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Monty Hall

MOTIVATION

Our motivations for studying the Monty Hall problem are focused on gaining a better understanding of the correct approach to the problem, demonstrating the role of probability in decision-making, and exploring variations of the problem. By studying this classic probability puzzle, we hope to enhance our problem-solving skills, challenge our intuitions, and apply the principles to real-life scenarios. Exploring variations of the problem can also deepen your understanding of conditional probability and its relevance in decision-making.

CONTRIBUTIONS

All group members (Carlos Mercado, Edgar Renteria, Maria Guimaraes, Jordan Nicholls) contributed to research and the implementation of the simulation of this problem.

PROBLEM STATEMENT

In the game, a contestant chooses one of three doors, behind one of which is a car, while behind the other two are goats. After the contestant has made their initial selection, the host, who knows what is behind each door, opens one of the other two doors to reveal a goat. The contestant is then given the option to switch their choice to the remaining door(s) or stick with their original choice. This game is extended to 6, 9, 20 and 100 doors.

APPROACH

The team's approach to simulating this problem starts by first "building the stage". The doors are represented as a list where each cell has a goat inside of it, except for one which has the grand prize (a car). After the stage is set, the contestant is free to make their choice. This is represented in the simulation as a random choice of a cell in a list. After the contestant makes their choice the host takes away a door, specifically a door that DOES NOT contain the car, and is not the door that the contestant chose, this is just represented by marking a cell in the list of doors with an 'X'. Now, the contestant has two choices, stay with their original choice or switch to one of the remaining doors. If they end up with the door holding the car at the end of the game they win. For each door count we did ~500,000 runs to get a good representation of the win rate.

RESULTS AND DISCUSSION

In the three doors case there was ~33 percent chance of picking the car if you did not switch and stuck with your original prediction, however if you decided to switch your odds doubled to ~66 percent. In the 6 and 9 doors case, the difference was not as great but still obvious $1/6 \rightarrow 5/24$ and $\sim 0.111 \rightarrow \sim 0.1270$ respectively. In the last two cases (20 and 100 doors), the difference was very little $\sim 0.05 \rightarrow \sim 0.05277777777778$ and $\sim 0.01 \rightarrow \sim 0.0101020408163$.

PROBLEM STATEMENT 2

In this variant of the game, once the contestant has selected one of the doors, the host slips on a banana peel and accidentally pushes open another door (that could be the car). This variant is also extended to 6, 9, 20 and 100 doors. Note that if the host slips and opens the door which holds the car that trial does not count.

APPROACH 2

The approach is identical except for the case mentioned previously where the host opens the car door.

RESULTS AND DISCUSSION

The results of the simulations were consistent across all door counts. Specifically, in all door count cases the win rate of stay and the win rate of switch was the same. The behavior was not only the same across the door counts but also across door counts. What I mean is that the win rate was roughly equal to $1/(n\text{Doors} - 1)$.

CONCLUSION

In conclusion, our study of the Monty Hall problem has provided valuable insights into the correct approach to the problem, the role of probability in decision-making, and the variations of the problem. Through the implementation of our simulation, we found that switching doors in the classic three-door case dramatically increased the chances of winning the grand prize. We also explored a variant of the problem where the host accidentally opens another door, and found that the win rate of stay and switch was consistent across all door counts. Our research has practical applications in fields such as game theory, statistics, and decision-making. Overall, this study has expanded our understanding of conditional probability and its relevance in real-world scenarios.