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Lab: Efficient Algorithms
for selected Problems
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Problem Set 1

1 Hello World!

Task: We wish to write the most basic program; it should simply output "Hello world!" on a single line, no matter what the input is.

Input: A random String.

Output: Hello world!

2 Grid snapping

Task: For geometric algorithms, it is often helpful to discretize the input points. In this task, we consider a standard grid in $\mathbb{R}^2$. Given the grid width and a set of points, compute the lower left corner of the grid cell that each point falls into.

Input: The first line contains the number of points n, which is at most 100. The second line contains the grid width with two digits after the decimal point. The coordinates are between -10000 and 10000. The following n lines each contain a point, consisting of two floating point numbers with two digits after the decimal point, separated by a space.

Output: A line for each input point, containing the lower left corner of the grid cell that it falls into, with two digits after the decimal point.

Sample Input:

```
3
0.75
0.00 0.00
-0.75 0.00
1.00 2.15
```

Sample Output:

```
0.00 0.00
-0.75 0.00
0.75 1.50
```

3 Matrix Multiplication

Task: Let $n \in \mathbb{N}$. You are given an $n \times n$-matrix A, and a vector v of size n. Most of the entries in A and v will be equal to zero. Compute the vector Av.

Input: The first line contains the dimension n of the matrix A. You may assume that $1 \leq n \leq 100000$. The second line contains the number m of non-zero entries in A. Then, m lines follow: Each of the following lines contain three integers i, j, and a_{ij}, which means that in row i and column j the matrix holds the value a_{ij}. You may assume that $0 \leq i, j \leq n-1$, and that $-5 \leq a_{ij} \leq 5$. The values of the matrix are given in increasing lexicographical order.

After the m lines, a line follows which contains the number b of non-zero entries in v. Then, each of the following b lines contains a number i with $0 \leq i \leq n-1$, and the value v_i.

Output: Output the values of Av that are different from zero. That is, if the i-th value of Av is non-zero, then print a line containing at first i and then the corresponding value of Av.

Sample Input:

```
3
4
0 0 1
0 1 -5
0 2 -4
1 0 -3
2
0 3
2 3
```

Sample Output:

```
0 -9
1 -9
```

4 Interval

You are given an interval on the non-negative integral numbers and a positive integer k. You can delete a number x from an interval if there exists a number y in the current interval such that $k \cdot x = y$. Return the maximum amount of numbers you can delete with the optimal strategy.

Input: The input consists of three integers a, b, k, where $10^{11} \geq a, b \geq 0$, $k > 0$. The numbers a and b describe the left and right bounds of the interval, i.e., the set $\{a, a+1, ..., b-1, b\}$.

Output: The output is a single integer r, which is the maximum amount of numbers you can delete when following the optimal strategy.

Sample Input:

```
0 10 3
```

Sample Output:

```
4
```

5 Area of a Polygon

Task: You are given the n vertices $p_0, \ldots, p_{n-1}$ of a non-self-intersecting polygon P. Compute the area A of P.

Input: The first line of the input contains the number n of vertices. Then, n lines follow describing the vertices in clockwise order along the boundary of the polygon. Each line contains the x and y coordinate of one vertex. You may assume that all coordinates are integers, and that we have $-1000 \leq x, y \leq 1000$.

Output: Output the area A of the described polygon.

Sample Input:

```
5
-1 0
0 1
2 0
1 -1
0 -2
```

Sample Output:

```
4.5
```