Verification of Precipitation Extremes with the SLX score

Structure of Local eXtremes - Sass (2021)

Carlos Peralta

2025-07-14

Motivation

Modern high-resolution NWP models provide detailed precipitation forecasts, but traditional verification suffers from the "double penalty" problem when extremes are slightly displaced. When it comes to precipitation **extremes**, typically what we want to know is: - Where will the **heaviest rain** fall? - Where will it stay **completely dry**?

SLX (Structure of Local Extremes) by Sass (2021) evaluates the capability of high resolution models to predict extremes by using neighbourhood verification focused specifically on extremes.

The SLX Method

The SLX (Structure of Local EXtremes) method is a spatial verification scheme designed to evaluate how well numerical weather prediction models forecast the spatial structure of local extremes (both maxima and minima) in precipitation fields.

SLX computes four neighbourhood-based scores:

Component	What it measures
SLX_ob_max	How well forecast captures observed maxima locations
SLX_fc_max	How well observed field captures forecast maxima locations
SLX_ob_min	How well forecast captures observed minima locations

Component	What it measures
SLX_fc_min	How well observed field captures forecast minima locations

where the final score is defined as:

$$\mathrm{SLX} = \frac{1}{4}(\mathrm{SLX}_{\mathrm{ob\_max}} + \mathrm{SLX}_{\mathrm{fc\_max}} + \mathrm{SLX}_{\mathrm{ob\_min}} + \mathrm{SLX}_{\mathrm{fc\_min}})$$

Neighborhood Approach: For each extreme point, the method looks in a square neighborhood of width L around that point to find the corresponding extreme in the other field.

Score Function: Uses a piecewise linear function S that: Returns 1 for perfect matches

Penalizes over-forecasting more than under-forecasting (asymmetric)

Has a tolerance parameter k (default 0.1 kg/m²) for small values

---

How the calculation works

1. Identify Local Extremes

- Local maxima: Points in the field that are higher than all their immediate neighbors (within a small tolerance).
- Local minima: Points that are lower than all their immediate neighbors.

This is done for both the analysis (observation) field and the forecast field.

2. Define a Neighborhood

- For each extreme point, define a square neighborhood of width L (so, for L=2, you look at a 5x5 grid centered on the point).
- This allows for some spatial "fuzziness"—if the forecasted extreme is close but not exactly at the observed location, it can still be matched.

3. Compare Extremes Across Fields

- For each observed maximum, find the maximum value in the forecast field within the neighborhood around that point.
- For each observed minimum, find the minimum value in the forecast field within the neighborhood.

- Do the same in reverse: for each forecasted maximum/minimum, look for the corresponding extreme in the analysis field.

4. Score Each Pair Using the Score Function

- For each pair (observed extreme, forecasted value in the neighborhood), compute a score using a special function (see below).

5. Average the Scores

- For each type (observed maxima, observed minima, forecast maxima, forecast minima), average the scores over all relevant points.
- The final SLX score is the mean of these four component scores.

Why is the Score Function Needed?

The Problem:

Simply comparing the values (e.g., "is the forecasted max equal to the observed max?") is not enough.

- Forecasts are rarely perfect.
- Small errors should be penalized less than large errors.
- Over-forecasting and under-forecasting may have different practical impacts.
- Zero (dry) values need special treatment.
- The Solution: The Score Function The score function $S(ob, \phi)$, chosen as a piecewise linear function that:
- Returns 1 for a perfect match (forecast matches observation within a small tolerance).
- Decreases linearly as the forecast deviates from the observation.
- Penalizes over-forecasting more gently than under-forecasting (asymmetry), reflecting the idea that a "false alarm" is less bad than a "miss" in some applications.
- For very small observed values (quasi-dry), uses a different branch to avoid dividing by zero or over-penalizing small errors.

Mathematically:

- If the observed value is large, and the forecast is close (within a tolerance k), the score is 1.
- If the forecast is much less than the observed, the score drops linearly to zero.
- If the forecast is much more than the observed, the score also drops, but with a different slope (controlled by parameter A).
- For very small observed values, the function is symmetric and ensures that small errors are not over-penalized.

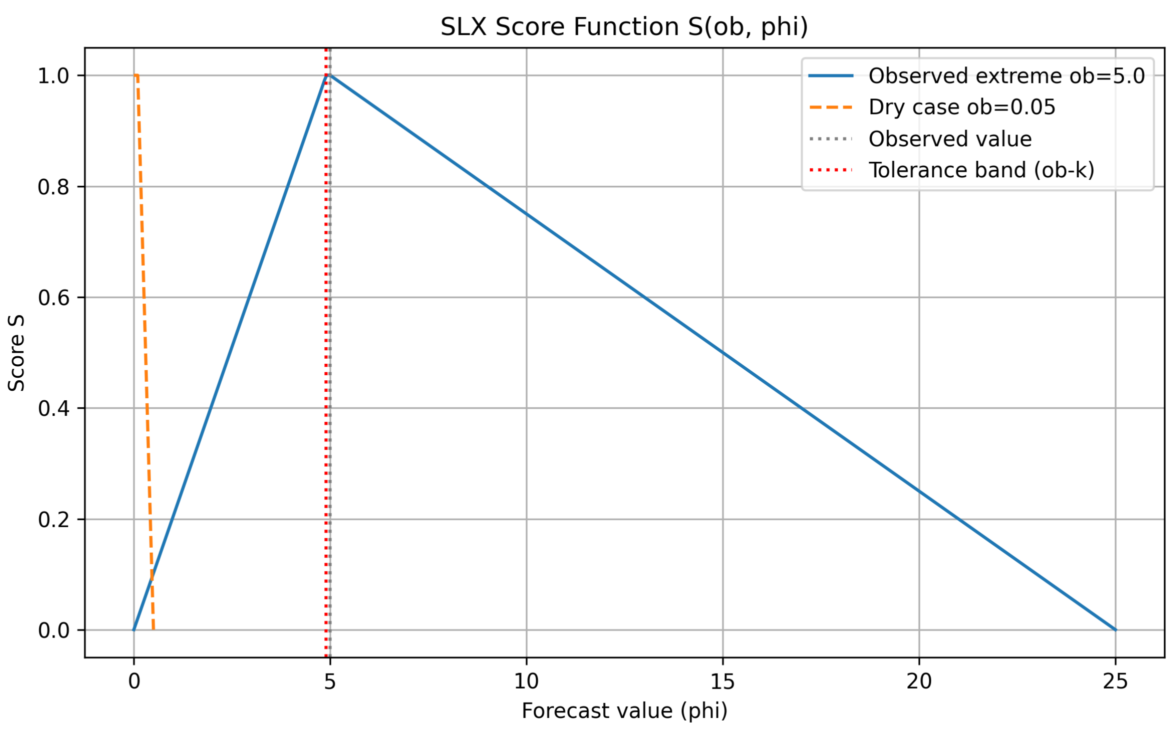
This function ensures:

- Scores are always between 0 and 1.
- Small errors are not punished too harshly.
- The score is interpretable: 1 = perfect, 0 = completely wrong.

Example

```python
import numpy as np
import matplotlib.pyplot as plt

# SLX score function from Sass (2021)
def S_score(ob, phi, k=0.1, A=4):
    if ob > k:
        if phi < ob - k:
            return phi / (ob - k)
        elif phi <= ob:
            return 1.0
        else:
            return max(1 - (phi - ob) / (A * ob), 0.0)
    else:  # ob <= k
        if phi <= k:
            return 1.0
        else:
            return max(1 - (phi - k) / (A * k), 0.0)

# Example: observed extreme value
ob = 5.0
phis = np.linspace(0, 25, 500)
```

```
scores = [S_score(ob, phi) for phi in phis]

# For small observed value (dry case)
ob_dry = 0.05
phis_dry = np.linspace(0, 0.5, 200)
scores_dry = [S_score(ob_dry, phi) for phi in phis_dry]

plt.figure(figsize=(8,5))
plt.plot(phis, scores, label=f"Observed extreme ob={ob}")
plt.plot(phis_dry, scores_dry, label=f"Dry case ob={ob_dry}", linestyle='--')
plt.axvline(ob, color='gray', linestyle=':', label="Observed value")
plt.axvline(ob-0.1, color='red', linestyle=':', label='Tolerance band (ob-k)')
plt.title("SLX Score Function S(ob, phi)")
plt.xlabel("Forecast value (phi)")
plt.ylabel("Score S")
plt.ylim(-0.05, 1.05)
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
```



In the example above For observed extreme value

```
ob = 5.0
phis = np.linspace(0, 25, 500)
scores = [S_score(ob, phi) for phi in phis]
```

For small observed value (dry case)

```
ob_dry = 0.05
phis_dry = np.linspace(0, 0.5, 200)
scores_dry = [S_score(ob_dry, phi) for phi in phis_dry]
```

How to Read the Plot

X-axis: Forecast value (ϕ), the value predicted by the model at or near the observed extreme.

Y-axis: Score S, which ranges from 0 (worst) to 1 (perfect match).

Solid blue curve: Case where the observed extreme is 5.0 (e.g., heavy rain).

Dashed orange curve: Case where the observed extreme is very small (0.05, a "dry" case).

Vertical gray line: The observed value (ob).

Vertical red line: The lower tolerance bound (ob−k), where k=0.1.

How the Score Function Works

1. Perfect Match (Score = 1)

- If the forecast value (ϕ) is within the tolerance band ($ob - k \leq \phi \leq ob$), the score is 1.
- This means small under-forecasts (up to k below the observed value) are not penalized.

2. Under-forecasting ($\phi < ob - k$)

- The score decreases linearly as the forecast drops below the tolerance band.
- At $\phi = 0$, the score is 0 (if the observed value is much higher).

3. Over-forecasting ($\phi > ob$)

- The score decreases linearly as the forecast exceeds the observed value.
- The slope is gentler (controlled by parameter A), so over-forecasting is penalized less harshly than under-forecasting.

- The score reaches 0 when the forecast is much larger than the observed value (specifically, at $\phi = ob + 4 \times ob = 5 \times ob$).

4. Dry Case ($ob \leq k$)

- For very small observed values, the function is symmetric and forgiving: as long as the forecast is also small ($phi \leq k$), the score is 1.
- If the forecast is larger, the score drops linearly to 0 as ϕ increases.

Why This Design? - Tolerance for small errors: Small under-forecasts are not penalized, reflecting uncertainty in observations and the practical irrelevance of tiny differences.

- Asymmetry: Over-forecasting is penalized less than under-forecasting, which is often desirable in weather warnings (better to have a false alarm than a miss).
- Dry case: Ensures that small errors in dry areas don't lead to large penalties.

Example Walkthrough

Suppose the observed extreme is 5.0:

If the forecast is 4.9, the score is 1 (within tolerance).

If the forecast is 4.0, the score is $4.0/(5.0 - 0.1) \approx 0.82$

If the forecast is 7.0, the score is $1 - (7.0 - 5.0)/(4 \times 5.0) = 0.9$

If the forecast is 0, the score is 0 (total miss).

Suppose the observed extreme is 0.05 (dry):

If the forecast is also ≤ 0.1, the score is 1.

If the forecast is 0.2, the score is $1 - (0.2 - 0.1)/(4 \times 0.1) = 0.75$

If the forecast is 0.5, the score is 0.

Algorithm Steps (Following Sass 2021)

Step 1: Extrema Detection

Local extremes are identified using a tolerance parameter δ (default $\delta \approx 0 kg/m^2$): - **obmax(K1)**: Observed local maximum points (M1 total)

- **obmin(K2)**: Observed local minimum points (M2 total)
- **fcmax(K3)**: Forecast local maximum points (M3 total)
- **fcmin(K4)**: Forecast local minimum points (M4 total)

Step 2: Neighbourhood Definition

For each extreme point, define a square neighbourhood of width L:

- Neighbourhood size: $(2L + 1)^2$ grid points
- L = 0 means point-to-point comparison
- Internal points only (boundary zone of width Lmax excluded)

Step 3: Neighbourhood Extrema Calculation

For each observed/forecast extreme, find the corresponding extreme in the other field's neighbourhood:

- max(L,K1) = Max{ (i,j)} in forecast neighbourhood around obmax(K1)
- min(L,K2) = Min{ (i,j)} in forecast neighbourhood around obmin(K2)
- Ψmax(L,K3) = Max{Ψ(i,j)} in observed neighbourhood around fcmax(K3)
- Ψmin(L,K4) = Min{Ψ(i,j)} in observed neighbourhood around fcmin(K4)

Step 4: Score Function Application

Apply the SLX score function $S(\phi, ob)$ with parameters k = 0.1 kg/m² and A = 4:

If ob > k:

- If $\phi < ob - k : S = \phi/(ob - k)$
- If $ob - k \leq \phi \leq ob : S = 1$
- If $\phi > ob : S = Max1 - (\phi - ob)/(A \times ob), 0$

If $ob \leq k$**:**

- If $\phi \leq k : S = 1$
- If $\phi > k : S = Max1 - (\phi - k)/(A \times k), 0$

Step 5: Component Score Calculation

Average individual scores for each component:

$$\mathrm{SLX}_{\mathrm{ob\_max}} = \frac{1}{M1} \sum_{K1=1}^{M1} S_{\mathrm{ob\_max}}(K1)$$

$$\mathrm{SLX}_{\mathrm{ob\_min}} = \frac{1}{M2} \sum_{K2=1}^{M2} S_{\mathrm{ob\_min}}(K2)$$

$$\mathrm{SLX}_{\mathrm{fc\_max}} = \frac{1}{M3} \sum_{K3=1}^{M3} S_{\mathrm{fc\_max}}(K3)$$

$$\mathrm{SLX}_{\mathrm{fc\_min}} = \frac{1}{M4} \sum_{K4=1}^{M4} S_{\mathrm{fc\_min}}(K4)$$

Python Implementation

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.ndimage import maximum_filter, minimum_filter

def find_local_extrema_sass_corrected(arr, mode='max', tolerance=0.0):
    """
    Find local extrema as described in Sass (2021)

    Key correction: For maxima, only consider non-zero values as potential maxima.
    This prevents every zero-valued dry point from being classified as a maximum.

    Key points from paper:
    - "Zero-valued dry areas will often exist... multiple points of zero value
      will be automatically selected as minima" (NOT maxima)
    - Default tolerance  = 0 kg/m²
    - All selected points contribute with equal weight
    """
    if mode == 'max':
        # Local maxima should exclude zeros unless they're true peaks
```

```
        # Only points with precipitation > tolerance can be maxima
        filtered = maximum_filter(arr, size=3)
        mask = (arr == filtered) & (arr > tolerance)
    else:  # mode == 'min'
        # Local minima: points that are <= all neighbors within tolerance
        # Paper explicitly states zeros are automatically selected as minima
        filtered = minimum_filter(arr, size=3)
        mask = (arr <= filtered + tolerance) & (arr == filtered)

    indices = np.where(mask)
    return [(i, j, arr[i, j]) for i, j in zip(indices[0], indices[1])]

def score_function_sass(phi, ob, k=0.1, A=4.0):
    """
    Exact SLX similarity function from Sass (2021) equations (2a)-(2c), (3a)-(3b)
    """
    if ob > k:
        if phi < ob - k:
            return phi / max(ob - k, 1e-9)  # Guard against division by zero
        elif phi <= ob:
            return 1.0
        else:  # phi > ob
            return max(1 - (phi - ob) / (A * ob), 0.0)
    else:  # ob <= k
        if phi <= k:
            return 1.0
        else:  # phi > k
            return max(1 - (phi - k) / (A * k), 0.0)

def get_neighbourhood_extreme_sass(arr, i, j, L, mode='max'):
    """
    Get max/min value in (2L+1)×(2L+1) neighbourhood around point (i,j)
    Following Sass (2021) equations (1a)-(1d)
    """
    # Define neighbourhood bounds: [i-L, i+L] × [j-L, j+L]
    i_min, i_max = max(0, i-L), min(arr.shape[0], i+L+1)
    j_min, j_max = max(0, j-L), min(arr.shape[1], j+L+1)
    neighbourhood = arr[i_min:i_max, j_min:j_max]
    return neighbourhood.max() if mode == 'max' else neighbourhood.min()

def calculate_slx_sass_corrected(obs, forecast, neighbourhood_sizes=None,
                                 tolerance=0.0, k=0.1, A=4.0):
```

```python
"""
Calculate SLX scores following Sass (2021) methodology

Key correction: Uses corrected extrema detection that doesn't classify
all zeros as maxima, which was causing SLX to decrease with neighbourhood size.

Parameters match paper specifications:
- tolerance:   parameter (default   0 kg/m²)
- k: dry threshold (default 0.1 kg/m²)
- A: penalty parameter (default 4.0)
"""
if neighbourhood_sizes is None:
    neighbourhood_sizes = [0, 1, 3, 5, 9]

results = {}

# Step 1: Find local extrema
obs_maxima = find_local_extrema_sass_corrected(obs, 'max', tolerance)
obs_minima = find_local_extrema_sass_corrected(obs, 'min', tolerance)
fc_maxima = find_local_extrema_sass_corrected(forecast, 'max', tolerance)
fc_minima = find_local_extrema_sass_corrected(forecast, 'min', tolerance)

for L in neighbourhood_sizes:
    scores_ob_max = []
    scores_ob_min = []
    scores_fc_max = []
    scores_fc_min = []

    # Step 2-4: Calculate component scores following equations (4)-(7)
    # SLX_ob_max: Equation (4)
    for i, j, ob_val in obs_maxima:
        fc_neighbourhood_max = get_neighbourhood_extreme_sass(forecast, i, j, L, 'max')
        scores_ob_max.append(score_function_sass(fc_neighbourhood_max, ob_val, k, A))

    # SLX_ob_min: Equation (5)
    for i, j, ob_val in obs_minima:
        fc_neighbourhood_min = get_neighbourhood_extreme_sass(forecast, i, j, L, 'min')
        scores_ob_min.append(score_function_sass(fc_neighbourhood_min, ob_val, k, A))

    # SLX_fc_max: Equation (6)
    for i, j, fc_val in fc_maxima:
        obs_neighbourhood_max = get_neighbourhood_extreme_sass(obs, i, j, L, 'max')
```

```python
            scores_fc_max.append(score_function_sass(fc_val, obs_neighbourhood_max, k, A))

        # SLX_fc_min: Equation (7)
        for i, j, fc_val in fc_minima:
            obs_neighbourhood_min = get_neighbourhood_extreme_sass(obs, i, j, L, 'min')
            scores_fc_min.append(score_function_sass(fc_val, obs_neighbourhood_min, k, A))

        # Step 5: Calculate component averages
        slx_ob_max = np.mean(scores_ob_max) if scores_ob_max else 0.0
        slx_ob_min = np.mean(scores_ob_min) if scores_ob_min else 0.0
        slx_fc_max = np.mean(scores_fc_max) if scores_fc_max else 0.0
        slx_fc_min = np.mean(scores_fc_min) if scores_fc_min else 0.0

        # Overall SLX score: Equation (8)
        slx_total = 0.25 * (slx_ob_max + slx_ob_min + slx_fc_max + slx_fc_min)

        results[L] = {
            'SLX': slx_total,
            'SLX_ob_max': slx_ob_max,
            'SLX_ob_min': slx_ob_min,
            'SLX_fc_max': slx_fc_max,
            'SLX_fc_min': slx_fc_min,
            'n_obs_max': len(obs_maxima),
            'n_obs_min': len(obs_minima),
            'n_fc_max': len(fc_maxima),
            'n_fc_min': len(fc_minima)
        }

    return results
```

Creating Synthetic Test Data

```python
def create_synthetic_radar_obs(nx=100, ny=100):
    """Create synthetic radar observation field"""
    np.random.seed(42)
    obs = np.zeros((ny, nx))

    # Add some convective cells (local maxima)
```

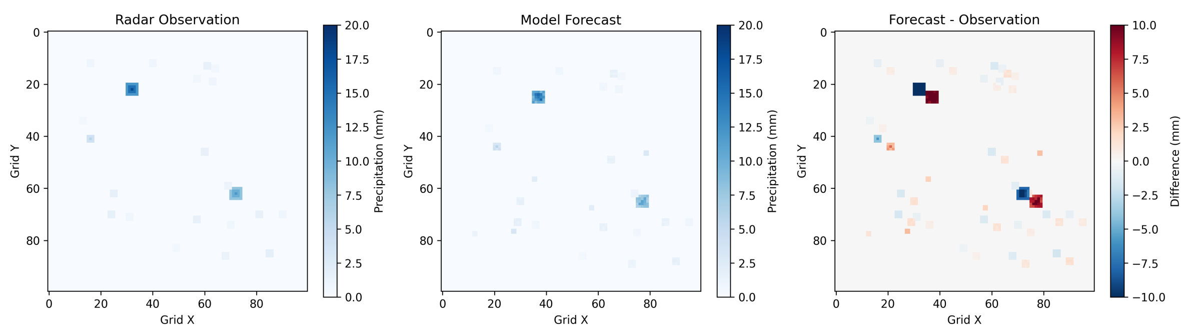
```
    # Cell 1: Strong convection
    obs[20:25, 30:35] = 12.0
    obs[21:24, 31:34] = 15.0
    obs[22, 32] = 18.0  # Peak

    # Cell 2: Moderate convection
    obs[60:65, 70:75] = 8.0
    obs[61:64, 71:74] = 10.0
    obs[62, 72] = 12.0  # Peak

    # Cell 3: Weak convection
    obs[40:43, 15:18] = 4.0
    obs[41, 16] = 6.0  # Peak

    # Add some light background precipitation
    for _ in range(20):
        i, j = np.random.randint(10, ny-10), np.random.randint(10, nx-10)
        if obs[i, j] == 0:  # Only add where it's currently dry
            obs[i:i+3, j:j+3] = np.random.uniform(0.5, 2.0)

    # Ensure non-negative values
    obs = np.maximum(obs, 0)
    return obs

def create_synthetic_model_forecast(obs_field, displacement=(3, 5), intensity_bias=0.9):
    """Create synthetic model forecast with displacement and bias"""
    ny, nx = obs_field.shape
    forecast = np.zeros_like(obs_field)

    # Apply spatial displacement and intensity bias
    dy, dx = displacement
    for i in range(ny):
        for j in range(nx):
            if obs_field[i, j] > 0:
                # Apply displacement
                new_i = i + dy
                new_j = j + dx
                # Check bounds
                if 0 <= new_i < ny and 0 <= new_j < nx:
                    # Apply intensity bias and some random noise
                    forecast[new_i, new_j] = obs_field[i, j] * intensity_bias * np.random.uni
```

```python
    # Add some forecast-specific features (false alarms)
    np.random.seed(123)
    for _ in range(5):
        i, j = np.random.randint(10, ny-10), np.random.randint(10, nx-10)
        if forecast[i, j] == 0:  # Only add where forecast is currently dry
            forecast[i:i+2, j:j+2] = np.random.uniform(1.0, 4.0)

    # Ensure non-negative values
    forecast = np.maximum(forecast, 0)
    return forecast

# Create synthetic observation and forecast fields
obs_field = create_synthetic_radar_obs()
fc_field = create_synthetic_model_forecast(obs_field)

print(f"Observation field shape: {obs_field.shape}")
print(f"Max precipitation: {obs_field.max():.1f} mm")
print(f"Min precipitation: {obs_field.min():.1f} mm")
print(f"Fraction of dry points: {(obs_field == 0).mean():.2f}")

print(f"\nForecast field shape: {fc_field.shape}")
print(f"Max precipitation: {fc_field.max():.1f} mm")
print(f"Min precipitation: {fc_field.min():.1f} mm")
print(f"Fraction of dry points: {(fc_field == 0).mean():.2f}")
```

```
Observation field shape: (100, 100)
Max precipitation: 18.0 mm
Min precipitation: 0.0 mm
Fraction of dry points: 0.98

Forecast field shape: (100, 100)
Max precipitation: 15.7 mm
Min precipitation: 0.0 mm
Fraction of dry points: 0.98
```

Visualizing the Fields

```
fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(15, 4))

# Plot observation
im1 = ax1.imshow(obs_field, cmap='Blues', vmin=0, vmax=20)
ax1.set_title('Radar Observation')
ax1.set_xlabel('Grid X')
ax1.set_ylabel('Grid Y')
plt.colorbar(im1, ax=ax1, label='Precipitation (mm)')

# Plot forecast
im2 = ax2.imshow(fc_field, cmap='Blues', vmin=0, vmax=20)
ax2.set_title('Model Forecast')
ax2.set_xlabel('Grid X')
ax2.set_ylabel('Grid Y')
plt.colorbar(im2, ax=ax2, label='Precipitation (mm)')

# Plot difference
diff = fc_field - obs_field
im3 = ax3.imshow(diff, cmap='RdBu_r', vmin=-10, vmax=10)
ax3.set_title('Forecast - Observation')
ax3.set_xlabel('Grid X')
ax3.set_ylabel('Grid Y')
plt.colorbar(im3, ax=ax3, label='Difference (mm)')

plt.tight_layout()
plt.show()
```



Applying SLX Algorithm

```python
# Calculate SLX scores using the Sass (2021) methodology
neighbourhood_sizes = [0, 1, 3, 5, 7, 9]
slx_results = calculate_slx_sass_corrected(obs_field, fc_field, neighbourhood_sizes)

# Display results
print("SLX Results:")
print("=" * 70)
print(f"{'L':<3} {'SLX':<6} {'ob_max':<7} {'ob_min':<7} {'fc_max':<7} {'fc_min':<7}")
print("-" * 70)
for L in neighbourhood_sizes:
    r = slx_results[L]
    print(f"{L:<3} {r['SLX']:<6.3f} {r['SLX_ob_max']:<7.3f} {r['SLX_ob_min']:<7.3f} "
          f"{r['SLX_fc_max']:<7.3f} {r['SLX_fc_min']:<7.3f}")

print("\n" + "=" * 70)
print("Extrema counts:")
r = slx_results[0]  # Use L=0 for counts
print(f"Observed maxima: {r['n_obs_max']} (only non-zero precipitation peaks)")
print(f"Observed minima: {r['n_obs_min']} (includes zeros as per paper)")
print(f"Forecast maxima: {r['n_fc_max']} (only non-zero precipitation peaks)")
print(f"Forecast minima: {r['n_fc_min']} (includes zeros as per paper)")

print("Expected intuitively - larger neighbourhoods")
print("are more tolerant to spatial displacement, leading to higher scores.")
```

```
SLX Results:
======================================================================
L   SLX    ob_max  ob_min  fc_max  fc_min
----------------------------------------------------------------------
0   0.493  0.018   0.974   0.001   0.977
1   0.526  0.069   0.994   0.044   0.996
3   0.683  0.398   0.998   0.337   0.999
5   0.878  0.852   0.998   0.663   0.999
7   0.854  0.758   0.998   0.662   0.999
9   0.838  0.696   0.998   0.660   0.999

======================================================================
Extrema counts:
Observed maxima: 168 (only non-zero precipitation peaks)
```

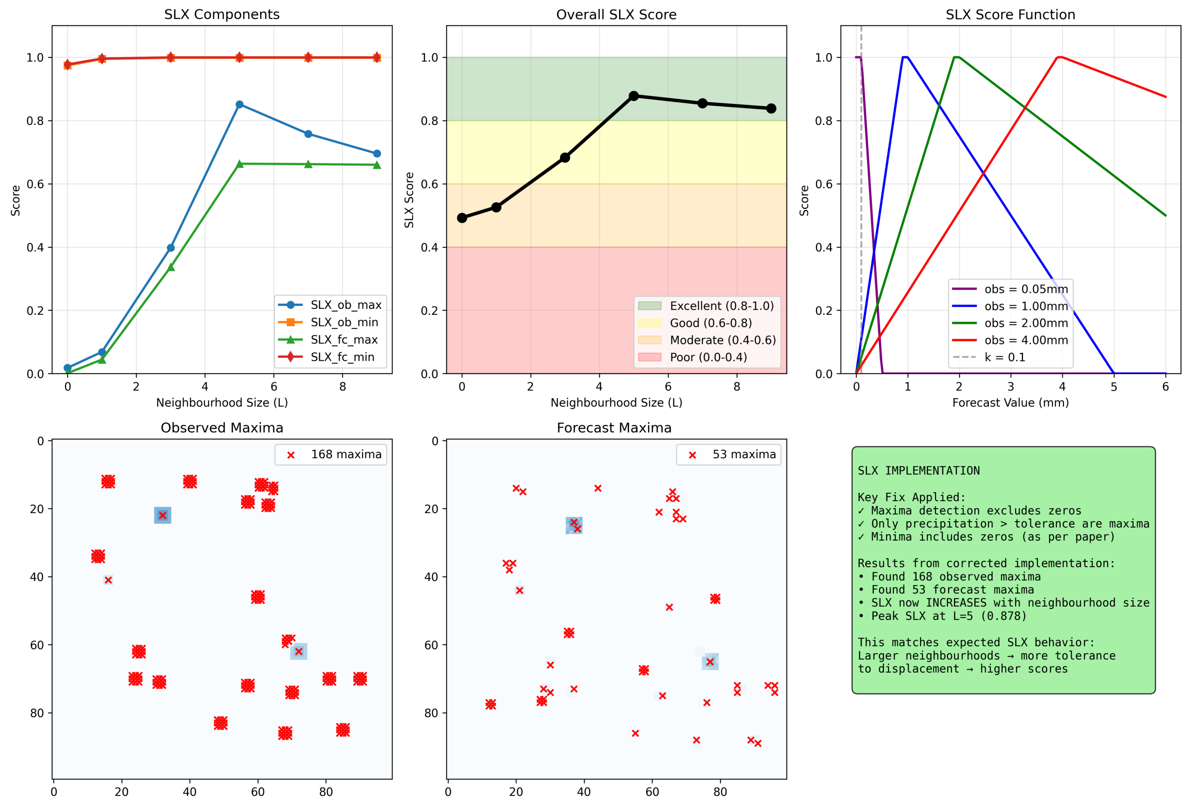
```
Observed minima: 9790 (includes zeros as per paper)
Forecast maxima: 53 (only non-zero precipitation peaks)
Forecast minima: 9756 (includes zeros as per paper)
Expected intuitively - larger neighbourhoods
are more tolerant to spatial displacement, leading to higher scores.
```

Visualizing SLX Components

```
fig, axes = plt.subplots(2, 3, figsize=(15, 10))

# Extract data for plotting
L_values = list(slx_results.keys())
slx_total = [slx_results[L]['SLX'] for L in L_values]
slx_ob_max = [slx_results[L]['SLX_ob_max'] for L in L_values]
slx_ob_min = [slx_results[L]['SLX_ob_min'] for L in L_values]
slx_fc_max = [slx_results[L]['SLX_fc_max'] for L in L_values]
slx_fc_min = [slx_results[L]['SLX_fc_min'] for L in L_values]

# Plot individual components
axes[0,0].plot(L_values, slx_ob_max, 'o-', label='SLX_ob_max', linewidth=2, markersize=6)
axes[0,0].plot(L_values, slx_ob_min, 's-', label='SLX_ob_min', linewidth=2, markersize=6)
axes[0,0].plot(L_values, slx_fc_max, '^-', label='SLX_fc_max', linewidth=2, markersize=6)
axes[0,0].plot(L_values, slx_fc_min, 'd-', label='SLX_fc_min', linewidth=2, markersize=6)
axes[0,0].set_xlabel('Neighbourhood Size (L)')
axes[0,0].set_ylabel('Score')
axes[0,0].set_title('SLX Components')
axes[0,0].legend()
axes[0,0].grid(True, alpha=0.3)
axes[0,0].set_ylim(0, 1.1)

# Plot total SLX
axes[0,1].plot(L_values, slx_total, 'ko-', linewidth=3, markersize=8)
axes[0,1].set_xlabel('Neighbourhood Size (L)')
axes[0,1].set_ylabel('SLX Score')
axes[0,1].set_title('Overall SLX Score')
axes[0,1].grid(True, alpha=0.3)
axes[0,1].set_ylim(0, 1.1)
```

```
# Add interpretation zones
axes[0,1].axhspan(0.8, 1.0, alpha=0.2, color='green', label='Excellent (0.8-1.0)')
axes[0,1].axhspan(0.6, 0.8, alpha=0.2, color='yellow', label='Good (0.6-0.8)')
axes[0,1].axhspan(0.4, 0.6, alpha=0.2, color='orange', label='Moderate (0.4-0.6)')
axes[0,1].axhspan(0.0, 0.4, alpha=0.2, color='red', label='Poor (0.0-0.4)')
axes[0,1].legend(loc='lower right')

# Show score function
phi_range = np.linspace(0, 6, 200)
ob_values = [0.05, 1.0, 2.0, 4.0]  # Include case where ob   k
colors = ['purple', 'blue', 'green', 'red']
for ob_val, color in zip(ob_values, colors):
    scores = [score_function_sass(phi, ob_val) for phi in phi_range]
    axes[0,2].plot(phi_range, scores, color=color, linewidth=2,
                   label=f'obs = {ob_val:.2f}mm')
axes[0,2].axvline(x=0.1, color='gray', linestyle='--', alpha=0.7, label='k = 0.1')
axes[0,2].set_xlabel('Forecast Value (mm)')
axes[0,2].set_ylabel('Score')
axes[0,2].set_title('SLX Score Function')
axes[0,2].legend()
axes[0,2].grid(True, alpha=0.3)
axes[0,2].set_ylim(0, 1.1)

# Show extrema locations
axes[1,0].imshow(obs_field, cmap='Blues', vmin=0, vmax=20, alpha=0.7)
obs_maxima = find_local_extrema_sass_corrected(obs_field, 'max')
if obs_maxima:
    max_i, max_j = zip(*[(i, j) for i, j, val in obs_maxima])
    axes[1,0].scatter(max_j, max_i, c='red', s=30, marker='x', label=f'{len(obs_maxima)} maxi
axes[1,0].set_title('Observed Maxima')
axes[1,0].legend()

axes[1,1].imshow(fc_field, cmap='Blues', vmin=0, vmax=20, alpha=0.7)
fc_maxima = find_local_extrema_sass_corrected(fc_field, 'max')
if fc_maxima:
    max_i, max_j = zip(*[(i, j) for i, j, val in fc_maxima])
    axes[1,1].scatter(max_j, max_i, c='red', s=30, marker='x', label=f'{len(fc_maxima)} maxim
axes[1,1].set_title('Forecast Maxima ')
axes[1,1].legend()

# Summary text
axes[1,2].axis('off')
```

```python
summary_text = f"""
SLX IMPLEMENTATION

Key Fix Applied:
  Maxima detection excludes zeros
  Only precipitation > tolerance are maxima
  Minima includes zeros (as per paper)

Results from corrected implementation:
• Found {r['n_obs_max']} observed maxima
• Found {r['n_fc_max']} forecast maxima
• SLX now INCREASES with neighbourhood size
• Peak SLX at L={L_values[np.argmax(slx_total)]} ({max(slx_total):.3f})

This matches expected SLX behavior:
Larger neighbourhoods → more tolerance
to displacement → higher scores
"""
axes[1,2].text(0.05, 0.95, summary_text, transform=axes[1,2].transAxes,
               fontsize=10, verticalalignment='top', fontfamily='monospace',
               bbox=dict(boxstyle="round,pad=0.5", facecolor='lightgreen', alpha=0.8))

plt.tight_layout()
plt.show()
```



Score Function Properties

- **Perfect match** → S = 1
- **Severe over-forecast** (>5×) → S = 0
- **Asymmetric**: Designed to avoid under-forecasting of warning conditions
- **Piecewise linear**: Simple but effective for operational use
- **Uncertainty aware**: k parameter accounts for observation uncertainty

Advantages of SLX

- **Extreme-focused**: Specifically designed for precipitation extremes
- **Neighborhood-based**: Addresses double penalty problem
- **Comprehensive**: Evaluates both maxima and minima
- **Scale-aware**: Tests multiple neighborhood sizes
- **Operationally practical**: Fast computation, daily output

- **Complements existing methods** like FSS (Fractions Skill Score) and SAL (Structure-Amplitude-Location)

Limitations of SLX

- **Parameter sensitivity**: Results depend on k, A, , and L choices
- **Score function complexity**: Piecewise linear function may need refinement
- **Extreme definition**: Tolerance parameter affects extreme selection
- **Computational scaling**: May need optimization for very large domains

---

References & Resources

Primary Reference: Sass, B.H. (2021). A scheme for verifying the spatial structure of extremes in numerical weather prediction: exemplified for precipitation. *Meteorological Applications*, 28, e2015.

Related Methods: - Roberts & Lean (2008): Fractions Skill Score (FSS) - Wernli et al. (2008): SAL verification - Gilleland et al. (2010): Spatial verification overview

Code Repository: - This corrected presentation is available in this repository