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Lab: Efficient Algorithms
for selected Problems
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Problem Set 3

Tasks that are marked with * are optional for groups with only two members and will not count into their score.

1 Basic Math

Task: You are given $U, V, W \in \{1, \ldots, 5 \cdot 10^{18}\}$. We define

$$S = \big\{(x, y, z) \in \mathbb{Z}^3 \,:\, (x \neq y) \wedge (x \neq z) \wedge (y \neq z) \\ \wedge (x + y + z = U) \wedge (xyz = V) \wedge (x^2 + y^2 + z^2 = W\big\}\,.$$

Output the lexicographically smallest element in S. In case that S is empty, print "empty set". A triple $s := (x, y, z)$ is lexicographically smaller than a triple $s' := (x', y', z')$ if the first coordinate where s and s' differ is smaller in s. More precisely, s is lex. smaller than s' if:

- $x < x'$,
- otherwise $x = x'$, but $y < y'$
- otherwise $x = x'$ and $y = y'$, but $z < z'$.

For this task there are 3 sets of testcases on the server with increasing difficulty.

Input: There will be several test cases concatenated in one file. The first line contains a positive integer n that specifies the number of test cases. In each of the following n lines a test case will be described. Each test case will be described by three integers $U, V, W \in \{1, \ldots, 400000\}$.

Output: For each test case print in a single line x, y and z. If S is empty, print "empty set".

Sample Input:

```
3
1 1 1
88 1136 5298
42 420 842
```

Sample Output:

```
empty set
1 16 71
1 20 21
```

2 Looking for oil*

Task: You know that there are some oil fields in a large plot of land which you would like to buy. The owner is forthcoming and does not insist on selling the entire plot, however, since he runs a large farming operation and is worried about his efficiency, he is only willing to sell you a single rectangular piece of his land. So, in order to find a good rectangle, you subdivide the quadratic plot into $n \times n$ quadratic parcels. For each parcel you estimate the difference between buying it and the selling value of the pumped up oil (the profit). Now you search for an optimal rectangle composed of parcels.

What is the maximum profit you can achieve?

Bonus. The server holds a second version of this problem with more challenging test cases. You may solve the more difficult version as a bonus problem.

Input: The first line contains n, where you can assume that $1 \leq n \leq 300$. Then, a single line follows. It contains the individual profits of each parcel, i.e., there are n^2 many integer values with values in $\{-100, \ldots, 100\}$.

We go down from north to south and from west to east, i.e., the first n numbers describe the profits in the parcels, which are furthest in the north, from west to east.

Output: Output the maximum profit you could obtain.

Sample Input:

```
3
-1 4 -1 -2 1 2 -2 -1 -3
```

Sample Output:

```
6
```

3 Köln-Deutz*

Deutsche Bahn wants to improve its infrastructure. We call a track section *critical* if closing down means that there exist two stations that are not connected any more. As a first step, they want to identify all track sections which are critical.

Input: You are given a simplified version of a railway network as a graph, i.e., the stations are the vertices and there is an unweighted undirected edge between two vertices if there is a track between the stations. The first line contains two integers: At first the number of vertices n, and then the number of edges m. The vertices are numbered from 0 to $n-1$. Then, m lines containing integers a_i and b_i follow, which encode an edge between vertices a_i and b_i.

Since there are construction works, you cannot assume that every station is connected to every other station. There may be parallel tracks between the stations.

Output: Output the number of critical track sections.

Sample Input:

```
7 6
0 1
1 2
3 4
3 5
4 5
5 6
```

Sample Output:

```
3
```

4 The Fast and the Furious

Task: You take part in a illegal street racing competition in Manhattan. The rules are as follows: You and your opponent are given the coordinates of a number of crossroads in Manhattan a few minutes before the race starts. The goal is to visit all these crossroads and then return to the start.

Your opponent is a local racer, so it is very likely that he will choose the optimal tour. Bad for you, but your face lights up when you take a look at his car: It is a pimped Nissan Skyline, and you are sure that your Lamborghini will be more than twice as fast as his car.

What is the length of the tour you choose?

Input: The first line contains the number n of crossroads you have to pass (including the point where you start). Then, in the following n lines the coordinates x_i and y_i of the crossroads are given. All coordinates will be integers. Since you are in Manhattan, the distance between two crossroads p_1 and p_2 is given by $\|p_1 - p_2\|_1$.

Output: Output the length of the route you will take.

Sample Input:

```
4
0 0
1 0
1 1
0 1
```

Sample Output:

```
4
```

5 Lazy Teacher

Task: The students in the Lab are unhappy with the workload. Some ask for more difficult tasks while others want easier exercises. To accommodate the students, you decide to let them vote on how the next exercise sheet should look like. Towards this end you select a group of students that will form a committee of size k and vote on the matter. Every member of this committee has to either vote increase or decrease the workload.

What the students do not know is that you have already prepared the next tasks and because you are lazy, you prefer to not change anything about it. So the best outcome for you is if the vote ends in a tie. You know that every student $i \in [n]$ will vote to increase the workload with probability p_i and vote to decrease workload with probability $(1 - p_i)$.

Choose a subset of the students $S \subseteq N$ with $|S| = k$ such that the probability to get a tie is maximized.

Input: The first line contains the number of candidates n and the size of the committee k. Here, k is guaranteed to be even. After that, each line contains the probability p as a floating point number that the corresponding candidate votes to increase the workload.

Output: For each test case, the output is a single line containing the maximal probability to achieve a tie rounded to 3 decimal places.

Sample Input 1:

```
4 2
0.4
0.8
0.3
0.5
```

Sample Output 1:

```
0.62
```

Sample Input 2:

```
8 4
0.1
0.3
0.9
0.6
0.5
0.3
0.9
0.5
```

Sample Output 2:

```
0.572
```

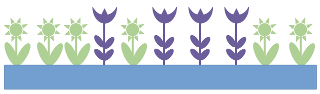
6 Royal Flowers

Task: You are the royal gardener at the summer residence of the king. Since summer is close and the return of the king is imminent you want to prepare the flower bed with the kings favourite arrangement of flowers. However, due to his eccentric personality, the king is prone to moodswings and his favourite arrangement might have changed since his last visit. The only thing you know for sure is that all of the arrangements have to consist of exactly N flowers. To make sure that the kings favourite flower sequence is represented in the garden, you want to plant the flowers in a way that each possible arrangement is represented. Since your time and resources are limited, you want to use as few flowers as possible.

Input: The input is a single line containing two integers, the size of the Kings favourite flower sequence $1 \leq n \leq 6$, and the number of different flower types at your disposal $1 \leq k \leq 10$.

Output: The output is a single string, consisting of k different types of characters representing the flowers, such that regardless of the kings preferences, his favourite flower arrangement can be found in your answer.

Sample Input 1:	**Sample Output 1:**
3 2	aaababbbaa



7 k-island clustering

Task: In this task, we consider a clustering problem that is a maximization problem. We call it the *k-island clustering* problem. Given are n points P in a metric space, and a number $k \geq 2$. The goal is to find a clustering with k clusters that *maximizes* the *minimum* distance between any two clusters in the partitioning, i.e., a partitioning $(P_i)_{i=1}^{k}$ that maximizes

$$\min_{i,j \in [k]} \min_{x \in P_i, y \in P_j} d(x, y).$$

The distances are given by a graph metric, i.e., the input contains an undirected weighted graph $G = (P, E)$, and the distance between $i, j \in P$ is the length of a shortest path between i and j in G. The input graph is connected, so all distances are finite.

Input: The first line contains n, the second line contains k. We adopt the names $0, \ldots, n-1$ for the n vertices. The third line contains m, the number of edges. Each edge is then given by a line containing three integers, the vertices i and j that it connects, and the (integral) weight w that the edge has. For every pair of vertices, there is at most one edge specified.

Output: The value of an optimum solution.

Sample Input:

```
10
2
12
0 1 1
0 4 5
1 2 1
2 3 1
3 4 1
4 5 1
5 6 1
6 7 1
7 8 2
8 9 1
1 3 6
6 9 4
```

Sample Output:

```
2
```