Information in Sequential Evaluations: the Good and the Bad

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updated paper online soon!

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Motivation

- Many risky opportunities need only *one taker*, but can be offered to *many*.
  - The seller of an asset & fin. derivative can ask many interested buyers
  - Debt seekers have many banks to apply to for credit
  - Entrepreneurs contact many VCs or angel investors for (seed) funding
- Whoever gets an offer must *evaluate* whether opportunity is good or bad

So: more information $\longrightarrow$ better evaluations $\longrightarrow$ better off evaluators?

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  So: more information $\longrightarrow$ better evaluations $\longrightarrow$ better off evaluators?

- But: evaluators have an **adverse selection** problem:

  Am I getting opportunity after everyone rejected it? Does this mean it is bad?

- Answer depends on what other evaluators know & do: **info. shapes adverse selection**

  Maybe: more information $\longrightarrow$ more adverse selection $\xrightarrow{??}$ worse off evaluators?

Motivation

When does more info. leave evaluators better off? (read: improve selection quality)

Important policy question, e.g. credit scoring.

- Each bank wants better scoring algorithm. Regulators unsure if that's good...

Basel II: allowed Internal Ratings Based systems instead of standardised scoring to

"reward stronger and more accurate risk measurement"

"provide a more risk-sensitive approach to measuring credit risk"

"Regulation Guide: An Introduction", *Moody's Analytics*

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Basel III banned it back after the 2008 crisis:

"CVA is a complex risk ... cannot be modelled by banks in a robust and prudent manner. The revised framework removes the use of an internally modelled approach ..."

"High-Level Summary of Basel III Reforms", *Bank of Intl. Settlements*

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Important policy question, e.g. credit scoring.

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Banks thought this a mistake **for the same reason**:

"[Internal assessments] by banks allow for the most accurate measurement of risk ... Relying on regulators' assessments ... will be rough approximations at best ... a major step backwards"

Kenneth Bentsen, CEO of Global Financial Markets Association

Contribution today: characterise when more information $\rightarrow$ better / worse treatment of risk

The Model

- APPLICANT with unknown quality $\theta \in \{L, H\}$ **seeks approval** from **one evaluator**.
    - Everyone has prior belief $\rho \in (0, 1)$ that applicant is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORS, at random order τ.
    - τ is a permutation of $\{1, 2, ..., n\}$, chosen **privately** and **uniformly at random** by applicant.
- EV **approves** $\longrightarrow$ payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0, 1)$. Game ends, other EV.s get 0 payoff.

  EV **rejects** $\longrightarrow$ 0 payoff. Applicant keeps applying. If no EV left $\rightarrow$ game ends.

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- He sequentially visits $n \geq 2$ EVALUATORS, at random order τ.
  - τ is private and $\mathbb{P}(\tau(i) = j) = \frac{1}{n}$ for all $i, j \in \{1, 2, ..., n\}$.
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The Model

- EVs **do not know** θ or τ. But: receive **private IID signals** x about **quality** θ:

$$\mathcal{X} \mid \theta \overset{\text{IID}}{\sim} p_\theta \qquad x \in \{s_1, s_2, ..., s_m\} \subset [0,1] \qquad s_i = \frac{p_H(s_i)}{p_H(s_i) + p_L(s_i)} < s_{i+1}$$

- **Equilibrium:** symmetric strategy and interim belief profile (σ^*, ψ^*) for EVs such that:
  - $\psi^* = \mathbb{P}\left(\theta = H \mid \text{applicant visited me}\right)$ – **consistent** with strategy profile σ^*

$$\psi = \frac{\rho \times \sum_{k=0}^{n-1} \mathbb{P}(\text{applicant got } k \text{ rejections} \mid \theta = H)}{\sum_{k=0}^{n-1} \mathbb{P}(\text{applicant got } k \text{ rejections})}$$

  - $\sigma^* : \{s_1, s_2, ..., s_m\} \to [0,1]$ – **optimal** given ψ^*:

$$\text{approve when } \mathbb{P}\left(\theta = H \mid x, \psi\right) > c$$

Main Question and Takeaway

How does more information affect evaluators' equilibrium payoffs?

- Synonymous with **selection quality**: eqm. payoffs $\uparrow$ when false positives & negatives $\downarrow$.
- **Main difficulty:** More information affects extent of adverse selection; this unintended effect might curb payoffs – selection quality.
- **Main result:** Characterise effect of arbitrary **Blackwell improvements** of EVs signals.
- **Main takeaway:** Effect depends on the **kind** of improvement. Roughly:
  - improving **favourable** evaluations: **good!**
  - improving **unfavourable** evaluations: eventually **bad**

affect different applicants

$\downarrow$

have different payoff effects

Literature

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- Giving EVs information about **order** τ
- EV makes *take-it-or-leave-it* price offer to applicant
- EVs compete on **application costs**

ask me after talk!

Literature

Equilibria: Selective and Embracive

Circularity between σ and ψ means properties of equilibria are not automatic.

Proposition

The set of equilibrium strategies is non-empty and compact, and pointwise totally ordered.

Furthermore:

1. all equilibrium strategies are monotone; $\sigma^*(s_i) > 0$ implies $\sigma^*(s_{i+1}) = 1$. [when $p_H \neq p_L$]
2. all equilibria exhibit adverse selection : $\psi^* \leq \rho$.

- Compact and totally ordered $\rightarrow$ we can talk about:
  - the *highest* (most embracive) equilibrium,
  - the *lowest* (most selective) equilibrium.

Equilibria: Selective and Embracive

- What do *selective* and *embracive* equilibria mean for payoffs?
- (the sum of) Evaluators' equilibrium payoffs:

$$\Pi(\sigma) := \rho \times (1 - c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1 - \rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- Different virtues: **selective** $\rightarrow$ filter *Low* quality approvals

  embracive $\rightarrow$ secure *High* quality approvals

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- Nonetheless, trade-off is always resolved in favour of selective equilibria:

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embracive* eqm. strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

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Proof Sketch:

- Take eqm. strategy σ^*, and consider *marginally more embracive* σ^ε: $||\sigma^\varepsilon - \sigma^*|| = \varepsilon$
- Fix the signals all EVs would see if app. visited them all: $\{x_1, x_2, ..., x_n\}$.
- Only app. whose outcome changes: rejected by all under σ^*, approved by some under σ^ε.
- If ε is small, he was a.s. rejected by all under σ^*, approved by one under σ^ε.
- Bad news: approving is suboptimal for this last EV
- Last step: payoffs are **single crossing** in embraciveness; where $\sigma'' > \sigma' > \sigma$:

$$\Pi(\sigma') \leq \Pi(\sigma) \implies \Pi(\sigma'') \leq \Pi(\sigma')$$

Information and Equilibrium Payoffs

How does more information change payoffs in these equilibria?

$$\Pi(\sigma) := \rho \times (1-c) \times \mathbb{P}\left(\text{one EV approves} \mid \theta = H, \sigma\right) +$$
$$-(1-\rho) \times c \times \mathbb{P}\left(\text{all EVs reject} \mid \theta = L, \sigma\right)$$

- With *one* evaluator, **Blackwell more informative** signal $\overset{(\Longleftarrow)}{\Longrightarrow}$ $\uparrow$ payoffs for all c, ρ
- Reason: affords higher true positives $[\text{approve} \mid \theta = H]$ & negatives $[\text{reject} \mid \theta = L]$

Information and Equilibrium Payoffs

- What could go wrong with many EVs? Rewrite EV i's payoff $\pi_i \left(= \frac{\Pi}{n}\right)$:

$$\pi_i(\sigma) = \mathbb{P}\left(\text{applicant visits } i\right) \\ \times \left[\psi \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) - (1-\psi) \times c \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]$$

- The blue terms are shaped by adverse selection: depend on *others'* strategies
- Other EVs do not internalise the adverse selection they imposes onto EV i
- Maximising individual selection quality $\neq$ maximising **overall** selection quality

Binary Signals: Information and Eqm. Payoffs

- Start from **binary signals**: main intuition and building block to full characterisation
- $x \in S = \{\overset{\cancel{s_1}}{s_L}, \overset{\cancel{s_2}}{s_H}\}$ **more informative** when $s_L \downarrow$: **stronger evidence for *Low* quality**

  and $s_H \uparrow$: **stronger evidence for *High* quality**

Theorem

Let EVs have a binary signal, $x \in S = \{s_L, s_H\}$. EVs equilibrium payoffs in the most selective (embracive) equilibria are weakly:

- increasing with stronger evidence for $\theta = H$ ($\uparrow s_H$),
- increasing with stronger evidence for $\theta = L$ ($\downarrow s_L$) when s_L is above a threshold,
- decreasing with stronger evidence for $\theta = L$ ($\downarrow s_L$) when s_L is below that threshold

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in the paper

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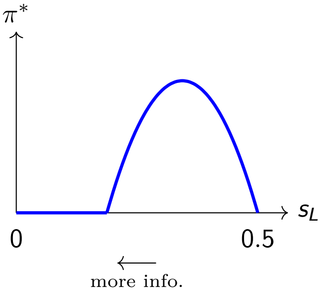
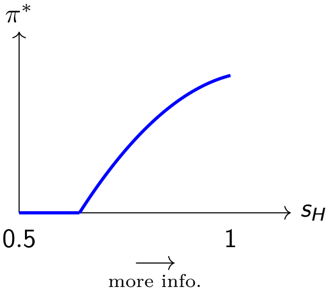
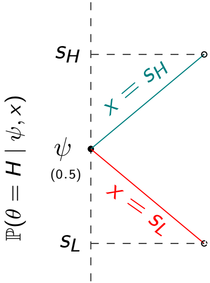




Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Consider decreasing s_L marginally. Which applicant gets affected?



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

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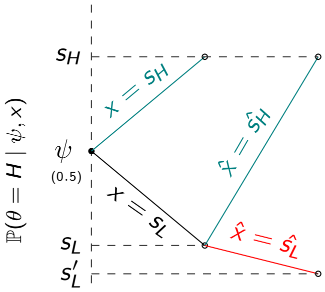
- How to implement this Blackwell improvement? Construct an auxiliary signal $\hat{\mathcal{X}}$.



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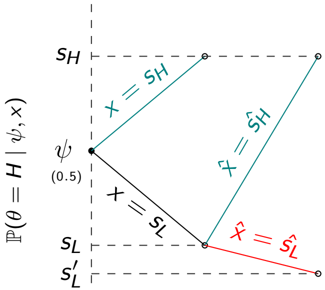

- Fix strat.s: high signal $\to$ approve & low signal $\to$ reject
- Fix signal pairs $(x, \hat{x})$ EVs would observe for applicant.
- We created **marginal admits**: rejected by all in old signal structure, approved by some in new.
- All info. on marginal admits' quality $\longrightarrow$ number of $\hat{x} = \hat{s_H}$ signals.

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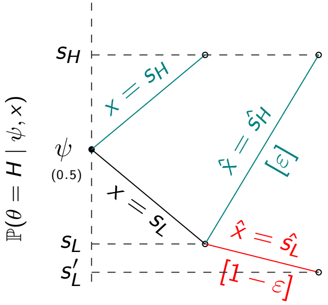

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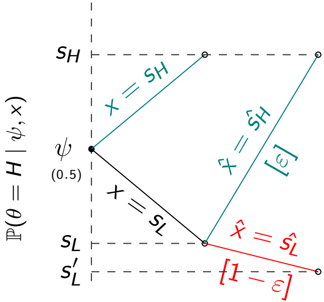
- Fix signal pairs $(x, \hat{x})$ EVs would observe for applicant.
- We created **marginal admits**: rejected by all in old signal structure, approved by some in new.
- All info. on marginal admits' quality $\longrightarrow$ number of $\hat{x} = \hat{s_H}$ signals. **Answer: only one!**
- For **marginal** decrease in s_L, $\mathbb{P}\left(\hat{x} = \hat{s_H}\right) \propto \varepsilon \rightarrow 0$.
- **Multiple** $\hat{x} = \hat{s_H}$ has negligible probability.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Consider decreasing s_L marginally. Which applicant gets affected?
- How to implement this Blackwell improvement? Construct an auxiliary signal $\hat{\mathcal{X}}$.



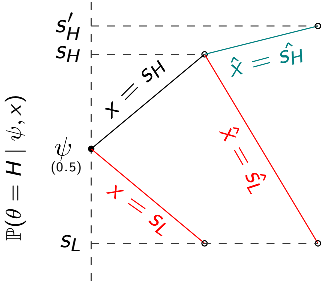
- Whether the **marginal admit** is profitable:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} \lessgtr \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- Now consider increasing s_H marginally. Which applicant gets affected?
- How to implement this Blackwell improvement? Construct an auxiliary signal $\hat{x}$.



- This time **marginal rejects**: approved by some before, rejected by all now.
- All EVs must have seen low signals.
- **Marginal reject** is always good to push out:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^n}_{n \text{ low signals}} < \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

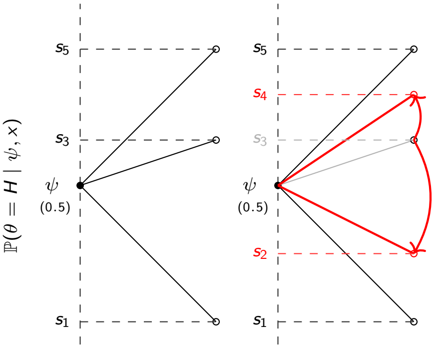
[actual pf: "strong low signals" is the only relevant case]

General Blackwell Improvements and Eqm. Payoffs

- Generalise the construction of the auxiliary signal $\hat{\mathcal{X}}$.

Take two signal str.s, $\mathcal{X} \mid \theta \overset{IID}{\sim} p_\theta$ and $\mathcal{X}' \mid \theta \overset{IID}{\sim} p'_\theta$. Joint supp $S \cup S' = \{s_1, s_2, ..., s_M\}$.

$\mathcal{X}'$ differs from $\mathcal{X}$ by a local MPS at s_i if:



- p_θ places no mass at s_{i-1} or s_{i+1}.
- p'_θ places no mass at s_i.
- p_θ and p'_θ place equal mass to all points except $\{s_{i-1}, s_i, s_{i+1}\}$.
- *p and p′ have equal normalised means*:

$$\sum_{j=1}^{M} s_j \times \left(\frac{p_L(s_j) + p_H(s_j)}{2} \right) = \sum_{j=1}^{M} s_j \times \left(\frac{p'_L(s_j) + p'_H(s_j)}{2} \right)$$

General Blackwell Improvements and Eqm. Payoffs

Local mean preserving spreads *characterise* Blackwell improvements

$$\mathcal{X} + \text{[finitely many local MPS]} \;=\; \mathcal{X}' \quad \iff \quad \mathcal{X}' \text{ is Blackwell more informative than } \mathcal{X}$$

- Only slight refinement of Rothschild and Stiglitz, 1970.
- I will characterise the effect of a local MPS. Local allows to control equilibrium evolution.

General Blackwell Improvements and Eqm. Payoffs

Theorem

Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i, and σ and σ' be the most embracive (selective) equilibria under $\mathcal{X}'$ and $\mathcal{X}'$. EVs' expected payoffs are:

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1. *weakly higher* under $(\sigma', \mathcal{X}')$ if $x = s_i$ leads to approvals under σ; $\sigma(s_i) = 1$,

General Blackwell Improvements and Eqm. Payoffs

Theorem

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1. *weakly higher* under $(\sigma', \mathcal{X}')$ if $x = s_i$ leads to approvals under σ; $\sigma(s_i) = 1$,
2. *weakly lower* under $\mathcal{X}'$, if:
   1. $x = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and
   2. **adverse selection condition for signal s_{i+1}** $\longrightarrow$ **in the paper**

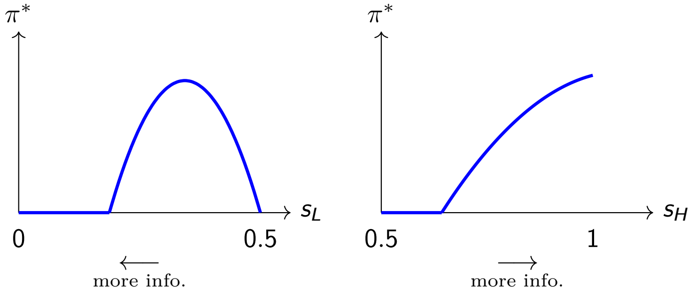
- Unpleasant: Theorem requires knowing equilibrium structure.
  - All eqa. can be located in $\leq 2(m+2)$ steps
- Stronger sufficient condition that relies only on the local MPS performed

in paper!

Thank You!

and please check back soon for (substantially) updated paper!

especially if you are hiring! :)



Related Literature

Main Question & Takeaway