1 The Model

An *applicant* (he) with either *High* or *Low* quality, $\theta \in \{H, L\}$, seeks an approval from any one of n *evaluators* (she). To obtain the approval he seeks, he sequentially visits these n evaluators, each at most once. Any evaluator who receives his visit decides whether to *approve* or *reject* the applicant. Once he is approved or has visited all n evaluators, the applicant stops his visits. Otherwise he visits the evaluator labeled $\tau(k)$ after his $k-1^{\text{th}}$ rejection, where $\tau(.)$ is a permutation of the set of evaluators' labels $\{1, 2, ..., n\}$.

If an evaluator approves the applicant, she pays a fixed cost of $c \in [0, 1]$. She also receives a benefit of 1 whenever the applicant has *High* quality. With her approval, the game ends; the applicant stops and the remaining evaluators walk away with a payoff of 0. On the other hand, if she rejects the applicant, she receives a payoff of 0. The applicant then proceeds to visit the remaining evaluators, unless none do.

At the outset of the game, the applicant and all n evaluators share the *prior belief* that the applicant has *High* quality with probability $\rho \in (0, 1)$. Moreover, the evaluators commonly believe that they are equally likely to be anywhere in the applicant's visit order; i.e. that $\mathbb{P}(\tau(k) = i) = \frac{1}{n}$ for all $k, i \in \{1, 2, ..., n\}$. No further information about the applicant's visits is disclosed to evaluators: neither the order $\tau(.)$ he follows, nor the number of evaluators who rejected him already.

Crucially however, an evaluator receiving a visit understands that the applicant was rejected in all his previous visits, however many there might have been. With this information the applicant's visit conveys, she updates her prior belief ρ about the applicant's quality to an *interim belief* ψ.

Subsequently, she observes a costless and private signal S about the applicant's quality. This signal S she observes is the outcome of a Blackwell experiment $\mathcal{E} = (\mathbf{S}, p_L, p_H)$; the signal takes a value s from the finite set $\mathbf{S}$ and has distribution p_θ over this set given the applicant's quality θ. Conditional on the applicant's quality, signals different evaluators observe are IID. After she observes the signal $S = s \in \mathbf{S}$, the evaluator updates her interim belief ψ that the applicant has *High* quality to a *posterior belief*, which I denote as $\mathbb{P}_\psi(\theta = H \mid S = s)$.

An evaluator's strategy $\sigma : \mathbf{S} \to [0, 1]$ prescribes a probability of approval $\sigma(s)$ to every possible realisation $s \in \mathbf{S}$ of her signal. I call a strategy σ *optimal against the evaluator's interim belief* ψ if, given this interim belief ψ, it maximises her expected payoff. Under such a strategy σ, the evaluator approves after any signal $s \in \mathbf{S}$ which raises her posterior belief that the applicant has *High* quality above c. Likewise, she rejects whenever this posterior belief sinks below c:

$$\sigma(s) = \begin{cases} 0 & \mathbb{P}_\psi(\theta = H \mid S = s) < c \\ 1 & \mathbb{P}_\psi(\theta = H \mid S = s) > c \end{cases}$$

A signal that sets her posterior belief exactly equal to c leaves her indifferent between approving and rejecting the applicant. Any approval probability her strategy dictates after such a signal realisation is consistent with its optimality.

I focus on the *symmetric Bayesian Nash Equilibria* of this game. Hereafter, I reserve the word "equilibrium" for such equilibria unless I state otherwise. A strategy and belief pair (σ^*, ψ^*) is an *equilibrium* of this game if and only if it satisfies the two conditions below:

1. The interim belief ψ^* is *consistent* with every evaluator having the strategy σ^*. That is, an evaluator receiving a visit forms the interim belief ψ^* given others' strategies are σ^*.
2. The strategy σ^* is optimal given the interim belief ψ^*.

I call any strategy σ^* which constitutes part of an equilibrium an *equilibrium strategy*.

2 Belief Formation and Equilibria

Before her verdict, the evaluator who receives a visit must assess the probability that she faces a *High* quality applicant. Her privately observed signal about this applicant's quality plays a crucial part in this assessment. But she obtains her first piece of information even earlier, through *her mere receipt of the applicant's visit.*

i don't quite know how to introduce this

The applicant visits this evaluator only if he was rejected by every evaluator he visited earlier. Any such rejections are themselves bad news about the applicant's quality, as they reveal his past evaluators' negative assessments. No information about the number of these past rejections is disclosed to our evaluator. Nonetheless, she is aware of the adverse selection problem she faces: the likelier her peers are to reject the applicant, the likelier she is to be visited by him. Therefore, she interprets the applicant's mere visit as bad news about his quality already.

In particular, when all her peers have the strategies σ, our evaluator understands that an applicant with quality θ faces a probability $r_\theta(\sigma; \mathcal{E})$ of getting rejected from any of his visits, where this probability is given by:

$$r_\theta(\sigma; \mathcal{E}) = 1 - \sum_{j=1}^{m} p_\theta(s_j)\sigma(s_j)$$

She – ex-ante – believes she is equally likely to be anywhere in the applicant's visit order $\tau(.)$. So, she believes that an applicant with quality θ will visit her with probability $\nu_\theta(\sigma; \mathcal{E})$ before any of her peers approves him:

$$\nu_\theta(\sigma; \mathcal{E}) = \frac{1}{n} \times \sum_{k=1}^{n} r_\theta(\sigma; \mathcal{E})^{k-1}$$

Our evaluator's *interim belief* ψ that the applicant who visited her has *High* quality must be *consistent* with these beliefs she holds. Through Bayes Rule, this consistency requirement pins her interim belief down uniquely:

$$\begin{aligned}\psi := \mathbb{P}\left(\theta = H \mid \text{visit received}\right) &= \frac{\mathbb{P}\left(\text{visit received} \mid \theta = H\right) \times \mathbb{P}(\theta = H)}{\mathbb{P}\left(\text{visit received}\right)} \\ &= \frac{\rho \times \nu_H\left(\sigma; \mathcal{E}\right)}{\rho \times \nu_H\left(\sigma; \mathcal{E}\right) + (1-\rho) \times \nu_L\left(\sigma; \mathcal{E}\right)}\end{aligned}$$

After the evaluator updates her prior belief to this interim belief, she observes the realisation $s \in \mathbf{S}$ of her private signal S. From her signal, she distils further information about the applicant's quality and updates her interim belief ψ to a final *posterior belief* $\mathbb{P}_\psi\left(\theta = H \mid S = s\right)$:

$$\mathbb{P}_\psi\left(\theta = H \mid S = s\right) = \frac{\psi \times p_H(s)}{\psi \times p_H(s) + (1-\psi) \times p_L(s)}$$

The information packed in a signal realisation $s \in \mathbf{S}$ is determined exclusively by its conditional probabilities, $p_H(s)$ and $p_L(s)$. So for notational convenience, I label each possible realisation of an evaluator's signal after a ratio of its conditional probabilities:

$$s = \frac{p_H(s)}{p_H(s) + p_L(s)}$$

I merge signal realisations for which these ratios are equal. Likewise without loss, I enumerate these signal realisations $\{s_1, s_2, ..., s_m\} = \mathbf{S}$ so that they are strictly increasing in their indices; $s_1 < s_2 < ... < s_m$.

Using this notation, we can re-express the evaluator's posterior belief upon observing the signal $s_j \in \mathbf{S}$ as simply:

$$\mathbb{P}_\psi\left(\theta = H \mid S = s_j\right) = \frac{\psi \times s_j}{\psi \times s_j + (1-\psi) \times (1-s_j)}$$

As this expression clarifies, an evaluator's posterior belief is also increasing in the index of the signal $s_i \in \mathbf{S}$ she observes. Note that an evaluator's posterior belief equals *exactly* the label of her observed signal s_i when her interim belief assigns equal probability to either quality; $\psi = 0.5$.

There is a unique interim belief ψ^* that is consistent with all evaluators using any given strategy σ^*. Whenever this strategy σ^* is optimal against this interim belief ψ^*, the pair (σ^*, ψ^*) forms an equilibrium. In principle, there might be multiple such pairs, or none. I set the ground in Proposition 1 by ruling this last possibility out: an equilibrium is always guaranteed to exist. Also in Proposition 1, I describe some properties of these equilibria that are fundamental to the rest of our analysis.

Proposition 1. Where Σ is the set of evaluators' equilibrium strategies:

1. An equilibrium always exists; $\Sigma \neq \emptyset$.

2. The set Σ is compact.

3. Any equilibrium strategy is *monotone*; for any equilibrium strategy $\sigma^* \in \Sigma$ and signal realisations $s_{j'}, s_j \in \mathbf{S}$:

$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad \text{whenever } s_{j'} > s_j$$

4. All equilibria exhibit adverse selection; $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy σ^*.

Removed totally orderedness to proposition 2 where it's more natural to discuss it.

Also changed eqm. existence proof. Previously was a direct proof with algorithm using IVT. Now Kakutani FPT.

I relegate the full proof of Proposition 1 to Section 8. Instead, I discuss its proof in broad strokes here. To establish the existence of an equilibrium, I construct a *best response correspondence* Φ for evaluators. Φ maps any strategy σ to the set of all strategies optimal against the unique interim belief consistent with σ. Put differently, $\Phi(\sigma)$ gives the set of strategies maximising an evaluator's expected payoff when all her peers use the strategy σ. Note that a strategy σ^* is an equilibrium strategy if and only if it is a fixed point of this best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I show that Φ indeed has a fixed point, by a routine application of Kakutani's Fixed Point Theorem. To this end, I prove that Φ is upper semi-continuous, which establishes the compactness of the set of equilibrium strategies as well.

Equilibrium strategies must be monotone simply because they must be optimal against some interim belief. As I remarked earlier, higher signal realisations induce higher posterior beliefs. Hence, if an evaluator weakly prefers approving an applicant at a low signal realisation, she strictly prefers it at a higher one. Monotonicity also implies that a *Low* quality applicant is likelier to receive a rejection in any equilibrium; as his evaluators are likelier to observe lower signals. Therefore, visits an evaluator receives are *adversely selected*: her interim belief ψ always lies below her prior belief ρ.

Though an equilibrium is guaranteed to exist, it need not be unique. A simple example suffices to illustrate this. Consider two evaluators who have a prior belief $\rho = 0.5$ about the applicant's quality, an approval cost $c < 0.5$, and a signal with two possible realisations; $s \in \{c, 1-c\}$. One equilibrium strategy for these evaluators is to approve every applicant who visits them. This strategy eradicates adverse selection; no evaluator receives an applicant with a past rejection. Thus, evaluators' interim belief ψ equals their prior $\rho = 0.5$, and observing the low signal $s = c$ leaves them indifferent between approving and rejecting the applicant.

There is, however, another equilibrium where evaluators approve the applicant only upon the high signal, $s = 1-c$. This triggers adverse selection: evaluators risk being visited by a past reject, thus revise their interim belief ψ below their prior $\rho = 0.5$. Consequently, the posterior belief induced by the low signal $s = c$ falls strictly below c, justifying the applicant's rejection. Note that approving the applicant upon the high signal $s = 1-c$ remains optimal: even an evaluator who is sure she saw this signal after her peer observed the low signal expects a positive payoff from approving the applicant.

In this latter equilibrium, an applicant – regardless of his quality – faces higher rejection chances in any of his visits. His evaluators are *more selective*; any signal they might observe leads to a (weakly) higher chance of rejection. It is natural in general to try and compare equilibria in their *selectivity*, whenever we face multiple.

Definition 1. Where σ' and σ are two strategies for evaluators, σ' is *more selective than* σ (or, σ is *less selective than* σ') if $\sigma'(s) \leq \sigma(s)$ for all $s \in \mathbf{S}$.

While natural, the *selectivity* (or *pointwise*) order might initially appear restrictive. This impression is misleading. In fact, the set of equilibrium strategies is *totally ordered* (or, a *chain*) under this order; any two equilibrium strategies can be compared under it. Furthermore, this set has both a *most selective* and *least selective* element, marking its extremes. I refer to them as the *extreme equilibria* in the sequel.

Lemma 1. The set of evaluators' equilibrium strategies Σ is *totally ordered* under the selectivity order. Moreover, Σ contains a *most selective* and *least selective* strategy, $\sigma^{\text{mos}} \in \Sigma$ and $\sigma^{\text{les}} \in \Sigma$ respectively, such that:

$$\sigma^{\text{mos}}(s) \leq \sigma^{*}(s) \leq \sigma^{\text{les}}(s) \qquad \text{for all } s \in \mathbf{S}$$

Proof. By Proposition 1, the set of equilibrium strategies Σ is a subset of the set of monotone strategies. The latter is a chain under the *selectivity* order; for two monotone strategies σ and σ', we have:

$$\sigma'(s_j) > \sigma(s_j) \implies 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'}) \qquad \text{for any } s_{j'} > s_j \in \mathbf{S}$$

Since any subset of a chain is also a chain, Σ is a chain too.

By Proposition 1, Σ is a compact set. Since it is also a chain, by a suitably general Extreme Value Theorem (see, for instance, Theorem 27.4 in Munkres, 2000) it has a *minimum* and *maximum* element, σ^{mos} and $\sigma^{\text{les}} \in \Sigma$ respectively, with respect to this order:

$$\sigma^{\text{mos}}(s) \leq \sigma^{*}(s) \leq \sigma^{\text{les}}(s) \qquad \text{for all } s \in \mathbf{S}$$

□

The applicant – regardless of his quality – is worse off with more selective evaluators. His evaluators grow more reluctant to approve him at any signal they might observe, so he faces a higher rejection risk in any of his visits. How moving to more selective equilibria affects evaluators' payoffs is less clear. Their payoffs are determined by how they balance their two key objectives: identifying and approving a *High* quality applicant, and rejecting a *Low* quality one. The expression $\Pi(\sigma;\mathcal{E})$, the *sum* of evaluators' payoffs when each use the strategy σ, highlights this:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) := \rho \times (1-c) \times \mathbb{P}\,(\text{some ev. approves when all use strategies } \sigma \mid \theta = H)\\ -(1-\rho) \times c \times [1 - \mathbb{P}\,(\text{all ev.s reject when all use strategies } \sigma \mid \theta = L)]\end{aligned} \tag{2.1}$$

Each evaluator expects simply $\left(\frac{1}{n}\right)^{\text{th}}$ of this sum of course, as the equilibrium is symmetric.

Increased selectivity has counteracting effects on these two objectives. It mitigates their risk of approving a *Low* quality applicant when they face one, sparing them a cost of c. However, this comes at the expense of curbing the approval chances of a *High* quality applicant too, which means forsaking a payoff of $1-c$. In principle, increased selectivity can therefore both be a vice and a virtue.

Our previous example, where we had identified two equilibria, illustrates these competing effects of increased selectivity. In the less selective equilibrium we identified, evaluators approve all applicants; either *High* quality, or *Low*. Their payoffs payoffs, therefore, sum to:

$$\Pi(\sigma^{\text{les}};\mathcal{E}) = 0.5 \times [(1-c) - c]$$

In the more selective equilibrium, an evaluator rejects her applicant upon observing the low signal, $s = c$. She observes this signal with probability $p_H(c) = c$ if her applicant has *High* quality, and with probability $p_L(c) = 1-c$ if he has *Low* quality; as the reader can verify. This depresses the approval chances of any applicant. A *High* quality applicant faces a probability $p_H^2(c) = c^2$ of getting rejected by both his evaluators. For a *Low* quality applicant, this probability is higher: $p_L^2(c) = (1-c)^2$. These more selective evaluators' payoffs sum to:

$$\Pi(\sigma^{\text{mos}};\mathcal{E}) = 0.5 \times \left[(1-c) \times (1-c^2) - c \times \left(1-(1-c)^2\right)\right]$$

Despite reducing the approval chances of *any* applicant, selectivity pays off for our evaluators in this example. Their payoffs in this latter equilibrium exceed those in the former:

is my "most" and "least" selective eqm. notation confusing given i didn't explain that these are the *only* possible eqa?

$$\Pi(\sigma^{\text{les}};\mathcal{E}) < \Pi(\sigma^{\text{mos}};\mathcal{E}) \iff (1-c) \times c^2 < c \times (1-c)^2 \iff c < (1-c)$$

Why increased selectivity ends up helping evaluators is clear in this example. Only applicants for whom *both* evaluators see low signals are pushed out in the more selective equilibrium. The probability that such an applicant has *High* quality is less than c; hence his rejection raises evaluators' payoffs.

In a more intricate example, increased selectivity might have compromised *High* quality applicants too often, and hurt evaluators. Consider, for instance, setting $c = 0.2$ in our running example, and equipping our two evaluators with a richer experiment. Their signal might now assume three values, $s \in \{0.2, 0.4, 0.8\}$, and has the distribution:

$$p_L(s) = \begin{cases} 0.48 & s = 0.2 \\ 0.36 & s = 0.4 \\ 0.16 & s = 0.8 \end{cases} \qquad p_H(s) = \begin{cases} 0.12 & s = 0.2 \\ 0.24 & s = 0.4 \\ 0.64 & s = 0.8 \end{cases}$$

How evaluators treat the signal $s = 0.4$ depends on the equilibrium they play. The strategies σ and σ' defined below, the former more selective than the latter, are *both* equilibrium strategies:

$$\sigma(s) := \begin{cases} 0 & s \in \{0.2, 0.4\} \\ 1 & s = 0.8 \end{cases} \qquad \sigma'(s) := \begin{cases} 0 & s = 0.2 \\ 1 & s \in \{0.4, 0.8\} \end{cases}$$

should I detail what interim belief they induce, and verify that this is an eqm?

It is no longer clear whether switching from the less selective strategies σ' to the more selective σ will benefit evaluators. After this switch, an applicant for whom both evaluators observe the signal $s = 0.4$ gets rejected. The probability that he has *High* quality is greater than 0.3; so his approval against the cost $c = 0.2$ would benefit evaluators. On the other hand, consider another applicant approved unknowingly by his second evaluator upon the signal $s = 0.4$, despite his first rejection upon $s = 0.2$. He has a probability less than 0.15 of having *High* quality. This shift pushes him out too, to evaluators' benefit.

This latter effect nonetheless dominates: here too, the more selective equilibrium leaves evaluators better off. Proposition 2 establishes that this phenomenon is in fact general: evaluators' trade-off between more and less selective equilibria is *always* resolved in favour of the former. Notably, this brings the welfare of the applicant, unambiguously harmed by selectivity, to a confict with the evaluators'.

Proposition 2. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**}; \mathcal{E}) \geq \Pi(\sigma^*; \mathcal{E})$.

Selectivity thus offers a very powerful comparison between equilibria, besides a very natural one. We can use it to compare any two equilibria, and to determine both the applicant's and evaluators' relative welfare. *Extreme* equilibria deserve particular focus. The *most selective* equilibrium maximises evaluators payoffs across all equilibria while minimising the applicant's approval chances. The *least selective* equilibrium, vice versa. They will remain under our spotlight in the remainder of this paper.

Proposition 2 follows as a corollary to Lemma 2, which establishes that in fact deviating to *any* less selective strategy hurts evaluators' payoffs when they start from an equilibrium strategy.

Lemma 2. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*; \mathcal{E}) \geq \Pi(\sigma; \mathcal{E})$.

Proof. See Section 8. $\square$

Besides birthing Proposition 2, Lemma 2 highlights an important contrast between the problem a *single* evaluator with no peers faces, and the one *multiple* evaluators do. A single evaluator with no peers faces no adverse selection; no evaluator could have received her applicant earlier. So her interim belief *is* her prior belief, regardless of the strategy she chooses. In equilibrium, her strategy must be optimal against this belief. This equilibrium strategy is unique – up to how she breaks ties when indifferent. Any equilibrium strategy gives her the same expected payoff, and deviation can leave her only worse off.

With multiple evaluators, a non-trivial multiplicity of equilibria becomes possible. There might be multiple strategies which motivate an interim belief they are optimal against. Crucially, evaluators' payoffs vary between these different equilibria; joint deviations to *more selective* equilibria benefit them. Lemma 2 highlights that only increased selectivity might pay off, though. Deviating – individually or jointly – from any equilibrium to a *less selective strategy* still hurts an evaluator's payoffs, just as it would had she no peers.

As the earlier examples illustrate, two kinds of applicants are pushed out when evaluators move to more selective equilibria. Some fall through the cracks: all evaluators reject them, although none have convincing evidence that they have *Low* quality. Other applicants, on the other hand, face decisive rejections from most evaluators, and – once they become more selective – lose the benefit of doubt a few unsuspecting evaluators would grant them. I prove Lemma 2 by showing that the overall effect is always driven by the latter group of applicants, whose rejection benefits evaluators.

3 More Informative Experiments and Equilibrium Payoffs

Evaluators' payoffs, as we discussed, depend on how well they can distinguish between a *High* quality and a *Low* quality applicant to approve the former but not the latter.

a *High* quality applicant to approve him, and a *Low* quality one to reject him.

Evaluators' payoffs, as we discussed, are determined by how well they can distinguish

and approve a *High* quality applicant, and reject

As I explained in the previous section, evaluators' payoffs are determined by how well they can distinguish between *High* and *Low* quality applicants in their decisions. The information they obtain about the applicant's quality from their experiment $\mathcal{E}$ critical for this exercise. Indeed, it is perhaps natural to hypothesise that an experiment which carries more precise

is section capitalised?

information about the applicant's quality would leave evaluators better off. assessments about the applicant's quality, this *more informative* experiment could

This *more informative* experiment

With this *more informative* experiment, evaluators could approve him more frequently when he has *High* quality, and less frequently otherwise. Indeed, a *perfectly informative* experiment which fully reveals the applicant's quality would leave evaluators best off. They would always approve the applicant when he has *High* quality, but otherwise never.

This intuition would in fact be true if we had a single evaluator, as formalised by Blackwell, 1953 in his classic result.

Blackwell's classic result (1953) formalises our intuition above, and establishes that indeed

validates this reasoning for single person decision problems. He shows that among two decision makers facing the same decision problem, the one with a *(Blackwell) more informative* experiment is always guaranteed a weakly higher expected payoff. Conversely, this decision maker remains better off in *any* decision problem *only if* her experiment is *(Blackwell) more informative* than her peer's.

If we had a *single* evaluator, she would be facing such a decision problem. It follows that she would be *guaranteed* a higher expected payoff with a Blackwell more informative experiment[1]. This is precisely because such a signal structure relaxes the trade-off she faces between wanting to ensure she approves when a *High* quality applicant comes along, but rejects a *Low* quality applicant[2].[3]

Nevertheless, the conjecture that this reasoning would carry over to our current setting turns out to be naïve. To showcase what goes wrong, re-express an individual evaluator i's expected payoff, $\pi_i(\sigma;\mathcal{E})$:

$$\begin{aligned}\pi_i(\sigma;\mathcal{E}) = {} & \utilde{\mathbb{P}\left(\text{applicant visits } i\right)} \\ & \times \left[\utilde{\psi} \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) + \utilde{(1-\psi)} \times (-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]\end{aligned}$$

Evaluator i seeks to better distinguish and approve *High* quality applicants while keeping *Low* quality ones out. However, this pursuit is constrained by the extent of *adverse selection* she faces: the applicant might not visit at all, and might be very unlikely to have *High* quality if he does, given he was not approved until he did. These two probabilities depend on *other* evaluators' strategies. While evaluating their applicants, these evaluators do not account for

[1] In fact, Blackwell's condition retains its necessity, too: she would be better off with $\mathcal{E}'$ rather than $\mathcal{E}$ regardless of her prior belief ψ and approval cost c *only if* $\mathcal{E}'$ is Blackwell more informative than $\mathcal{E}$. For completeness, I present a self contained proof of this in Section 8.2, Lemma 3.

[2] The reader can refer to Blackwell and Girshick, 1954's Theorems 12.2.2 and 12.4.2 for a textbook exposition of these classic results.

[3] I illustrate this in the context of binary signal structures in Figure 1 in the following subsection, where I discuss Blackwell improvements of binary signal structures in further depth.

the adverse selection they impose on evaluator i; just as she disregards the adverse selection she imposes on them. Giving evaluators more information might exacerbate this adverse selection externality. Evaluators might eventually leave worse off; their improved ability to evaluate an applicant being eclipsed by their poor expectations about the applicant they do receive.

How does, then, a *Blackwell improvement* of evaluators' signals about quality affect their equilibrium payoffs? This question is at the heart of my paper, and in this Section, I answer it. The answer depends on the *kind* of improvement in evaluators' signals. Broadly speaking, I show that *more confident approvals* benefits evaluators. In contrast, *more confident rejections* hurts them *eventually*. I develop this idea by first focusing on evaluators with *binary* signals, in Section 3.1. I then generalise the insight I develop there, in Section 3.2.

i am not sure if "eventually" is the right word. See Thm 2.

3.1 Binary Signals

Consider evaluators who either observe a high signal $s = s_H$, or a low signal $s = s_L$. The former carries evidence for *High* quality, and the latter for *Low* quality. How does receiving a more informative binary signal instead affect their equilibrium payoffs? The answer to this question will be the building block and key intuition for Section 3.2, where we will think of evaluators with arbitrary discrete signal structures.

Take two binary signal structures $\mathcal{E}'$ and $\mathcal{E}$, with respective supports $\{s'_L, s'_H\}$ and $\{s_L, s_H\}$. $\mathcal{E}'$ is *(Blackwell) more informative than* (or *(Blackwell) improves on*) $\mathcal{E}$ if its signals carry stronger evidence both for *Low* and *High* quality[4]:

$$s'_L \leq s_L \qquad\qquad s'_H \geq s_H$$

Recall that these signals are labeled after the *normalised posterior beliefs* they induce. Hence, an evaluator who observes $s' = s'_H$ grows more confident that $\theta = H$ than one who observes $s = s_H$, whenever they have the same interim beliefs. In other words, $\mathcal{E}'$ offers stronger evidence for *High* quality than $\mathcal{E}$. Similarly, observing $s' = s'_L$ induces more confidence that $\theta = L$ than observing $s = s_L$; i.e. $\mathcal{E}'$ offers *stronger evidence for Low quality* than $\mathcal{E}$.

The more informative signal structure $\mathcal{E}'$ allows a better evaluation of any applicant as Figure 1 illustrates. The shaded area whose right vertex is labeled $\mathcal{E}$ covers the probabilities of rejecting the *Low* quality applicant the evaluator can secure against any given probability of approving the *High* quality applicant. Both increasing s_H and decreasing s_L expand this region, relaxing the evaluator's trade-off between these objectives; albeit in different ways.

I removed the example in prev. draft.

Theorem 1 reveals that the response of evaluators' equilibrium payoffs to a more informative binary signal hinges precisely on whether evidence for *High* or *Low* quality gets stronger; i.e the *kind* of information evaluators get. While stronger evidence for *High* quality always increases

[4] See Section 12.5 in Blackwell and Girshick, 1954 for a textbook exposition of this classic result.

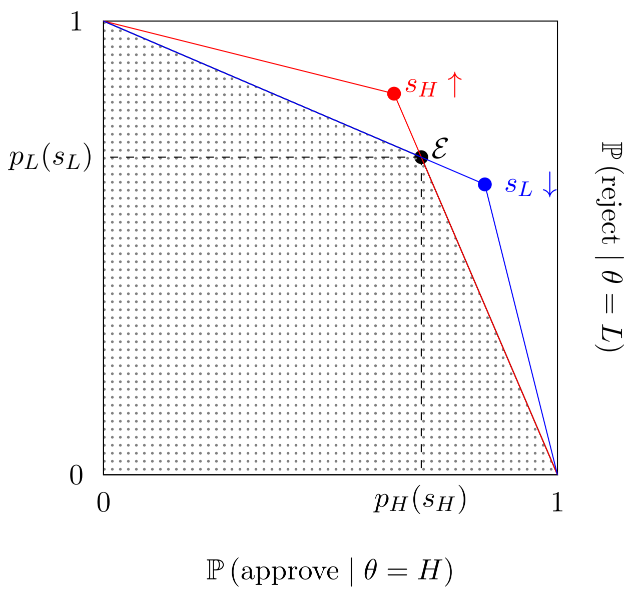


Figure 1: Improving Binary Signals and Probabilities of Mistakes

their equilibrium payoffs, stronger evidence for *Low* quality *eventually harms* them. Once it is too strong, signal structures with stronger evidence for *Low* quality can only hurt evaluators.

Theorem 1. Let $\pi(\sigma^*; \mathcal{E})$ be an evaluator's payoff in an extreme equilibrium under the binary signal structure $\mathcal{E}$ with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. $\pi(\sigma^*; \mathcal{E})$ is weakly:

a) increasing with the strength of evidence for $\theta = H$ (s_H),

b) increasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is above a threshold,

c) decreasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is below that threshold.

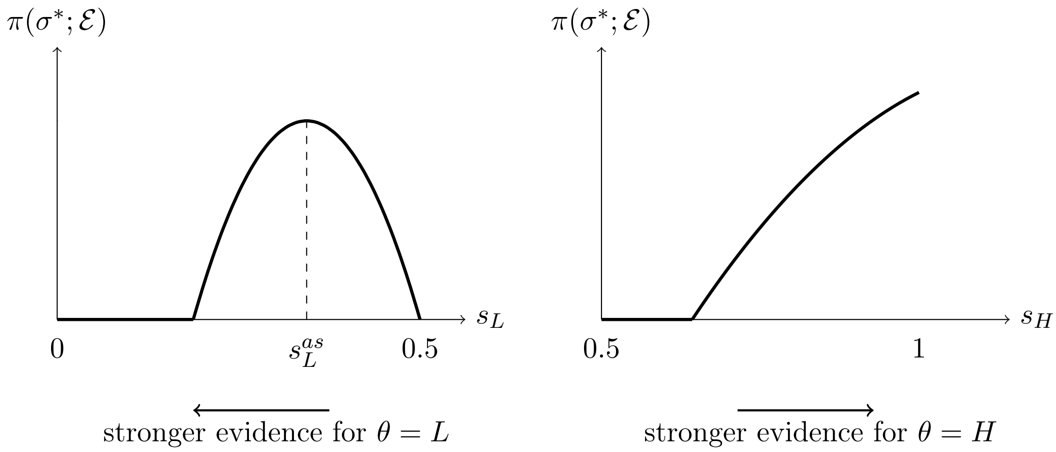


Figure 2: Theorem 1 Illustrated

Figure 2 illustrates Theorem 1 for the particular case where the approval cost c exceeds the prior belief ρ, which forces a unique equilibrium. In this unique equilibrium, evaluators approve upon a high signal and reject upon a low one if they can secure positive payoffs when they all do so. If the adverse selection they face is too strong to prevent this, they reject with positive probability upon high signals as well. They then expect equilibrium payoffs of zero. Although this illustration brushes equilibrium multiplicity away, evaluators' equilibrium payoffs respond qualitatively the same way to an improvement in evidence for either quality, as Theorem 1 asserts.

Do I need a proof for this?

In the ensuing discussion, I explain the discrepancy Theorem 1 highlights between strengthening the two pieces of evidence evaluators might receive. The full proof, which I relegate to Section 8, builds on the forces this discussion lays out. Theorem 1 is notably silent about the threshold after which stronger evidence for *Low* quality starts harming evaluators' payoffs. I elaborate on this threshold as I clarify the forces behind Theorem 1. I then characterise it in Proposition 3.

Carlos: I actually have a nice result here. I can prove it using another one in the appendix. "Equilibrium adverse selection always increases with the informativeness of the signal".
This is why the reader should care about what's to come next anyway: because one's first instinct towards a proof will be fruitless. I don't know how to insert this yet. It yields another nice result:
"The expected quality of an approved applicant falls when s_L falls, in equilibrium."

Evaluators' payoffs under any signal structure depend on the applicants they eventually approve in the equilibrium it induces. Any change in this signal structure has the potential to affect some of these applicants' eventual outcomes. A High quality applicant who otherwise would only receive rejections might now get approved by some, for instance. Likewise, a Low quality applicant previously approved by some might now be unanimously rejected. Both of these would leave evaluators better off. The same outcome reversals experienced by an applicant of the opposing quality, in contrast, would harm them. The distinct consequences of strengthening evidence for *Low* or *High* quality stems precisely from their distinct effects on applicants' eventual outcomes.

To illustrate this, let us set evaluators' strategic responses aside and say they each approve upon a high signal and reject upon a low one in both signal structures. To begin with, consider giving evaluators a signal structure with *marginally* stronger evidence for *Low* quality, but the same strength of evidence for *High* quality. Specifically, their new signal structure $\mathcal{E}'$ will have normalised posterior beliefs:

$$s'_L = s_L - \delta \qquad\qquad s'_H = s_H$$

where $\delta > 0$ is small. Whose eventual outcomes does this affect?

We can see this most clearly by interpreting this improvement in evaluators' information as showing them an additional *auxiliary signal* $\hat{S}$ about the applicant's quality, rather than replacing $\mathcal{E}$ wholesale with $\mathcal{E}'$. We engineer $\hat{S}$ carefully so that it *completes* the information $\mathcal{E}$ provides to evaluators to what $\mathcal{E}'$ can. An evaluator observes the realisation of $\hat{s}$ *only* she first observes $s = s_L$. In turn, the new signal $\hat{s}$ is also binary, with possible realisations $\{\hat{s_L}, \hat{s_H}\}$. Conditional on θ, it is independent from s and has distribution:

"with distribution" right language?

$$\hat{p_H}(\hat{s_H}) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p_L}(\hat{s_H}) = \varepsilon \times \frac{s_L}{1 - s_L}$$

ε, like δ, is a small positive number. It is related intimately to δ, as I explain shortly.

If the evaluator observes the signal $s = s_L$ followed by $\hat{s} = \hat{s_H}$, her belief that the applicant has *High* quality jumps to what it would be had she observed $s = s_H$ straightaway. Note this from the likelihood ratio for this signal pair:

$$\frac{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_H} \mid \theta = L\right)} = \frac{s_L}{1 - s_L} \times \frac{\frac{s_H}{1-s_H}}{\frac{s_L}{1-s_L}} = \frac{s_H}{1 - s_H}$$

If she observes $\hat{s} = \hat{s_L}$ however, she grows yet more pessimistic that the applicant has *High* quality. Specifically, the joint observation $(s, \hat{s}) = (s_L, \hat{s_L})$ has the likelihood ratio:

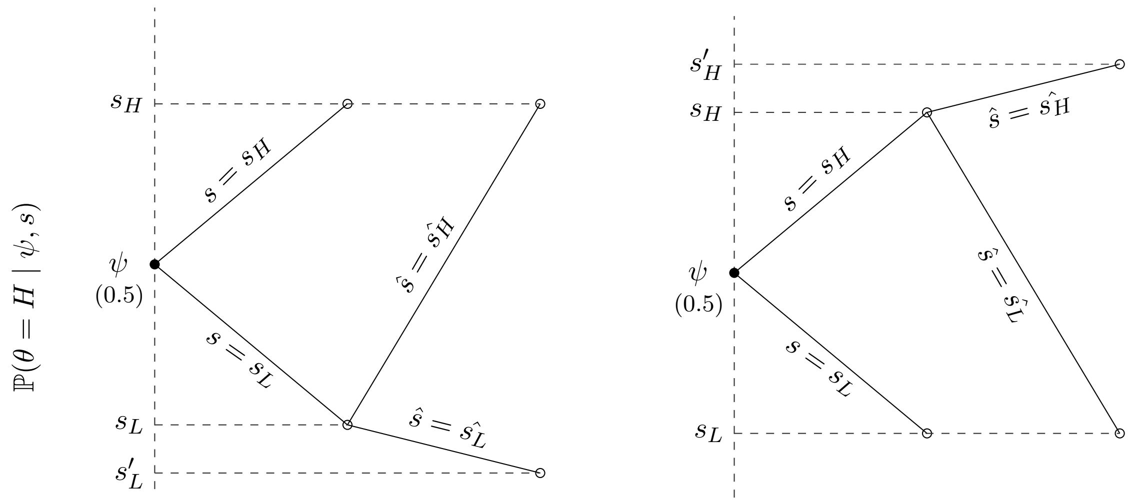
$$\frac{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_L} \mid \theta = H\right)}{\mathbb{P}\left(s = s_L, \hat{s} = \hat{s_L} \mid \theta = L\right)} = \underbrace{\frac{s_L}{1 - s_L} \times \frac{1 - \frac{s_H}{1-s_H} \times \varepsilon}{1 - \frac{s_L}{1-s_L} \times \varepsilon}}_{(L,\hat{L})} < \frac{s_L}{1 - s_L}$$

This likelihood ratio labeled $(L, \hat{L})$ above continuously and monotonically decreases as ε varies from 0 to $\frac{1-s_H}{s_H}$. We can thus choose ε to equate $(L, \hat{L})$ with the likelihood ratio for the signal $s' = s'_L$, labeled (L') below:

$$\frac{\mathbb{P}\left(s' = s'_L \mid \theta = H\right)}{\mathbb{P}\left(s' = s'_L \mid \theta = L\right)} = \underbrace{\frac{s_L - \delta}{1 - (s_L - \delta)}}_{(L')}$$

Note that the value of ε which equates the likelihood ratios $(L, \hat{L})$ and (L') is then continuous and strictly increasing as a function of δ.

When the likelihood ratios $(L, \hat{L})$ and (L') are equal, the information an evaluator can extract about the applicant's quality from the signal pair $(s, \hat{s})$ is equivalent to the information $\mathcal{E}'$ would provide. Eventually observing a high signal in the former – either $s = s_H$ or $(s, \hat{s}) = (s_L, \hat{s_H})$ – carries the same evidence for *High* quality as observing $s' = s'_H$ does. Similarly, observing only low signals – the pair $(s, \hat{s}) = (s_L, \hat{s_L})$ – carries the same evidence for *Low* quality as $s' = s'_L$. Thus for a given interim belief, the distribution of an evaluator's posterior beliefs about θ are the same whether she observes the signal tuple $(s, \hat{s})$ or just s'. This construction is illustrated in the left panel of Figure 3 .



(a) improving $\theta = L$ evidence

(b) improving $\theta = H$ evidence

Figure 3: Improving Binary Signals

We wish to compare applicants' eventual outcomes when evaluators approve upon high signals and reject upon low ones under both signal structures. Receiving the information contained in $\mathcal{E}'$ through this procedure is no obstacle to implementing an equivalent decision rule. Whenever they observe an equivalent of $s' = s'_H$ – either $s = s_H$ or $(s, \hat{s}) = (s_L, \hat{s_H})$, evaluators simply approve. Upon an equivalent of $s' = s'_L$ though – $(s, \hat{s}) = (s_L, \hat{s_L})$ – they reject.

One can interpret this reformulation of evaluators' improved information as a "second inspection" of their initial "rejection" pile. Now, an evaluator re-assesses applicants initially placed in the "rejection" pile upon a low signal. Upon this second assessment, she either concludes that she erred and should have placed him in the "approval" pile, or reinforces her verdict that the applicant must have *Low* quality.

This reformulation exposes whose eventual outcomes vary between $\mathcal{E}$ and $\mathcal{E}'$, or when evaluators observe the auxiliary signal $\hat{s}$ after s. Applicants approved by some evaluator under $\mathcal{E}$ are also approved under the pair $(s, \hat{s})$, after the same $s = s_H$ signal. Applicants rejected by all n evaluators under $\mathcal{E}$ however, get *n second chances.* These *marginal admits* reverse some of their evaluators' initial rejection verdicts by receiving a $\hat{s} = \hat{s_H}$ signal in their second assessment, saving themselves of a rejection.

How approving these *marginal admits* under $(s, \hat{s})$ affects evaluators' payoffs depends on how likely such an applicant is to have *High* quality. Any and all information evaluators could hope to scour about this is contained in the collection of signal pairs $\{(s_i, \hat{s}_i)\}_{i=1}^n$ they would observe if he visited them all. He was initially rejected in unison; so all evaluators observed $s = s_L$ initially. As he later reversed some of these rejection verdicts, at least one of them later observed a high signal, $\hat{s} = \hat{s_H}$. How many?

For small δ, and therefore ε, the answer is *almost surely, just the one.* The probability that any k evaluators observe $\hat{s} = \hat{s_H}$ is proportional to ε^k. So, as ε shrinks to 0, the probability that any $k > 1$ evaluators observed $\hat{s} = \hat{s_H}$ in their second assessment vanishes in favour of the probability that only one did. He is the *most adversely selected admit*; approved only by his *last* evaluator.

Whether approving this marginal admit raises or hurts evaluators' payoffs then depends on whether this single $\hat{s} = \hat{s_H}$ signal is strong enough to justify his approval despite the remaining $n-1$ $\hat{s} = \hat{s_L}$ signals:

$$
\begin{aligned}
&\frac{\mathbb{P}\left(\theta = H \mid\ n-1\ \hat{s} = \hat{s_L} \text{ signals and one } \hat{s} = \hat{s_H}\right)}{\mathbb{P}\left(\theta = L \mid\ n-1\ \hat{s} = \hat{s_L} \text{ signals and one } \hat{s} = \hat{s_H}\right)} \\
&= \lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \left(\frac{\hat{s_L}}{1-\hat{s_L}}\right)^{n-1} \times \frac{s_H}{1-s_H} \\
&= \frac{\rho}{1-\rho} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{a single high signal}} \lesseqgtr \frac{c}{1-c}
\end{aligned}
$$

When the LHS above exceeds the RHS, this marginal admit is sufficiently likely to have *High* quality, despite his $n-1$ low signals. *Adverse selection poses no threat* in this case: an evaluator who observes a high signal is happy to approve the applicant with no interest in his predecessors' evaluations. Strengthening evidence for *Low* quality – decreasing s_L – however, eventually sinks the LHS below the RHS. The evidence $n-1$ low signals carry for *Low* quality becomes too daunting to justify an approval at cost c thereafter, despite the single high signal an evaluator observed. I denote this tipping point for s_L below which adverse selection starts posing a threat, and such marginal admits start hurting evaluators, as s_L^{as}.

Definition 2. For a binary signal structure $\mathcal{E}$ with given strength of evidence for *High* quality s_H, s_L^{as} is the *strongest* possible evidence for *Low* quality where adverse selection poses no threat:

$$
\frac{\rho}{1-\rho} \times \left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}
$$

Note that increasing the approval cost c or the number of evaluators n brings this tipping point forward; adverse selection becomes a threat earlier. Increasing evaluators' prior ρ and s_H pushes it back.

As the reader might be anticipating, s_L^{as} plays an important role in characterising the threshold Theorem 1 pointed at. Indeed, it is no coincidence that Figure 2 depicts evaluators' payoffs as decreasing precisely when s_L falls below s_L^{as}. Recall that in the unique equilibria whose payoffs it tracks, evaluators approve upon high signals, and reject upon low ones granted this yields positive payoffs. Hence, precisely these marginal admits determine how their payoffs respond to stronger evidence for *Low* quality.

Equilibrium dynamics might differ in general, distinguishing this threshold from s_L^{as}. If, for instance, evaluators approve all applicants anyway in the original equilibrium, stronger evidence for *Low* quality cannot bring about marginal admits. In contrast, it might push evaluators to a more selective equilibrium, even. Such equilibria where evaluators approve anyone exist whenever evidence for *Low* quality is too weak: absent adverse selection, evaluators have no reason to reject an applicant upon a low signal. I denote this second tipping point for s_L as s_L^{mute}.

Definition 3. For a binary signal structure $\mathcal{E}$, s_L^{mute} is the strongest possible evidence for *Low* quality where all applicants are approved in the most embracive equilibrium:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

s_L^{mute} rises with the approval cost c and falls with evaluators' prior ρ. Note that it need not be below 0.5; there might be no feasible value for s_L where the most embracive equilibrium sees all applicants approved. In fact, it never is when $c \geq \rho$.

Once this equilibrium ceases to be the most or least selective one, the effect of decreasing s_L further on the payoffs at these equilibria is indeed determined by the *marginal admit*. The most selective equilibrium switches earlier than the least selective equilibrium. The least selective equilibrium remains to see all applicants approved until s_L finally falls below s_L^{mute}.

Proposition 3. Let $\mathcal{E}$ be a binary signal structure with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. Evaluators' payoffs across the most embracive equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < \min\{s_L^{\text{mute}}, s_L^{as}\}$.

Similarly, there exists a threshold $s_L^{\dagger} \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$, such that their payoffs across the most selective equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq s_L^{\dagger}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < s_L^{\dagger}$.

Can you pin this threshold down? If not, why?

We can interpret a marginal strengthening of evidence for *High* quality analogously. This time, consider the signal structure $\mathcal{E}'$ with normalised posterior beliefs:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small $\delta > 0$. This time, evaluators will observe the realisation of an auxiliary signal $\hat{s}$ *only if* they first observe $s = s_H$. The complementary signal $\hat{s}$, conditionally independent from s as before, has the distribution:

$$\mathbb{P}(\hat{s} = \hat{s_L} \mid \theta = H) = \varepsilon \times \frac{s_L}{1-s_L} \qquad\qquad \mathbb{P}(\hat{s} = \hat{s_L} \mid \theta = L) = \varepsilon \times \frac{s_H}{1-s_H}$$

We choose ε so that the pair $(s, \hat{s})$ provides the same information about quality as $\mathcal{E}'$:

$$\frac{\mathbb{P}\left(s = s_H, \hat{s} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(s = s_H, \hat{s} = \hat{s_H} \mid \theta = L\right)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \frac{s_L}{1 - s_L}}{1 - \varepsilon \times \frac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - s_H - \delta}$$

As before, ε is a continuous and increasing function of δ. The right panel of Figure 3 illustrates.

Under this signal pair $(s, \hat{s})$, observing consecutive high signals $(s_H, \hat{s_H})$ elevates the evaluator's belief about the applicant's quality as observing $s' = s'_H$ would. The evaluator then approves. If he sees any low signal however, be it s_L or $\hat{s_L}$, his belief sinks as it would had he observed $s' = s'_L$. He rejects.

As before, we can interpret evaluators' observations of the auxiliary signal as a "second inspection", this time of their initial "approval" pile. Each evaluator re-assesses applicants she initially placed in her approval pile upon a high signal. Consecutively, she either concludes that she erred and that her applicant should have been in the "rejection" pile instead, or reinforces her conviction that the applicant likely has *High* quality.

This reformulation exposes the new group of applicants whose eventual outcomes switch when we strengthen evidence for *High* quality. An applicant who would be rejected by all evaluators under $\mathcal{E}$ faces the same fate under $(s, \hat{s})$: his initial $s = s_L$ signals still lead to rejections. But an applicant who previously would be approved by some evaluator faces a renewed threat of rejection by all. Any initial $s = s_H$ signal he had can be overturned by a $\hat{s} = \hat{s_L}$ signal now. Unlucky enough, and he might overturn all his initial high signals, being left with nothing but rejections.

How pushing this *marginal reject* out affects evaluators' payoffs depends again on how likely he is to have *High* quality. As before, all and any information about this is contained in the signals his evaluators would observe for him if he visited them all. Inferring what these signals must be is now easier. As all evaluators eventually rejected him, they all must have observed low signals; either $s = s_L$ or $\hat{s} = \hat{s_L}$.

If low signals indeed lead to a rejection in equilibrium, pushing this marginal reject out is sure to raise evaluators' payoffs. Intuitively, equilibrium behaviour shows that the *fear* of adverse selection suffices to keep an evaluator from an applicant upon a low signal. So it can certainly not be optimal to approve an applicant who, in fact, *is* the most adversely selected one, upon a low signal.

If a low signal *already* leads to a rejection in equilibrium, this marginal reject is *sure* to raise evaluators' expected payoffs. When evaluators find rejecting upon a low signal optimal, learning that *all* evaluators saw low signals can only strengthen this conviction.

3.2 General Discrete Signals

The previous section uncovered how evaluators' equilibrium payoffs vary across binary signal structures. Such a signal structure is *more informative* whenever it provides *stronger evidence* either for *Low* or *High* quality. Theorem 1 showed that while stronger evidence for *High* quality always benefits evaluators, stronger evidence for *Low* quality *eventually* harms them.

In many settings of interest, however, the evaluators in concern have richer signal structures. Traders of financial assets and derivatives, for instance, might get recommendations of varying levels of strength, such as "Strong Sell", "Sell", "Buy", and "Strong Buy". Similarly, a bank's credit scoring algorithm might output varying probabilities that the loan seeker will default, rather than simply summarising this information as "Good" or "Bad". These highlight the importance of extending our characterisation in Theorem 1 to improvements of such richer signal structures. ~~This is precisely the present section's aim. Its main result, Theorem TWO, characterises the effect of more information on evaluators' equilibrium payoffs, for any *arbitrary* discrete signal structure they might hold.~~

Evidence for this

When studying improvements of binary signals, I introduced the idea of auxiliary signals engineered to replicate any given improvement. This construction helped pin down the applicants whose eventual outcomes a given improvement affects. I now formalise and generalise this idea, using *local mean preserving spreads* of a signal structure. Local mean preserving spreads are the key to identify who the applicants affected by an *arbitrary* Blackwell improvement are.

Definition 4 (Local Mean Preserving Spread)**.** Let p and p' be the normalised posterior densities for the signal structures $\mathcal{E}$ and $\mathcal{E}'$. Additionally, let $s_1 < s_2 < ... < s_M$ be the elements of $S \cup S'$; the joint support of $\mathcal{E}$ and $\mathcal{E}'$. If there exists some $i \in \{2, ..., M-1\}$ such that:

$$p'(s_{i-1}) \geq p(s_{i-1}) \qquad 0 = p'(s_i) \leq p(s_i) \qquad p'(s_{i+1}) \geq p(s_{i+1})$$

$$p'(s_j) = p(s_j) \quad \text{for all } j \notin \{i-1, i, i+1\}$$

$$\sum_{i=1}^{M} s_i \times p'(s_i) = \sum_{i=1}^{M} s_i \times p(s_i)$$

I say *$\mathcal{E}'$ differs from $\mathcal{E}$ by a local mean preserving spread (at s_i).*

the original definition has F not $\mathcal{E}$, is that a problem?

Much like an ordinary mean preserving spread (Rothschild and Stiglitz, 1970[5]), a *local mean preserving spread* distributes probability away from an *origin* point to two *destination* points,

[5]Rothschild and Stiglitz, 1970 describe mean preserving spreads through *four* points in the support of the distribution. Here, I describe them through *three*. This is without loss of generality. In fact, mean preserving spreads were first characterised by Muirhead, 1900 in the context of majorisation (transformations T), with *three* points. Rasmusen and Petrakis, 1992 show formally that these the three or four point characterisations of MPS are in fact equivalent.

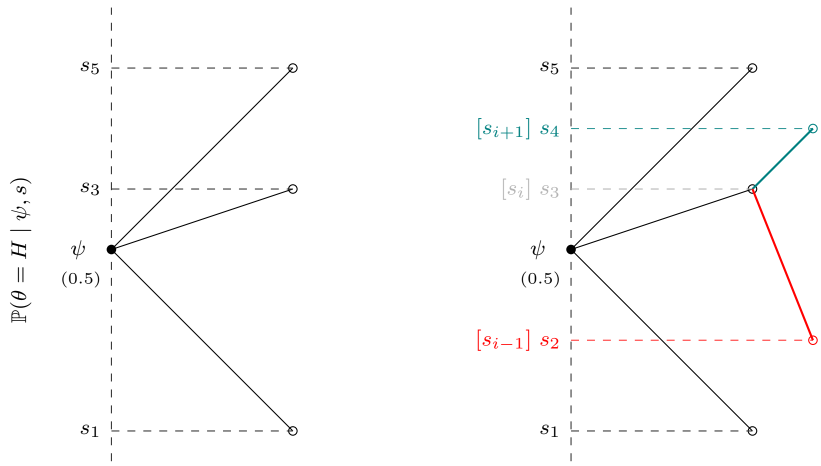


Figure 4: A Local Mean Preserving Spread

one above and one below it. It does so while preserving the mean of the original distribution. Crucially however, a mean preserving spread is *local* if and only if the destination points are the immediate neighbours of the origin point[6]. In other words, neither the original nor the resulting distribution assign positive probability to any other point between the origin and the two destination points[7].

The auxiliary signals I introduced in the previous section create such local mean preserving spreads. To strengthen evidence for *Low* quality, for instance, the auxiliary signal spreads all the probability mass assigned to the origin point s_L to the neighbouring destination points s_H and s'_L, where $s_H > s_L > s'_L$.

Local mean preserving spreads are simple ways to Blackwell improve signal structures. Nonetheless, they are powerful enough to characterise *any* Blackwell improvement of a signal structure, too. Remark 1, slightly refining the classic result in Rothschild and Stiglitz, 1970[8], states that we can decompose *any* Blackwell improvement of a discrete signal structure into a sequence of finitely many *local* mean preserving spreads. So, I focus on the effect of such local mean preserving spreads on evaluators' equilibrium payoffs.

Remark 1. [Müller and Stoyan, 2002, Theorem 1.5.29] $\mathcal{E}'$ is Blackwell more informative than $\mathcal{E}$ if and only if there is a finite sequence $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_k$ such that $\mathcal{E}_1 = \mathcal{E}$, $\mathcal{E}_k = \mathcal{E}'$, and $\mathcal{E}_{i+1}$ differs from $\mathcal{E}_i$ by a local mean preserving spread.

As before, we can interpret a local MPS of s_i as a re-evaluation of the applicant upon that signal realisation; I illustrate this in Figure **??**. This re-evaluation might change an evaluator's initial verdict upon s_i. As a result, whether applicants face a renewed rejection threat or get a

[6]The reader will notice that this statement is ill-defined unless the signal structure is discrete. To the best of the author's knowledge, no counterpart for *local mean preserving spreads* exist for, say, atomless signal structures.

[7]The attentive reader will also realise that this definition also requires that *all* probability mass be spread away from the origin point. This difference is insignificant in our current setting.

[8]This result appeared in earlier work related to majorisation. See Muirhead, 1900, whose textbook exposition appears in Hardy et al., 1952.

second approval chance hinges on what this initial verdict would have been.

If the signal s_i originally led to an approval, applicants who initially owed an approval to this signal might get rejected upon this re-evaluation. Consequently, an applicant who would initially be approved by some evaluator might now get rejected by all. As Theorem 2 states, pushing such applicants out always benefits evaluators. Intuitively, upon his re-evaluation, this applicant falls into the rejection region of *every* evaluator. Given her signal, *no* evaluator can justify approving him under the *threat* of adverse selection. The evaluator's fear of adverse selection and the rejection it motivates are, in fact, valid. Her approval would indeed stand alone among *all* her peers' rejections.

If the signal s_i originally led to a rejection, however, this re-evaluation presents an applicant who were rejected with it a *second chance*. Some of his initial rejections owing to s_i might be overturned upon a positive revision of his signal to s_{i+1}. This positive re-evaluation is good news about his quality, but it still comes against the potential backdrop of some – unchanged – poor evaluations. The evaluator who approves upon her re-assessment benefits if the signal s_{i+1} is strong enough to counter this adverse selection threat.

If the signal s_{i+1} is strong enough to overwhelm rejections even by *all* $n-1$ evaluators, our evaluator faces no such threat. She would find it beneficial to approve the applicant upon the signal s_{i+1}, regardless of the number of his past rejections. For a fixed strategy σ all evaluators use, I say *adverse selection poses no threat at signal* s_{i+1} when so.

Definition 5. Fix the signal structure $\mathcal{E}$ and a monotone strategy σ. I say *adverse selection poses no threat at signal s* if:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{s}{1-s} > \frac{c}{1-c}$$

~~If adverse selection poses no threat at signal s_{i+1}, the applicant approved upon a revision of s_i to s_{i+1} increases evaluators' payoffs. How many rejections other evaluators would issue is irrelevant; these are insufficient to overwhelm the good news s_{i+1} carries. Strikingly however, this is also *necessary* for this admit to improve evaluators' payoffs. *Whenever* adverse selection poses a threat at signal s_{i+1}, evaluators are worse off due to these admits brought about with the local spread of s_i.~~

Strikingly, *any* threat of adverse selection at s_{i+1} suffices for evaluators to be worse off with the admit s_i's spread brings about. The argument parallels the one describing how *marginal admits* affect evaluators' payoffs in the binary case. Consider, again, a "small" spread which revises the signal s_i to s_{i+1} with a vanishingly small probability. The probability that our applicant, previously rejected by all evaluators, overturned *multiple* s_i signals positively is vanishingly small. Hence, the evaluator revising her rejection to an approval *does* in fact suffer from the most severe form of adverse selection.

Theorem 2. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $s = s_i$ leads to approvals under σ.
2. *weakly lower* than under σ:

    i if $s = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

    ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{E}$ and σ.

Whether adverse selection poses a threat at s_{i+1} depends on evaluators' precise equilibrium strategy. This might concern an analyst with no knowledge of this strategy when she wants to judge whether a given spread guarantees to harm evaluators. This is less alarming than it first appears; the analyst can locate both the most and least selective equilibrium strategies precisely with the algorithm I present to prove equilibrium existence in Proposition 1. Nonetheless, Proposition 4 offers a stronger sufficient condition for the most selective equilibrium. It strengthens the notion of adverse selection threat at s_{i+1} Theorem 2 uses to one which depends only on the signals s_i and s_{i+1}.

Proposition 4. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proposition 4 still requires knowing that s_i is a rejection signal in the most selective equilibrium. This too, can be strengthened to a sufficient condition that relies only on the signal realisation. Recall from the previous subsection that signal realisations below s_L^{mute} ought to be rejected in any equilibrium. Corollary 5 uses this fact:

Corollary 5. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

4 Evaluators with Regulated Strategies

This section will be about a social planner who can dictate evaluators' strategies to maximise their total payoffs can achieve.

1. First result: a planner who wishes to dictate the same strategy to every evaluator can *never benefits* from evaluators having information that would be harmful in equilibrium.

   Reason: dictating the same strategy to every evaluator $\equiv$ giving each a Blackwell inferior experiment. So at best, you can do as well as you did with a Blackwell inferior experiment.

   In the binary case, you can do only as well as the equilibrium outcome of some inferior experiment. I don't yet know if this is true in general.

2. Result 1.5: with binary signals, an evaluator can simply raise the approval cost to implement this symmetric optimum.

3. Second result: a planner who can dictate *different* strategies to evaluators can break this. Example: binary signal with $s_L < s_L^{\text{mute}}$. Optimal solution: k evaluators approve upon s_H and reject upon s_L, remaining $n-k$ reject always. Evaluators' payoffs are then monotone with any improvement in information.

4. Third result: the above is equivalent to giving evaluators' full information about history. In general, not optimal (I can maybe cite Makarov and Plantin, 2023 on this, or give my own example).

5. Fourth result (follow up to two): conjectured. In general, evaluators' payoffs are not monotone in information even when they have full history information. I will cook an example for this.

5 Information with Arbitrary History Signals

Highest priority!

This Section is about the generalisation to arbitrary history signals that I wish to make. I conjecture that Theorem 1 is going to generalise in some form to *any* history signal. I do not wish to get a fully general result in the spirit of Theorem 2; i.e. will restrict myself to binary in this section. This is because with partially (or fully) observed past decisions, Blackwell improvements might behave weirdly simply because decisions are censored data à la classic social learning anyway.

6 Take-It-Or-Leave-It Price Offers

Nothing changes when evaluators offer take it or leave it prices to applicants. Diamond's paradox kicks in, every evaluator offers max acceptable price.

7 Competing in Application Costs

Evaluators post application costs (potentially negative). Applicant applies from lowest to highest cost. Turns the game into an all-pay auction to evade adverse selection, à la Broecker, 1990. Equilibrium strategies of application costs are mixed with no atoms, so ex-post order of applications are perfectly known.

8 Proof Appendix

8.1 Useful Definitions and Notation

In what follows, I occasionally express beliefs in *likelihood form* for convenience. The reader can easily verify the identities:

$$\frac{\psi}{1-\psi} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})} \qquad \frac{\mathbb{P}_\psi(\theta = H \mid S = s_i)}{1-\mathbb{P}_\psi(\theta = H \mid S = s_i)} = \frac{\psi}{1-\psi} \times \frac{s_i}{1-s_i}$$

Through similar reasoning, the reader can verify that it is optimal to approve the applicant when:

$$\frac{\mathbb{P}_\psi(\theta = H \mid S = s_i)}{1-\mathbb{P}_\psi(\theta = H \mid S = s_i)} > \frac{c}{1-c}$$

Some strategies require evaluators to randomise when approving their applicant upon observing a particular signal realisation. To facilitate the discussion, I assume that each evaluator observes the realisation of a *tie-breaking signal* $u \sim U[0,1]$ alongside the outcome of her experiment. This signal is not informative about the applicant's quality as its distribution is independent of it. I denote the outcome of evaluator i's experiment as s^i and her tie-breaking signal as u^i. Without loss, evaluator i approves the applicant if and only if $\sigma(s^i) \leq u^i$; where σ is her strategy. I call the pair (s^i, u^i) the *score* evaluator i observes for the applicant.

Definition 6. The *score* evaluator i observes for the applicant is the tuple $Z^i = (s^i, u^i)$, where $u^i \overset{IID}{\sim} U[0,1]$. The applicant's *score profile* $Z^\otimes$ is the set of scores each evaluator would observe if he visits all; $Z^\otimes = \{(s^i, u^i)\}_{i=1}^n$. Analogously, the applicant's *signal profile* $S^\otimes = \{s^i\}_{i=1}^n$ is the set of outcomes each evaluator would observe for her experiment.

8.2 Omitted Results

Lemma 3. Let there be a *single* evaluator who observes the outcome of an experiment before she decides whether to approve the applicant. The evaluator's expected payoff is greater when she observes the outcome of $\mathcal{E}'$ than when she observes the outcome of $\mathcal{E}$ regardless of her approval cost $c \in [0,1]$ and prior belief $\rho \in [0,1]$ if and only if $\mathcal{E}' \underset{B}{\succeq} \mathcal{E}$.

Proof. Sufficiency is due to Blackwell's Theorem (Blackwell and Girshick, 1954, Theorem 12.2.2). I show necessity by fixing a prior belief ρ for the evaluator.

Let q_j be the posterior belief the evaluator forms about the applicant's quality upon observing the signal $S = s_j \in \mathbf{S}$:

$$q_j = \frac{\rho \times s_j}{\rho \times s_j + (1-\rho) \times (1-s_j)}$$

Furthermore, let $F(.)$ and $F'(.)$ be the distributions of posterior beliefs $\mathcal{E}$ and $\mathcal{E}'$ induce, respectively, for this prior belief ρ:

$$F(q_j) = (1-\rho) \times \sum_{l=1}^{j} p_L(s_l) + \rho \times \sum_{l=1}^{j} p_H(s_l)$$
$$F'(q_j) = (1-\rho) \times \sum_{l=1}^{j} p'_L(s_l) + \rho \times \sum_{l=1}^{j} p'_H(s_l)$$

The evaluator's expected payoff under $\mathcal{E}$ is then:

$$\int_c^1 (q-c)dF(q) = \int_c^1 qdF(q) - c \times (1 - F(c)) = (1-c) - \int_c^1 F(q)dq$$

Of course, an analogous expression gives her expected payoff under $\mathcal{E}'$. Therefore, for her expected payoffs under $\mathcal{E}'$ to exceed those under $\mathcal{E}$ for any $c \in [0,1]$, we must have:

$$\int_c^1 \left(F(q) - F'(q)\right) dq \geq 0$$

which is equivalent to $\mathcal{E}'$ being Blackwell more informative than $\mathcal{E}$[9].

□

8.3 Omitted Proofs

Proposition 1. Where Σ is the set of evaluators' equilibrium strategies:

1. An equilibrium always exists; $\Sigma \neq \emptyset$.
2. The set Σ is compact.
3. Any equilibrium strategy is *monotone*; for any equilibrium strategy $\sigma^* \in \Sigma$ and signal realisations $s_{j'}, s_j \in \mathbf{S}$:

$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad\qquad \text{whenever } s_{j'} > s_j$$

4. All equilibria exhibit adverse selection; $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy σ^*.

Proof. It will be convenient to treat each strategy $\sigma : \mathbf{S} \to [0,1]$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$. Since this is a finite dimensional vector space, I endow it with the metric induced

[9]See Müller and Stoyan, 2002, Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica, 2016.

by the taxicab norm without loss of generality (see Kreyszig, 1978 Theorem 2.4-5):

$$||\sigma' - \sigma|| = \sum_{j=1}^{m} |\sigma'(s_j) - \sigma(s_j)| \qquad \text{for any two strategies } \sigma' \text{ and } \sigma$$

For further convenience, I denote the function which maps a strategy σ to the interim belief consistent with it as $\Psi(.;\mathcal{E}) : [0,1]^n \to [0,1]$:

$$\Psi(\sigma;\mathcal{E}) = \frac{\rho \times \nu_H(\sigma;\mathcal{E})}{\rho \times \nu_H(\sigma;\mathcal{E}) + (1-\rho) \times \nu_L(\sigma;\mathcal{E})}$$

Note that $\Psi(.;\mathcal{E})$ is a continuous function of evaluators' strategies.

I begin by proving statements 3 and 4 of the Proposition. Following that, I prove statements 1 and 2.

3. Any equilibrium strategy is monotone.

Any equilibrium strategy σ^* must be monotone; i.e.:

$$\sigma^*(s_j) > 0 \implies \sigma^*(s_{j'}) = 1 \qquad \text{for any } s_{j'} > s_j$$

This is simply because any equilibrium strategy must be optimal against the interim belief ψ^* it induces. Whenever $\rho \in (0,1)$, $\psi^* = \Psi(\sigma^*;\mathcal{E}) \in (0,1)$, and:

$$\mathbb{P}_{\psi^*}\left(\theta = H \mid S = s_{j'}\right) > \mathbb{P}_{\psi^*}\left(\theta = H \mid S = s_j\right)$$

5. Selectivity is a total order on Σ.

Any two monotone strategies σ' and σ are comparable under the *selectiveness* order since:

$$\sigma'(s_j) > \sigma(s_j) \implies 1 = \sigma'(s_{j'}) \geq \sigma(s_{j'})$$

for any $j' > j$. Since Σ must be a subset of the set of monotone strategies, Σ is also totally ordered under the *selectivity* relation.

4. All equilibria exhibit adverse selection.

This follows from the stronger fact that $\Psi(\sigma;\mathcal{E}) \leq \rho$ for any monotone strategy σ. To see this, note that $p_H(.)$ first order stochastically dominates $p_L(.)$ since it likelihood ratio dominates it[10]. Therefore, $r_L(\sigma;\mathcal{E}) \geq r_H(\sigma;\mathcal{E})$ and $\Psi(\sigma;\mathcal{E}) \leq \rho$.

1. An equilibrium always exists.

[10]Theorem 1.C.1 in Shaked and Shanthikumar, 2007.

Define $\Phi(.) : [0,1]^m \to 2^{[0,1]^m}$ to be the evaluators' *best response correspondence*. $\Phi(.)$ maps any strategy σ to the set of strategies that are optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ induced by this strategy σ:

$$\Phi(\sigma) = \left\{\sigma' \in [0,1]^m : \sigma' \text{ is optimal against } \Psi(\sigma;\mathcal{E})\right\}$$

A strategy σ^* is an equilibrium strategy if and only if it is a fixed point of evaluators' best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I establish that the correspondence Φ has at least such fixed point through Kakutani's Fixed Point Theorem.

Φ is trivially non-empty; every interim belief has some strategy optimal against it. Moreover, Φ is convex valued. Unless there is a signal $s_{j^*} \in \mathbf{S}$ such that $\mathbb{P}_{\Psi(\sigma;\mathcal{E})}(\theta = H \mid S = s_{j^*}) = c$, the strategy optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ is unique. Otherwise, a strategy σ is optimal if and only if:

$$\sigma(s_j) = \begin{cases} 0 & j < j^* \\ \in [0,1] & j = j^* \\ 1 & j > j^* \end{cases}$$

The set of these strategies is convex.

Now take an arbitrary sequence of strategies $\{\sigma_n\}$ such that $\sigma_n \to \sigma_\infty$. Denote the interim beliefs consistent with these strategies as $\psi_n := \Psi(\sigma_n;\mathcal{E})$. Since $\Psi(.;\mathcal{E})$ is continuous in evaluators' strategies, we also have $\psi_n \to \psi_\infty$ where $\psi_\infty = \Psi(\sigma_\infty;\mathcal{E})$. Take now a sequence of strategies $\{\sigma_n^*\}$ from the image of this correspondence; $\sigma_n^* \in \Phi(\sigma_n)$. Note that every strategy in the sequence $\{\sigma_n^*\}$ is monotone, since any strategy that is optimal against an interim belief $\psi \in (0,1)$ must be monotone.

We want to show that Φ is upper semi-continuous; i.e.:

$$\sigma_n^* \to \sigma_\infty^* \implies \sigma_\infty^* \in \Phi(\sigma_\infty)$$

The upper semi-continuity of $\Phi(.)$ implies the existence of a fixed point for this correspondence through Kakutani's Fixed Point Theorem. This establishes the existence of equilibria.

By the Monotone Subsequence Theorem, the sequence $\{\sigma_n^*\}$ has a subsequence of strategies $\sigma_{n_k}^* \to \sigma_\infty^*$ whose norms $||\sigma_{n_k}^*||$ are monotone in their indices n_k. Here I take the case where these norms are increasing, the proof is analogous for the opposite case. Since σ_∞^* is the limit of a subsequence of monotone strategies whose norms are increasing, σ_∞^* must also be a monotone strategy.

this is kind of obvious but maybe i should write one more sentence on it.

Let s_{j^*} be the highest signal for which $\sigma_\infty^*(s_{j^*}) > 0$. Then, there is some $N \in \mathbb{N}$ such that for all $n_k \geq N$ we have $\sigma_\infty^*(s_{j^*}) > \sigma_{n_k}^*(s_{j^*}) > 0$, too. For such $n_k \geq N$, we must have:

$$\frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{s_{j^*}}{1-s_{j^*}} \geq \frac{c}{1-c}$$

since $\sigma^*_{n_k}$ are optimal against the interim beliefs ψ_{n_k}. Furthermore, the continuity of the interim beliefs implies:

$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s_{j^*}}{1-s_{j^*}} \geq \frac{c}{1-c}$$

If $s_{j^*} = s_m$, this implies that $\sigma^*_\infty \in \Phi(\sigma_\infty)$ and therefore concludes our proof. Otherwise, the monotonicity of the subsequence $\{\sigma^*_{n_k}\}$ and $\sigma^*_\infty(s_{j^*+1}) = 0$ implies that $\sigma^*_{n_k}(s_{j^*+1}) = 0$. By the optimality of these strategies against the interim beliefs ψ_{n_k}, we have:

$$\frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{s_{j^*+1}}{1-s_{j^*+1}} \leq \frac{c}{1-c}$$

and by the continuity of interim beliefs:

$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s_{j^*+1}}{1-s_{j^*+1}} \leq \frac{c}{1-c}$$

These observations conclude our proof.

2. The set of equilibrium strategies Σ is compact.

Σ is a subset of $[0,1]^n$ and therefore bounded, hence it suffices to show that is closed. This follows immediately from the upper semi-continuity of evaluators' best response correspondence $\Phi(.)$.

□

The following Lemma, of independent interest itself, will be useful to prove Proposition 2.

Lemma 4. Take three monotone strategies $\sigma'' > \sigma' > \sigma$. If $\Pi(\sigma'; \mathcal{E}) \leq \Pi(\sigma; \mathcal{E})$, then $\Pi(\sigma''; \mathcal{E}) \leq \Pi(\sigma'; \mathcal{E})$.

Proof. For the three strategies $\sigma'' > \sigma' > \sigma$, consider three sets $A, A', A'' \subset (S \times [0,1])^n$ where the applicant's score profile might lie:

$$\begin{aligned}
Z^{\otimes} \in A & \qquad \text{if } Z^{\otimes} \text{ is eventually approved under } \sigma'' \text{ but not } \sigma \\
Z^{\otimes} \in A' & \qquad \text{if } Z^{\otimes} \text{ is eventually approved under } \sigma' \text{ but not } \sigma \\
Z^{\otimes} \in A'' & \qquad \text{if } Z^{\otimes} \text{ is eventually approved under } \sigma'' \text{ but not } \sigma'
\end{aligned}$$

Notice that $A' \cap A'' = \emptyset$ and $A' \cup A'' = A$. We can write the difference between the sum of evaluators' payoffs under different strategies as:

$$\Pi(\sigma'; \mathcal{E}) - \Pi(\sigma; \mathcal{E}) = \mathbb{P}\left(Z^{\otimes} \in A'\right) \times \left[\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A'\right) - c\right]$$

and:

$$\Pi(\sigma''; \mathcal{E}) - \Pi(\sigma'; \mathcal{E}) = \mathbb{P}\left(Z^{\otimes} \in A''\right) \times \left[\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right) - c\right]$$

$\Pi(\sigma';\mathcal{E}) \leq \Pi(\sigma;\mathcal{E})$ implies $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A') \leq c$. But then we must have $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A'') \leq c$, since $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A)$ is a convex combination of $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A')$ and $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A'')$, and:

$$\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) \geq \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A \cap A''\right) = \mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A''\right)$$

Therefore, we have $\mathbb{P}(\theta = H \mid Z^{\otimes} \in A'') \leq \mathbb{P}(\theta = H \mid Z^{\otimes} \in A) \leq \mathbb{P}(\theta = H \mid Z^{\otimes} \in A') \leq c$.

□

Lemma 2. Let σ^* and σ be two monotone strategies, where σ^* is more selective than σ. If σ^* is an equilibrium strategy, then $\Pi(\sigma^*;\mathcal{E}) \geq \Pi(\sigma;\mathcal{E})$.

Proof. Let $Z^{\otimes}$ be the applicant's *score profile*, as in Definition 6. Take an equilibrium strategy σ^* and a more embracive strategy σ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & \underline{s} \\ 0 & s \neq \underline{s} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. Now, let $A \subset (S \times [0,1])^n$ be the set of all score profiles which lead to rejections by all evaluators under σ^*, but an eventual approval under σ:

$$Z^{\otimes} \in A \iff \begin{cases} \sigma^*(s^i) > u^i & \text{for all } i \in \{1,2,...,n\} \\ \sigma(s^i) \leq u^i & \text{for some } i \in \{1,2,...,n\} \end{cases}$$

Furthermore, for a given score profile $Z^{\otimes}$, let $\#$ be the number of evaluators whose observed scores are such that $\sigma(s^i) \geq u^i > \sigma^*(s^i)$. These evaluators would approve the applicant under the strategy σ, but not under σ^*.

An applicant's eventual outcome differs between the strategy profiles σ and σ^* if and only if his score profile $Z^{\otimes}$ lies in A. Furthermore, his eventual outcome can only change from a rejection by all evaluators in σ^* to an approval by some evaluator in σ. Thus:

$$\begin{aligned} \Pi(\sigma;\mathcal{E}) - \Pi(\sigma^*;\mathcal{E}) &= \left[\mathbb{P}(\theta = H \mid Z^{\otimes} \in A) - c\right] \times \mathbb{P}(Z^{\otimes} \in A) \\ &\propto \mathbb{P}(\theta = H \mid Z^{\otimes} \in A) - c \end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the applicant's signal profile is in A:

$$\mathbb{P}\left(\theta = H \mid Z^{\otimes} \in A\right) = \sum_{i=1}^{n} \mathbb{P}\left(\theta = H \mid \# = i\right) \times \frac{\mathbb{P}\left(\# = i\right)}{\mathbb{P}(Z^{\otimes} \in A)}$$

Now note:

$$\mathbb{P}\left(\# = i \mid \theta\right) = \left(p_\theta(\underline{s})\right)^i \times \left(1 - p_\theta(\underline{s})\right)^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}(\# = i) \propto \varepsilon^i$. Since $\mathbb{P}(Z^\otimes \in A) = \sum_{i=1}^{n} \mathbb{P}(\# = i)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\#=i)}{\mathbb{P}(Z^\otimes \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid Z^\otimes \in A\right) - \mathbb{P}(\theta = H \mid \# = 1) = 0$$

I conclude the proof by showing that $\mathbb{P}(\theta = H \mid \# = 1) < c$, and invoking Lemma 4.

$$\begin{aligned}
\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H \mid \# = 1)}{\mathbb{P}(\theta = L \mid \# = 1)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \frac{\mathbb{P}(\# = 1 \mid \theta = H)}{\mathbb{P}(\# = 1 \mid \theta = L)} \\
&= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma; \mathcal{E})}{r_L(\sigma; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underset{\sim}{s})}{p_L(\underset{\sim}{s})} \\
&= \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma^*; \mathcal{E})}{r_L(\sigma^*; \mathcal{E})}\right)^{n-1} \times \frac{p_H(\underset{\sim}{s})}{p_L(\underset{\sim}{s})} \\
&\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underset{\sim}{s})}{p_L(\underset{\sim}{s})} \leq \frac{c}{1 - c}
\end{aligned}$$

where $\psi^* = \Psi(\sigma^*; \mathcal{E})$ is the interim belief of the evaluators induced by σ^*. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{1 + r_H^* + \ldots + (r_H^*)^{n-1}}{1 + r_L^* + \ldots + (r_L^*)^{n-1}} \leq \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*; \mathcal{E})$. The last inequality is due to the fact that $\underset{\sim}{s} \in S$ is optimally rejected under σ^*.

□

Proposition 2. Where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*, evaluators' expected payoffs under σ^{**} exceed those under σ^*; $\Pi(\sigma^{**}; \mathcal{E}) \geq \Pi(\sigma^*; \mathcal{E})$.

Proof. This is an immediate corollary to Lemma 2. □

Theorem 1. Let $\pi(\sigma^*; \mathcal{E})$ be an evaluator's payoff in an extreme equilibrium under the binary signal structure $\mathcal{E}$ with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. $\pi(\sigma^*; \mathcal{E})$ is weakly:

a) increasing with the strength of evidence for $\theta = H$ (s_H),

b) increasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is above a threshold,

c) decreasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is below that threshold.

I will use the five lemmata below, possibly of independent interest, to prove Theorem 1. insert numbers! Throughout, I denote the most selective equilibrium under the signal structure $\mathcal{E}$ as $\sigma_{\mathcal{E}}^{\text{sel}*}$. Similarly, $\sigma_{\mathcal{E}}^{\text{em}*}$ is the most embracive equilibrium. The subscript is dropped when the signal structure in question is obvious.

Lemma 5. Let $\mathcal{E}$ be binary. $\Psi(\sigma; \mathcal{E})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

is strictly true here?

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

Proof. Part i:

Let $\sigma(F) \in (0, 1)$ and $\sigma(S) = 1$. The interim belief ψ is then given by:

$$\begin{aligned}
\Psi(\sigma; \mathcal{E}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right) \\
&= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] \\
&= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]
\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every s_L signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}
\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) = &\mathbb{P}\left(\text{ev. was (i+1)}^{\text{th}} \text{ in order } \mid \text{applicant got i rejections}\right) \\
&\times \mathbb{P}\left(\text{applicant got i rejections}\right) \\
=&\frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i
\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus ψ increases, in $\sigma(s_L)$.

Part ii:

Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma; \mathcal{E}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma; \mathcal{E}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}
\Psi(\sigma; \mathcal{E}) &\propto \frac{1 + r_H + ... + r_H^{n-1}}{1 + r_L + ... + r_L^{n-1}} \\
&= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_H}{1 - r_L} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)} \\
&\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - (1 - p_H(s_H)\sigma(s_H))^n}{1 - (1 - p_L(s_H)\sigma(s_H))^n}
\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1-s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1-(r_H)^n}{1-(r_L)^n}$$

The positive term is the likelihood ratio of one s_H signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since approvals only happen with s_H signals, the negative term strictly exceeds the positive term. Thus, $\Psi(\sigma;\mathcal{E})$ decreases in $\sigma(s_H)$. $\square$

The Corollary below follows from Lemma 5: if $\mathcal{E}' \underset{B}{\succeq} \mathcal{E}$ where both signal structures are binary, adverse selection is stronger under $\mathcal{E}'$, if evaluators always (i) approve upon the high signal, and (ii) reject upon the low signal, under both signal structures.

Corollary 6. Let $\mathcal{E}'$ be more informative than $\mathcal{E}$, the strategies $\sigma'_{(0,1)}$ and σ_1 be $\sigma'_{(0,1)}(s'_L) = \sigma_1(s_L) = 0$ and $\sigma'_{(0,1)}(s'_H) = \sigma_1(s_H) = 1$. Then, $\Psi(\sigma';\mathcal{E}') \leq \Psi(\sigma;\mathcal{E})$.

Proof. I will only prove that the assertion holds when $s_L = s'_L$ but $s'_H > s_H$. The mirror case, which establishes the second part of the corollary, is analogous.

The proof will show that the outcome induced by σ under signal structure $\mathcal{E}$ can be replicated by $\tilde{\sigma}$ under signal structure $\mathcal{E}'$, where $\tilde{\mathcal{E}}(s_L) > 0$ and $\tilde{\mathcal{E}}(s_H) = 1$. Then, the conclusion follows from Lemma 5.

Take the pair $(\sigma, \mathcal{E})$. The probabilities that the applicant is rejected or approved upon a visit, conditional on θ, is given by:

$$\frac{\mathbb{P}(\sigma \text{ rejects } \mid \theta = H)}{\mathbb{P}(\sigma \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\sigma \text{ approves } \mid \theta = H)}{\mathbb{P}(\sigma \text{ approves } \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma}, \mathcal{E}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}(\tilde{\sigma} \text{ rejects} \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = L)} = \frac{p'_H(s_H) + \tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H) + \tilde{\sigma}(s_L)p'_L(s_L)}$$

where the family of distributions $\{p'_\theta\}$ belong to $\mathcal{E}'$. It is easy to verify that this last fraction on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma;\mathcal{E})$.

This proves the corollary. $\square$

Lemma 6. Let $\mathcal{E}$ be binary. There is no mixing at $s = s_L$ neither in $\sigma^{\text{sel}*}$ nor in $\sigma^{\text{emb}*}$; i.e. $\sigma^{\text{sel}*}(s_L), \sigma^{\text{em}*}(s_L) \in \{0, 1\}$.

Proof. I start by showing $\sigma^{\text{em}*}(s_L) \in \{0,1\}$. Where s_L^{mute} is as it was defined in Definition **??**, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium. This is because $\psi = \rho$

at this induced equilibrium, thus approving upon the low signal is optimal. This is the most embracive equilibrium, since there is no strategy that's more embracive. When $s_L < s_L^{\text{mute}}$, any equilibrium σ must feature $\sigma(s_L) = 0$, since $\psi \leq \rho$.

Now consider $\sigma^{\text{sel}*}$. For contradiction, let $\sigma^{\text{sel}*}(s_L) > 0$. By Lemma 5, and an argument used while proving equilibrium existence in Proposition 1, there is then another equilibrium σ clarify! where $\sigma(s_L) = 0$.

□

Lemma 7 characterises evaluators' decisions upon seeing the low signal in the extreme equilibria, given how informative $\mathcal{E}$ is. Broadly, more informative signal structures push evaluators to reject upon the low signal under both equilibria.

Lemma 7. Let $\mathcal{E}$ be binary, with signal realisations s_L and s_H. Then:

$$\sigma^{\text{em}*}(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \sigma^{\text{sel}*}(s_L) = \begin{cases} 1 & s_H < s_H^{\dagger}(s_L) \\ 0 & s_H \geq s_H^{\dagger}(s_L) \end{cases}$$

where $s_H^{\dagger}(.)$ is an increasing function, and $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

Proof. Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1-s_L} \geq \frac{c}{1-c}$$

which proves the part for the most embracive equilibrium, combined with Lemma 6.

Let the strategy σ_1 be such that $\sigma_1(s_L) = 0$ and $\sigma_1(s_H) = 1$. The following is a necessary and sufficient condition for an equilibrium σ^* where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

Necessity follows from $\Psi(\sigma_1; \mathcal{E}) \geq \Psi(\sigma^*; \mathcal{E})$ due to Lemma 5. Sufficiency follows from the fact that an equilibrium always exists, and the condition above implies s_L must always be rejected in it. Due to Corollary 6, we know that this condition holds when s_H is weakly above some threshold $s_H^{\dagger}(s_L)$, increasing with s_L. The necessary and sufficient condition holds whenever s_L^{mute}, therefore $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

□

Proof, Theorem 1: I prove Theorem 1 by establishing four facts:

1. The expected payoff in an equilibrium σ^* where $\sigma^*(s_L) = 0$ is higher than the expected payoff of approving all applicants.

   This follows directly from Proposition **??**.

2. There is at most one equilibrium where $\sigma^*(s_L) = 0$.

   Let $\{\sigma_\alpha\}_{\alpha \in [0,1]}$ be the family of strategies where the low signal is rejected: $\sigma_\alpha(s_L) := 0$ and $\sigma_\alpha(s_H) := \alpha$. If:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_H}{1 - s_H} \geq \frac{c}{1-c}$$

   σ_1 is the only equilibrium candidate among this family; the interim belief is higher under any lower α by Lemma 5. Otherwise, again by Lemma 5, there is at most one $\alpha \in [0,1]$ for which:

$$\frac{\Psi(\sigma_\alpha; \mathcal{E})}{1 - \Psi(\sigma_\alpha; \mathcal{E})} \times \frac{s_H}{1 - s_H} - \frac{c}{1-c} = 0$$

   When such an α exists, σ_α is the only equilibrium candidate in this family. Under higher α, approving upon $s = s_H$ is not optimal. Under lower α, rejecting upon $s = s_L$ is not optimal. If the expression above is strictly negative for *any* α, then the only equilibrium candidate where the low signal is rejected is σ_0.

3. When an equilibrium $\sigma^* \in \{\sigma_\alpha\}_{\alpha \in [0,1]}$ where all low signals are rejected exists, the expected payoff in this equilibrium is given by $\pi_i(\sigma^*; \mathcal{E}) = \max\{0, \pi_i(\sigma_1; \mathcal{E})\}$.

   Above we showed that evaluators expect positive expected payoff (necessarily from approving an applicant) only when $\alpha = 1$. Otherwise, they either approve no applicant or are indifferent to rejecting those they do.

Theorem 1 then follows from our fourth claim:

Below, I use s_L^{as} but do not reiterate what it means.

make sure the notation is either Π or π

4. $\max\{0, \Pi(\sigma_1; \mathcal{E})\}$ is:

   i weakly increasing in s_H whenever σ_α is an equilibrium strategy for some $\alpha \in [0,1]$,

   ii hump-shaped in s_L. As s_L falls, it is:

   - weakly increasing when $s_L \geq s_L^{as}$,
   - weakly decreasing when $s_L \leq s_L^{as}$

   where s_L^{as} is implicitly defined as:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1 - s_L}\right)^{n-1} \times \frac{s_H}{1 - s_H} = \frac{c}{1-c}$$

   for the signal structure $\mathcal{E}$.

Due to Lemma 7, both the most embracive and most selective equilibria shift once from the equilibrium where *all* s_L signals are approved to the one where *none* are approved, as the binary signal structure $\mathcal{E}$ becomes more informative. Due to the first fact laid out in the proof of this Theorem, this induces an increase in evaluators' expected payoff. Therefore, this last assertion about the shape of evaluators' payoffs in the equilibrium where the low signal is rejected concludes the proof.

Proof for the fourth claim:

Part i: Increasing s_H; i.e. the strength of evidence for $\theta = H$.

Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary signal structures with respective signal realisations $\{s_L, s_H\}$ and $\{s'_L, s'_H\}$. Let $s'_L = s_L$ and $s'_H = s_H + \delta$ for $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma_1; \mathcal{E}') > \Pi(\sigma_1; \mathcal{E})$.

Step 1: Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

Rather than having evaluators observe one draw from the signal structure $\mathcal{E}'$, say an evaluator potentially observes *two* signal realisations; s and $\hat{s}$. She first observes s, a single draw from $\mathcal{E}$. If this signal realises as $s = s_L$, she observes no further information. If instead $s = s_H$, she observes another signal $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from the signal structure $\hat{\mathcal{E}}$. $\hat{s}$ has the following distribution, and is independent from s, conditional on θ:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(s, \hat{s})$ is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{8.1}$$

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{8.2}$$

Note that the likelihood ratio 8.1 increases continuously with ε. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - (s_H + \delta)} \tag{8.3}$$

for our chosen (δ, ε). Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ. Furthermore, by varying ε between 0 and $\frac{1 - s_H}{s_H}$, the equivalent of *any* signal structure $\mathcal{E}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$ can be obtained.

Step 2: $\pi(\sigma_1; \mathcal{E}') > \pi(\sigma_1; \mathcal{E}')$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(s, \hat{s})$ instead of s'. To do so, the evaluator approves if and only if the pair $(s, \hat{s}) = (s_H, \hat{s_H})$ is observed. Note that, conditional on the visiting applicant's quality, the probability that the evaluator approves him is the same under these two policies. This is due to the identical informational content of these signals, as laid out in equations 8.2 and 8.3. Thus, evaluators' payoffs are also identical under these policies.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(s_i, \hat{s}_i)\}_{i=1}^n$. An applicant is a *marginal reject* if he has no $(s_i, \hat{s}_i) = (s_H, \hat{s_H})$ signals. The difference between evaluators' payoffs under $(\mathcal{E}, \hat{\mathcal{E}})$ and $\mathcal{E}$ is determined by these *marginal rejects*: they are *eventually rejected* under $(\mathcal{E}, \hat{\mathcal{E}})$ but *eventually approved* under $\mathcal{E}$. So:

$$\Pi(\sigma_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[c - \mathbb{P}\left(\theta = H \mid \text{marginal reject}\right)\right]}_{(1)}$$

A marginal reject only has signal realisations $(s, \hat{s}) = (s_H, \hat{s_L})$ or $s = s_L$. These carry equivalent information about θ. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid s_1 = \ldots = s_n = s_L\right]$$

In the relevant region where $s = s_L$ leads to a rejection, the expression above must be weakly positive. Therefore, $\Pi(\sigma_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) \geq 0$.

Maybe a brief explainer.

This concludes the first part of the claim that $\max\{0, \pi(\sigma_1; \mathcal{E})\}$ is weakly increasing in s_H.

I should probably just focus on the π not the whole thing, correct!

Part ii: Decreasing s_L; i.e. increasing the strength of evidence for $\theta = L$.

Now I show that replacing $\mathcal{E}$ with $\mathcal{E}'$ when $s'_L = s_L - \delta$ and $s'_H = s_H$:

i increases $\pi(\sigma_1; \mathcal{E})$ when $s_L \leq s_L^{as}$,

ii decreases $\pi(\sigma_1; \mathcal{E})$ when $s_L > s_L^{as}$

for $\delta > 0$ arbitrarily small. The desired assertion follows.

I should probably use the sum of evaluators' payoffs here.

Step 1: Replicating $\mathcal{E}'$ in two signals.

As before, let the evaluator potentially observe *two* signal realisations, s and $\hat{s}$. She first observes s, a single draw from $\mathcal{E}$. If this signal realises as $s = s_H$, she receives no further information. If it realises as $s = s_L$, she observes another signal $\hat{s} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from a signal structure we construct now, $\hat{\mathcal{E}}$. $\hat{s}$ is distributed independently from s conditional on θ, as follows:

$$\mathbb{P}\left(\hat{s} = s_H \mid \theta = H\right) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad \mathbb{P}\left(\hat{s} = s_H \mid \theta = L\right) = \varepsilon \times \frac{s_L}{1 - s_L}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(s, \hat{s})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)} = \frac{s_H}{1 - s_H} \tag{8.4}$$

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)} = \frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} \tag{8.5}$$

Note that 8.5 is continuously and strictly decreasing with ε, taking values between $\frac{s_L}{1-s_L}$ and 0 as ε varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_L}{1-s_L} \times \frac{1-\varepsilon \times \dfrac{s_H}{1-s_H}}{1-\varepsilon \times \dfrac{s_L}{1-s_L}} = \frac{s_L-\delta}{1-(s_L-\delta)}$$

Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ.

Step 2: $\pi(\sigma_1;\mathcal{E})$ increases (decreases) with a marginal decrease in s_L, whenever $s_L \geq s_L^{as}$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(s, \hat{s})$ instead of s'. To do so, the evaluator rejects if and only if the pair $(s, \hat{s}) = (s_L, \hat{s_L})$ is observed.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(s_i, \hat{s}_i)\}_{i=1}^n$. An applicant is a *marginal admit* if: (i) he has *no* $s = s_H$ signals, and (ii) he has *at least one* $\hat{s} = \hat{s_L}$ signal. The difference between evaluators' payoffs under $(\mathcal{E}, \hat{\mathcal{E}})$ and $\mathcal{E}$ is determined by these *marginal admits*, who are *eventually rejected* under $\mathcal{E}$, but *eventually approved* under $(\mathcal{E}, \hat{\mathcal{E}})$. So:

$$\Pi(\sigma_1;\mathcal{E}') - \Pi(\sigma_1;\mathcal{E}) = \mathbb{P}\left(\text{marginal admit}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

For a marginal admit, $(s_i, \hat{s}_i) \in \{(s_L, \hat{s_H}), (s_L, \hat{s_L})\}$, and $(s_j, \hat{s}_j) = (s_L, \hat{s_H})$ for at least one evaluator j. Denote the number of evaluators who observe $(s_L, \hat{s_H})$ as $\#$. Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}{\sum\limits_{j=1}^{n} \mathbb{P}\left(j \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}}_{(3)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$

where:

$$\mathbb{P}\left(i \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ...s_n = s_L\right) = \binom{n}{i} \times (k \times \varepsilon)^i \times (1 - k \times \varepsilon)^{n-1}$$

for $k = \mathbb{P}\left(\theta = H \mid s_1 = ... = s_n = s_L\right)$. The limit of expression (3) as $\varepsilon \to 0$ (thus $\delta \to 0$) is:

$$\lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(i \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)}{\sum\limits_{j=1}^{n} \mathbb{P}\left(j \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)} = \mathbb{P}\left(1 \ \hat{s} = \hat{s_H} \text{ signals } \mid s_1 = ... = s_n = s_L\right)$$

and thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c = \lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \# = 1\right) - c$$

This expression is strictly positive (negative) when the expression below is strictly positive (negative):

there is a limit below as well, add.

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} - \frac{c}{1-c}$$

proving the claim.

□

Proposition 3. Let $\mathcal{E}$ be a binary signal structure with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. Evaluators' payoffs across the most embracive equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < \min\{s_L^{\text{mute}}, s_L^{as}\}$.

Similarly, there exists a threshold $s_L^{\dagger} \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$, such that their payoffs across the most selective equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq s_L^{\dagger}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < s_L^{\dagger}$.

Proof. I start with the most embracive equilibrium. When $s_L \geq s_L^{\text{safe}}$, the strategy $\sigma_{(1,1)}$ which approves everyone; i.e. $\sigma_{(1,1)}(s_L) = \sigma_{(1,1)}(s_H) = 1$, is an equilibrium. This owes to $\Psi(\sigma_{(1,1)}; \mathcal{E}) = \rho$ as it can be easily checked, and to the definition of s_L^{safe}. Since no strategy is more embracive, $\sigma^{\text{em}*} = \sigma_{(1,1)}$. In this parameter region, $\pi(\sigma^{\text{em}*}; \mathcal{E})$ does not vary as every applicant is approved. When $s_L < s_L^{\text{safe}}$, this equilibrium is no longer possible, and evaluators' equilibrium payoffs are thus given by $\pi(\sigma^{\text{em}*}; \mathcal{E}) = \max\{0, \pi(\sigma_1; \mathcal{E})\}$; as it was explained in the second fact under Theorem 1's proof. As s_L decreases, this increases (decreases) when $s_L \geq s_L^{as}$ ($s_L < s_L^{as}$). This establishes the first part of Proposition 3.

For the most selective equilibrium to have $\sigma^*(\text{sel}*) = 0$, a necessary and sufficient condition is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1-\Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

This owes to Lemma 5, which establishes that the interim belief *increases* in $\sigma(s_H)$.

Clearly, this condition is satisfied when $s_L \leq s_L^{\text{safe}}$. Thus, the most selective equilibrium becomes one where s_L leads to a rejection once s_L falls below some threshold $s_L^{\text{thr}} \geq s_L^{\text{safe}}$. Evaluators' equilibrium payoffs then start falling with stronger evidence for $\theta = L$ once $s_L \leq \min\{s_L^{\text{thr}}, s_L^{as}\}$.

□

Theorem 2. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $s = s_i$ leads to approvals under σ.

2. *weakly lower* than under σ:

   i if $s = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

   ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{E}$ and σ.

Proof. Following the notation introduced in Definition 4, let $S \cup S'$ be the joint support of the signal structures $\mathcal{E}$ and $\mathcal{E}'$, and $s_1 < s_2 < ... < s_M$ be its elements. I begin by noting that the outcome the monotone strategy $\sigma : S \to [0,1]$ generates under $\mathcal{E}$ can be replicated under $\mathcal{E}'$ by another monotone strategy $\tilde{\sigma}' : S' \to [0,1]$ provided $\sigma(s_i) \in \{0,1\}$[11]:

$$\tilde{\sigma}'(s_j) = \begin{cases} \sigma(s_i) & j \in \{i-1, i+1\} \\ \sigma(s_j) & j \notin \{i-1, i+1\} \end{cases}$$

Part 1:

Now suppose s_i leads to approvals under σ; $\sigma(s_i) = 1$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 1$. I argue below that $\tilde{\sigma} \geq \sigma'$; evaluators *reject* more when s_i is spread. From Proposition ??, it follows that $\pi(\sigma; \mathcal{E}) = \pi(\tilde{\sigma}'; \mathcal{E}') \leq \pi(\sigma'; \mathcal{E}')$.

If $s_{i-1} = \min S \cup S'$ or $\sigma'(s_{i-2}) = 0$, we necessarily have $\tilde{\sigma} \geq \sigma'$ and are done. So, for contradiction, let $\sigma'(s_{i-2}) > 0$, and $\sigma' > \tilde{\sigma}'$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{E}$ such that:

i $\tilde{\sigma}$ replicates the outcome σ' induces in $\mathcal{E}'$,

ii $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ if and only if σ' is an equilibrium strategy under $\mathcal{E}'$,

iii $\tilde{\sigma} > \sigma$, so σ cannot be the most embracive equilibrium under $\mathcal{E}$.

So, define the strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{E}$ as simply:

$$\tilde{\sigma}(s_j) := \begin{cases} 1 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of σ'. Furthermore, σ' is an equilibrium under $\mathcal{E}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}$: they induce the same interim belief ψ as the latter replicates the former, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}\left(\theta = H \mid \psi, s = s_{i-2}\right) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

[11]The characterisation of $\tilde{'}\sigma$ is otherwise the same, but it ceases to be *monotone* by our definition.

Lastly, since $\sigma' > \tilde{\sigma}'$, it must be that $\tilde{\sigma} > \sigma$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Recall that $\tilde{\sigma}$ and σ induce the same interim belief ψ under their respective signal structures; $\Psi(\tilde{\sigma}; \mathcal{E}') = \Psi(\sigma; \mathcal{E}) = \psi$. Therefore, if $\mathbb{P}(\theta = H \mid s = s_{i-1}, \psi) \geq c$, $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}'$. This implies that $\sigma' \leq \tilde{\sigma}$, since σ' is the most selective equilibrium under $\mathcal{E}'$. If $\mathbb{P}(\theta = H \mid s = s_{i-1}, \psi) < c$ otherwise, there is an equilibrium $\sigma' < \tilde{\sigma}$ under $\mathcal{E}'$ due to the intermediate value argument presented when equilibrium existence was established in Proposition 1.

make this reference clear, we are using it a lot.

Part 2:

Now suppose s_i leads to rejections under σ; $\sigma(s_i) = 0$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 0$. I will establish Theorem 2's claim in two steps:

1. $\sigma' \geq \tilde{\sigma}'$; evaluators *approve* more when s_i is spread,

2. $\pi(\sigma'; \mathcal{E}') \leq \pi(\tilde{\sigma}'; \mathcal{E}') = \pi(\sigma; \mathcal{E})$ when adverse selection poses a threat at signal s_{i+1} for signal structure $\mathcal{E}$ and strategy σ.

Step 1:

If $s_{i+1} = \max S \cup S'$ or $\sigma'(s_{i+1}) > 0$, we necessarily have $\sigma' \geq \tilde{\sigma}$. So, let $s_{i+1} < \max S \cup S'$ and $\sigma'(s_{i+1}) = 0$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Recall that $\Psi(\sigma; \mathcal{E}) = \Psi(\tilde{\sigma}'; \mathcal{E}') = \psi$ since $\tilde{\sigma}'$ replicates the outcome of σ. Thus, if:

$$\mathbb{P}(\theta = H \mid s = s_{i+1}, \psi) \leq c$$

$\tilde{\sigma}'$ must be an equilibrium strategy under $\mathcal{E}'$; the optimality conditions for all signals below s_{i+1} are satisfied *a fortiori*, and those for the signals above s_{i+1} are satisfied since σ is an equilibrium strategy in $\mathcal{E}$. Then, $\sigma' \geq \tilde{\sigma}'$, since σ' is the most embracive equilibrium. If

$$\mathbb{P}(\theta = H \mid s = s_{i+1}, \psi) > c$$

on the other hand, by the intermediate value argument we used to establish equilibrium existence in Proposition 1, there is an equilibrium strategy $\sigma' > \tilde{\sigma}$ under $\mathcal{E}'$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{E}$ and $\mathcal{E}'$, respectively.

Since $\sigma'(s_{i+1})$, its outcome under $\mathcal{E}'$ can be replicated $\mathcal{E}$ with a strategy $\tilde{\sigma}$, defined as:

$$\tilde{\sigma}(s_j) = \begin{cases} 0 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

I will show that this necessarily implies that $\sigma' \geq \tilde{\sigma}'$, in two steps:

i $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ if and only if σ' is an equilibrium strategy under $\mathcal{E}'$,

ii $\tilde{\sigma} \leq \sigma$, and therefore $\tilde{\sigma} = \sigma$ since σ is the most selective equilibrium under $\mathcal{E}$.

(i) follows trivially, since both strategies have the same optimality condition for every signal realisation above s_{i+1}. Now, since σ is the most selective equilibrium under $\mathcal{E}$, we must have $\sigma \leq \tilde{\sigma}$; as $\tilde{\sigma}$ is an equilibrium strategy by (i). However, this means $\tilde{\sigma}' \leq \sigma'$. Since $\tilde{\sigma}'$ must also be an equilibrium in $\mathcal{E}'$, we must have $\tilde{\sigma}' = \sigma'$ and therefore $\sigma \leq \tilde{\sigma}$.

Step 2:

Now I establish the second part. The case where $\tilde{\sigma}' = \sigma'$ is trivial, so I focus on the case $\sigma' > \tilde{\sigma}'$. As we showed when establishing Case 2 in the first step, we must then have $\sigma'(s_{i+1}) > 0$.

Now take a strategy σ^{ε} for $\mathcal{E}'$, defined as $\sigma^{\varepsilon}(s_{i+1}) := \varepsilon$. We take ε small enough so that $\sigma' > \sigma^{\varepsilon} > \tilde{\sigma}'$. I will now show that when adverse selection poses a threat at signal s_{i+1} for $(\sigma; \mathcal{E})$, we have:

$$\pi(\sigma^{\varepsilon}; \mathcal{E}') \leq \pi(\tilde{\sigma}'; \mathcal{E}') = \pi(\sigma; \mathcal{E})$$

Proposition **??** then coins the result.

I show this slightly circuitously. Construct a ternary signal $\mathcal{E}^{\text{re}}$ which we will use to replicate the outcomes σ^{ε} and $\tilde{\sigma}'$ generate. This signal admits the realisations $s^{\text{re}} \in \{s_L^{\text{re}}, s_{\varepsilon}^{\text{re}}, s_H^{\text{re}}\}$ and has distribution:

$$\mathbb{P}\left(s^{\text{re}} = s \mid \theta\right) = \begin{cases} 1 - r_\theta(\sigma; \mathcal{E}) & s = s_H^{\text{re}} \\ \varepsilon \times p'_\theta(s_{i+1}) & s = s_{\varepsilon}^{\text{re}} \\ r_\theta(\sigma; \mathcal{E}) - \varepsilon \times p'_\theta(s_{i+1}) & s = s_L^{\text{re}} \end{cases}$$

Clearly, as defined below, the strategies σ^{re} and $\sigma^{\text{re-}\varepsilon}$ for $\mathcal{E}^{\text{re}}$ replicate the outcomes of $\tilde{\sigma}$ and σ^{ε} under $\mathcal{E}'$:

$$\sigma^{\text{re}}(s) = \begin{cases} 1 & s = s_H^{\text{re}} \\ 0 & s = s_{\varepsilon}^{\text{re}} \\ 0 & s = s_L^{\text{re}} \end{cases} \qquad \sigma^{\text{re}}(s) = \begin{cases} 1 & s = s_H^{\text{re}} \\ 1 & s = s_{\varepsilon}^{\text{re}} \\ 0 & s = s_L^{\text{re}} \end{cases}$$

This makes it clear that the difference in evaluators' payoffs between $\tilde{\sigma}$ and σ^{ε} will be the *marginal admits* whose evaluators will observe:

i no s_H^{re} signal realisation,

ii at least one $s_{\varepsilon}^{\text{re}}$ signal realisation.

if they visit all evaluators. Thus, we have:

$$\Pi(\sigma^{\varepsilon}; \mathcal{E}') - \Pi(\tilde{\sigma}; \mathcal{E}') = \mathbb{P}\left(\text{marginal admits}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

where (2) then equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_{L}^{\text{re}} \text{ signals}\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ s_{\varepsilon}^{\text{re}} \text{ and } n-j\ s_{L}^{\text{re}} \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_{L}^{\text{re}} \text{ signals}\right)$$

COMPLETE! □

Proposition 4. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proof. □

Corollary 5. Let $\mathcal{E}'$ differ from $\mathcal{E}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

why does this appear with the incorrect number?

Proof. □

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