Tecnológico de Monterrey
Escuela de Ingeniería y Ciencias

Monterrey Campus

Autonomy of Unmanned Aerial Vehicles

MR3003C.601

Report:

Practice 01 Mathematical Modelling of DC Motor Velocity

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1. Complete Block Diagram of the Mathematical Model

The procedure in Figure 1 depicts the mathematical model of the QUBE-Servo 3 model in terms of the input voltage and the angular velocity.

$$i_m(t) = \frac{V_m(t)}{R_m} - \frac{(k_m + u)\,\omega_m(t)}{R_m} \quad , \; eq.1$$

$$J_{eq}\,\dot{\omega}_m(t) = \tau_m(t)$$

$$J_{eq}\,\dot{\omega}_m(t) = k_t\, i_m(t)$$

$$i_m(t) = \frac{J_{eq}\,\dot{\omega}_m(t)}{k_t} \quad , \; eq.2$$

eq. 1 and eq. 2

$$\frac{J_{eq}\,\dot{\omega}_m(t)}{k_t} = \frac{V_m(t)}{R_m} - \frac{(k_m + u)\,\omega_m(t)}{R_m}$$

$$\dot{\omega}_m(t) = \left[\frac{V_m(t)}{R_m} - \frac{(k_m + u)\,\omega_m(t)}{R_m}\right]\frac{k_t}{J_{eq}}$$

$$\dot{\omega}_m(t) = \left[\frac{V_m(t) - (k_m + u)\,\omega_m(t)}{R_m}\right]\frac{k_t}{J_{eq}}$$

Figure 1 Mathematical Model Procedure

Then, the Simulink model is developed as follows:

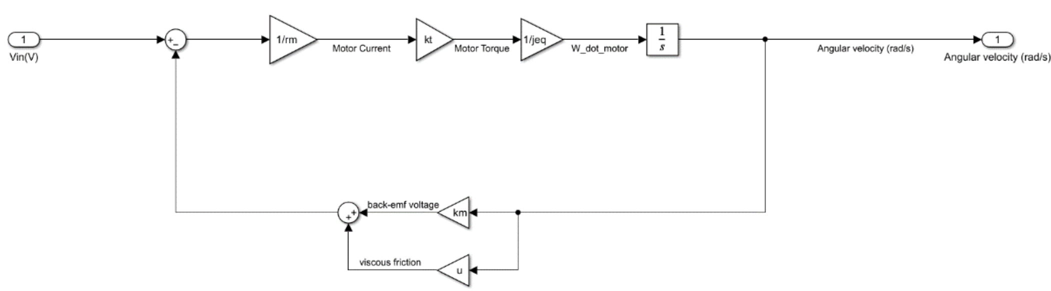


Figure 2 Simulink Model Block Diagram

2. Code With the Parameters of The System

All the parameters of the system were preloaded to the Simulink environment via the following script depicted in Figure 3.

```
%Instituto Tecnológico y de Estudios Superiores de Monterrey
%Name: Carlos Hernán Auquilla Larriva
%Course: Autonomy of Unmanned Aerial Vehicles
%Last update: August 12th 2024
clc; clear;

%%Practice 1
%Parameters

rm = 7.5;
kt = 0.0422;
km = 0.0422;
jm = 1.4e-6;
lm = 1.15e-3;
mh = 0.0106;
rh = 0.0111;
md = 0.053;
rd = 0.0248;
u = -2e-3;  %Qube servo, SN: 56856

jh = 0.5 * mh * rh^2;
jd = 0.5 * md * rd^2;

jeq = jm + jh + jd;
```

Figure 3 Matlab Parameter Script

3. Comparation Between Model and Measurements

Since mathematical modelling takes in consideration some assumptions like neglecting the inductance term, the model is not going to match the system exactly because in some cases such assumptions could be important. In the case of the DC motor model, in low velocities some error could be observed.

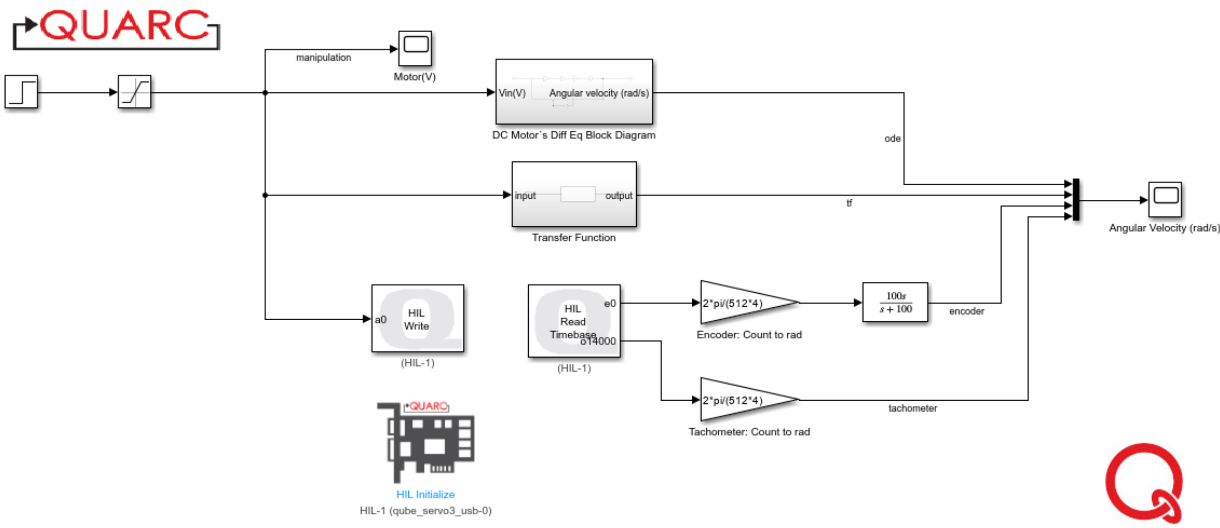


Figure 4 Simulation of the system

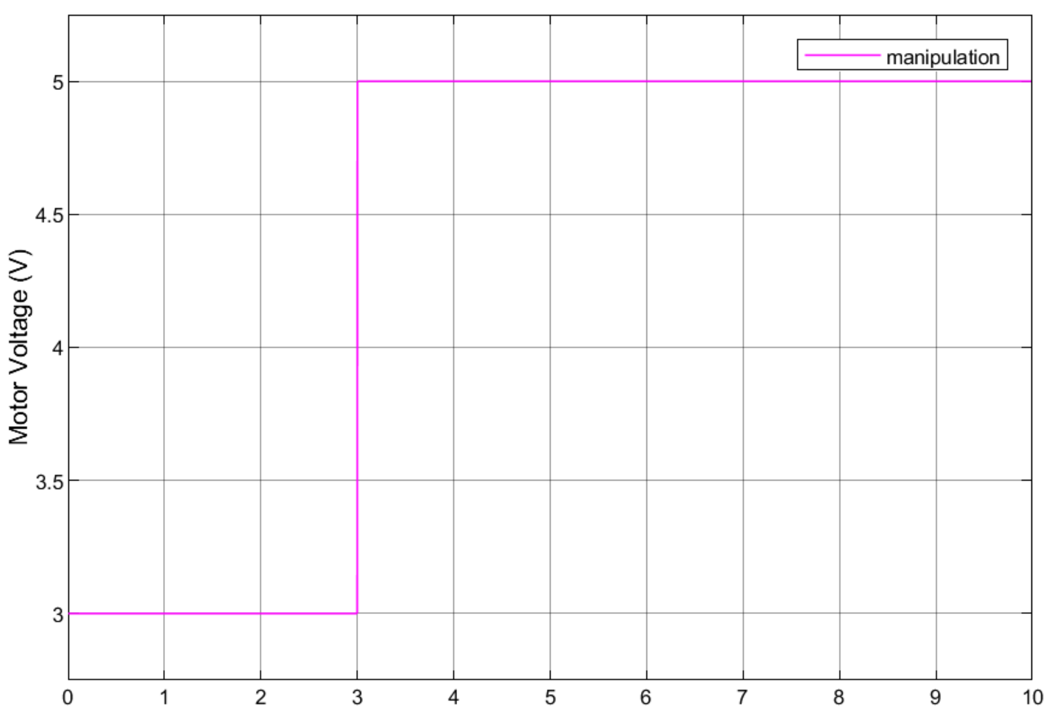


Figure 5 Input Voltage in time

For example, as you will see in Figure 6, for low values of velocity the model of the system does not match exactly with the measured system between 0 to 3 seconds. Nevertheless, for high velocities the system could be more accurate.

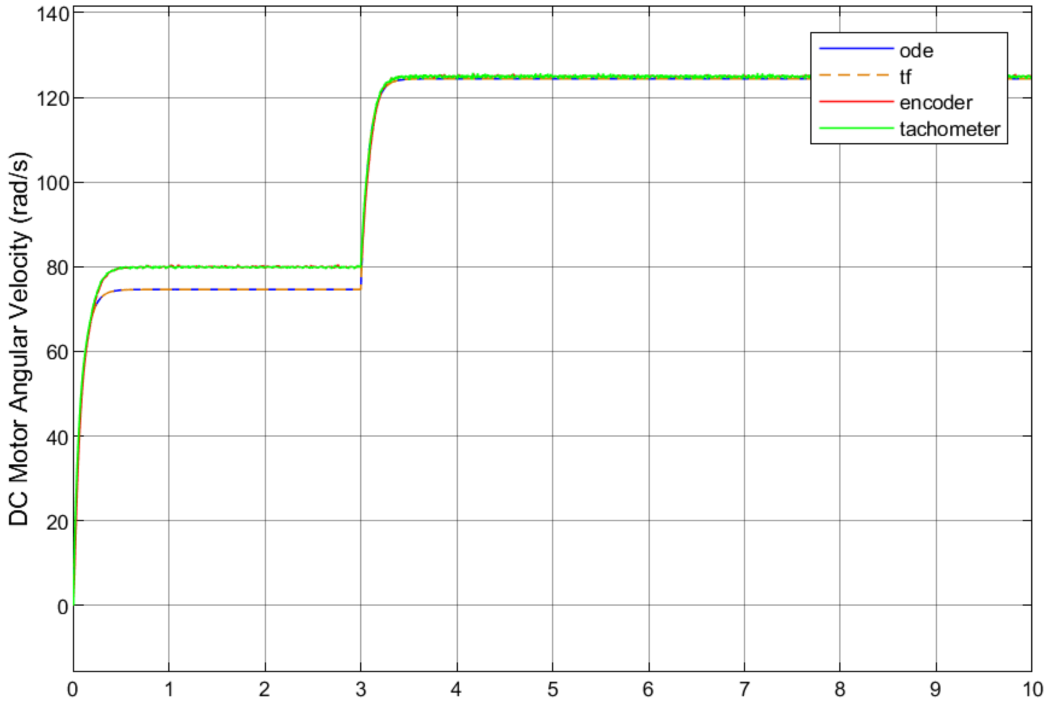


Figure 6 Angular Velocity in time

4. Mathematical Procedure to Obtain the Transfer Function

R.4

$$\dot{\omega}_m(t) = \left[\frac{V_m(t) - (K_m + u)\,\omega_m(t)}{R_m}\right]\frac{K_t}{J_{eq}}$$

$$\mathcal{L}\left\{\dot{\omega}_m(t)\right\} = \mathcal{L}\left\{\left[\frac{V_m(t) - (K_m + u)\,\omega_m(t)}{R_m}\right]\frac{K_t}{J_{eq}}\right\}$$

$$s\,\Omega_m(s) = \left(\frac{K_t}{J_{eq}}\right)\mathcal{L}\left\{\left(\frac{V_m(t)}{R_m}\right) - \left[\frac{(K_m + u)\,\omega_m(t)}{R_m}\right]\right\}$$

$$s\,\Omega_m(s) = \left(\frac{K_t}{J_{eq}}\right)\left[\left(\frac{V_m(s)}{R_m} - \frac{(K_m + u)\,\Omega_m(s)}{R_m}\right)K(t)\right]$$

$$s\,\Omega_m(s) = \frac{K_t}{R_m J_{eq}}\left[V_m(s) - (K_m + u)\,\Omega_m(s)\right]$$

$$s\,\Omega_m(s)\,(R_m J_{eq}) = V_m(s) - (K_m + u)\,\Omega_m(s)$$

$$s\,\Omega_m(s)\left(\frac{R_m J_{eq}}{K_t}\right) + (K_m + u)\,\Omega_m(s) = V_m(s)$$

$$\Omega_m(s)\left[\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + u)\right] = V_m(s)$$

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + u)}$$

Figure 7 Procedure to obtain the Transfer Function

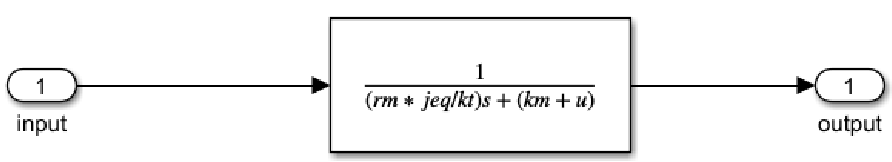


Figure 8 Simulink Block Diagram

5. Final Value of the Angular Velocity $\omega_m(t)$ at Steady State

Considering an input step from 3[V] to 5[V], the final value is going to be 124.3781 (rad/s). However, it is important to note that the initial value of velocity was added, it means that the previous velocity obtained from 0 to 3 seconds will be taken in consideration.

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)}$$

$$\Omega_m(s) = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} \quad V_m(s)$$

$$\Omega_m(s) = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} \quad \left(\frac{5-3}{s}\right)$$

$$\lim_{t \to \infty} W(t) = \lim_{s \to 0} s\,\Omega_m(s) + W(0)$$

$$\lim_{t \to \infty} w(t) = \lim_{s \to 0} s \left[\frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} \left(\frac{2}{s}\right)\right] + W_0$$

$$\lim_{t \to \infty} w(t) = \lim_{s \to 0} \frac{2}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} + W_0$$

$$\lim_{t \to \infty} w(t) = \frac{2}{K_m + \mu} + W_0$$

$$w(\infty) = \frac{2}{(0.0422) + (-2E-3)} + 74.6269$$

$$w(\infty) = 124.3781 \text{ rad/s}$$

Figure 9 Procedure to obtain the Final Value of the Angular Velocity

6. Angular Velocity $\omega_m(t)$ at any given time t

R.6

$$\Omega_m(s) = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} \left(\frac{5-3}{s}\right)$$

$$\Omega_m(s) = \frac{1}{\left(\frac{R_m J_{eq}}{K_t}\right)s + (K_m + \mu)} \left(\frac{2}{s}\right)$$

$$\Omega_m(s) = \frac{\frac{K_t}{R_m J_{eq}}}{s + \left(\frac{K_m + \mu}{R_m J_{eq}}\right)K_t} \left(\frac{2}{s}\right)$$

$$\left(\frac{K_m + \mu}{R_m J_{eq}}\right)K_t = a \qquad \frac{K_t}{R_m J_{eq}} = b \qquad \Omega_m(s) = \frac{b}{s + a}\left(\frac{2}{s}\right)$$

Poles

$p_1 = 0$

$p_2 = -a, \ a > 0$

$$\Omega_m(s) = \frac{c_1}{s} + \frac{c_2}{s + a}$$

$$c_1 = \left| \cancel{(s)} \ \frac{b}{s+a} \left(\frac{2}{\cancel{s}}\right) \right|_{s \to 0}$$

$$c_1 = \frac{2b}{a}$$

$$c_2 = \left| \cancel{(s+a)} \ \frac{b}{\cancel{s+a}} \left(\frac{2}{s}\right) \right|_{s \to -a}$$

$$c_2 = -\frac{2b}{a}$$

$$\Omega_m(s) = \frac{2b}{as} - \frac{2b}{a(s + a)}$$

Figure 10 Partial fraction Procedure

$$\mathcal{L}^{-1}\{\Omega_m(s)\} = \mathcal{L}^{-1}\left\{\frac{2b}{as} - \frac{2b}{a(s+a)}\right\}$$

$$\omega_m(t) = \frac{2b}{a}u(t) - \frac{2b}{a}e^{-at} + \omega_0$$

$$\omega_m(t) = \frac{2b}{a}\left(1-e^{-at}\right) + \omega_0$$

$$\left(\frac{K_m+\mu}{R_m J_{eq}}\right)K_t = a \qquad \frac{K_t}{R_m J_{eq}} = b$$

Qube - Servo 3   SN: 56856

$R_m = 7.5\ \Omega$

$J_{eq} = 1.8352\ E-5$ Kgm^2

$K_t = 0.0422$ Nm/A

$K_m = 0.0422$ Nm/A

$\mu = -2\ E-3$ V/(rad/s)

$a = 12.3255 \qquad b = 306.6041$

$$\omega_m(t) = \frac{2b}{a}\left(1-e^{-at}\right) + \frac{3}{K_m+\mu}$$, +Initial value

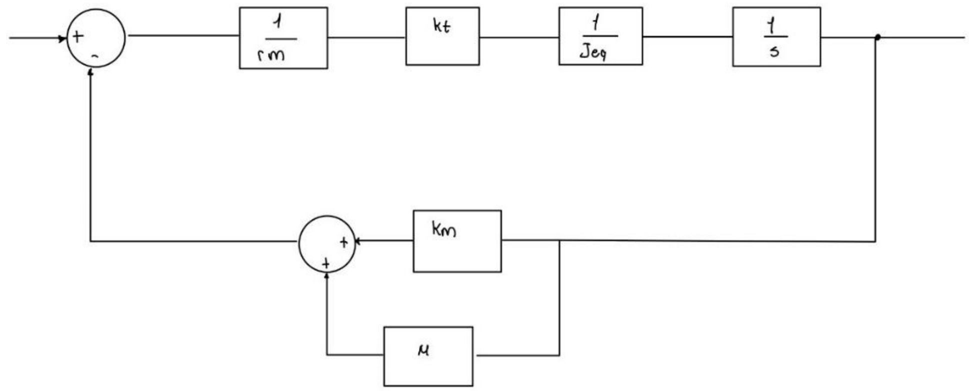
$$\omega_m(t) = 49.7512\left(1-e^{-12.3255t}\right) + 74.6269$$

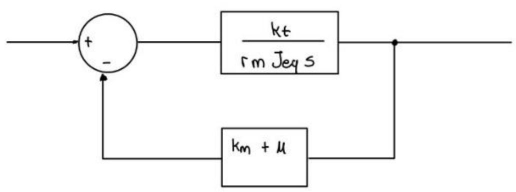
Figure 11 Inverse Laplace Procedure

7. Transfer Function Using Block Algebra and Mason’s Rule

R. 7

By Block Algebra





$$\frac{\frac{k_t}{r_m J_{eq} s}}{1+\left(\frac{k_t}{r_m J_{eq} s}\right)(k_m+\mu)}$$

$$\frac{1}{\frac{r_m J_{eq}}{k_t} s + (k_m+\mu)}$$

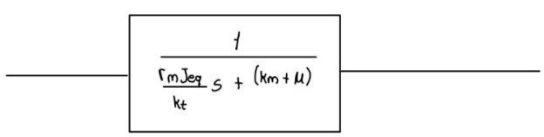


Figure 12 Block Algebra Procedure

By Mason's Rule

$$P_1 = \left(\frac{1}{r_m}\right)(k_t)\left(\frac{1}{J_{eq}}\right)\left(\frac{1}{s}\right)$$

$$P_1 = \frac{k_t}{r_m J_{eq} s}$$

$$L_1 = \left(\frac{1}{r_m}\right)(k_t)\left(\frac{1}{J_{eq}}\right)\left(\frac{1}{s}\right)(k_m + \mu)$$

$$\Delta = 1 - \sum L_i + \cancel{\sum L_i L_j} - \cancel{\sum L_i L_j}$$

$$\Delta = 1 - L_1$$

$$\Delta = 1 - \left(\frac{1}{r_m}\right)(k_t)\left(\frac{1}{J_{eq}}\right)\left(\frac{1}{s}\right)(k_m + \mu)$$

$$\Delta = 1 - \left(\frac{k_t}{r_m J_{eq} s}\right)(k_m + \mu)$$

$$\Delta_1 = 1 - \cancel{L_1}$$

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{1}{\Delta} P_1 \Delta_1$$

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{1}{1}\left(\frac{k_t}{r_m J_{eq} s}\right)\left[1 - \left(\frac{k_t}{r_m J_{eq} s}\right)(k_m + \mu)\right]$$

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{\left(\frac{k_t}{r_m J_{eq} s}\right)}{\left[1 - \left(\frac{k_t}{r_m J_{eq} s}\right)(k_m + \mu)\right]}$$

$$\frac{\Omega_m(s)}{V_m(s)} = \frac{1}{\frac{r_m J_{eq}}{k_t} s + (k_m + \mu)}$$

Figure 13 Mason´s Rule Procedure

Conclusion

The mathematical modeling of the DC motor's velocity was successfully developed and simulated using Simulink. The comparison between the model and the actual measurements highlighted that while the model provides a reasonable approximation, especially at higher velocities, some discrepancies arise at lower velocities due to assumptions like neglecting inductance. The final steady-state angular velocity was accurately predicted using the model, confirming the validity of the approach within the considered scope. The transfer functions were derived using both block algebra and Mason's Rule, demonstrating a comprehensive understanding of the system dynamics. This exercise reinforces the importance of considering practical limitations and assumptions in mathematical modeling, particularly in control systems for unmanned aerial vehicles.