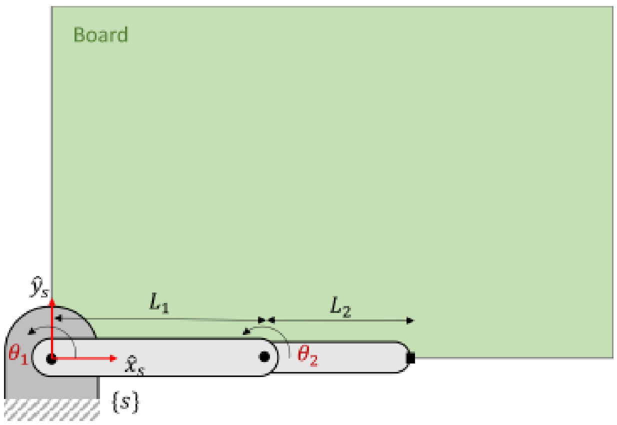
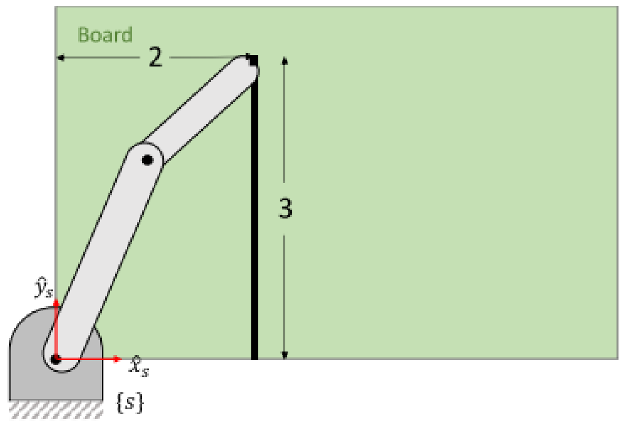
Robotic Manipulator Trajectory control / Feedforward PID feedback controller with torque input Design

Mechanics of Robotic Manipulators Graduate Research Project

Problem Statement & Goal:





For this project I considered an RR manipulator in its zero configuration, as shown above, with $L_1 = 3$ and $L_2 = 2$. My robot has a marker attached at the end-effector, which can draw on the board defined by the x-y plane. I was tasked to control the robot to draw a straight vertical line from (2, 3) to (2,0) within 3 seconds. The desired trajectories in the task space are given by:

$$\vec{x}_d(t) = \begin{bmatrix} 2 \\ 3-t \end{bmatrix} \text{ where } 0 \le t \le 3 \text{ [sec]}$$

1. I generated the desired positional trajectory ($\vec{\theta}_d$) in the joint space numerically by using the Newton-Raphson method. Plots of the joint trajectories in a single graph, with t along the x-axis and $\theta_1(t)$ and $\theta_2(t)$ on the y-axis can be shown in the report as Figure 1. I also plotted the calculated end effector trajectory given calculated $\theta_1(t)$ and $\theta_2(t)$ saved as Figure 2.

   ▪ For each figure, I used different colors & line types for individual variables. $h = 0.1$ [sec] was chosen for the time step size and $e_d = 0.001$ for the desired error.

   ▪ For the Newton-Raphson method, I used the initial guess $\vec{\theta}(0)^0 = [\pi/6, \pi/6]^T$; and for all following data point I used the previously calculated $\vec{\theta}$ values as the initial guess for the next approximation, such that $\vec{\theta}(t_i)^0 = \vec{\theta}(t_{i-1})^n$, where $\vec{\theta}(t_{i-1})^n$ is the final approximation at the previous time step.

2. Now, I was tasked to consider a constant speed control with the desired trajectory specified by $\vec{\theta}_d = [t, 0]^T$; $\dot{\vec{\theta}}_d = [1,0]^T$, and $\ddot{\vec{\theta}}_d = \vec{0}$. If the center of mass for

each link (considered as a point mass) is located at the distal tip of the link, and there is no friction. In this case, the dynamic model of this robot can be calculated as

$$\vec{\tau} = M(\vec{\theta})\ddot{\vec{\theta}} + V\left(\vec{\theta}, \dot{\vec{\theta}}\right) + G(\vec{\theta})$$

Where:

$$M(\vec{\theta}) = \begin{bmatrix} L_2^2 m_2 + 2L_1 L_2 m_2 \cos\theta_2 + L_1^2(m_1 + m_2) & L_2^2 m_2 + L_1 L_2 m_2 \cos\theta_2 \\ L_2^2 m_2 + L_1 L_2 m_2 \cos\theta_2 & L_2^2 m_2 \end{bmatrix}$$

$$V\left(\vec{\theta}, \dot{\vec{\theta}}\right) = \begin{bmatrix} -m_2 L_1 L_2 \sin\theta_2 \dot{\theta}_2^2 - 2m_2 L_1 L_2 \sin\theta_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 L_1 L_2 \sin\theta_2 \dot{\theta}_1^2 \end{bmatrix}$$

$$G(\vec{\theta}) = \begin{bmatrix} m_2 L_2 g \cos(\theta_1 + \theta_2) + (m_1 + m_2) L_1 g \cos\theta_1 \\ m_2 L_2 g \cos(\theta_1 + \theta_2) \end{bmatrix}$$

given $m_1 = m_2 = 1kg$, and $g = 9.8m/s\ 2$.

a) My task was to Design the feedforward plus PID feedback controller with torque input using $k_p = 10$, $k_d = 5$, and $k_i = 0.2$ for both joints.

b) Using MATLAB, I plotted the torque τ_1 and τ_2 obtained from the controller for t = 0: 0.1: 5[sec]. I used different colors or line types for τ_1 and τ_2 and saved it as Figure 3.

c) Using MATLAB, I plotted $\theta_1(t)$ and $\theta_2(t)$ for where $\theta_1(0) = 1$ and $\theta_2(0) = 1$ by numerically integrating the acceleration resulted from the PID controller and saved it as Figure 4.

Concepts learned while working in this project:

- **rigid-body motions**, forward and inverse **kinematics**, differential kinematics**, forward and inverse dynamics of robotic manipulators**, **motion planning**, and **control** theories
- **Assign frames of reference** to mathematically describe the locations of individual joints and end-effectors of the robot.
- Understand the relationships between **kinematic design, configuration space, and workspace.**
- Describe the **dynamic motions of a robotic manipulator** and apply analytical and/or numerical methods to **solve the equations of motion**.
- **Apply linear control laws** for controlling a robotic manipulator with a single or multiple degrees of freedom.