000
P4	0.000000	0.000000
PG22	0.000000	103.7037
PG13	360.0000	0.000000
PG32	0.000000	207.4074
PG23	0.000000	0.000000
PG33	140.0000	0.000000
PG42	0.000000	0.000000
PG43	0.000000	0.000000
PG31	0.000000	0.000000
PG41	500.0000	0.000000

Example 18 - Final solution (3)

Global optimal solution found at iteration: 6

Objective value: 29518.52

Variable	Value	Reduced Cost
PG12	500.0000	0.000000
DA2	0.000000	0.000000
DA3	0.000000	0.000000
DA4	0.000000	0.000000

Example 18 - Final solution (4)

Row	Slack or Surplus	Dual Price
1	29518.52	-1.000000
2	0.000000	5.000000
3	0.000000	6.000000
4	0.000000	8.000000
5	0.000000	10.00000
6	0.000000	-103.7037
7	0.000000	-103.7037
8	0.000000	-103.7037
9	0.000000	0.000000
10	2481.481	0.000000
11	0.000000	7.444444
12	0.000000	103.7037
13	0.000000	103.7037

Example 18 - Final solution (5)

Row	Slack or Surplus	Dual Price
14	0.000000	103.7037
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000
18	0.000000	-103.7037
19	0.000000	0.000000
20	0.000000	0.000000
21	518.5185	0.000000
22	0.000000	0.000000
23	0.000000	0.000000
24	0.000000	0.000000
25	0.000000	0.000000
26	0.000000	0.000000

Example 18 - Final solution (6)

Row	Slack or Surplus	Dual Price
27	3000.000	0.000000
28	0.000000	0.000000
29	518.5185	0.000000
30	0.000000	0.000000
31	0.000000	0.000000
32	360.0000	0.000000
33	0.000000	0.000000
34	0.000000	0.000000
35	0.000000	0.000000
36	140.0000	0.000000
37	0.000000	0.000000
38	0.000000	0.000000

Example 18 - Final analisis

According to LINGO's solution, the recycling plant must purchase **3000** raw material 1, **0** raw material 2 and **518.518** raw material 3, which is the same amount processed with de-inking. In contrast **3000** tons of materials will be processed using asphalt dispersion. **360** tons of pulp 1 and **140** of pulp 3 will be produced. Finally, there are **360** tons of pulp 1 for paper grade 3, **140** tons of pulp 3 for paper grade 3 and **500** tons of pulp 4 for paper grade 1. So the cost of meeting these demands is of **$29518.52**.

Example 19

Example 19 - Problem (1)

The mayor of Llanview is trying to determine the number of judges needed to handle the judicial caseload. During each month of the year it is estimated that the number of judicial hours needed is given in the following table.

Month	Hours
January	400
February	300
March	200
April	600
May	800
June	300
July	200
August	400
September	300
October	200
November	100
December	300

Example 19 - Problem (2)

Each judge works all 12 months, and can handle up to 120 hours per month of casework. To avoid creating a backlog, all cases must be handled by the end of December. Formulate an LP whose solution will determine how many judges Llanview needs.

Example 19 - Analisis

Llanview needs to determine the minimum number of judges needed to handle the judicial caseload, so letting X be the amount of judges needed it's all about minimize X with the given restrictions.

$X \longrightarrow$ amount of judges that Llanview requires

Example 19 - Function

Determine the minimum amount of judges needed to handle the judicial caseload of Llanview. Minimize:

$$Z = X$$

Example 19 - Restrictions

Se tiene que $120 * X \geq$ cantidad de horas necesarias por cada mes dadas en la tabla anterior, que todas se pueden resumir en que sea mayor o igual a la mas grande, esto asegura se mayor o igual a las demás: $120X \geq 800$. Por último se tiene la restricción trivial de que la cantidad de jueces debe ser mayor o igual a 0 pues no hay jueces negativos: $X \geq 0$.

Example 19 - Model

Minimize:

$$Z = X$$

Subject to:

$$120X \geq 800$$
$$X \geq 0$$

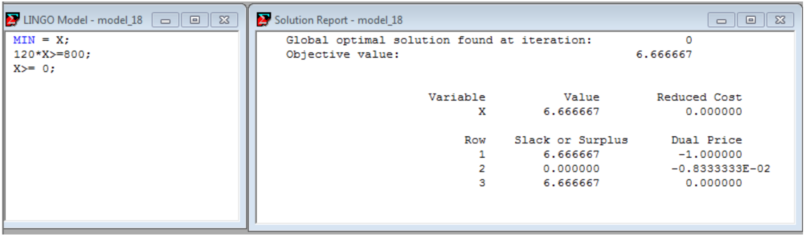

Figure: Example 19 model and solution in Lingo Software.

Example 19 - Final solution

Global optimal solution found at iteration: 0

Objective value: 6.666667

Variable	Value	Reduced Cost
X	6.666667	0.000000

Row	Slack or Surplus	Dual Price
1	6.666667	-1.000000
2	0.000000	-0.8333333E
3	6.666667	0.000000

Example 19 - Final analisis

The solution given by LINGO says that Llanview needs **6.666667** judges.

Example 20

Example 20 - Problem (1)

Olé Oil produces three products: heating oil, gasoline, and jet fuel. The average octane levels must be at least 4.5 for heating oil, 8.5 for gas and 7.0 for jet fuel. To produce these products Olé purchases two types of oil: crude 1 (at $12 per barrel) and crude 2 (at $10 per barrel). Each day, at most 10000 barrels of each type of oil can be purchased.

Example 20 - Problem (2)

Before crude can be used to produce products for sale it must be distilled. Each day, at most 15000 barrels of oil can be distilled. It cost 10$ to distill a barrel of oil. The result of distillation is as follows: (1) Each barrel of crude 1 yields 0.6 barrel of naphtha, 0.3 barrel of distilled 1, and 0.1 barrel of distilled 2. (2) Each barrel of crude 2 yields 0.4 barrel of naphtha, 0.2 barrel of distilled 1, and 0.4 barrel of distilled 2. Distilled naphtha can be used only to produce gasoline or jet fuel. Distilled oil can be used to produce heating oil or it can be sent through the catalytic cracker (at a cost of 15$ per barrel). Each day, at most 5000 barrels of distilled oil can be sent through the cracker.

Example 20 - Problem (3)

Each barrel of distilled 1 sent through the cracker yields 0.8 barrel of cracked 1and 0.2 barrel of cracked 2. Each barrel of distilled 2 sent through the cracker yields 0.7 barrel of cracked 1 and 0.3 barrel of cracked 2. Cracked oil can be used to produce gasoline and jet fuel but no to produce heating oil.

The octane level of each type of oil is as follows: naphtha, 8; distilled 1, 4; distilled 2, 5; cracked 1, 9; cracked 2, 6.

All heating oil produced can be sold at \$14 per barrel; all gasoline produced, \$18 per barrel; and all jet fuel produced \$16 per barrel. Marketing considerations dictate that at least 3000 barrels of each product must be produced daily. Formulate an LP to maximize Olé's daily profit.

Example 20 - Analisis

What we need to do is to maximize Olé's daily profit of producing heating oil, gasoline and jet fuel. So this is about maximizing the sum of selling each product - the production costs and the costs of send the product through the catalytic cracker.

Example 20 - Variables

$C1 \longrightarrow$ amount of crude 1 barrels bought for distillation
$C2 \longrightarrow$ amount of crude 2 barrels bought for distillation
$NG \longrightarrow$ amount of naphtha barrels produced for gasoline
$NJF \longrightarrow$ amount of naphtha barrels produced for jet fuel
$X_{d1ho} \longrightarrow$ amount of distillated 1 oil barrels produced for heating oil
$X_{d1c} \longrightarrow$ amount of distillated 1 oil barrels produced for cracker
$X_{d2ho} \longrightarrow$ amount of distillated 2 oil barrels produced for heating oil
$X_{d2c} \longrightarrow$ amount of distillated 2 oil barrels produced for cracker
$X_{co1g} \longrightarrow$ amount of crack oil 1 barrels produced for gasoline
$X_{co1ho} \longrightarrow$ amount of crack oil 1 barrels produced for heating oil
$X_{co1j} \longrightarrow$ amount of crack oil 1 barrels produced for jet fuel
$X_{co2g} \longrightarrow$ amount of crack oil 2 barrels produced for gasoline
$X_{co2ho} \longrightarrow$ amount of crack oil 2 barrels produced for heating oil
$X_{co2j} \longrightarrow$ amount of crack oil 2 barrels produced for jet fuel

Example 20 - Function

Maximize Olé's daily profit of producing heating oil, gasoline and jet fuel:

$$\begin{aligned} Z = & 17.75X_{co1g} + 15.75X_{co1j} + \\ & 17.75X_{co2g} + 15.75X_{co2j} + \\ & 17.9NG + 15.9NJF + \\ & 13.9X_{d1ho} + 13.9X_{d2ho} - \\ & 12C1 - 10C2 \end{aligned}$$

Example 20 - Restrictions

$$
\begin{aligned}
C1 &\leq 10000 \\
C2 &\leq 10000 \\
C1 + C2 &\leq 15000 \\
NG + NJF - 0.6C1 - 0.4C2 &= 0 \\
X_{d1c} + X_{d1ho} - 0.3C1 - 0.2C2 &= 0 \\
X_{d2c} + X_{d2ho} - 0.1C1 - 0.4C2 &= 0 \\
X_{d1c} + X_{d2c} &\leq 5000 \\
X_k &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
X_{co1g} + X_{co1ho} - 0.8X_{d1c} - 0.7X_{d2c} &= 0 \\
X_{co1g} + X_{co1ho} - 0.3X_{d1c} - 0.2X_{d2c} &= 0 \\
9X_{co1g} + 6X_{co2g} + 8NG &\geq 8.5 \\
4X_{d1ho} + 5X_{d2ho} &\geq 4.5 \\
9X_{co1j} + 6X_{co2j} + 8NJF &\geq 7.0 \\
X_{co1g} + X_{co2g} + NG &\geq 3000 \\
X_{d1ho} + X_{d2ho} &\geq 3000 \\
X_{co1j} + X_{co2j} + NJF &\geq 3000
\end{aligned}
$$

Example 20 - Model

Maximize:

$$
\begin{aligned}
Z =& 17.75X_{co1g} + 15.75X_{co1j} + \\
& 17.75X_{co2g} + 15.75X_{co2j} + \\
& 17.9NG + 15.9NJF + \\
& 13.9X_{d1ho} + 13.9X_{d2ho} - \\
& 12C1 - 10C2
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
C1 &\leq 10000 \\
C2 &\leq 10000 \\
C1 + C2 &\leq 15000 \\
NG + NJF - 0.6C1 - 0.4C2 &= 0 \\
X_{d1c} + X_{d1ho} - 0.3C1 - 0.2C2 &= 0 \\
X_{d2c} + X_{d2ho} - 0.1C1 - 0.4C2 &= 0 \\
X_{d1c} + X_{d2c} &\leq 5000 \\
X_k &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
X_{co1g} + X_{co1ho} - 0.8X_{d1c} - 0.7X_{d2c} &= 0 \\
X_{co1g} + X_{co1ho} - 0.3X_{d1c} - 0.2X_{d2c} &= 0 \\
9X_{co1g} + 6X_{co2g} + 8NG &\geq 8.5 \\
4X_{d1ho} + 5X_{d2ho} &\geq 4.5 \\
9X_{co1j} + 6X_{co2j} + 8NJF &\geq 7.0 \\
X_{co1g} + X_{co2g} + NG &\geq 3000 \\
X_{d1ho} + X_{d2ho} &\geq 3000 \\
X_{co1j} + X_{co2j} + NJF &\geq 3000
\end{aligned}
$$

Example 20 - Solution (1)

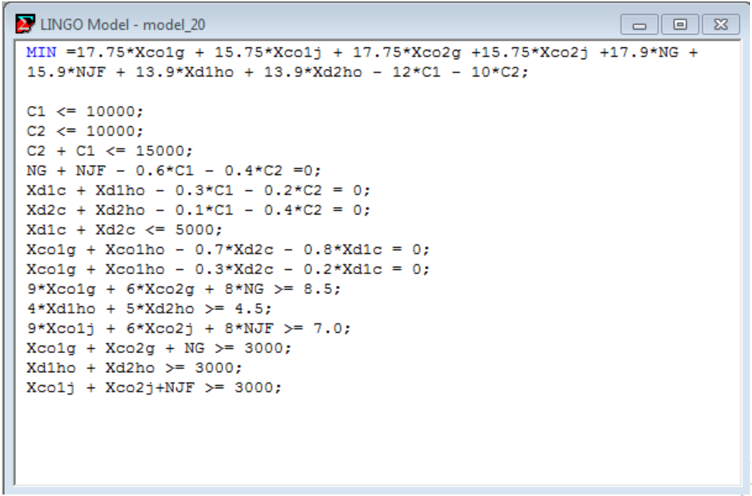

Figure: Example 20 model and solution in Lingo Software.

Example 20 - Solution (2)

Solution Report - model_20

Global optimal solution found at iteration: 9

Objective value: 37000.00

Variable	Value	Reduced Cost
XCO1G	0.000000	10.58333
XCO1J	0.000000	10.58333
XCO2G	0.000000	10.58333
XCO2J	0.000000	10.58333
NG	3000.000	0.000000
NJF	3000.000	0.000000
XD1HO	3000.000	0.000000
XD2HO	1000.000	0.000000
C1	10000.00	0.000000
C2	0.000000	2.633333
XD1C	0.000000	6.950000
XD2C	0.000000	0.000000
XCO1HO	0.000000	0.000000

Figure: Example 20 model and solution in Lingo Software.

Example 20 - Solution (3)

Solution Report - model_20

Row	Slack or Surplus	Dual Price
1	37000.00	-1.000000
2	0.000000	0.000000
3	10000.00	0.000000
4	5000.000	0.000000
5	0.000000	-10.73333
6	0.000000	-13.90000
7	0.000000	-13.90000
8	5000.000	0.000000
9	0.000000	-34.75000
10	0.000000	34.75000
11	23991.50	0.000000
12	16995.50	0.000000
13	23993.00	0.000000
14	0.000000	-7.166667
15	1000.000	0.000000
16	0.000000	-5.166667

Figure: Example 20 model and solution in Lingo Software.

Example 20 - Final solution (1)

Global optimal solution found at iteration: 9

Objective value: 37000.00

Variable	Value	Reduced Cost
XCO1G	0.000000	10.58333
XCO1J	0.000000	10.58333
XCO2G	0.000000	10.58333
XCO2J	0.000000	10.58333
NG	3000.000	0.000000
NJF	3000.000	0.000000
XD1HO	3000.000	0.000000
XD2HO	1000.000	0.000000
C1	10000.00	0.000000
C2	0.000000	2.633333
XD1C	0.000000	6.950000
XD2C	0.000000	0.000000
XCO1HO	0.000000	0.000000

Example 20 - Final solution (2)

Row	Slack or Surplus	Dual Price
1	37000.00	-1.000000
2	0.000000	0.000000
3	10000.00	0.000000
4	5000.000	0.000000
5	0.000000	-10.73333
6	0.000000	-13.90000
7	0.000000	-13.90000
8	5000.000	0.000000

Example 20 - Final solution (3)

Row	Slack or Surplus	Dual Price
9	0.000000	-34.75000
10	0.000000	34.75000
11	23991.50	0.000000
12	16995.50	0.000000
13	23993.00	0.000000
14	0.000000	-7.166667
15	1000.000	0.000000
16	0.000000	-5.166667

Example 20 - Final analisis

The optimal solution given by LINGO says that no crack oil barrels for gasoline nor for heating oil must be produced, otherwise it will decrease the profits by \$10.58333 per barrel produced. Also it says that there are needed **3000** barrels of Naphtha for gasoline, **3000** barrels of Naphtha for Jet Fuel, **3000** barrels of distillated oil 1 to produce heating oil, **1000** barrels of distillated oil 2 to produce heating oild. Also **10000** barrels of crude 1 for distilation must be bought. If the company buys crude 2 the profit will decrease by \$2.63333, same happens with barrels of distillate 1 produced for cracker, they would decrease profit by \$6.95.