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MEEN 612

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Special Assignment

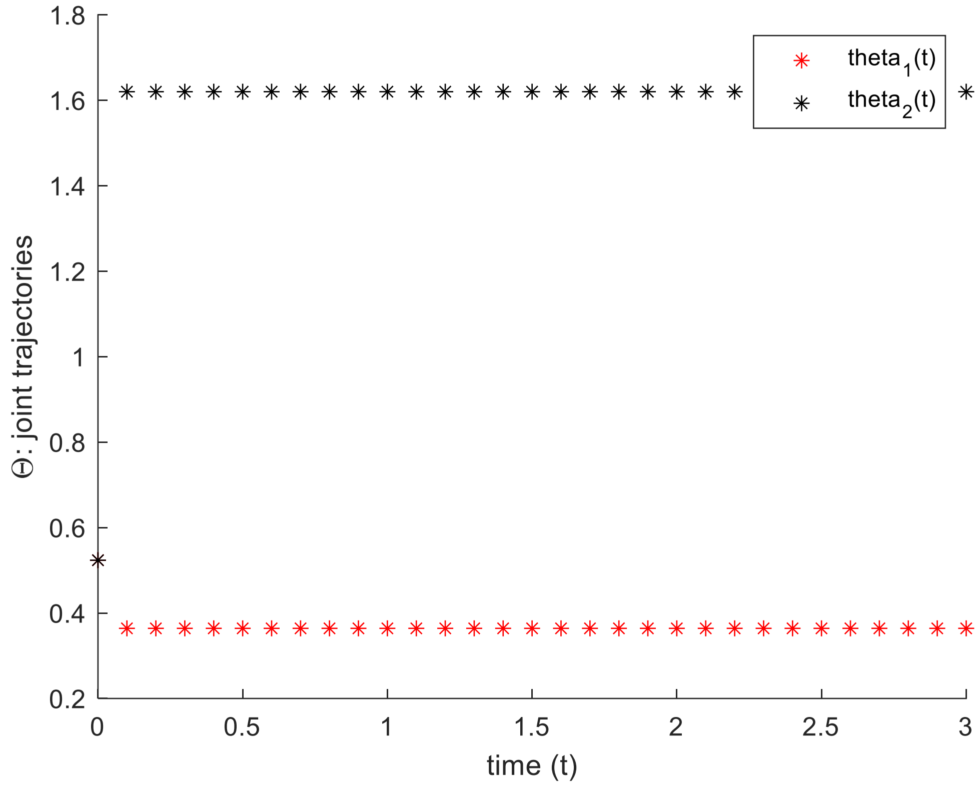
1.

$\boxed{\text{Forward Kinematics}}$

$$\begin{bmatrix} L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) \\ L_1 \sin(\theta_1) + L_2 \sin(\theta_1+\theta_2) \end{bmatrix} = \vec{X}_e = \begin{bmatrix} f_x(\vec{\theta}) \\ f_y(\vec{\theta}) \end{bmatrix}$$

$\boxed{\text{Jacobian}}$

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial \theta_1} & \frac{\partial f_x}{\partial \theta_2} \\ \frac{\partial f_y}{\partial \theta_1} & \frac{\partial f_y}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -L_1 \sin(\theta_1) - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) & L_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$



2.

a)

Design of the feedforward plus PID feedback controller with torque input using kp=10, kd=5 and ki=0.2 for both joints.

$m_1 = m_2 = 1\,kg$

$g = 9.8\ m/s^2$

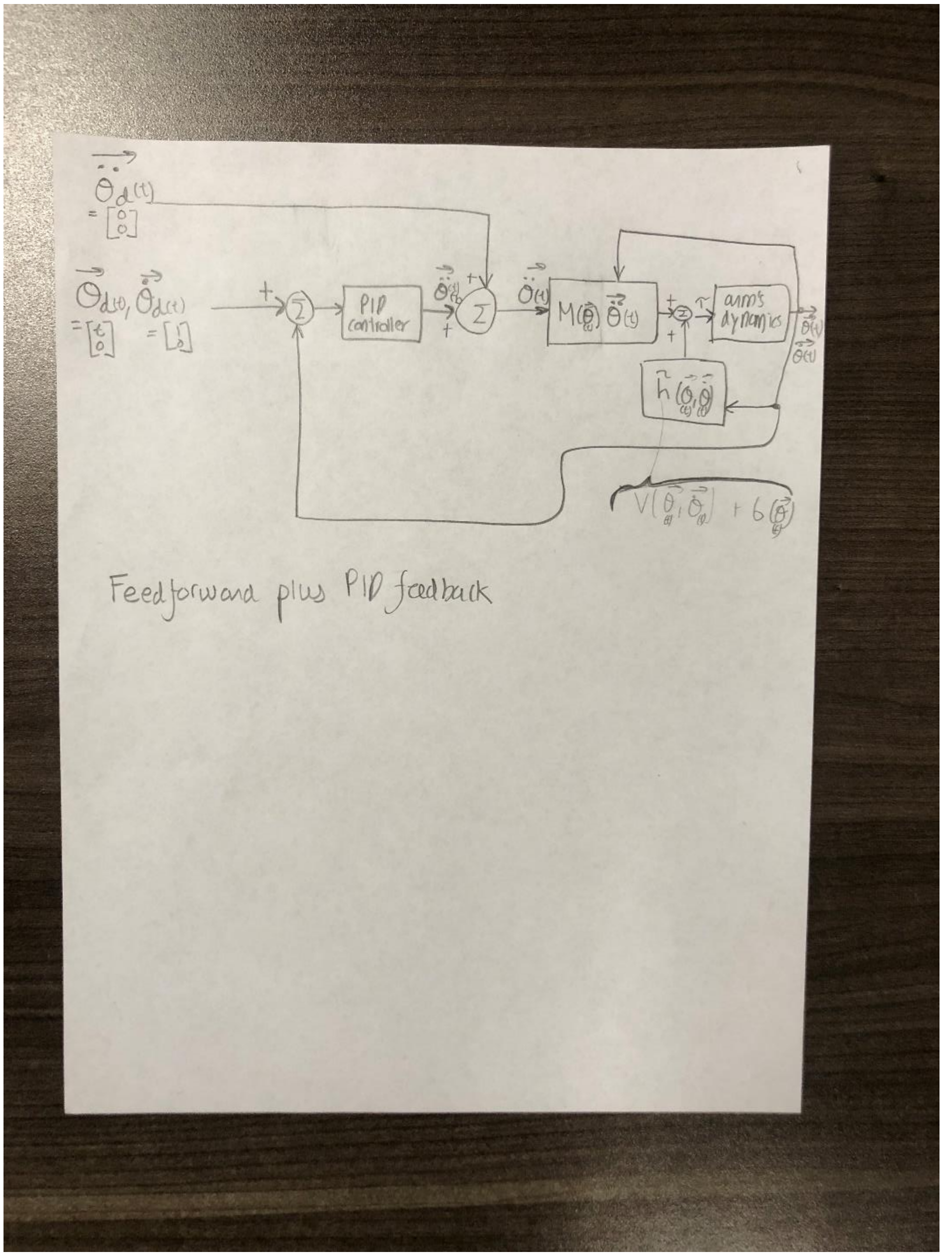
$$\theta_e(t) = \underbrace{\theta_d(t)}_{\begin{bmatrix} t \\ 0 \end{bmatrix}} - \theta(t)$$

$$\dot{\theta}_e = \underbrace{\dot{\theta}_d(t)}_{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} - \dot{\theta}(t)$$

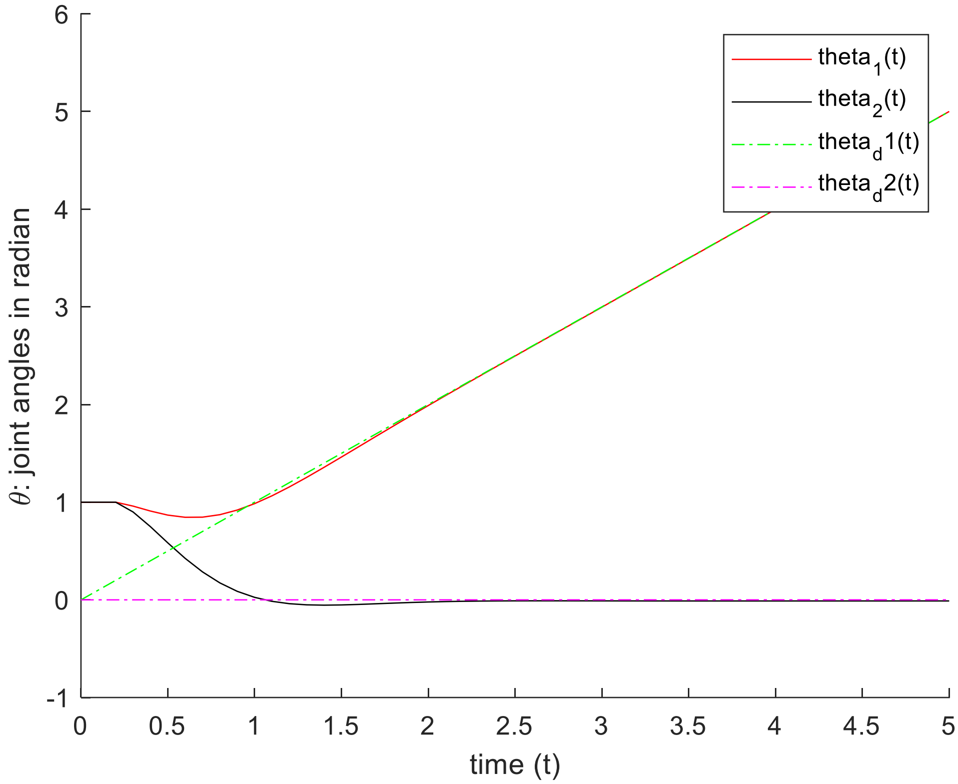
$$\ddot{\theta}_d(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\ddot{\theta} = \underbrace{\ddot{\theta}_d}_{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} + \underbrace{K_d}_{5} \underbrace{\dot{\theta}_e}_{\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \dot{\theta}(t)} + \underbrace{K_p}_{10} \underbrace{\theta_e}_{\begin{bmatrix} t \\ 0 \end{bmatrix} - \theta(t)} + \underbrace{K_i}_{0.2} \int \underbrace{\theta_e(t)}_{\begin{bmatrix} t \\ 0 \end{bmatrix} - \theta(t)} dt$$

$$\tau(t) = M(\vec{\theta})\ddot{\vec{\theta}} + V(\vec{\theta}, \dot{\vec{\theta}}) + G(\vec{\theta})$$


PID controller
arm's dynamics
Feedforward plus PID feedback

b) Fig Number 3



c) Fig Number 4