Information in Sequential Evaluations: the Good and the Bad

D. Carlos Akkar
Nuffield College, University of Oxford

ES North American Meetings, Nashville
June 2024

Motivation

- Many risky opportunities are offered *sequentially* to *multiple* parties *until one takes it*:
  - The seller of an asset & fin. derivative can contact many interested buyers
  - Debt seekers have many banks to apply to for credit
  - Entrepreneurs contact many VCs or angel investors for (seed) funding
- Whoever gets an offer must *evaluate* whether opportunity is profitable or not

  So: more information $\longrightarrow$ better judgements of opportunity $\longrightarrow$ better off evaluators?

Motivation

- Many risky opportunities are offered *sequentially* to *multiple* parties *until one takes it*:
  - The seller of an asset & fin. derivative can contact many interested buyers
  - Debt seekers have many banks to apply to for credit
  - Entrepreneurs contact many VCs or angel investors for (seed) funding
- Whoever gets an offer must *evaluate* whether opportunity is profitable or not

  So: more information $\longrightarrow$ better judgements of opportunity $\longrightarrow$ better off evaluators?
- But: evaluators have an **adverse selection** problem:

  Did I receive an applicant with many rejections?
- **Information shapes adverse selection:**

  Maybe: more information $\longrightarrow$ more adverse selection $\xrightarrow{??}$ worse off evaluators?

Motivation

When does more info. leave evaluators better off? (read: improve selection quality)

Important policy question, e.g. credit scoring.

- Each bank wants better scoring algorithm: competitiveness, quality lending...
- Regulators want them to have better information, too!

Basel II: allowed Internal Ratings Based systems instead of standardised scoring to

"provide a more risk-sensitive approach to measuring credit risk"

"reward stronger and more accurate risk measurement"

"Regulation Guide: An Introduction", *Moody's Analytics*

Motivation

When does more info. leave evaluators better off? (read: improve selection quality)

Important policy question, e.g. credit scoring.

- Each bank wants better scoring algorithm: competitiveness, quality lending...
- Regulators want them to have better information, too!

Basel III banned it back after the 2008 crisis:

"CVA is a complex risk ... cannot be modelled by banks in a robust and prudent manner. The revised framework removes the use of an internally modelled approach ..."

"High-Level Summary of Basel III Reforms", *Bank of Intl. Settlements*

Motivation

When does more info. leave evaluators better off? (read: improve selection quality)

Important policy question, e.g. credit scoring.

- Each bank wants better scoring algorithm: competitiveness, quality lending...
- Regulators want them to have better information, too! But sometimes not!

Banks disagree with Basel III... **for the same reason**:

"[Internal assessments] by banks allow for the most accurate measurement of risk ... Relying on regulators' assessments ... will be rough approximations at best ... a major step backwards"

Kenneth Bentsen, CEO of Global Financial Markets Association

The Model

- APPLICANT with unknown quality $\theta \in \{L, H\}$ **seeks approval** from **one evaluator**.
  - Prior belief $\rho \in (0, 1)$ that applicant is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORs, at random order τ.
  - τ is a permutation of $\{1, 2, ..., n\}$, chosen **privately** and **uniformly at random** by applicant.
- EV **approves** $\longrightarrow$ payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0, 1)$. Game ends, other EV.s get 0 payoff.

  EV **rejects** $\longrightarrow$ 0 payoff. Applicant keeps applying. If no EV left $\rightarrow$ game ends.

The Model

- APPLICANT with unknown quality $\theta \in \{L, H\}$ **seeks approval** from **one evaluator**.
  - Prior belief $\rho \in (0,1)$ that applicant is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORs, at random order τ.
  - τ is private and $\mathbb{P}(\tau(i) = j) = \frac{1}{n}$ for all $i, j \in \{1, 2, ..., n\}$.
- EV **approves** $\longrightarrow$ payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0,1)$. Game ends, other EV.s get 0 payoff.

  EV **rejects** $\longrightarrow$ 0 payoff. Applicant keeps applying. If no EV left $\rightarrow$ game ends.

The Model

- APPLICANT with unknown quality $\theta \in \{L, H\}$ **seeks approval** from **one evaluator**.
  - Prior belief $\rho \in (0,1)$ that applicant is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORS, at random order τ.
  - τ is private and $\mathbb{P}(\tau(i) = j) = \frac{1}{n}$ for all $i, j \in \{1, 2, ..., n\}$.
- EV **approves** $\longrightarrow$ payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0,1)$. Game ends, other EV.s get 0 payoff.
  EV **rejects** $\longrightarrow$ 0 payoff. Applicant keeps applying. If no EV left $\rightarrow$ game ends.
- EVs **do not know** θ or τ. But: receive **private IID signals** x about **quality** θ:

$$\mathcal{X} \mid \theta \overset{\text{IID}}{\sim} p_\theta \qquad x \in \{s_1, s_2, ..., s_m\} \subset [0,1] \qquad s_i = \frac{p_H(s_i)}{p_H(s_i) + p_L(s_i)} < s_{i+1}$$

- **Equilibrium:** symmetric strategy and interim belief profile (σ^*, ψ^*) for EVs such that:
  - $\psi^* = \mathbb{P}(\theta = H \mid \text{applicant visited me})$ – **consistent** with strategy profile σ^*
  - $\sigma^* : \{s_1, s_2, ..., s_m\} \rightarrow [0,1]$ – **optimal** given ψ^*; approve when $\mathbb{P}(\theta = H \mid x, \psi) > c$

Main Question and Takeaway

How does more information affect evaluators' equilibrium payoffs?

- Synonymous with **selection quality**: eqm. payoffs $\uparrow$ when false positives & negatives $\downarrow$.
- **Main problem:** More information affects adverse selection; i.e. the interim belief ψ^*.

  This indirect effect might backfire on payoffs – selection quality.
- **Main result:** Characterise effect of arbitrary **Blackwell improvements** of EVs signals.
- **Main takeaway:** Effect on payoffs depends on the **kind** of improvement. Roughly:
  - higher **approval confidence**: **good!**
  - higher **rejection confidence**: eventually **bad**

affect different applicants

$\downarrow$

have different payoff effects

Literature

Main Question and Takeaway

How does more information affect evaluators' equilibrium payoffs?

- Synonymous with **selection quality**: eqm. payoffs $\uparrow$ when false positives & negatives $\downarrow$.
- **Main problem:** More information affects adverse selection; i.e. the interim belief ψ^*.
  This indirect effect might backfire on payoffs – selection quality.
- **Main result:** Characterise effect of arbitrary **Blackwell improvements** of EVs signals.
- **Main takeaway:** Effect on payoffs depends on the **kind** of improvement. Roughly:
  - higher **approval confidence**: **good!**
  - higher **rejection confidence**: eventually **bad**

affect different applicants

$\downarrow$

have different payoff effects

- Giving EVs information about **order** τ
- EV makes *take-it-or-leave-it* price offer to applicant
- EVs compete on **application costs**

ask me after talk!

Literature

Interim Beliefs and Equilibria

- Prior belief ρ that applicant is born with $\theta = H$ is a primitive.
- Interim belief $\psi = \mathbb{P}(\theta = H \mid \text{applicant visited me})$ is **endogenous**:
  - What is the chance applicant visited me after k rejections?
  - What do those rejections mean about his quality?
- ψ is *consistent* with the strategy σ for all evaluators, iff:

$$\psi = \frac{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k}{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k + (1-\rho) \times \sum_{k=1}^{n} r_L(\sigma)^k} \qquad \text{where} \quad r_\theta(\sigma) = 1 - \sum_{i=1}^{m} p_\theta(s_i)\sigma(s_i)$$

Interim Beliefs and Equilibria

- Prior belief ρ that applicant is born with $\theta = H$ is a primitive.
- Interim belief $\psi = \mathbb{P}\left(\theta = H \mid \text{applicant visited me}\right)$ is **endogenous**:
  - What is the chance applicant visited me after k rejections?
  - What do those rejections mean about his quality?
- ψ is *consistent* with the strategy σ for all evaluators, iff:

$$\psi = \frac{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k}{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k + (1-\rho) \times \sum_{k=1}^{n} r_L(\sigma)^k} \quad \text{where} \quad \underbrace{r_\theta(\sigma) = 1 - \sum_{i=1}^{m} p_\theta(s_i)\sigma(s_i)}_{\text{rejection prob. given quality}}$$

Interim Beliefs and Equilibria

- Prior belief ρ that applicant is born with $\theta = H$ is a primitive.
- Interim belief $\psi = \mathbb{P}(\theta = H \mid \text{applicant visited me})$ is **endogenous**:
  - What is the chance applicant visited me after k rejections?
  - What do those rejections mean about his quality?
- ψ is *consistent* with the strategy σ for all evaluators, iff:

$$\psi = \frac{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k}{\underbrace{\rho \times \sum_{k=1}^{n} r_H(\sigma)^k}_{\text{prob. High quality visit}} + \underbrace{(1-\rho) \times \sum_{k=1}^{n} r_L(\sigma)^k}_{\text{prob. Low quality visit}}} \quad \text{where} \quad \underbrace{r_\theta(\sigma) = 1 - \sum_{i=1}^{m} p_\theta(s_i)\sigma(s_i)}_{\text{rejection prob. given quality}}$$

Interim Beliefs and Equilibria

- Endogeneity means existence, number, or properties of equilibria are not automatic.

Proposition

Let $p_H \neq p_L$. The set of equilibrium strategies is non-empty and compact, and pointwise totally ordered. Furthermore:

1. all equilibrium strategies are monotone; $\sigma^*(s_i) > 0$ implies $\sigma^*(s_{i+1}) = 1$.
2. all equilibria exhibit adverse selection : $\psi^* \leq \rho$.

- Compact and totally ordered $\rightarrow$ we can talk about:
  - the *highest* (most embracive) equilibrium,
  - the *lowest* (most selective) equilibrium.

Interim Beliefs and Equilibria

- What do *selective* and *embracive* equilibria mean for payoffs?
- (the sum of) Evaluators' equilibrium payoffs:

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- Different virtues: **selective** $\rightarrow$ filter *Low* quality approvals but miss out *High* quality
  embracive $\rightarrow$ secure *High* quality but overlook *Low* quality approvals

Interim Beliefs and Equilibria

- What do *selective* and *embracive* equilibria mean for payoffs?
- (the sum of) Evaluators' equilibrium payoffs:

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- Different virtues: **selective** $\rightarrow$ filter *Low* quality approvals but miss out *High* quality
  embracive $\rightarrow$ secure *High* quality but overlook *Low* quality approvals
- Nonetheless, trade-off is always resolved in favour of selective equilibria:

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* eqm. strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

Interim Beliefs and Equilibria

- What do *selective* and *embracive* equilibria mean for payoffs?
- (the sum of) Evaluators' equilibrium payoffs:

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- Different virtues: **selective** $\rightarrow$ filter *Low* quality approvals but miss out *High* quality
  embracive $\rightarrow$ secure *High* quality but overlook *Low* quality approvals
- Nonetheless, trade-off is always resolved in favour of selective equilibria:

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* ~~eqm.~~ monotone strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

Interim Beliefs and Equilibria

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* ~~eqm.~~ monotone strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

Proof Sketch:

- Take eqm. strategy σ^*, and consider *marginally more embracive* σ^ε: $||\sigma^\varepsilon - \sigma^*|| = \varepsilon$
- Fix the signals all EVs would see if app. visited them all: $\{x_1, x_2, ..., x_n\}$.
- Only app. whose outcome changes: rejected by all under σ^*, approved by some under σ^ε.
- If ε is small, he was a.s. rejected by all under σ^*, approved by one under σ^ε.
- Bad news: approving is suboptimal for this last EV
- Last step: payoffs are **single crossing** in embraciveness; where $\sigma'' > \sigma' > \sigma$:

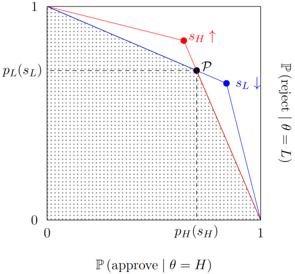
$$\Pi(\sigma') \leq \Pi(\sigma) \implies \Pi(\sigma'') \leq \Pi(\sigma')$$

Information and Equilibrium Payoffs

How do payoffs in these equilibria change with more information?

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- With *one* evaluator, **Blackwell more informative** signal $\overset{(\Longleftarrow)}{\Longrightarrow}$ $\uparrow$ payoffs for all c, ρ
- Reason: affords lower false positives & negatives



Example: binary signal.

$$S = \{s_1, s_2\}$$

A Blackwell imp. is:

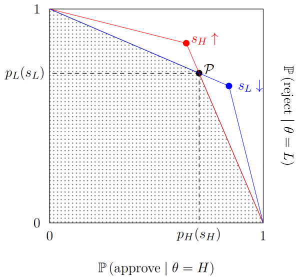
$$\downarrow s_1 \qquad \uparrow s_2$$

Information and Equilibrium Payoffs

How do payoffs in these equilibria change with more information?

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- With *one* evaluator, **Blackwell more informative** signal $\overset{(\Longleftarrow)}{\Longrightarrow}$ $\uparrow$ payoffs for all c, ρ
- Reason: affords lower false positives & negatives



Example: binary signal.

$$S = \{s_L, s_H\}$$

A Blackwell imp. is:

$$\downarrow s_L \qquad \uparrow s_H$$

Information and Equilibrium Payoffs

- What could go wrong with many EVs? Rewrite EV i's payoff $\pi_i \left(= \frac{\Pi}{n}\right)$:

$$\pi_i(\sigma) = \mathbb{P}\,(\text{applicant visits } i)$$
$$\times \left[\psi \times (1-c) \times \mathbb{P}\,(i \text{ approves} \mid \theta = H) - (1-\psi) \times c \times \mathbb{P}\,(i \text{ approves} \mid \theta = L)\right]$$

- The blue terms are shaped by adverse selection: depend on *others'* strategies
- An EV does not internalise the adverse selection she imposes onto others
- Maximising individual selection quality $\neq$ maximising **overall** selection quality

Binary Signals: Information and Eqm. Payoffs

- Start from **binary signals**: main intuition and building block to full characterisation
- $x \in S = \{s_L, s_H\}$ **more informative** when $s_L \downarrow$: **stronger evidence for *Low* quality**

  and $s_H \uparrow$: **stronger evidence for *High* quality**

Theorem

Let EVs have a binary signal, $x \in S = \{s_L, s_H\}$. Their equilibrium payoffs in the most selective (embracive) equilibria are weakly:

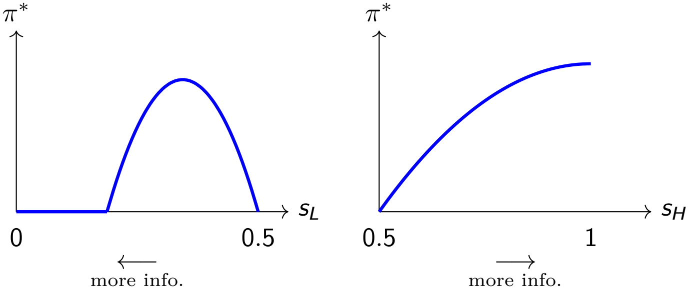
- increasing with stronger evidence for $\theta = H$ ($\uparrow s_H$),
- increasing with stronger evidence for $\theta = L$ ($\downarrow s_L$) when s_L is above a threshold,
- decreasing with stronger evidence for $\theta = L$ ($\downarrow s_L$) when s_L is below that threshold.

Binary Signals: Information and Eqm. Payoffs

- Start from **binary signals**: main intuition and building block to full characterisation
- $x \in S = \{s_L, s_H\}$ **more informative** when $s_L \downarrow$: **stronger evidence for *Low* quality**

  and $s_H \uparrow$: **stronger evidence for *High* quality**

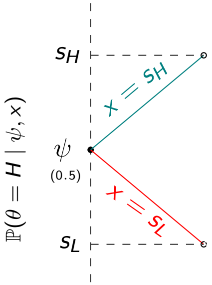
Theorem:



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

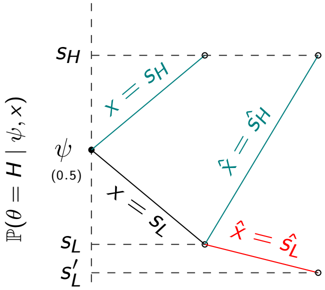
- Consider decreasing s_L marginally. How to implement this Blackwell improvement?
- Fix strategies [handle in actual proof]: approve with high signal & reject with low signal



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Consider decreasing s_L marginally. How to implement this Blackwell improvement?
- Fix strategies [handle in actual proof]: approve with high signal & reject with low signal
- Construct an auxiliary signal $\hat{\mathcal{X}}$.

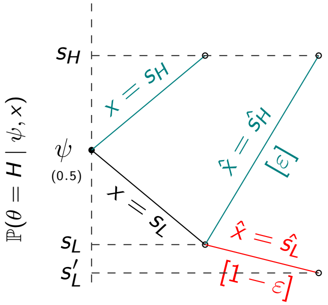


- Like before, fix signal pairs $(x, \hat{x})$ all EVs would see.
- We created **marginal admits**: rejected by all in old signal structure, approved by some in new.
- All info. on marginal admits' quality $\longrightarrow$ number of $\hat{x} = \hat{s_H}$ signals.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Consider decreasing s_L marginally. How to implement this Blackwell improvement?
- Fix strategies [handle in actual proof]: approve with high signal & reject with low signal
- Construct an auxiliary signal $\hat{\mathcal{X}}$.

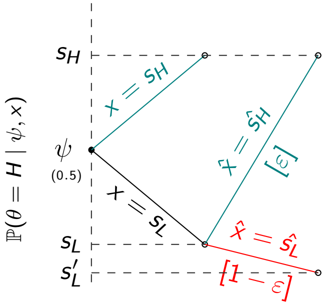


- Like before, fix signal pairs $(x, \hat{x})$ all EVs would see.
- We created **marginal admits**: rejected by all in old signal structure, approved by some in new.
- All info. on marginal admits' quality $\longrightarrow$ number of $\hat{x} = \hat{s_H}$ signals. **Answer: only one!**
- For **marginal** decrease in s_L, $\mathbb{P}\left(\hat{x} = \hat{s_H}\right) \propto \varepsilon \to 0$.
- **Multiple** $\hat{x} = \hat{s_H}$ has negligible probability.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Consider decreasing s_L marginally. How to implement this Blackwell improvement?
- Fix strategies [handle in actual proof]: approve with high signal & reject with low signal
- Construct an auxiliary signal $\hat{\mathcal{X}}$.



- Whether the **marginal admit** is profitable:

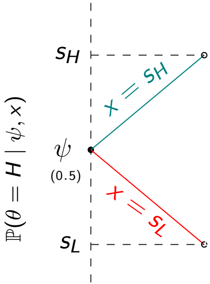
$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} \lessgtr \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- **Lesson:** Whenever there is a **threat** of adverse selection, stronger evidence for $\theta = L$ hurts.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

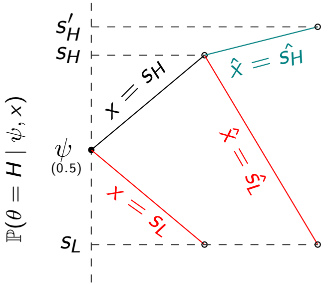
- Now consider increasing s_H marginally. How to implement this Blackwell improvement?
- Fix strategies as before: approve with high signal & reject with low signal



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Now consider increasing s_H marginally. How to implement this Blackwell improvement?
- Fix strategies as before: approve with high signal & reject with low signal
- Construct an auxiliary signal $\hat{x}$.



- This time **marginal rejects**: approved by some before, rejected by all now.
- All EVs must have seen low signals.
- **Marginal reject** is always good to push out:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^n}_{n \text{ low signals}} < \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

[actual pf: "strong low signals" is the only relevant case]

Binary Signals: Information and Eqm. Payoffs

- Main lesson:
  - stronger evidence for $\theta = H \longrightarrow$ marginal rejects
  - stronger evidence for $\theta = L \longrightarrow$ marginal admits

Binary Signals: Information and Eqm. Payoffs

- Main lesson:
  - stronger evidence for $\theta = H \longrightarrow$ marginal rejects benefits EVs
  - stronger evidence for $\theta = L \longrightarrow$ marginal admits

Binary Signals: Information and Eqm. Payoffs

- Main lesson:
  - stronger evidence for $\theta = H \longrightarrow$ marginal rejects benefits EVs
  - stronger evidence for $\theta = L \longrightarrow$ marginal admits hurts EVs **when $\mathbf{s_L} <$ some threshold**

Binary Signals: Information and Eqm. Payoffs

- Main lesson:
  - stronger evidence for $\theta = H \longrightarrow$ marginal rejects benefits EVs
  - stronger evidence for $\theta = L \longrightarrow$ marginal admits hurts EVs **when $\mathbf{s_L} <$ some threshold**
- A candidate for the threshold came from sketch proof:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1}}_{n-1\text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- $\mathbf{s_L^{\text{as}}}$: strongest evidence for $\theta = L$ where **adverse selection poses no threat**.
- Sketch proof showed: **marginal admit hurts** when $s_L < s_L^{\text{as}}$.

Binary Signals: Information and Eqm. Payoffs

- The actual threshold depends more subtly on equilibrium dynamics.
- EVs might be stuck in eqa. where all applicants are approved when info. is too weak:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}}\right)}_{\text{one low signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

when $s_L \geq s_L^{\text{mute}}$, *always* an eqm: approve all $\rightarrow$ no adverse selection $\rightarrow$ approve all

Binary Signals: Information and Eqm. Payoffs

- The actual threshold depends more subtly on equilibrium dynamics.
- EVs might be stuck in eqa. where all applicants are approved when info. is too weak:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}}\right)}_{\text{one low signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

when $s_L \geq s_L^{\text{mute}}$, *always* an eqm: approve all $\rightarrow$ no adverse selection $\rightarrow$ approve all

Proposition

Let EVs have a binary signal $x \in \{s_L, s_H\}$. The threshold below which lower s_L weakly decreases their equilibrium payoffs is:

- $s_L^{\text{as}} \geq \min\{s_L^{\text{mute}}, s_L^{\text{as}}\}$ for the most embracive equilibrium.
- $s_L^{\text{as}} \geq s_L^{\dagger} \geq \min\{s_L^{\text{mute}}, s_L^{\text{as}}\}$ for the most selective equilibrium.

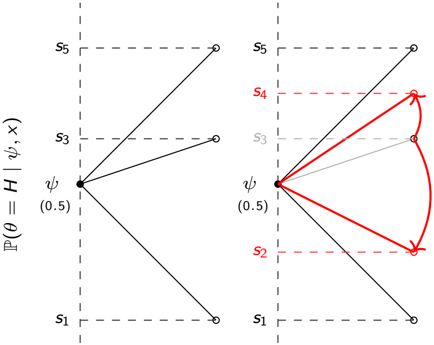
General Blackwell Improvements and Eqm. Payoffs

- In many settings, EVs of risky opportunities have richer signal structures:
  - Analyst reports for stock traders: "Strong Sell", "Sell", "Buy", "Strong Buy".
  - Consumer credit scoring: multi-class scorecard might only eventually be aggregated to binary.
- Important to generalise from binary to Blackwell improvements of any discrete signal.
- Previously: auxiliary signal to spread belief further after initial $x = s_H$ or $x = s_L$.
- Now generalise this idea: **local mean preserving spreads**.

General Blackwell Improvements and Eqm. Payoffs

Take two signals, $\mathcal{X} \mid \theta \overset{IID}{\sim} p_\theta$ and $\mathcal{X}' \mid \theta \overset{IID}{\sim} p'_\theta$. Joint support $S \cup S' = \{s_1, s_2, ..., s_M\}$.

$\mathcal{X}'$ differs from $\mathcal{X}$ by a local MPS at s_i if:



- p_θ places no mass at s_{i-1} or s_{i+1}.
- p'_θ places no mass at s_i.
- p_θ and p'_θ place equal mass to all points except $\{s_{i-1}, s_i, s_{i+1}\}$.
- *p and p' have equal normalised means*:

$$\sum_{j=1}^{M} s_j \times \left(\frac{p_L(s_j) + p_H(s_j)}{2}\right) = \sum_{j=1}^{M} s_j \times \left(\frac{p'_L(s_j) + p'_H(s_j)}{2}\right)$$

General Blackwell Improvements and Eqm. Payoffs

Local mean preserving spreads *characterise* Blackwell improvements:

Remark

If $\mathcal{X}'$ differs from $\mathcal{X}$ by a local MPS, $\mathcal{X}'$ is *Blackwell more informative than* $\mathcal{X}$. Furthermore, if $\mathcal{X}'$ is *Blackwell more informative than* $\mathcal{X}$, there is a finite sequence $\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_k$ such that:

- $\mathcal{X}_1 = \mathcal{X}$ and $\mathcal{X}_k = \mathcal{X}'$,
- $\mathcal{X}_{i+1}$ differs from $\mathcal{X}_i$ by a local MPS.

- Only *slight* refinement of classic Rothschild and Stiglitz, 1970 result.
- I will characterise the effect of a local MPS.

General Blackwell Improvements and Eqm. Payoffs

For a fixed signal structure $\mathcal{X}$ and strategy σ, *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

General Blackwell Improvements and Eqm. Payoffs

For a fixed signal structure $\mathcal{X}$ and strategy σ, *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

Theorem

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i, and σ and σ' be the most embracive (selective) equilibria under $\mathcal{X}'$ and $\mathcal{X}'$. EVs' expected payoffs are:

General Blackwell Improvements and Eqm. Payoffs

For a fixed signal structure $\mathcal{X}$ and strategy σ, *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

Theorem

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i, and σ and σ' be the most embracive (selective) equilibria under $\mathcal{X}'$ and $\mathcal{X}'$. EVs' expected payoffs are:

1. *weakly higher* under $(\sigma', \mathcal{X}')$ if $x = s_i$ leads to approvals under σ; $\sigma(s_i) = 1$,

General Blackwell Improvements and Eqm. Payoffs

For a fixed signal structure $\mathcal{X}$ and strategy σ, *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

Theorem

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i, and σ and σ' be the most embracive (selective) equilibria under $\mathcal{X}'$ and $\mathcal{X}'$. EVs' expected payoffs are:

1. *weakly higher* under $(\sigma', \mathcal{X}')$ if $x = s_i$ leads to approvals under σ; $\sigma(s_i) = 1$,
2. *weakly lower* under $\mathcal{X}'$, if:
   1. $x = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and
   2. adverse selection poses a threat at signal s_{i+1} under $(\sigma, \mathcal{X})$.

General Blackwell Improvements and Eqm. Payoffs

- Local MPS necessary, helps pin down equilibrium response to spread.
- Unpleasant: Thm. requires knowing equilibrium structure.
  - All eqa. can be located in $\leq 2(m+2)$ steps.
- Still, we have a stronger sufficient condition that relies only on the local MPS performed:

Proposition

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. EVs expected payoffs in the most selective equilibrium are lower under $\mathcal{X}'$ whenever s_i is a rejection signal under $\mathcal{X}$ and:

General Blackwell Improvements and Eqm. Payoffs

- Local MPS necessary, helps pin down equilibrium response to spread.
- Unpleasant: Thm. requires knowing equilibrium structure.
  - All eqa. can be located in $\leq 2(m+2)$ steps.
- Still, we have a stronger sufficient condition that relies only on the local MPS performed:

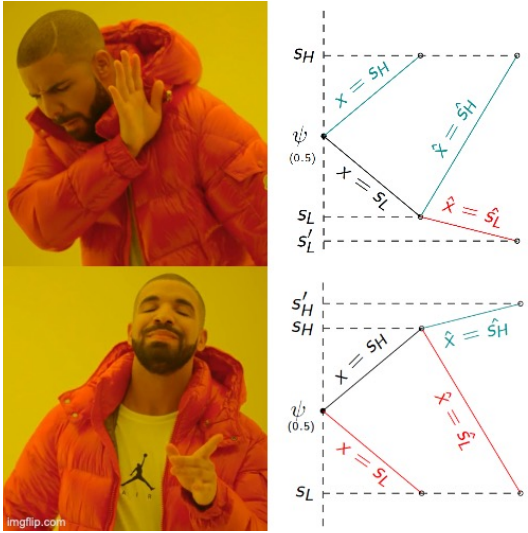
Proposition

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. EVs expected payoffs in the most selective equilibrium are lower under $\mathcal{X}'$ whenever ~~s_i is a rejection signal under $\mathcal{X}$~~ and:

a fortiori: $s_L < s_L^{\text{mute}}$:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Thank You!



s_H
$x = s_H$
$\hat{x} = \hat{s}_H$
ψ
(0.5)
$x = s_L$
$\hat{x} = \hat{s}_L$
s_L
s'_L
s'_H
s_H
$\hat{x} = \hat{s}_H$
$x = s_H$
ψ
(0.5)
$\hat{x} = \hat{s}_L$
$x = s_L$
s_L
imgflip.com

Related Literature

Main Question & Takeaway