# The (Mis)use of Information in Decentralised Markets

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**Abstract**

I study whether allocative efficiency in a decentralised market for a common value asset increases with (i) more buyers, each with a signal, (ii) better-informed buyers. Both increase the information available in the market; but also the adverse selection buyers are exposed to. With more buyers, surplus from trade eventually increases and converges to the full-information upper bound if buyers' signals have unbounded likelihood ratio at the top. Otherwise, it eventually decreases and converges to the no-information lower bound. With better-informed buyers, surplus from trade decreases if information helps buyers reconsider trading and unless adverse selection is irrelevant. It increases if information helps them reconsider rejecting. For binary signals, this yields a sharp characterisation: stronger good news increase, but stronger bad news eventually decrease efficiency.

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# 1 Introduction

In this paper, I study when *more information* in a *decentralised market* for a *common value* asset improves or harms allocative efficiency. The setting is parsimonious: a seller sequentially visits $n$ buyers until one accepts to trade, or all reject. He trades with the first buyer who pays his known reservation value—a simplifying assumption I relax later[1]. Whether trade is efficient depends on buyers' common value for the asset, or the asset's *quality*, which is either High or Low. Each buyer holds a private signal about the asset's quality. Conditional on quality, their signals are IID. A buyer accepts trade when she expects positive surplus from doing so; she rejects otherwise. I ask:

1. Does efficiency (the expected trade surplus) increase with *more* buyers, each with an additional signal?

2. Does efficiency increase with *better* informed buyers, each with a more informative signal?

In this decentralised market, more information—through either channel—is a double-edged sword. On the one hand, buyers need information to judge whether trading is efficient. On the other hand, more information in the market might exacerbate buyers' *adverse selection* problem: when there are more buyers, more may have already rejected the seller; when each buyer has better information, each of those rejections carry—possibly—worse news. This worsening of adverse selection may push buyers to worse trades—even though there is richer information in the market to screen the asset's quality with.

The overarching insight I draw is that the *kind*, not the *amount*, of information in the market determines efficiency. In my first main result (Theorem 1), I answer how increasing the number of buyers influences efficiency. The answer hinges on whether the buyers' signal structure has an unbounded likelihood ratio at the top. If it does, efficiency is eventually increasing in the number of buyers, and converges to the full-information benchmark; the upper bound for efficiency in the market. If it does not, efficiency is eventually decreasing in the number of buyers, and converges to the no-information benchmark; the lower bound for efficiency in the market. This is the level of efficiency that would be attained buyers had no information about the asset's value.

Theorem 1 shows that increasing the number of buyers is either a blessing or a curse for efficiency, depending on buyers' signal structure. The condition necessary for a large market to reveal the asset's quality is the same as that for a large sealed bid auction[2]. However, when this condition is violated, the outcome in a large auction may still approximate quality well[3]. In contrast, here, efficiency converges to its lower bound. My result is also reminiscent of Lauermann and Wolinsky, 2016[4]. There the outcome in a large market (generically) either fully reveals or becomes completely

[1] In Section 7, I show that this is without loss in a setting where buyers make ultimatum price offers. This assumption is natural for my settings of interest and standard in the literature (Glosten and Milgrom, 1985, Zhu, 2012, Glode and Opp, 2019).

[2] See Wilson, 1977 and Milgrom, 1979.

[3] See, for instance, Section IV in Lauermann and Wolinsky, 2017.

[4] Their model relates to mine, as I explain in Section 1.1.

uninformative about the asset's quality. Here, these possibilities arise but are not unique: the execution of trade can still be partially informative about the asset's quality in a large market—even though efficiency sits at the no-information lower bound.

In Theorems 2 and 3, I answer how *better* information for each existing buyer influences efficiency. Theorem 2 shows that when buyers' signal structure is binary, strengthening good news (increasing the likelihood ratio at the top) always increases efficiency, but strengthening bad news (decreasing the likelihood ratio at the bottom) eventually decreases it. Theorem 3 generalises the insight behind it to arbitrary finite signal structures: giving a buyer additional information where she would have accepted trade—a *negative override*—increases efficiency. In contrast, giving her additional information where she would have rejected—a *positive override*—decreases it unless *adverse selection is irrelevant* in the appropriate sense (Definition 4).

To understand the main insight behind Theorems 2 and 3, consider buyers with a binary signal structure: each buyer receives either a good, or a bad signal. For simplicity, ignore their equilibrium beliefs and incentives—let buyers simply accept upon a good signal and reject upon a bad signal. Now consider revealing additional information to each buyer—another binary signal. This could be useful for a buyer for two reasons. If it is revealed after an initial good signal—it is a *negative override*—the additional signal could lead the buyer to revise her decision to a rejection. If it is revealed after an initial bad signal—a *positive override*—it could lead her to revise her decision to an acceptance. In this simple binary-on-binary setting, a negative override *strengthens good news*: a buyer can rely on two good signals to accept. A positive override *strengthens bad news*: a buyer can rely on two bad signals to reject.

A negative override increases efficiency. With it, it becomes harder for the seller to trade; in particular, a seller who would be accepted by some buyer before the negative override became available might now be rejected by every buyer. But this would reveal that the expected surplus from this trade is negative anyway: each buyer concluded so after observing a bad signal, despite not knowing (but suspecting) that all other buyers did so too.

In contrast, a positive override might decrease efficiency. With it, it becomes easier for the seller to trade; a seller every buyer would have rejected otherwise might now be accepted by some buyer. However, due to adverse selection, the expected surplus from this trade might be negative: though some buyer received a good received a good signal about the asset's quality, others might have received bad news. If too many did, a good signal would no longer justify accepting the seller. However, the number of those who did is hidden from every buyer. So, the buyer who saw a good signal might have accepted the seller when she should not have.

I show that this obstacle adverse selection places on positive overrides' ability to improve efficiency is severe: unless *adverse selection is irrelevant*, i.e., a buyer need not care about the number of previous refusals the seller received, a positive override decreases efficiency.

This insight yields my sharp characterisation for binary signal structures in Theorem 2. To study arbitrary finite signal structures, I formalise positive overrides as *local (mean preserving) spreads*[5] of an acceptance signal and negative overrides as local spreads of a rejecion signal. Studying informativeness at the local spread level is key to this exercise; it provides the necessary tractability but comes at no cost of generality—any Blackwell improvement is a combination of finitely many local spreads.

I view my main contributions to be twofold. First, I contribute to a vast literature on information aggregation in markets. Most of this literature asks when the outcome in a large market fully reveals its participants' information. Instead, I ask whether the outcome in a *finite* decentralised market becomes more efficient with *more information* in the market. I study both an increase in information through more buyers, each with additional information; and through buyers who have a more informative signal. While Riordan, 1993 explored the former for a first price auction, this is the first paper to explore it in decentralised market. It is also the first to explore the latter channel.

Second, my findings have important policy implications for markets where trades are negotiated bilaterally and privately, and with little to no trade transparency: such as over-the-counter markets[6], credit markets[7], and the housing market[8]. Recent technological advances have allowed participants in these markets to enjoy increasingly greater access to information[9]. It is commonly presumed that the "more efficient processing of information, for example in credit markets, financial markets, [...] contribute to a more efficient financial system" (Financial Stability Board, 2017). My Theorems 2 and 3 show that this presumption—which ignores the adverse selection problem in these markets—is misleading. My Theorem 1 shows that, likewise, more information brought by increased competition might hurt efficiency too—validating a fact empirically recognised by regulators and industry leaders[10].

Existing regulation in credit markets already limits the information lenders can use to assess borrowers[11]. Directly resonating with Theorems 2 and 3, European Central Bank, 2024 states that "institutions should be more restrictive with positive overrides than with negative ones"[12]. I offer a novel justification[13] for these policies, rooted in the adverse selection problem that is inherent to

[5] See Definition 1.5.28 in Müller and Stoyan, 2002 or Section 5.2 of this paper.

[6] OTC markets are characterised by sequential contacts and little transparency (Duffie, 2012, Zhu, 2012). Liquidity providers typically make ultimatum offers that only last "as long as the breath is warm" (Bessembinder and Maxwell, 2008).

[7] In the US and the UK, credit scores mask borrowers' recent applications, and borrowers exercise little bargaining power against lenders (Agarwal et al., 2024 and Consumer Rights, 2024).

[8] In the housing market, "buyers and sellers must search for each other" (Han and Strange, 2015). Sellers frequently relist, making it difficult to infer how many viewings resulted in no trade: RE/MAX, 2024 advises "if a property has been sitting on the market and going stale, there is no harm in relisting it so that it appears fresh and new".

[9] Hedge funds and broker-dealers use increasingly sophisticated data and algorithms to assess trades' profitability (Financial Stability Board, 2017); lenders use cutting-edge ML technology in credit scoring (Financial Stability Board, 2017); algorithmic traders in housing markets analyse and execute trades faster than traditional investors (Raymond, 2024).

[10] Regulators (partially) blamed adverse selection for the collapse of a British bank, HBOS: "the borrowers who came through its doors inevitably included many whom better established banks had turned away" (Kay, 2024).

[11] For instance, following the 2008 crash, the Basel III Accord severely limited the use of "advanced internal ratings systems" to determine credit risk exposure. This overturned the conventions set in Basel II. See BCBS, 2017.

[12] A positive (negative) override is an upwards (downwards) revision of an algorithmic credit decision by a human, where she believes the initial decision to inadequately reflects existing information (BCBS, 2004, par.s 417 and 428).

[13] Currently, these policies are mostly justified by a distrust in the "robustness and prudence" of lenders' abilities

the structure of these markets.

The remainder is organised as follows: in Section 1.1, I review the relevant literature. Section 2 presents the model. Section 3 explains how buyers form beliefs about the asset's quality, how equilibria are formed, and how efficiency varies among them. Section 4 presents Theorem 1. Section 5 presents Theorems 2 and 3; the former is discussed in Section 5.1 and the latter in Section 5.2. Unless given in the main text, all proofs appear in Section 8.

### 1.1 Literature Review

As mentioned earlier, I contribute to the literature on information aggregation in markets. Most of this literature[14] investigates when a large market fully aggregates information. To the best of my knowledge, my first question—whether efficiency in the market increases with an *additional* participant—has not been addressed for a decentralised market. It was addressed only by Riordan, 1993[15] for a centralised exchange (a first price auction). There, the adverse selection problem is simply the winner's curse—the bank (analogous to the buyer here) who accepts the borrower (analogous to the seller here) understands that it had the highest signal. Consequently, his sufficient condition[16] for efficiency to be increasing or decreasing in the number of bidders differs from the necessary and sufficient condition in my Theorem 1.

Moreover, both Riordan, 1993 (and Broecker, 1990, whose model he studies) and my paper speak to adverse selection in credit markets. However, I lift their assumption that banks (here, buyers) can publicly commit to their loan prices. Theorem 1 echoes Riordan, 1993's conclusion that increased competition can hurt efficiency.

My second question—whether efficiency increases when buyers are *better* informed—is novel in this literature. The most related papers, to the best of my knowledge, are Levin, 2001 and Glode and Opp, 2019. Levin, 2001 asks whether in Akerlof, 1970's model, where inefficiency is caused by the informational asymmetry between the seller and buyer, giving the seller more information necessarily hurts efficiency. He finds that the right kind of information can increase efficiency. Since his environment is different than mine, so is his characterisation. Glode and Opp, 2019 show that an OTC market can be more efficient than a limit-order market because it provides buyers with incentives to acquire more information. Buyers' signal structure is not general: they invest in their probability of getting a fully revealing signal. My Theorems 2 and 3 show that in general, efficiency can *decrease* when buyers in a market are better informed.

to screen borrowers (BCBS, 2017).

[14]Prominent and related examples areWilson, 1977, Milgrom, 1979, Riordan, 1993 and Lauermann and Wolinsky, 2017 for centralised exchanges (auctions) and Wolinsky, 1990, Blouin and Serrano, 2001, Zhu, 2012, Golosov et al., 2014, and Lauermann and Wolinsky, 2016 for decentralised markets.

[15]Relatedly, Di Tillio et al., 2021 study whether the winning bid in a first price common value auction becomes more informative about the asset's value when there are more bidders. Efficiency is of no direct concern in their setting: trade is always efficient and always materialises.

[16]Where $F_\theta(s)$ is the CDF of the signal distribution for quality $\theta \in \{L, H\}$, he finds that the sign of the expression $\frac{f_H(.)/F_H(.)}{f_L(.)/F_L(.)} - \frac{F_H(.)}{F_L(.)}$ over the support is a sufficient condition to determine this.

My model follows Glosten and Milgrom, 1985 in spirit[17] and is closest to Zhu, 2012. Importantly, Zhu, 2012 assumes that trade is efficient for any asset quality, the reservation value of a Low quality seller is 0—otherwise our models are identical. He (Proposition 4) also shows that efficiency in large market might not converge to the full-information benchmark. My Theorem 1 shows that in fact—where trading with a Low quality seller reduces surplus—efficiency in a large market converges either to the full- or no-information benchmark, its lower bound. Interestingly, he (Proposition 6) also shows that trade is always likelier in his OTC market than in an auction. In this model this implies greater efficiency. In mine it does not—more frequent trade with a Low quality seller reduces efficiency. This question remains open in my setting, I leave it to future research.

Lauermann and Wolinsky, 2016's model is also similar to mine, but they focus on a large market where the buyer (analogous to the seller here) has small visit costs and partial bargaining power, so can keep searching for a good price. Their results are driven by this force and so are different in nature.

My model can also be interpreted as social learning model á la Bikhchandani et al., 1992 and Smith and Sørensen, 2000; as in Herrera and Hörner, 2013, later decision makers (buyers) are called to decide only if those before them reject, and no decision maker sees her position in the queue. This literature mostly focuses on whether full learning attains with a large number of decision makers. Instead, my results speak to how—with finitely many decision makers—more information, either through more decision makers or better informed ones, influences welfare.

Lastly, Cavounidis, 2022 studies a variant of my model where buyers' (there, appraisers) information acquisition decisions are costly and endogenous. He asks how the maximum number of buyers the market can accommodate is influenced by the cost of acquiring information.

## 2 Model

A *seller* (he) seeks to sell an asset of either High or Low quality, $\theta \in \{H, L\}$. There are $n \in \mathbb{N}$ prospective buyers. He sequentially visits these buyers, until one *accepts* to trade or they all *refuse*. He does so at a uniformly random order: his next visit is equally likely to be to any buyer he has not visited yet.

The seller's reservation value for the asset is $c \in [0, 1]$. He is willing to trade with the first buyer who accepts to pay it[18]. If a buyer he visits refuses trade, no transaction takes place. If the buyer accepts, the buyer pays the seller his value $c$ and enjoys a return of 1 for a High quality asset and 0 for a Low quality one.

The asset's quality is unknown. At the outset of the game, each player shares the *prior belief* that it is High with probability $\rho \in (0, 1)$. Each buyer obtains additional *private information* about the asset's quality through the outcome of a Blackwell experiment $\mathcal{E} = (\mathbf{S}, p_L, p_H)$. The outcome

[17] There, the prices are posted by the uninformed seller (specialist) who must be mindful that the informed buyer (investor) will adversely select into trade.

[18] In Section 7, I relax this assumption.

$s$ of the experiment—the buyer's *signal*—is drawn from the finite set $\mathbf{S}$ with a distribution $p_\theta$. Conditional on the asset's quality, buyers' signals are IID.

Buyers receive no information about the seller's *previous* visits. Nonetheless, each buyer that the seller visits deduces that he was refused by every buyer he previously visited. This conveys additional information about the asset's value.

Using these two pieces of information, each buyer the seller visits forms a *posterior belief* about the asset's quality. First, she uses the information conveyed by the seller's visit to revise her prior belief $\rho$ to an *interim belief* $\psi$. Then, she uses the information conveyed by her private signal to revise her interim belief to a *posterior belief* $\mathbb{P}_\psi(\theta = H \mid S = s)$.

A buyer's *strategy* $\sigma : \mathbf{S} \to [0, 1]$ maps every signal $s \in \mathbf{S}$ she might observe to a probability $\sigma(s)$ with which she accepts to trade. Her strategy $\sigma$ is *optimal against the interim belief* $\psi$ if, given this interim belief and her signal, the buyer accepts (refuses) to trade whenever her expected payoff from trading with the seller is positive (negative):

$$\sigma(s) = \begin{cases} 0 & \mathbb{P}_\psi(\theta = H \mid s) < c \\ 1 & \mathbb{P}_\psi(\theta = H \mid s) > c \end{cases}$$

She may accept to trade with any probability when she expects zero surplus from trading.

I focus on *symmetric Bayesian Nash Equilibria* of this game. Hereafter, I reserve the term *equilibrium* for such equilibria unless I state otherwise. An *equilibrium* is a strategy and interim belief pair $(\sigma^*, \psi^*)$ such that:

1. The interim belief $\psi^*$ is *consistent* with the strategy $\sigma^*$; i.e., it is the interim belief of a buyer who receives the seller's visit *and* believes every other buyer uses the strategy $\sigma^*$.

2. The strategy $\sigma^*$ is optimal given the interim belief $\psi^*$.

I call any strategy $\sigma^*$ that constitutes part of an equilibrium an *equilibrium strategy.*

# 3 Buyers' Beliefs, Equilibria, and Market Efficiency

This section lays the necessary groundwork to discuss my main results. I first present how buyers form their interim beliefs, and the fundamental properties of the set of equilibria. Then, I define *market efficiency*, and show how it varies across different equilibria.

## 3.1 Buyers' Beliefs and Equilibria

No buyer is given information about how many others the seller visited previously. However, the seller's visit reveals that he was refused in all those previous visits, however many there may have been. How does a buyer interpret this information?

The probability that a buyer he visits refuses the seller depends on the quality of his asset. When each buyer uses a strategy $\sigma$, the seller faces a probability $r_\theta(\sigma;\mathcal{E})$ of getting refused in any of his visits, where:

$$r_\theta(\sigma;\mathcal{E}) := 1 - \sum_{j=1}^{m} p_\theta\left(s_j\right) \times \sigma\left(s_j\right)$$

Any buyer understands that the seller is equally likely to try any number $k \in \{0, 1, ..., n-1\}$ of buyers before visiting her. The seller visits her if and only if each of those buyers refuse him. This happens with probability $r_\theta(\sigma;\mathcal{E})^k$. So, each buyer assigns a probability $\nu_\theta\left(\sigma;\mathcal{E}\right)$ to being visited by the seller before he trades with another:

$$\nu_\theta\left(\sigma;\mathcal{E}\right) := \frac{1}{n} \times \sum_{k=0}^{n-1} r_\theta\left(\sigma;\mathcal{E}\right)^k$$

The probability that any given buyer is visited by the seller varies with the quality of the asset. Thus, the seller's visit is informative about the asset's quality. Using this information, each buyer the seller visits forms an *interim belief* $\psi$ about the asset's quality:

$$\begin{aligned}\psi = \mathbb{P}\left(\theta = H \mid \text{visit received}\right) &= \frac{\mathbb{P}\left(\text{visit received} \mid \theta = H\right) \times \mathbb{P}(\theta = H)}{\mathbb{P}\left(\text{visit received}\right)} \\ &= \frac{\rho \times \nu_H\left(\sigma;\mathcal{E}\right)}{\rho \times \nu_H\left(\sigma;\mathcal{E}\right) + (1-\rho) \times \nu_L\left(\sigma;\mathcal{E}\right)}\end{aligned}$$

This is the unique interim belief that is *consistent* with every buyer using the strategy $\sigma$.

Buyers receive private information about the asset's quality, too. To form her *posterior belief*, a buyer updates her interim belief using the information her *private signal* $s \in \mathbf{S}$ conveys—a more familiar exercise. The buyer's posterior belief about the asset's quality then becomes:

$$\mathbb{P}_\psi\left(\theta = H \mid s_i\right) = \frac{\psi \times p_H(s)}{\psi \times p_H(s) + (1-\psi) \times p_L(s)}$$

Note that the informational content of a signal is distilled by the ratio $\frac{p_H(s)}{p_H(s)+p_L(s)}$. For notational convenience, I simply use the signal's label $s \in \mathbf{S}$ to refer to this ratio. The *likelihood ratio*, $\frac{p_H(s)}{p_L(s)}$ of a signal, simply becomes $\frac{s}{1-s}$. Furthermore, I enumerate the elements of $\mathbf{S} := \{s_1, s_2, ..., s_m\}$ so that $s_1 \leq s_2 \leq ... \leq s_m$. Note that higher (indexed) signals induce higher posterior beliefs for the same interim belief.

Whenever a strategy $\sigma^*$ is optimal against the unique interim belief $\psi^*$ consistent with it, the pair $(\sigma^*, \psi^*)$ forms an equilibrium. There might be many such pairs in principle, or none. Proposition 1 sets the ground by ruling the latter possibility out: equilibrium existence is guaranteed. It also establishes some fundamental properties of the set of equilibria.

**Proposition 1.** Let $\Sigma$ be the set of equilibrium strategies. Then:

1. $\Sigma$ is non-empty and compact.

2. Any equilibrium strategy $\sigma^*$ is *monotone*; for any $\sigma^* \in \Sigma$, $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.

3. All equilibria exhibit *adverse selection*: $\psi^* \leq \rho$ for any interim belief $\psi^*$ consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

*Proof outline:* To establish the existence of an equilibrium, I construct a *best response correspondence* $\Phi$ for buyers. $\Phi$ maps any strategy $\sigma$ to the set of all strategies optimal against the unique interim belief consistent with $\sigma$; i.e. those that maximise a buyer's expected payoff when her all her peers use the strategy $\sigma$. Note that $\sigma^*$ is an equilibrium strategy if and only if it is a fixed point of this best response correspondence; i.e., $\sigma^* \in \Phi(\sigma^*)$. I show that $\Phi$ indeed has a fixed point, by a routine application of Kakutani's Fixed Point Theorem. To this end, I prove that $\Phi$ is upper semicontinuous, which establishes the compactness of the set of equilibrium strategies as well.

Monotonicity is a straightforward necessity for a strategy to be optimal: higher signals induce higher posterior beliefs, so buyers (weakly) prefer to trade upon higher signals. A crucial consequence of monotonicity is that a *Low* quality seller is likelier to be refused in any of his visits, as buyers are likelier to observe lower signals for him. Thus, the seller is *adversely selected* through his past rejections. This pushes buyers' interim beliefs below their prior.

$\square$

Though an equilibrium is guaranteed to exist, it need not be unique. The following example, which I will revisit on occasion, illustrates this. Say there are two prospective buyers who share the prior belief $\rho = 0.5$, and that the seller's value is $c = 0.2$. Buyers' experiment $\mathcal{E}$ is binary, $\mathbf{S} = \{0.2, 0.8\}$. Given the asset's quality, its outcome has the distribution:

$$p_L(s) = \begin{cases} 0.8 & s = 0.2 \\ 0.2 & s = 0.8 \end{cases} \qquad\qquad p_H(s) = \begin{cases} 0.2 & s = 0.2 \\ 0.8 & s = 0.8 \end{cases}$$

In this example, one equilibrium strategy for buyers is to approve the seller whenever he visits them. Under this strategy, there is no adverse selection. The first buyer he visits approves the seller. So, the buyer the seller visits is certain that she was the first one he visited. Thus, buyers' interim belief equals their prior, $\psi = \rho = 0.5$. If a buyer observes the low signal $s = 0.2$, she updates this to a posterior belief of 0.2. At this belief, the buyer still finds it optimal to trade with the seller; her expected surplus from trade is zero.

This equilibrium, however, is not the only one. There is another one, where buyers only trade if they receive the high signal $s = 0.8$. Buyers' selectivity triggers adverse selection: each buyer understands that she need not be the first buyer the seller visited. Thus, her interim belief $\psi$ lies below her prior belief:

$$\psi = \frac{1 + 0.2}{(1 + 0.2) + (1 + 0.8)} = 0.4$$

Therefore, the posterior belief a buyer forms upon the low signal $s = 0.2$ is also lower in this equilibrium; it equals $1/7$. At this posterior belief, the buyer expects a negative surplus from trading with the seller at the price $c = 0.2$. So, she refuses trade. In contrast, the posterior belief she forms upon the high signal $s = 0.8$ leads to a posterior belief of 0.7. At this posterior belief, she still expects a positive surplus from trading with the seller. So, she accepts to trade.

In this latter equilibrium, the seller is likelier to be refused in any of his visits. The buyers are *more selective*—they are (weakly) likelier to refuse trade at any signal they might observe.

**Definition 1.** Where $\sigma'$ and $\sigma$ are two strategies, $\sigma'$ is *more selective than* $\sigma$ (or, $\sigma$ is *less selective than* $\sigma'$) if $\sigma'(s) \leq \sigma(s)$ for all $s \in \mathbf{S}$.

*Selectivity* offers a natural way to rank buyers' equilibrium strategies. Proposition 2 shows that it offers a complete ranking among them, too.

**Proposition 2.** *Selectivity* is a *complete order* over the set of equilibrium strategies $\Sigma$. Moreover, $\Sigma$ contains a *most selective* and *least selective* strategy, $\hat{\sigma} \in \Sigma$ and $\check{\sigma} \in \Sigma$ respectively:

$$\hat{\sigma}(s) \leq \sigma^*(s) \leq \check{\sigma}(s) \qquad \text{for all } s \in \mathbf{S} \text{ and } \sigma^* \in \Sigma$$

*Proof.* By Proposition 1, the set of equilibrium strategies $\Sigma$ is a subset of the set of monotone strategies. The latter is a chain under the *selectivity* order; for two monotone strategies $\sigma$ and $\sigma'$, we have:

$$\sigma'(s_j) > \sigma(s_j) \implies \begin{array}{ll} 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'}) & \text{for any } s_{j'} > s_j \in \mathbf{S} \\ \sigma'(s_{j'}) \geq \sigma(s_{j'}) = 0 & \text{for any } s_{j'} < s_j \in \mathbf{S} \end{array}$$

Since any subset of a chain is also a chain, $\Sigma$ is a chain too.

By Proposition 1, $\Sigma$ is a compact set. Since it is also a chain, by applying a suitably general Extreme Value Theorem (see Theorem 27.4 in Munkres, 2000) to the identity mapping on $\Sigma$, one verifies that $\Sigma$ has a *minimum* and *maximum* element with respect to this order; i.e. there are two strategies $\hat{\sigma}, \check{\sigma} \in \Sigma$ such that for any other strategy $\sigma^* \in \Sigma$ we have $\hat{\sigma}(s) \leq \sigma^*(s) \leq \check{\sigma}(s)$ for all $s \in \mathbf{S}$.

□

## 3.2 Market Efficiency

My core object of analysis is the expected total surplus in the market, *market efficiency* for short. This also equals buyers' surplus from trade; the seller expects no surplus since he is either left with the asset, or paid his reservation value for it.

Market efficiency hinges on buyers' ability to screen the asset's quality; trading with the seller generates a surplus of $1 - c$ when the asset has High quality, but $-c$ when it has Low quality. If buyers had *full information* about the asset's quality, all gains from trade would be realised: the seller would trade whenever he had a High quality asset, but never when he had a Low quality asset. Market efficiency in this full information benchmark would equal $\Pi^f := \rho \times (1-c)$.

If buyers instead had *no information* about the asset's quality, their decisions would be guided solely by their prior beliefs. There would be no adverse selection: buyers' decisions to refuse the seller reveal no information about the asset's quality since buyers themselves have no information. Thus, buyers trade when they expect positive surplus given their prior beliefs, and not otherwise. Market efficiency in this no information benchmark would then equal $\Pi^\emptyset := \max\{0, \rho - c\}$.

When buyers' Blackwell experiment $\mathcal{E}$ is partially informative about the asset's quality, market efficiency depends on how well they use this information to screen the buyer with. When they use the strategies $\sigma$, market efficiency becomes:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) &:= (1-c) \times \mathbb{P}_{\sigma;\mathcal{E}}\left(\theta = H \cap \text{some buyer trades}\right) - c \times \mathbb{P}_{\sigma;\mathcal{E}}\left(\theta = L \cap \text{some buyer trades}\right) \\ &= (1-c) \times \rho \times [1 - r_H(\sigma;\mathcal{E})^n] - c \times (1-\rho) \times [1 - r_L(\sigma;\mathcal{E})^n]\end{aligned}$$

As Proposition 2 showed, there need not be a unique way for buyers to use their information in equilibrium. There may be multiple equilibria, differing in how selective buyers are when trading with the seller. *A priori*, how market efficiency varies among these equilibria is unclear. Under a more selective strategy, any buyer is less likely to trade with the seller. When the asset has Low quality, this may save a social loss of $c$. However, when the asset has High quality, it jeopardises a social gain of $1-c$. Thus, in principle, increased selectivity may both be a vice and a virtue.

Our previous example illustrates these competing effects. There, we identified two equilibria; these were, in fact, the most and least selective equilibria in that example[19] . In the least selective equilibrium, the seller always trades. So, market efficiency equals:

$$\Pi(\check{\sigma};\mathcal{E}) = 0.5 \times (1-c) - 0.5 \times c = 0.3$$

Notably, this equals market efficiency in the no information benchmark; though buyers have partial information about the asset's quality, they are unable to utilise it in this equilibrium.

In contrast, in the most selective equilibrium, the seller does not trade unless some buyer receives the high signal. So, he does not trade with a probability $p_H^2(0.2) = 0.04$ when the asset has High quality, and with a probability $p_L^2(0.2) = 0.64$ when it has Low quality. On balance, market efficiency increases:

$$\Pi(\hat{\sigma};\mathcal{E}) = 0.5 \times \left[(1-c) \times (1 - p_H(0.2)^2) - c \times \left(1 - p_L(0.2)^2\right)\right] = 0.348$$

Note that it still remains below market efficiency in the full information benchmark: 0.6.

[19] One can see this by noting that in any equilibrium where trade happens with *partial* probability, the interim belief is strictly below 0.5 and strictly above 0.4. Thus, there are no equilibria where buyers do not trade upon the high signal. Nor there are any equilibria where they trade with *partial* probability upon the low signal.

Proposition 3 shows that this, in fact, illustrates a general pattern.

**Proposition 3.** Equilibrium market efficiency is bounded above by the *full information* benchmark $\Pi^f$ and below by the *no information* benchmark $\Pi^\emptyset$. Furthermore, market efficiency is higher under more selective equilibrium strategies; where $\sigma^*$ and $\sigma^{**}$ are two equilibrium strategies such that $\sigma^{**}$ is more selective than $\sigma^*$:

$$\max\{0, \rho - c\} = \Pi^\emptyset \leq \Pi(\sigma^{**}; \mathcal{E}) \leq \Pi(\sigma^*; \mathcal{E}) \leq \Pi^f = \rho \times (1 - c)$$

*Proof.* The second part of the Proposition follows as a corollary to Lemma 4. All surplus from trade is realised when buyers have full information; hence $\Pi^f$ bounds market efficiency from above. Since market efficiency equals buyers' surplus from trade, it is bounded below by 0. Thus, when $\rho \leq c$, $\Pi^\emptyset = 0$ bounds market efficiency from below. Now let $\rho > c$, and assume for contradiction that there is an equilibrium strategy $\sigma^*$ such that $\Pi(\sigma^*; \mathcal{E}) < \Pi^\emptyset = \rho - c$. Then:

$$\begin{aligned}
&\mathbb{P}_{\sigma^*;\mathcal{E}}(\text{some buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}(\theta = H \mid \text{some buyer trades}) - c] < \\
&\mathbb{P}_{\sigma^*;\mathcal{E}}(\text{some buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}(\theta = H \mid \text{some buyer trades}) - c] + \\
&\qquad\qquad \mathbb{P}_{\sigma^*;\mathcal{E}}(\text{no buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}(\theta = H \mid \text{no buyer trades}) - c] = \Pi^n
\end{aligned}$$

So, $\mathbb{P}_{\sigma^*;\mathcal{E}}(\theta = H \mid \text{no buyer trades}) > c$. However, $\sigma^*$ then cannot be an equilibrium strategy: each buyer has a profitable deviation to trade with the seller whenever he visits. □

The second part of Proposition 3 follows as a corollary to Lemma 4.

**Lemma 4.** Let $\sigma^*$ and $\sigma$ be two monotone strategies, where $\sigma^*$ is more selective than $\sigma$. If $\sigma^*$ is an equilibrium strategy, then $\Pi(\sigma^*; \mathcal{E}) \geq \Pi(\sigma; \mathcal{E})$.

Lemma 4 strengthens Proposition 3: in fact, *any* deviation from an equilibrium strategy to a less selective one decreases efficiency. This highlights an important contrast between the problem of a *single* buyer who confronts the seller, and a decentralised market with multiple buyers the seller can visit. A single buyer faces no adverse selection; there is no one else the seller may visit. Her equilibrium strategy maximises her expected surplus given her prior belief, and a deviation can only (weakly) decrease it. In contrast, in a decentralised market, there is a non-trivial equilibrium multiplicity. Market efficiency—thus, buyers' surplus—varies across equilibria: deviations to more selective equilibria increase market efficiency. However, Lemma 4 shows that, just as it is in the single buyer case, joint deviations to *less* selective strategies still hurt market efficiency.

# 4 Market Efficiency with More Buyers

In this Section, I discuss how market efficiency changes as the number of buyers in the market increases. My first main result, Theorem 1, answers that question.

**Theorem 1.** Let $\Pi^n(\hat{\sigma}; \mathcal{E})$ be the efficiency of a market with $n$ buyers and under the most selective equilibrium. If $\mathcal{E}$ has an outcome that fully reveals High quality ($s_m = 1$), the sequence $\{\Pi^n(\hat{\sigma}; \mathcal{E})\}_{n=1}^{\infty}$ is eventually increasing and converges to full information efficiency. Otherwise, it is eventually decreasing and converges to no information efficiency.

The more buyers there are, the more information there is in the market about the asset's quality. As the market becomes arbitrarily large, buyers' collective information becomes sufficient to perfectly reveal the asset's quality, unless buyers' signals are uninformative about it.

As it grows, whether the market can incorporate buyers' information and reach full information efficiency depends on the *kind* of information each buyer has. If buyers have a signal that fully reveals High quality, in a large enough market, each buyer refuses the seller unless she observes that fully revealing signal. The severity of adverse selection justifies this: only a fully revealing signal can dissipate a buyer's doubt about the seller. Hence, a Low quality seller never trades. A High quality seller does, however; and the larger the market, the likelier he is to trade, and the higher market efficiency is. In an arbitrarily large market, he trades almost surely.

Otherwise, the market experiences an *informational breakdown*. When $\rho > c$, in a large enough market, there is at least one signal upon which a buyer always trades with the seller. Eventually, increasing the number of buyers who may observe that signal benefits a Low quality seller more than it does a High quality one. Market efficiency falls. In an arbitrarily large market, the seller trades almost surely—just as he would if buyers had no information about the asset's quality. When $\rho \leq c$, such behaviour would lead to negative total surplus; so, each buyer ought to trade with the seller only with a small probability, if at all. This requires buyers to mix between accepting and refusing to trade when they observe the highest signal. They are incentivised to do so: these strategies breed severe adverse selection, so buyers expect at most zero surplus from trading.

It is worth noting that even with a signal that fully reveals High quality, the market need not converge to full information efficiency in *every* equilibrium. To see this, consider a slightly modified version of our running example. Buyers' prior belief is $\rho = 0.5$, and the seller's reservation value is $c = 0.2$. A buyer's signal is the outcome the outcome of the experiment $\mathcal{E}^g$, which has a distribution:

$$p_L^g(s) = \begin{cases} 1 & s = 0.2 \\ 0 & s = 1 \end{cases} \qquad\qquad p_H^g(s) = \begin{cases} 0.25 & s = 0.2 \\ 0.75 & s = 1 \end{cases}$$

Hence, $\mathcal{E}^g$ has an outcome that fully reveals High quality. Note that, regardless of the number of buyers, a buyer trades with the seller after any signal in the least selective equilibrium. Thus, the seller always trades, and market efficiency always equals the no information efficiency, $\Pi^{\emptyset} = 0.3$. On the other hand, buyers only trade upon the high signal $s = 1$ in the most selective equilibrium. Hence, a Low quality seller never trades. A High quality seller trades with probability $1 - (0.25)^n$ in a market with $n$ buyers. In an arbitrarily large market, he trades almost surely; market efficiency converges to $0.5 \times [1 - 0.2]$.

Notably, whether a classic common value auction reveals the value of the item through the winning bid, too, depends on whether buyers' signal has an unbounded likelihood ratio at the top (Wilson, 1977 and Milgrom, 1979). However, even when this condition is violated, a large common value auction may aggregate information "well" (Lauermann and Wolinsky, 2017). Indeed, in a setting where trade is efficient if buyers' common value is High but not Low, Riordan, 1993 identifies a different sufficient condition[20] under which efficiency increases or decreases with more bidders. In my setting, a bounded likelihood ratio at the top causes market efficiency to eventually decrease in the number of buyers. The decrease is severe: in the limit, the market reaches the lower bound of efficiency—an informational breakdown.

My informational breakdown result is reminiscent of Lauermann and Wolinsky, 2016. Their model is a variant of mine, the seller (*buyer*, in their language) pays a small cost for each visit and has bargaining power over the price; so he keeps searching until he finds a good enough price to take. They show that in a large market, "generically" the transaction price either fully reflects the value of the asset, or is independent from it. Though they explore a similar adverse selection problem as me, sellers' ability to mimic each others' search behaviour drives their results. In their model, this has no bearing on efficiency—trade is always efficient.

Finally, Zhu, 2012 studies a model very similar to mine, but in a setting where trading is always efficient. He finds that in a large market, trade always happens when buyers' signals have unbounded likelihood ratio at the top—the market reaches full efficiency—but may not otherwise. In contrast, trade is efficient only for a High quality asset in my model. I find that the same condition guarantees full efficiency in my model too, but because buyers can distinguish and trade with High quality, but not Low quality sellers. Otherwise, the market destroys *all* surplus from trade, even when trade does happen.

# 5 Market Efficiency with Better Informed Buyers

In this Section, I answer how giving each buyer *better* information—a Blackwell more informative experiment—affects market efficiency. I thus take the number of buyers $n$ to be a primitive not a parameter. Market efficiency attains its extremes in the *most* and *least selective* equilibria, so I focus on them. My main results cover both of these equilibria. For brevity, I write "equilibrium$^\star$", where the reader may read "most selective equilibrium" or "least selective equilibrium". I denote the equilibrium$^\star$ strategy for an experiment $\mathcal{E}$ as $\sigma^\star_{\mathcal{E}}$.

If there were a single buyer in the market—exposed to no adverse selection—a Blackwell improvement of her experiment would be necessary and sufficient for market efficiency to rise regardless of the seller's reservation value[21]. The reason is simple: with better information, the buyer can

[20] He works with continuous signal distributions. Where the signal distribution has CDF $F_\theta$ and PDF $f_\theta$, efficiency increases (decreases) with more buyers if the reverse hazard ratio of $F_H$ to $F_L$ is always above (below) $\frac{F_H}{F_L}$.

[21] In general, a Blackwell improvement is sufficient, but not necessary for a decision maker to extract higher value from a decision problem (see Blackwell, 1953). However, it is necessary for the decision maker to extract higher value

better screen the asset and target efficient trades.

In a market with multiple buyers, buyers' ability to screen the seller is shaped both by the quality of their private signals *and* the extent of adverse selection each face in the market. So, better information becomes a double-edged sword. On the one hand, it allows each buyer to screen the seller more effectively. On the other hand, it might exacerbate adverse selection: previous refusals might become likelier, and each might carry worse news about the quality of the asset. Each buyer becomes better capable to screen the seller, but they might end up executing worse trades simply because each receive a more adversely selected seller. Thus, better information might lead to lower market efficiency.

How a Blackwell improvement of buyers' experiment shapes market efficiency depends on the *kind* of this improvement. In the Introduction, I discussed two possible kinds of improvements in buyers' information. The first was a *negative override*: providing a buyer additional information about a seller she would have traded with. The second was a *positive override*: additional information about a seller she would have refused.

This Section gives a precise definition for negative and positive overrides (Definition 3). In Theorem 3, I show that the *kind* of override buyers receive determines how market efficiency evolves: a negative override increases efficiency, but a positive override decreases it unless an "irrelevance" condition for adverse selection holds. The next subsection lays the groundwork for this discussion: it studies how market efficiency responds to better information when buyers are restricted to binary experiments. The intuition behind the main result there, Theorem 2, guides the discussion surrounding Theorem 3.

## 5.1 Binary Experiments

A binary experiment $\mathcal{E}$ has two possible outcomes $s_1, s_2 \in \mathbf{S}$, which I relabel as $s_L, s_H \in \mathbf{S}$ for convenience. The low outcome $s_L \in [0, 0.5]$ decreases a buyer's interim belief about the quality of the asset, while a high outcome $s_H \in [0.5, 1]$ increases it.

Comparing two binary experiments, $\mathcal{E}'$ and $\mathcal{E}$, on their Blackwell informativeness is simple. Where the former has the possible outcomes $s'_L, s'_H \in \mathbf{S}'$, the experiment $\mathcal{E}'$ is Blackwell more informative than $\mathcal{E}$ if and only if it delivers both *stronger bad news*, $s'_L \leq s_L$, and *stronger good news*, $s'_H \geq s_H$[22].

Theorem 2 answers how market efficiency evolves when buyers' binary experiment becomes Blackwell more informative—delivers either stronger good news, or stronger bad news.

**Theorem 2.** Let buyers' experiment $\mathcal{E}$ be binary. Then, equilibrium$^\star$ market efficiency is increasing in the strength of good news ($s_H$) but is quasiconcave and eventually decreasing in the strength of bad news ($s_L$).

from *any* decision problem where the unknown state is binary—such our buyers' screening problem. I present a self contained proof of this fact in Section 8.2, Lemma 12 for completeness.

[22]See Section 12.5 in Blackwell and Girshick, 1954 for a textbook exposition of this classic result

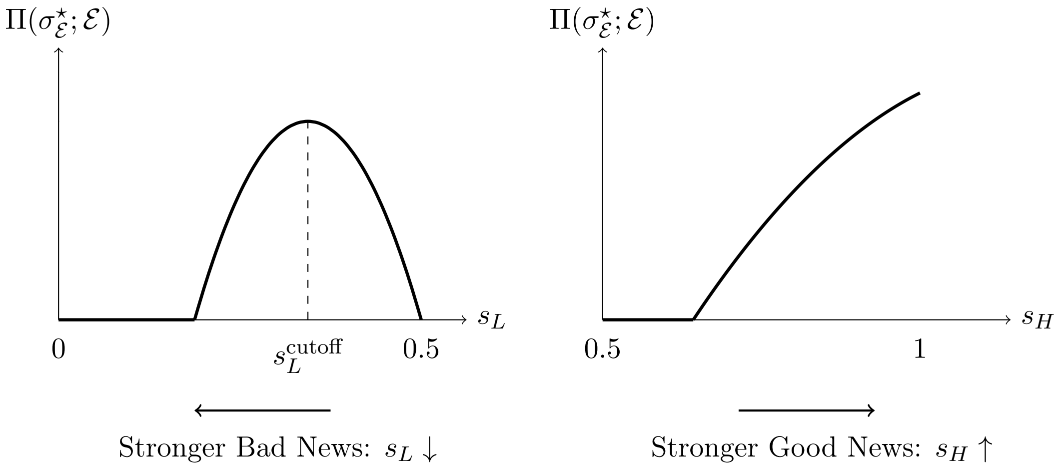



Figure 5.1: Theorem 2 illustrated

Both improvements in buyers' experiment exacerbate the adverse selection problem: with stronger bad news, each previous refusal becomes worse news; and with stronger good news, each buyer refuses more often, thus previous refusals become likelier. So, equilibrium$^\star$ interim beliefs fall. Nonetheless, market efficiency always increases with stronger good news. In contrast, it eventually decreases with stronger bad news.

In the remainder of this section, I first discuss the intuition behind Theorem 2. This intuition is at the heart of this paper, and is shared by Theorem 3. Then, I present Proposition 5, which characterises *when* stronger bad news start decreasing market efficiency.

How changing buyers' experiment $\mathcal{E}$ affects market efficiency depends on its influence on the probability that a seller, either of High or Low quality, trades. Let us start by considering a binary experiment $\mathcal{E}' = (\mathbf{S}', p'_L, p'_H)$ which delivers *stronger good news* than $\mathcal{E}$; where $\mathbf{S}' = \{s'_L, s'_H\}$:

$$\frac{p'_H(s'_L)}{p'_H(s'_L) + p'_L(s'_L)} = s'_L = s_L < s_H < s'_H = \frac{p'_H(s'_L)}{p'_H(s'_L) + p'_L(s'_L)}$$

To simplify intuition, say that in their equilibrium$^\star$ strategies for both experiments, buyers accept to trade upon a high signal, $s_H$ or $s'_H$, but refuse upon a low signal, $s_L$ or $s'_L$. How does this Blackwell improvement affect the seller's chances of trading?

The answer is clearest when we reinterpret the stronger good news buyers receive as an additional piece of evidence they might observe after an initial high signal, $s_H$, from the original experiment $\mathcal{E}$. After an initial low signal, $s_L$, a buyer observes no further evidence. However, after an initial high signal $s_H$, she observes the outcome of an *additional* binary experiment $\mathcal{E}^a = (\{s^a_L, s^a_H\}, p^a_L, p^a_H)$. Conditional on the asset's quality, $\mathcal{E}^a$ is independent from $\mathcal{E}$ and IID across buyers. We engineer it carefully so that when appended to $\mathcal{E}$, it mimics the improvement in information that $\mathcal{E}'$ offers:

$$\frac{s_H}{1-s_H} \times \frac{s^a_H}{1-s^a_H} = \frac{s'_H}{1-s'_H} \qquad\qquad \frac{s_H}{1-s_H} \times \frac{s^a_L}{1-s^a_L} = \frac{s'_L}{1-s'_L}$$

Thus, the signals $(s_H, s_H^a)$ contain information equivalent to the one the stronger good news signal $s_H'$ from experiment $\mathcal{E}'$ does; and $(s_H, s_L^a)$ contain information equivalent to the one that the bad new signal $s_L'$ does.

A buyer who receives initial bad news through the signal $s_L$ observes no additional evidence. She refuses the seller, as before. However, a buyer who receives initial good news through the signal $s_H$ observes additional evidence through $\mathcal{E}^a$. Absent this additional evidence, she would have traded. But a low signal $s_L^a$ from $\mathcal{E}^a$ negatively overrides her initial verdict: she now refuses the seller.

So, stronger good news jeopardises trade: a seller who, before this improvement, would have traded with some buyer may now be refused by every buyer. This raises market efficiency. The seller is refused by every buyer because each observe a low signal, either $s_L$ or $s_L^a$. Upon this signal, each of those buyers—mindful that the seller *may* have received previous refusals—expect negative surplus from trade. If any of those buyers knew that the seller, in fact, *was* refused by *every* buyer, her expectation would only deteriorate:

$$\mathbb{P}\left(\theta = H \mid s_L, n-1 \text{ refusals before visit}\right) < \sum_{k=0}^{n-1} \begin{bmatrix} \mathbb{P}\left(\theta = H \mid s_L, k \text{ refusals before visit}\right) \\ \times \ \mathbb{P}\left(k \text{ refusals before visit} \mid \text{visit}\right) \end{bmatrix}$$

Now, let us turn to an experiment $\mathcal{E}'$ which delivers *stronger bad news* than $\mathcal{E}$. To elucidate the threshold Theorem 2 identifies, I focus on a small improvement of buyers' information—the signal $s_L'$ delivers marginally stronger bad news:

$$s_L' = \frac{p_H'(s_L')}{p_H'(s_L') + p_L'(s_L')} = s_L - \delta < s_H = s_H' = \frac{p_H'(s_H')}{p_H'(s_L') + p_L'(s_L')} \qquad \text{for } \delta \downarrow 0$$

As before, I reinterpret this improvement as an additional piece of evidence buyers observe; this time, after an initial low signal $s_L$. The additional binary experiment $\mathcal{E}^a$ that they observe, is engineered to mimic the improvement $\mathcal{E}'$ offers over $\mathcal{E}$:

$$\frac{s_L}{1-s_L} \times \frac{s_H^a}{1-s_H^a} = \frac{s_H}{1-s_H} \qquad\qquad \frac{s_L}{1-s_L} \times \frac{s_L^a}{1-s_L^a} = \frac{s_L'}{1-s_L'} = \frac{s_L - \delta}{1-(s_L-\delta)}$$

She trades with the seller. However, a buyer who receives initial bad news through the signal $s_L$ observes additional evidence through $\mathcal{E}^a$. Absent this additional evidence, she would have refused the seller. But a high signal $s_H^a$ from $\mathcal{E}^a$ positively overrides her initial verdict: she now trades.

So, stronger bad news encourages trade: a seller who, before this improvement, would be refused by every buyer might now trade with one. The effect this has on market efficiency, however, is less clear. What can we infer about the quality of the seller's asset given stronger bad news allowed her to trade?

Initially, the seller was refused by every buyer; so, all buyers initially observed low signals, $s_L$. After the additional experiment $\mathcal{E}^a$ was introduced, some buyer accepted to trade; so, at least one buyer observed the high signal, $s^a_H$. However, the rest observed the low signal, $s^a_L$, from this additional experiment. Whether trade will raise surplus in expectation depends *how many* buyers observed the high signal:

$$\mathbb{P}(\theta = H \mid \geq 1 \text{ buyer observed } s^a_H) = \sum_{k=1}^{n} \begin{bmatrix} \mathbb{P}(\theta = H \mid k \text{ buyers observed } s^a_H) \\ \times \mathbb{P}(k \text{ buyers observed } s^a_H \mid \geq 1 \text{ buyer obs. } s^a_H) \end{bmatrix}$$

However, we may deduce that almost surely only one buyer observed the high signal $s^a_H$. To see this, note the likelihood ratios of the signals a buyer may observe from the experiment $\mathcal{E}^a$:

$$\frac{s^a_L}{1 - s^a_L} = \frac{\frac{s_L - \delta}{1-(s_L-\delta)}}{\frac{s_L}{1-s_L}} \qquad\qquad \frac{s^a_H}{1 - s^a_H} = \frac{\frac{s_H}{1-s_H}}{\frac{s_L}{1-s_L}}$$

While the likelihood ratio for the high signal $s^a_H$ is constant, that for the low signal $s^a_L$ converges to 1 as $\delta \downarrow 0$—the improvement in buyers' information becomes "smaller". Due to the martingale property of likelihood ratios, these likelihood ratios must average to 1; so $\mathbb{P}(s^a_H)$ must vanish as $\delta \downarrow 0$.

So, the expected surplus from this trade is non-negative if and only if:

$$\lim_{\delta \downarrow 0} \mathbb{P}(\theta = H \mid \geq 1 \text{ buyer observed } s^a_H) - c = \mathbb{P}(\theta = H \mid 1 \text{ buyer observed } s^a_H) - c \geq 0$$
$$\iff \frac{\rho}{1-\rho} \times \left[\frac{s_L}{1-s_L}\right]^{n-1} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c}$$

The interpretation is that marginally stronger bad news allows the most adversely selected seller to trade. Before buyers' information improved, he was refused by every buyer. After, he is accepted by only one. The RHS of the expression above reflects this: trading with this seller increases surplus if and only if the high signal $s^a_H$ observed by that one buyer overpowers the low signals $s^a_L$[23] observed by the remaining $n-1$ buyers:

This condition is stark. We can interpret it as the *irrelevance of adverse selection*: a buyer who observes the high signal need not be concerned about any previous refusals the seller may have received. It is also closely linked to the cutoff Theorem 2 identifies.

**Proposition 5.** Where buyers' experiment is binary, equilibrium$^\star$ market efficiency weakly decreases with stronger bad news (lower $s_L$) when:

$$\frac{\rho}{1-\rho} \times \max\left\{\frac{s_L}{1-s_L}, \left[\frac{s_L}{1-s_L}\right]^{n-1} \times \frac{s_H}{1-s_H}\right\} \leq \frac{c}{1-c}$$

This condition is also necessary in the least selective equilibrium.

**Corollary 6.** Where buyers' experiment is binary and $\rho \leq c$, equilibrium$^\star$ market efficiency weakly decreases with stronger bad news (lower $s_L$) when $\left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} \leq \frac{c}{1-c}$.

[23]Recall that $s^a_L \to s_L$ as $\delta \downarrow 0$.

Market efficiency falls with stronger bad news once bad news is strong ($s_L$ is low) enough to:

- violate the "irrelevance of adverse selection" condition we identified, and
- for buyers to refuse the seller with positive probability in their equilibrium$^\star$ strategies.

For some parameter constellations, the first condition is met before the second; bad news may need to get stronger before buyers refuse the seller with positive probability. Once bad news hits this critical threshold, market efficiency experiences a one time upward jump. Thereafter, stronger bad news decreases market efficiency. When $\rho < c$, there is no equilibrium where the seller always trades; we are left with the first condition.

### 5.2 Finite Experiments

How does a Blackwell improvement of buyers' experiment affect market efficiency when the experiment has finitely but arbitrarily many outcomes? I answer this question here, in Theorem 3.

The ideas we developed in Theorem 2 cannot be deployed immediately here: first, for non-binary experiments, the ideas of stronger good and bad news lose meaning; second, Blackwell improvements of such experiments are complex—they cannot be described by simple movements of likelihood ratios. Nonetheless, the core idea behind Theorem 2 supplies the answer: whether market efficiency improves depends on whether an improvement is a *positive* or *negative override.*

Before I state Theorem 2, I formalise two ideas we developed in the previous section. Definition 3 formalises positive and negative overrides by using *local mean preserving spreads* (Definition 2). Definition 4 formalises the "irrelevance of adverse selection".

**Definition 2.** Enumerate the joint outcome set of the experiments $\mathcal{E}' = (\mathbf{S}', p'_L, p'_H)$ and $\mathcal{E} = (\mathbf{S}, p_L, p_H)$ as $\mathbf{S}' \cup \mathbf{S} = \{s_1, s_2, ..., s_M\}$. The experiment $\mathcal{E}'$ differs from $\mathcal{E}$ by a *local mean preserving spread* (or, *local spread*) *at* $s_j \in \mathbf{S}$ if:

$$p'_\theta(s_j) = 0 \qquad p_\theta(s_{j+1}) = p_\theta(s_{j-1}) = 0 \qquad p'_\theta(s_{j+1}) + p'_\theta(s_{j-1}) = p_\theta(s_j)$$

and $p'_\theta(s) = p_\theta(s)$ for any $s \in \mathbf{S}' \cup \mathbf{S} \setminus \{s_j\}$.

A local spread moves all the probability mass experiment $\mathcal{E}$ places on a particular signal to two new signals—one better news about the asset's quality, one worse. In this sense, we can think of it as providing additional information to a buyer who observes the original signal $s_j \in \mathbf{S}$.

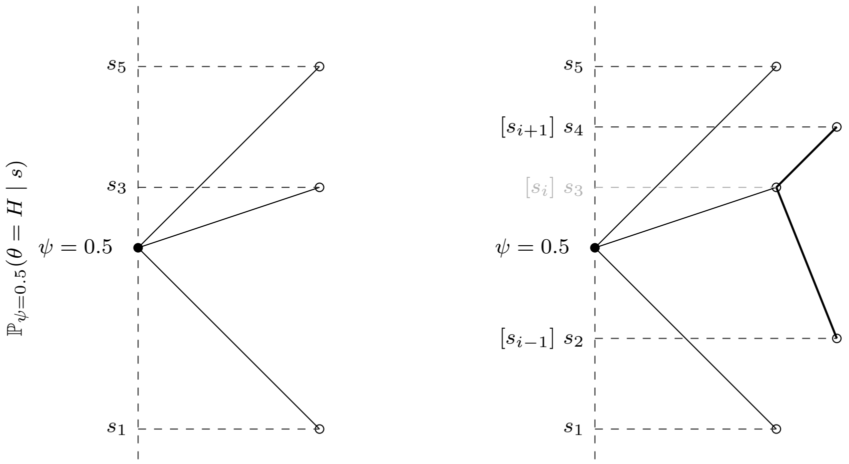
Every local spread is an ordinary *mean preserving spread*[24]. The converse is not true; local spreads can move the probability mass on a signal only to its neighbouring signals: the mass on

[24]Specifically, a "3-part MPS", in the language of Rasmusen and Petrakis, 1992. Mean preserving spreads are originally due to Muirhead, 1900 and were popularised in Economics by the seminal paper Rothschild and Stiglitz, 1970.

$s_j$ is spread to $s_{j+1}$ and $s_{j-1}$. In contrast, the destination signals are arbitrary for a regular mean preserving spread. I visualise the construction of a local spread in Figure 5.2, where I fix $\psi = 0.5$ for convenience.

Though local spreads are a strict subset of ordinary mean preserving spreads, they are without loss for experiments with finite outcome sets[25]—every Blackwell improvement, and *a fortiori*, ordinary mean preserving spread can be constructed through a finite number of local spreads. Remark 1 establishes this.

**Remark 1.** [Müller and Stoyan, 2002, Theorem 1.5.29] An experiment $\mathcal{E}'$ is Blackwell more informative than another, $\mathcal{E}$, if and only if there is a finite sequence of experiments $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_k$ such that $\mathcal{E}_1 = \mathcal{E}$, $\mathcal{E}_k = \mathcal{E}'$, and $\mathcal{E}_{i+1}$ differs from $\mathcal{E}_i$ by a local spread.




(a) Before local spread (b) After local spread

Figure 5.2: A Local Mean Preserving Spread

Having introduced local spreads, we are ready to formally define positive and negative overrides.

**Definition 3.** Let experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j$ and $\sigma^\star_{\mathcal{E}}$ denote the equilibrium$^\star$ strategy under experiment $\mathcal{E}$. I say this local spread is a *negative override* under equilibrium$^\star$ if $\sigma^\star_{\mathcal{E}}(s_j) = 1$, and a *positive override* if $\sigma^\star_{\mathcal{E}}(s_j) = 0$ instead.

Both positive and negative overrides are local spreads of a buyer's experiment, but they differ in *which* seller they inform the buyer about. A negative override is a local spread of a signal upon which the buyer would trade in the equilibrium$^\star$ of her original experiment. A positive override is a local spread of a signal upon which the buyer would refuse trade in the equilibrium$^\star$ of her original experiment.

Lastly before I introduce Theorem 3, I formalise the "irrelevance of adverse selection" condition we identified in the previous section, and extend it to arbitrary finite experiments.

[25]However, local mean preserving spreads are only defined for finite experiments; see Muller and Scarsini, 2001 and Müller and Stoyan, 2002.

**Definition 4.** Where $\sigma$ is a monotone strategy for a fixed experiment $\mathcal{E}$, I say *adverse selection is $\sigma$-irrelevant for signal $s \in \mathbf{S}$* if:

$$\frac{\rho}{1-\rho} \times \left[\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right]^{n-1} \times \frac{s}{1-s} \geq \frac{c}{1-c}$$

When adverse selection is $\sigma$-irrelevant for a signal $s \in \mathbf{S}$, a buyer finds it optimal to trade upon the signal $s \in \mathbf{S}$ even if every other buyer refused the seller—provided those buyers use the strategies $\sigma$.

**Theorem 3.** Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j \in \mathbf{S}$. Equilibrium$^\star$ market efficiency is:

1. weakly greater under $\mathcal{E}'$ if the local spread is a negative override under equilibrium$^\star$.

2. weakly less under $\mathcal{E}'$ if the local spread is a positive override under equilibrium$^\star$, unless adverse selection is $\sigma^\star_{\mathcal{E}}$-irrelevant for signal $s_{j+1}$.

Theorem 3 shows that the effect of a local spread on market efficiency depends on the *kind* of the spread. Negative overrides always increase market efficiency. Positive overrides decrease it—unless adverse selection is irrelevant for a buyer who receives the override.

When proving Theorem 3, a negative override indeed pushes buyers to reject the seller more often in the new equilibrium$^\star$. A positive override pushes buyers to trade more often with him. This is a difficult exercise. It is complicated by the fact that pinpointing movements of buyers' interim beliefs with changes in their experiments is infeasible beyond the simplest cases. Hence, this exercise is infeasible with ordinary mean preserving spreads. Instead, studying "local" spread is crucial to its success. They provide the necessary tractability by allowing equilibrium comparative statics without needing to pinpoint changes in interim beliefs. Furthermore, there is no cost to generality: "local" spreads, like ordinary spreads, form the basis of every Blackwell improvement.

Unlike Theorem 2, Theorem 3 requires knowledge of buyers' equilibrium$^\star$ strategies to identify the effect of an improvement in information on efficiency. In practice, an analyst might want to remain agnostic about equilibrium$^\star$ strategies. To alleviate this concern, Proposition 7 offers a sufficient condition for a positive override to decrease market efficiency in the most selective equilibrium.

**Proposition 7.** Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j$. Market efficiency in the most selective equilibrium is lower under $\mathcal{E}'$ if the following conditions hold:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right) \leq \frac{c}{1-c} \qquad \text{and} \qquad \frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

The sufficient conditions in Proposition 7 strengthen those supplied by Theorem 3. The condition on the left ensures that the local spread is a negative override; it ensures that a buyer rejects the seller upon $s_j$ in the most selective equilibrium, since interim beliefs must always lie below the prior belief $\rho$. The condition on the right strengthens the irrelevance condition for adverse selection. The latter requires trade to be optimal upon signal $s_{j+1}$ even if the seller received $n-1$ rejections earlier. The sufficient condition in Proposition 7, in contrast, requires that it be optimal even if the seller received $n-1$ rejections by buyers who observed the best signal below $s_{j+1}$: $s_j$.

# 6 Maximising Efficiency Through Coarsened Information

Section 5 showed that efficiency might be lower in a market where buyers are better informed; more information in the market accentuates the adverse selection problem each buyer faces, counteracting each buyer's improved ability to screen the asset's quality. This invites a question: if a regulator could coarsen buyers' information—perhaps through banning the use and dissemination of certain data—could this raise efficiency? How should a regulator who can use this tool to maximise efficiency in the market go about this exercise?

In this Section, I consider the problem of a regulator who wishes to garble buyers' experiment $\mathcal{E}$ in order to maximise (the most selective) equilibrium market efficiency. As in the previous section, I take the number of buyers $n$ to be a primitive not a parameter. The regulator can choose any finite garbling $\mathcal{E}^G = \left(\mathbf{S}^G, p_L^G, p_H^G\right)$ of buyers' experiment $\mathcal{E}$; i.e. any finite set of outcomes $\mathbf{S}^G = \left\{s_1^G, s_2^G, ..., s_R^G\right\}$ and probability mass functions $p_\theta^G(.)$ over it such that for some Markov matrix $\mathbf{T}_{m\times R}$:

$$\underbrace{\begin{bmatrix} p_L(s_1) & \cdots & p_L(s_m) \\ p_H(s_1) & \cdots & p_H(s_m) \end{bmatrix}}_{=\mathbf{P}} \times \mathbf{T} = \underbrace{\begin{bmatrix} p_L^G(s_1^G) & \cdots & p_L^G(s_R^G) \\ p_H^G(s_1^G) & \cdots & p_H^G(s_R^G) \end{bmatrix}}_{=\mathbf{P^G}}$$

We can interpret this as a coarsening of the original data available to each buyer, generated with the process that matrix $\mathbf{P}$ summarises. The regulator transforms this data, using the Markov matrix $\mathbf{T}$, into a set of summary statistics. The buyer only observes this set of summary statistics, whose data generating process is now described by the matrix $\mathbf{P^G}$.

Once the regulator chooses the garbled experiment $\mathcal{E}^G$, the game proceeds as before—only, buyers' experiment $\mathcal{E}$ is replaced by $\mathcal{E}^G$. An equilibrium, as before, is a pair $\left(\sigma^G, \psi^G\right)$ such that the strategy $\sigma^G : \mathbf{S}^G \to [0,1]$ is optimal given the interim belief $\psi^G$; and the interim belief $\psi^G$ is consistent with the strategy $\sigma^G$. I call this the game induced by the garbling $\mathcal{E}^G$. I call a garbling $\mathcal{E}^G$ regulator-optimal if efficiency in the most selective equilibrium of the game it induces weakly exceeds equilibrium efficiency in the game induced by any other garbling of $\mathcal{E}$.

In Proposition 9, I show that the regulator coarsens buyers' experiment to a recommender signal which transforms the original signal the buyer would have observed to an "acceptance" or "rejection" recommendation. The regulator wishes to recommend a rejection following every signal a buyer could observe unless adverse selection is irrelevant at that outcome. Before I describe the regulator-optimal garbling in Proposition 9, I describe the special class of garblings in which it is contained—monotone binary garblings which provide optimal recommendations. I then show how we can use the idea of the "irrelevance of adverse selection"—which I used to describe equilibria previously—to describe garblings.

I call a garbling $\mathcal{E}^G$ *monotone binary* if the garbling has two possible outcomes, $|\mathbf{S}^G| = 2$, and there is a cutoff signal $s_{i^*} \in \mathbf{S}$ such that the entries $\{t_{ij}\}$ of matrix $\mathbf{T}_{m\times 2}$ for which $\mathbf{P} \times \mathbf{T} = \mathbf{P^G}$ are:

$$t_{i1} = \begin{cases} 1 & i < i^* \\ \in [0,1] & i = i^* \\ 0 & i > i^* \end{cases} \qquad\qquad t_{i2} = 1 - t_{i1}$$

When defining a monotone binary garbling for which "adverse selection is irrelevant", it will be convenient to refer to the signal $s_{i^*} \in \mathbf{S}$ as the *threshold signal* of the garbling.

A monotone binary garbling gives the buyer an "acceptance recommendation" $s_2^G$ when her original signal realises above a threshold signal $s_{i^*} \in \mathbf{S}$, and a "rejection recommendation" $s_1^G$ whenever it lies below it. Following these recommendations—accepting trade upon the signal $s_2^G$, and rejecting when it upon the signal $s_1^G$—need not be an equilibrium strategy in the game induced by the coarsened experiment $\mathcal{E}^G$. When it does, I say that the garbling $\mathcal{E}^G$ *produces incentive compatible recommendations.*

**Definition 5.** A monotone binary garbling $\mathcal{E}^G$ *produces incentive compatible (IC) recommendations* if the strategy $\sigma^G$, defined below, is an equilibrium strategy in the game induced by $\mathcal{E}^G$:

$$\sigma^G(s^G) := \begin{cases} 0 & s^G = s_1^G \\ 1 & s^G = s_2^G \end{cases}$$

Lemma 8 establishes that the regulator can restrict herself to monotone binary garblings that produce IC recommendations.

**Lemma 8.** Where it exists, the regulator-optimal garbling is monotone binary and produces IC recommendations.

The reason that the regulator can restrict herself to binary garblings is closely connected to a fundamental principle in information design. A buyer ultimately distils the information relayed by the garbled experiment into an action recommendation; she either accepts or rejects. The regulator can distil that information herself—supplying only recommendations to a buyer[26]. Monotone garblings align buyers' decisions better with the regulator's goal of maximising efficiency—it raises

[26]Note that coarsening buyers' experiment $\mathcal{E}$ may also create new equilibrium outcomes, some of which yield lower payoff than the previous least selective equilibrium. Our focus on the most selective equilibrium—besides being the appropriate focus for this exercise—frees us from the need to worry about this complication and utilise this fundamental principle.

the chance that the seller is accepted when the asset has High, but not Low quality. The non-trivial conclusion Lemma 8 establishes that the regulator faces no trade-off between a monotone garbling and one whose recommendations are IC.

Since monotone garblings, like monotone strategies, divide the set of outcomes the experiment $\mathcal{E}$, $\mathbf{S}^G$, into an acceptance and rejection region, we can adopt the "selectivity" order for them as well.

**Definition 6.** A monotone garbling $\mathcal{E}^G$ of $\mathcal{E}$ is *more selective* than another, $\mathcal{E}^{G'}$, if $p_\theta^G\left(s_2^G\right) \leq p_\theta^{G'}\left(s_2^{G'}\right)$ for all $\theta \in \{L, H\}$.

Like monotone strategies, and for the same reason, selectivity is a complete order over the set of monotone binary garblings.

The last idea needed to describe the regulator-optimal garbling is the "irrelevance of adverse selection" for a garbling.

**Definition 7.** Let $\mathcal{E}^G$ be a monotone binary garbling of $\mathcal{E}$, with the threshold signal $s^* \in \mathbf{S}$. *Adverse selection is irrelevant* under $\mathcal{E}^G$ either if:

$$\frac{\rho}{1-\rho} \times \frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{p_H^G(s_1^G)}{p_L^G(s_1^G)}\right)^{n-1} \geq \frac{c}{1-c}$$

or either of the following two conditions hold:

1. $\mathcal{E}^G$ recommends no acceptances; i.e. $p_\theta(s_2^G) = 0$.
2. $\mathcal{E}^G$ recommends no rejections; i.e. $p_\theta(s_1^G) = 0$ and $\frac{\rho}{1-\rho} \times \left(\frac{s_1}{1-s_1}\right)^n \geq \frac{c}{1-c}$.

If this condition is violated, I say that adverse selection is *not* irrelevant under $\mathcal{E}^G$.

This condition translates the corresponding idea we developed to describe strategies to describe garblings—*adverse selection is irrelevant* under a garbling if a buyer who receives an "acceptance" recommendation need not be concerned about the number of buyers who received "rejection" recommendations. This property is also satisfied when the garbling never recommends an "acceptance", or when accepting the seller would yield positive expected surplus regardless of the signals the $n$ buyers could observe under their original experiment.

**Proposition 9.** If the least selective monotone binary garbling under which adverse selection is irrelevant produces IC recommendations, it is the regulator-optimal garbling. Otherwise, the regulator-optimal garbling is either:

- the least selective garbling under which adverse selection is irrelevant, or
- the most selective garbling under which adverse selection is not irrelevant

among monotone binary garblings which produce IC recommendations.

**Corollary 10.** When the seller's reservation value $c$ weakly exceeds the prior belief $\rho$, the regulator-optimal garbling is the least selective monotone binary garbling under which adverse selection is irrelevant.

Proposition 9 reveals that the solution to the regulator's problem takes a striking form: although the regulator wishes to maximise a buyer's *expected* contribution to trade surplus, she focuses on the "worst case" where a buyer is the last to receive the seller.

Proposition 9 is intimately connected to the insight the previous section delivers: unless "adverse selection is irrelevant", information which pushes buyers to accept trade more often can harm efficiency. The regulator whishes to censor such information by coarsening buyers' experiment: if she were not bound by buyers' incentives to follow her recommendations, her optimal garbling would bundle every outcome of the original experiment $\mathcal{E}$ into a "rejection recommendation" unless adverse selection is irrelevant at that outcome. Corollary 10 establishes that in a substantial parameter region—when the seller's reservation value $c$ weakly exceeds buyers' prior belief $\rho$—such recommendations are IC, and hence are adopted by the regulator.

# 7 Ultimatum Price Offers by Buyers

In this Section, I relax the assumption that the seller trades with the first buyer willing to pay his reservation value $c$. I show that this behaviour emerges endogenously in the extended model here, where each buyer the seller visits makes him an ultimatum (take-it-or-leave-it) price offer. Specifically, I show that in the only equilibrium that is robust to the seller's information about the asset's quality, a buyer either offers to pay the seller his reservation value $c$ or nothing—unless a seller of some quality can guarantee trade with the first buyer he visits[27]. To simplify notation, in this extended model I assume that the seller knows the quality of the asset[28]. Proposition 11 establishes that in this extended model, the aforementioned equilibrium is unique. The argument it presents straightforwardly reveals that the proposed equilibrium survives regardless of what the seller knows about the asset's quality.

As before, the seller visits $n$ prospective buyers in a random order. Each buyer he visits makes him a take-it-or-leave-it price offer; the strategy $\omega : \mathbf{S} \to \Delta(\{0\} \cup [c, 1])$ maps every signal a buyer might observe to a distribution over possible price offers. Without loss of generality, I exclude offers in the interval $(0, c)$; since these prices are below the seller's reservation value $c$, they will always be rejected—the buyer might as well offer 0 instead. The seller either takes the offered price $o$, or leaves it and visits the next buyer. If he eventually takes an offer of $o$, he enjoys a payoff

[27] In this latter circumstance, a multiplicity of equilibria with no relevance to our exercise might emerge. In all such equilibria, either some buyer almost surely trades with the seller, or a buyer trades with the seller unless she receives conclusive evidence that the quality is Low (at any signal a High quality seller might generate). In the latter case, buyers might coordinate on some fixed price $c$ that they offer a seller who has not revealed himself Low quality. These have no bearing on trading patterns besides shifting surplus to the seller.

[28] Otherwise, the notation gets more involved since the seller will also consider what a buyer's offer reveals about the asset's quality and therefore what offers he should expect should he continue his visits.

of $o-c$. If he trades with no buyer, he enjoys his reservation value for the asset, $c$. Given the quality $\theta$ of his asset and for each of the $k \in \{1,2,...,n\}$ visits he might make, the seller's strategy $\chi_{k,\theta} : [0,1] \to [0,1]$ is a measurable mapping from his $k^{\text{th}}$ offer to a probability that he takes the offer. I assume that the seller accepts his current offer if and only if his payoff from doing so weakly exceeds his expected continuation payoff; so, each $\chi_{k,\theta} : [0,1] \to [0,1]$ will be a cutoff strategy in equilibrium[29].

Given the strategies $\omega^*$ for each buyer, we can recursively calculate the continuation value the seller expects right before he pays his $k^{\text{th}}$ buyer, $V^*_{k;\theta}$. Denoting the seller's value of not selling the asset as $V^*_{n+1;\theta} := c$, we get:

$$V^*_{k;\theta} := \sum_{s\in\mathbf{S}} p_\theta(s) \times \int_{\{0\}\cup[c,1]} \max\left\{V^*_{k+1;\theta}, m-c\right\} d\omega(s)(m)$$

Furthermore, given the cutoff strategies $\chi^*_{k,\theta} : [0,1] \to [0,1]$ for the seller and the strategy $\omega^*$ for each buyer, we can calculate the probability that a seller of quality $\theta$ does not trade in his $k^{\text{th}}$ visit as:

$$r^e_{k,\theta}(\omega^*;\mathcal{E}) = \sum_{s\in\mathbf{S}} p_\theta(s) \times \int_{\{0\}\cup[c,1]} \left(1-\chi^*_{k,\theta}(m)\right) d\omega(s)(m)$$

Through these probabilities and by using Bayes Rule, the probability that the seller is in his $k^{\text{th}}$ visit by the time he visited a given buyer can be easily calculated; call it $\kappa_{\omega^*}(k)$. Likewise, denote the probability a buyer assigns to the asset having High quality given her observed the signal $s\in\mathbf{S}$ and that the seller is in his $k^{\text{th}}$ visit and as $\iota_{\omega^*}(s)$.

The strategies $\omega^*$ and $\left\{\chi^*_{k,\theta}\right\}_{k=1}^{n}$ for each quality $\theta \in \{L,H\}$ form an equilibrium in this extended model if they maximise the buyers' and the seller's expected payoffs, respectively, given their equilibrium beliefs:

$$\chi^*_{k,\theta}(o) = \begin{cases} 1 & o-c \geq V^*_{k+1,\theta} \\ 0 & o-c < V^*_{k+1,\theta} \end{cases} \qquad \operatorname{supp}\,\omega^*(s) \subseteq \underset{o\in\{0\}\cup[c,1]}{\arg\max} \sum_{k=1}^{n} \kappa_{\omega^*}(k) \times \begin{bmatrix} \iota_{\omega^*}(s)\times(1-o)\times\chi_{k,H}(o) \\ -(1-\iota_{\omega^*}(s))\times o\times\chi_{k,L}(o) \end{bmatrix}$$

**Proposition 11.** Unless a seller of some quality almost surely trades with the first buyer he visits, in the unique equilibrium of the extended model:

- a buyer offers either the seller's reservation value $c$ or nothing; $\operatorname{supp}\,\omega^* \subseteq \{0,c\}$.
- the seller trades with the first buyer who offers his reservation value; $\chi_{k,\theta}(o) = \mathbb{1}\{o \geq c\}$.

*Proof.* The proposed tuple of strategies obviously form an equilibrium. If, almost surely, every buyer offers a price weakly below $c$, no buyer has an incentive to offer anything above it. The seller's continuation value always equals $c$, so he takes the first offer of $c$.

[29]Hence, assuming measurability is without loss.

Now, I prove that this is the unique equilibrium unless a seller of some quality can guarantee with the first buyer he visits. Note that the continuation value $V^*_{k,\theta}$ must be weakly decreasing in $k$ in every equilibrium. Furthermore, it must always lie weakly above $c$. Denote $\theta^* \in \arg\max_{\theta \in \{L,H\}} V^*_{1,\theta}$. If $V_{1,\theta^*} = c$, then the seller is almost surely offered the price 0 or $c$ in any of his visits; we are done. If $V_{1,\theta^*} > c$, any offer $o > V_{1,\theta^*} + c$ is strictly dominated for a buyer: the seller's payoff from taking the offer $o$ is strictly greater than $V_{1,\theta^*}$ so he will also take an offer slightly below $o$. So, the seller can expect to be offered at most $V_{1,\theta^*} + c$. Since his continuation value at the first visit is $V_{1,\theta^*}$, the seller almost surely expects the offer $V_{1,\theta^*} + c$. But this reveals that a seller of quality $\theta^*$ almost surely trades with the first buyer he visits: for all $s \in \text{supp } P_{\theta^*}$, $\omega^*(s) = \delta_{V_{1,\theta^*}+c}$ and the seller immediately trades.

$\square$

# 8 Proof Appendix

## 8.1 Useful Definitions and Notation

In what follows, I occasionally operate with the likelihood ratios of beliefs for convenience. The reader can easily verify the identities:

$$\frac{\psi}{1-\psi} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})} \qquad\qquad \frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} = \frac{\psi}{1-\psi} \times \frac{s_i}{1-s_i}$$

Through similar reasoning, the reader can verify that it is optimal for a buyer to accept trade when:

$$\frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} > \frac{c}{1-c}$$

Some strategies require buyers to randomise upon observing a particular outcome. To facilitate technical discussion, where it is warranted I assume that each buyer observes the realisation of a *tie-breaking signal* $u \sim U[0,1]$ alongside the outcome of her experiment. This signal is not informative about the asset's quality: it is distributed independently from it conditional on the experiment's outcome. I denote the outcome of buyer $i$'s experiment as $s^i$ and her tie-breaking signal as $u^i$. Without loss, buyer $i$ accepts trade if and only if $\sigma(s^i) \leq u^i$; where $\sigma$ is her strategy. I call the pair $(s^i, u^i)$ the *score* buyer $i$ observes for the seller.

**Definition 8.** The tuple $Z^i = (s^i, u^i)$, where $u^i \overset{IID}{\sim} U[0,1]$ is the *score* buyer $i$ observes for the seller. The seller's *score profile* $\mathbf{z}$ is the set of scores each buyer observes; $\mathbf{z} = \{(s^i, u^i)\}_{i=1}^n$. Analogously, the seller's *signal profile* $\mathbf{s} = \{s^i\}_{i=1}^n$ is the set of outcomes of each buyer's experiment.

Some proofs in Section 8.3 require comparing interim beliefs across pairs of strategies and experiments; $(\sigma, \mathcal{E})$. For convenience, I define the mapping from such a pair to the interim belief consistent with them as $\Psi(.;\mathcal{E}) : [0,1]^n \to [0,1]$:

$$\Psi(\sigma;\mathcal{E}) := \frac{\rho \times \nu_H(\sigma;\mathcal{E})}{\rho \times \nu_H(\sigma;\mathcal{E}) + (1-\rho) \times \nu_L(\sigma;\mathcal{E})}$$

Wherever necessary, I treat each strategy $\sigma : \mathbf{S} \to [0,1]$ for an experiment $\mathcal{E}$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$. This is a finite dimensional vector space, so I endow it with the metric induced by the taxicab norm without loss of generality (see Kreyszig, 1978 Theorem 2.4-5):

$$||\sigma' - \sigma|| = \sum_{j=1}^{m} |\sigma'(s_j) - \sigma(s_j)| \qquad\qquad \text{for any two strategies } \sigma' \text{ and } \sigma$$

Note that the interim belief function $\Psi(.;\mathcal{E})$ is thus a continuous function of buyers' strategies[30].

[30] $r_\theta$ is a continuous function of $\sigma$, thus both the nominator and denominator are strictly positive continuous functions of $\sigma$.

**Definition 9.** Where the experiment $\mathcal{E}$ is binary, $s_L^{\text{mute}}$ is the *strongest* level of bad news for which there is an equilibrium where a buyer trades regardless of her signal:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

## 8.2 Omitted Results

**Lemma 12.** Suppose there is a single buyer, $n = 1$. Equilibrium market efficiency under experiment $\mathcal{E}'$ exceeds that under $\mathcal{E}$ *regardless of the seller's reservation value* $c \in [0,1]$ *and buyer's prior belief* $\rho \in [0,1]$ if and only if $\mathcal{E}'$ is (Blackwell) more informative than $\mathcal{E}$.

*Proof.* The sufficiency part of this Lemma follows from Blackwell's Theorem (Blackwell and Girshick, 1954, Theorem 12.2.2). To show necessity, I fix an arbitrary prior belief $\rho$ for the evaluator.

Let $q_j$ be the posterior belief the buyer forms about the asset's quality upon observing the outcome $s_j \in \mathbf{S}$:

$$q_j = \frac{\rho \times s_j}{\rho \times s_j + (1-\rho) \times (1-s_j)}$$

Furthermore, let $F(.)$ and $F'(.)$ be the CDFs of the posterior beliefs $\mathcal{E}$ and $\mathcal{E}'$ induce, respectively, for this prior belief $\rho$:

$$F(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p_H(s)$$
$$F'(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p'_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p'_H(s)$$

Equilibrium market efficiency (and the buyer's expected payoff) under $\mathcal{E}$ is given by:

$$\int_c^1 (q-c)dF(q) = \int_c^1 qdF(q) - c \times (1-F(c)) = (1-c) - \int_c^1 F(q)dq$$

An analogous expression gives equilibrium market efficiency under $\mathcal{E}'$. For the former to exceed the latter for any $c \in [0,1]$, we must have:

$$\int_c^1 \big(F(q) - F'(q)\big)\, dq \geq 0$$

which is equivalent to $\mathcal{E}'$ being Blackwell more informative than $\mathcal{E}$[31].

□

Lemma 13 proves useful when proving Theorem 3, the main result of Section 5.2. This Lemma can also be used towards an alternative and direct proof for the equilibrium existence claim of

[31] See Müller and Stoyan, 2002, Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica, 2016.

Proposition 1.

**Lemma 13.** For each $j \in \{1, 2, ..., m\}$, let $\sigma_j$ be the strategy defined as:

$$\sigma_j(s) = \begin{cases} 0 & s < s_j \\ 1 & s \geq s_j \end{cases}$$

and $\psi_j$ be the interim belief consistent with this strategy. Unless $\sigma_j$ is itself an equilibrium strategy:

i There is an equilibrium strategy $\sigma^*$ that is *more selective than* $\sigma_j$ if $\mathbb{P}_{\psi_j}(\theta = H \mid s_j) < c$.

ii There is an equilibrium strategy $\sigma^*$ that is *less selective than* $\sigma_j$ if $\mathbb{P}_{\psi_j}(\theta = H \mid s_{j-1}) > c$.

*Proof.* Abusing notation slightly, I add two fully revealing outcomes $s_0$ and $s_{m+1}$ to the set $\mathbf{S}$ (duplicating $s_1$ and $s_m$ if either of them are already fully revealing), and denote the strategy which *never* accepts trade as $\sigma_{m+1}$:

$$\frac{s_{m+1}}{1 - s_{m+1}} = \infty \qquad \frac{s_0}{1 - s_0} = 0 \qquad \frac{\psi_{m+1}}{1 - \psi_{m+1}} = \frac{\rho}{1 - \rho}$$

**Claim i.**

The strategy $\sigma_{m+1}$ is the most selective strategy buyers can adopt, and is an equilibrium strategy unless:

$$\frac{s_m}{1 - s_m} \times \frac{\psi_{m+1}}{1 - \psi_{m+1}} > \frac{c}{1 - c}$$

So, assume this condition is satisfied. Likewise, the strategy $\sigma_k$ for $k \geq j$ is an equilibrium if the following inequality is satisfied:

$$\frac{s_k}{1 - s_k} \times \frac{\psi_k}{1 - \psi_k} \geq \frac{c}{1 - c} \geq \frac{s_{k-1}}{1 - s_{k-1}} \times \frac{\psi_k}{1 - \psi_k} \tag{8.1}$$

So, assume inequality 8.1 is violated for every $k \geq j$. This gives us:

$$\frac{s_{m+1}}{1 - s_{m+1}} \times \frac{\psi_{m+1}}{1 - \psi_{m+1}} > \frac{c}{1 - c} > \frac{s_j}{1 - s_j} \times \frac{\psi_j}{1 - \psi_j} \tag{8.2}$$

where the last part of this inequality is by hypothesis.

Now, let $k^* \in \{j, j+1, ..., m\}$ be the first index for which the following inequality is satisfied:

$$\frac{s_{k^*+1}}{1 - s_{k^*+1}} \times \frac{\psi_{k^*+1}}{1 - \psi_{k^*+1}} \geq \frac{c}{1 - c} \geq \frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*}}{1 - \psi_{k^*}}$$

such a $k^*$ must exist due to inequality 8.2. But since inequality 8.1 is violated, we must have:

$$\frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*+1}}{1 - \psi_{k^*+1}} > \frac{c}{1 - c} \geq \frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*}}{1 - \psi_{k^*}}$$

But since the function $\Psi(\sigma;\mathcal{E})$ is continuous in buyers' strategy $\sigma$, we can then find some strategy $\sigma^*$:

$$\sigma^*(s) = \begin{cases} 1 & s > s_{k^*} \\ \in [0,1] & s = s_{k^*} \\ 0 & s < s_{k^*} \end{cases}$$

such that:

$$\frac{s_{k^*+1}}{1-s_{k^*+1}} \times \frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})} \geq \frac{c}{1-c} = \frac{s_{k^*}}{1-s_{k^*}} \times \frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})}$$

The strategy $\sigma^*$ is thus an equilibrium strategy. It is clearly more selective than $\sigma_j$; since it is more selective than $\sigma_{k^*}$, where $k^* \geq j$.

**Claim ii.**

For any $k \in \{1, 2, ..., j\}$, the strategy $\sigma_k$ is an equilibrium if the inequality 8.1 is satisfied. So, as earlier, assume 8.1 is violated for every such $k$. Then, we get:

$$\frac{s_j}{1-s_j} \times \frac{\psi_j}{1-\psi_j} \geq \frac{s_{j-1}}{1-s_{j-1}} \times \frac{\psi_j}{1-\psi_j} > \frac{c}{1-c} > \frac{s_1}{1-s_1} \times \frac{\psi_1}{1-\psi_1}$$

where the second inequality in the chain follows by hypothesis and the last inequality follows from the violation of inequality 8.1 for $k = 1$. We can now repeat the argument we constructed after inequality 8.2 to prove Claim i, to prove the existence of an equilibrium strategy $\sigma^*$ that is less selective than $\sigma_j$.

$\square$

## 8.3 Omitted Proofs

**Proposition 1.** Let $\Sigma$ be the set of equilibrium strategies. Then:

1. $\Sigma$ is non-empty and compact.

2. Any equilibrium strategy $\sigma^*$ is *monotone*; for any $\sigma^* \in \Sigma$, $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.

3. All equilibria exhibit *adverse selection*: $\psi^* \leq \rho$ for any interim belief $\psi^*$ consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

*Proof.* In what follows, I treat each strategy $\sigma : \mathbf{S} \to [0,1]$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$, endowed with the taxicab metric (see the end of Section 8.1). I start by proving that any equilibrium strategy must be monotone and all equilibria exhibit adverse selection. Using these observations, I prove that the set of equilibrium strategies is non-empty and compact.

2. Any equilibrium strategy is monotone.

Any equilibrium strategy $\sigma^*$ must be optimal against the interim belief $\psi^*$ consistent with it. Whenever $\rho \in (0,1)$, $\psi^* = \Psi(\sigma^*;\mathcal{E}) \in (0,1)$, and so $\mathbb{P}_{\psi^*}(\theta = H \mid s') > \mathbb{P}_{\psi^*}(\theta = H \mid S = s)$ for $s', s \in \mathbf{S}$ such that $s' > s$.

3. All equilibria exhibit adverse selection.

*A fortiori*, $\Psi(\sigma;\mathcal{E}) \leq \rho$ for any monotone strategy $\sigma$. To see this, note that $p_H(.)$ first order stochastically dominates $p_L(.)$ since it likelihood ratio dominates it[32]. Therefore, $\nu_L(\sigma;\mathcal{E}) \geq \nu_H(\sigma;\mathcal{E})$. The result then follows since $\frac{\Psi(\sigma;\mathcal{E})}{1-\Psi(\sigma;\mathcal{E})} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})}$.

1. The set of equilibrium strategies is non-empty and compact.

i The set of equilibrium strategies is non-empty.

Define $\Phi(.) : [0,1]^m \to 2^{[0,1]^m}$ to be the buyers' *best response correspondence.* $\Phi(.)$ maps any strategy $\sigma$ to the set of strategies that are optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ it induces:

$$\Phi(\sigma) = \left\{\sigma' \in [0,1]^m : \sigma' \text{ is optimal against } \Psi(\sigma;\mathcal{E})\right\}$$

A strategy $\sigma^*$ is an equilibrium strategy if and only if it is a fixed point of buyers' best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I establish that the correspondence $\Phi$ has at least such fixed point through Kakutani's Fixed Point Theorem.

$\Phi$ is trivially non-empty; every interim belief has some strategy optimal against it. It is also convex valued; if two distinct approval probabilities are optimal after some outcome $s \in \mathbf{S}$, *any* approval probability is optimal upon that outcome.

The only task that remains is to prove that $\Phi$ is upper-semi continuous. For this, take an arbitrary sequence of strategies $\{\sigma_n\}$ such that $\sigma_n \to \sigma_\infty$. Denote the interim beliefs consistent with these strategies as $\psi_n := \Psi(\sigma_n;\mathcal{E})$. Since $\Psi(.;\mathcal{E})$ is continuous in buyers' strategies, we also have $\psi_n \to \psi_\infty$ where $\psi_\infty = \Psi(\sigma_\infty;\mathcal{E})$. Now, take a sequence of strategies $\{\sigma_n^*\}$ where $\sigma_n^* \in \Phi(\sigma_n)$. Note that every $\sigma_n^*$ is monotone since optimality against any interim belief $\psi \in (0,1)$ requires monotonicity. We want to show that $\Phi$ is upper semicontinuous; i.e.:

$$\sigma_n^* \to \sigma_\infty^* \implies \sigma_\infty^* \in \Phi(\sigma_\infty)$$

By the Monotone Subsequence Theorem, the sequence $\{\sigma_n^*\}$ has a subsequence $\sigma_{n_k}^* \to \sigma_\infty^*$ of strategies whose norms $||\sigma_{n_k}^*||$ are monotone in their indices $n_k$. Here I take the case where these norms are increasing, the proof is analogous for the opposite case. Since $\sigma_\infty^*$ is the limit of a subsequence of monotone strategies, it must be a monotone strategy too. Assuming otherwise leads to a contradiction; for any $s, s' \in \mathbf{S}$ such that $s' > s$:

$$\sigma_\infty^*(s) > 0 \ \&\ \sigma_\infty^*(s') < 1 \quad \implies \quad \exists N \in \mathbb{N} \text{ s.t. } \forall\, n_k \geq N \quad \sigma_{n_k}^*(s) > 0 \ \&\ \sigma_{n_k}^*(s') < 1$$

[32] Theorem 1.C.1 in Shaked and Shanthikumar, 2007.

Now let $\bar{s}$ be the highest outcome for which $\sigma^*_\infty(\bar{s}) > 0$. I show that:

- If $\sigma^*_\infty(\bar{s}) \in (0,1)$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

- If $\sigma^*_\infty(\bar{s}) = 1$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s}{1-s} \begin{cases} \leq \frac{c}{1-c} & s < \bar{s} \\ \geq \frac{c}{1-c} & s \geq \bar{s} \end{cases}$$

The first case easily follows by noting that:

$$\sigma^*_\infty(\bar{s}) \in (0,1) \implies \sigma^*_{n_k}(\bar{s}) \in (0,1) \implies \frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c} \implies \frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

for all $n_k \geq N' \in \mathbb{N}$. The second case follows similarly, by noting that $\sigma^*_\infty(\bar{s}) = 1$ and $\sigma^*_\infty(s') = 0$ for all $s' < \bar{s}$ implies $\sigma^*_{n_k}(\bar{s}) > 0$ and $\sigma^*_{n_k}(s') = 0$ for all $n_k \geq N'' \in \mathbb{N}$.

ii The set of equilibrium strategies is compact.

$\Sigma$ is a subset of $[0,1]^m$ and therefore bounded, hence it suffices to show that is closed. Let $\{\sigma^*_n\}$ be a sequence of equilibrium strategies. Note that this means $\sigma^*_n \in \Phi(\sigma^*_n)$. Since $\Phi(.)$ is upper semicontinuous, $\sigma^*_n \to \sigma_\infty$ implies $\sigma_\infty \in \Phi(\sigma_\infty)$, and therefore an equilibrium strategy itself.

□

**Proposition 3.** Equilibrium market efficiency is bounded above by the *full information* benchmark $\Pi^f$ and below by the *no information* benchmark $\Pi^\emptyset$. Furthermore, market efficiency is higher under more selective equilibrium strategies; where $\sigma^*$ and $\sigma^{**}$ are two equilibrium strategies such that $\sigma^{**}$ is more selective than $\sigma^*$:

$$\max\{0, \rho - c\} = \Pi^\emptyset \leq \Pi(\sigma^{**}; \mathcal{E}) \leq \Pi(\sigma^*; \mathcal{E}) \leq \Pi^f = \rho \times (1-c)$$

*Proof.* This is an immediate corollary to Lemmas 4 and 14 below; both of independent interest. □

**Lemma 14.** Take three monotone strategies $\sigma'', \sigma'$ and, $\sigma$, ordered from the least selective to the most. If $\Pi(\sigma'; \mathcal{E}) \leq \Pi(\sigma; \mathcal{E})$, then $\Pi(\sigma''; \mathcal{E}) \leq \Pi(\sigma'; \mathcal{E})$.

*Proof.* For the three strategies $\sigma'', \sigma'$, and $\sigma$, consider three sets $Z, Z', Z'' \subset (S \times [0,1])^n$ where the seller's score profile $\mathbf{z}$ might lie:

$$\mathbf{z} \in \begin{cases} Z & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma'' \text{ but not } \sigma \\ Z' & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma' \text{ but not } \sigma \\ Z'' & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma'' \text{ but not } \sigma' \end{cases}$$

Notice that $Z' \cap Z'' = \emptyset$ and $Z' \cup Z'' = Z$. We can write the difference between market efficiency under different strategies as:

$$\Pi(\sigma'; \mathcal{E}) - \Pi(\sigma; \mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z'\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) - c\right]$$

and:

$$\Pi(\sigma''; \mathcal{E}) - \Pi(\sigma'; \mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z''\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) - c\right]$$

Therefore we want to prove that:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \leq c \implies \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \leq c$$

Now, note that $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z)$ is a convex combination of $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z')$ and $\mathbb{P}(\theta = H \mid \mathbf{z} \in Z'')$. Furthermore:

$$\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) \geq \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z \cap Z''\right) = \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right)$$

which then implies:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \leq \mathbb{P}(\theta = H \mid \mathbf{z} \in Z) \leq \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \leq c$$

$\square$

**Lemma 4.** Let $\sigma^*$ and $\sigma$ be two monotone strategies, where $\sigma^*$ is more selective than $\sigma$. If $\sigma^*$ is an equilibrium strategy, then $\Pi(\sigma^*; \mathcal{E}) \geq \Pi(\sigma; \mathcal{E})$.

*Proof.* Let $\mathbf{z}$ be the seller's *score profile.* Take an equilibrium strategy $\sigma^*$ and a less selective strategy $\sigma$ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & s = \underline{s} \\ 0 & \text{otherwise} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. I show that:

$$\lim_{\varepsilon \to 0} \Pi(\sigma; \varepsilon) - \Pi(\sigma^*; \varepsilon) \leq 0$$

By Lemma 14, this establishes the result.

Now, let $Z \subset (S \times [0,1])^n$ be the set of score profiles under which some buyer trades under $\sigma$, but all buyers reject the seller under $\sigma^*$:

$$\mathbf{z} \in Z \iff \begin{array}{rl} \sigma^*(s^i) > u^i & \text{for all } i \in \{1, 2, \ldots, n\}, \\ & \text{and} \\ \sigma(s^i) \leq u^i & \text{for some } i \in \{1, 2, \ldots, n\}. \end{array}$$

Furthermore, for a given score profile $\mathbf{z}$, let $\#$ be the number of buyers whose observed scores are such that $\sigma(s^i) \geq u^i > \sigma^*(s^i)$. These buyers would accept trade under the strategy $\sigma$, but not under $\sigma^*$.

The seller's eventual outcome differs between the strategy profiles $\sigma$ and $\sigma^*$ if and only if his score profile $\mathbf{z}$ lies in $Z$. Furthermore, his eventual outcome can only change from a rejection by all buyers in $\sigma^*$ to an approval by some buyer in $\sigma$. Thus:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) - \Pi(\sigma^*;\mathcal{E}) &= [\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c] \times \mathbb{P}(\mathbf{z} \in Z) \\ &\propto \mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c\end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the seller's signal profile lies in $Z$:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z\right) = \sum_{i=1}^{n} \mathbb{P}\left(\theta = H \mid \# = i\right) \times \frac{\mathbb{P}\left(\# = i\right)}{\mathbb{P}(\mathbf{z} \in Z)}$$

Now note:

$$\mathbb{P}\left(\# = i \mid \theta\right) = \left(p_\theta(\underline{s})\right)^i \times \left(1 - p_\theta(\underline{s})\right)^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}\left(\# = i\right) \propto \varepsilon^i$. Since $\mathbb{P}(\mathbf{z} \in A) = \sum_{i=1}^{n} \mathbb{P}\left(\# = i\right)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\#=i)}{\mathbb{P}(\mathbf{z} \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \mathbf{z} \in A\right) - \mathbb{P}\left(\theta = H \mid \# = 1\right) = 0$$

I conclude the proof by showing that $\mathbb{P}\left(\theta = H \mid \# = 1\right) \leq c$ as $\varepsilon \to 0$:

$$\begin{aligned}\lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H \mid \# = 1\right)}{\mathbb{P}\left(\theta = L \mid \# = 1\right)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \frac{\mathbb{P}\left(\# = 1 \mid \theta = H\right)}{\mathbb{P}\left(\# = 1 \mid \theta = L\right)} \\ &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\ &= \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \left(\frac{r_H(\sigma^*;\mathcal{E})}{r_L(\sigma^*;\mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\ &\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \leq \frac{c}{1 - c}\end{aligned}$$

where $\psi^* = \Psi(\sigma^*;\mathcal{E})$ is the interim belief consistent with $\sigma^*$. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{\rho}{1 - \rho} \times \frac{1 + r_H^* + ... + (r_H^*)^{n-1}}{1 + r_L^* + ... + (r_L^*)^{n-1}} \leq \frac{\rho}{1 - \rho} \times \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*;\mathcal{E})$. The last inequality is due to the fact that $\underline{s} \in S$ is optimally rejected under $\sigma^*$.

□

**Theorem 1.** Let $\Pi^n\left(\hat{\sigma};\mathcal{E}\right)$ be the efficiency of a market with $n$ buyers and under the most selective equilibrium. If $\mathcal{E}$ has an outcome that fully reveals High quality ($s_m = 1$), the sequence $\{\Pi^n(\hat{\sigma}; \mathcal{E})\}_{n=1}^{\infty}$ is eventually increasing and converges to full information efficiency. Otherwise, it is eventually decreasing and converges to no information efficiency.

*Proof.* For each $j \in \{1, 2, ..., m\}$, define $\sigma_j$ to be the strategy:

$$\sigma_j(s) := \begin{cases} 0 & s < s_j \\ 1 & s \geq s_j \end{cases}$$

Moreover, let $F_\theta(x) = \sum_{s \in \mathbf{S}: s \leq x} p_\theta(x)$. In a market with $n$ buyers, the interim belief $\psi_{j:n}$ consistent with buyers using the strategies $\sigma_j$ is then implicitly given by:

$$\frac{\psi_{j:n}}{1 - \psi_{j:n}} = \frac{\sum_{k=0}^{n-1} F_H(s_{j-1})^k}{\sum_{k=0}^{n-1} F_L(s_{j-1})^k}$$

Note that for all $j > 1$, the RHS is bounded and strictly decreasing in $n$, so the sequence $\{\psi_{j:n}\}$ is convergent.

**Case 1:** $s_m = 1$.

To prove the Theorem's statement for this case, I first show that $\psi_{m:n} \xrightarrow{n} 0$. Let $X_n$ be the random variable that is uniformly distributed over the set $\left\{F_H(s_{m-1})^k\right\}_{k=0}^{n-1}$. Then, note that:

$$\frac{\psi_{m:n}}{1 - \psi_{m:n}} = \frac{\sum_{k=0}^{n-1} F_H(s_{m-1})^k}{\sum_{k=0}^{n-1} F_L(s_{m-1})^k} = \mathbb{E}[X_n]$$

Now, fix any $x > 0$. Since $F_H(s_{m-1})^k$ is strictly decreasing in $k$, for any $\delta < 1$ of our choice, we can find some $N_{x;\delta} \in \mathbb{N}$ such that for all $n \geq N_{x;\delta}$ implies $\mathbb{P}(X_n \leq x) \geq \delta$ and $\mathbb{E}[X_n] \leq \delta x + (1 - \delta)$. Fixing $x = \frac{\varepsilon}{2\delta}$ for some $\varepsilon > 0$ small, we have $\mathbb{E}[X_n] \leq \frac{\varepsilon}{2} + 1 - \delta$. Since we can take $\delta$ arbitrarily close to 1, this shows that $\mathbb{E}[X_n] \to 0$, proving this first claim.

Since buyers must always accept to trade upon observing $s_m = 1$ in equilibrium, this implies that there is some $N \in \mathbb{N}$ for which $\sigma_m$ is the most selective equilibrium strategy for all $n \geq N$. So, for $n \geq N$, a seller with a Low quality asset never trades. Moreover, every additional buyer increases the probability that a seller with a High quality asset trades. As $n \to \infty$, such a seller trades almost surely. We thus prove that $\Pi^n(\hat{\sigma}; \mathcal{E}) \to \Pi^f$.

**Case 2:** $s_m < 1$

The case where buyers' experiment $\mathcal{E}$ is uninformative is trivial; it always yields the no-information benchmark. So, I assume that $s_m \neq s_1$.

For any $j \in \{1, 2, ..., m\}$, the sequence $\left\{\frac{\psi_{j;n}}{1-\psi_{j;n}} \times \frac{s_j}{1-s_j}\right\}_{n=1}^{\infty}$ is bounded and monotone decreasing, thus convergent. Let $\mathcal{L}_j$ be the limit of this sequence. If $\mathcal{L}_m < \frac{c}{1-c}$, by Lemma 13, there is some $N' \in \mathbb{N}$ such that for all $n \geq N'$, the most selective equilibrium is more selective than $\sigma_m$. So, buyers must be indifferent when they trade—expected trade surplus must be 0. Since market efficiency is bounded below by $\Pi^\emptyset$, we conclude that $\Pi^n(\hat{\sigma}; \mathcal{E}) = \Pi^\emptyset = 0$ for all $n \geq N'$.

Now consider the case $\mathcal{L}_m \geq \frac{c}{1-c}$. Since $\psi_{j:n}$ is decreasing in $n$, the most selective equilibrium with $n$ buyers, $\hat{\sigma}_n$, must get weakly more selective with $n$. So, the sequence $\{r_\theta(\hat{\sigma}_n;\mathcal{E})\}_{n=1}^{\infty}$ is weakly increasing in $n$, convergent, and Cauchy. If for any $N \in \mathbb{N}$, $\hat{\sigma}_N$ is more selective than $\sigma_m$, we are done by the argument in the preceding paragraph. Otherwise, the sequence $\{r_\theta(\hat{\sigma}_n;\mathcal{E})\}_{n=1}^{\infty}$ converges to a number below 1. If they both converge to 0, we are done—market efficiency is equal to the no-information benchmark along the sequence. Otherwise, $r_L(\hat{\sigma}_n;\mathcal{E}) > r_H(\hat{\sigma}_n;\mathcal{E})$ along the sequence. Since both sequences are Cauchy, there exists some $N \in \mathbb{N}$ and $M \geq N$ such that for all $m \geq M$, we have:

$$\begin{aligned}
&\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_m;\mathcal{E})]^m - (1-\rho) \times c \times [1 - r_L(\hat{\sigma}_m;\mathcal{E})^m] \\
\approx&\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_N;\mathcal{E})]^m - (1-\rho) \times c \times [1 - r_L(\hat{\sigma}_N;\mathcal{E})^m] \\
>&\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_N;\mathcal{E})]^{m+1} - (1-\rho) \times c \times \left[1 - r_L(\hat{\sigma}_N;\mathcal{E})^{m+1}\right] \\
\approx&\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_{m+1};\mathcal{E})]^{m+1} - (1-\rho) \times c \times \left[1 - r_L(\hat{\sigma}_{m+1};\mathcal{E})^{m+1}\right]
\end{aligned}$$

This proves that market efficiency is eventually decreasing.

Furthermore, since there is at least one outcome of the experiment $\mathcal{E}$ where a buyer surely trades with the seller, as $n \to \infty$, the seller trades almost surely regardless of quality. Hence, market efficiency converges to $\rho - c$. Since market efficiency can never be negative, it must be that $\Pi^{\emptyset} = \rho - c$ in this case.

□

**Theorem 2.** Let buyers' experiment $\mathcal{E}$ be binary. Then, equilibrium$^\star$ market efficiency is increasing in the strength of good news ($s_H$) but is quasiconcave and eventually decreasing in the strength of bad news ($s_L$).

I will use Lemmas 15, 17, and 18 below, possibly of independent interest, to prove Theorem 2. Throughout, I denote the most and least selective equilibrium strategies under the experiment $\mathcal{E}$ as $\hat{\sigma}^*_{\mathcal{E}}$ and $\check{\sigma}^*_{\mathcal{E}}$, respectively. I drop the subscript whenever the experiment in question is obvious.

**Lemma 15.** Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. $\Psi(\sigma;\mathcal{E})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

*Proof. Part i:*

Let $\sigma(s_L) \in (0,1)$ and $\sigma(s_H) = 1$. The interim belief $\Psi(\sigma;\mathcal{E})$ is then given by:

$$\begin{aligned}\Psi(\sigma;\mathcal{E}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right)\\ &= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]\\ &= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every $s_L$ signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) =& \,\mathbb{P}\left(\text{buyer was (i+1)}^{\text{st}} \text{ in order } \mid \text{seller got i rejections}\right)\\ &\times \mathbb{P}\left(\text{seller got i rejections}\right)\\ =& \frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus $\Psi(\sigma;\mathcal{E})$ increases, in $\sigma(s_L)$.

*Part ii:*

Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma;\mathcal{E}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma;\mathcal{E}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}\Psi(\sigma;\mathcal{E}) &\propto \frac{1 + r_H + ... + r_H^{n-1}}{1 + r_L + ... + r_L^{n-1}}\\ &= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_L}{1 - r_H} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)}\\ &\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - (1 - p_H(s_H)\sigma(s_H))^n}{1 - (1 - p_L(s_H)\sigma(s_H))^n}\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1 - s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1 - (r_H)^n}{1 - (r_L)^n}$$

The positive term is the likelihood ratio of one $s_H$ signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since acceptances only happen with $s_H$

signals, the negative term strictly exceeds the positive term. This can be verified directly, too:

$$\frac{1-(r_H)^n}{1-(r_L)^n} > \frac{s_H}{1-s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} \iff \frac{1-(r_H)^n}{1-(r_L)^n} \times \frac{1-r_L}{1-r_H} > \left(\frac{r_H}{r_L}\right)^{n-1}$$
$$\iff \frac{1+...+(r_H)^{n-1}}{1+...+(r_L)^{n-1}} > \left(\frac{r_H}{r_L}\right)^{n-1}$$

The last inequality can be verified easily. Thus, $\Psi(\sigma;\mathcal{E})$ decreases in $\sigma(s_H)$.

□

The Corollary below follows from Lemma 15. Let both $\mathcal{E}'$ and $\mathcal{E}$ are binary experiments, where the former is Blackwell more informative than the latter. If, under both experiments, every buyer accepts upon "good news" and rejects upon "bad news", the interim belief under $\mathcal{E}'$ is lower.

**Corollary 16.** Let $\mathcal{E}'$ and $\mathcal{E}$ be two binary experiments, where the former is Blackwell more informative than the latter. Let the strategies $\sigma'$ and $\sigma$ for these respective experiments be defined as:

$$\sigma'(s') := \begin{cases} 0 & s' = s'_L \\ 1 & s' = s'_H \end{cases} \qquad \sigma(s) := \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

Then, $\Psi(\sigma';\mathcal{E}') \leq \Psi(\sigma;\mathcal{E})$.

*Proof.* Establishing that this holds for a pair $(\mathcal{E}',\mathcal{E})$ for which either (i) $s'_H > s_H$ and $s_L = s'_L$, or (ii) $s'_H = s_H$ and $s_L > s'_L$ suffices. I will only prove the first case, the second is analogous. Below I show that the outcome induced by $\sigma$ under experiment $\mathcal{E}$ can be replicated by some strategy $\tilde{\sigma}$ under experiment $\mathcal{E}'$, where $\tilde{\sigma}(s_L) > 0$ and $\tilde{\sigma}(s_H) = 1$. Then, the desired conclusion follows from Lemma 15.

Take the pair $(\sigma,\mathcal{E})$. The probabilities that a buyer accepts or rejects trade, conditional on $\theta$, is given by:

$$\frac{\mathbb{P}_\sigma(\text{rejected} \mid \theta = H)}{\mathbb{P}_\sigma(\text{rejected} \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}_\sigma(\text{accepted} \mid \theta = H)}{\mathbb{P}_\sigma(\text{accepted} \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma},\mathcal{E}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}_{\tilde{\sigma}}(\text{rejected} \mid \theta = H)}{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = H)}{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = L)} = \frac{p'_H(s_H)+\tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H)+\tilde{\sigma}(s_L)p'_L(s_L)}$$

where $\{p'_L, p'_H\}$ are the distributions for the experiment $\mathcal{E}'$. It is easy to verify that the expression on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma;\mathcal{E})$.

□

**Lemma 17.** Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. Let $\sigma^\star$ be buyers' equilibrium strategy, either in the most or least selective equilibrium. Then, $\sigma^\star \in \{0, 1\}$.

*Proof.* I start by proving this for the least selective equilibrium; i.e. $\check{\sigma}^*(s_L) \in \{0, 1\}$. For $s_L^{\text{mute}}$ defined in Definition 9, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium; so we must have $\check{\sigma}^*(s_L) = 1$. The strategy $\sigma$ defined by $\sigma(s_H) = \sigma(s_L) = 1$ gives rise to the interim belief $\Psi(\sigma; \mathcal{E}) = \rho$, which in turn renders approving upon the outcome $s_L$ optimal. In turn, if $s_L < s_L^{\text{mute}}$, we must have $\sigma^*(s_L) = 1$ for any equilibrium strategy; since the equilibrium interim belief always lies below the prior belief (Proposition 1).

Now consider the most selective equilibrium strategy; $\hat{\sigma}^*$. For contradiction, let $1 > \hat{\sigma}^*(s_L) > 0$ and $\hat{\sigma}^*(s_H) = 1$. Lemma 15 establishes that the interim belief falls as $\sigma(s_L)$ falls; which implies there must be another, more selective equilibrium strategy $\sigma^*$ such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) = 1$. ☐

Lemma 17 establishes that when their experiment $\mathcal{E}$ is binary, buyers *never* mix upon seeing "bad news", $s = s_L$, neither in the most nor the least selective equilibrium. Following up, Lemma 18 establishes that a more informative binary experiment pushes buyers to reject upon bad news in both equilibria.

**Lemma 18.** Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. Buyers' acceptance probabilities upon "bad news", $s = s_L$, in the least and most selective equilibrium strategies are given by:

$$\check{\sigma}^*(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \hat{\sigma}^*(s_L) = \begin{cases} 1 & s_L < s_L^\dagger(s_H) \\ 0 & s_L \geq s_L^\dagger(s_H) \end{cases}$$

where $s_L^\dagger(.)$ is an increasing function of $s_H$, and $s_L^\dagger(s_H) \geq s_L^{\text{safe}}$.

*Proof.* Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1-s_L} \geq \frac{c}{1-c}$$

which, combined with Lemma 17, proves the part of the Lemma for the selective equilibrium.

Now, define the strategies $\sigma_0$ as $\sigma_1$ as:

$$\sigma_0(s) = \begin{cases} 0 & s = s_L \\ 0 & s = s_H \end{cases} \qquad \sigma_1(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

A necessary and sufficient condition for an equilibrium $\sigma^*$ where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

Sufficiency follows since either:

$$\frac{\Psi(\sigma_0; \mathcal{E})}{1 - \Psi(\sigma_0; \mathcal{E})} \times \frac{s_H}{1 - s_H} \leq \frac{c}{1 - c}$$

which implies $\sigma_0$ is an equilibrium, or there is an equilibrium strategy $\sigma^*$ such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) > 0$ by Lemma 15. The condition is necessary, since any strategy that is less selective than $\sigma_1$ induces a higher interim belief, by Lemma 15.

By Corollary 16, whenever this necessary and sufficient condition holds for an experiment $\mathcal{E}$, it also holds for a (Blackwell) more informative experiment $\mathcal{E}'$. Moreover, whenever the low signals are rejected in the least selective equilibrium, they must be in the most selective equilibrium. This concludes the proof.

□

*Proof, Theorem 2:* By Lemma 18, Blackwell improving buyers' experiment shifts both their least selective and most selective equilibrium strategies once from *always* accepting trade to rejecting upon the low signal. By Lemma 14, this shift in buyers' strategy increases efficiency—and therefore each buyer's expected surplus.

Let $\{\sigma_\alpha\}_{\alpha \in [0,1]}$ be the family of strategies where buyers reject upon the low signal:

$$\sigma_\alpha(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

By Lemma 15, the interim belief $\psi_\alpha$ that the strategy $\sigma_\alpha$ induces is strictly decreasing in $\alpha$. Thus, at most one of these can be an equilibrium strategy for a given experiment. Furthermore, whenever buyers' expected surplus from $\sigma_1$ is weakly positive, this must be the equilibrium strategy; decreasing $\alpha$ can only make approving upon the high signal *more* profitable. Hence, whenever buyers reject upon the low signal in equilibrium, efficiency is given by: $\Pi(\sigma^*; \mathcal{E}) = \max\{0, \Pi(\sigma_1; \mathcal{E})\}$. The Theorem then follows from the Claim below:

**Claim.** $\max\{0, \Pi(\sigma_1; \mathcal{E})\}$ is:

i weakly increasing in $s_H$ whenever there is some equilibrium strategy $\sigma^*$ s.t. $\sigma^*(s_L) = 0$.

ii hump-shaped in $s_L$. As $s_L$ falls, it:

- weakly increases when $s_L \geq s_L^{as}$,
- weakly decreases when $s_L \leq s_L^{as}$

where $s_L^{as}$ is defined implicitly as:

$$\frac{\rho}{1 - \rho} \times \left( \frac{s_L^{\text{as}}}{1 - s_L^{\text{as}}} \right)^{n-1} \times \frac{s_H}{1 - s_H} = \frac{c}{1 - c}$$

*Proof of the Claim.*

**Part i.** Increasing the strength of good news; i.e. $s_H$.

Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary experiments with outcome sets $\mathbf{S} = \{s_L, s_H\}$ and $\mathbf{S}' = \{s'_L, s'_H\}$. The experiment $\mathcal{E}'$ carries *marginally stronger good news* than experiment $\mathcal{E}$:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small $\delta$ such that $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma'_1; \mathcal{E}') > \Pi(\sigma_1; \mathcal{E})$; where $\sigma'_1$ is defined analogously to $\sigma_1$ for experiment $\mathcal{E}'$.

**Step 1.** Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

Rather than observing the outcome of experiment $\mathcal{E}'$, say a buyer initially observes her original signal $s$, and then potentially an additional auxilliary signal $\hat{s}$. The first signal she receives, $s$, records the outcome of $\mathcal{E}$. If the low outcome $s_L$ materialises, the buyer observes no more information. If, however, the high outcome $s_H$ materialises, she then observes the additional auxiliary signal $\hat{s}$. This auxiliary signal records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. The outcome of $\hat{\mathcal{E}}$ is independent both from $s$ and anything else any other buyer observes. Conditional on the asset's quality $\theta$, the distribution over its outcomes is given by the pmf $p_\theta(.)$:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the buyer's beliefs when she observes this signal pair is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{8.3}$$

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{8.4}$$

Note that the likelihood ratio 8.3 increases continuously with $\varepsilon$.

The information from observing the pair $(s, \hat{s})$ as such is equivalent to observing the outcome of experiment $\mathcal{E}'$, when:

$$\frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - (s_H + \delta)} \tag{8.5}$$

for our chosen $(\delta, \varepsilon)$. I choose $\varepsilon$ to satisfy this equality for our $\delta$. As such, $\varepsilon$ becomes a continuously increasing function of $\delta$. Furthermore, note that by varying $\varepsilon$ between 0 and $\frac{1 - s_H}{s_H}$, we can replicate *any* experiment $\mathcal{E}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$.

**Step 2.** $\pi(\sigma_1'; \mathcal{E}') \geq \pi(\sigma_1; \mathcal{E})$.

The buyer who observes the signal pair $(s, \hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma} : \{s_L, (s_H, \hat{s_H}), (s_H, \hat{s_L})\} \to [0,1]$ for this signal pair that replicates the outcome of the strategy $\sigma_1'$ for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_H, \hat{s_H}) = 1 \qquad\qquad \tilde{\sigma}(s_L) = \tilde{\sigma}(s_H, \hat{s_L}) = 0$$

and replicates the likelihood ratios of an acceptance and rejection signal under $\mathcal{E}'$.

Now, fix the seller's *signal profile* $\mathbf{s} = \{(s^i, \hat{s}^i)\}_{i=1}^n$ (defined in Section 8.1). I call a seller a *marginal admit* if his score profile is such that:

i for at least one $i \in \{1, 2, ..., n\}$, $s^i = s_H$, and

ii for *every* $i \in \{1, 2, ..., n\}$, either $s^i = s_L$, or $\hat{s}^i = \hat{s}_L$.

These marginal admits drive the wedge between efficiency under $\mathcal{E}'$ and $\mathcal{E}$: while some buyer trades under $\mathcal{E}$, they *all* reject him under $\hat{\mathcal{E}}$. So:

$$\Pi(\sigma_1'; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\,(\text{marginal admit}) \times \underbrace{[c - \mathbb{P}\,(\theta = H \mid \text{marginal admit})]}_{(1)}$$

A marginal admit only has signal realisations $(s, \hat{s}) = (s_H, \hat{s_L})$ or $s = s_L$. These carry equivalent information about $\theta$. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right]$$

In the relevant region where there is an equilibrium strategy that leads to rejections after the low outcome $s_L$, the expression above must be weakly positive:

$$\begin{aligned} c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right] &\propto \frac{c}{1-c} \times \frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^n \\ &\leq \frac{c}{1-c} - \frac{\rho}{1-\rho} \times \frac{\sum_{k=0}^{n-1} p_H(s_L)^k}{\sum_{k=0}^{n-1} p_L(s_L)^k} \times \frac{s_L}{1-s_L} \\ &= \frac{c}{1-c} - \frac{\Psi(\sigma_1; \mathcal{E})}{1-\Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq 0 \end{aligned}$$

where the last inequality follows from the necessary and sufficient condition the proof of Lemma 18 introduced for such an equilibrium to exist.

**Part ii.** Increasing the strength of bad news; i.e. decreasing $s_L$.

Now, let the experiment $\mathcal{E}'$ carry *marginally stronger bad news* than experiment $\mathcal{E}$ instead; for some arbitrarily small $\delta \in [0, s_L]$:

$$s'_L = s_L - \delta \qquad\qquad s'_H = s_H$$

Where $\sigma'_1$ and $\sigma_1$ are defined as before, I show that:

i $\Pi(\sigma'_1; \mathcal{E}) - \Pi(\sigma_1; \mathcal{E}) \geq 0$ when $s_L \geq s_L^{as}$, and

ii $\Pi(\sigma'_1; \mathcal{E}) - \Pi(\sigma_1; \mathcal{E}) \leq 0$ when $s_L \leq s_L^{as}$

**Step 1.** Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

As before, let each buyer observe *two* signals, potentially: $s$ and $\hat{s}$. She first observes $s$, which records the outcome of $\mathcal{E}$. If the high outcome $s_H$ materialises, she receives no further information. If, however, the low outcome $s_L$ materialises, she then observes the additional auxiliary signal $\hat{s}$, which records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. As before, the outcome of this experiment is independent both from $s$ and anything observed by any other buyer. Its distribution conditional on the asset's quality $\theta$ is given by the pmf $p_\theta(.)$:

$$\hat{p}_H\left(\hat{s}_H\right) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p}_L\left(\hat{s}_H\right) = \varepsilon \times \frac{s_L}{1 - s_L}$$

The evolution of the buyer's beliefs upon seeing the signal pair $(s, \hat{s})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)} = \frac{s_H}{1 - s_H} \tag{8.6}$$

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)} = \frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} \tag{8.7}$$

Note that 8.7 is continuously and strictly decreasing with $\varepsilon$, taking values between $\frac{s_L}{1-s_L}$ and 0 as $\varepsilon$ varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} = \frac{s_L - \delta}{1 - (s_L - \delta)}$$

I choose $\varepsilon$ to satisfy this equality. As before, $\varepsilon$ then becomes a continuously increasing function of $\delta$.

**Step 2.** $\Pi(\sigma'_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) \begin{cases} \geq 0 & s_L \geq s_L^{\text{as}} \\ \leq 0 & s_L \leq s_L^{\text{as}} \end{cases}$

The buyer who observes the signal pair $(s, \hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma} : \{(s_L, \hat{s_H}), (s_L, \hat{s_L}), s_H\} \to [0,1]$ for this signal pair that replicates the outcome of the strategy $\sigma_1'$ for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_L, \hat{s_H}) = \tilde{\sigma}(s_H) = 1 \qquad\qquad \tilde{\sigma}(s_L, \hat{s_L}) = 0$$

and replicates the likelihood ratios of an approval and rejection signal under $\mathcal{E}'$.

Now, fix the seller's *score profile*: $\mathbf{s} = \{(s^i, \hat{s}^i)\}_{i=1}^n$. I call a seller a *marginal reject* if:

i for every $i \in \{1, 2, ..., n\}$, $s^i = s_L$, and

ii for at least one $i \in \{1, 2, ..., n\}$, $\hat{s}^i = \hat{s}_H$.

Marginal rejects drive the wedge between efficiency under $\mathcal{E}'$ and $\mathcal{E}$: while *no* buyer trades under $\mathcal{E}$, *at least one* buyer does under $\mathcal{E}'$. So:

$$\Pi(\sigma_1'; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal reject}\right) - c\right]}_{(2)}$$

For a marginal reject, buyers observe either $(s^i, \hat{s}^i) = (s_L, \hat{s}_L)$, or $(s^i, \hat{s}^i) = (s_L, \hat{s}_H)$. Denote the number of buyers who observed the latter as $\#$. Since the seller is a marginal reject, $\# \geq 1$. Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i \ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\sum\limits_{j=1}^{n} \mathbb{P}\left(j \ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}}_{(3)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$

where:

$$\begin{aligned}\mathbb{P}\left(i \ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right) = k \times & \binom{n}{i} \times \left(\frac{s_H}{1 - s_H} \times \varepsilon\right)^i \times \left(1 - \frac{s_H}{1 - s_H} \times \varepsilon\right)^{n-i} \\ + (1-k) \times & \binom{n}{i} \times \left(\frac{s_L}{1 - s_L} \times \varepsilon\right)^i \times \left(1 - \frac{s_L}{1 - s_L} \times \varepsilon\right)^{n-i}\end{aligned}$$

and $k = \mathbb{P}\left(\theta = H \mid s^1 = ... = s^n = s_L\right)$. Thus, the limit of expression (3) as $\varepsilon \to 0$ (and therefore, $\delta \to 0$) for any $i > 1$ is:

$$\lim_{\varepsilon \to 0} \frac{\frac{1}{\varepsilon} \times \mathbb{P}\left(i \ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\frac{1}{\varepsilon} \times \sum\limits_{j=1}^{n} \mathbb{P}\left(j \ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)} = 0 \tag{8.8}$$

Therefore, we get:

$$\lim_{\varepsilon\to 0}\sum_{i=1}^{n}\frac{\mathbb{P}\left(i\ \hat{s_H}\text{ signals }\mid s^1=\ldots=s^n=s_L\right)}{\sum_{j=1}^{n}\mathbb{P}\left(j\ \hat{s_H}\text{ signals }\mid s^1=\ldots=s^n=s_L\right)}\times\mathbb{P}\left(\theta=H\mid\#=i\right)-c$$

$$=\mathbb{P}\left(\theta=H\mid\#=1\right)-c$$

$$\propto\frac{\rho}{1-\rho}\times\left(\frac{s_L}{1-s_L}\right)^{n-1}\times\frac{s_H}{1-s_H}-\frac{c}{1-c}$$

proving the claim. □

□

**Proposition 5.** Where buyers' experiment is binary, equilibrium$^\star$ market efficiency weakly decreases with stronger bad news (lower $s_L$) when:

$$\frac{\rho}{1-\rho}\times\max\left\{\frac{s_L}{1-s_L},\left[\frac{s_L}{1-s_L}\right]^{n-1}\times\frac{s_H}{1-s_H}\right\}\leq\frac{c}{1-c}$$

This condition is also necessary in the least selective equilibrium.

*Proof.*

i The least selective equilibrium:

By Lemma 17, the probability that the seller trades upon the low outcome in the least selective equilibrium is:

$$\check{\sigma}^*(s_L)=\begin{cases}1 & s_L\geq s_L^{\text{mute}}\\ 0 & s_L<s_L^{\text{mute}}\end{cases}$$

Thus, efficiency equals (i) the expected surplus from always approving the applicant when $s_L\geq s_L^{\text{mute}}$, and (ii) $\max\left\{0,\Pi\left(\sigma_1;\mathcal{E}\right)\right\}$ when $s_L<s_L^{\text{mute}}$ (established in the proof of Theorem 2):

$$\Pi(\check{\sigma}^*;\mathcal{E})=\begin{cases}\rho-c & s_L\geq s_L^{\text{mute}}\\ \max\left\{0,\Pi\left(\sigma_1;\mathcal{E}\right)\right\} & s_L<s_L^{\text{mute}}\end{cases}$$

Since always trading is always feasible, we have $\max\left\{0,\Pi\left(\sigma_1;\mathcal{E}\right)\right\}\geq\rho-c$ when $s_L<s_L^{\text{mute}}$ by Lemma 14. Furthermore, the final Claim in Theorem 2's proof establishes that as $s_L$ falls, the expression $\max\left\{0,\Pi\left(\sigma_1;\mathcal{E}\right)\right\}$ weakly increases (decreases) when $s_L\geq s_L^{\text{as}}$ ($s_L\leq s_L^{\text{as}}$). Thus the desired conclusion is established.

ii The most selective equilibrium:

By Lemma 18, the most selective equilibrium shifts from one where a buyer always trades to one where she rejects upon the low signal when $s_H\geq s_H^{\dagger}(s_L)$, where $s_H^{\dagger}(.)$ is an increasing function of $s_L$. Following the arguments made for the least selective equilibrium then, efficiency:

- weakly increases as $s_L$ decreases, when $s_L \geq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$
- weakly decreases as $s_L$ decreases, when $s_L \leq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$.

The desired result follows by noting that $s_L^{\dagger}(s_H) \geq s_L^{\text{safe}}$, and therefore $\min\left\{s_L^{\dagger}, s_L^{\text{as}}(s_H)\right\} \geq \min\left\{s_L^{\text{safe}}, s_L^{\text{as}}(s_H)\right\}$.

□

**Theorem 3.** Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j \in \mathbf{S}$. Equilibrium$^\star$ market efficiency is:

1. weakly greater under $\mathcal{E}'$ if the local spread is a negative override under equilibrium$^\star$.
2. weakly less under $\mathcal{E}'$ if the local spread is a positive override under equilibrium$^\star$, unless adverse selection is $\sigma^\star_{\mathcal{E}}$-irrelevant for signal $s_{j+1}$.

*Proof.* The Theorem focuses either on the least, or the most selective equilibrium strategies under both experiments. In the discussion below, I let $\sigma^*$ and $\sigma^{*'}$ denote whichever equilibria we are focusing on under the respective experiments $\mathcal{E}$ and $\mathcal{E}'$. When I need to distinguish between the least and most selective equilibria, I denote them as $(\check{\sigma}, \check{\sigma}')$ and $(\hat{\sigma}, \hat{\sigma}')$, respectively. Following the notation introduced in Definition 2, let $\mathbf{S} \cup \mathbf{S}' = \{s_1, s_2, ..., s_M\}$ be the joint support of the experiments $\mathcal{E}$ and $\mathcal{E}'$, with elements increasing in their indices as usual. Since $\mathcal{E}'$ is obtained by a *local* mean preserving spread of $\mathcal{E}$, there is a monotone strategy $\sigma' : \mathbf{S}' \to [0, 1]$ whose outcome under $\mathcal{E}'$ replicates the outcome of $\sigma^*$ under $\mathcal{E}$:

$$\sigma'(s) = \begin{cases} \sigma^*(s_j) & s \in \{s_{j-1}, s_{j+1}\} \\ \sigma^*(s) & s \notin \{s_{j-1}, s_{j+1}\} \end{cases}$$

**Claim 1.** Efficiency under the most (least) selective equilibrium of $\mathcal{E}'$ weakly exceeds that under $\mathcal{E}$ when $\hat{\sigma}(s_j) = 1$ ($\check{\sigma}(s_j) = 1$).

Now suppose $s_j$ leads to trade under $\sigma^*$; $\sigma^*(s_j) = 1$. Therefore, $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 1$. Below, I show that $\sigma^{*'}$ is *more selective than* $\sigma'$. By Lemma 4, it follows that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \geq \Pi(\sigma'; \mathcal{E}') = \Pi(\sigma; \mathcal{E})$.

If $s_{j-1} = \min \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j-2}) = 0$, $\sigma^{*'}$ must necessarily be more selective than $\sigma'$; and we are done. So, for contradiction, I assume the following:

- $s_{j-1} > \min \mathbf{S} \cup \mathbf{S}'$
- $\sigma^{*'}(s_{j-2}) > 0$
- $\sigma^{*'}$ is *less* selective than $\sigma'$, where the two strategies differ.

Case i. $\sigma^*$ and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for experiment $\mathcal{E}$, such that:

i $\tilde{\sigma}$ replicates the outcome $\check{\sigma}'$ induces in $\mathcal{E}'$,

ii That $\check{\sigma}'$ is an eqm. strategy under $\mathcal{E}'$ implies that $\tilde{\sigma}$ is an eqm. strategy under $\mathcal{E}$,

iii But $\tilde{\sigma}$ is less selective than $\check{\sigma}$, contradicting that $\check{\sigma}$ is the least selective equilibrium strategy under $\mathcal{E}$.

I define the strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for $\mathcal{E}$ as:

$$\tilde{\sigma}(s) := \begin{cases} 1 & s = s_i \\ \sigma'(s) & s \neq s_i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of $\check{\sigma}'$. Furthermore, $\check{\sigma}'$ is an equilibrium under $\mathcal{E}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}$: they induce the same interim belief $\psi$, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}_\psi\left(\theta = H \mid s_{j-2}\right) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

The strategy $\tilde{\sigma}$ under experiment $\mathcal{E}$ replicates the outcome of $\check{\sigma}'$ under experiment $\mathcal{E}'$, and $\sigma'$ under $\mathcal{E}'$ replicates the outcome of $\check{\sigma}$ under experiment $\mathcal{E}$. Since we assumed that $\check{\sigma}'$ is less selective than $\sigma'$, it must be that $\tilde{\sigma}$ is less selective than $\check{\sigma}$.

Case ii. $\sigma^*$ and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

Since strategy $\sigma'$ for experiment $\mathcal{E}'$ replicates the outcome of $\hat{\sigma}$ for experiment $\mathcal{E}$, the two strategies induce the same interim belief $\psi$. Therefore, if $\mathbb{P}_\psi\left(\theta = H \mid s_{j-1}\right) \geq c$, $\sigma'$ is an equilibrium under $\mathcal{E}'$; meaning $\hat{\sigma}'$ must be more selective than $\sigma'$.

Otherwise, say $\mathbb{P}_\psi\left(\theta = H \mid s_{j-1}\right) < c$. Then, by Lemma 13, there must be an equilibrium strategy that is more selective than $\sigma'$ under $\mathcal{E}'$.

**Claim 2.** Efficiency under the most (least) selective equilibrium of $\mathcal{E}'$ falls weakly below that under $\mathcal{E}$ if:

i. $s_j$ leads to rejections under $\mathcal{E}$; i.e. $\hat{\sigma}(s_j) = 0$ ($\check{\sigma}(s_j) = 0$), and

ii. the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

Now, suppose $s_j$ leads to rejections under $\sigma^*$; $\sigma^*(s_j) = 0$. Consequently, we have $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 0$. I establish Claim 2 in two steps:

Step 1. $\sigma^{*'}$ is less selective than $\sigma'$; trade is likelier when when $s_j$ is locally spread.

Step 2. This efficiency when the condition in Claim 2 is met; $\Pi(\sigma^{*'}; \mathcal{E}') \leq \Pi(\sigma'; \mathcal{E}') = \Pi(\sigma^*; \mathcal{E})$.

**Step 1.**

If $s_{j+1} = \max \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j+1}) > 0$, it must be the case that $\sigma^{*'}$ is less selective than $\sigma'$, and we are done. So instead, I assume that $s_{j+1} < \max \mathbf{S} \cup \mathbf{S}'$ and $\sigma^{*'}(s_{j+1}) = 0$.

Case i. $\sigma^*$ and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$

Since $\sigma'$ replicates the outcome of $\check{\sigma}$, we have $\Psi(\check{\sigma}; \mathcal{E}) = \Psi(\sigma'; \mathcal{E}') = \psi$. Thus, $\sigma'$ must be an equilibrium strategy under $\mathcal{E}'$ if $\mathbb{P}_\psi\left(\theta = H \mid s_{j+1}\right) \leq c$: the optimality conditions for all signals below $s_{j+1}$ are satisfied *a fortiori*, and those for the signals above $s_{j+1}$ are satisfied since $\check{\sigma}$ has the same optimality conditions under $\mathcal{E}$. So, $\check{\sigma}'$ must be less selective than $\sigma'$, since the former is the least selective equilibrium. If on the other hand, $\mathbb{P}_\psi\left(\theta = H \mid s_{j+1}\right) > c$, there must be an equilibrium strategy under experiment $\mathcal{E}'$ that is *less* selective than $\sigma'$, by Lemma 13.

Case ii. $\sigma^*$ and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

$\hat{\sigma}'$ is the most selective equilibrium strategy under experiment $\mathcal{E}'$, and we assumed that $\hat{\sigma}'(s_{j+1}) = 0$. The strategy $\tilde{\sigma}$ defined below for experiment $\mathcal{E}$ replicates the outcome $\hat{\sigma}'$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \hat{\sigma}'(s) & s > s_j \end{cases}$$

Note that $\tilde{\sigma}$ must be an equilibrium under experiment $\mathcal{E}$, since the interim belief it induces is the same as the one $\hat{\sigma}'$ does, and its optimality constraints are a subset of the latter's. But since $\hat{\sigma}$ is the *most* selective equilibrium strategy under $\mathcal{E}$, $\tilde{\sigma}$ must be less selective than it.

**Step 2.**

The statement is trivially true when $\sigma' = \sigma^{*'}$, so I focus on the case where these two strategies differ. As Step 1 established, $\sigma^{*'}$ must be less selective than $\sigma'$. This implies that $\sigma^{*'}(s_{j+1}) > 0$. To see why, say we had $\sigma^{*'}(s_{j+1}) = 0$ instead. We can then construct a strategy $\tilde{\sigma}$ for experiment $\mathcal{E}$, which replicates the outcome $\sigma^{*'}$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \sigma^{*'}(s) & s > s_j \end{cases}$$

As they induce the same interim belief and the optimality constraints of the latter are a subset of the former's, $\tilde{\sigma}$ must be an equilibrium under $\mathcal{E}$. This contradicts with $\sigma^*$ and $\sigma^{*'}$ being the least selective strategies; since $\sigma^{*'}$ being less selective than $\sigma'$ implies that $\tilde{\sigma}$ must be less selective than $\sigma^*$. It also contradicts with $\sigma^*$ and $\sigma^{*'}$ being the most selective strategies; since it would imply that $\sigma'$, more selective than $\sigma^{*'}$, should be an equilibrium under $\mathcal{E}'$.

Given that $\sigma^{*'}(s_{j+1}) > 0$, I now take another strategy $\sigma^{\delta}_{\mathcal{E}'} : \mathbf{S}' \to [0,1]$ for experiment $\mathcal{E}'$:

$$\sigma^{\delta}_{\mathcal{E}'}(s) = \begin{cases} 1 & s > s_{j+1} \\ \delta & s = s_{j+1} \\ 0 & s < s_{j+1} \end{cases}$$

where $\delta > 0$ is small enough so that $\sigma^{\delta}_{\mathcal{E}'}$ is more selective than $\sigma^{*'}$, but less selective than $\sigma'$. I will show that, when the condition stated in Claim 2 holds, we have $\Pi\left(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}'\right) \leq \Pi\left(\sigma'; \mathcal{E}'\right)$ for $\delta \to 0$. Lemma 14 then implies that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \leq \Pi\left(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}'\right)$, which coins the result.

To show this, I construct another experiment $\mathcal{E}^{\text{re}}$ under which I will use compare two strategies, $\sigma_{\text{re}}$ and $\sigma^{\delta}_{\text{re}}$, that replicate the outcomes of the strategies $\sigma'$ and $\sigma^{\delta}_{\mathcal{E}'}$, respectively. The experiment $\mathcal{E}^{\text{re}}$ has three possible outcomes, $\{s^{\text{re}}_L, s^{\text{re}}_{\delta}, s^{\text{re}}_H\}$. Conditional on the applicant's quality $\theta$, its outcome distribution is independent from any other information any evaluator sees, and is given by the following pmf $p^{\text{re}}_{\theta}$:

$$p_{\theta}(s^{\text{re}}) = \begin{cases} 1 - r_{\theta}(\sigma^*; \mathcal{E}) & s = s^{\text{re}}_H \\ \delta \times p'_{\theta}(s_{j+1}) & s = s^{\text{re}}_{\delta} \\ r_{\theta}\left(\sigma^*; \mathcal{E}\right) - \delta \times p'_{\theta}(s_{j+1}) & s = s^{\text{re}}_L \end{cases}$$

Define the strategies $\sigma_{\text{re}}$ and $\sigma^{\delta}_{\text{re}}$ for this experiment as follows:

$$\sigma_{\text{re}}(s) = \begin{cases} 1 & s = s^{\text{re}}_H \\ 0 & s = s^{\text{re}}_{\delta} \\ 0 & s = s^{\text{re}}_L \end{cases} \qquad \sigma^{\delta}_{\text{re}} = \begin{cases} 1 & s = s^{\text{re}}_H \\ 1 & s = s^{\text{re}}_{\delta} \\ 0 & s = s^{\text{re}}_L \end{cases}$$

Now note that these two strategies replicate the outcomes of the strategies $\sigma'$ and $\sigma^{\delta}_{\mathcal{E}'}$, respectively. Under $\sigma_{\text{re}}(s)$, the probability that a buyer trades, conditional on the seller's quality, is the same as it is under strategy $\sigma'$ (or $\sigma^*$, which it replicates), and under $\sigma^{\delta}_{\text{re}}$, it is the same as it is under $\sigma^{\delta}_{\mathcal{E}'}$.

So, the difference between efficiency under these two strategies is determined by the *marginal reject* who:

- is rejected by *every* buyer under the strategy $\sigma_{\text{re}}$.

- is accepted by *at least one* buyer under the strategy $\sigma^{\delta}_{\text{re}}$.

Where $\mathbf{s^{re}} = \left\{s^1, ..., s^n\right\}$ is the seller's signal profile under the experiment $\mathcal{E}^{\text{re}}$, he has:

- *no* $s^{\text{re}}_H$ signals; $s^i \neq s^{\text{re}}_H$ for all $i \in \{1, 2, ..., n\}$ and
- *at least one* $s^{\text{re}}_\delta$ signal; there exists some $i \in \{1, 2, ..., n\}$ such that $s^i = s^{\text{re}}_H$.

Thus we have:

$$\begin{aligned}\Pi(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}') - \Pi(\sigma'; \mathcal{E}') &= \Pi(\sigma^{\delta}_{\text{re}}; \mathcal{E}^{\text{re}}) - \Pi(\sigma_{\text{re}}; \mathcal{E}^{\text{re}}) \\ &= \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal reject}\right) - c\right]}_{(2)}\end{aligned}$$

The expression labelled (2) above equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s^{\text{re}}_\delta \text{ and } n-i\ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k\ s^{\text{re}}_\delta \text{ and } n-k\ s^{\text{re}}_L \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s^{\text{re}}_\delta \text{ and } n-i\ s^{\text{re}}_L \text{ signals}\right) - c$$

Since the probability that a buyer observes the $s^{\text{re}}_\delta$ signal is proportional to $\delta$, we have[33]:

$$\lim_{\delta \to 0} \frac{\mathbb{P}\left(i\ s^{\text{re}}_\delta \text{ and } n-i\ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k\ s^{\text{re}}_\delta \text{ and } n-k\ s^{\text{re}}_L \text{ signals}\right)} = 0$$

Therefore, we get:

$$\begin{aligned}&\lim_{\delta \to 0} \sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s^{\text{re}}_\delta \text{ and } n-i\ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k\ s^{\text{re}}_\delta \text{ and } n-k\ s^{\text{re}}_L \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s^{\text{re}}_\delta \text{ and } n-i\ s^{\text{re}}_L \text{ signals}\right) - c \\ &\lim_{\delta \to 0} \mathbb{P}\left(\theta = H \mid \text{one } s^{\text{re}}_\delta \text{ signal and } n-1\ s^{\text{re}}_L \text{ signals}\right) - c \\ &\propto \lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \frac{p'_H(s_{j+1})}{p'_L(s_{j+1})} \times \left(\frac{r_H(\sigma^*; \mathcal{E}) - \delta \times p'_H(s_{j+1})}{r_L(\sigma^*; \mathcal{E}) - \delta \times p'_L(s_{j+1})}\right)^{n-1} - \frac{c}{1-c} \\ &= \frac{\rho}{1-\rho} \times \frac{s_{j+1}}{1-s_{j+1}} \left(\frac{r_H(\sigma^*; \mathcal{E})}{r_L(\sigma^*; \mathcal{E})}\right)^{n-1} - \frac{c}{1-c}\end{aligned}$$

□

**Proposition 7.** Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j$. Market efficiency in the most selective equilibrium is lower under $\mathcal{E}'$ if the following conditions hold:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right) \leq \frac{c}{1-c} \qquad \text{and} \qquad \frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

[33]See expression 8.8 and the surrounding discussion in the proof of Theorem 2 for a more detailed explanation of this.

*Proof.* First, I let $\hat{\sigma}(s_{j+2}) < 1$. I show that this implies $\hat{\sigma}$ and $\hat{\sigma}'$ induce equivalent outcomes under their respective experiments. The strategy $\sigma' : \mathbf{S}' \to [0,1]$ which replicates the outcome of $\hat{\sigma}'$ under experiment $\mathcal{E}'$:

$$\sigma'(s) = \begin{cases} \hat{\sigma}(s) & s \geq s_{j+2} \\ 0 & s < s_{j+2} \end{cases}$$

must then be an equilibrium strategy under experiment $\mathcal{E}'$. This is because these strategies induce the same interim belief, that $\hat{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ ensures that the optimality conditions of $\sigma'$ for signals below $s_{j+2}$ are satisfied, and for signals above $s_{j+2}$, the optimality conditions are the same as those for $\sigma'$. This means that $\hat{\sigma}'$ must be more selective than $\sigma'$. However, when proving Theorem 3, we established that $\sigma'$ must be more selective than $\hat{\sigma}'$. So it must be that $\sigma' = \hat{\sigma}'$, and we are done.

So instead, let $\hat{\sigma}(s_{j+2}) = 1$. But then, it is easily established that:

$$\frac{r_H(\hat{\sigma};\mathcal{E})}{r_L(\hat{\sigma};\mathcal{E})} \leq \frac{s_j}{1-s_j}$$

since $r_\theta(\hat{\sigma};\mathcal{E}) = \sum_{k=1}^{j} p_\theta(s_k)$. So, the condition Proposition 7 supplies is sufficient for the one Theorem 3 does.

□

**Lemma 8.** Where it exists, the regulator-optimal garbling is monotone binary and produces IC recommendations.

*Proof.* To prove this statement, I take some garbling $\mathcal{E}^G$ and an equilibrium $\sigma^G : \mathbf{S}^G \to [0,1]$ it supports. I then construct a monotone binary garbling $\mathcal{E}^{G*}$ which produces IC recommendations, and show that efficiency under $\mathcal{E}^{G*}$ and the strategy which obeys its recommendations, $\sigma^{G*}$, are higher than those under $\mathcal{E}^G$ and $\sigma^G$.

For the monotone binary garbling $\mathcal{E}^{G*} = (\mathbf{S}^G, \mathbf{P}^{G*})$ and the garbling $\mathcal{E}^G = (\mathbf{S}^G, \mathbf{P}^G)$ in question:

$$\mathbf{P} \times \mathbf{T} = \mathbf{P}^G \qquad\qquad \mathbf{P} \times \mathbf{T}^* = \mathbf{P}^{G*}$$

define the expressions:

$$f^*(s) := p_L(s) \times t^*_{i1} \qquad\qquad f(s) := p_L(s) \times \sum_{s_j^G \in \mathbf{S}^G} t_{ij} \times \left(1 - \sigma^G(s_j^G)\right)$$

for each $s \in \mathbf{S}$. Given the asset has Low quality, $f^*(s)$ is the probability that (i) a buyer would have observed the signal $s \in \mathbf{S}$ in her original experiment, *and* (ii) the garbling $\mathcal{E}^{G*}$ issues her a "rejection recommendation". Similarly, $f(s)$ is the probability that (i) a buyer would observe the signal $s \in \mathbf{S}$ in her original experiment, *and* (ii) he would be rejected under the equilibrium strategies $\sigma^G$. For this Low quality seller, $r_L^{G*}$ below is the probability that the buyer receives a rejection recommendation under $\mathcal{E}^{G*}$; and $r_L^G$ is the probability that the buyer rejects him under $(\mathcal{E}^G, \sigma^G)$:

$$r_L^{G*} := \sum_{s \in \mathbf{S}} f^*(s) \qquad\qquad r_L^G := \sum_{s \in \mathbf{S}} f(s)$$

Now, take the least selective monotone binary garbling $\mathcal{E}^{G*}$ such that $r_L^{G*} = r_L^G$. Evidently, this garbling exists.

Clearly, one can treat $f^*$ and $f$ as probability density functions over $\mathbf{S}$ when normalised. Furthermore, the distribution the former describes is first order stochastically dominated by the one the latter does; $\frac{f^*(s_j)}{\sum\limits_{s \in \mathbf{S}} f^*(s)}$ crosses $\frac{f(s)}{\sum\limits_{s \in \mathbf{S}} f(s)}$ once from below. Therefore we get:

$$\begin{aligned} r_H^* := \sum_{s \in \mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f^*(s)}{\sum\limits_{s \in \mathbf{S}} f^*(s)} \\ \leq \sum_{s \in \mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f(s)}{\sum\limits_{s \in \mathbf{S}} f(s)} =: r_H \end{aligned}$$

where $r_H^*$ and $r_H$ are the probabilities that a High quality seller is rejected from a visit under the strategies $\sigma^{G*}$ and $\sigma^G$, respectively.

Since $r_H^* \geq r_H$ and $r_L^* = r_L$, efficiency is higher under $\sigma^*$ than it is under $\sigma$. It only remains to show that the strategy $\sigma^*$ is optimal against the interim belief $\psi^*$ consistent with it.

The interim belief $\psi^*$ consistent with $\mathcal{E}^{G*}$ and $\sigma^{G*}$ lies below $\psi$—the interim belief consistent with $\mathcal{E}^G$ and $\sigma^G$:

$$\frac{\psi^*}{1-\psi^*} = \frac{\sum\limits_{k=0}^{n-1} \left(r_H^*\right)^k}{\sum\limits_{k=0}^{n-1} \left(r_L^*\right)^k} = \frac{\sum\limits_{k=0}^{n-1} (r_H)^k}{\sum\limits_{k=0}^{n-1} (r_L)^k} \leq \frac{\psi}{1-\psi}$$

Under the interim belief $\psi^*$, it is optimal for a buyer upon the signal $s_L^{G*}$ if and only if:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{c}{1-c}$$

But this inequality must hold; since $\frac{r_H^*}{r_L^*} \leq \frac{r_H}{r_L}$, $\psi^* \leq \psi$, and $\sigma^G$ is optimal against $\psi$:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{\psi}{1-\psi} \times \frac{r_H}{r_L} \leq \frac{c}{1-c}$$

Furthermore, that $\Pi(\sigma^{G*}; S^{G*}) \geq \Pi\left(\sigma; S^G\right) \geq 0$ suggests that the expected suprlus from accepting a seller upon the "approve" recommendation must be weakly positive; hence optimal. Thus, the strategy $\sigma^{G*}$ is optimal against $\psi^*$.

$\square$

**Proposition 9.** If the least selective monotone binary garbling under which adverse selection is irrelevant produces IC recommendations, it is the regulator-optimal garbling. Otherwise, the regulator-optimal garbling is either:

- the least selective garbling under which adverse selection is irrelevant, or
- the most selective garbling under which adverse selection is not irrelevant

among monotone binary garblings which produce IC recommendations.

*Proof.* **Step 1:** The following are well-defined:

- the least selective monotone binary garbling under which adverse selection is irrelevant,
- the least (most) selective monotone binary garbling under which adverse selection is (not) irrelevant among those which produce IC recommendations.

I first show the least selective monotone binary garbling under which adverse selection is irrelevant is well defined. For any monotone binary garbling $\mathcal{E}^G = (\mathbf{S}^G, \mathbf{P}^G)$, let $\mathbf{P} \times \mathbf{T} = \mathbf{P}^G$ and define $d(\mathcal{E}^G) := \sum_{i=1}^{m} t_{i2}$. Evidently, $d(.)$ is a bijection between the space of monotone binary garblings of $\mathcal{E}$ and $[0, m]$. Also, where both are monotone binary garblings of $\mathcal{E}$, $\mathcal{E}^G$ is more selective than $\mathcal{E}^{G'}$ if and only if $d(\mathcal{E}^G) \leq d(\mathcal{E}^{G'})$. Thus, we seek the monotone binary garbling $d^{-1}(D^*)$ where $D^* := \max\{D \in [0, m] :$ a.s. is irrelevant under $d^{-1}(D)\}$. We must only show that $D^*$ is well defined. To that end, define the Real valued function $F$ over the space of monotone binary garblings, where:

$$F(S^G) = \begin{cases} \frac{\rho}{1-\rho}\frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{r_H^G}{r_L^G}\right)^{n-1} - \frac{c}{1-c} & d(\mathcal{E}^G) \in (0, m) \\ \lim\limits_{D \downarrow 0} F \circ d^{-1}(D) & d(\mathcal{E}^G) = 0 \\ +\infty & d(\mathcal{E}^G) = m \end{cases} \qquad r_\theta^G := \sum_{s \in \mathbf{S}} p_s^G(s_L^G) \times p_\theta(s)$$

where $s^*$ is the threshold signal of this garbling.

So, we seek $D^* := \max\{D \in [0, m] : F \circ d^{-1}(D) \geq 0\}$. But this maximiser exists because the function $F \circ d^{-1}$ is upper semicontinuous: $F \circ d^{-1}$ is a decreasing function, and for any $\bar{D} \in [0, m]$ and $\varepsilon > 0$, we can find some $\delta_\varepsilon$ such that $D \in (\bar{D} - \delta_\varepsilon, \bar{D}) \cap [0, m]$ implies $d^{-1}(\bar{D})$ and $d^{-1}(D)$ have the same threshold signal $s^*$ and thus $F \circ d^{-1}(D) < F \circ d^{-1}(\bar{D}) + \varepsilon$ since $r_H^G / r_L^G$ is continuous in $d(S^G)$.

Now say this garbling does not provide IC recommendations. Denote the interim belief that is consistent with evaluators following $\mathcal{E}^G$'s recommendations as $\psi^G$. The garbling $\mathcal{E}^G$ provides IC recommendations if:

$$\underbrace{\frac{r_H^G}{r_L^G} \times \frac{\psi^G}{1-\psi^G}}_{:=f_1(d(\mathcal{E}^G))} \leq \frac{c}{1-c} \qquad\qquad \underbrace{\frac{1-r_H^G}{1-r_L^G} \times \frac{\psi^G}{1-\psi^G}}_{:=f_2(d(\mathcal{E}^G))} \geq \frac{c}{1-c}$$

As defined above, both $f_1(.)$ and $f_2(.)$ are continuous. Therefore, the set of monotone binary garblings with optimal recommendations—$\left\{D \in [0,m] : f_1(D) \leq \frac{c}{1-c} \text{ and } f_2(D) \geq \frac{c}{1-c}\right\}$—is compact. Thus, both objects below are well-defined:

$$\max\left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has IC rec.s } : F \circ d^{-1}(D) \geq 0\right\}$$
$$\min\left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has IC rec.s } : F \circ d^{-1}(D) \leq 0\right\}$$

Among those with IC recommendations, the former gives us the least selective garbling under which adverse selection is irrelevant. The latter gives us the most selective garbling under which adverse selection is not irrelevant among such garblings, since the least-selective garbling under which adverse selection is irrelevant does *not* have IC recommendations (the minimiser of this set *must* have $F \circ d^{-1}(D) < 0$).

**Step 2:** Proving the statement of Proposition 9.

Efficiency under a monotone binary garbling $S^G$ and strategies $\sigma^G$ that obey its recommendations is given by:

$$\Pi\left(\sigma^G; S^G\right) = \rho - c - \rho \times \left(r_H^G\right)^n \times (1-c) + (1-\rho) \times \left(r_L^G\right)^n \times c$$

As a function of $d^{-1}(.)$, efficiency is continuous and therefore attains its maximum over the set $[0,m]$. I show that this maximum is attained with the least selective garbling under which adverse selection is irrelevant.

For the garbling $\mathcal{E}^G$, define $\mathcal{E}^G_{+\varepsilon} := d^{-1}(d(\mathcal{E}^G)+\varepsilon)$ and $S^G_{-\varepsilon} := d^{-1}(d(S^G)-\varepsilon)$. Likewise, let $s^*_{+\delta}$ and $s^*_{-\delta}$ be the threshold signals of these experiments, and $r^*_{\theta;+\delta}$, $r^*_{\theta;-\delta}$ be the probability that a seller of quality $\theta$ is rejected in a visit, under each garbling. From our earlier reasoning about the impact of making evaluators strategies marginally more (less) selective, we observe that:

$$\lim_{\delta\to 0} \Pi\left(\sigma^G_{+\delta}; \mathcal{E}^G_{+\delta}\right) - \Pi\left(\sigma^G; \mathcal{E}^G\right) \propto \lim_{\delta\to 0} \frac{\rho}{1-\rho} \times \frac{p_H(s^*_{+\delta})}{p_L(s^*_{+\delta})} \times \left(\frac{r^G_{H;+\delta}}{r^G_{L;+\delta}}\right)^{n-1} - \frac{c}{1-c} \leq 0$$

where the last inequality follows since $\mathcal{E}^G$ is the least selective garbling under which adverse selection is irrelevant. We conclude that giving evaluators a marginally less selective garbling, and therefore (Lemma 14) any garbling that is less selective than $S^G$, cannot improve their payoffs. Likewise, for a marginally more selective garbling we have:

$$\lim_{\delta\to 0}\Pi\left(\sigma^G_{+\delta};\mathcal{E}^G_{-\delta}\right)-\Pi\left(\sigma^G;\mathcal{E}^G\right)\propto-\lim_{\delta\to 0}\frac{\rho}{1-\rho}\times\frac{p_H(s^*)}{p_L(s^*)}\times\left(\frac{r^G_{H;+\delta}}{r^G_{L;+\delta}}\right)^{n-1}-\frac{c}{1-c}\geq 0$$

where the term on the RHS is now negative because the probability of trade *decreases* when strategies become more selective. By a reasoning similar to that behind Lemma 14, this reveals that *no* garbling that is more selective can improve efficiency either.

This also proves that among those with optimal recommendations, the least selective garbling under which adverse selection is irrelevant cannot be improved with a more selective garbling and the most selective garbling under which adverse selection is not irrelevant cannot be improved with a less selective garbling.

□

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