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Maximizing Power Coefficient Output of a Vertical Wind Turbine

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Victor de la Parra, Carlos Navarro, Karthik Moorthy, & Namrata Chandravadan Thakkar

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Introduction

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- Wind turbine are responsible for:
  - Roughly 5% of the worlds power
  - 8 % of United States power
- Over the past ten years, the Americas' wind energy capacity has tripled
- Goal of a wind turbine is to produce the largest amount of electrical energy possible by extracting as much energy as it can from the wind
- Efficiency is measured by the power coefficient (Cp)
- The higher Cp the better energy output
- Average range of Cp for a 1 m diameter blade is from 0.35 to 0.45.

Objective

- Maximize a wind turbine power coefficient (Cp) by changing blade geometric parameters (Outside Radius of the Blade , Tip Speed Ratio, Root Chord, Tip Chord, Root Alpha, Tip Alpha, Aerofoil) while satisfying the given constraints.

$$J_1 = \max(C_p)$$

$$\text{Maximize } J(x,p)$$

$$\text{Subject to } g(x,p) \leq 0$$

$$x_{i,LB} \leq x_i \leq x_{i,UB} \quad (i = 1, \ldots, 6)$$

Design Variables

- The blade profile was kept constant which was the S822 blade
- Additional Design Variables

$x_1 =$ Turbine radius (R)

$x_2 =$ Tip chord length ($tipchord$)

$x_3 =$ Root chord length ($rootChord$)

$x_4 =$ Desired root angle of attack ($rootAlpha$)

$x_5 =$ Desired tip angle of attack ($tipAlpha$)

$x_6 =$ ($tipSpeedRatio$)

$x_7 =$ Blade profile data ($S822$)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} R \\ Tipchord \\ rootChord \\ rootAlpha \\ tipAlpha \\ tipSpeedRatio \end{bmatrix}$$

Design Vector

Parameters

Design Constants:

- Wind speed ; p_1= 3m/s
- Number of Blades; p_2= 3
- Inside radius of the blade aka (hubRadius); p_3= 0.12 m
- Number of elements along the blade length (elementsPerBlade); p_4= 30

Physical constants:

- Density of air at 20 ℃, 1 bar; p_5 = 1.225 (kg/m^3)
- Dynamic viscosity of air at 20 ℃, 1 bar; p_6= 1.82053-5 Pa*s

Module Identification

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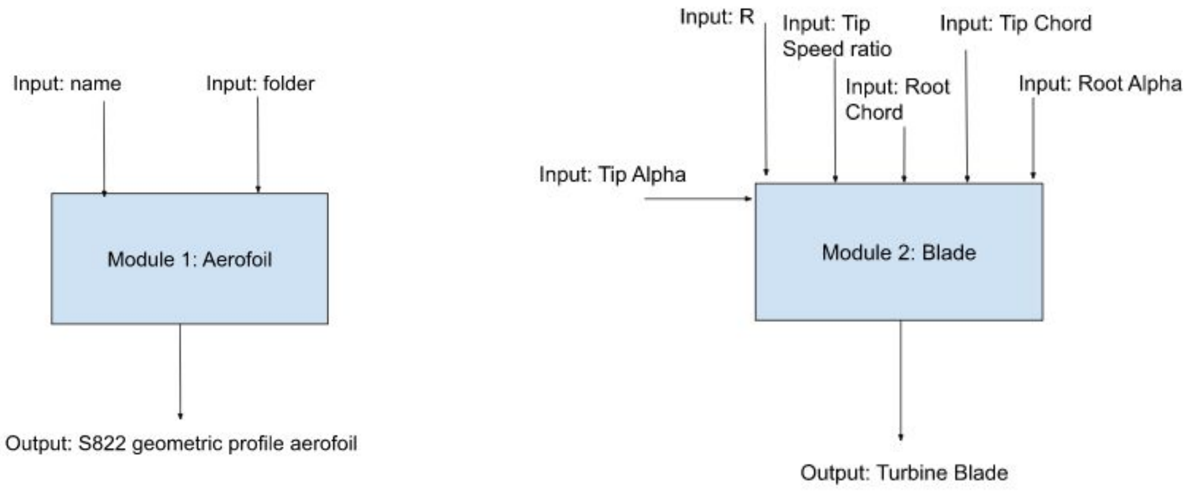

Module 1: AEROFOIL Represents aerofoil profile. **Loads data, lift/drag coefficient lookup & blade geometry**. **Creates** a **new aerofoil** with the given name and reads the relevant data from the given folder directory.
Module 2: BLADE t**akes care of the wind turbine blade design**. It **acquires all variables necessary** to define the blade geometry and **creates a new turbine blade with the given parameters**.
Module 3: SOLVE **Solves** all **elements** of this blade design **for pitch angle**. **Calculates the resulting output power coefficient**.

Equations

$$local\ speed\ ratio = \frac{(tip\ speed\ ratio)(r)}{R_o}$$

$$local\ Blade\ Solidity = \frac{(numBlades)(chord)}{2\pi r}$$

$$inflow\ angle = \arctan\left(\frac{1-ax\ Ind}{local\ SpeedRatio(1+angInd)}\right)$$

$$V_{rel} = \frac{windspeed(1-axInd)}{\sin(inFlowAngle)}$$

$$Re = \frac{(\rho V_{rel})(chord)}{\mu}$$

$$C_x = C_L\cos(inflowAngle) + C_D\sin(inflowAngle)$$

$$C_\theta = C_L\sin(inflowAngle) + C_D\cos(inflowAngle)$$

$$axInd = \frac{1}{\frac{4\sin(inflow\ Angle)^2}{C_x(localBladeSolidity)} + 1}$$

$$angInd = \frac{1}{\frac{4\sin(inflow\ Angle)\cos(inflowAngle)}{C_\theta(localBladeSolidity)} - 1}$$

$$pitch = inflowAngle - \alpha$$

$$F_\theta = 0.5\rho V_{rel}^{\ 2}(chord)(r)(C_\theta)$$

$$T = \int F_\theta dr \rightarrow Total\ torque\ on\ blade$$

$$C_p = \frac{P_{blade}}{P_{wind}} = \frac{(rotation\ speed)(T)(\#\ of\ blades)}{\frac{1}{2}\rho * V_{wind}^{\ 3} * Area}$$

N-2 Diagram

In	name, folder	tipAlpha, R , tipSpeedRatio, rootChord, tipchord, rootAlpha		
	Aerofoil	S822 geometric profile aerofoil		
		Blade	Turbine Blade	
			Solve	Cp
				Out

Feasibility

Our **algorithm** is a **very realistic** one and therefore presents **upper and lower bounds** for every single design variable.

- Turbine radius (R): $0.3 \leq x_1 \leq 1$ [m]
- Tip chord length (tipchord): $0.2 \leq x_2 \leq 0.5$ [m]
- Root chord length (rootChord): $0.2 \leq x_3 \leq 1$ [m]
- Desired root angle of attack (rootAlpha): $0 \leq x_4 \leq 12$ degrees
- Desired tip angle of attack (tipAlpha): $5 \leq x_5 \leq 10$ degrees
- (tipSpeedRatio) : $1 \leq x_6 \leq 10$

Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
0.475	0.2	0.8	0.017453293	0.087266463	2	0.353229322

The simulation returned a coefficient of power of 0.35229. This is a very positive sign that our algorithm works and that we are getting a realistic output. The average wind turbine falls in the **35-45% range**

Design of Experiments

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One-at-a-Time

Levels (l) = 5

Factors (n) = 6

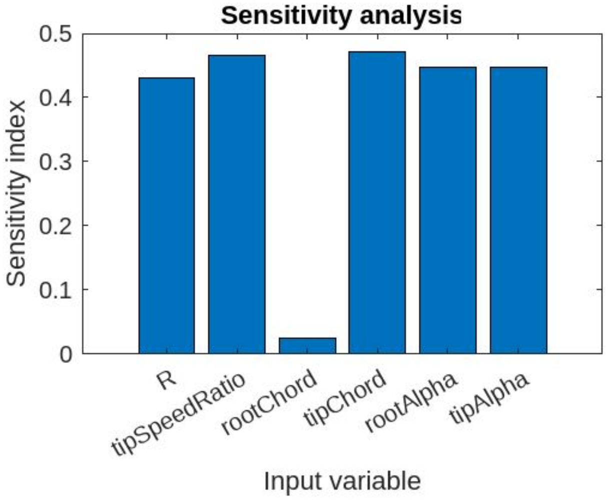
Experiments = 1+n(l-1) → 25 experiments

Experiment #	Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
1	0.3	0.2	0.2	0.017453293	0.087266463	2	0.038392307
2	0.475	0.2	0.2	0.017453293	0.087266463	2	0.044529733
3	0.65	0.2	0.2	0.017453293	0.087266463	2	0.039306392
4	0.825	0.2	0.2	0.017453293	0.087266463	2	0.035335239
5	1	0.2	0.2	0.017453293	0.087266463	2	0.031430548
6	0.475	0.275	0.2	0.017453293	0.087266463	2	0.143633189
7	0.475	0.35	0.2	0.017453293	0.087266463	2	0
8	0.475	0.425	0.2	0.017453293	0.087266463	2	0
9	0.475	0.5	0.2	0.017453293	0.087266463	2	0
10	0.475	0.2	0.4	0.017453293	0.087266463	2	0.19465744
11	0.475	0.2	0.6	0.017453293	0.087266463	2	0.274269051
12	0.475	0.2	0.8	0.017453293	0.087266463	2	0.353229322
13	0.475	0.2	1	0.017453293	0.087266463	2	0.410049907
14	0.475	0.2	0.2	0.034906585	0.087266463	2	0.075347948
15	0.475	0.2	0.2	0.052359878	0.087266463	2	0.106064441
16	0.475	0.2	1	0.06981317	0.087266463	2	0
17	0.475	0.2	1	0.087266463	0.087266463	2	0
18	0.475	0.2	1	0.017453293	0.109083078	2	0
19	0.475	0.2	1	0.017453293	0.130899694	2	0
20	0.475	0.2	1	0.017453293	0.15271631	2	0
21	0.475	0.2	1	0.017453293	0.174532925	2	0
22	0.475	0.2	1	0.017453293	0.087266463	4	0
23	0.475	0.2	1	0.017453293	0.087266463	6	0
24	0.475	0.2	1	0.017453293	0.087266463	8	0
25	0.475	0.2	1	0.017453293	0.087266463	10	0

Experiment #	Turbine radius (R)	Tip chord length (tipchord)	Root chord length (rootChord)	Desired root angle of attack (rootAlpha)	Desired tip angle of attack (tipAlpha)	(tipSpeedRatio)	Observation (Coefficient of Power)
13	0.475	0.2	1	0.017453293	0.087266463	2	0.410049907

Sensitivity Analysis

Design Variable	Sensitivity Index	Importance
Turbine Radius (R)	0.4313	More important than Root Chord Length, but less important than other variables
Tip Speed Ratio	0.4660	Important
Root Chord Length	0.0244	Least important
Tip Chord Length	0.4717	Most Important
Root angle of attack	0.4481	Equally Important
Tip angle of attack	0.4481	



Multi-start SQP

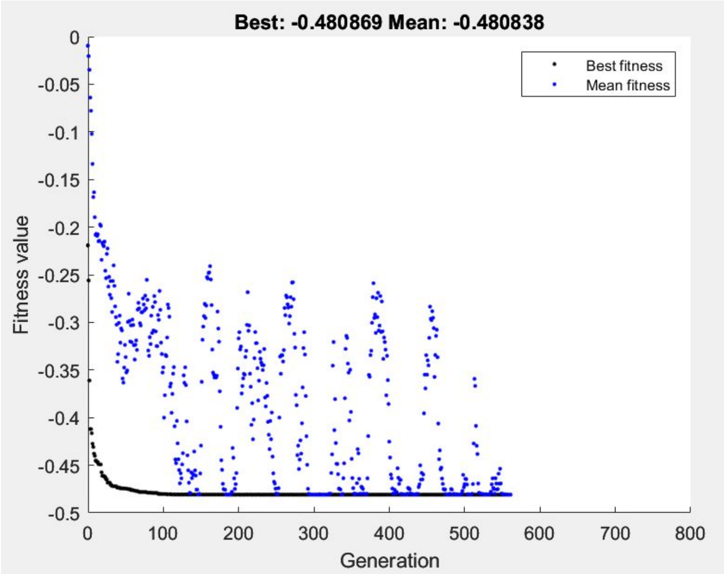
- 10 random design vectors
  - Used as initial guesses for the SQP optimizer.
- Converges at the locally maximum Cp
- SQP was implemented using the ‘fmincon’ function in Matlab
- Results:
  - All initial design vector inputs converged to the same Cp = 0.4809
- Note
  - First order optimality value from ‘fmincon’ function were non-zero & in the order of 10^{-4}
  - Most likely due to non-smoothness of the objective function
  - We are not absolute certainty that the design vector is a global minimum

SQP Results

Iteration	Cp	Time [s]
1	0.4809	10.4
2	0.4809	4.2
3	0.4809	5.4
4	0.4809	8.2
5	0.4809	6.4
6	0.4809	6
7	0.4809	4.3
8	0.4809	4.4
9	0.4809	7.1
10	0.4809	5.6

Genetic Algorithm

- 10 trials of the GA using a population size of 50, with a maximum of 800 generations.
- The crossover rate = 0.5 (balance between retaining good design vectors across future generations while also diversifying the population enough to prevent any convergence to locally minimum objective)
- Results:
  - Approx. 0.4808
  - Average run time for GA is more than SQP
  - Hence, a costly approach



GA Convergence Plot (Population Size = 50; Crossover Fraction = 0.5; Runtime = 574 s)

GA-SQP Hybrid

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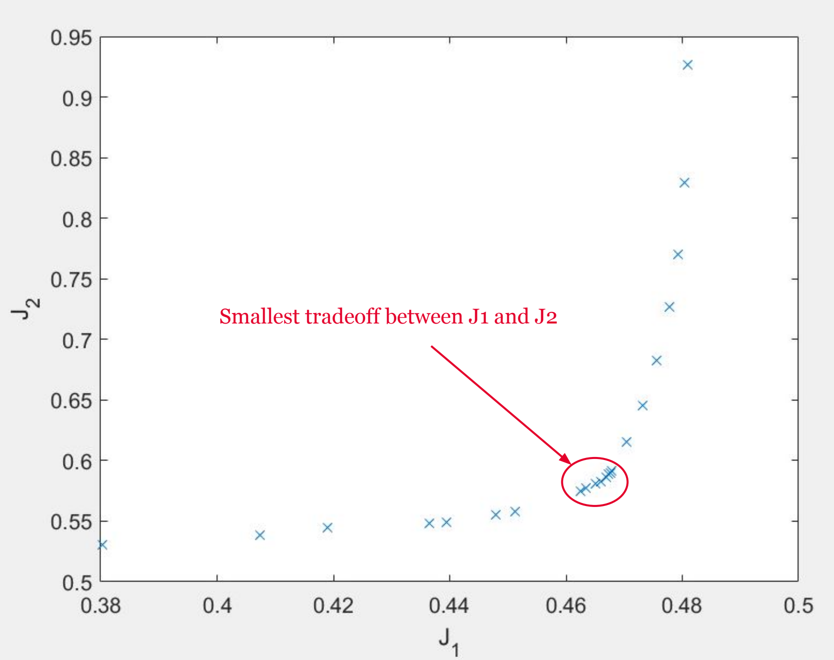
- Utilized GA first, then SQP optimizer using result from GA.
- Both multi-start SQP and hybrid GA-SQP converged to the same design vector.
- GA-SQP validates Multi-Start SQP.
- Final design for single objective (Cp):
  - **X*** = [R, tipChord, rootAlpha, tipAlpha, tipSpeedRatio]*
  - = **[0.841, 0.086, 9, 9, 4.14]**

GA-SQP Results

Trial #	C_p (1st pass: GA)	C_p (2nd pass: SQP)
1	0.480858	0.4809
2	0.480843	0.4809
3	0.480839	0.4809
4	0.480869	0.4809
5	0.480854	0.4809

Multiobjective Optimization

- $J1 = Cp(X)$
  - X = [R tipChord rootAlpha tipAlpha tipSpeedRatio]
- J2 = tip chord length + R
- Goal is to maximize J1 while minimizing J2 subject to the inequality constraints $g(X) \leq 0$.
- J1 and J2 are normalized appropriately:
  - $J1^* = J1/(-0.4809)$
  - $J2^* = J2/(2.2)$
- New objective function is created using the weighted sum method:
  - $J3 = \lambda(J1^*)+(1-\lambda)J2^*$
  - $g(X) \leq 0$
  - $\lambda \in [0,1]$
- $J1 \in [0.462, 0.468]$ and $J2 \in [0.575, 0.591]$ in the generated pareto front attains small increases in J2 for the higher J1



Pareto Front for Multiobjective optimization using weighted sum method

Future Work

- Finalize a post optimality analysis.
- Use a different blade profile.
- Maximize the Coefficient of Power (Cp) for larger wind turbines.
- Scale blade radius and tip chord length in the multiobjective optimization, corresponding to how much material each dimension provides to the blade.

References

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Thank you! Questions?