Agreement Scales for Convective-Scale Ensemble Verification

Spatial Predictability Assessment - Dey et al. (2016)

Based on Dey et al. (2016) Methods

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Motivation

Modern convective-scale ensembles provide detailed precipitation forecasts, but traditional verification suffers from the **"double penalty"** problem when features are slightly displaced.

When it comes to convective precipitation, what we want to know is:

- Where will **precipitation occur** with confidence?
- What are the **spatial uncertainties** in ensemble forecasts?
- How do we assess **location-dependent predictability**?

Agreement Scales by Dey et al. (2016) evaluates the spatial predictability of convective-scale ensembles by calculating neighborhood-based agreement between forecasts at each grid point.

The Agreement Scales Method

Agreement Scales compute location-dependent measures of spatial agreement:

Component	What it measures
SA(mm)_ij	Agreement scales between ensemble member pairs
SA(mo)_ij	Agreement scales between members and observations
Spatial Spread	How ensemble members differ spatially
Spatial Skill	How well ensemble captures observed spatial patterns

Spatial Spread-Skill Relationship:

$$\text{Compare } SA_{ij}^{(mm)} \text{ with } SA_{ij}^{(mo)}$$

Agreement Scale Calculation

The similarity criterion compares forecast fields f and f :

$$D_{ij}^S = \begin{cases} \frac{(f_{1ij}^S - f_{2ij}^S)^2}{(f_{1ij}^S)^2 + (f_{2ij}^S)^2}, & \text{if } f_{1ij}^S > 0 \text{ or } f_{2ij}^S > 0 \\ 1, & \text{if } f_{1ij}^S = 0 \text{ and } f_{2ij}^S = 0 \end{cases}$$

Agreement Criterion:

$$D_{ij}^S \leq D_{crit,ij}^S = \alpha + (1-\alpha)\frac{S}{S_{lim}}$$

Default parameters: = 0.5, S_lim = 80 grid points

Python Implementation

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import maximum_filter, minimum_filter
import xarray as xr

def calculate_similarity_criterion(f1, f2, alpha=0.5):
    """Calculate similarity criterion D between two fields"""
    if f1 > 0 or f2 > 0:
        return (f1 - f2)**2 / (f1**2 + f2**2)
    else:
        return 1.0

def agreement_criterion(scale, alpha=0.5, s_lim=80):
    """Calculate agreement criterion threshold"""
    return alpha + (1 - alpha) * scale / s_lim
```

```python
def calculate_agreement_scale(field1, field2, i, j, alpha=0.5, s_lim=80):
    """Calculate agreement scale at grid point (i,j) between two fields"""
    ny, nx = field1.shape

    for scale in range(s_lim + 1):
        # Define neighborhood bounds
        i_min = max(0, i - scale)
        i_max = min(ny, i + scale + 1)
        j_min = max(0, j - scale)
        j_max = min(nx, j + scale + 1)

        # Calculate neighborhood averages
        f1_avg = np.mean(field1[i_min:i_max, j_min:j_max])
        f2_avg = np.mean(field2[i_min:i_max, j_min:j_max])

        # Calculate similarity
        d_ij = calculate_similarity_criterion(f1_avg, f2_avg, alpha)
        d_crit = agreement_criterion(scale, alpha, s_lim)

        # Check if agreement criterion is met
        if d_ij <= d_crit:
            return scale

    return s_lim
```

Agreement Scale Maps Function

```python
def calculate_agreement_scale_map(field1, field2, alpha=0.5, s_lim=80):
    """Calculate agreement scale map between two precipitation fields"""
    ny, nx = field1.shape
    agreement_map = np.zeros((ny, nx))

    for i in range(ny):
        for j in range(nx):
            agreement_map[i, j] = calculate_agreement_scale(
                field1, field2, i, j, alpha, s_lim
            )
```

```
    return agreement_map

def calculate_ensemble_agreement_scales(ensemble_fields, alpha=0.5, s_lim=80):
    """Calculate SA(mm) from ensemble member pairs"""
    n_members = len(ensemble_fields)
    ny, nx = ensemble_fields[0].shape

    # Calculate agreement scales for all member pairs
    agreement_scales = []

    for i in range(n_members):
        for j in range(i + 1, n_members):
            scale_map = calculate_agreement_scale_map(
                ensemble_fields[i], ensemble_fields[j], alpha, s_lim
            )
            agreement_scales.append(scale_map)

    # Average over all pairs
    sa_mm = np.mean(agreement_scales, axis=0)
    return sa_mm

def calculate_member_obs_agreement_scales(ensemble_fields, observations, alpha=0.5, s_lim=80)
    """Calculate SA(mo) between ensemble members and observations"""
    agreement_scales = []

    for member_field in ensemble_fields:
        scale_map = calculate_agreement_scale_map(
            member_field, observations, alpha, s_lim
        )
        agreement_scales.append(scale_map)

    # Average over all members
    sa_mo = np.mean(agreement_scales, axis=0)
    return sa_mo
```

Creating Synthetic Ensemble Data

```python
def create_synthetic_precipitation_ensemble(nx=100, ny=100, n_members=12):
    """Create synthetic ensemble precipitation fields"""
    np.random.seed(42)
    ensemble = []

    # Base precipitation pattern
    base_field = np.zeros((ny, nx))

    # Add convective cells with spatial uncertainty
    cell_centers = [(25, 35), (65, 75), (45, 20)]
    cell_intensities = [15.0, 10.0, 8.0]

    for member in range(n_members):
        field = np.zeros((ny, nx))

        for (cy, cx), intensity in zip(cell_centers, cell_intensities):
            # Add spatial displacement for each member
            dy = np.random.randint(-5, 6)
            dx = np.random.randint(-5, 6)

            # Add intensity variation
            member_intensity = intensity * np.random.uniform(0.7, 1.3)

            # Create precipitation cell
            y_center = max(5, min(ny-5, cy + dy))
            x_center = max(5, min(nx-5, cx + dx))

            # Gaussian-like precipitation pattern
            y_indices, x_indices = np.ogrid[:ny, :nx]
            distances = np.sqrt((y_indices - y_center)**2 + (x_indices - x_center)**2)
            cell_pattern = member_intensity * np.exp(-distances**2 / (2 * 3**2))

            # Only keep significant precipitation
            cell_pattern[cell_pattern < 0.5] = 0
            field += cell_pattern

        ensemble.append(field)

    return ensemble

# Create synthetic ensemble
```

```python
ensemble_fields = create_synthetic_precipitation_ensemble()
print(f"Created ensemble with {len(ensemble_fields)} members")
print(f"Field shape: {ensemble_fields[0].shape}")
```

```
Created ensemble with 12 members
Field shape: (100, 100)
```

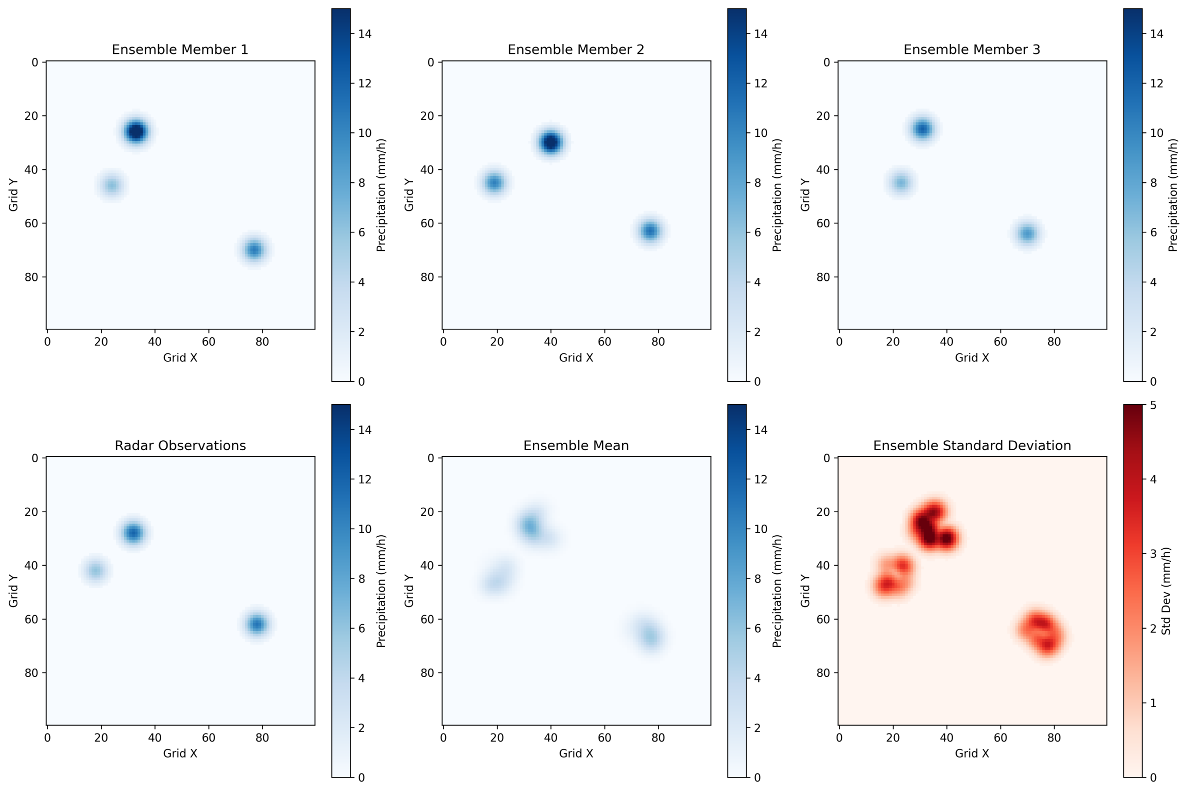
Creating Synthetic Observations

```python
def create_synthetic_observations(nx=100, ny=100):
    """Create synthetic radar observations"""
    np.random.seed(123)  # Different seed for observations
    obs_field = np.zeros((ny, nx))

    # True precipitation locations (with some displacement from ensemble mean)
    true_centers = [(28, 32), (62, 78), (42, 18)]
    true_intensities = [12.0, 11.0, 6.0]

    for (cy, cx), intensity in zip(true_centers, true_intensities):
        # Create precipitation cell
        y_indices, x_indices = np.ogrid[:ny, :nx]
        distances = np.sqrt((y_indices - cy)**2 + (x_indices - cx)**2)
        cell_pattern = intensity * np.exp(-distances**2 / (2 * 3**2))

        # Only keep significant precipitation
        cell_pattern[cell_pattern < 0.5] = 0
        obs_field += cell_pattern

    return obs_field

# Create synthetic observations
observations = create_synthetic_observations()
print(f"Observation field shape: {observations.shape}")
print(f"Max precipitation: {observations.max():.1f} mm/h")
```

```
Observation field shape: (100, 100)
Max precipitation: 12.0 mm/h
```

Visualizing Ensemble and Observations

```
fig, axes = plt.subplots(2, 3, figsize=(15, 10))

# Plot first 3 ensemble members
for i in range(3):
    im = axes[0, i].imshow(ensemble_fields[i], cmap='Blues', vmin=0, vmax=15)
    axes[0, i].set_title(f'Ensemble Member {i+1}')
    axes[0, i].set_xlabel('Grid X')
    axes[0, i].set_ylabel('Grid Y')
    plt.colorbar(im, ax=axes[0, i], label='Precipitation (mm/h)')

# Plot observations and ensemble mean
obs_im = axes[1, 0].imshow(observations, cmap='Blues', vmin=0, vmax=15)
axes[1, 0].set_title('Radar Observations')
axes[1, 0].set_xlabel('Grid X')
axes[1, 0].set_ylabel('Grid Y')
plt.colorbar(obs_im, ax=axes[1, 0], label='Precipitation (mm/h)')

# Ensemble mean
ensemble_mean = np.mean(ensemble_fields, axis=0)
mean_im = axes[1, 1].imshow(ensemble_mean, cmap='Blues', vmin=0, vmax=15)
axes[1, 1].set_title('Ensemble Mean')
axes[1, 1].set_xlabel('Grid X')
axes[1, 1].set_ylabel('Grid Y')
plt.colorbar(mean_im, ax=axes[1, 1], label='Precipitation (mm/h)')

# Ensemble standard deviation
ensemble_std = np.std(ensemble_fields, axis=0)
std_im = axes[1, 2].imshow(ensemble_std, cmap='Reds', vmin=0, vmax=5)
axes[1, 2].set_title('Ensemble Standard Deviation')
axes[1, 2].set_xlabel('Grid X')
axes[1, 2].set_ylabel('Grid Y')
plt.colorbar(std_im, ax=axes[1, 2], label='Std Dev (mm/h)')

plt.tight_layout()
plt.show()
```



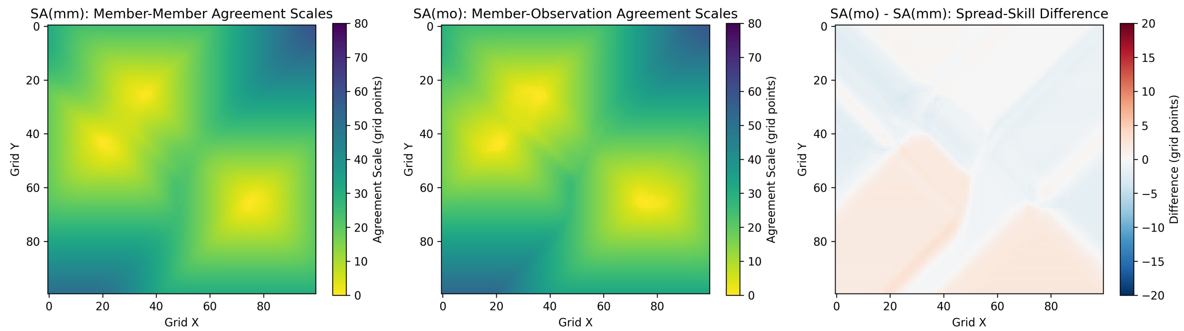
Calculating Agreement Scales

```
# Calculate SA(mm) - agreement between ensemble members
print("Calculating SA(mm) - ensemble member agreement scales...")
sa_mm = calculate_ensemble_agreement_scales(ensemble_fields)

# Calculate SA(mo) - agreement between members and observations
print("Calculating SA(mo) - member-observation agreement scales...")
sa_mo = calculate_member_obs_agreement_scales(ensemble_fields, observations)

print("Agreement scale calculations completed!")
print(f"SA(mm) range: {sa_mm.min():.1f} to {sa_mm.max():.1f} grid points")
print(f"SA(mo) range: {sa_mo.min():.1f} to {sa_mo.max():.1f} grid points")
print(f"Domain average SA(mm): {sa_mm.mean():.1f} grid points")
print(f"Domain average SA(mo): {sa_mo.mean():.1f} grid points")
```

```
Calculating SA(mm) - ensemble member agreement scales...
Calculating SA(mo) - member-observation agreement scales...
Agreement scale calculations completed!
SA(mm) range: 0.1 to 59.4 grid points
SA(mo) range: 0.0 to 58.2 grid points
Domain average SA(mm): 21.2 grid points
Domain average SA(mo): 21.4 grid points
```

Visualizing Agreement Scale Maps

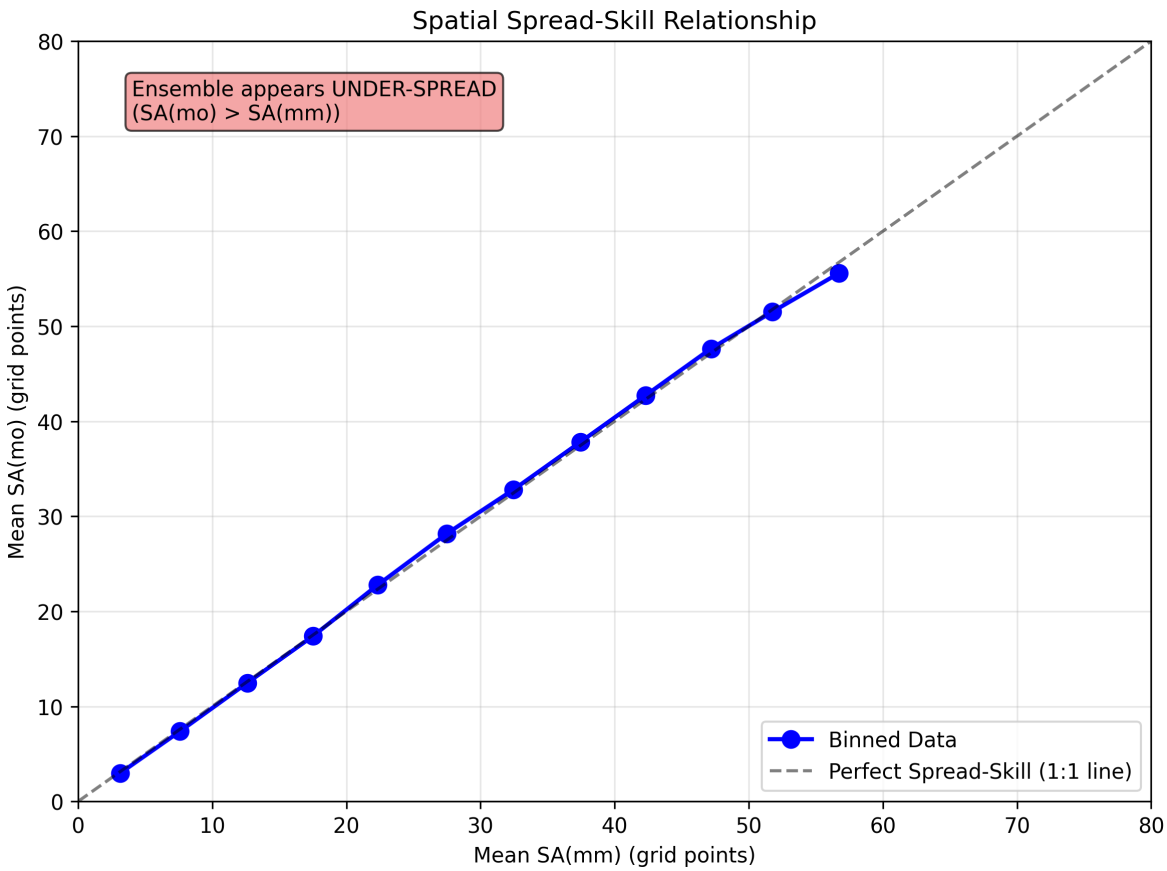
```python
fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(15, 4))

# Plot SA(mm)
im1 = ax1.imshow(sa_mm, cmap='viridis_r', vmin=0, vmax=80)
ax1.set_title('SA(mm): Member-Member Agreement Scales')
ax1.set_xlabel('Grid X')
ax1.set_ylabel('Grid Y')
plt.colorbar(im1, ax=ax1, label='Agreement Scale (grid points)')

# Plot SA(mo)
im2 = ax2.imshow(sa_mo, cmap='viridis_r', vmin=0, vmax=80)
ax2.set_title('SA(mo): Member-Observation Agreement Scales')
ax2.set_xlabel('Grid X')
ax2.set_ylabel('Grid Y')
plt.colorbar(im2, ax=ax2, label='Agreement Scale (grid points)')

# Plot difference (SA(mo) - SA(mm))
diff = sa_mo - sa_mm
im3 = ax3.imshow(diff, cmap='RdBu_r', vmin=-20, vmax=20)
ax3.set_title('SA(mo) - SA(mm): Spread-Skill Difference')
ax3.set_xlabel('Grid X')
ax3.set_ylabel('Grid Y')
plt.colorbar(im3, ax=ax3, label='Difference (grid points)')

plt.tight_layout()
plt.show()
```



Spatial Spread-Skill Analysis

```
def create_binned_scatter_plot(sa_mm, sa_mo, bin_size=5):
    """Create binned scatter plot for spread-skill analysis"""
    # Flatten arrays
    sa_mm_flat = sa_mm.flatten()
    sa_mo_flat = sa_mo.flatten()

    # Create bins
    max_scale = max(sa_mm_flat.max(), sa_mo_flat.max())
    bins = np.arange(0, max_scale + bin_size, bin_size)

    bin_centers = []
    sa_mm_binned = []
    sa_mo_binned = []

    for i in range(len(bins) - 1):
        # Find points in this bin based on SA(mm)
        mask = (sa_mm_flat >= bins[i]) & (sa_mm_flat < bins[i + 1])

        if np.sum(mask) > 0:  # Only include bins with data
            bin_centers.append((bins[i] + bins[i + 1]) / 2)
            sa_mm_binned.append(np.mean(sa_mm_flat[mask]))
            sa_mo_binned.append(np.mean(sa_mo_flat[mask]))

    return np.array(bin_centers), np.array(sa_mm_binned), np.array(sa_mo_binned)

# Create binned scatter plot
```

```python
bin_centers, sa_mm_binned, sa_mo_binned = create_binned_scatter_plot(sa_mm, sa_mo)

# Plot results
plt.figure(figsize=(8, 6))
plt.plot(sa_mm_binned, sa_mo_binned, 'bo-', linewidth=2, markersize=8, label='Binned Data')
plt.plot([0, 80], [0, 80], 'k--', alpha=0.5, label='Perfect Spread-Skill (1:1 line)')
plt.xlabel('Mean SA(mm) (grid points)')
plt.ylabel('Mean SA(mo) (grid points)')
plt.title('Spatial Spread-Skill Relationship')
plt.legend()
plt.grid(True, alpha=0.3)
plt.xlim(0, 80)
plt.ylim(0, 80)

# Add interpretation text
if np.mean(sa_mo_binned) > np.mean(sa_mm_binned):
    plt.text(0.05, 0.95, 'Ensemble appears UNDER-SPREAD\n(SA(mo) > SA(mm))',
             transform=plt.gca().transAxes, verticalalignment='top',
             bbox=dict(boxstyle='round', facecolor='lightcoral', alpha=0.7))
else:
    plt.text(0.05, 0.95, 'Ensemble appears OVER-SPREAD\n(SA(mo) < SA(mm))',
             transform=plt.gca().transAxes, verticalalignment='top',
             bbox=dict(boxstyle='round', facecolor='lightblue', alpha=0.7))

plt.tight_layout()
plt.show()
```



Statistical Summary

```
# Calculate domain statistics
domain_sa_mm = np.mean(sa_mm)
domain_sa_mo = np.mean(sa_mo)
correlation = np.corrcoef(sa_mm.flatten(), sa_mo.flatten())[0, 1]

# Calculate spread-skill metrics
spread_skill_diff = sa_mo - sa_mm
mean_diff = np.mean(spread_skill_diff)
rmse_diff = np.sqrt(np.mean(spread_skill_diff**2))

print("=== Agreement Scales Summary ===")
```

```python
print(f"Domain Average SA(mm): {domain_sa_mm:.2f} grid points")
print(f"Domain Average SA(mo): {domain_sa_mo:.2f} grid points")
print(f"Mean Difference (SA(mo) - SA(mm)): {mean_diff:.2f} grid points")
print(f"RMSE of Difference: {rmse_diff:.2f} grid points")
print(f"Spatial Correlation: {correlation:.3f}")
print()

# Interpretation
if mean_diff > 2:
    print("  INTERPRETATION: Ensemble appears UNDER-SPREAD")
    print("   → Ensemble members are too similar to each other")
    print("   → Real spatial uncertainty is larger than ensemble suggests")
elif mean_diff < -2:
    print("  INTERPRETATION: Ensemble appears OVER-SPREAD")
    print("   → Ensemble members are too different from each other")
    print("   → Ensemble overestimates spatial uncertainty")
else:
    print("  INTERPRETATION: Ensemble appears WELL-SPREAD")
    print("   → Good balance between ensemble spread and skill")

print()
print("=== Precipitation Coverage Analysis ===")
# Calculate precipitation coverage
precip_threshold = 0.5  # mm/h
obs_coverage = np.sum(observations > precip_threshold) / observations.size * 100
ensemble_coverage = [np.sum(field > precip_threshold) / field.size * 100
                    for field in ensemble_fields]
mean_ensemble_coverage = np.mean(ensemble_coverage)

print(f"Observed precipitation coverage: {obs_coverage:.1f}%")
print(f"Mean ensemble precipitation coverage: {mean_ensemble_coverage:.1f}%")
print(f"Coverage bias: {mean_ensemble_coverage - obs_coverage:.1f}%")
```

```
=== Agreement Scales Summary ===
Domain Average SA(mm): 21.24 grid points
Domain Average SA(mo): 21.38 grid points
Mean Difference (SA(mo) - SA(mm)): 0.13 grid points
RMSE of Difference: 1.40 grid points
Spatial Correlation: 0.993

  INTERPRETATION: Ensemble appears WELL-SPREAD
   → Good balance between ensemble spread and skill
```

```
=== Precipitation Coverage Analysis ===
Observed precipitation coverage: 4.9%
Mean ensemble precipitation coverage: 5.2%
Coverage bias: 0.3%
```

Interpretation Guidelines

Agreement Scale Ranges:

- **0-10 grid points**: High spatial predictability, precipitation location well constrained
- **10-30 grid points**: Moderate spatial uncertainty, neighborhood approach needed
- **30-50 grid points**: Large spatial uncertainty, broad areas of possible precipitation
- **50+ grid points**: Very low spatial predictability or dry regions

Spread-Skill Relationship:

- **SA(mo) > SA(mm)**: **Under-spread** ensemble - members too similar, underestimates uncertainty
- **SA(mo) SA(mm)**: **Well-spread** ensemble - good representation of spatial uncertainty
- **SA(mo) < SA(mm)**: **Over-spread** ensemble - members too different, overestimates uncertainty

Physical Interpretation: - Small agreement scales near precipitation indicate **high spatial predictability**

- Large agreement scales indicate **low spatial predictability** or distance from precipitation
- Topographic effects often reduce agreement scales (higher predictability)

Practical Applications

Operational Verification:

- Assess spatial ensemble performance routinely
- Identify systematic biases in precipitation placement
- Compare different ensemble configurations
- Monitor ensemble calibration over time

Forecast Applications:

- Provide location-dependent uncertainty estimates
- Support probabilistic precipitation forecasts
- Guide forecast interpretation and decision-making
- Identify high/low confidence regions

Research Applications:

- Understand ensemble behavior in different weather regimes
- Evaluate impact of model changes on spatial predictability
- Study relationship between physical processes and predictability
- Develop improved ensemble perturbation strategies

Key Advantages of Agreement Scales

1. **Location-dependent** - preserves spatial information unlike domain-wide metrics
2. **Scale-aware** - identifies appropriate neighborhood sizes for verification
3. **Physically meaningful** - links to meteorological processes and predictability
4. **Ensemble-specific** - designed for convective-scale ensemble evaluation
5. **Flexible** - applicable to different precipitation thresholds and variables

Complements existing methods like FSS (Fractions Skill Score) and traditional ensemble verification

Addresses double penalty problem through neighborhood-based approach

References & Resources

Primary References:

- Dey, S.R.A., et al. (2016). A new method for the characterization and verification of local spatial predictability for convective-scale ensembles. *Q.J.R. Meteorol. Soc.*, 142, 1982-1996.
- Dey, S.R.A., et al. (2016). Assessing spatial precipitation uncertainties in a convective-scale ensemble. *Q.J.R. Meteorol. Soc.*, 142, 2935-2948.

Related Methods:

- Roberts & Lean (2008): Fractions Skill Score (FSS)
- Clark et al. (2011): Ensemble verification at convective scales
- Johnson et al. (2014): Multiscale ensemble characteristics