Displacement and Amplitude Score (DAS) for Precipitation Verification

Optical Flow-Based Spatial Verification - Keil & Craig (2009)

Based on Keil & Craig (2009) Methods

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Motivation

Modern high-resolution numerical weather prediction models forecast weather with great detail, but traditional verification suffers from the **"double penalty"** problem.

When convective precipitation is slightly displaced:

- **Traditional metrics**: Poor categorical scores (many misses and false alarms)
- **Large RMSE**: Even for correct intensity, size, and timing
- **Model value**: Difficult to prove using gridpoint-based verification

What we need:

- Separate **displacement errors** from **amplitude errors**
- Account for **spatial uncertainty** in precipitation forecasts
- Provide **meaningful verification** for high-resolution models

Displacement and Amplitude Score (DAS) by Keil & Craig (2009) uses optical flow techniques to quantify both location and intensity errors simultaneously.

The DAS Methodology

DAS combines distance and amplitude errors using an **optical flow algorithm** that morphs one field to match another:

Component	What it measures
Displacement Error	Distance to corresponding forecast object
Amplitude Error	Intensity difference after morphing
Optical Flow	Vector field that deforms forecast to match observations
Pyramidal Matching	Multi-scale algorithm for robust feature matching

Key Innovation:

- Morphing in **both directions**: forecast→observation AND observation→forecast
- Handles **false alarms** and **missed events** properly
- **Maximum search distance** parameter controls matching range

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Optical Flow Algorithm

The pyramidal image-matching algorithm computes displacement vectors by minimizing amplitude differences:

Step 1: Multi-scale Decomposition

- Apply spatial filtering at multiple resolutions
- Start with coarsest scale, refine progressively
- Each scale contributes to final displacement field

Step 2: Vector Field Computation

$$\vec{d}(x, y) = \text{argmin}_{\vec{d}} \sum |F(x + d_x, y + d_y) - O(x, y)|^2$$

Step 3: Morphing Process

- Apply displacement vectors to deform fields
- Create morphed forecast: $F'(x, y) = F(x + d_x, y + d_y)$
- Calculate residual differences after morphing

DAS Error Components

Observation Space Analysis (morphing forecast onto observation):

$$DIS_{obs}(x,y) = |\vec{d}_{obs}(x,y)|$$

$$AMP_{obs}(x,y) = \sqrt{(O(x,y) - F'(x,y))^2}$$

Forecast Space Analysis (morphing observation onto forecast):

$$DIS_{fct}(x,y) = |\vec{d}_{fct}(x,y)|$$

$$AMP_{fct}(x,y) = \sqrt{(F(x,y) - O'(x,y))^2}$$

Combined Metrics:

$$DIS = \frac{1}{n_{obs} + n_{fct}}(n_{obs} \cdot DIS_{obs} + n_{fct} \cdot DIS_{fct})$$

$$AMP = \frac{1}{n_{obs} + n_{fct}}(n_{obs} \cdot AMP_{obs} + n_{fct} \cdot AMP_{fct})$$

Final DAS Score

The displacement and amplitude components are normalized and combined:

$$DAS = \frac{DIS}{D_{max}} + \frac{AMP}{I_0}$$

Normalization Parameters:

- D_{max}: Maximum search distance (e.g., 360 km)
- I_0: Characteristic intensity (RMS of observed precipitation)

Interpretation:

- **DAS = 0**: Perfect forecast
- **DAS 1**: Either displacement error = D_{max} OR amplitude error = I_0
- **No upper limit**: Combines both error types

Physical Meaning:

Displacement error of D_{max} is equivalent to amplitude error from unmatched features (miss + false alarm).

Python Implementation

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import gaussian_filter
from scipy.optimize import minimize

def pyramidal_optical_flow(forecast, observation, max_search_distance=90, levels=4):
    """
    Simplified pyramidal optical flow implementation
    """
    ny, nx = forecast.shape
    displacement_field = np.zeros((ny, nx, 2))

    # Multi-scale processing
    for level in range(levels):
        scale_factor = 2**(levels - level - 1)

        # Downsample fields
        if scale_factor > 1:
            f_scaled = gaussian_filter(forecast, sigma=scale_factor)[::scale_factor, ::scale_
            o_scaled = gaussian_filter(observation, sigma=scale_factor)[::scale_factor, ::sca
        else:
            f_scaled = forecast.copy()
            o_scaled = observation.copy()

        # Calculate displacement at this scale
```

```
        level_displacement = calculate_displacement_field(
            f_scaled, o_scaled, max_search_distance // scale_factor
        )

        # Upsample and add to total displacement
        if scale_factor > 1:
            level_displacement = np.repeat(np.repeat(level_displacement, scale_factor, axis=
                                            scale_factor, axis=1)
            level_displacement = level_displacement[:ny, :nx] * scale_factor

        displacement_field += level_displacement

    return displacement_field

def calculate_displacement_field(forecast, observation, max_search):
    """Calculate displacement field between two fields"""
    ny, nx = forecast.shape
    displacement = np.zeros((ny, nx, 2))

    for i in range(ny):
        for j in range(nx):
            if observation[i, j] > 0:  # Only calculate for precipitation areas
                best_match = find_best_match(forecast, observation[i, j], i, j, max_search)
                displacement[i, j] = [best_match[0] - i, best_match[1] - j]

    return displacement

def find_best_match(field, target_value, center_i, center_j, max_search):
    """Find best matching location within search radius"""
    ny, nx = field.shape
    best_error = float('inf')
    best_location = (center_i, center_j)

    for di in range(-max_search, max_search + 1):
        for dj in range(-max_search, max_search + 1):
            i, j = center_i + di, center_j + dj
            if 0 <= i < ny and 0 <= j < nx:
                error = abs(field[i, j] - target_value)
                if error < best_error:
                    best_error = error
                    best_location = (i, j)
```

```python
    return best_location
```

DAS Calculation Functions

```python
def apply_displacement_field(field, displacement_field):
    """Apply displacement field to morph a field"""
    ny, nx = field.shape
    morphed_field = np.zeros_like(field)

    for i in range(ny):
        for j in range(nx):
            di, dj = displacement_field[i, j]
            source_i = int(i - di)
            source_j = int(j - dj)

            if 0 <= source_i < ny and 0 <= source_j < nx:
                morphed_field[i, j] = field[source_i, source_j]

    return morphed_field

def calculate_das(forecast, observation, max_search_distance=90, threshold=1.0):
    """
    Calculate Displacement and Amplitude Score (DAS)
    """
    # Apply threshold
    forecast_thresh = np.where(forecast > threshold, forecast, 0)
    observation_thresh = np.where(observation > threshold, observation, 0)

    # Calculate characteristic intensity
    I0 = np.sqrt(np.mean(observation_thresh[observation_thresh > 0]**2))

    # Observation space: morph forecast onto observation
    disp_field_obs = pyramidal_optical_flow(forecast_thresh, observation_thresh, max_search_d
    morphed_forecast = apply_displacement_field(forecast_thresh, disp_field_obs)

    # Calculate errors in observation space
    obs_mask = observation_thresh > 0
    dis_obs = np.sqrt(np.sum(disp_field_obs**2, axis=2))
```

```
    dis_obs = dis_obs * obs_mask  # Only where observations exist
    amp_obs = np.abs(observation_thresh - morphed_forecast) * obs_mask

    # Forecast space: morph observation onto forecast
    disp_field_fct = pyramidal_optical_flow(observation_thresh, forecast_thresh, max_search_d
    morphed_observation = apply_displacement_field(observation_thresh, disp_field_fct)

    # Calculate errors in forecast space
    fct_mask = forecast_thresh > 0
    dis_fct = np.sqrt(np.sum(disp_field_fct**2, axis=2))
    dis_fct = dis_fct * fct_mask  # Only where forecasts exist
    amp_fct = np.abs(forecast_thresh - morphed_observation) * fct_mask

    # Combine observation and forecast space errors
    n_obs = np.sum(obs_mask)
    n_fct = np.sum(fct_mask)

    if n_obs + n_fct == 0:
        return 0.0, 0.0, 0.0  # No precipitation in either field

    DIS = (n_obs * np.mean(dis_obs[obs_mask]) + n_fct * np.mean(dis_fct[fct_mask])) / (n_obs
    AMP = (n_obs * np.mean(amp_obs[obs_mask]) + n_fct * np.mean(amp_fct[fct_mask])) / (n_obs

    # Normalize and combine
    DIS_norm = DIS / max_search_distance
    AMP_norm = AMP / I0 if I0 > 0 else 0
    DAS = DIS_norm + AMP_norm

    return DAS, DIS_norm, AMP_norm

def create_synthetic_precipitation_case(case_type="displacement", nx=100, ny=100):
    """Create synthetic test cases similar to ICP geometric cases"""

    # Create base elliptical precipitation feature
    y_indices, x_indices = np.ogrid[:ny, :nx]
    center_y, center_x = ny//2, nx//2

    # Elliptical feature (50x200 points as in paper)
    a, b = 25, 10  # Semi-major and semi-minor axes
    ellipse = ((x_indices - center_x)/a)**2 + ((y_indices - center_y)/b)**2 <= 1

    # Create observation field
```

```
    observation = np.zeros((ny, nx))
    observation[ellipse] = 15.0  # 15 mm/h intensity

    # Create forecast field based on case type
    forecast = np.zeros((ny, nx))

    if case_type == "displacement":
        # Pure displacement (50 points to the right)
        shifted_ellipse = ((x_indices - (center_x + 20))/a)**2 + ((y_indices - center_y)/b)**
        forecast[shifted_ellipse] = 15.0

    elif case_type == "amplitude":
        # Same location, different amplitude
        forecast[ellipse] = 22.5  # 50% higher intensity

    elif case_type == "mixed":
        # Displacement + amplitude error
        shifted_ellipse = ((x_indices - (center_x + 10))/a)**2 + ((y_indices - center_y)/b)**
        forecast[shifted_ellipse] = 22.5

    return forecast, observation
```

Creating Test Cases

```
# Create synthetic test cases
cases = {
    "Pure Displacement": create_synthetic_precipitation_case("displacement"),
    "Pure Amplitude": create_synthetic_precipitation_case("amplitude"),
    "Mixed Error": create_synthetic_precipitation_case("mixed")
}

# Calculate DAS for each case
results = {}
for case_name, (forecast, observation) in cases.items():
    das, dis_norm, amp_norm = calculate_das(forecast, observation, max_search_distance=50)
    results[case_name] = {
        'DAS': das,
        'DIS/Dmax': dis_norm,
```

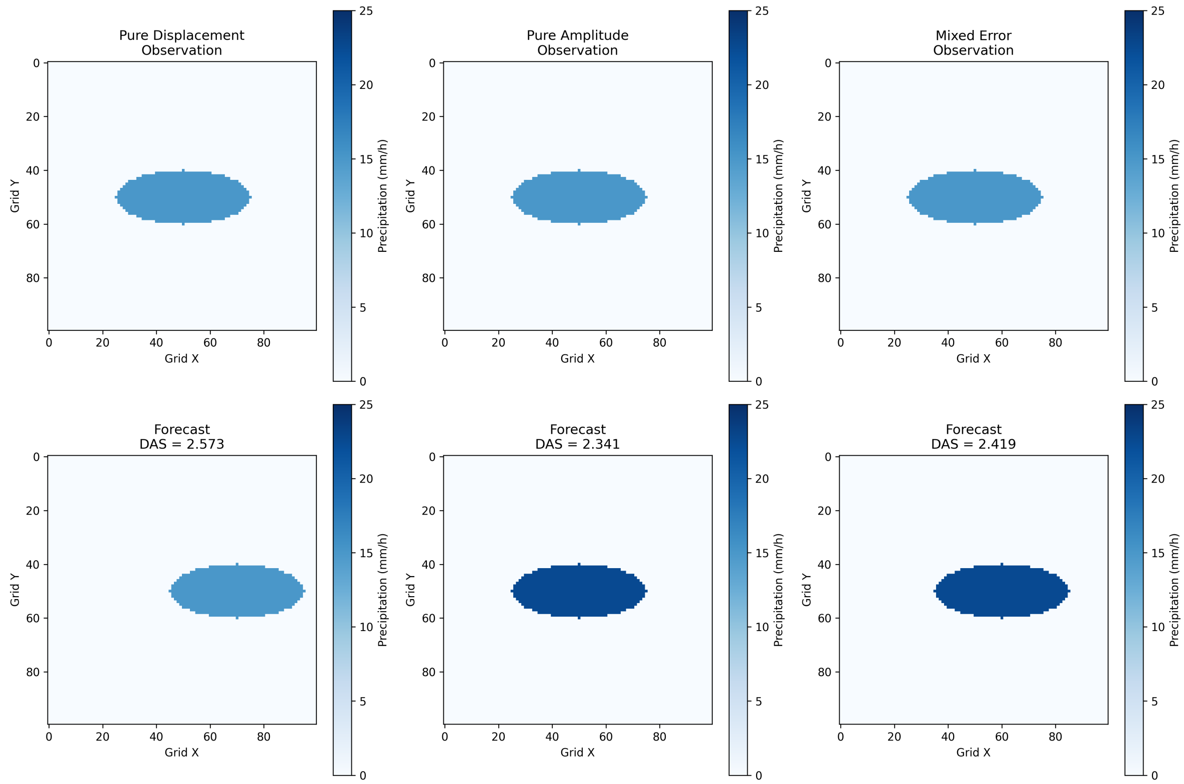
```
        'AMP/IO': amp_norm
    }

# Display results
print("=== DAS Results for Synthetic Cases ===")
print(f"{'Case':<20} {'DAS':<8} {'DIS/Dmax':<10} {'AMP/IO':<10}")
print("-" * 50)
for case_name, metrics in results.items():
    print(f"{case_name:<20} {metrics['DAS']:<8.3f} {metrics['DIS/Dmax']:<10.3f} {metrics['AMI
```

```
=== DAS Results for Synthetic Cases ===
Case                 DAS      DIS/Dmax   AMP/IO
--------------------------------------------------
Pure Displacement    2.573    1.623      0.950
Pure Amplitude       2.341    1.103      1.237
Mixed Error          2.419    1.187      1.232
```

Visualizing Test Cases

```
fig, axes = plt.subplots(2, 3, figsize=(15, 10))

case_names = list(cases.keys())
for i, case_name in enumerate(case_names):
    forecast, observation = cases[case_name]

    # Plot observation
    im1 = axes[0, i].imshow(observation, cmap='Blues', vmin=0, vmax=25)
    axes[0, i].set_title(f'{case_name}\nObservation')
    axes[0, i].set_xlabel('Grid X')
    axes[0, i].set_ylabel('Grid Y')
    plt.colorbar(im1, ax=axes[0, i], label='Precipitation (mm/h)')

    # Plot forecast
    im2 = axes[1, i].imshow(forecast, cmap='Blues', vmin=0, vmax=25)
    axes[1, i].set_title(f'Forecast\nDAS = {results[case_name]["DAS"]:.3f}')
    axes[1, i].set_xlabel('Grid X')
    axes[1, i].set_ylabel('Grid Y')
```

```
    plt.colorbar(im2, ax=axes[1, i], label='Precipitation (mm/h)')

plt.tight_layout()
plt.show()
```



Real Case Application: SPC2005

The paper demonstrates DAS on real Weather Research and Forecasting (WRF) model forecasts from the 2005 Storm Prediction Center Spring Program:

Case Study: 13 May 2005

- **Main Issue**: Squall line with underestimated north-south extension
- **False Alarms**: Spurious precipitation in southeast
- **DAS Components**: Large amplitude error dominates displacement error

- **Expert Ranking**: Correctly identified as worst among three model configurations

Key Findings:

- **wrf4ncep**: DAS = 1.38 (worst performance)
- **wrf4ncar**: DAS = 1.12 (best performance)
- **wrf2caps**: DAS = 1.18 (intermediate)

Comparison with Traditional Metrics:

- DAS ranking agreed with expert subjective evaluation
- Traditional ETS and BIAS showed less consistent ranking
- DAS captured both displacement and amplitude errors effectively

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Advantages and Limitations

Advantages:

1. **Separates error types**: Distinguishes displacement from amplitude errors
2. **Handles false alarms**: Proper treatment through bidirectional morphing
3. **Scale-aware**: Pyramidal algorithm works across multiple scales
4. **Physically meaningful**: Relates to meteorological feature displacement
5. **Computationally efficient**: ~10 seconds per image pair

Limitations:

1. **Complex field matching**: Difficulty with overlapping or merging features
2. **Parameter sensitivity**: Maximum search distance affects results
3. **Interpolation errors**: Small residual errors from morphing process
4. **Ambiguity**: Complex fields may have multiple valid interpretations
5. **Single time**: Does not account for temporal displacement

Comparison with Other Methods:

- More comprehensive than traditional categorical scores
- Complements object-based verification methods

- Less intuitive than feature-based approaches for complex cases

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Parameter Selection Guidelines

Maximum Search Distance (Dmax):

- Should reflect **dynamical scales** (e.g., radius of deformation)
- Typical values: 200-400 km for synoptic features
- Smaller values for convective-scale verification
- Too large: Everything gets matched (loses discrimination)
- Too small: Valid displacements treated as false alarms

Intensity Threshold:

- Remove background noise and light precipitation
- Typical: 1 mm/h for precipitation verification
- Should match forecast system capabilities
- Affects both displacement and amplitude calculations

Characteristic Intensity (I0):

- RMS of observed precipitation above threshold
- Can use climatological values for consistency
- Affects relative weighting of amplitude vs displacement errors
- Should represent typical precipitation intensities

Operational Applications

Model Development:

- Identify systematic displacement biases
- Evaluate impact of model changes on spatial accuracy
- Compare different physics parameterizations
- Assess ensemble member diversity

Forecast Verification:

- Routine assessment of high-resolution model performance
- Complement traditional verification metrics
- Provide spatial error characteristics
- Support model ranking and selection

Research Applications:

- Study predictability of different weather regimes
- Evaluate ensemble perturbation strategies
- Understand scale-dependent forecast errors
- Develop improved verification frameworks

Future Extensions:

- Multi-time analysis for temporal displacement
- Application to other meteorological variables
- Integration with probabilistic verification
- Real-time forecast guidance systems

Key Insights from Keil & Craig (2009)

Methodological Contributions:

1. **Optical flow adaptation**: Successfully applied computer vision techniques to meteorology
2. **Bidirectional morphing**: Proper handling of false alarms and misses
3. **Error decomposition**: Meaningful separation of spatial and amplitude errors
4. **Normalization scheme**: Physically-based combination of error components

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References & Resources

Primary Reference:

- Keil, C., & Craig, G. C. (2009). A displacement and amplitude score employing an optical flow technique. *Weather and Forecasting*, 24(5), 1297-1308.

Related Spatial Verification Methods:

- Gilleland et al. (2009): Intercomparison of spatial forecast verification methods
- Davis et al. (2006): Object-based verification (MODE)
- Roberts & Lean (2008): Fractions Skill Score (FSS)
- Ebert & McBride (2000): Contiguous rain area analysis

Technical Background: - Zinner et al. (2008): Pyramidal image matching algorithm

- Marzban et al. (2008): Alternative optical flow applications
- Ahijevych et al. (2009): Spatial Verification Methods ICP

Applications:

- Kain et al. (2008): SPC2005 Spring Program
- Various ensemble verification studies
- Operational model evaluation systems