Maximizing Power Coefficient Output of a Vertical Wind Turbine

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I. Abstract

This paper demonstrates how the results from the Turbine Blade algorithm, as shown in the optimizer section, can be used to generate the best geometry for a S822 aerofoil wind turbine blade. A single objective multidisciplinary design optimization was conducted considering geometry, solid mechanics, fluid principles, and dynamic requirements. MATLAB optimization toolbox was the main tool used all throughout our optimization process. The work in this paper shows that the coefficient of power of a wind turbine can be heavily improved by optimizing a series of blade geometry variables in that turbine.

II. Introduction & Motivation

A tiny but rapidly expanding portion of power is produced by wind energy. Wind turbines are responsible for roughly 5% of the world's power and 8% of the electricity used in the United States [2]. Globally, wind energy capacity exceeds 743 gigawatts, which is more than can be obtained from solar energy [2]. Most of the wind turbines are located at wind farms in China, the United States, Germany, India, and Spain. Over the past ten years, the Americas' wind energy capacity has tripled. With enough wind turbines to provide more than 100 megawatts of electricity (equivalent to 29 million typical homes) [2]. Wind turbines are the leading renewable energy source in America. Over the past ten years, it has become more affordable, enabling it to compete with solar energy and natural gas. We can use solar energy in the morning and switch to wind energy in the evening and at night. Particularly at higher latitudes, wind energy provides advantages in regions where it is too overcast or dark for robust solar energy generation.

The goal of a wind turbine is to produce the largest amount of electrical energy possible by extracting as much energy as it can from the wind [3]. To evaluate how efficient a wind turbine is, engineers use the power coefficient as an indicator of a turbine's performance. This power coefficient (Cp) is the ratio of actual electric power produced by a wind turbine divided by the total wind power flowing into the turbine's blades at a specific wind speed. The Cp for a particular turbine is measured or calculated by the manufacturer and is usually provided at various wind speeds [3]. If we know the Cp at a given wind speed for a specific turbine it can be used to estimate the electrical power output. The average range of coefficient of power for a 1m diameter blade wind turbine falls in the 35-45% (cp = 0.35-0.45) range [1].

A wind turbine can be divided into 5 big modules: foundation, tower, (rotor, hub & blades), nacelle, and generator (see Figure 2). There is a lot of research done to improve the efficiency of each of these components separately. Every single one of these areas contributes to the overall power coefficient of the turbine. In this paper, we focus on optimizing the blade's module, more specifically the geometric parameters that are used to design the (rotor, hub & blades) (see Figure 1). The blade geometry can be optimized to maximize the aerodynamic performance of the turbine. Such optimization can be done by using Multidisciplinary System Analysis and Design Optimization techniques to come up with an optimal set of design variables that translates into a high cp. The optimized blade shape allows for greater use of wind energy. The aerodynamic performance of a wind turbine is an important factor in its design and has a huge impact in maximizing the cp.

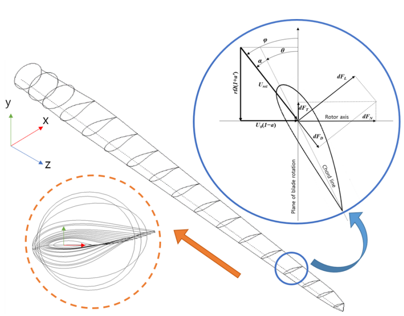

Figure 1: Overview of the Blade Geometry Variables in a Wind Turbine's Blade Profile

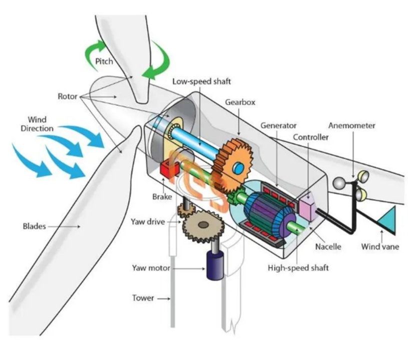

Figure 2: Main Components of a Wind Turbine

III. Problem Formulation

A. Problem Statement & Goal

Maximize a wind turbine's power coefficient (Cp) by changing blade geometric parameters (Outside Radius of the Blade, Tip Speed Ratio, Root Chord, Tip Chord, Root Alpha, Tip Alpha, Aerofoil) while satisfying the given constraints: (allowed chord length, the maximum possible value of the power coefficient, convergence threshold, and number of iterations) at the given parameters: (Wind speed, Number of Blades, Inside radius of the blade, Number of elements along the blade length, Density of air, Dynamic viscosity of air, mutationChance, mutationAmount).

$$\text{Maximize } \mathbf{J}(\mathbf{x},\mathbf{p})$$
$$\text{Subject to } \mathbf{g}(\mathbf{x},\mathbf{p}) \leq 0$$
$$x_{i,LB} \leq x_i \leq x_{i,UB} \quad (i= 1, \ldots, 6)$$

B. Design Variables

The design space of the wind turbine consists of the following design variables, which will be considered to achieve the desired result. The last design variable was kept constant during all our optimization process since it refers to the blade model aerofoil data. For our project, we are always using the S822 blade type which can be changed for other available types if the user wanted to.

$x_{1} =$ Turbine radius (R)
$x_{2} =$ Tip chord length (*tipchord*)
$x_{3} =$ Root chord length (*rootChord*)
$x_{4} =$ Root angle of attack (*rootAlpha*)
$x_{5} =$ Tip angle of attack (*tipAlpha*)
$x_{6} =$ (*tipSpeedRatio*)

C. Constraints/Design Variables Upper & Lower Bounds

The following will be considered as upper and lower bounds. Moreover, these constraints will be implemented in the simulation.

- Turbine radius (R) [m]: $0.5 \leq x_1 \leq 2$
- Tip chord length (tipchord) [m]: $0 \leq x_2 \leq 0.2$
- Root chord length (rootChord) [m]: $0.2 \leq x_3 \leq 1$
- Desired root angle of attack (rootAlpha) [°]: $0 \leq x_4 \leq 12$
- Desired tip angle of attack (tipAlpha) [°]: $5 \leq x_5 \leq 10$
- (tipSpeedRatio): $1 \leq x_6 \leq 10$
- $x_2 \leq x_3$
- $x_4 \leq x_5$

The maximum blade radius is bounded by 2 meters to model smaller wind turbines. The S822 aerofoil properties table can only input angles of attack between 0° and 12° and hence the angle of attack design variables are bounded by those values. Also, the angle of attack at the root of the blade x_4 is constrained to be less than or equal to the angle of attack at the tip of the blade x_5. The blade's chord length at the root of the blade x_3 is constrained to be greater than or equal to the chord length at the tip of the blade x_4.

D. Parameters

Design Constants:

Wind speed; p_1= 3m/s
Number of Blades; p_2= 3
Inside radius of the blade aka (hubRadius); p_3= 0.12
Number of elements along the blade length (elementsPerBalde); p_4= 30

Physical constants:

Density of air at 20 ℃, 1 bar; p_5 = 1.225 (kg/m^3)

Dynamic viscosity of air at 20 ℃, 1 bar; p_6= 1.82053-5 Pa*s

E. Objective

Maximizing the power coefficient output of a vertical wind turbine power system by finding the best geometry of an S822 aerofoil. The power coefficient is the result of the ratio of actual electric power produced by a wind turbine divided by the total wind power going towards the turbine blades at a specific wind speed

$$Max\ \ J_1(x_1, x_2\, x_3\, x_4, x_5\, x_6, \boldsymbol{p})$$

F. Governing Equations

The blade is split into elements where the force at each element is calculated and numerically integrated across the blade to calculate its torque. Then, the corresponding power and power coefficient (C_p) are calculated. The coefficients of lift, C_L, and coefficients of drag C_D are linearly interpolated from the S822 aerofoil properties table by inputting the calculated Reynolds number and angle of attack on the blade element. Also, the angular induction, $angInd$, axial induction, $axInd$, and in-flow angle, $inflow\ angle$, are coupled and therefore require iterative convergence of each of those variables. The variables r, $chord$, and α represent the element's radial distance from the hub, the chord length of the blade element, and the angle of attack of the element, respectively. The angle of attack, radius, and chord length parameters of each element linearly increases from the root to the tip of the blade, starting from the root angle of attack to the tip angle of attack, the hub radius to the radius of the blade, and the root chord length to the tip chord length, respectively. All respective equations are in the appendix section A.

G. Master table

Table 1. Master Table

Symbol	Description	Units	Type
R	Outside Radius of the Blade	m	Design Variable
tipSpeedRatio	Tip Speed Ratio	m/s	Design Variable
rootChord	Root Chord	m	Design Variable After Sensitivity Analysis it becomes a Design Parameter.
tipChord	Tip Chord	m	Design Variable
rootAlpha	Root Alpha	Radian	Design Variable
tipAlpha	Tip Alpha	Radian	Design Variable
S822	Blade profile data	Not applicable	Design Variable
maxChord	Maximum allowed chord length	m	Constraints
Cps	Maximum possible value of Power Coefficient	Unitless	Constraints
windSpeed	$Velocity\ of\ the\ wind$	m/s	Constant
numBlades	Number of Blades	Not applicable	Constant

hubRadius	Inside Radius of the Blade	m	Constant
elementsPerBlade	Number of elements along the blade length	Not applicable	Constant
rho	Density of air at 20 ℃, 1 bar	kg/m^3	Constant
mu	Dynamic viscosity of air at 20 ℃, 1 bar	Pa*s	Constant
mutationChance	Chance of given blade parameter being mutated	Not applicable	Constant
betzLimit	Maximum possible value of Cp	Not applicable	Constant
mutationAmount	Fractional change in values when mutated	Not applicable	Constant

IV. Model and Simulation Process

A. Module Identification

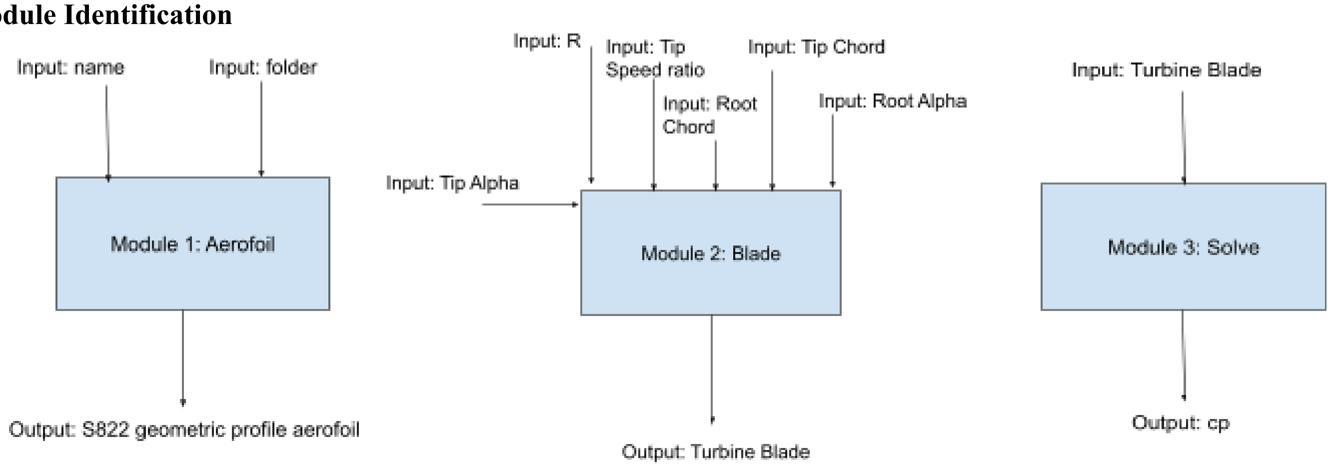

Figure 3: Module Identification

Module 1: AEROFOIL Represents an aerofoil profile. Deals with loading the data, lift/drag coefficient lookup and storing the blade geometry. The folder input is a path to a directory containing .mat files for CL lookup, Re values, alpha values, and coordinate points. Creates a new aerofoil with the given name and reads the relevant data from the given directory.

Module 2: BLADE takes care of the wind turbine blade design. It acquires all variables (R, Tip Alpha, Tip Speed ratio, Root Chord, Tip Chord, Root Alpha) necessary to define the blade geometry. It creates a new turbine blade with the given parameters.

Module 3: SOLVE Solves all elements of this blade design for pitch angle and calculates the resulting output power coefficient.

B. Model Validation

The four modules were validated primarily by comparing module inputs and outputs to data found in the literature [5]. The main algorithm used by the three modules was retrieved from a design engineering graduate paper and validated by the department of Mechanical Engineering at the University of Bath. During the early stages of the project, our team decided to work on performing Multidisciplinary Design Optimization to models that were as realistic as possible to the ones used in the wind turbine industry. Although the codes retrieved for each module seemed quite complex, they account for variables and aerodynamic aspects that bring our algorithm as close as possible to a real-life industry model.

C. Module ordering: N2 Diagram

Table 2: N2 Diagram

In	name, folder	tipAlpha, R , tipSpeedRatio, rootChord, tipchord, rootAlpha		
	Aerofoil	S822 geometric profile aerofoil		
		Blade	Turbine Blade	
			Solve	Cp
				Out

D. Simulation/Feasibility

A simulation was conducted using the constraints & design variables for the upper & lower bounds established in Problem Formulation, Section C. The objective function used in the initial simulation was J_1 = max(Cp) with the following assumptions: the wind turbine used for optimization is less or equal to 2 meters in diameter and the S822 blade profile was used as the blade model. The design vector used is presented in Figure 4:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} R \\ Tipchord \\ rootChord \\ rootAlpha \\ tipAlpha \\ tipSpeedRatio \end{bmatrix}$$

Figure 4: Design Vector

Our algorithm is a very realistic one and therefore presents upper and lower bounds for every single design variable and multiple constraints to address the problem trying to solve as realistically as possible. To start implementing simulation in our algorithm we must provide an initial feasible design vector that satisfies these ranges for each design variable. We have considered the constraints mentioned before in the simulation section.

Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
0.475	0.2	0.8	0.017453293	0.087266463	2	0.353229322

Figure 5: First Simulation Results

Our first simulation before performing DOE was feasible and satisfied every upper & lower bound range, constraints, g, &h. The simulation returned a coefficient of power of 0.35229. This is a very positive sign that our algorithm works and that we are getting a realistic output. The average wind turbine falls in the 35-45% range [1]. Another element that we must consider in order to check for feasibility is the maximum cp that a wind turbine can have which is Cp = 0.593.

E. DOE

We carried out an initial exploration of the design space by using the One at a Time DOE Technique. This method was chosen because it gives us a great opportunity to initially sample our space without the cost of running a full factorial. In total, we chose 6 factors or design variables for our DOE: Turbine radius (R), Tip chord length (tipchord), Root chord length (rootChord), Desired root angle of attack (rootAlpha), Desired tip angle of attack (tipAlpha), (tipSpeedRatio). We chose these factors because these design variables present a wide range of values that we can choose from when selecting our initial set of values for design vector **x**. The DOE will help us make a more educated guess on the starting values by minimizing the pool of options for **x**. Understanding what approximate values our variables should take will ease our way into getting a high coefficient of power. Our DOE provides us with a higher chance that our algorithm will not get stuck in local maximums. It will also help my team look for potentially interesting areas that our optimizer should explore and search. We are doing 5 levels for each of the factors. Creating a total of 25 experiments and consequently 25 different observations.

- Levels (l) = 5
- Factors (n) = 6
- # Experiments = $1 + n(l - 1) \rightarrow$ 25 experiments

The more levels we add, the more expensive our DOE will end up turning. However, our algorithm had multiple constraints as listed earlier in the paper. This means that there is a high chance that many of our experiments will not give us a reasonable coefficient of power or simply violate some constraint. The following DOE table generated from our Matlab code helps us visualize the observations obtained from each experiment. When our observation (Coefficient of power) is 0 this means that constraints were violated, and our code produced an error as a result.

Table 3: One-at-a-Time Design of Experiments

Experiment #	Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
1	0.3	0.2	0.2	0.017453293	0.087266463	2	0.038392307
2	0.475	0.2	0.2	0.017453293	0.087266463	2	0.044529733
3	0.65	0.2	0.2	0.017453293	0.087266463	2	0.039306392
4	0.825	0.2	0.2	0.017453293	0.087266463	2	0.035335239
5	1	0.2	0.2	0.017453293	0.087266463	2	0.031430548
6	0.475	0.275	0.2	0.017453293	0.087266463	2	0.143633189
7	0.475	0.35	0.2	0.017453293	0.087266463	2	0
8	0.475	0.425	0.2	0.017453293	0.087266463	2	0
9	0.475	0.5	0.2	0.017453293	0.087266463	2	0
10	0.475	0.2	0.4	0.017453293	0.087266463	2	0.19465744
11	0.475	0.2	0.6	0.017453293	0.087266463	2	0.274269051
12	0.475	0.2	0.8	0.017453293	0.087266463	2	0.353229322
13	0.475	0.2	1	0.017453293	0.087266463	2	0.410049907
14	0.475	0.2	0.2	0.034906585	0.087266463	2	0.075347948
15	0.475	0.2	0.2	0.052359878	0.087266463	2	0.106064441
16	0.475	0.2	1	0.06981317	0.087266463	2	0
17	0.475	0.2	1	0.087266463	0.087266463	2	0
18	0.475	0.2	1	0.017453293	0.109083078	2	0
19	0.475	0.2	1	0.017453293	0.130899694	2	0
20	0.475	0.2	1	0.017453293	0.15271631	2	0
21	0.475	0.2	1	0.017453293	0.174532925	2	0
22	0.475	0.2	1	0.017453293	0.087266463	4	0
23	0.475	0.2	1	0.017453293	0.087266463	6	0
24	0.475	0.2	1	0.017453293	0.087266463	8	0
25	0.475	0.2	1	0.017453293	0.087266463	10	0

During One at a Time, we change the first factor and keep all others at base value. If the output is improved, we will keep the new level for that factor. We then move on to the next factor and repeat. We can see that even though we ran all these experiments, almost half of them violated some kind of constraint and therefore were infeasible (Power Coefficient = 0). Based on this analysis we are going to initially start our numerical optimization with point x0:

Experiment #	Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
13	0.475	0.2	1	0.017453293	0.087266463	2	0.410049907

Figure 6: Design variables combination resulting in the highest Cp value

The coefficient of power of a wind turbine is a measurement of how efficiently the wind turbine converts the energy in the wind into electricity. This **x0** set gives us a Power Coefficient of 0.41. This DOE gave us a very satisfactory output. The average range of coefficient of power for a 1m diameter blade wind turbine falls in the 35-45% range [1]. Just with the x0 that we obtained from the DOE we are already at a very optimal place in terms of achieving the highest power coefficient possible.

V. Optimization

A. Sensitivity Analysis

We ran a sensitivity analysis for the results obtained from the One-at-a-Time Design of Experiments. It was a linear regression model that gave the sensitivity indices for each of the design variables namely R, tipSpeedRatio, rootChord, tipChord, rootAlpha and tipAlpha. The results obtained are shown in the table below:

Table 4: Sensitivity Analysis Results

Design Variable	Sensitivity Index	Importance
Turbine Radius (R)	0.4313	More important than Root Chord Length, but less important than other variables
Tip Speed Ratio	0.4660	Important
Root Chord Length	0.0244	Least important
Tip Chord Length	0.4717	Most Important
Root angle of attack	0.4481	Equally Important
Tip angle of attack	0.4481	

The result of the sensitivity analysis is shown in the picture below:

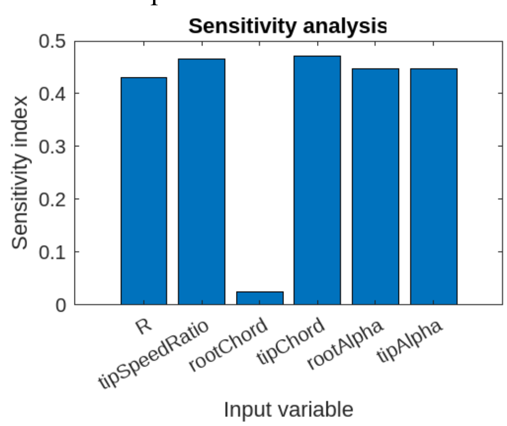

Figure 7: Sensitivity Analysis

From the table and the graph above, it is seen that the Root Chord Length is the least important design variable and hence can be treated as a design parameter. The value for the same is set at 0.2 meters to model smaller wind turbine blades for further optimization. Moreover, the most important design variables are the tip speed ratio, the angles of attack, followed by the turbine radius.

B. Single Objective Optimization

B.1 Multi-start SQP Approach

The function to calculate the coefficient of power, Cp, is non-smooth because it uses a linear interpolation of the coefficient of lift and coefficient of drag values from the S822 aerofoil properties table. There will be 3 different approaches to this optimization problem addressed in this paper. The first approach utilizes a multi-start gradient-based algorithm which involves the creation of 10 random design vectors which will be used as initial guesses to a Sequential Quadratic Program (SQP) optimizer. The SQP will converge to a design point corresponding to a local maximum Cp value which will also serve as a candidate for the potential global maximum. The 10 randomly generated design vectors are first scaled by a factor equal to their upper bound, such that they are normalized. The constraints are also scaled by the same factor. After all 10 randomly generated design vectors are input to the SQP algorithm, the converged design vector corresponding to the maximum Cp value is selected as the global maximum of the objective. The SQP was implemented using the 'fmincon' function in MATLAB and the objective function was negated so that the function will converge to the maximum value of the objective. The results for the multi-start approach can be observed in Table 5 below. It can be observed that all initial design vectors input to the SQP algorithm converged to the same point yielding a Cp value of 0.4809. It should be noted that the first-order optimality values from the 'fmincon' function were non-zero and in the order of 10^{-2} at most. This is most likely due to the non-smoothness of the objective function and therefore we cannot state with absolute certainty that the design vector is a global minimum. The design vector corresponding to the maximum Cp value was:

$$[R,\ tipChord, rootchord,\ rootAlpha,\ tipAlpha,\ tipSpeedRatio] = [\,0.841,\ 0.086, 0.2,\ 9,\ 9,\ 4.14]$$

Table 5: Multi-Start SQP

Iteration	Cp	Time [s]
1	0.4809	10.4
2	0.4809	4.2
3	0.4809	5.4
4	0.4809	8.2
5	0.4809	6.4
6	0.4809	6
7	0.4809	4.3
8	0.4809	4.4
9	0.4809	7.1
10	0.4809	5.6

B2. Genetic Algorithm Approach

The second approach used to accomplish the optimization problem is the Genetic Algorithm (GA). This heuristic will be used to locate the general area where the global minimum is located. 10 trials of the GA were conducted in MATLAB using a population size of 50, with a maximum of 800 generations to validate the precision of the heuristic and compare to results obtained from the multi-start algorithm discussed before. One trial of the GA can be observed in Figure 4 below. Over 500 generations were required in this trial for convergence. The crossover rate was tuned to be 0.5 to achieve a balance between retaining good design vectors across future generations while also diversifying the population enough to prevent any convergence to a local minimum objective. It can be observed that the best possible Cp value is approximately 0.4808 which is close to but not exactly equal to the optimal value obtained from the multi-start approach. All GA trials converged to design points corresponding to a Cp of approximately 0.4808. It can also be seen that the average runtime for one trial of the GA is higher than the average runtime for each iteration of SQP in the multi-start approach which indicates that it is a more costly approach.

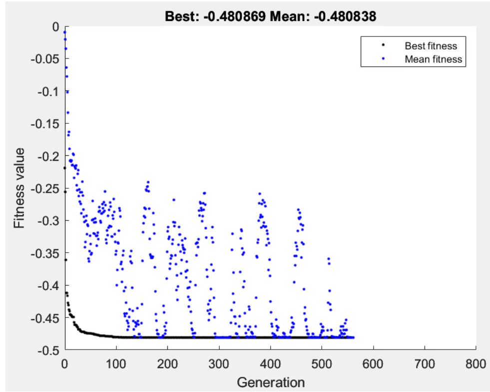

Figure 8: GA Convergence Plot (Population Size = 50; Crossover Fraction = 0.5; Runtime = 574 s)

B3. GA-SQP Hybrid Approach

The third approach used to accomplish the optimization problem is a hybrid approach of utilizing GA followed by running SQP on an initial design vector provided by the output of the GA. This approach is used to obtain a more accurate solution from the results output from the GA heuristic. As discussed before, the GA heuristic was implemented with 800 allowable generations at maximum. The results of this approach are in Table 6 below. These results show that the globally optimal design vector corresponds to a Cp value of 0.4809. Both the multi-start SQP approach and the hybrid GA-SQP approach converged to the same design vector which indicates that the same design vector output from both approaches corresponds to the globally optimal design for the optimization problem.

Table 6: Results of GA-SQP Hybrid Approach

Iteration #	C_p (1^{st} pass: GA)	C_p (2^{nd} pass: SQP)
1	0.480858	0.4809
2	0.480843	0.4809
3	0.480839	0.4809
4	0.480869	0.4809
5	0.480854	0.4809

C. Multiobjective Optimization

Another objective function J_2 was introduced representing the amount of material used to create the wind turbine blade. Objective functions J_1 and J_2 can be represented below:

$$J_1 = C_p(X)$$
$$J_2 = tipChord \ + \ R$$

C_p is a function of X which is equal to the design vector:

$$X \ = \ [R, \ tipChord, \ rootAlpha, \ tipAlpha, \ tipSpeedRatio].$$

The motivation for creating another objective function J_2 is to determine a maximum Coefficient of Power J_1 while minimizing the amount of material used in the turbine J_2, which we are defining as the radius of the turbine R and the chord length at the tip of the blade $tipChord$. Specifically, the objective functions can be written as follows:

$$max_X(J_1)$$

$$min_{[X(1), X(2)]}(J_2)$$
$$\text{subject to } g(X) \leq 0$$

The weighted sum approach was used to maximize J_1 while minimizing J_2 by creating a new objective J_3 which is minimized as shown below:

$$min_X(J_3 = \lambda\overline{J_1} + (1 - \lambda)\overline{J_2})$$
$$\text{subject to } g(X) \leq 0$$
$$\text{for } \lambda \in [0, 1]$$

$\overline{J_1}$ and $\overline{J_2}$ are the objective functions J_1 and J_2 scaled by the maximum value achievable by the individual objective J_i in order to normalize the objective function so that they are equally weighted. The objective functions are therefore scaled by the following factors, as shown below:

$$\overline{J_1} = \frac{J_1}{-0.4809}$$

$$\overline{J_2} = \frac{J_2}{2.2}$$

J_1 is scaled by the negated value of the maximum C_p achievable for the constraints $g(X) \leq 0$ so that minimizing J_3 will lead to a maximization of J_1. The weighted sum approach was implemented for 25 λ values ranging from 0 to 1 which produced the following pareto front in Figure 9 below:

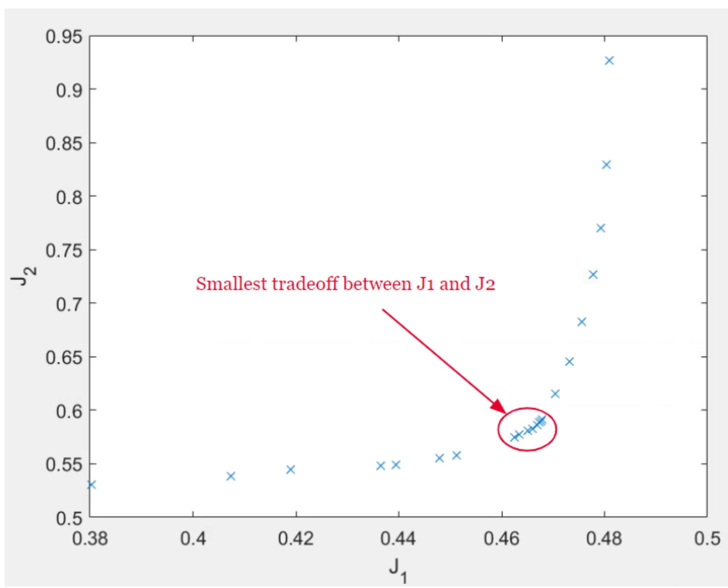

Figure 9: Pareto Front for Multiobjective optimization using Weighted Sum Method

It was observed that the selection of points in the range of $J_1 \in [0.462, 0.468]$ and $J_2 \in [0.575, 0.591]$ in the pareto front has the smallest increase in J_2 for the largest values in J_1. In other words, a design with a sufficiently high C_p with minimal increase in tip chord length and blade radius can be chosen from that selection of points.

C1. Post-Optimality Analysis

To measure the sensitivity of the objective function to changes in design parameters, the design parameters and constraints were first scaled, as done in the multi-start SQP approach, to normalize the design variables. Then, the gradient was numerically calculated at the final design point in MATLAB. For maximizing the single objective, C_p, the gradient located at the optimal design point was found to be:

$$[\frac{\partial J_1}{\partial \bar{x}_1} \frac{\partial J_1}{\partial \bar{x}_2} \frac{\partial J_1}{\partial \bar{x}_4} \frac{\partial J_1}{\partial \bar{x}_5} \frac{\partial J_1}{\partial \bar{x}_6}] = [10^{-5}, 10^{-6}, -0.05, 0.04, -10^{-4}]$$

$\bar{x}$ represents the scaled design vector. The most sensitive variables are x_4 and x_5 because they have the highest magnitude of gradient in the objective function. This was verified to be true as the root and tip angles of attack contributed to the highest perturbation of C_p for some constant arbitrary perturbation to each design variable separately. In the multi-objective optimization, the design variables and constraints were also scaled and the gradient of the design vector corresponding to J_1=0.468 and J_2 = 0.591 m was numerically calculated and shown below:

$$[\frac{\partial J_3}{\partial \bar{x}_1} \frac{\partial J_3}{\partial \bar{x}_2} \frac{\partial J_3}{\partial \bar{x}_4} \frac{\partial J_3}{\partial \bar{x}_5} \frac{\partial J_3}{\partial \bar{x}_6}] = [-10^{-4}, -10^{-3}, -10^{-3}, -10^{-3}, 10^{-3}]$$

It can be observed from this gradient that the most sensitive variables are the tip chord length, angle of attack at the root, and angle of attack at the tip. It should be noted that the gradients located at the optimal design points are not zero, most likely due to the non-smoothness of the function.

VI. Conclusion

For small wind turbine blades of a 0.2-meter root chord length, the optimal design for maximizing the coefficient of power C_p in a 3-bladed wind turbine, assuming a constant wind speed of 3 m/s, is a blade of a 0.841-meter radius with a constant angle of attack of 9° across its length, with an operating tip speed ratio of 4.14. To maximize the coefficient of power while also minimizing the radius of the blade and its tip chord length, in an effort to minimize the amount of material used in the design, it was found that a selection of design points indicated in the Pareto front from Figure 5 has maximum possible C_p and the minimum possible sum of radius and tip chord length for the smallest tradeoff between the two objectives. In a post-optimality analysis, it was found that the gradient values at the final design points considered in our single objective and multiobjective analysis were close to zero with the root and tip angles of attack being the most sensitive variables. For future work, different aerofoil profiles can be considered for the optimization of C_p with larger wind turbines which were not bound by the upper bound constraints in our analysis. Also, the multiobjective optimization of maximizing C_p while minimizing the amount of material used can be improved by assigning scale factors to the dimensions of the wind turbine which represent how much material each dimension can provide to the turbine.

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Appendix

A. Governing Equations

$$local\ speed\ ratio = \frac{(tip\ speed\ ratio)(r)}{R_o}$$

$$local\ Blade\ Solidity = \frac{(numBlades)(chord)}{2\pi r}$$

$$inflow\ angle = arctan\left(\frac{1 - ax\ Ind}{local\ SpeedRatio(1 + angInd)}\right)$$

$$V_{rel} = \frac{windspeed(1 - axInd)}{sin(inFlowAngle)}$$

$$Re = \frac{(\rho V_{rel})(chord)}{\mu}$$

$$C_x = C_L cos(inflowAngle) + C_D sin(inflowAngle)$$
$$C_\theta = C_L sin(inflowAngle) + C_D cos(inflowAngle)$$

$$axInd = \frac{1}{\frac{4sin(inflow\ Angle)^2}{C_x(localBladeSolidity)} + 1}$$
$$angInd = \frac{1}{\frac{4sin(inflow\ Angle)\ cos(inflowAngle)}{C_\theta(localBladeSolidity)} - 1}$$

$$angInd,\ axInd,\ inflowAngle\ converges$$
$$pitch = inflowAngle - \alpha$$
$$F_\theta = 0.5\rho V_{rel}^2(chord)(r)(C_\theta)$$

$$T = \int F_\theta dr \rightarrow Total\ torque\ on\ blade$$

$$C_p = \frac{P_{blade}}{P_{wind}} = \frac{(rotation\ speed)(T)(\#\ of\ blades)}{\frac{1}{2}\rho * V_{wind}^3 * Area}$$