1 The Model

An *applicant* (he) with either *High* or *Low* quality, $\theta \in \{H, L\}$, seeks an approval from any one of n *evaluators* (she). To obtain the approval he seeks, he sequentially visits these n evaluators, each at most once. Any evaluator who receives his visit decides whether to *approve* or *reject* the applicant. Once he is approved or has visited all n evaluators, the applicant stops his visits. Otherwise he visits the evaluator labelled $\tau(k)$ after his $k-1^{\text{th}}$ rejection, where $\tau(.)$ is a permutation of the set of evaluators' labels $\{1, 2, ..., n\}$.

If an evaluator approves the applicant, she pays a fixed cost of $c \in [0, 1]$. She also receives a benefit of 1 whenever the applicant has *High* quality. With her approval, the game ends; the applicant stops and the remaining evaluators walk away with a payoff of 0. On the other hand, if she rejects the applicant, she receives a payoff of 0. The applicant then proceeds to visit the remaining evaluators, unless none do.

At the outset of the game, the applicant and all n evaluators share the *prior belief* that the applicant has *High* quality with probability $\rho \in (0, 1)$. Moreover, the evaluators commonly believe that they are equally likely to be anywhere in the applicant's visit order; i.e. that $\mathbb{P}(\tau(k) = i) = \frac{1}{n}$ for all $k, i \in \{1, 2, ..., n\}$. No further information about the applicant's visits is disclosed to evaluators: neither the order $\tau(.)$ he follows, nor the number of evaluators who rejected him already.

Crucially however, an evaluator receiving a visit understands that the applicant was rejected in all his previous visits, however many there might have been. With this information the applicant's visit conveys, she updates her prior belief ρ about the applicant's quality to an *interim belief* ψ.

Subsequently, she observes a costless and private signal S about the applicant's quality. This signal S she observes is the outcome of a Blackwell experiment $\mathcal{E} = (\mathbf{S}, p_L, p_H)$; the signal takes a value s from the finite set $\mathbf{S}$ and has distribution p_θ over this set given the applicant's quality θ. Conditional on the applicant's quality, signals different evaluators observe are IID. After she observes the signal $S = s \in \mathbf{S}$, the evaluator updates her interim belief ψ that the applicant has *High* quality to a *posterior belief*, which I denote as $\mathbb{P}_\psi(\theta = H \mid S = s)$.

An evaluator's strategy $\sigma : \mathbf{S} \to [0, 1]$ prescribes a probability of approval $\sigma(s)$ to every possible realisation $s \in \mathbf{S}$ of her signal. I call a strategy σ *optimal against the evaluator's interim belief* ψ if, given this interim belief ψ, it maximises her expected payoff. Under such a strategy σ, the evaluator approves after any signal $s \in \mathbf{S}$ which raises her posterior belief that the applicant has *High* quality above c. Likewise, she rejects whenever this posterior belief sinks below c:

$$\sigma(s) = \begin{cases} 0 & \mathbb{P}_\psi(\theta = H \mid S = s) < c \\ 1 & \mathbb{P}_\psi(\theta = H \mid S = s) > c \end{cases}$$

A signal that sets her posterior belief exactly equal to c leaves her indifferent between approving and rejecting the applicant. Any approval probability her strategy dictates after such a signal realisation is consistent with its optimality.

I focus on the *symmetric Bayesian Nash Equilibria* of this game. Hereafter, I reserve the word "equilibrium" for such equilibria unless I state otherwise. A strategy and belief pair (σ^*, ψ^*) is an *equilibrium* of this game if and only if it satisfies the two conditions below:

1. The interim belief ψ^* is *consistent* with every evaluator having the strategy σ^*. That is, an evaluator receiving a visit forms the interim belief ψ^* given others' strategies are σ^*.

2. The strategy σ^* is optimal given the interim belief ψ^*.

I call any strategy σ^* which constitutes part of an equilibrium an *equilibrium strategy.*

2 Belief Formation and Equilibria

Before her verdict, the evaluator who receives a visit must assess the probability that she faces a *High* quality applicant. Her privately observed signal about this applicant's quality plays a crucial part in this assessment. But she obtains her first piece of information even earlier, through *her mere receipt of the applicant's visit.*

i don't quite know how to introduce this

The applicant visits this evaluator only if he was rejected by every evaluator he visited earlier. Any such rejections are themselves bad news about the applicant's quality, as they reveal his past evaluators' negative assessments. No information about the number of these past rejections is disclosed to our evaluator. Nonetheless, she is aware of the adverse selection problem she faces: the likelier her peers are to reject the applicant, the likelier she is to be visited by him. Therefore, she interprets the applicant's mere visit as bad news about his quality already.

In particular, when all her peers have the strategies σ, our evaluator understands that an applicant with quality θ faces a probability $r_\theta(\sigma; \mathcal{E})$ of getting rejected from any of his visits, where this probability is given by:

$$r_\theta(\sigma; \mathcal{E}) = 1 - \sum_{j=1}^{m} p_\theta(s_j)\sigma(s_j)$$

She – ex-ante – believes she is equally likely to be anywhere in the applicant's visit order $\tau(.)$. So, she believes that an applicant with quality θ will visit her with probability $\nu_\theta\left(\sigma; \mathcal{E}\right)$ before any of her peers approves him:

$$\nu_\theta\left(\sigma; \mathcal{E}\right) = \frac{1}{n} \times \sum_{k=1}^{n} r_\theta(\sigma; \mathcal{E})^{k-1}$$

Our evaluator's *interim belief* ψ that the applicant who visited her has *High* quality must be *consistent* with these beliefs she holds. Through Bayes Rule, this consistency requirement pins her interim belief down uniquely:

$$\psi := \mathbb{P}\left(\theta = H \mid \text{visit received}\right) = \frac{\mathbb{P}\left(\text{visit received} \mid \theta = H\right) \times \mathbb{P}(\theta = H)}{\mathbb{P}\left(\text{visit received}\right)}$$
$$= \frac{\rho \times \nu_H\left(\sigma; \mathcal{E}\right)}{\rho \times \nu_H\left(\sigma; \mathcal{E}\right) + (1-\rho) \times \nu_L\left(\sigma; \mathcal{E}\right)}$$

After the evaluator updates her prior belief to this interim belief, she observes the realisation $s \in \mathbf{S}$ of her private signal S. From her signal, she distils further information about the applicant's quality and updates her interim belief ψ to a final *posterior belief* $\mathbb{P}_\psi\left(\theta = H \mid S = s\right)$:

$$\mathbb{P}_\psi\left(\theta = H \mid S = s\right) = \frac{\psi \times p_H(s)}{\psi \times p_H(s) + (1-\psi) \times p_L(s)}$$

The information packed in a signal realisation $s \in \mathbf{S}$ is determined exclusively by its conditional probabilities, $p_H(s)$ and $p_L(s)$. So for notational convenience, I label each possible realisation of an evaluator's signal after a ratio of its conditional probabilities:

$$s = \frac{p_H(s)}{p_H(s) + p_L(s)}$$

I call this ratio the *normalised belief* signal s induces. I merge signal realisations whose normalised beliefs are equal; there is no reason to distinguish between them. Likewise without loss, I enumerate these signal realisations $\{s_1, s_2, ..., s_m\} = \mathbf{S}$ so that their indices are strictly increasing in the normalised beliefs they induce; $s_1 < s_2 < ... < s_m$.

Using this notation, we can re-express the evaluator's posterior belief upon observing the signal $s_j \in \mathbf{S}$ as simply:

$$\mathbb{P}_\psi\left(\theta = H \mid S = s_j\right) = \frac{\psi \times s_j}{\psi \times s_j + (1-\psi) \times (1-s_j)}$$

As this expression clarifies, an evaluator's posterior belief is also increasing in the index of the signal $s_i \in \mathbf{S}$ she observes; it is obtained by scaling the *normalised belief* the evaluator's signal induces with the her interim belief. Note that the normalised belief equals the posterior belief when the evaluator's interim belief assigns equal probability to either quality; $\psi = 0.5$.

There is a unique interim belief ψ^* that is consistent with all evaluators using any given strategy σ^*. Whenever this strategy σ^* is optimal against this interim belief ψ^*, the pair (σ^*, ψ^*) forms an equilibrium. In principle, there might be multiple such pairs, or none. I set the ground in Proposition 1 by ruling this last possibility out: an equilibrium is always guaranteed to exist. Also in Proposition 1, I describe some properties of these equilibria that are fundamental to the rest of our analysis.

Proposition 1. Where Σ is the set of evaluators' equilibrium strategies:

1. An equilibrium always exists; $\Sigma \neq \emptyset$.
2. The set Σ is compact.
3. Any equilibrium strategy is *monotone*; for any equilibrium strategy $\sigma^* \in \Sigma$ and signal realisations $s_{j'}, s_j \in \mathbf{S}$:
$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad \text{whenever } s_{j'} > s_j$$
4. All equilibria exhibit adverse selection; $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy σ^*.

I relegate the full proof of Proposition 1 to Section 8. Instead, I discuss its proof in broad strokes here. To establish the existence of an equilibrium, I construct a *best response correspondence* Φ for evaluators. Φ maps any strategy σ to the set of all strategies optimal against the unique interim belief consistent with σ. Put differently, $\Phi(\sigma)$ gives the set of strategies maximising an evaluator's expected payoff when all her peers use the strategy σ. Note that a strategy σ^* is an equilibrium strategy if and only if it is a fixed point of this best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I show that Φ indeed has a fixed point, by a routine application of Kakutani's Fixed Point Theorem. To this end, I prove that Φ is upper semi-continuous, which establishes the compactness of the set of equilibrium strategies as well.

Removed totally orderedness to proposition 2 where it's more natural to discuss it.

Also changed eqm. existence proof. Previously was a direct proof with algorithm using IVT. Now Kakutani FPT.

Equilibrium strategies must be monotone simply because they must be optimal against some interim belief. As I remarked earlier, higher signal realisations induce higher posterior beliefs. Hence, if an evaluator weakly prefers approving an applicant at a low signal realisation, she strictly prefers it at a higher one. Monotonicity also implies that a *Low* quality applicant is likelier to receive a rejection in any equilibrium; as his evaluators are likelier to observe lower signals. Therefore, visits an evaluator receives are *adversely selected*: her interim belief ψ always lies below her prior belief ρ.

Though an equilibrium is guaranteed to exist, it need not be unique. I illustrate this with a simple example which I will revisit on occasion throughout this paper. Consider two evaluators who have the prior belief $\rho = 0.5$ about the applicant's quality, and a cost $c = 0.2$ of approving him. The experiment $\mathcal{E}$ whose outcome they observe is binary; $\mathbf{S} = \{0.2, 0.8\}$. This outcome has the distribution:

$$p_L(s) = \begin{cases} 0.8 & s = 0.2 \\ 0.2 & s = 0.8 \end{cases} \qquad\qquad p_H(s) = \begin{cases} 0.2 & s = 0.2 \\ 0.8 & s = 0.8 \end{cases}$$

One equilibrium strategy for evaluators in this example is to approve every applicant who visits them. Doing so eliminates adverse selection: evaluators never receive an applicant with a past rejection, so their interim belief ψ always equals their prior $\rho = 0.5$. However, at this interim belief, the low signal $s = 0.2$ still implies a 20% probability that the applicant has *High* quality, rendering his approval against the cost 0.2 optimal.

This equilibrium, however, is not unique. There is yet another equilibrium where evaluators approve the applicant only upon the high signal, $s = 0.8$. Their selectivity triggers adverse selection: each evaluator risks being visited by a past reject, thus revises her interim belief ψ below her prior $\rho = 0.5$:

$$\psi = \frac{1 + 0.2}{(1 + 0.2) + (1 + 0.8)} = 0.4$$

Consequently, she places a 1/7 probability on the applicant having *High* quality upon observing the low signal $s = 0.2$; justifying his rejection. She still finds approving the applicant optimal upon the high signal $s = 0.8$, though. Even at this interim belief, she places a probability greater than 70% on him having *High* quality when she observes this signal.

In this latter equilibrium, an applicant – regardless of his quality – faces higher rejection chances in any of his visits. His evaluators are *more selective*; any signal they might observe leads to a (weakly) higher chance of rejection. It is natural in general to try and compare equilibria in their *selectivity*, whenever we face multiple.

Definition 1. Where σ' and σ are two strategies for evaluators, σ' is *more selective than* σ (or, σ is *less selective than* σ') if $\sigma'(s) \leq \sigma(s)$ for all $s \in \mathbf{S}$.

While natural, the *selectivity* (or *pointwise*) order might initially appear restrictive. This impression is misleading. In fact, the set of equilibrium strategies is *totally ordered* (or, a *chain*) under this order; any two equilibrium strategies can be compared under it. Furthermore, this set has both a *most selective* and *least selective* element, marking its extremes. I refer to them as the *extreme equilibria* in the sequel. The two equilibria we identified in our example earlier were, in fact, its extreme equilibria.

Lemma 1. The set of evaluators' equilibrium strategies Σ is *totally ordered* under the selectivity order. Moreover, Σ contains a *most selective* and *least selective* strategy, $\sigma^{\text{mos}} \in \Sigma$ and $\sigma^{\text{les}} \in \Sigma$ respectively, such that:

$$\sigma^{\text{mos}}(s) \leq \sigma^*(s) \leq \sigma^{\text{les}}(s) \qquad \text{for all } s \in \mathbf{S}$$

Proof. By Proposition 1, the set of equilibrium strategies Σ is a subset of the set of monotone strategies. The latter is a chain under the *selectivity* order; for two monotone strategies σ and

σ', we have:

$$\sigma'(s_j) > \sigma(s_j) \implies 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'}) \qquad \text{for any } s_{j'} > s_j \in \mathbf{S}$$

Since any subset of a chain is also a chain, Σ is a chain too.

By Proposition 1, Σ is a compact set. Since it is also a chain, by a suitably general Extreme Value Theorem (see, for instance, Theorem 27.4 in Munkres, 2000) it has a *minimum* and *maximum* element, σ^{mos} and $\sigma^{\text{les}} \in \Sigma$ respectively, with respect to this order:

$$\sigma^{\text{mos}}(s) \leq \sigma^*(s) \leq \sigma^{\text{les}}(s) \qquad \text{for all } s \in \mathbf{S}$$

□

The applicant – regardless of his quality – is worse off with more selective evaluators. His evaluators grow more reluctant to approve him at any signal they might observe, so he faces a higher rejection risk in any of his visits. How moving to more selective equilibria affects evaluators' payoffs is less clear. Their payoffs are determined by how they balance their two key objectives: identifying and approving a *High* quality applicant, and rejecting a *Low* quality one. The expression $\Pi(\sigma; \mathcal{E})$, the *sum* of evaluators' payoffs when each use the strategy σ, highlights this:

$$\begin{aligned} \Pi(\sigma; \mathcal{E}) := \rho \times (1-c) \times \mathbb{P}\left(\text{some ev. approves when all use strategies } \sigma \mid \theta = H\right) \\ -(1-\rho) \times c \times \left[1 - \mathbb{P}\left(\text{all ev.s reject when all use strategies } \sigma \mid \theta = L\right)\right] \end{aligned} \tag{2.1}$$

Each evaluator expects simply $\left(\frac{1}{n}\right)^{\text{th}}$ of this sum of course, as the equilibrium is symmetric.

Increased selectivity has counteracting effects on these two objectives. It mitigates their risk of approving a *Low* quality applicant when they face one, sparing them a cost of c. However, this comes at the expense of curbing the approval chances of a *High* quality applicant too, which means forsaking a payoff of $1-c$. In principle, increased selectivity can therefore both be a vice and a virtue.

Our previous example, where we had identified two equilibria, illustrates these competing effects of increased selectivity. In the least selective equilibrium, evaluators approve all applicants; either *High* quality, or *Low*. Their payoffs payoffs, therefore, sum to:

$$\Pi(\sigma^{\text{les}}; \mathcal{E}) = 0.5 \times [(1-c) - c] = 0.3$$

In the most selective equilibrium on the other hand, an evaluator rejects an applicant for whom she observes the low signal $s = 0.2$. This depresses the approval chances of any applicant. A *High* quality applicant faces a probability $p_H^2(0.2) = 0.04$ of getting rejected by both his evaluators.

This probability is higher for a *Low* quality applicant, $p_L^2(c) = 0.64$. Under these more selective strategies, evaluators' payoffs sum to:

$$\Pi(\sigma^{\text{mos}}; \mathcal{E}) = 0.5 \times \left[(1-c) \times (1 - p_H(0.2)^2) - c \times \left(1 - p_L(0.2)^2\right)\right] = 0.348$$

Despite reducing the approval chances of both *Low* and *High* quality applicants, selectivity pays off for our evaluators in this example.

Why increased selectivity ends up helping evaluators is clear in this example. When they switch to the more selective equilibrium, evaluators push out only applicants for whom *both* of them saw low signals. The probability that such an applicant has *High* quality is less than 6%. By rejecting him, they save a cost of 0.2 against an expected benefit of 0.06, which raises their payoffs.

In more intricate examples, it becomes less clear whether evaluators will benefit from increased selectivity; as it can lead them to reject desirable applicants. Consider, for instance, a richer experiment $\mathcal{E}'$ with three possible outcomes $\mathbf{S}' = \{0.2, 0.4, 0.8\}$ for the evaluators in our running example. The outcome of $\mathcal{E}'$ has the distribution:

$$p_L'(s) = \begin{cases} 0.48 & s = 0.2 \\ 0.36 & s = 0.4 \\ 0.16 & s = 0.8 \end{cases} \qquad p_H'(s) = \begin{cases} 0.12 & s = 0.2 \\ 0.24 & s = 0.4 \\ 0.64 & s = 0.8 \end{cases}$$

Again, evaluators treat the signal $s = 0.4$ differently in different equilibria. Both σ and σ' defined below are equilibrium strategies. In the former, more selective equilibrium, evaluators reject upon the signal $s = 0.4$. In the latter less selective one, they approve.

$$\sigma(s) := \begin{cases} 0 & s \in \{0.2, 0.4\} \\ 1 & s = 0.8 \end{cases} \qquad \sigma'(s) := \begin{cases} 0 & s = 0.2 \\ 1 & s \in \{0.4, 0.8\} \end{cases}$$

should I detail what interim belief they induce, and verify that this is an eqm?

It is no longer as clear whether switching from the less selective strategies σ' to the more selective σ will benefit evaluators. As an unpleasant consequence of this switch, an applicant for whom both evaluators observe the signal $s = 0.4$ gets rejected. The probability that he has *High* quality exceeds 0.3; so his approval against the cost $c = 0.2$ would have benefited evaluators. On the other hand, consider another applicant approved unknowingly by his second evaluator upon the signal $s = 0.4$, despite his first rejection upon $s = 0.2$. He has a probability less than 0.15 of having *High* quality. To their benefit, evaluators successfully push this applicant out too, upon this switch.

Nonetheless, if we calculated evaluators' payoffs under both equilibria, we would find that the overall effect is driven by this latter applicant; their increased selectivity – yet again – leaves evaluators better off. The conclusion these examples share in fact demonstrates a general phenomenon I establish in Proposition 2. Evaluators' trade-off between more and less selective equilibria is *always* resolved in favour of the former. Notably, this brings the welfare of the applicant, unambiguously harmed by selectivity, into conflict with the evaluators'.

Proposition 2. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**}; \mathcal{E}) \geq \Pi(\sigma^*; \mathcal{E})$.

Selectivity thus offers a very powerful comparison between equilibria, besides a very natural one. We can use it to compare any two equilibria, and to determine both the applicant's and evaluators' relative welfare. *Extreme* equilibria deserve particular focus. The *most selective* equilibrium maximises evaluators payoffs across all equilibria while minimising the applicant's approval chances. The *least selective* equilibrium, vice versa. They remain under my spotlight in the remainder of this paper.

Proposition 2 follows as a corollary to Lemma 2, which establishes that in fact deviating to *any* less selective strategy hurts evaluators' payoffs when they start from an equilibrium strategy.

Lemma 2. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*; \mathcal{E}) \geq \Pi(\sigma; \mathcal{E})$.

Proof. See Section 8. □

Besides birthing Proposition 2, Lemma 2 highlights an important contrast between the problem a *single* evaluator with no peers faces, and the one *multiple* evaluators do. A single evaluator with no peers faces no adverse selection; no evaluator could have received her applicant earlier. So her interim belief *is* her prior belief, regardless of the strategy she chooses. In equilibrium, her strategy must be optimal against this belief. This equilibrium strategy is unique – up to how she breaks ties when indifferent. Any equilibrium strategy gives her the same expected payoff, and deviation can leave her only worse off.

With multiple evaluators, a non-trivial multiplicity of equilibria becomes possible. There might be multiple strategies which motivate an interim belief they are optimal against. Crucially, evaluators' payoffs vary between these different equilibria; joint deviations to *more selective* equilibria benefit them. Lemma 2 highlights that only increased selectivity might pay off, though. Deviating – individually or jointly – from any equilibrium to a *less selective strategy* still hurts an evaluator's payoffs, just as it would had she no peers.

As the earlier examples illustrate, two kinds of applicants are pushed out when evaluators move to more selective equilibria. Some fall through the cracks: all evaluators reject them, although none have convincing evidence that they have *Low* quality. Other applicants, on the other hand, face decisive rejections from most evaluators, and – once they become more selective

– lose the benefit of doubt a few unsuspecting evaluators would grant them. I prove Lemma 2 by showing that the overall effect is always driven by the latter group of applicants, whose rejection benefits evaluators.

3 More Informative Experiments and Equilibrium Payoffs

As we discussed in the previous section, evaluators' payoffs are determined fundamentally by how well they can distinguish and approve a *High* quality applicant while rejecting a *Low* quality one. The information they obtain about the applicant's quality from their experiment $\mathcal{E}$ lies at the heart of this exercise. One might, perhaps naturally, hypothesise that a *more informative* experiment is the key to improving evaluators' welfare. Eventually, more information helps evaluators identify their applicant's quality better. This ought to ease the tension between their two fundamental objectives, allowing them to boost their payoffs.

This hypothesis would indeed be correct if we only had *one* evaluator. With no peers, she faces a simple decision problem: given her fixed prior belief about her applicant's quality, she must choose whether to approve him given the outcome of her experiment $\mathcal{E}$. As Blackwell's seminal result (1953) establishes, observing instead the outcome of an experiment $\mathcal{E}'$ that is *(Blackwell) more informative* than $\mathcal{E}$ would indeed leave her better off. In fact – absent further particulars about her decision problem (namely, her approval cost c and prior belief ρ) – only a Blackwell more informative experiment can guarantee her a higher expected payoff[1]. This is precisely because more information relaxes our evaluator's key trade-off. She can reject *Low* quality applicants more frequently without compromising *High* quality applicants more often, and (or) vice versa [2]. I illustrate this in Figure 1 in the next subsection, in the context of *binary* experiments (where $\mathbf{S} = \{s_1, s_2\}$).

Nevertheless, this naïve hypothesis fails in our current setting. Re-expressing an individual evaluator i's payoff $\pi_i(\sigma; \mathcal{E})$ showcases what goes wrong:

$$\pi_i(\sigma;\mathcal{E}) = \underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{\mathbb{P}\left(\text{applicant visits } i\right)}$$
$$\times \left[\underset{\sim}{\psi} \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) + \underset{\sim\sim\sim\sim}{(1-\psi)} \times (-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]$$

Evaluator i's payoff, like others', is determined by how well she can tailor her decisions to the applicant's quality. However, she is also affected by the extent of *adverse selection* she faces.

[1] Blackwell's Theorem (1953) is in fact weaker. It states that an experiment $\mathcal{E}'$ offers the decision maker a higher expected payoff than $\mathcal{E}$ *regardless of the decision problem she faces* if and only if $\mathcal{E}'$ is (Blackwell) more informative than $\mathcal{E}$. This establishes that (Blackwell) more informativeness is *sufficient* to secure our evaluator a higher expected payoff. However, it does not establish its *necessity*; our evaluator does not just face *any* decision problem, but a two state - two action one. Nonetheless, Blackwell's Theorem retains its sufficiency for this class of decision problems too. I present a self contained proof for this in Section 8.2, Lemma 3 for completeness.

[2] See Blackwell and Girshick, 1954's Theorems 12.2.2 and 12.4.2 for a textbook exposition of these classic results.

The applicant might not visit her at all, and if he does, he might be very unlikely to have *High* quality given no other evaluator approved him so far.

The extent of adverse selection evaluator i faces is shaped by *other* evaluators' strategies. These evaluators do not account for the adverse selection their decisions' impose on her; just as she disregards the adverse selection she imposes on them. This adverse selection externality can transform more information into a threat. By Pushing evaluators to improve their selection quality, more information might accentuate the adverse selection they impose on each other. This might eclipse their improved ability to evaluate the applicant, and leave all evaluators worse off.

thoughts on this par?

Our simple example illustrates this possibility. To simplify this illustration, I take evaluators' approval cost to be $c = 0.5$ instead. The least selective equilibrium earlier does not survive this increase in the approval cost; even without adverse selection, evaluators find it optimal to reject their applicant upon the low signal $s = 0.2$. In fact, the most selective equilibrium we identified earlier now becomes the unique one: evaluators approve the applicant upon the high signal $s = 0.8$, but reject him upon the low signal, $s = 0.2$. With this modified cost, evaluators' payoff in this equilibrium becomes:

$$0.5 \times \left[(1-c) \times (1 - p_H(0.2)^2) - c \times \left(1 - p_L(0.2)^2\right)\right] \Bigg|_{c=0.5} = 0.15$$

Now, consider swapping our evaluators' experiment $\mathcal{E}$ with a more informative binary experiment $\mathcal{E}^g$ whose possible outcomes are in the set $\mathbf{S}^g = \{c, 1\}$. The high signal in $\mathcal{E}^g$ carries *conclusive good news*; since no evaluator observes it otherwise, any evaluator to observe it definitively concludes that the applicant has *High* quality. The evidence the low signal carries for *Low* quality, however, is no stronger than before. For a fixed interim belief, an evaluator who observes the low signal in $\mathcal{E}^{\text{good}}$ forms the same posterior belief as the one who observes the low signal in $\mathcal{E}$ would. The outcome of experiment $\mathcal{E}^g$ has the distribution:

$$p_L^g(s) = \begin{cases} 1 & s = 0.2 \\ 0 & s = 1 \end{cases} \qquad\qquad p_H^g(s) = \begin{cases} 0.25 & s = 0.2 \\ 0.75 & s = 1 \end{cases}$$

Under this more informative experiment, the unique equilibrium remains the one where evaluators approve upon the high signal $s = 1$, but reject upon the low signal $s = 0.2$. Thus, evaluators manage to avoid approving *any Low* quality applicant, albeit forsaking *High* quality applicants more often than earlier. This improvement in their information pays off; evaluators' payoffs now sum up to:

$$0.5 \times \left[(1-c) \times \left(1 - p_H^g(0.2)^2\right)\right] \Big|_{c=0.5} \approx 0.23$$

surpassing their payoffs of 0.15 under their original experiment $\mathcal{E}$.

Now consider another binary experiment, $\mathcal{E}^b$, again more informative than evaluators' original experiment $\mathcal{E}$. The possible outcomes of $\mathcal{E}^b$ lie in the set $\mathbf{S}^b = \{0, 0.8\}$ this time, with distribution:

$$p_L^b(s) = \begin{cases} 0.75 & s = 0 \\ 0.25 & s = 0.8 \end{cases} \qquad\qquad p_H^b(s) = \begin{cases} 0 & s = 0 \\ 1 & s = 0.8 \end{cases}$$

It is the low signal $s = 0$ which carries *conclusive bad news* this time; any evaluator concludes that the applicant has *Low* quality upon observing it. Would our evaluators benefit if we swapped their experiment not with $\mathcal{E}^g$ but with $\mathcal{E}^b$ instead?

The answer is no this time, it turns out. Still, evaluators approve upon the high signal $s = 0.8$ and reject upon the low, $s = 0$, in the unique equilibrium. This now guarantees that they *always* approve a *High* quality applicant, though at the expense of approving *Low* quality applicants more often than before. Their payoffs now sum up to:

$$0.5 \times \left[(1-c) - c \times \left(1 - p_L(0)^2\right)\right] \Bigg|_{c=0.5} \approx 0.14$$

falling behind their payoffs under the original experiment $\mathcal{E}$.

The pattern in this example is a general one: while *stronger good news*, in the appropriate sense, always benefit evaluators, *stronger bad news* eventually hurts them. In the remainder of this section, I uncover and explore this general pattern. I start by restricting evaluators to binary experiments in Section 3.1. Theorem 1 there characterises how giving evaluators a Blackwell more informative binary experiment affects their payoffs, and discusses the intuition behind this. The general result I present in Theorem 2 in Section 3.2 builds on this groundwork. It characterises how an *arbitrary* Blackwell improvement of evaluators' experiment affects their payoffs, and shows the intuition laid out by Theorem 2 fully generalises. Throughout, I focus on evaluators' payoffs across *extreme equilibria*. They delineate the boundaries of both the evaluators' and the applicant's welfare across equilibria, hence command the highest importance.

is this the right language?

3.1 Evaluators with Binary Experiments

In this section, I consider evaluators who observe the outcome of a *binary* experiment. Such an experiment has two possible outcomes, $\mathbf{S} = \{s_1, s_2\}$, which I respectively rename $\mathbf{S} = \{s_L, s_H\}$ for notational convenience. The low outcome induces the normalised belief $s_L \in [0, 0.5]$, carrying *bad news* about the applicant's quality. In contrast, the high outcome induces the normalised belief $s_H \in [0.5, 1]$, carrying *good news* about the applicant's quality.

How do evaluators' equilibrium payoffs change when we swap this experiment with a more informative binary experiment $\mathcal{E}'$? Here, I answer this question. This answer also lays the building block and key intuition for the next section, where I ask how an *arbitrary* Blackwell improvement affects evaluators' payoffs.

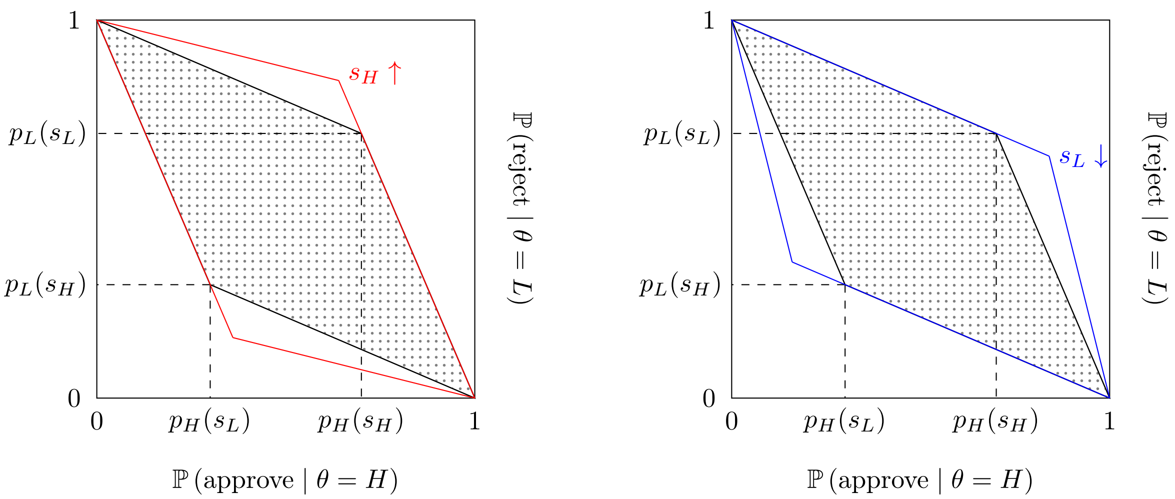
Where $\mathcal{E}$ and $\mathcal{E}'$ are two experiments, with possible outcomes in $\mathbf{S} = \{s_L, s_H\}$ and $\mathbf{S}' = \{s'_L, s'_H\}$ respectively, $\mathcal{E}'$ is *(Blackwell) more informative than* (or *(Blackwell) improves on*) $\mathcal{E}$ if and only if it carries both *stronger good news* and *stronger bad news* than $\mathcal{E}$[3]; i.e.:

$$s'_L \leq s_L \qquad\qquad s'_H \geq s_H$$

For any fixed interim belief $\psi \in [0,1]$, the experiment $\mathcal{E}'$ helps an evaluator form more confident assessments of the applicant's quality. Its low outcome leaves her more confident that her applicant has *Low* quality, and its high outcome that he has *High*:

$$\mathbb{P}_\psi\left(\theta = H \mid s' = s'_L\right) \leq \mathbb{P}_\psi\left(\theta = H \mid s = s_L\right) \quad \mathbb{P}_\psi\left(\theta = H \mid s' = s'_H\right) \geq \mathbb{P}_\psi\left(\theta = H \mid s = s_H\right)$$

Figure 1 illustrates how switching to the more informative experiment $\mathcal{E}'$ transforms the key trade-off each evaluator faces. This figure depitcs a unit square whose points represent the outcomes of all possible strategies an evaluator might adopt. Their horizontal coordinate stands for an evaluator's probability of approving the applicant when he has *High* quality, and the vertical coordinate for her probability of rejecting him when he has *Low*. An evaluator who observes the outcome of $\mathcal{E}$ is restricted to outcomes in the dotted region of this unit square. By varying her strategy, she can achieve every outcome in this region, but nothing more. With an experiment $\mathcal{E}'$ which offers *stronger good news* (right panel) or *stronger bad news* (left panel), the evaluator can achieve a wider region of outcomes.



(a) Receiving stronger good news: $s_H \uparrow$ (b) Receiving stronger bad news: $s_L \downarrow$

Figure 1: Evaluators' Trade-Offs with More Informative Experiments

[3]See Section 12.5 in Blackwell and Girshick, 1954 for a textbook exposition of this classic result.

How does shifting to the more informative experiment $\mathcal{E}'$ affect evaluators' payoffs? As we foreshadowed, Theorem 1 reveals that this hinges precisely on whether $\mathcal{E}'$ carries *stronger good news* or *stronger bad news*. Stronger good news – higher s_H – always increase evaluators' payoffs, both in the *most selective* equilibria and the *least*. The effect of stronger bad news – lower s_L – in contrast, is more delicate. Initially, evaluators benefit from stronger bad news. However, once s_L falls below a cutoff, their payoffs fall as bad news get even stronger. This effect occurs across both the most and the least extreme equilibria; though the cutoff for these two equilibria may differ.

Theorem 1. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. As s_H increases, evaluators' payoffs across the extreme equilibria weakly improve. In contrast, evaluators' payoffs for the most (least) selective equilibria:

1. weakly improve as s_L falls, while s_L remains below a cutoff s_L^{mos} (s_L^{les}),
2. weakly deteriorate as s_L falls, once s_L falls below this cutoff.

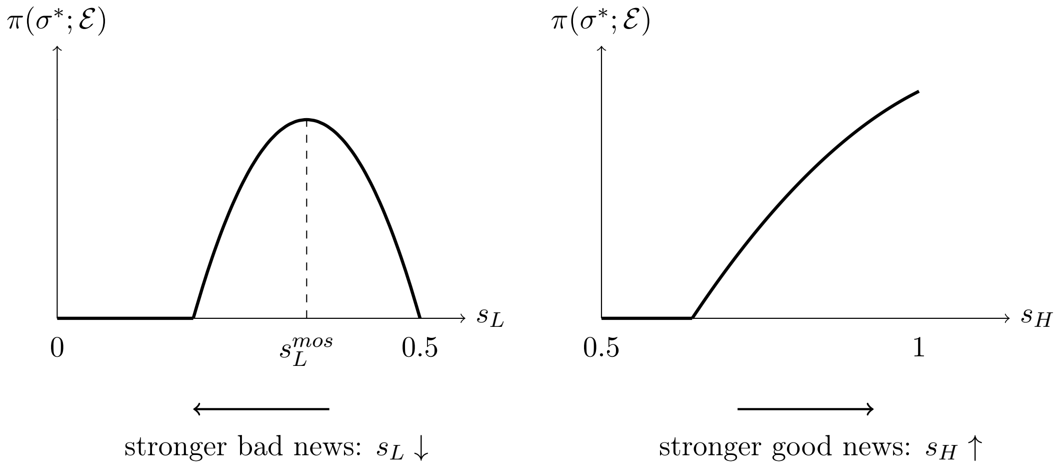


Figure 2: Theorem 1 illustrated

Figure 2 illustrates Theorem 1 for an example where evaluators' approval cost c weakly exceeds their prior belief ρ. This conveniently forces a unique equilibrium across experiments, whose strategies I indiscriminately label σ^*. Examples with multiple equilibria would produce qualitatively the same plots; both for the most selective equilibria different experiments induce, and the least.

In the ensuing discussion, I explain the discrepancy between the effect of strengthening good news and strengthening bad news on evaluators' payoffs. I relegate the full proof of Theorem 1 to Section 8, which builds on the forces I lay out here. The eager reader will notice that Theorem 1 does not elaborate on the cutoff below which stronger bad news harm evaluators' payoffs. As I clarify the forces driving Theorem 1, I cast light on this cutoff as well. Building on that discussion, I characterise this cutoff in Proposition 3.

Carlos: I actually have a nice result here. I can prove it using another one in the appendix. "Equilibrium adverse selection always increases with the informativeness of the signal". This is why the reader should care about what's to come next anyway: because one's first instinct towards a proof will be fruitless. I don't know how to insert this yet. It yields another nice result:
"The expected quality of an approved applicant falls when s_L falls, in equilibrium."

Switching evaluators' experiment ultimately affects who they approve and who they reject. A *High* quality applicant they would otherwise reject might this time receive an evaluator's approval, or a *Low* quality applicant who would otherwise slip through their net might instead face a wall of rejections. Other changes might be less welcome: evaluators might inadvertently dismiss more *High* quality applicants, or fail to do so with *Low* quality ones. It is the applicants they affect who determine how stronger good news or stronger bad news influence evaluators' payoffs.

Let us start with the case of *stronger bad news.* Consider switching evaluators' experiment from $\mathcal{E}$ to $\mathcal{E}'$, whose possible outcomes $\mathbf{S}' = \{s'_L, s'_H\}$ induce the normalised beliefs:

$$s'_L = \frac{p'_H(s'_L)}{p'_H(s'_L) + p'_L(s'_L)} = s_L - \delta \qquad\qquad s'_H = \frac{p'_H(s'_H)}{p'_H(s'_H) + p'_L(s'_H)} = s_H$$

for some small $\delta > 0$. Experiment $\mathcal{E}'$ thus offers *marginally* stronger bad news than $\mathcal{E}$, but the same the strength of good news as the latter. Evaluators' equilibrium strategies will of course react to this switch. Nevertheless, ignoring this strategic response allows for a clearer intuition. So instead, let us simply assume that under both experiments, evaluators approve whenever they observe the "high" outcome, and reject whenever they observe the "low" outcome. Which applicants' outcomes does the switch from $\mathcal{E}$ to $\mathcal{E}'$ affect?

The clearest way to answer this question is by reinterpreting this improvement in evaluators' information. Imagine, instead of replacing their original experiment $\mathcal{E}$ wholesale, that evaluators observe an *auxiliary signal* $\hat{S}$ in addition to their original S. We will construct this auxiliary signal $\hat{S}$ carefully so that it completes the information evaluators garner from their original experiment $\mathcal{E}$ to the one they could from $\mathcal{E}'$. I illustrate this construction in Figure 3. The reader might benefit from referring to it throughout the ensuing discussion.

This auxiliary signal $\hat{S}$ evaluators observe is also binary, with possible realisations $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$. Conditional on the applicant's quality θ, its outcome is independent both from the evaluator's original signal S and anything other evaluators observe. It has a distribution:

$$\hat{p_H}(\hat{s_H}) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p_L}(\hat{s_H}) = \varepsilon \times \frac{s_L}{1 - s_L}$$

ε, like δ, is a small positive number. It is related intimately to δ, as I explain shortly.

An evaluator observes the realisation of this auxiliary signal $\hat{s}$ *only* if the initial signal she observes is low, $s = s_L$. If she observes $\hat{s} = \hat{s_H}$ following this initial low signal, her belief that the applicant has *High* quality jumps to what it would be had she observed $s = s_H$ straightaway. This is most visible from the likelihood ratio for this signal pair:

$$\frac{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_H} \mid \theta = L\right)} = \frac{p_H(s_L)}{p_L(s_L)} \times \frac{\hat{p_H}(\hat{s_H})}{\hat{p_L}(\hat{s_H})} = \frac{s_L}{1 - s_L} \times \frac{\frac{s_H}{1-s_H}}{\frac{s_L}{1-s_L}} = \frac{s_H}{1 - s_H}$$

If she instead observes $\hat{s} = \hat{s_L}$ though, she grows yet more confident that the applicant has *Low* quality. Again note this from the likelihood ratio for this signal pair, labelled $(L, \hat{L})$:

$$\frac{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_L} \mid \theta = H\right)}{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_L} \mid \theta = L\right)} = \frac{p_H(s_L)}{p_L(s_L)} \times \frac{\hat{p_H}(\hat{s_L})}{\hat{p_L}(\hat{s_L})} = \underbrace{\frac{s_L}{1 - s_L} \times \frac{1 - \frac{s_H}{1-s_H} \times \varepsilon}{1 - \frac{s_L}{1-s_L} \times \varepsilon}}_{(L,\hat{L})} < \frac{s_L}{1 - s_L}$$

The likelihood ratio $(L, \hat{L})$ decreases continuously and monotonically as ε rises from 0 to $\frac{1-s_H}{s_H}$. We can thus choose ε so that this likelihood ratio equals the one for the low outcome of experiment $\mathcal{E}'$, $s' = s'_L$. This latter likelihood ratio is labelled (L') below:

it is a confusing jump from likelihood ratios to "normalised beliefs"?

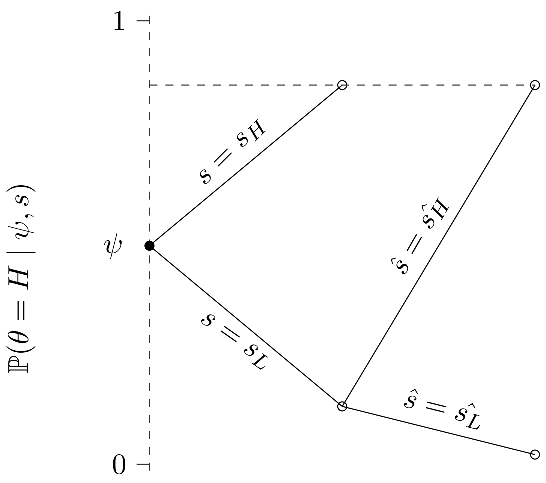
$$\frac{\mathbb{P}\left(s' = s'_L \mid \theta = H\right)}{\mathbb{P}\left(s' = s'_L \mid \theta = L\right)} = \frac{p'_H(s'_L)}{p'_L(s_L)} = \underbrace{\frac{s_L - \delta}{1 - (s_L - \delta)}}_{(L')}$$

Note that the value of ε equating these likelihood ratios is a continuous and strictly increasing function of δ.

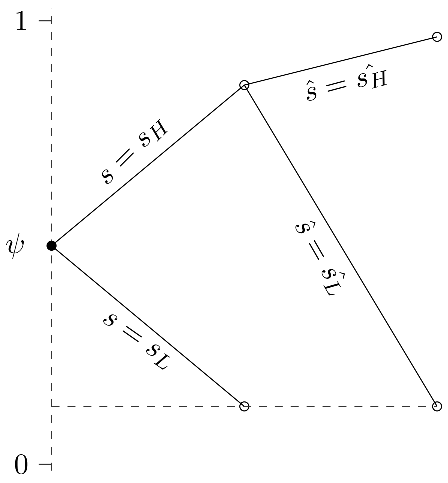
When the likelihood ratios $(L, \hat{L})$ and (L') are equal, the information an evaluator obtains from observing the outcome of $\mathcal{E}'$ is equivalent to the one she does by observing the signal pair $(S, \hat{S})$. Receiving a high signal, either $s = s_H$ or $\hat{s} = \hat{s_H}$, carries the same information as the high outcome $s' = s'_H$ from experiment $\mathcal{E}'$. Receiving only the low signals $s = s_L$ and $\hat{s} = \hat{s_L}$ on the other hand, carries the same information as observing $s' = s'_L$ from $\mathcal{E}'$. Our evaluator can easily replicate the outcome of her original strategy with this signal pair; she approves the applicant if she receives a high signal, but rejects him otherwise.

We can interpret this auxiliary signal as an evaluator's "re-evaluation" of her initial rejection decisions. Her initial approvals remain final; so this re-evaluation does not affect applicants who received some evaluator's approval anyway. Instead, it affects the applicants who were initially rejected by *all* n evaluators. Each of these n re-evaluations might overturn an evaluator's negative verdict, and grant this applicant the approval he seeks.

How likely is this applicant, approved upon at least one of his evaluators' positive re-evaluation, to have *High* quality? Any information about this, of course, is contained in the signals evaluators observe about it. All these evaluators' initial signals were low, which led the



(a) Stronger Bad News



(b) Stronger Good News

Figure 3: Improving Binary Signals

applicant's initial rejections. If they all re-evaluated him, how many of them would receive a high signal $\hat{s} = \hat{s_H}$ and revise their verdict?

The answer is, for small δ and therefore ε, *almost surely just one*. Recall from our construction that the probability any evaluator observes the signal $\hat{s} = \hat{s_H}$ for an applicant is proportional to ε; and so the probability that any k evaluators observe it, to ε^k. As ε shrinks to 0, the probability that multiple evaluators observed this signal vanishes in favour of the probability that just one of them did. Hence, that one evaluator who observes the signal $\hat{s} = \hat{s_H}$ approves the applicant against the backdrop of $n-1$ $\hat{s} = \hat{s_L}$ signals her peers observed. The stronger the bad news those signals carry, the less likely he is to have *High* quality. Inspecting his signals' likelihood ratios reveals precisely when bad news are *too strong* for evaluators to benefit from this applicant:

$$\begin{aligned}
&\lim_{\delta \to 0} \ \mathbb{P}(\theta = H \mid \ n-1 \ \hat{s} = \hat{s_L} \text{ signals and one } \hat{s} = \hat{s_H}) \geq c \\
\iff &\lim_{\delta \to 0} \frac{\mathbb{P}(\theta = H \mid \ n-1 \ \hat{s} = \hat{s_L} \text{ signals and one } \hat{s} = \hat{s_H})}{\mathbb{P}(\theta = L \mid \ n-1 \ \hat{s} = \hat{s_L} \text{ signals and one } \hat{s} = \hat{s_H})} \geq \frac{c}{1-c} \\
\iff &\lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \left(\frac{\hat{s_L}}{1-\hat{s_L}}\right)^{n-1} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c} \\
\iff &\frac{\rho}{1-\rho} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{a single high signal}} \geq \frac{c}{1-c}
\end{aligned}$$

When the LHS above exceeds the RHS, *adverse selection poses no threat* to evaluators: even after $n-1$ low signals, a high signal suffices to justify an applicant's approval. This threat subsides when evaluators have a lower bar for rejecting the applicant; either due to a favourable prior belief ρ about his quality, or a low approval cost c. Having less evaluators helps too, capping the number of low signals the applicant can accumulate. But fundamentally, whether adverse selection poses a threat depends on the strength of good and bad news. As bad news get stronger against good news, the applicant's $n-1$ low signals increasingly dominate over the single high signal he received. Once the strength of bad news exceeds a threshold, this high signal no longer vindicates the applicant. I denote the normalised belief which marks this threshold as s_L^{as}.

Definition 2. For a binary experiment $\mathcal{E}$ with given strength of good news s_H, s_L^{as} is the *strongest* bad news can get before adverse selection poses no threat:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

This threshold is intimately related to the cutoff Theorem 1 points at. Indeed, unless they already approve *every* applicant, stronger bad news always pushes evaluators to approve their marginal admits. Whenever adverse selection poses a threat, these approvals hurt their payoffs. The example in the beginning of this section illustrates this. There, we have no equilibria where *every* applicant is approved; a low signal always warrants a rejection as evaluators' prior belief ρ is already below their approval cost c. Adverse selection poses a threat beyond the threshold $s_L^{\text{as}} = 0.2$; precisely the strength of bad news in their original experiment $\mathcal{E}$. Consequently, switching to $\mathcal{E}^b$ hurts evaluators' payoffs with each marginal admit it pushes them to approve.

The exception is equilibria where bad news are too weak to convince evaluators oblivious about the applicant's past to reject him. Until the strength of bad news exceeds a threshold s_L^{mute} I introduce below, its weakness causes evaluators to approve *every* applicant in the least selective equilibrium. This threshold is in general higher for the most selective equilibrium, where adverse selection might push evaluators to reject low signals earlier.

Definition 3. For a binary experiment $\mathcal{E}$, s_L^{mute} is the *strongest* bad news can get before all equilibria where every applicant is approved vanish:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

Pushing evaluators away from such equilibria might, in general, require strengthening bad news beyond the threshold s_L^{as}. Once they do so however, further strengthening bad news is followed with marginal admits, as before.

Proposition 3. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Evaluators' payoffs in the least selective equilibrium deteriorate with lower s_L when s_L is below the cutoff $s_L^{\text{les}} := \min\{s_L^{\text{mute}}, s_L^{as}\}$. This threshold is higher for the most selective equilibrium; $s_L^{\text{mos}} \geq s_L^{\text{les}}$.

3.1.1 old text

We can interpret a marginal strengthening of evidence for *High* quality analogously. This time, consider the signal structure $\mathcal{E}'$ with normalised posterior beliefs:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small $\delta > 0$. This time, evaluators will observe the realisation of an auxiliary signal $\hat{s}$ *only if* they first observe $s = s_H$. The complementary signal $\hat{s}$, conditionally independent from s as before, has the distribution:

$$\mathbb{P}\left(\hat{s} = \hat{s_L} \mid \theta = H\right) = \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \mathbb{P}\left(\hat{s} = \hat{s_L} \mid \theta = L\right) = \varepsilon \times \frac{s_H}{1 - s_H}$$

We choose ε so that the pair $(s, \hat{s})$ provides the same information about quality as $\mathcal{E}'$:

$$\frac{\mathbb{P}\left(s = s_H, \hat{s} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(s = s_H, \hat{s} = \hat{s_H} \mid \theta = L\right)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \frac{s_L}{1 - s_L}}{1 - \varepsilon \times \frac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - s_H - \delta}$$

As before, ε is a continuous and increasing function of δ. The right panel of Figure 3 illustrates.

Under this signal pair $(s, \hat{s})$, observing consecutive high signals $(s_H, \hat{s_H})$ elevates the evaluator's belief about the applicant's quality as observing $s' = s'_H$ would. The evaluator then approves. If he sees any low signal however, be it s_L or $\hat{s_L}$, his belief sinks as it would had he observed $s' = s'_L$. He rejects.

As before, we can interpret evaluators' observations of the auxiliary signal as a "second inspection", this time of their initial "approval" pile. Each evaluator re-assesses applicants she initially placed in her approval pile upon a high signal. Consecutively, she either concludes that she erred and that her applicant should have been in the "rejection" pile instead, or reinforces her conviction that the applicant likely has *High* quality.

This reformulation exposes the new group of applicants whose eventual outcomes switch when we strengthen evidence for *High* quality. An applicant who would be rejected by all evaluators under $\mathcal{E}$ faces the same fate under $(s, \hat{s})$: his initial $s = s_L$ signals still lead to rejections. But an applicant who previously would be approved by some evaluator faces a renewed threat of rejection by all. Any initial $s = s_H$ signal he had can be overturned by a $\hat{s} = \hat{s_L}$ signal now. Unlucky enough, and he might overturn all his initial high signals, being left with nothing but rejections.

How pushing this *marginal reject* out affects evaluators' payoffs depends again on how likely he is to have *High* quality. As before, all and any information about this is contained in the signals his evaluators would observe for him if he visited them all. Inferring what these signals must be is now easier. As all evaluators eventually rejected him, they all must have observed low signals; either $s = s_L$ or $\hat{s} = \hat{s_L}$.

If low signals indeed lead to a rejection in equilibrium, pushing this marginal reject out is sure to raise evaluators' payoffs. Intuitively, equilibrium behaviour shows that the *fear* of adverse selection suffices to keep an evaluator from an applicant upon a low signal. So it can certainly not be optimal to approve an applicant who, in fact, *is* the most adversely selected one, upon a low signal.

If a low signal *already* leads to a rejection in equilibrium, this marginal reject is *sure* to raise evaluators' expected payoffs. When evaluators find rejecting upon a low signal optimal, learning that *all* evaluators saw low signals can only strengthen this conviction.

3.2 General Discrete Signals

The previous section uncovered how evaluators' equilibrium payoffs vary across binary signal structures. Such a signal structure is *more informative* whenever it provides *stronger evidence* either for *Low* or *High* quality. Theorem 1 showed that while stronger evidence for *High* quality always benefits evaluators, stronger evidence for *Low* quality *eventually* harms them.

In many settings of interest, however, the evaluators in concern have richer signal structures. Traders of financial assets and derivatives, for instance, might get recommendations of varying levels of strength, such as "Strong Sell", "Sell", "Buy", and "Strong Buy". Similarly, a bank's credit scoring algorithm might output varying probabilities that the loan seeker will default, rather than simply summarising this information as "Good" or "Bad". These highlight the importance of extending our characterisation in Theorem 1 to improvements of such richer signal structures. ~~This is precisely the present section's aim. Its main result, Theorem TWO, characterises the effect of more information on evaluators' equilibrium payoffs, for any *arbitrary* discrete signal structure they might hold.~~

Evidence for this

When studying improvements of binary signals, I introduced the idea of auxiliary signals engineered to replicate any given improvement. This construction helped pin down the applicants whose eventual outcomes a given improvement affects. I now formalise and generalise this idea, using *local mean preserving spreads* of a signal structure. Local mean preserving spreads are the key to identify who the applicants affected by an *arbitrary* Blackwell improvement are.

Definition 4 (Local Mean Preserving Spread)**.** Let p and p' be the normalised posterior densities for the signal structures $\mathcal{E}$ and $\mathcal{E}'$. Additionally, let $s_1 < s_2 < ... < s_M$ be the elements of $S \cup S'$; the joint support of $\mathcal{E}$ and $\mathcal{E}'$. If there exists some $i \in \{2, ..., M-1\}$ such that:

$$p'(s_{i-1}) \geq p(s_{i-1}) \qquad 0 = p'(s_i) \leq p(s_i) \qquad p'(s_{i+1}) \geq p(s_{i+1})$$

$$p'(s_j) = p(s_j) \quad \text{for all } j \notin \{i-1, i, i+1\}$$

$$\sum_{i=1}^{M} s_i \times p'(s_i) = \sum_{i=1}^{M} s_i \times p(s_i)$$

I say $\mathcal{E}'$ *differs from* $\mathcal{E}$ *by a local mean preserving spread* (*at* s_i).

the original definition has F not $\mathcal{E}$, is that a problem?

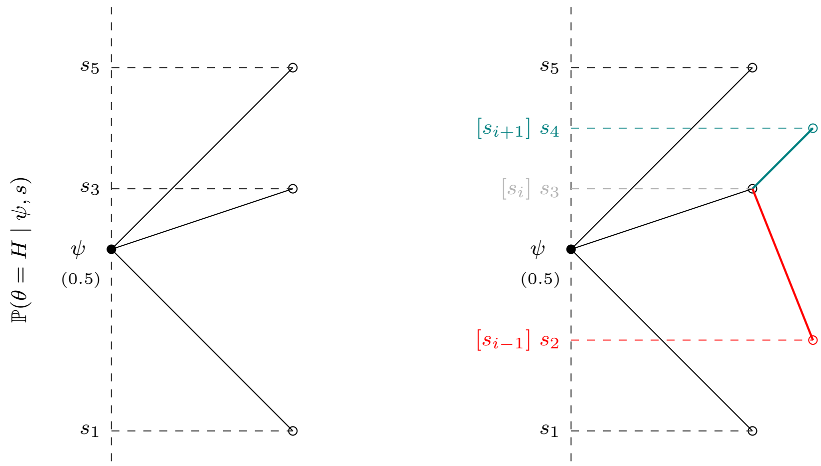


Figure 4: A Local Mean Preserving Spread

Much like an ordinary mean preserving spread (Rothschild and Stiglitz, 1970[4]), a *local mean preserving spread* distributes probability away from an *origin* point to two *destination* points, one above and one below it. It does so while preserving the mean of the original distribution. Crucially however, a mean preserving spread is *local* if and only if the destination points are the immediate neighbours of the origin point[5]. In other words, neither the original nor the resulting distribution assign positive probability to any other point between the origin and the two destination points[6].

The auxiliary signals I introduced in the previous section create such local mean preserving spreads. To strengthen evidence for *Low* quality, for instance, the auxiliary signal spreads all the probability mass assigned to the origin point s_L to the neighbouring destination points s_H and s'_L, where $s_H > s_L > s'_L$.

Local mean preserving spreads are simple ways to Blackwell improve signal structures. Nonetheless, they are powerful enough to characterise *any* Blackwell improvement of a signal

[4]Rothschild and Stiglitz, 1970 describe mean preserving spreads through *four* points in the support of the distribution. Here, I describe them through *three*. This is without loss of generality. In fact, mean preserving spreads were first characterised by Muirhead, 1900 in the context of majorisation (transformations T), with *three* points. Rasmusen and Petrakis, 1992 show formally that these the three or four point characterisations of MPS are in fact equivalent.

[5]The reader will notice that this statement is ill-defined unless the signal structure is discrete. To the best of the author's knowledge, no counterpart for *local mean preserving spreads* exist for, say, atomless signal structures.

[6]The attentive reader will also realise that this definition also requires that *all* probability mass be spread away from the origin point. This difference is insignificant in our current setting.

structure, too. Remark 1, slightly refining the classic result in Rothschild and Stiglitz, 1970[7], states that we can decompose *any* Blackwell improvement of a discrete signal structure into a sequence of finitely many *local* mean preserving spreads. So, I focus on the effect of such local mean preserving spreads on evaluators' equilibrium payoffs.

Remark 1. [Müller and Stoyan, 2002, Theorem 1.5.29] $\mathcal{E}'$ is Blackwell more informative than $\mathcal{E}$ if and only if there is a finite sequence $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_k$ such that $\mathcal{E}_1 = \mathcal{E}$, $\mathcal{E}_k = \mathcal{E}'$, and $\mathcal{E}_{i+1}$ differs from $\mathcal{E}_i$ by a local mean preserving spread.

As before, we can interpret a local MPS of s_i as a re-evaluation of the applicant upon that signal realisation; I illustrate this in Figure **??**. This re-evaluation might change an evaluator's initial verdict upon s_i. As a result, whether applicants face a renewed rejection threat or get a second approval chance hinges on what this initial verdict would have been.

If the signal s_i originally led to an approval, applicants who initially owed an approval to this signal might get rejected upon this re-evaluation. Consequently, an applicant who would initially be approved by some evaluator might now get rejected by all. As Theorem 2 states, pushing such applicants out always benefits evaluators. Intuitively, upon his re-evaluation, this applicant falls into the rejection region of *every* evaluator. Given her signal, *no* evaluator can justify approving him under the *threat* of adverse selection. The evaluator's fear of adverse selection and the rejection it motivates are, in fact, valid. Her approval would indeed stand alone among *all* her peers' rejections.

If the signal s_i originally led to a rejection, however, this re-evaluation presents an applicant who were rejected with it a *second chance*. Some of his initial rejections owing to s_i might be overturned upon a positive revision of his signal to s_{i+1}. This positive re-evaluation is good news about his quality, but it still comes against the potential backdrop of some – unchanged – poor evaluations. The evaluator who approves upon her re-assessment benefits if the signal s_{i+1} is strong enough to counter this adverse selection threat.

If the signal s_{i+1} is strong enough to overwhelm rejections even by *all* $n-1$ evaluators, our evaluator faces no such threat. She would find it beneficial to approve the applicant upon the signal s_{i+1}, regardless of the number of his past rejections. For a fixed strategy σ all evaluators use, I say *adverse selection poses no threat at signal* s_{i+1} when so.

Definition 5. Fix the signal structure $\mathcal{E}$ and a monotone strategy σ. I say *adverse selection poses no threat at signal* s if:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{s}{1-s} > \frac{c}{1-c}$$

[7] This result appeared in earlier work related to majorisation. See Muirhead, 1900, whose textbook exposition appears in Hardy et al., 1952.

~~If adverse selection poses no threat at signal s_{i+1}, the applicant approved upon a revision of s_i to s_{i+1} increases evaluators' payoffs. How many rejections other evaluators would issue is irrelevant; these are insufficient to overwhelm the good news s_{i+1} carries. Strikingly however, this is also *necessary* for this admit to improve evaluators' payoffs. *Whenever* adverse selection poses a threat at signal s_{i+1}, evaluators are worse off due to these admits brought about with the local spread of s_i.~~

Strikingly, *any* threat of adverse selection at s_{i+1} suffices for evaluators to be worse off with the admit s_i's spread brings about. The argument parallels the one describing how *marginal admits* affect evaluators' payoffs in the binary case. Consider, again, a "small" spread which revises the signal s_i to s_{i+1} with a vanishingly small probability. The probability that our applicant, previously rejected by all evaluators, overturned *multiple* s_i signals positively is vanishingly small. Hence, the evaluator revising her rejection to an approval *does* in fact suffer from the most severe form of adverse selection.

Theorem 2. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $s = s_i$ leads to approvals under σ.
2. *weakly lower* than under σ:

   i if $s = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

   ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{E}$ and σ.

Whether adverse selection poses a threat at s_{i+1} depends on evaluators' precise equilibrium strategy. This might concern an analyst with no knowledge of this strategy when she wants to judge whether a given spread guarantees to harm evaluators. This is less alarming than it first appears; the analyst can locate both the most and least selective equilibrium strategies precisely with the algorithm I present to prove equilibrium existence in Proposition 1. Nonetheless, Proposition 4 offers a stronger sufficient condition for the most selective equilibrium. It strengthens the notion of adverse selection threat at s_{i+1} Theorem 2 uses to one which depends only on the signals s_i and s_{i+1}.

Proposition 4. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proposition 4 still requires knowing that s_i is a rejection signal in the most selective equilibrium. This too, can be strengthened to a sufficient condition that relies only on the signal realisation. Recall from the previous subsection that signal realisations below s_L^{mute} ought to be rejected in any equilibrium. Corollary 5 uses this fact:

Corollary 5. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

4 Evaluators with Regulated Strategies

This section will be about a social planner who can dictate evaluators' strategies to maximise their total payoffs can achieve.

1. First result: a planner who wishes to dictate the same strategy to every evaluator can *never benefits* from evaluators having information that would be harmful in equilibrium.

   Reason: dictating the same strategy to every evaluator $\equiv$ giving each a Blackwell inferior experiment. So at best, you can do as well as you did with a Blackwell inferior experiment.

   In the binary case, you can do only as well as the equilibrium outcome of some inferior experiment. I don't yet know if this is true in general.

2. Result 1.5: with binary signals, an evaluator can simply raise the approval cost to implement this symmetric optimum.

3. Second result: a planner who can dictate *different* strategies to evaluators can break this. Example: binary signal with $s_L < s_L^{\text{mute}}$. Optimal solution: k evaluators approve upon s_H and reject upon s_L, remaining $n-k$ reject always. Evaluators' payoffs are then monotone with any improvement in information.

4. Third result: the above is equivalent to giving evaluators' full information about history. In general, not optimal (I can maybe cite Makarov and Plantin, 2023 on this, or give my own example).

5. Fourth result (follow up to two): conjectured. In general, evaluators' payoffs are not monotone in information even when they have full history information. I will cook an example for this.

5 Information with Arbitrary History Signals

Highest priority!

This Section is about the generalisation to arbitrary history signals that I wish to make. I conjecture that Theorem 1 is going to generalise in some form to *any* history signal. I do not wish to get a fully general result in the spirit of Theorem 2; i.e. will restrict myself to binary in this section. This is because with partially (or fully) observed past decisions, Blackwell improvements might behave weirdly simply because decisions are censored data à la classic social learning anyway.

6 Take-It-Or-Leave-It Price Offers

Nothing changes when evaluators offer take it or leave it prices to applicants. Diamond's paradox kicks in, every evaluator offers max acceptable price.

7 Competing in Application Costs

Evaluators post application costs (potentially negative). Applicant applies from lowest to highest cost. Turns the game into an all-pay auction to evade adverse selection, à la Broecker, 1990. Equilibrium strategies of application costs are mixed with no atoms, so ex-post order of applications are perfectly known.

8 Proof Appendix

8.1 Useful Definitions and Notation

In what follows, I occasionally express beliefs in *likelihood form* for convenience. The reader can easily verify the identities:

$$\frac{\psi}{1-\psi} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})} \qquad \frac{\mathbb{P}_\psi(\theta = H \mid S = s_i)}{1-\mathbb{P}_\psi(\theta = H \mid S = s_i)} = \frac{\psi}{1-\psi} \times \frac{s_i}{1-s_i}$$

Through similar reasoning, the reader can verify that it is optimal to approve the applicant when:

$$\frac{\mathbb{P}_\psi(\theta = H \mid S = s_i)}{1-\mathbb{P}_\psi(\theta = H \mid S = s_i)} > \frac{c}{1-c}$$

Some strategies require evaluators to randomise when approving their applicant upon observing a particular signal realisation. To facilitate the discussion, I assume that each evaluator observes the realisation of a *tie-breaking signal* $u \sim U[0,1]$ alongside the outcome of her experiment. This signal is not informative about the applicant's quality as its distribution is independent of it. I denote the outcome of evaluator i's experiment as s^i and her tie-breaking signal as u^i. Without loss, evaluator i approves the applicant if and only if $\sigma(s^i) \leq u^i$; where σ is her strategy. I call the pair (s^i, u^i) the *score* evaluator i observes for the applicant.

Definition 6. The *score* evaluator i observes for the applicant is the tuple $Z^i = (s^i, u^i)$, where $u^i \overset{IID}{\sim} U[0,1]$. The applicant's *score profile* $Z^\otimes$ is the set of scores each evaluator would observe if he visits all; $Z^\otimes = \{(s^i, u^i)\}_{i=1}^n$. Analogously, the applicant's *signal profile* $S^\otimes = \{s^i\}_{i=1}^n$ is the set of outcomes each evaluator would observe for her experiment.

8.2 Omitted Results

Lemma 3. Let there be a *single* evaluator who observes the outcome of an experiment before she decides whether to approve the applicant. The evaluator's expected payoff is greater when she observes the outcome of $\mathcal{E}'$ than when she observes the outcome of $\mathcal{E}$ regardless of her approval cost $c \in [0,1]$ and prior belief $\rho \in [0,1]$ if and only if $\mathcal{E}' \underset{B}{\succeq} \mathcal{E}$.

Proof. Sufficiency is due to Blackwell's Theorem (Blackwell and Girshick, 1954, Theorem 12.2.2). I show necessity by fixing a prior belief ρ for the evaluator.

Let q_j be the posterior belief the evaluator forms about the applicant's quality upon observing the signal $S = s_j \in \mathbf{S}$:

$$q_j = \frac{\rho \times s_j}{\rho \times s_j + (1-\rho) \times (1-s_j)}$$

Furthermore, let $F(.)$ and $F'(.)$ be the distributions of posterior beliefs $\mathcal{E}$ and $\mathcal{E}'$ induce, respectively, for this prior belief ρ:

$$F(q_j) = (1-\rho) \times \sum_{l=1}^{j} p_L(s_l) + \rho \times \sum_{l=1}^{j} p_H(s_l)$$
$$F'(q_j) = (1-\rho) \times \sum_{l=1}^{j} p'_L(s_l) + \rho \times \sum_{l=1}^{j} p'_H(s_l)$$

The evaluator's expected payoff under $\mathcal{E}$ is then:

$$\int_c^1 (q-c)dF(q) = \int_c^1 qdF(q) - c \times (1 - F(c)) = (1-c) - \int_c^1 F(q)dq$$

Of course, an analogous expression gives her expected payoff under $\mathcal{E}'$. Therefore, for her expected payoffs under $\mathcal{E}'$ to exceed those under $\mathcal{E}$ for any $c \in [0,1]$, we must have:

$$\int_c^1 \left(F(q) - F'(q)\right) dq \geq 0$$

which is equivalent to $\mathcal{E}'$ being Blackwell more informative than $\mathcal{E}$[8].

□

8.3 Omitted Proofs

Proposition 1. Where Σ is the set of evaluators' equilibrium strategies:

1. An equilibrium always exists; $\Sigma \neq \emptyset$.

2. The set Σ is compact.

3. Any equilibrium strategy is *monotone*; for any equilibrium strategy $\sigma^* \in \Sigma$ and signal realisations $s_{j'}, s_j \in \mathbf{S}$:

$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad\qquad \text{whenever } s_{j'} > s_j$$

4. All equilibria exhibit adverse selection; $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy σ^*.

Proof. It will be convenient to treat each strategy $\sigma : \mathbf{S} \to [0,1]$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$. Since this is a finite dimensional vector space, I endow it with the metric induced

[8] See Müller and Stoyan, 2002, Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica, 2016.

by the taxicab norm without loss of generality (see Kreyszig, 1978 Theorem 2.4-5):

$$||\sigma' - \sigma|| = \sum_{j=1}^{m} |\sigma'(s_j) - \sigma(s_j)| \qquad \text{for any two strategies } \sigma' \text{ and } \sigma$$

For further convenience, I denote the function which maps a strategy σ to the interim belief consistent with it as $\Psi(.;\mathcal{E}) : [0,1]^n \to [0,1]$:

$$\Psi(\sigma;\mathcal{E}) = \frac{\rho \times \nu_H(\sigma;\mathcal{E})}{\rho \times \nu_H(\sigma;\mathcal{E}) + (1-\rho) \times \nu_L(\sigma;\mathcal{E})}$$

Note that $\Psi(.;\mathcal{E})$ is a continuous function of evaluators' strategies.

I begin by proving statements 3 and 4 of the Proposition. Following that, I prove statements 1 and 2.

3. Any equilibrium strategy is monotone.

Any equilibrium strategy σ^* must be monotone; i.e.:

$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad \text{for any } s_{j'} > s_j$$

This is simply because any equilibrium strategy must be optimal against the interim belief ψ^* it induces. Whenever $\rho \in (0,1)$, $\psi^* = \Psi(\sigma^*;\mathcal{E}) \in (0,1)$, and:

$$\mathbb{P}_{\psi^*}\left(\theta = H \mid S = s_{j'}\right) > \mathbb{P}_{\psi^*}\left(\theta = H \mid S = s_j\right)$$

5. Selectivity is a total order on Σ.

Any two monotone strategies σ' and σ are comparable under the *selectiveness* order since:

$$\sigma'(s_j) > \sigma(s_j) \implies 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'})$$

for any $j' > j$. Since Σ must be a subset of the set of monotone strategies, Σ is also totally ordered under the *selectivity* relation.

4. All equilibria exhibit adverse selection.

This follows from the stronger fact that $\Psi(\sigma;\mathcal{E}) \leq \rho$ for any monotone strategy σ. To see this, note that $p_H(.)$ first order stochastically dominates $p_L(.)$ since it likelihood ratio dominates it[9]. Therefore, $r_L(\sigma;\mathcal{E}) \geq r_H(\sigma;\mathcal{E})$ and $\Psi(\sigma;\mathcal{E}) \leq \rho$.

1. An equilibrium always exists.

[9] Theorem 1.C.1 in Shaked and Shanthikumar, 2007.

Define $\Phi(.) : [0,1]^m \to 2^{[0,1]^m}$ to be the evaluators' *best response correspondence*. $\Phi(.)$ maps any strategy σ to the set of strategies that are optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ induced by this strategy σ:

$$\Phi(\sigma) = \left\{\sigma' \in [0,1]^m : \sigma' \text{ is optimal against } \Psi(\sigma;\mathcal{E})\right\}$$

A strategy σ^* is an equilibrium strategy if and only if it is a fixed point of evaluators' best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I establish that the correspondence Φ has at least such fixed point through Kakutani's Fixed Point Theorem.

Φ is trivially non-empty; every interim belief has some strategy optimal against it. Moreover, Φ is convex valued. Unless there is a signal $s_{j^*} \in \mathbf{S}$ such that $\mathbb{P}_{\Psi(\sigma;\mathcal{E})}\left(\theta = H \mid S = s_{j^*}\right) = c$, the strategy optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ is unique. Otherwise, a strategy σ is optimal if and only if:

$$\sigma(s_j) = \begin{cases} 0 & j < j^* \\ \in [0,1] & j = j^* \\ 1 & j > j^* \end{cases}$$

The set of these strategies is convex.

Now take an arbitrary sequence of strategies $\{\sigma_n\}$ such that $\sigma_n \to \sigma_\infty$. Denote the interim beliefs consistent with these strategies as $\psi_n := \Psi(\sigma_n;\mathcal{E})$. Since $\Psi(.;\mathcal{E})$ is continuous in evaluators' strategies, we also have $\psi_n \to \psi_\infty$ where $\psi_\infty = \Psi\left(\sigma_\infty;\mathcal{E}\right)$. Take now a sequence of strategies $\{\sigma_n^*\}$ from the image of this correspondence; $\sigma_n^* \in \Phi(\sigma_n)$. Note that every strategy in the sequence $\{\sigma_n^*\}$ is monotone, since any strategy that is optimal against an interim belief $\psi \in (0,1)$ must be monotone.

We want to show that Φ is upper semi-continuous; i.e.:

$$\sigma_n^* \to \sigma_\infty^* \implies \sigma_\infty^* \in \Phi(\sigma_\infty)$$

The upper semi-continuity of $\Phi(.)$ implies the existence of a fixed point for this correspondence through Kakutani's Fixed Point Theorem. This establishes the existence of equilibria.

By the Monotone Subsequence Theorem, the sequence $\{\sigma_n^*\}$ has a subsequence of strategies $\sigma_{n_k}^* \to \sigma_\infty^*$ whose norms $||\sigma_{n_k}^*||$ are monotone in their indices n_k. Here I take the case where these norms are increasing, the proof is analogous for the opposite case. Since σ_∞^* is the limit of a subsequence of monotone strategies whose norms are increasing, σ_∞^* must also be a monotone strategy.

this is kind of obvious but maybe i should write one more sentence on it.

Let s_{j^*} be the highest signal for which $\sigma_\infty^*(s_{j^*}) > 0$. Then, there is some $N \in \mathbb{N}$ such that for all $n_k \geq N$ we have $\sigma_\infty^*(s_{j^*}) > \sigma_{n_k}^*(s_{j^*}) > 0$, too. For such $n_k \geq N$, we must have:

$$\frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{s_{j^*}}{1-s_{j^*}} \geq \frac{c}{1-c}$$

since $\sigma^*_{n_k}$ are optimal against the interim beliefs ψ_{n_k}. Furthermore, the continuity of the interim beliefs implies:

$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s_{j^*}}{1-s_{j^*}} \geq \frac{c}{1-c}$$

If $s_{j^*} = s_m$, this implies that $\sigma^*_\infty \in \Phi(\sigma_\infty)$ and therefore concludes our proof. Otherwise, the monotonicity of the subsequence $\{\sigma^*_{n_k}\}$ and $\sigma^*_\infty(s_{j^*+1}) = 0$ implies that $\sigma^*_{n_k}(s_{j^*+1}) = 0$. By the optimality of these strategies against the interim beliefs ψ_{n_k}, we have:

$$\frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{s_{j^*+1}}{1-s_{j^*+1}} \leq \frac{c}{1-c}$$

and by the continuity of interim beliefs:

$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s_{j^*+1}}{1-s_{j^*+1}} \leq \frac{c}{1-c}$$

These observations conclude our proof.

2. The set of equilibrium strategies Σ is compact.

Σ is a subset of $[0,1]^n$ and therefore bounded, hence it suffices to show that is closed. This follows immediately from the upper semi-continuity of evaluators' best response correspondence $\Phi(.)$.

□

The following Lemma, of independent interest itself, will be useful to prove Proposition 2.

Lemma 4. Take three monotone strategies $\sigma'' > \sigma' > \sigma$. If $\Pi(\sigma';\mathcal{E}) \leq \Pi(\sigma;\mathcal{E})$, then $\Pi(\sigma'';\mathcal{E}) \leq \Pi(\sigma';\mathcal{E})$.

Proof. For the three strategies $\sigma'' > \sigma' > \sigma$, consider three sets $A, A', A'' \subset (S \times [0,1])^n$ where the applicant's score profile might lie:

$$\begin{aligned} &Z^\otimes \in A && \text{if } Z^\otimes \text{ is eventually approved under } \sigma'' \text{ but not } \sigma \\ &Z^\otimes \in A' && \text{if } Z^\otimes \text{ is eventually approved under } \sigma' \text{ but not } \sigma \\ &Z^\otimes \in A'' && \text{if } Z^\otimes \text{ is eventually approved under } \sigma'' \text{ but not } \sigma' \end{aligned}$$

Notice that $A' \cap A'' = \emptyset$ and $A' \cup A'' = A$. We can write the difference between the sum of evaluators' payoffs under different strategies as:

$$\Pi(\sigma';\mathcal{E}) - \Pi(\sigma;\mathcal{E}) = \mathbb{P}\left(Z^\otimes \in A'\right) \times \left[\mathbb{P}\left(\theta = H \mid Z^\otimes \in A'\right) - c\right]$$

and:

$$\Pi(\sigma'';\mathcal{E}) - \Pi(\sigma';\mathcal{E}) = \mathbb{P}\left(Z^\otimes \in A''\right) \times \left[\mathbb{P}\left(\theta = H \mid Z^\otimes \in A''\right) - c\right]$$

$\Pi(\sigma';\mathcal{E}) \leq \Pi(\sigma;\mathcal{E})$ implies $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A'\right) \leq c$. But then we must have $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right) \leq c$, since $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right)$ is a convex combination of $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A'\right)$ and $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right)$, and:

$$\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) \geq \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A \cap A''\right) = \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right)$$

Therefore, we have $\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right) \leq \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) \leq \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A'\right) \leq c$.

□

Lemma 2. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*;\mathcal{E}) \geq \Pi(\sigma;\mathcal{E})$.

Proof. Let $Z^{\otimes}$ be the applicant's *score profile*, as in Definition 6. Take an equilibrium strategy σ^* and a more embracive strategy σ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & \underline{s} \\ 0 & s \neq \underline{s} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. Now, let $A \subset (S \times [0,1])^n$ be the set of all score profiles which lead to rejections by all evaluators under σ^*, but an eventual approval under σ:

$$Z^{\otimes} \in A \iff \begin{cases} \sigma^*(s^i) > u^i & \text{for all } i \in \{1, 2, ..., n\} \\ \sigma(s^i) \leq u^i & \text{for some } i \in \{1, 2, ..., n\} \end{cases}$$

Furthermore, for a given score profile $Z^{\otimes}$, let $\#$ be the number of evaluators whose observed scores are such that $\sigma(s^i) \geq u^i > \sigma^*(s^i)$. These evaluators would approve the applicant under the strategy σ, but not under σ^*.

An applicant's eventual outcome differs between the strategy profiles σ and σ^* if and only if his score profile $Z^{\otimes}$ lies in A. Furthermore, his eventual outcome can only change from a rejection by all evaluators in σ^* to an approval by some evaluator in σ. Thus:

$$\begin{aligned} \Pi(\sigma;\mathcal{E}) - \Pi(\sigma^*;\mathcal{E}) &= \left[\mathbb{P}(\theta = H \mid Z^{\otimes} \in A) - c\right] \times \mathbb{P}(Z^{\otimes} \in A) \\ &\propto \mathbb{P}(\theta = H \mid Z^{\otimes} \in A) - c \end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the applicant's signal profile is in A:

$$\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) = \sum_{i=1}^{n} \mathbb{P}\left(\theta = H \mid \# = i\right) \times \frac{\mathbb{P}\left(\# = i\right)}{\mathbb{P}(Z^{\otimes} \in A)}$$

Now note:

$$\mathbb{P}\left(\# = i \mid \theta\right) = \left(p_\theta(\underline{s})\right)^i \times \left(1 - p_\theta(\underline{s})\right)^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}(\# = i) \propto \varepsilon^i$. Since $\mathbb{P}(Z^{\otimes} \in A) = \sum_{i=1}^{n} \mathbb{P}(\# = i)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\#=i)}{\mathbb{P}(Z^{\otimes} \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) - \mathbb{P}(\theta = H \mid \# = 1) = 0$$

I conclude the proof by showing that $\mathbb{P}(\theta = H \mid \# = 1) < c$, and invoking Lemma 4.

$$\begin{aligned}
\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H \mid \# = 1)}{\mathbb{P}(\theta = L \mid \# = 1)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \frac{\mathbb{P}(\# = 1 \mid \theta = H)}{\mathbb{P}(\# = 1 \mid \theta = L)} \\
&= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma; \mathcal{E})}{r_L(\sigma; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&= \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma^*; \mathcal{E})}{r_L(\sigma^*; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \leq \frac{c}{1 - c}
\end{aligned}$$

where $\psi^* = \Psi(\sigma^*; \mathcal{E})$ is the interim belief of the evaluators induced by σ^*. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{1 + r_H^* + \ldots + (r_H^*)^{n-1}}{1 + r_L^* + \ldots + (r_L^*)^{n-1}} \leq \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*; \mathcal{E})$. The last inequality is due to the fact that $\underline{s} \in S$ is optimally rejected under σ^*.

$\square$

Proposition 2. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**}; \mathcal{E}) \geq \Pi(\sigma^*; \mathcal{E})$.

Proof. This is an immediate corollary to Lemma 2. $\square$

Theorem 1. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. As s_H increases, evaluators' payoffs across the extreme equilibria weakly improve. In contrast, evaluators' payoffs for the most (least) selective equilibria:

1. weakly improve as s_L falls, while s_L remains below a cutoff s_L^{mos} (s_L^{les}),

2. weakly deteriorate as s_L falls, once s_L falls below this cutoff.

I will use the five lemmata below, possibly of independent interest, to prove Theorem 1. Throughout, I denote the most selective equilibrium under the signal structure $\mathcal{E}$ as $\sigma_{\mathcal{E}}^{\text{sel}*}$. Similarly, $\sigma_{\mathcal{E}}^{\text{em}*}$ is the most embracive equilibrium. The subscript is dropped when the signal structure in question is obvious.

insert numbers!

Lemma 5. Let $\mathcal{E}$ be binary. $\Psi(\sigma; \mathcal{E})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

is strictly true here?

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

Proof. Part i:
Let $\sigma(F) \in (0,1)$ and $\sigma(S) = 1$. The interim belief ψ is then given by:

$$\begin{aligned}
\Psi(\sigma;\mathcal{E}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right) \\
&= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] \\
&= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]
\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every s_L signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}
\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) =& \mathbb{P}\left(\text{ev. was (i+1)}^{\text{th}} \text{ in order } \mid \text{applicant got i rejections}\right) \\
& \times \mathbb{P}\left(\text{applicant got i rejections}\right) \\
=& \frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i
\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus ψ increases, in $\sigma(s_L)$.
Part ii:
Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma;\mathcal{E}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma;\mathcal{E}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}
\Psi(\sigma;\mathcal{E}) &\propto \frac{1 + r_H + ... + r_H^{n-1}}{1 + r_L + ... + r_L^{n-1}} \\
&= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_H}{1 - r_L} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)} \\
&\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - (1 - p_H(s_H)\sigma(s_H))^n}{1 - (1 - p_L(s_H)\sigma(s_H))^n}
\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1 - s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1 - (r_H)^n}{1 - (r_L)^n}$$

The positive term is the likelihood ratio of one s_H signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since approvals only happen with s_H signals, the negative term strictly exceeds the positive term. Thus, $\Psi(\sigma;\mathcal{E})$ decreases in $\sigma(s_H)$. $\square$

The Corollary below follows from Lemma 5: if $\mathcal{E}' \underset{B}{\succeq} \mathcal{E}$ where both signal structures are binary, adverse selection is stronger under $\mathcal{E}'$, if evaluators always (i) approve upon the high signal, and (ii) reject upon the low signal, under both signal structures.

Corollary 6. Let $\mathcal{E}'$ be more informative than $\mathcal{E}$, the strategies $\sigma'_{(0,1)}$ and σ_1 be $\sigma'_{(0,1)}(s'_L) = \sigma_1(s_L) = 0$ and $\sigma'_{(0,1)}(s'_H) = \sigma_1(s_H) = 1$. Then, $\Psi(\sigma';\mathcal{E}') \leq \Psi(\sigma;\mathcal{E})$.

Proof. I will only prove that the assertion holds when $s_L = s'_L$ but $s'_H > s_H$. The mirror case, which establishes the second part of the corollary, is analogous.

The proof will show that the outcome induced by σ under signal structure $\mathcal{E}$ can be replicated by $\tilde{\sigma}$ under signal structure $\mathcal{E}'$, where $\tilde{\mathcal{E}}(s_L) > 0$ and $\tilde{\mathcal{E}}(s_H) = 1$. Then, the conclusion follows from Lemma 5.

Take the pair $(\sigma,\mathcal{E})$. The probabilities that the applicant is rejected or approved upon a visit, conditional on θ, is given by:

$$\frac{\mathbb{P}(\sigma \text{ rejects } \mid \theta = H)}{\mathbb{P}(\sigma \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\sigma \text{ approves } \mid \theta = H)}{\mathbb{P}(\sigma \text{ approves } \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma},\mathcal{E}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}(\tilde{\sigma} \text{ rejects } \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = L)} = \frac{p'_H(s_H) + \tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H) + \tilde{\sigma}(s_L)p'_L(s_L)}$$

where the family of distributions $\{p'_\theta\}$ belong to $\mathcal{E}'$. It is easy to verify that this last fraction on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma;\mathcal{E})$.

This proves the corollary. $\square$

Lemma 6. Let $\mathcal{E}$ be binary. There is no mixing at $s = s_L$ neither in $\sigma^{\text{sel}*}$ nor in $\sigma^{\text{emb}*}$; i.e. $\sigma^{\text{sel}*}(s_L), \sigma^{\text{em}*}(s_L) \in \{0,1\}$.

Proof. I start by showing $\sigma^{\text{em}*}(s_L) \in \{0,1\}$. Where s_L^{mute} is as it was defined in Definition **??**, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium. This is because $\psi = \rho$ at this induced equilibrium, thus approving upon the low signal is optimal. This is the most embracive equilibrium, since there is no strategy that's more embracive. When $s_L < s_L^{\text{mute}}$, any equilibrium σ must feature $\sigma(s_L) = 0$, since $\psi \leq \rho$.

Now consider $\sigma^{\text{sel}*}$. For contradiction, let $\sigma^{\text{sel}*}(s_L) > 0$. By Lemma 5, and an argument used while proving equilibrium existence in Proposition 1, there is then another equilibrium σ where $\sigma(s_L) = 0$. clarify!

$\square$

Lemma 7 characterises evaluators' decisions upon seeing the low signal in the extreme equilibria, given how informative $\mathcal{E}$ is. Broadly, more informative signal structures push evaluators to reject upon the low signal under both equilibria.

Lemma 7. Let $\mathcal{E}$ be binary, with signal realisations s_L and s_H. Then:

$$\sigma^{\text{em}*}(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \sigma^{\text{sel}*}(s_L) = \begin{cases} 1 & s_H < s_H^{\dagger}(s_L) \\ 0 & s_H \geq s_H^{\dagger}(s_L) \end{cases}$$

where $s_H^{\dagger}(.)$ is an increasing function, and $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

Proof. Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1-s_L} \geq \frac{c}{1-c}$$

which proves the part for the most embracive equilibrium, combined with Lemma 6.

Let the strategy σ_1 be such that $\sigma_1(s_L) = 0$ and $\sigma_1(s_H) = 1$. The following is a necessary and sufficient condition for an equilibrium σ^* where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1;\mathcal{E})}{1-\Psi(\sigma_1;\mathcal{E})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

Necessity follows from $\Psi(\sigma_1;\mathcal{E}) \geq \Psi(\sigma^*;\mathcal{E})$ due to Lemma 5. Sufficiency follows from the fact that an equilibrium always exists, and the condition above implies s_L must always be rejected in it. Due to Corollary 6, we know that this condition holds when s_H is weakly above some threshold $s_H^{\dagger}(s_L)$, increasing with s_L. The necessary and sufficient condition holds whenever s_L^{mute}, therefore $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

$\square$

Proof, Theorem 1: I prove Theorem 1 by establishing four facts:

1. The expected payoff in an equilibrium σ^* where $\sigma^*(s_L) = 0$ is higher than the expected payoff of approving all applicants.

   This follows directly from Proposition **??**.

2. There is at most one equilibrium where $\sigma^*(s_L) = 0$.

Let $\{\sigma_\alpha\}_{\alpha\in[0,1]}$ be the family of strategies where the low signal is rejected: $\sigma_\alpha(s_L) := 0$ and $\sigma_\alpha(s_H) := \alpha$. If:

$$\frac{\Psi(\sigma_1;\mathcal{E})}{1-\Psi(\sigma_1;\mathcal{E})} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c}$$

σ_1 is the only equilibrium candidate among this family; the interim belief is higher under any lower α by Lemma 5. Otherwise, again by Lemma 5, there is at most one $\alpha \in [0,1]$ for which:

$$\frac{\Psi(\sigma_\alpha;\mathcal{E})}{1-\Psi(\sigma_\alpha;\mathcal{E})} \times \frac{s_H}{1-s_H} - \frac{c}{1-c} = 0$$

When such an α exists, σ_α is the only equilibrium candidate in this family. Under higher α, approving upon $s = s_H$ is not optimal. Under lower α, rejecting upon $s = s_L$ is not optimal. If the expression above is strictly negative for *any* α, then the only equilibrium candidate where the low signal is rejected is σ_0.

3. When an equilibrium $\sigma^* \in \{\sigma_\alpha\}_{\alpha\in[0,1]}$ where all low signals are rejected exists, the expected payoff in this equilibrium is given by $\pi_i(\sigma^*;\mathcal{E}) = \max\{0, \pi_i(\sigma_1;\mathcal{E})\}$.

   Above we showed that evaluators expect positive expected payoff (necessarily from approving an applicant) only when $\alpha = 1$. Otherwise, they either approve no applicant or are indifferent to rejecting those they do.

Theorem 1 then follows from our fourth claim:

Below, I use s_L^{as} but do not reiterate what it means.

4. $\max\{0, \Pi(\sigma_1;\mathcal{E})\}$ is:

   i weakly increasing in s_H whenever σ_α is an equilibrium strategy for some $\alpha \in [0,1]$,

   ii hump-shaped in s_L. As s_L falls, it is:

   - weakly increasing when $s_L \geq s_L^{as}$,
   - weakly decreasing when $s_L \leq s_L^{as}$

make sure the notation is either Π or π

   where s_L^{as} is implicitly defined as:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

   for the signal structure $\mathcal{E}$.

Due to Lemma 7, both the most embracive and most selective equilibria shift once from the equilibrium where *all* s_L signals are approved to the one where *none* are approved, as the binary signal structure $\mathcal{E}$ becomes more informative. Due to the first fact laid out in the proof of this Theorem, this induces an increase in evaluators' expected payoff. Therefore, this last assertion about the shape of evaluators' payoffs in the equilibrium where the low signal is rejected concludes the proof.

Proof for the fourth claim:

Part i: Increasing s_H; i.e. the strength of evidence for $\theta = H$.

Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary signal structures with respective signal realisations $\{s_L, s_H\}$ and $\{s'_L, s'_H\}$. Let $s'_L = s_L$ and $s'_H = s_H + \delta$ for $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma_1; \mathcal{E}') > \Pi(\sigma_1; \mathcal{E})$.

Step 1: Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

Rather than having evaluators observe one draw from the signal structure $\mathcal{E}'$, say an evaluator potentially observes *two* signal realisations; s and $\hat{s}$. She first observes s, a single draw from $\mathcal{E}$. If this signal realises as $s = s_L$, she observes no further information. If instead $s = s_H$, she observes another signal $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from the signal structure $\hat{\mathcal{E}}$. $\hat{s}$ has the following distribution, and is independent from s, conditional on θ:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(s, \hat{s})$ is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{8.1}$$

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{8.2}$$

Note that the likelihood ratio 8.1 increases continuously with ε. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - (s_H + \delta)} \tag{8.3}$$

for our chosen (δ, ε). Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ. Furthermore, by varying ε between 0 and $\frac{1 - s_H}{s_H}$, the equivalent of *any* signal structure $\mathcal{E}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$ can be obtained.

Step 2: $\pi(\sigma_1; \mathcal{E}') > \pi(\sigma_1; \mathcal{E}')$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(s, \hat{s})$ instead of s'. To do so, the evaluator approves if and only if the pair $(s, \hat{s}) = (s_H, \hat{s_H})$ is observed. Note that, conditional on the visiting applicant's quality, the probability that the evaluator approves him is the same under these two policies. This is due to the identical informational content of these signals, as laid out in equations 8.2 and 8.3. Thus, evaluators' payoffs are also identical under these policies.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(s_i, \hat{s}_i)\}_{i=1}^{n}$. An applicant is a *marginal reject* if he has no $(s_i, \hat{s}_i) = (s_H, \hat{s_H})$ signals. The difference between evaluators' payoffs under $(\mathcal{E}, \hat{\mathcal{E}})$ and $\mathcal{E}$ is determined by these *marginal rejects*: they are *eventually rejected* under $(\mathcal{E}, \hat{\mathcal{E}})$ but *eventually approved* under $\mathcal{E}$. So:

$$\Pi(\sigma_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[c - \mathbb{P}\left(\theta = H \mid \text{marginal reject}\right)\right]}_{(1)}$$

A marginal reject only has signal realisations $(s, \hat{s}) = (s_H, \hat{s_L})$ or $s = s_L$. These carry equivalent information about θ. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid s_1 = \ldots = s_n = s_L\right]$$

In the relevant region where $s = s_L$ leads to a rejection, the expression above must be weakly positive. Therefore, $\Pi(\sigma_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) \geq 0$.

Maybe a brief explainer.

This concludes the first part of the claim that $\max\{0, \pi(\sigma_1; \mathcal{E})\}$ is weakly increasing in s_H.

I should probably just focus on the π not the whole thing, correct!

Part ii: Decreasing s_L; i.e. increasing the strength of evidence for $\theta = L$.

Now I show that replacing $\mathcal{E}$ with $\mathcal{E}'$ when $s'_L = s_L - \delta$ and $s'_H = s_H$:

i increases $\pi(\sigma_1; \mathcal{E})$ when $s_L \leq s_L^{as}$,

ii decreases $\pi(\sigma_1; \mathcal{E})$ when $s_L > s_L^{as}$

for $\delta > 0$ arbitrarily small. The desired assertion follows.

I should probably use the sum of evaluators' payoffs here.

Step 1: Replicating $\mathcal{E}'$ in two signals.

As before, let the evaluator potentially observe *two* signal realisations, s and $\hat{s}$. She first observes s, a single draw from $\mathcal{E}$. If this signal realises as $s = s_H$, she receives no further information. If it realises as $s = s_L$, she observes another signal $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from a signal structure we construct now, $\hat{\mathcal{E}}$. $\hat{s}$ is distributed independently from s conditional on θ, as follows:

$$\mathbb{P}\left(\hat{s} = s_H \mid \theta = H\right) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad \mathbb{P}\left(\hat{s} = s_H \mid \theta = L\right) = \varepsilon \times \frac{s_L}{1 - s_L}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(s, \hat{s})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)} = \frac{s_H}{1 - s_H} \tag{8.4}$$

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)} = \frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} \tag{8.5}$$

Note that 8.5 is continuously and strictly decreasing with ε, taking values between $\frac{s_L}{1 - s_L}$ and 0 as ε varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_L}{1-s_L} \times \frac{1-\varepsilon \times \dfrac{s_H}{1-s_H}}{1-\varepsilon \times \dfrac{s_L}{1-s_L}} = \frac{s_L-\delta}{1-(s_L-\delta)}$$

Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ.

Step 2: $\pi(\sigma_1;\mathcal{E})$ increases (decreases) with a marginal decrease in s_L, whenever $s_L \geq s_L^{as}$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(s, \hat{s})$ instead of s'. To do so, the evaluator rejects if and only if the pair $(s, \hat{s}) = (s_L, \hat{s_L})$ is observed.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(s_i, \hat{s}_i)\}_{i=1}^n$. An applicant is a *marginal admit* if: (i) he has *no* $s = s_H$ signals, and (ii) he has *at least one* $\hat{s} = \hat{s_L}$ signal. The difference between evaluators' payoffs under $(\mathcal{E}, \hat{\mathcal{E}})$ and $\mathcal{E}$ is determined by these *marginal admits*, who are *eventually rejected* under $\mathcal{E}$, but *eventually approved* under $(\mathcal{E}, \hat{\mathcal{E}})$. So:

$$\Pi(\sigma_1;\mathcal{E}') - \Pi(\sigma_1;\mathcal{E}) = \mathbb{P}\left(\text{marginal admit}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

For a marginal admit, $(s_i, \hat{s}_i) \in \{(s_L, \hat{s_H}), (s_L, \hat{s_L})\}$, and $(s_j, \hat{s}_j) = (s_L, \hat{s_H})$ for at least one evaluator j. Denote the number of evaluators who observe $(s_L, \hat{s_H})$ as $\#$. Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}{\sum\limits_{j=1}^{n} \mathbb{P}\left(j\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}}_{(3)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$

where:

$$\mathbb{P}\left(i\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ...s_n = s_L\right) = \binom{n}{i} \times (k \times \varepsilon)^i \times (1 - k \times \varepsilon)^{n-1}$$

for $k = \mathbb{P}\left(\theta = H \mid s_1 = ... = s_n = s_L\right)$. The limit of expression (3) as $\varepsilon \to 0$ (thus $\delta \to 0$) is:

$$\lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(i\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}{\sum\limits_{j=1}^{n} \mathbb{P}\left(j\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)} = \mathbb{P}\left(1\ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)$$

and thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c = \lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \# = 1\right) - c$$

This expression is strictly positive (negative) when the expression below is strictly positive (negative):

there is a limit below as well, add.

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} - \frac{c}{1-c}$$

proving the claim.

$\square$

Proposition 3. Suppose the experiment $\mathcal{E}$ is binary with outcomes inducing the normalised beliefs $s_L \in [0, 0.5]$ and $s_H \in [0.5, 1]$. Evaluators' payoffs in the least selective equilibrium deteriorate with lower s_L when s_L is below the cutoff $s_L^{\text{les}} := \min\{s_L^{\text{mute}}, s_L^{as}\}$. This threshold is higher for the most selective equilibrium; $s_L^{\text{mos}} \geq s_L^{\text{les}}$.

Proof. I start with the most embracive equilibrium. When $s_L \geq s_L^{\text{safe}}$, the strategy $\sigma_{(1,1)}$ which approves everyone; i.e. $\sigma_{(1,1)}(s_L) = \sigma_{(1,1)}(s_H) = 1$, is an equilibrium. This owes to $\Psi(\sigma_{(1,1)}; \mathcal{E}) = \rho$ as it can be easily checked, and to the definition of s_L^{safe}. Since no strategy is more embracive, $\sigma^{\text{em}*} = \sigma_{(1,1)}$. In this parameter region, $\pi(\sigma^{\text{em}*}; \mathcal{E})$ does not vary as every applicant is approved. When $s_L < s_L^{\text{safe}}$, this equilibrium is no longer possible, and evaluators' equilibrium payoffs are thus given by $\pi(\sigma^{\text{em}*}; \mathcal{E}) = \max\{0, \pi(\sigma_1; \mathcal{E})\}$; as it was explained in the second fact under Theorem 1's proof. As s_L decreases, this increases (decreases) when $s_L \geq s_L^{as}$ ($s_L < s_L^{as}$). This establishes the first part of Proposition 3.

For the most selective equilibrium to have $\sigma^*(\text{sel}*) = 0$, a necessary and sufficient condition is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1 - s_L} \leq \frac{c}{1 - c}$$

This owes to Lemma 5, which establishes that the interim belief *increases* in $\sigma(s_H)$.

Clearly, this condition is satisfied when $s_L \leq s_L^{\text{safe}}$. Thus, the most selective equilibrium becomes one where s_L leads to a rejection once s_L falls below some threshold $s_L^{\text{thr}} \geq s_L^{\text{safe}}$. Evaluators' equilibrium payoffs then start falling with stronger evidence for $\theta = L$ once $s_L \leq \min\{s_L^{\text{thr}}, s_L^{as}\}$.

$\square$

Theorem 2. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $s = s_i$ leads to approvals under σ.

2. *weakly lower* than under σ:

   i if $s = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

   ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{E}$ and σ.

Proof. Following the notation introduced in Definition 4, let $S \cup S'$ be the joint support of the signal structures $\mathcal{E}$ and $\mathcal{E}'$, and $s_1 < s_2 < ... < s_M$ be its elements. I begin by noting that the outcome the monotone strategy $\sigma : S \to [0,1]$ generates under $\mathcal{E}$ can be replicated under $\mathcal{E}'$ by another monotone strategy $\tilde{\sigma}' : S' \to [0,1]$ provided $\sigma(s_i) \in \{0,1\}$[10]:

$$\tilde{\sigma}'(s_j) = \begin{cases} \sigma(s_i) & j \in \{i-1, i+1\} \\ \sigma(s_j) & j \notin \{i-1, i+1\} \end{cases}$$

Part 1:

Now suppose s_i leads to approvals under σ; $\sigma(s_i) = 1$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 1$. I argue below that $\tilde{\sigma} \geq \sigma'$; evaluators *reject* more when s_i is spread. From Proposition **??**, it follows that $\pi(\sigma; \mathcal{E}) = \pi(\tilde{\sigma}'; \mathcal{E}') \leq \pi(\sigma'; \mathcal{E}')$.

If $s_{i-1} = \min S \cup S'$ or $\sigma'(s_{i-2}) = 0$, we necessarily have $\tilde{\sigma} \geq \sigma'$ and are done. So, for contradiction, let $\sigma'(s_{i-2}) > 0$, and $\sigma' > \tilde{\sigma}'$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{E}$ such that:

i $\tilde{\sigma}$ replicates the outcome σ' induces in $\mathcal{E}'$,

ii $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ if and only if σ' is an equilibrium strategy under $\mathcal{E}'$,

iii $\tilde{\sigma} > \sigma$, so σ cannot be the most embracive equilibrium under $\mathcal{E}$.

So, define the strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{E}$ as simply:

$$\tilde{\sigma}(s_j) := \begin{cases} 1 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of σ'. Furthermore, σ' is an equilibrium under $\mathcal{E}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}$: they induce the same interim belief ψ as the latter replicates the former, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}\left(\theta = H \mid \psi, s = s_{i-2}\right) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

Lastly, since $\sigma' > \tilde{\sigma}'$, it must be that $\tilde{\sigma} > \sigma$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Recall that $\tilde{\sigma}$ and σ induce the same interim belief ψ under their respective signal structures; $\Psi\left(\tilde{\sigma}; \mathcal{E}'\right) = \Psi\left(\sigma; \mathcal{E}\right) = \psi$. Therefore, if $\mathbb{P}\left(\theta = H \mid s = s_{i-1}, \psi\right) \geq c$, $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}'$. This implies that $\sigma' \leq \tilde{\sigma}$, since σ' is the most selective equilibrium under $\mathcal{E}'$. If

[10]The characterisation of $\tilde{\sigma}$ is otherwise the same, but it ceases to be *monotone* by our definition.

$\mathbb{P}(\theta = H \mid s = s_{i-1}, \psi) < c$ otherwise, there is an equilibrium $\sigma' < \tilde{\sigma}$ under $\mathcal{E}'$ due to the intermediate value argument presented when equilibrium existence was established in Proposition 1.

make this reference clear, we are using it a lot.

Part 2:

Now suppose s_i leads to rejections under σ; $\sigma(s_i) = 0$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 0$. I will establish Theorem 2's claim in two steps:

1. $\sigma' \geq \tilde{\sigma}'$; evaluators *approve* more when s_i is spread,
2. $\pi(\sigma'; \mathcal{E}') \leq \pi(\tilde{\sigma}'; \mathcal{E}') = \pi(\sigma; \mathcal{E})$ when adverse selection poses a threat at signal s_{i+1} for signal structure $\mathcal{E}$ and strategy σ.

Step 1:

If $s_{i+1} = \max S \cup S'$ or $\sigma'(s_{i+1}) > 0$, we necessarily have $\sigma' \geq \tilde{\sigma}$. So, let $s_{i+1} < \max S \cup S'$ and $\sigma'(s_{i+1}) = 0$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Recall that $\Psi(\sigma; \mathcal{E}) = \Psi(\tilde{\sigma}'; \mathcal{E}') = \psi$ since $\tilde{\sigma}'$ replicates the outcome of σ. Thus, if:

$$\mathbb{P}(\theta = H \mid s = s_{i+1}, \psi) \leq c$$

$\tilde{\sigma}'$ must be an equilibrium strategy under $\mathcal{E}'$; the optimality conditions for all signals below s_{i+1} are satisfied *a fortiori*, and those for the signals above s_{i+1} are satisfied since σ is an equilibrium strategy in $\mathcal{E}$. Then, $\sigma' \geq \tilde{\sigma}'$, since σ' is the most embracive equilibrium. If

$$\mathbb{P}(\theta = H \mid s = s_{i+1}, \psi) > c$$

on the other hand, by the intermediate value argument we used to establish equilibrium existence in Proposition 1, there is an equilibrium strategy $\sigma' > \tilde{\sigma}$ under $\mathcal{E}'$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Since $\sigma'(s_{i+1})$, its outcome under $\mathcal{E}'$ can be replicated $\mathcal{E}$ with a strategy $\tilde{\sigma}$, defined as:

$$\tilde{\sigma}(s_j) = \begin{cases} 0 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

I will show that this necessarily implies that $\sigma' \geq \tilde{\sigma}'$, in two steps:

i $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ if and only if σ' is an equilibrium strategy under $\mathcal{E}'$,

ii $\tilde{\sigma} \leq \sigma$, and therefore $\tilde{\sigma} = \sigma$ since σ is the most selective equilibrium under $\mathcal{E}$.

(i) follows trivially, since both strategies have the same optimality condition for every signal realisation above s_{i+1}. Now, since σ is the most selective equilibrium under $\mathcal{E}$, we must have

$\sigma \leq \tilde{\sigma}$; as $\tilde{\sigma}$ is an equilibrium strategy by (i). However, this means $\tilde{\sigma}' \leq \sigma'$. Since $\tilde{\sigma}'$ must also be an equilibrium in $\mathcal{E}'$, we must have $\tilde{\sigma}' = \sigma'$ and therefore $\sigma \leq \tilde{\sigma}$.

Step 2:

Now I establish the second part. The case where $\tilde{\sigma}' = \sigma'$ is trivial, so I focus on the case $\sigma' > \tilde{\sigma}'$. As we showed when establishing Case 2 in the first step, we must then have $\sigma'(s_{i+1}) > 0$.

Now take a strategy σ^{ε} for $\mathcal{E}'$, defined as $\sigma^{\varepsilon}(s_{i+1}) := \varepsilon$. We take ε small enough so that $\sigma' > \sigma^{\varepsilon} > \tilde{\sigma}'$. I will now show that when adverse selection poses a threat at signal s_{i+1} for $(\sigma; \mathcal{E})$, we have:

$$\pi(\sigma^{\varepsilon}; \mathcal{E}') \leq \pi(\tilde{\sigma}'; \mathcal{E}') = \pi(\sigma; \mathcal{E})$$

Proposition **??** then coins the result.

I show this slightly circuitously. Construct a ternary signal $\mathcal{E}^{\text{re}}$ which we will use to replicate the outcomes σ^{ε} and $\tilde{\sigma}'$ generate. This signal admits the realisations $s^{\text{re}} \in \{s_L^{\text{re}}, s_{\varepsilon}^{\text{re}}, s_H^{\text{re}}\}$ and has distribution:

$$\mathbb{P}\left(s^{\text{re}} = s \mid \theta\right) = \begin{cases} 1 - r_{\theta}(\sigma; \mathcal{E}) & s = s_H^{\text{re}} \\ \varepsilon \times p'_{\theta}(s_{i+1}) & s = s_{\varepsilon}^{\text{re}} \\ r_{\theta}(\sigma; \mathcal{E}) - \varepsilon \times p'_{\theta}(s_{i+1}) & s = s_L^{\text{re}} \end{cases}$$

Clearly, as defined below, the strategies σ^{re} and $\sigma^{\text{re-}\varepsilon}$ for $\mathcal{E}^{\text{re}}$ replicate the outcomes of $\tilde{\sigma}$ and σ^{ε} under $\mathcal{E}'$:

$$\sigma^{\text{re}}(s) = \begin{cases} 1 & s = s_H^{\text{re}} \\ 0 & s = s_{\varepsilon}^{\text{re}} \\ 0 & s = s_L^{\text{re}} \end{cases} \qquad \sigma^{\text{re}}(s) = \begin{cases} 1 & s = s_H^{\text{re}} \\ 1 & s = s_{\varepsilon}^{\text{re}} \\ 0 & s = s_L^{\text{re}} \end{cases}$$

This makes it clear that the difference in evaluators' payoffs between $\tilde{\sigma}$ and σ^{ε} will be the *marginal admits* whose evaluators will observe:

i no s_H^{re} signal realisation,

ii at least one $s_{\varepsilon}^{\text{re}}$ signal realisation.

if they visit all evaluators. Thus, we have:

$$\Pi(\sigma^{\varepsilon}; \mathcal{E}') - \Pi(\tilde{\sigma}; \mathcal{E}') = \mathbb{P}\left(\text{marginal admits}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

where (2) then equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ s_{\varepsilon}^{\text{re}} \text{ and } n-j\ s_L^{\text{re}} \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_L^{\text{re}} \text{ signals}\right)$$

COMPLETE! ☐

Proposition 4. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proof. ☐

Corollary 5. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

why does this appear with the incorrect number?

Proof. ☐

References

Blackwell, D. (1953). Equivalent comparisons of experiments. The Annals of Mathematical Statistics, 24(2), 265–272. Retrieved October 4, 2022, from http://www.jstor.org/stable/2236332

Blackwell, D., & Girshick, M. A. (1954). Theory of games and statistical decisions. John Wiley & Sons.

Broecker, T. (1990). Credit-worthiness tests and interbank competition. Econometrica, 58(2), 429–452. Retrieved May 31, 2023, from http://www.jstor.org/stable/2938210

Gentzkow, M., & Kamenica, E. (2016). A rothschild-stiglitz approach to bayesian persuasion. The American Economic Review, 106(5), 597–601. Retrieved May 27, 2024, from http://www.jstor.org/stable/43861089

Hardy, G., Littlewood, J., & Pólya, G. (1952). Inequalities. Cambridge University Press. https://books.google.co.uk/books?id=t1RCSP8YKt8C

Kreyszig, E. (1978). Introductory functional analysis with applications. John Wiley & Sons.

Makarov, I., & Plantin, G. (2023). Sequential credit markets [Working Paper]. Retrieved March 14, 2024, from https://personal.lse.ac.uk/makarov1/index_files/SequentialCreditMarkets.pdf

Muirhead, R. F. (1900). Inequalities relating to some algebraic means. Proceedings of the Edinburgh Mathemati 19, 36–45. https://doi.org/10.1017/S0013091500032594

Müller, A., & Stoyan, D. (2002). Comparison methods for stochastic models and risks. Wiley. https://books.google.co.uk/books?id=a8uPRWteCeUC

Munkres, J. (2000). Topology. Prentice Hall, Incorporated. https://books.google.co.uk/books?id=XjoZAQAAIAAJ

Rasmusen, E., & Petrakis, E. (1992). Defining the mean-preserving spread: 3-pt versus 4-pt. In J. Geweke (Ed.), Decision making under risk and uncertainty: New models and empirical findings (pp. 53–58). Springer Netherlands. https://doi.org/10.1007/978-94-011-2838-4_7

Rothschild, M., & Stiglitz, J. E. (1970). Increasing risk: I. a definition. Journal of Economic Theory, 2(3), 225–243. https://doi.org/https://doi.org/10.1016/0022-0531(70)90038-4

Shaked, M., & Shanthikumar, J. (2007). Stochastic orders. Springer New York.