Information in Sequential Evaluations: the Good and the Bad

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updated paper online soon!

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Motivation

- Many risky opportunities are offered to many before someone takes it:
  - The seller of a financial asset can ask many interested buyers
  - Debt seekers have many banks to apply to for credit
  - Entrepreneurs contact many VCs or angel investors for (seed) funding
- Whoever gets an offer must *evaluate* whether opportunity is good or bad

So: more information $\longrightarrow$ better evaluations $\longrightarrow$ better off evaluators?

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  So: more information $\longrightarrow$ better evaluations $\longrightarrow$ better off evaluators?

- But: evaluators face an **adverse selection** problem:

  Am I getting opportunity after everyone rejected it? Does this mean it is bad?

- Answer depends on what other evaluators know & do: **info. shapes adverse selection**

  Maybe: more information $\longrightarrow$ more adverse selection $\xrightarrow{??}$ worse off evaluators?

Motivation

When does more info. leave evaluators better off?

Tied to EVs aggregate selection quality. Important policy question: credit scoring.

- Regulators unsure and confused if allowing more info. $\rightarrow$ better risk evaluations

Basel II: allowed Internal Ratings Based systems instead of standardised scoring to

"reward stronger and more accurate risk measurement"

"provide a more risk-sensitive approach to measuring credit risk"

"Regulation Guide: An Introduction", *Moody's Analytics*

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Basel III banned it back:

"CVA is a complex risk ... cannot be modelled by banks in a robust and prudent manner. The revised framework removes the use of an internally modelled approach ..."

"High-Level Summary of Basel III Reforms", *Bank of Intl. Settlements*

Motivation

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Banks highlighted seeming paradox: wanting better risk evaluation $\times$ curbing use of info.

"[Internal assessments] by banks allow for the most accurate measurement of risk ... Relying on regulators' assessments ... will be rough approximations at best ... a major step backwards"

Kenneth Bentsen, CEO of Global Financial Markets Association

Contribution today: indeed sometimes: more info. $\rightarrow$ worse risk evaluations

I will characterise exactly when

The Model

- APPLICANT with quality $\theta \in \{L, H\}$ seeks approval from **one evaluator**.
  - Everyone has prior belief $\rho \in (0,1)$ that he is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORs, at random order τ.
  - τ is a permutation of $\{1, 2, ..., n\}$, chosen **privately** and **uniformly at random** by applicant.
- EV **approves** $\longrightarrow$ payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0,1)$. Game ends, other EV.s get 0 payoff.

  EV **rejects** $\longrightarrow$ 0 payoff. Applicant keeps applying. If no EV left $\rightarrow$ game ends.

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    - τ is private and $\mathbb{P}(\tau(i) = j) = \frac{1}{n}$ for all $i, j \in \{1, 2, ..., n\}$.
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The Model

- EVs do not know θ or τ. But: receive **private IID signals** x about quality θ:

$$\mathcal{X} \mid \theta \overset{\text{IID}}{\sim} p_\theta \qquad x \in \{s_1, s_2, ..., s_m\} \subset [0,1] \qquad s_i = \frac{p_H(s_i)}{p_H(s_i) + p_L(s_i)} < s_{i+1}$$

- **Equilibrium:** symmetric strategy and interim belief profile (σ^*, ψ^*) for EVs such that:
  - $\psi^* = \mathbb{P}(\theta = H \mid \text{applicant visited me})$ – **consistent** with strategy profile σ^*

$$\psi^* = \frac{\rho \times \mathbb{P}(\text{app. visits me before smo. else approves} \mid \theta = H, \sigma^*)}{\mathbb{P}(\text{app. visits me before smo. else approves} \mid \sigma^*)}$$

  - $\sigma^* : \{s_1, s_2, ..., s_m\} \to [0,1]$ – **optimal** given ψ^*:

$$\text{approve when } \mathbb{P}(\theta = H \mid x, \psi) > c$$

The Model

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$$\mathcal{X} \mid \theta \overset{\text{IID}}{\sim} p_\theta \qquad x \in \{s_1, s_2, ..., s_m\} \subset [0,1] \qquad \frac{\psi^*}{1-\psi^*} \times \frac{s_i}{1-s_i} = \frac{\text{posterior}}{1-\text{posterior}}$$

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Main Question and Takeaway

How does more information affect evaluators' equilibrium payoffs?

- Synonymous with selection quality: eqm. payoffs $\uparrow$ when false positives & negatives $\downarrow$.
- **Main difficulty:** More info. affects extent of **adverse selection** $\rightarrow$ can clawback on payoffs – selection quality.
- **Main result:** Characterise effect of arbitrary Blackwell improvements of EVs signals.
- **Main takeaway:** Effect depends on the **kind** of improvement. Roughly:
  - improving **favourable** evaluations: **good!**
  - improving **unfavourable** evaluations: eventually **bad**

affect different applicants

$\downarrow$

have different payoff effects

Literature

Main Question and Takeaway

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- Giving EVs information about **order** τ
- EV makes *take-it-or-leave-it* price offer to applicant
- EVs compete on **application costs**

ask me after talk!

Literature

Equilibria: Selectivity and Payoffs

- $\sigma^* \rightleftarrows \psi^*$ means possibly multiple eqa. But set of eqa. still well behaved.

Proposition

The set of equilibrium strategies is non-empty and compact. Furthermore:

1. all eqm. strategies are monotone; $\sigma^*(s_i) > 0 \implies \sigma^*(s_{i+1}) = 1$. [when $p_H \neq p_L$]
2. eqm. strategies are pointwise totally ordered;

$$\text{either } \sigma^{**}(s_i) \geq \sigma^*(s_i) \text{ or } \sigma^{**}(s_i) \leq \sigma^*(s_i) \text{ for all } s_i \in S$$

3. all eqa. exhibit adverse selection : $\psi^* \leq \rho$.

- Compact and totally ordered $\to$ the lowest (most selective) and highest (least selective) eqm. strategies.

Equilibria: Selectivity and Payoffs

Selective eqa. reduce approval chances for applicant. What do they mean for EV payoffs?

- (the sum of) Evaluators' equilibrium payoffs:

$$\Pi(\sigma) := (1-c) \times \rho \times \mathbb{P}\left(\text{some EV approves} \mid \theta = H, \sigma\right) + (-c) \times (1-\rho) \times \left[1 - \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)\right]$$

- Different virtues: more selective $\rightarrow$ filter *Low* quality approvals
  less selective $\rightarrow$ secure *High* quality approvals

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- Different virtues: more selective $\rightarrow$ filter *Low* quality approvals
  less selective $\rightarrow$ secure *High* quality approvals
- This trade-off is always resolved in favour of more selective eqa.:

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* eqm. strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

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Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* ~~eqm.~~ **monotone** strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

proof

Information and Equilibrium Payoffs

How does more information change payoffs in more/less selective eqa?

$$\Pi(\sigma) := (1-c) \times \rho \times \mathbb{P}\left(\text{some EV approves} \mid \theta = H, \sigma\right) + (-c) \times (1-\rho) \times \left[1 - \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)\right]$$

- With just one EV, classic Blackwell, 1953 result:

$$\text{Blackwell more informative signal} \overset{(\Longleftarrow)}{\Longrightarrow} \uparrow \text{payoffs for all } c, \rho$$

- Reason: more info. allows higher probabilities of: $[\text{approve} \mid \theta = H]$ & $[\text{reject} \mid \theta = L]$

Information and Equilibrium Payoffs

- What goes wrong with many EVs? Rewrite EV i's payoff $\pi_i \left(= \frac{\Pi}{n}\right)$:

$$\pi_i(\sigma) = \mathbb{P}\left(\text{applicant visits } i\right)$$
$$\times \left[\psi \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) - (1-\psi) \times c \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]$$

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Information and Equilibrium Payoffs

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- As before, EV i's payoffs depend on probabilities of $[\text{approve} \mid \theta = H]$ & $[\text{reject} \mid \theta = L]$
- However, also on the red terms shaped by adverse selection.
- Red terms are shaped by others' strategies.
- Other EVs do not internalise the adverse selection they impose onto EV i.
- Maximising individual selection quality $\neq$ maximising **overall** selection quality

Binary Signals: Information and Eqm. Payoffs

- Start from **binary signals**: main intuition and building block to full characterisation
- $x \in S = \{\overset{\cancel{s_1}}{s_L}, \overset{\cancel{s_2}}{s_H}\}$ **more informative** when $s_L \downarrow$: stronger evidence for $\theta = L$
  and $s_H \uparrow$: stronger evidence for $\theta = H$

Theorem (1)

With a binary signal $x \in \{s_L, s_H\}$, EVs payoffs in the most selective eqm. are weakly:

- increasing with stronger evidence for $\theta = H$; $\uparrow s_H$
- hump shaped in stronger evidence for $\theta = L$; $\downarrow s_L$:
  - increasing when s_L is above a threshold
  - decreasing when s_L is below that threshold

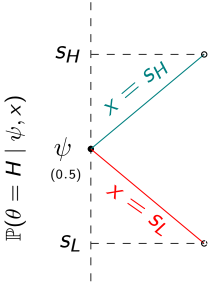
- For least selective eqm: same result, different threshold.

details: threshold

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

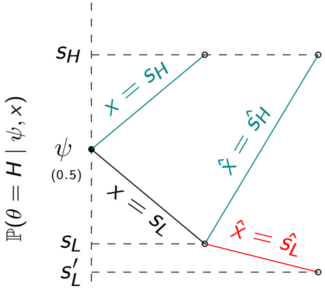
- How to think about $\downarrow s_L$ marginally?



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

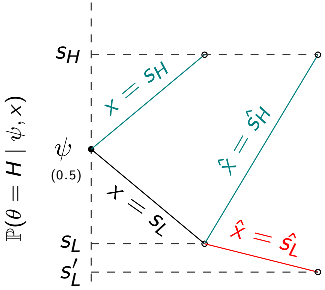
- How to think about $\downarrow s_L$ marginally? Construct an auxiliary signal $\hat{\mathcal{X}}$.



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- How to think about $\downarrow s_L$ marginally? Construct an auxiliary signal $\hat{\mathcal{X}}$.
- Fix EV strat.s: high $\rightarrow$ approve & low $\rightarrow$ reject. Which applicant gets affected?



- Fix signal pairs $(x, \hat{x})$ all EVs would observe for applicant.
- **Marginal admits**: rejected by all $(x = s_L)$ in old signal structure, approved by some $(\hat{x} = \hat{s_H})$ in new.
- Prob. marginal admit has $\theta = H$ depends on how many $\hat{x} = \hat{s_H}$ signals EVs would see.

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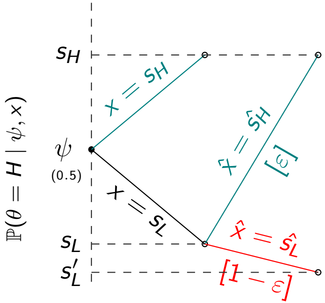

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- Prob. marginal admit has $\theta = H$ depends on how many $\hat{x} = \hat{s_H}$ signals EVs would see. **Answer: only one!**
- For **marginal** decrease in s_L, $\mathbb{P}(\hat{x} = \hat{s_H}) \propto \varepsilon \rightarrow 0$.
- Multiple $\hat{x} = \hat{s_H}$ has negligible probability.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

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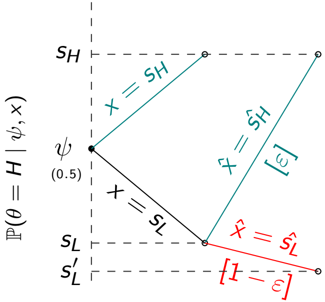
- Fix EV strat.s: high $\to$ approve & low $\to$ reject. Which applicant gets affected?



- Whether the marginal admit is profitable:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} \lessgtr \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- Whether s_H justifies approving the most ad. selected applicant – **adverse selection poses any threat**.
- $s_L \downarrow \implies$ the $n-1$ low signals $>>$ the single high signal

details: threshold

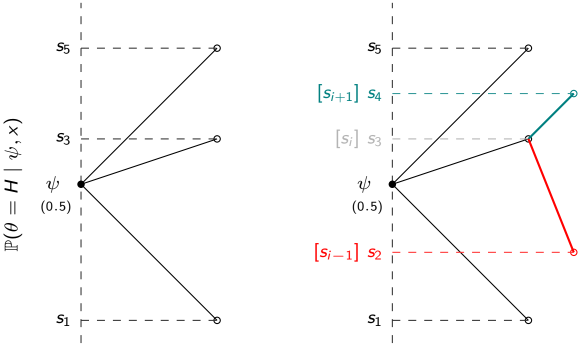
proof: $s_H \uparrow$

General Blackwell Improvements and Eqm. Payoffs

- Generalise the idea of replicating Blackwell improvement with auxiliary signal.

Take two signal str.s, $\mathcal{X} \mid \theta \overset{IID}{\sim} p_\theta$ and $\mathcal{X}' \mid \theta \overset{IID}{\sim} p'_\theta$. Joint supp $S \cup S' = \{s_1, s_2, \ldots, s_M\}$.

$\mathcal{X}'$ differs from $\mathcal{X}$ by a local MPS at s_i if:



- mean preserving spread
- **Local:** mass is spread only to neighboring points

General Blackwell Improvements and Eqm. Payoffs

Local mean preserving spreads characterise Blackwell improvements

$$\mathcal{X} + [\text{finitely many local MPS}] \;=\; \mathcal{X}' \qquad \Longleftrightarrow \qquad \mathcal{X}' \text{ is Blackwell more informative than } \mathcal{X}$$

- Slight refinement of Rothschild and Stiglitz, 1970 for **discrete signals**.
- Theorem 2 will characterise how payoffs move after a local MPS.
- **Local** allows to control how eqm. evolves with more info.

General Blackwell Improvements and Eqm. Payoffs

Theorem (2)

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. EVs' payoffs in the most selective eqm. are:

1. *weakly higher* under $\mathcal{X}'$ if $x = s_i$ leads to approvals under σ; $\sigma(s_i) = 1$,
2. *weakly lower* under $\mathcal{X}'$, if:
   1. $x = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and
   2. adverse selection poses a threat at signal s_{i+1}

details on this condition

General Blackwell Improvements and Eqm. Payoffs

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   2. adverse selection poses a threat at signal s_{i+1}

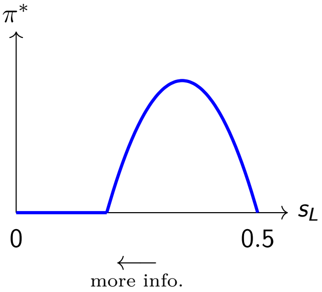
details on this condition

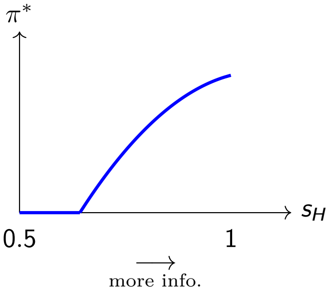
- Adverse selection likelier to pose a threat when s_{i+1} is lower
- Theorem requires knowing equilibrium structure.
  - All eqa. can be located in $\leq 2(m+2)$ steps
- Also: offer stronger sufficient condition that relies only on the local MPS performed

Thank You!

and please check back soon for (substantially) updated paper!

especially if you are hiring! :)





Related Literature

Main Question & Takeaway

Equilibria: Selectivity and Payoffs

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* ~~eqm.~~ monotone strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

Proof Sketch:

- Take eqm. strategy σ^*, and consider *marginally more embracive* σ^ε: $||\sigma^\varepsilon - \sigma^*|| = \varepsilon$
- Fix the signals all EVs would see if app. visited them all: $\{x_1, x_2, ..., x_n\}$.
- Only app. whose outcome changes: rejected by all under σ^*, approved by some under σ^ε.
- If ε is small, he was a.s. rejected by all under σ^*, approved by one under σ^ε.
- Bad news: approving is suboptimal for this last EV
- Last step: payoffs are **single crossing** in embraciveness; where $\sigma'' > \sigma' > \sigma$:

$$\Pi(\sigma') \leq \Pi(\sigma) \implies \Pi(\sigma'') \leq \Pi(\sigma')$$

back to result

When is $\downarrow s_L$ Harmful?

- A candidate for the threshold came from sketch proof:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\mathrm{as}}}{1-s_L^{\mathrm{as}}}\right)^{n-1}}_{n-1\text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- $\mathbf{s_L^{\mathrm{as}}}$: strongest evidence for $\theta = L$ where *adverse selection poses no threat*.

  EV happy to approve upon $x = s_H$ **even if she learned all prev. EVs observed** $x = s_L$

- Sketch proof showed: marginal admit hurts when $s_L < s_L^{\mathrm{as}}$.

back to Theorem 1

When is $\downarrow s_L$ Harmful?

- But until s_L low enough, EVs might be stuck in an eqm. approving all.

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}}\right)}_{\text{one low signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- When $s_L \geq s_L^{\text{mute}}$, *always* an eqm: approve all $\rightarrow$ no adverse selection $\rightarrow$ approve all
- We need to decrease s_L enough to eliminate these eqa.

Proposition

Let EVs have a binary signal $x \in \{s_L, s_H\}$. EVs payoffs decrease as $s_L \downarrow$ when:

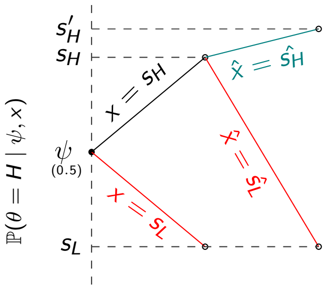
- $s_L \leq \min\{s_L^{\text{mute}}, s_L^{\text{as}}\}$ in the least selective eqm.
- $s_L \leq s_L^{\dagger}$ in the most selective eqm., where $s_L^{\text{as}} \geq s_L^{\dagger} \geq \min\{s_L^{\text{mute}}, s_L^{\text{as}}\}$.

back to Theorem 1

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- How to think about $\uparrow s_H$ marginally? Construct an auxiliary signal $\hat{\mathcal{X}}$.
- Fix EV strat.s: high $\to$ approve & low $\to$ reject. Which applicant gets affected?



- This time **marginal reject**: approved by some $(x = s_H)$ before, rejected by all $(\hat{x} = \hat{s}_L)$ now.
- All EVs must have seen low signals.
- **Marginal reject** is always good to push out:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^n}_{n \text{ low signals}} < \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

[actual pf: "strong low signals" is the only relevant case]

back to proof: s_L ↓

Adverse Selection Condition for Theorem 2

For a fixed signal structure $\mathcal{X}$ and strategy σ, *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- Sufficient condition for the most selective eqm. relies only on the local MPS performed:

Proposition

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. EVs expected payoffs in the most selective eqm. are lower under $\mathcal{X}'$ whenever s_i is a rejection signal under $\mathcal{X}$ (a fortiori: $s_L < s_L^{\text{mute}}$) and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

back to Theorem 2