

Information in Sequential Evaluations: the Good and the Bad

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updated paper online soon!

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Motivation

- Many risky opportunities are offered to **many** before someone takes it:
 - The seller of a financial asset can ask many interested buyers
 - Debt seekers have many banks to apply to for credit
 - Entrepreneurs contact many VCs or angel investors for (seed) funding
- Whoever gets an offer must *evaluate* whether opportunity is good or bad
 - So: more information → better evaluations → better off evaluators?

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So: more information \longrightarrow better evaluations \longrightarrow better off evaluators?

- But: evaluators face an **adverse selection** problem:

*Am I getting opportunity **after everyone rejected it**? Does this mean it is **bad**?*

- Answer depends on what other evaluators know & do: **info. shapes adverse selection**

Maybe: more information \longrightarrow more adverse selection $\xrightarrow{??}$ worse off evaluators?

Motivation

When does more info. leave evaluators better off?

Tied to EVs aggregate selection quality. Important policy question: [credit scoring](#).

- Regulators unsure and confused if allowing more info. → better risk evaluations

Basel II: allowed [Internal Ratings Based](#) systems instead of standardised scoring to

“reward stronger and more accurate risk measurement”

“provide a more risk-sensitive approach to measuring credit risk”

“Regulation Guide: An Introduction”, *Moody's Analytics*

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Basel III banned it back:

*“CVA is a *complex risk* ... cannot be modelled by banks in a robust and prudent manner. The revised framework removes the use of an internally modelled approach ...”*

“High-Level Summary of Basel III Reforms”, Bank of Intl. Settlements

Motivation

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Tied to EVs aggregate selection quality. Important policy question: **credit scoring**.

- Regulators unsure and confused if allowing more info. → better risk evaluations

Banks highlighted seeming paradox: wanting better risk evaluation × curbing use of info.

“[Internal assessments] by banks allow for the most accurate measurement of risk ... Relying on regulators’ assessments ... will be rough approximations at best ... a major step backwards”

Kenneth Bentsen, CEO of Global Financial Markets Association

Contribution today: indeed sometimes: more info. → worse risk evaluations

I will characterise exactly when

The Model

- APPLICANT with quality $\theta \in \{L, H\}$ seeks approval from **one evaluator**.
 - Everyone has prior belief $\rho \in (0, 1)$ that he is **born** with *High* quality.
- He sequentially visits $n \geq 2$ EVALUATORS, at random order τ .
 - τ is a permutation of $\{1, 2, \dots, n\}$, chosen **privately** and **uniformly at random** by applicant.
- EV **approves** \longrightarrow payoff $\mathbb{1}\{\theta = H\} - c$, $c \in (0, 1)$. Game ends, other EV.s get 0 payoff.
EV **rejects** \longrightarrow 0 payoff. Applicant keeps applying. If no EV left \rightarrow game ends.

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The Model

- EVs do not know θ or τ . But: receive **private IID signals** x about quality θ :

$$\mathcal{X} \mid \theta \stackrel{\text{IID}}{\sim} p_\theta \quad x \in \{s_1, s_2, \dots, s_m\} \subset [0, 1] \quad s_i = \frac{p_H(s_i)}{p_H(s_i) + p_L(s_i)} < s_{i+1}$$

- Equilibrium:** symmetric strategy and interim belief profile (σ^*, ψ^*) for EVs such that:

- $\psi^* = \mathbb{P}(\theta = H \mid \text{applicant visited me})$ – **consistent** with strategy profile σ^*

$$\psi^* = \frac{\rho \times \mathbb{P}(\text{app. visits me before smo. else approves} \mid \theta = H, \sigma^*)}{\mathbb{P}(\text{app. visits me before smo. else approves} \mid \sigma^*)}$$

- $\sigma^* : \{s_1, s_2, \dots, s_m\} \rightarrow [0, 1]$ – **optimal** given ψ^* :

approve when $\mathbb{P}(\theta = H \mid x, \psi) > c$

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Main Question and Takeaway

How does more information affect evaluators' equilibrium payoffs?

- Synonymous with selection quality: eqm. payoffs \uparrow when false positives & negatives \downarrow .
 - **Main difficulty:** More info. affects extent of **adverse selection** \rightarrow can clawback on payoffs – selection quality.
 - **Main result:** Characterise effect of arbitrary Blackwell improvements of EVs signals.
 - **Main takeaway:** Effect depends on the **kind** of improvement. Roughly:
 - improving **favourable** evaluations: **good!**
 - improving **unfavourable** evaluations: **eventually bad**
- affect different applicants
 \downarrow
have different payoff effects

Main Question and Takeaway

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} affect different applicants
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 - Giving EVs information about **order** τ
 - EV makes *take-it-or-leave-it* price offer to applicant
 - EVs compete on **application costs**
- } ask me after talk!

Equilibria: Selectivity and Payoffs

- $\sigma^* \rightleftarrows \psi^*$ means possibly multiple eqa. But set of eqa. still well behaved.

Proposition

The set of equilibrium strategies is **non-empty** and **compact**. Furthermore:

- ① all eqm. strategies are **monotone**; $\sigma^*(s_i) > 0 \implies \sigma^*(s_{i+1}) = 1$. [when $p_H \neq p_L$]
- ② eqm. strategies are **pointwise totally ordered**;

either $\sigma^{**}(s_i) \geq \sigma^*(s_i)$ or $\sigma^{**}(s_i) \leq \sigma^*(s_i)$ for all $s_i \in S$

- ③ all eqa. **exhibit adverse selection** : $\psi^* \leq \rho$.

- Compact and totally ordered \rightarrow the lowest (**most selective**) and highest (**least selective**) eqm. strategies.

Equilibria: Selectivity and Payoffs

Selective eqa. reduce approval chances for applicant. What do they mean for EV payoffs?

- (the sum of) Evaluators' equilibrium payoffs:

$$\begin{aligned}\Pi(\sigma) := & (1 - c) \times \rho \times \mathbb{P}(\text{some EV approves} \mid \theta = H, \sigma) + \\ & (-c) \times (1 - \rho) \times [1 - \mathbb{P}(\text{all EVs reject} \mid \theta = L, \sigma)]\end{aligned}$$

- Different virtues: more selective \rightarrow filter *Low* quality approvals
less selective \rightarrow secure *High* quality approvals

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- Different virtues: more selective \rightarrow filter *Low* quality approvals
less selective \rightarrow secure *High* quality approvals
- This trade-off is always resolved in favour of more selective eqa.:

Proposition

Let σ^* be an eqm. strategy, and σ^{**} be a less selective monotone strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

proof

Information and Equilibrium Payoffs

How does more information change payoffs in most/least selective eqa?

$$\Pi(\sigma) := (1 - c) \times \rho \times \mathbb{P}(\text{some EV approves} \mid \theta = H, \sigma) + \\ (-c) \times (1 - \rho) \times [1 - \mathbb{P}(\text{all EVs reject} \mid \theta = L, \sigma)]$$

- With just one EV, classic Blackwell, 1953 result:

Blackwell more informative signal $(\stackrel{\Leftarrow}{\Rightarrow}) \uparrow$ payoffs for all c, ρ

- Reason: more info. allows higher probabilities of: [approve $\mid \theta = H$] & [reject $\mid \theta = L$]

Information and Equilibrium Payoffs

- What goes wrong with many EVs? Rewrite EV i 's payoff $\pi_i (= \frac{\Pi}{n})$:

$$\begin{aligned}\pi_i(\sigma) = & \mathbb{P}(\text{applicant visits } i) \\ & \times [\psi \times (1 - c) \times \mathbb{P}(i \text{ approves} \mid \theta = H) - (1 - \psi) \times c \times \mathbb{P}(i \text{ approves} \mid \theta = L)]\end{aligned}$$

- As before, EV i 's payoffs depend on probabilities of [approve $\mid \theta = H$] & [reject $\mid \theta = L$]

Information and Equilibrium Payoffs

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- As before, EV i 's payoffs depend on probabilities of [approve $\mid \theta = H$] & [reject $\mid \theta = L$]
- However, also on the **red adverse selection terms** shaped by others' strategies.
- Other EVs do not internalise the adverse selection they impose onto EV i .
- Maximising individual selection quality \neq maximising **overall** selection quality

Binary Signals: Information and Eqm. Payoffs

- Start from **binary signals**: main intuition and building block to full characterisation
- $x \in S = \{\cancel{s_L^1}, \cancel{s_H^2}\}$ **more informative** when $s_L \downarrow$: stronger evidence for $\theta = L$
and $s_H \uparrow$: stronger evidence for $\theta = H$

Theorem (1)

With a binary signal $x \in \{s_L, s_H\}$, EVs payoffs in the most selective eqm. are weakly:

- increasing with stronger evidence for $\theta = H$; $\uparrow s_H$
- hump shaped in stronger evidence for $\theta = L$; $\downarrow s_L$:
 - increasing when s_L is above a threshold
 - decreasing when s_L is below that threshold

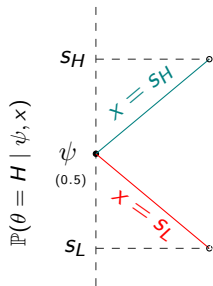
- For least selective eqm: same result, different threshold.

details: threshold

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

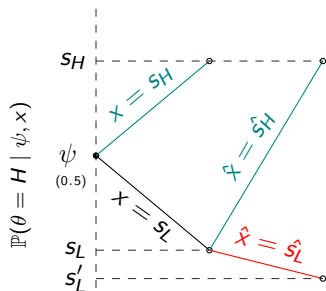
- How to think about $\downarrow s_L$ marginally?



Binary Signals: Information and Eqm. Payoffs

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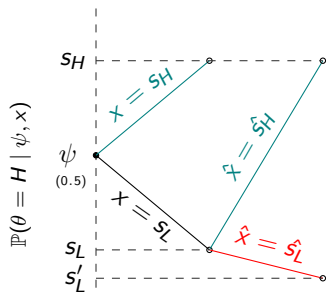
- How to think about $\downarrow s_L$ marginally? Construct an auxiliary signal \hat{x} .



Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- How to think about $\downarrow s_L$ marginally? Construct an **auxiliary signal** \hat{x} .
- Fix EV strat.s: **high** \rightarrow **approve** & **low** \rightarrow **reject**. Which applicant gets affected?

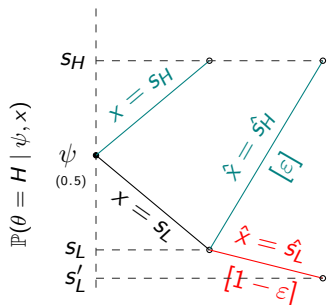


- Fix signal pairs (x, \hat{x}) all EVs would observe for applicant.
- **Marginal admits:** rejected by all $(x = s_L)$ in old signal structure, approved by some $(\hat{x} = \hat{s}_H)$ in new.
- Prob. marginal admit has $\theta = H$ depends on how many $\hat{x} = \hat{s}_H$ signals EVs would see.

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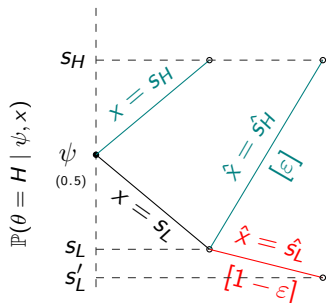


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- Prob. marginal admit has $\theta = H$ depends on how many $\hat{x} = \hat{s}_H$ signals EVs would see. **Answer: only one!**
- For **marginal** decrease in s_L , $\mathbb{P}(\hat{x} = \hat{s}_H) \propto \varepsilon \rightarrow 0$.
- Multiple $\hat{x} = \hat{s}_H$ has negligible probability.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- How to think about $\downarrow s_L$ marginally? Construct an auxiliary signal \hat{x} .
- Fix EV strat.s: high \rightarrow approve & low \rightarrow reject. Which applicant gets affected?



- Whether the marginal admit is profitable:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} \stackrel{<}{>} \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- Whether s_H justifies approving the most ad. selected applicant – **adverse selection poses any threat.**
- $s_L \downarrow \implies$ the $n-1$ low signals \gg the single high signal

details: threshold

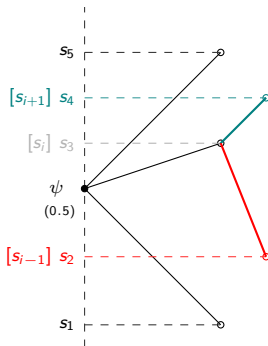
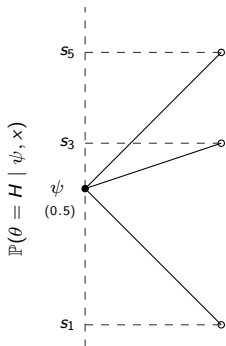
proof: $s_H \uparrow$

General Blackwell Improvements and Eqm. Payoffs

- Generalise the idea of replicating Blackwell improvement with **auxiliary signal**.

Take two signal str.s, $\mathcal{X} \mid \theta \stackrel{IID}{\sim} p_\theta$ and $\mathcal{X}' \mid \theta \stackrel{IID}{\sim} p'_\theta$. Joint supp $S \cup S' = \{s_1, s_2, \dots, s_M\}$.

\mathcal{X}' differs from \mathcal{X} by a local MPS at s_i if:



- mean preserving spread
- Local:** mass is spread only to neighboring points

General Blackwell Improvements and Eqm. Payoffs

Local mean preserving spreads characterise Blackwell improvements

$\mathcal{X} + [\text{finitely many local MPS}] = \mathcal{X}' \iff \mathcal{X}' \text{ is Blackwell more informative than } \mathcal{X}$

- Slight refinement of Rothschild and Stiglitz, 1970 for **discrete signals**.
- Theorem 2 will characterise how payoffs move after a local MPS.
- **Local** allows to control how eqm. evolves with more info.

General Blackwell Improvements and Eqm. Payoffs

Theorem (2)

Let \mathcal{X}' differ from \mathcal{X} by a local MPS at s_i . EVs' payoffs in the most selective eqm. are:

- ① *weakly higher* under \mathcal{X}' if $x = s_i$ leads to approvals under σ ; $\sigma(s_i) = 1$,
- ② *weakly lower* under \mathcal{X}' , if:
 - ① $x = s_i$ leads to rejections under σ ; $\sigma(s_i) = 0$, and
 - ② **adverse selection poses a threat at signal s_{i+1}**

details on this condition

General Blackwell Improvements and Eqm. Payoffs

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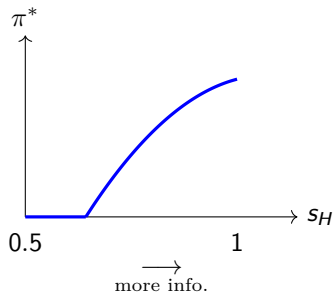
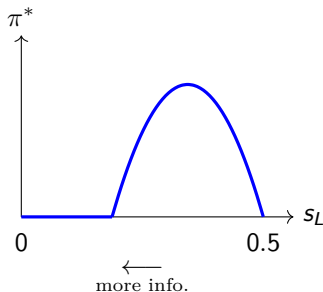
- ① *weakly higher* under \mathcal{X}' if $x = s_i$ leads to approvals under σ ; $\sigma(s_i) = 1$,
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details on this condition

- Adverse selection likelier to pose a threat when s_{i+1} is lower
- Theorem requires knowing equilibrium structure.
 - All eqa. can be located in $\leq 2(m+2)$ steps
- Also: offer stronger sufficient condition that relies only on the local MPS performed

Thank You!

and please check back soon for (substantially) updated paper!
especially if you are hiring! :)



Contribution to Literature

Observational Learning:

Bikhchandani, Hirshleifer, and Welch, 1992, Banerjee, 1992, Smith and Sørensen, 2000

- Add: imperfectly observed history + only rejections are passed on

Censored/Biased Information (Mis)Aggregation:

Broecker, 1990, Lockwood, 1991, Herrera and Hörner, 2013, Board, Meyer-ter-Vehn, and Sadzik, 2023, Cavounidis, 2022, Bobtcheff, Levy, and Mariotti, 2022

Information Aggregation and Sampling/Solicitation Curse in Search:

Lauermann and Wolinsky, 2016 and 2017, Ekmekci and Lauermann, 2019

Novel Q in all three lit.s: how does \uparrow DMs info. affect selection quality away from asymptote?

Equilibria: Selectivity and Payoffs

Proposition

Let σ^* be an eqm. strategy and σ^{**} be a *more embrative* eqm. **monotone** strategy; $\sigma^{**} > \sigma^*$.

Then: $\Pi(\sigma^{**}) \leq \Pi(\sigma^*)$

Proof Sketch:

- Take eqm. strategy σ^* , and consider *marginally more embrative* σ^ε : $\|\sigma^\varepsilon - \sigma^*\| = \varepsilon$
- Fix the signals all EVs would see if app. visited them all: $\{x_1, x_2, \dots, x_n\}$.
- Only app. whose outcome changes: **rejected by all under σ^* , approved by some under σ^ε** .
- If ε is small, he was a.s. **rejected by all under σ^* , approved by one under σ^ε** .
- Bad news: **approving is suboptimal for this last EV**
- Last step: payoffs are **single crossing** in embraciveness; where $\sigma'' > \sigma' > \sigma$:

$$\Pi(\sigma') \leq \Pi(\sigma) \implies \Pi(\sigma'') \leq \Pi(\sigma')$$

When is $\downarrow s_L$ Harmful?

- A candidate for the threshold came from sketch proof:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}} \right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{the only high signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- s_L^{as} : strongest evidence for $\theta = L$ where *adverse selection poses no threat*.

EV happy to approve upon $x = s_H$ **even if she learned all prev.** EVs **observed** $x = s_L$

- Sketch proof showed: marginal admit hurts when $s_L < s_L^{\text{as}}$.

When is $\downarrow s_L$ Harmful?

- But until s_L low enough, EVs might be stuck in an eqm. approving all.

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} \right)}_{\text{one low signal}} = \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- When $s_L \geq s_L^{\text{mute}}$, *always* an eqm: **approve all** \rightarrow **no adverse selection** \rightarrow **approve all**
- We need to decrease s_L enough to eliminate these eqa.

Proposition

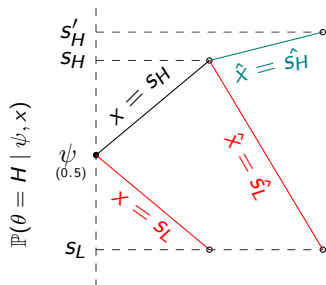
Let EVs have a binary signal $x \in \{s_L, s_H\}$. EVs payoffs decrease as $s_L \downarrow$ when:

- $s_L \leq \min \{s_L^{\text{mute}}, s_L^{\text{as}}\}$ in the least selective eqm.
- $s_L \leq s_L^\dagger$ in the most selective eqm., where $s_L^{\text{as}} \geq s_L^\dagger \geq \min \{s_L^{\text{mute}}, s_L^{\text{as}}\}$.

Binary Signals: Information and Eqm. Payoffs

Sketch Proof:

- How to think about $\uparrow s_H$ marginally? Construct an **auxiliary signal** \hat{x} .
- Fix EV strat.s: **high** \rightarrow **approve** & **low** \rightarrow **reject**. Which applicant gets affected?



- This time **marginal reject**: approved by some ($x = s_H$) before, rejected by all ($\hat{x} = \hat{s}_L$) now.
- All EVs must have seen low signals.
- **Marginal reject** is always good to push out:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^n}_{n \text{ low signals}} < \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

[actual pf: “strong low signals” is the only relevant case]

Adverse Selection Condition for Theorem 2

For a fixed signal structure \mathcal{X} and strategy σ , *adverse selection poses a threat at signal s if*:

$$\underbrace{\frac{\rho}{1-\rho}}_{\text{prior}} \times \underbrace{\left(\frac{r_H(\sigma)}{r_L(\sigma)}\right)^{n-1}}_{n-1 \text{ past rejections}} \times \underbrace{\frac{s}{1-s}}_{\text{last signal is } s} \leq \underbrace{\frac{c}{1-c}}_{\text{approval cost}}$$

- Sufficient condition for the most selective eqm. relies only on the local MPS performed:

Proposition

Let \mathcal{X}' differ from \mathcal{X} by a local MPS at s_i . EVs expected payoffs in the *most selective* eqm. are lower under \mathcal{X}' whenever s_i is a rejection signal under \mathcal{X} (a fortiori: $s_L < s_L^{\text{mute}}$) and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$