1 The Model

An *applicant* (he) seeks an approval from any one of $n \in \mathbb{N}_{\geq 2}$ *evaluators* (she), each with a distinct label $i \in \{1, 2, ..., n\}$. He is born with either *High* or *Low quality*; $\theta \in \{H, L\}$. Though θ is unknown both to himself[1] and the evaluators, all players correctly believe that he is born with *High* quality with probability $\rho \in (0, 1)$. To receive the approval he seeks, the applicant sequentially visits (applies to) all evaluators until either one *approves* him, or they all *reject* him. In the former case, I say he is *eventually approved*, and in the latter, *eventually rejected.* The order of his visits is described by the permutation $\tau(.)$ of the set of labels $\{1, 2, ..., n\}$. The applicant visits the evaluator labeled $\tau(i)$ after his $i - 1^{\text{st}}$ rejection.

No evaluator knows the applicant's order of visits $\tau(.)$, but they commonly believe that they are all equally likely to be anywhere in the order; i.e. $\mathbb{P}(\tau(i) = j) = \frac{1}{n}$ for all $i, j \leq n$[2]. Thus, when an evaluator receives the applicant, she understands that he was rejected from all his earlier visits, but she does not know how many of these occurred. Therefore, she revises her prior belief ρ about the applicant's quality to an *interim belief* ψ_i upon receiving a visit. This revision is based on what she *believes* about the number of these past rejections and what they imply for the applicant's quality. I explain how evaluators form this interim belief in greater detail in Section 2.

Before evaluator i decides whether to *approve* the applicant she received, she observes the realisation of a private and costless signal x_i. x_i takes values in a finite set $S = \{s_1, s_2, ..., s_m\} \subset [0, 1]$. Without loss, I denote higher elements of S with higher indices: wherever $i > j$; $s_i \geq s_j$. Where I deal with *binary signals* with two possible realisations, $S = \{s_1, s_2\}$, it is friendlier to denote the low signal s_1 as s_L and the high signal s_2 as s_H instead. I do so.

x_i's distribution over S depends on the applicant's quality θ, and is described by the discrete density function $p_\theta : S \to [0, 1]$. Conditional on θ, evaluators' signals are IID; the distribution of an evaluator's signal depends neither on her label, nor on her order in τ. Without loss of generality, I label signal realisations after the *normalised posterior beliefs* they induce:

$$s = \frac{p_H(s)}{p_L(s) + p_H(s)} \qquad \text{wherever } p_H(s) + p_L(s) > 0$$

Likewise, I define the *normalised posterior density* p induced by $\mathcal{X}$ as:

$$p(s) := \frac{p_L(s) + p_H(s)}{2} \qquad \text{wherever } p_H(s) + p_L(s) > 0$$

Both the *actual* posterior beliefs and distribution $\mathcal{X}$ induces over them depend on evaluators' interim beliefs, themselves endogenous in this model. Generically, these coincide with their

[1] This is without loss of generality in the baseline model. The applicant's knowledge of his quality has no relevance to the analysis.

[2] I study the implications of relaxing this assumption later in Section 5.

normalised counterparts only when the interim belief assigns equal probability to either quality. Nonetheless, referring to the normalised posterior beliefs and density will be helpful especially when comparing the relative informativeness of signal structures.

is "generically" a right word to use here?

After observing the realised signal x_i, evaluator i decides whether to *approve* or *reject* the applicant. To approve the applicant, she incurs a fixed cost $c \in (0,1)$; but whenever $\theta = H$, she also receives a benefit I normalise to 1. Upon an approval, the game ends and all other evaluators receive a payoff of 0. If the evaluator instead rejects her applicant, she receives a payoff of 0. The applicant then moves on to visit the evaluators he has not yet, unless none remain.

Evaluator i's *strategy* $\sigma_i : S \to [0,1]$ maps every possible signal realisation $s \in S$ she might observe to a corresponding probability that she will approve the applicant, $\sigma_i(s)$. This strategy σ_i is *optimal* if:

$$\sigma_i(s) = \begin{cases} 0 & \mathbb{P}\left(\theta = H \mid \psi_i, x_i = s\right) < c \\ \in [0,1] & \mathbb{P}\left(\theta = H \mid \psi_i, x_i = s\right) = c \\ 1 & \mathbb{P}\left(\theta = H \mid \psi_i, x_i = s\right) > c \end{cases}$$

where ψ_i is evaluator i's endogenously formed *interim belief* that the applicant has *High* quality, *given he visited her.*

interim belief? interim belief? what other name?

I focus on the *symmetric Bayesian Nash Equilibria* (henceforth just *equilibria*) of this game. An equilibrium is a strategy – belief pair (σ^*, ψ^*) such that:

1. each evaluator believes that $\theta = H$ with probability ψ^* given (i) the applicant visited her, and (ii) the strategies $\sigma_j = \sigma^*$ for all other evaluators $j \in \{1, 2, ..., n\}$,
2. the strategy σ^* is optimal given this belief ψ^*.

I call such a strategy σ^* an *equilibrium strategy*, and belief ψ^* an *equilibrium interim belief* under $\mathcal{X}$.

2 Interim Beliefs and Equilibria

Upon receiving a visit, an evaluator must assess the probability that she faces a *High* quality applicant. The signal x_i she observes is central to this assessment. However, she gleans crucial information about the applicant's quality from his mere visit, too.

The applicant visits our evaluator only if he was rejected by all other evaluators he visited earlier. Each such rejection itself is bad news about the applicant's quality. Our evaluator does not know how many such rejections occurred in the past. She nonetheless holds a belief about them. In particular, when all her peers use the strategy σ, she assigns a probability $r_\theta(\sigma; \mathcal{X})$ to the applicant receiving a rejection from any of his visits, conditional on having quality θ:

$$r_\theta(\sigma;\mathcal{X}) = 1 - \sum_{i=1}^{m} p_\theta(s_i)\sigma(s_i)$$

She believes she is – ex-ante – equally likely to be anywhere in the applicant's visit order $\tau(.)$. So she assigns a probability $\mathcal{R}_\theta(\sigma;\mathcal{X})$ to being visited before any of her peers approve the applicant:

$$\mathcal{R}_\theta(\sigma;\mathcal{X}) = \frac{1}{n} \times \sum_{k=1}^{n} r_\theta(\sigma;\mathcal{X})^{k-1}$$

The evaluator's interim belief ψ that the applicant who visits her has *High* quality is then a function $\Psi(.)$ of peer evaluators' strategies σ and the signal structure $\mathcal{X}$:

$$\psi = \Psi(\sigma;\mathcal{X}) := \frac{\rho \times \mathcal{R}_H(\sigma,\mathcal{X})}{\rho \times \mathcal{R}_H(\sigma,\mathcal{X}) + (1-\rho) \times \mathcal{R}_L(\sigma,\mathcal{X})}$$

When all evaluators use the strategy σ, they each hold the same interim belief ψ. This belief is determined *endogenously* in equilibrium; it depends on what evaluators' equilibrium strategies will be. However, the equilibrium strategies themselves must be optimal against the interim belief they induce. Thus, neither the existence nor the properties of equilibria are automatic.

Our first Proposition sets the ground by establishing some basic facts about equilibria. I exclude the uninteresting case of an uninformative signal structure, i.e. $p_H = p_L$. This is for brevity and without loss of interest: an equilibrium certainly exists in this case (either to approve *any* applicant, or *none*). In the knife edge case where we also have $\rho = c$, *any* strategy is an equilibrium; in any equilibrium evaluators are left indifferent between approving and rejecting the applicant regardless of the realised signal.

Proposition 1 first assures us that an equilibrium always exists. Further, any equilibrium strategy must be *monotone*: if a signal realisation s_i *might* lead to an approval, any better signal realisation $s_j > s_i$ *always* leads to an approval.

The monotonicty of equilibrium strategies critically implies that equilibria will always exhibit *adverse selection*. With a monotone strategy, a *Low* quality applicant is always likelier to get rejected than a *High* quality applicant. Thus, past rejections point to *Low* quality in any equilibrium. As no evaluator can rule out having possibly received the applicant after he has had many rejections already, evaluators view any visit to be *adversely selected*: an evaluator's interim belief ψ that the applicant who visited her has *High* quality is always weakly below her prior belief ρ.

Evaluators' interim beliefs are endogenous to their equilibrium strategies. So there might be multiple strategies which are optimal against the interim belief they induce; thus multiple equilibria. Nonetheless, Proposition 1 assures us that the set of equilibrium strategies is always *compact*. Compactness will be helpful to identify the *highest* and *lowest* equilibrium strategies, which play a special role in the sequel.

Equilibria will differ in the chances applicants stand to be approved. When $\sigma(s) \leq \sigma'(s)$ for every $s \in S$, or simply $\sigma \leq \sigma'$, I say σ is *more selective* than σ'. Conversely, I say σ' is *more embracive* than σ. A more selective (embracive) equilibrium offers applicants a higher probability of approval for any signal their evaluator might observe. Proposition 1 establishes that this ordering is *complete* over the set of equilibrium strategies: where σ and σ' both describe equilibrium strategy profiles, σ is either more selective or more embracive than σ'. This follows straightforwardly from the monotonicity of equilibrium strategies. In fact, this pointwise order is complete in the space of *all* monotone strategies.

should i retain "embracive" or not? "less selective" sounded derogatory.

Proposition 1. Let $p_H \neq p_L$. Where Σ is the set of equilibrium strategies:

1. *an equilibrium exists;* $\Sigma \neq \emptyset$,
2. *all equilibrium strategies are monotone;* for any $\sigma^* \in \Sigma$ and $s' > s$, $\sigma^*(s) > 0$ implies $\sigma^*(s') = 1$,
3. *all equilibria exhibit adverse selection;* $\psi^* \leq \rho$ for any ψ^* induced by an equilibrium strategy,
4. Σ is compact. Moreover, elements of Σ are pointwise totally ordered.

should i say anything about proof? quite standard.

Proof. See Section 6. $\square$

Carlos: Easy to prove that more evaluators = more adverse selection *holding strategies fixed*. This out of eqm result didn't sound too interesting, so I excluded it for now.

That the set of equilibrium strategies is compact and totally ordered is important mainly because it implies that the *most* embracive and *most* selective equilibria, respectively the highest and lowest elements of this set, are well defined. I will focus heavily on these equilibrium strategies in the remainder of this paper, referring to them jointly as the *extreme equilibria*.

Notwithstanding its assurance that we can order equilibria from the most selective to the most embracive, Proposition 1 is silent about why this order would be useful. That gap is filled by Proposition 2. There, I establish that whether evaluators are better off in an equilibrium or the other is determined precisely by which one is more selective.

A priori, how moving towards more selective equilibria affects evaluators' payoffs is not clear. How evaluators fare in an equilibrium depends on how well they can distinguish and (i) eventually approve a *High* quality applicant, and (ii) eventually reject a *Low* quality one. In particular, when all evaluators use the strategy σ, the *sum* of all evaluators' expected payoffs are given by:

$$\begin{aligned}\Pi(\sigma;\mathcal{X}) &:= \rho \times (1-c) \times \mathbb{P}\left(\text{eventually approved} \mid \theta = H, \sigma_i = \sigma\ \forall i\right) \\ &+ (1-\rho) \times (-c) \times \left[1 - \mathbb{P}\left(\text{eventually rejected} \mid \theta = L, \sigma_i = \sigma\ \forall i\right)\right]\end{aligned} \tag{2.1}$$

$\Pi(\sigma;\mathcal{X})$ is just a sum of the probabilities that (i) *High* quality applicants are eventually approved, and (ii) *Low* quality applicants are eventually rejected. These probabilities are weighted by the the benefit of either outcome to evaluators, and the probability that the applicant was born with *High* quality. Note that we can easily recover a single evaluator's expected payoff $\pi_i(\sigma;\mathcal{X})$ from $\Pi(\sigma;\mathcal{X})$. $\pi_i(\sigma;\mathcal{X})$ is simply $\left(\frac{1}{n}\right)^{\text{th}}$ of $\Pi(\sigma;\mathcal{X})$ as all evaluators are ex-ante identical, and the equilibrium is symmetric.

Thus, selective and embracive equilibrium strategies have different virtues for evaluators' payoffs. Selective strategies depress applicants' chances of approval. By doing so, they protect evaluators against approving *Low* quality applicants too frequently. However, the high bar they impose for approvals potentially forsakes *High* quality applicants in the process. Vice versa, more embracive strategies give *High* quality applicants generous approval chances, but might be too admissive to *Low* quality applicants in the meanwhile.

Nonetheless, I establish in Corollary 3 that this trade-off is always resolved in favour of more selective equilibria. I show this by proving Proposition 2, which establishes an indeed stronger fact: evaluators' payoffs decrease *whenever* they move from an equilibrium strategy profile to *any* monotone strategy that's more embracive than it.

Proposition 2. Let σ^* be an equilibrium strategy, and σ be any other monotone strategy more embracive than σ^*. Evaluators' expected payoffs under σ^* exceed those under σ; $\Pi(\sigma^*;\mathcal{X}) \geq \Pi(\sigma;\mathcal{X})$.

Proof. See Section 6. □

Corollary 3. Let σ^* and σ^{**} be two equilibrium strategies, where σ^{**} is more embracive than σ^*. Evaluators' expected payoffs under σ^* exceed those under σ^{**}; $\Pi(\sigma^*;\mathcal{X}) \geq \Pi(\sigma^{**};\mathcal{X})$.

Extreme equilibria thus have special importance: they delineate the boundaries of payoffs evaluators can achieve across equilibria.

To grasp the intuition behind Proposition 2, consider moving all evaluators from an equilibrium strategy σ^* to a *marginally* more embracive strategy σ. Specifically, say σ^* and σ differ only for the highest signal which possibly leads to a rejection in σ^*, and that σ assigns only a marginally higher approval probability to this signal:

$$\sigma(\underline{s}) - \sigma^*(\underline{s}) > \varepsilon \qquad \text{where } \underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$$

To further our thought experiment, fix the stream of signals $(x_1, x_2, ..., x_n)$ which evaluators will observe for this applicant if he visits them. Whether this applicant is eventually approved depends on what evaluators' strategies will instruct them to do upon the signal they observe.

Holding fixed this stream of signals, an applicant approved under the more selective strategy σ^* would also be approved under the more embracive strategy σ[3]. But the transition to σ might *also* lead to the eventual approval of some applicants who were rejected by every evaluator under σ^*. For such applicants, some of the signals their evaluators saw were only slightly below the mark under σ^* but still sufficient for an approval under σ.

is this foot-note clear? should i dwell on it?

With their eventual approval under σ, some such applicants might boost evaluators' expected payoffs. Consider for instance, an applicant who missed the mark only marginally in *every* evaluation he had. Under σ^*, all his evaluators lean on the cautious side, unable to rule out that he previously received multiple rejections, potentially with very low signals. If they knew that any previous rejections he had fell only marginally below the mark, they could have decided they were overestimating how adversely selected he was and revised their decision to an approval.

On the other hand, evaluators would rather keep some such applicants out, too. Consider, for instance, an applicant who faces a rejection by all but one of his evaluators under both strategy profiles. But under σ, say he scrapes through his *last* evaluation. Under σ^*, the bare *fear* of other evaluators' rejections, possibly with very low signals, had convinced this last evaluator to reject him. Her fear was in fact valid: the applicant was indeed rejected by all her peers. For all the worse, with signals that fell even below the more embracive standard σ imposes. Therefore, the approval of this applicant hurts evaluators' payoffs.

To prove Proposition 2, I identify which of these kinds of applicants the evaluators are likelier to approve on balance when their strategies get more embracive. I show that the probability of approving a *most* adversely selected applicant, of the latter kind, is overwhelmingly high. This is simply because when the difference between $\sigma(s_k)$ and $\sigma^*(s_k)$ shrinks, the probability that the applicant misses the mark under multiple evaluations vanishes rapidly. It thus becomes overwhelmingly likely that if he did miss the mark in some of his evaluations, he in fact did so in only *one*. He would fail all his other evaluations regardless of the strategy his evaluators used.

3 Equilibrium Payoffs with Better Informed Evaluators

As made evident by expression 2.1, evaluators' payoffs are determined by how well they can distinguish between *High* and *Low* quality applicants in their decisions. How *informative* their private signals x are about the applicant's quality is critical for this exercise. It is indeed

[3] For simplicity, I sideline discussing possible differences in tie-breaking between σ^* and σ, which might of course mean an applicant approved under σ^* is rejected under σ; simply because he got unlucky when his evaluators were randomising in the latter. This is a distraction, and is easily remedied by appending evaluators' signals with a uniformly drawn tie-breaking signal according to which they break ties.

natural to conjecture that a more informative signal would allow evaluators to approve *High* quality applicants more frequently and *Low* quality ones less so, and thereby leave them better off.

Blackwell's classic result (1953) validates this reasoning for single person decision problems. He shows that among two decision makers facing the same decision problem, the one with a *(Blackwell) more informative* signal structure is always guaranteed a weakly higher expected payoff. Conversely, this decision maker remains better off in *any* decision problem *only if* her signal structure is *(Blackwell) more informative* than her peer's. It follows that if we had a *single* evaluator, she would be *guaranteed* a higher expected payoff with a Blackwell more informative signal structure[4]. This is precisely because such a signal structure relaxes the trade-off she faces between wanting to ensure she approves when a *High* quality applicant comes along, but rejects a *Low* quality applicant[5].[6]

Nevertheless, the conjecture that this reasoning would carry over to our current setting turns out to be naïve. To showcase what goes wrong, re-express an individual evaluator i's expected payoff, $\pi_i(\sigma;\mathcal{X})$:

$$\begin{aligned}\pi_i(\sigma;\mathcal{X}) = &\ \underset{\sim}{\mathbb{P}\left(\text{applicant visits } i\right)} \\ &\times \left[\underset{\sim}{\psi} \times (1-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = H\right) + \underset{\sim}{(1-\psi)} \times (-c) \times \mathbb{P}\left(i \text{ approves} \mid \theta = L\right)\right]\end{aligned}$$

Evaluator i seeks to better distinguish and approve *High* quality applicants while keeping *Low* quality ones out. However, this pursuit is constrained by the extent of *adverse selection* she faces: the applicant might not visit at all, and might be very unlikely to have *High* quality if he does, given he was not approved until he did. These two probabilities depend on *other* evaluators' strategies. While evaluating their applicants, these evaluators do not account for the adverse selection they impose on evaluator i; just as she disregards the adverse selection she imposes on them. Giving evaluators more information might exacerbate this adverse selection externality. Evaluators might eventually leave worse off; their improved ability to evaluate an applicant being eclipsed by their poor expectations about the applicant they do receive.

How does, then, a *Blackwell improvement* of evaluators' signals about quality affect their equilibrium payoffs? This question is at the heart of my paper, and in this Section, I answer it. The answer depends on the *kind* of improvement in evaluators' signals. Broadly speaking, I show that *more confident approvals* benefits evaluators. In contrast, *more confident rejections*

[4] In fact, Blackwell's condition retains its necessity, too: she would be better off with $\mathcal{X}'$ rather than $\mathcal{X}$ regardless of her prior belief ψ and approval cost c *only if* $\mathcal{X}'$ is Blackwell more informative than $\mathcal{X}$. For completeness, I present a self contained proof of this in Section 6.2, Lemma 7.

[5] The reader can refer to Blackwell and Girshick, 1954's Theorems 12.2.2 and 12.4.2 for a textbook exposition of these classic results.

[6] I illustrate this in the context of binary signal structures in Figure 1 in the following subsection, where I discuss Blackwell improvements of binary signal structures in further depth.

hurts them *eventually*. I develop this idea by first focusing on evaluators with *binary* signals, in Section 3.1. I then generalise the insight I develop there, in Section 3.2.

i am not sure if "eventually" is the right word. See Thm 2.

3.1 Binary Signals

Consider evaluators who either observe a high signal $x = s_H$, or a low signal $x = s_L$. The former carries evidence for *High* quality, and the latter for *Low* quality. How does receiving a more informative binary signal instead affect their equilibrium payoffs? The answer to this question will be the building block and key intuition for Section 3.2, where we will think of evaluators with arbitrary discrete signal structures.

Take two binary signal structures $\mathcal{X}'$ and $\mathcal{X}$, with respective supports $\{s'_L, s'_H\}$ and $\{s_L, s_H\}$. $\mathcal{X}'$ is *(Blackwell) more informative than* (or *(Blackwell) improves on*) $\mathcal{X}$ if its signals carry stronger evidence both for *Low* and *High* quality[7]:

$$s'_L \leq s_L \qquad\qquad s'_H \geq s_H$$

Recall that these signals are labeled after the *normalised posterior beliefs* they induce. Hence, an evaluator who observes $x' = s'_H$ grows more confident that $\theta = H$ than one who observes $x = s_H$, whenever they have the same interim beliefs. In other words, $\mathcal{X}'$ offers stronger evidence for *High* quality than $\mathcal{X}$. Similarly, observing $x' = s'_L$ induces more confidence that $\theta = L$ than observing $x = s_L$; i.e. $\mathcal{X}'$ offers *stronger evidence for Low quality* than $\mathcal{X}$.

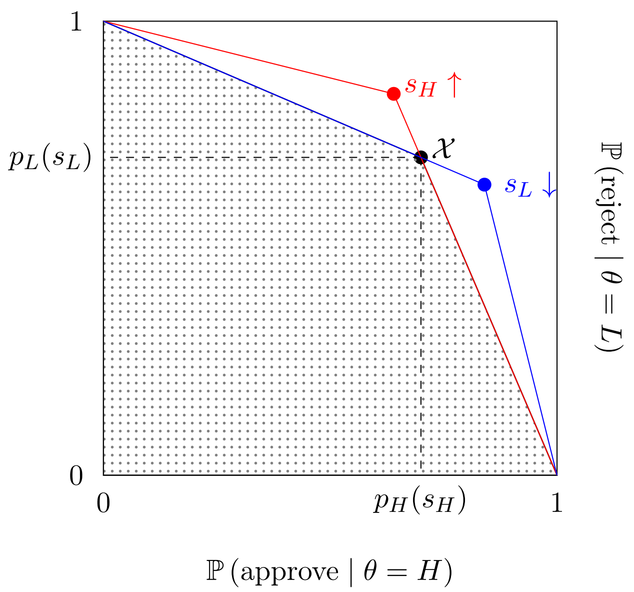


Figure 1: Improving Binary Signals and Probabilities of Mistakes

The more informative signal structure $\mathcal{X}'$ allows a better evaluation of any applicant as Figure 1 illustrates. The shaded area whose right vertex is labeled $\mathcal{X}$ covers the probabilities

[7] See Section 12.5 in Blackwell and Girshick, 1954 for a textbook exposition of this classic result.

of rejecting the *Low* quality applicant the evaluator can secure against any given probability of approving the *High* quality applicant. Both increasing s_H and decreasing s_L expand this region, relaxing the evaluator's trade-off between these objectives; albeit in different ways.

I removed the example in prev. draft.

Theorem 1 reveals that the response of evaluators' equilibrium payoffs to a more informative binary signal hinges precisely on whether evidence for *High* or *Low* quality gets stronger; i.e the *kind* of information evaluators get. While stronger evidence for *High* quality always increases their equilibrium payoffs, stronger evidence for *Low* quality *eventually harms* them. Once it is too strong, signal structures with stronger evidence for *Low* quality can only hurt evaluators.

Theorem 1. Let $\pi(\sigma^*; \mathcal{X})$ be an evaluator's payoff in an extreme equilibrium under the binary signal structure $\mathcal{X}$ with normalised posterior beliefs $0 \le s_L \le 0.5$ and $0.5 \le s_H \le 1$. $\pi(\sigma^*; \mathcal{X})$ is weakly:

a) increasing with the strength of evidence for $\theta = H$ (s_H),

b) increasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is above a threshold,

c) decreasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is below that threshold.

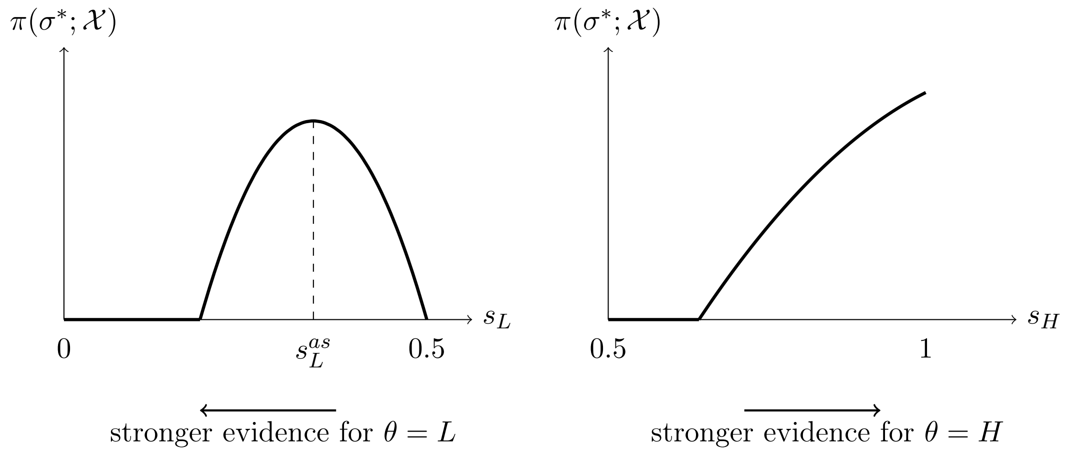


Figure 2: Theorem 1 Illustrated

Figure 2 illustrates Theorem 1 for the particular case where the approval cost c exceeds the prior belief ρ, which forces a unique equilibrium. In this unique equilibrium, evaluators approve upon a high signal and reject upon a low one if they can secure positive payoffs when they all do so. If the adverse selection they face is too strong to prevent this, they reject with positive probability upon high signals as well. They then expect equilibrium payoffs of zero. Although this illustration brushes equilibrium multiplicity away, evaluators' equilibrium payoffs respond qualitatively the same way to an improvement in evidence for either quality, as Theorem 1 asserts.

Do I need a proof for this?

In the ensuing discussion, I explain the discrepancy Theorem 1 highlights between strengthening the two pieces of evidence evaluators might receive. The full proof, which I relegate to

Section 6, builds on the forces this discussion lays out. Theorem 1 is notably silent about the threshold after which stronger evidence for *Low* quality starts harming evaluators' payoffs. I elaborate on this threshold as I clarify the forces behind Theorem 1. I then characterise it in Proposition 4.

Carlos: I actually have a nice result here. I can prove it using another one in the appendix. "Equilibrium adverse selection always increases with the informativeness of the signal".
This is why the reader should care about what's to come next anyway: because one's first instinct towards a proof will be fruitless. I don't know how to insert this yet. It yields another nice result:
"The expected quality of an approved applicant falls when s_L falls, in equilibrium."

Evaluators' payoffs under any signal structure depend on the applicants they eventually approve in the equilibrium it induces. Any change in this signal structure has the potential to affect some of these applicants' eventual outcomes. A High quality applicant who otherwise would only receive rejections might now get approved by some, for instance. Likewise, a Low quality applicant previously approved by some might now be unanimously rejected. Both of these would leave evaluators better off. The same outcome reversals experienced by an applicant of the opposing quality, in contrast, would harm them. The distinct consequences of strengthening evidence for *Low* or *High* quality stems precisely from their distinct effects on applicants' eventual outcomes.

To illustrate this, let us set evaluators' strategic responses aside and say they each approve upon a high signal and reject upon a low one in both signal structures. To begin with, consider giving evaluators a signal structure with *marginally* stronger evidence for *Low* quality, but the same strength of evidence for *High* quality. Specifically, their new signal structure $\mathcal{X}'$ will have normalised posterior beliefs:

$$s'_L = s_L - \delta \qquad\qquad s'_H = s_H$$

where $\delta > 0$ is small. Whose eventual outcomes does this affect?

We can see this most clearly by interpreting this improvement in evaluators' information as showing them an additional *auxiliary signal* $\hat{\mathcal{X}}$ about the applicant's quality, rather than replacing $\mathcal{X}$ wholesale with $\mathcal{X}'$. We engineer $\hat{\mathcal{X}}$ carefully so that it *completes* the information $\mathcal{X}$ provides to evaluators to what $\mathcal{X}'$ can. An evaluator observes the realisation of $\hat{x}$ *only* she first observes $x = s_L$. In turn, the new signal $\hat{x}$ is also binary, with possible realisations $\{\hat{s_L}, \hat{s_H}\}$. Conditional on θ, it is independent from x and has distribution:

"with distribution" right language?

$$\hat{p_H}(\hat{s_H}) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p_L}(\hat{s_H}) = \varepsilon \times \frac{s_L}{1 - s_L}$$

ε, like δ, is a small positive number. It is related intimately to δ, as I explain shortly.

If the evaluator observes the signal $x = s_L$ followed by $\hat{x} = \hat{s_H}$, her belief that the applicant has *High* quality jumps to what it would be had she observed $x = s_H$ straightaway. Note this from the likelihood ratio for this signal pair:

$$\frac{\mathbb{P}\left(x = s_L, \hat{x} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(x = s_L, \hat{x} = \hat{s_H} \mid \theta = L\right)} = \frac{s_L}{1 - s_L} \times \frac{\frac{s_H}{1 - s_H}}{\frac{s_L}{1 - s_L}} = \frac{s_H}{1 - s_H}$$

If she observes $\hat{x} = \hat{s_L}$ however, she grows yet more pessimistic that the applicant has *High* quality. Specifically, the joint observation $(x, \hat{x}) = (s_L, \hat{s_L})$ has the likelihood ratio:

$$\frac{\mathbb{P}\left(x = s_L, \hat{x} = \hat{s_L} \mid \theta = H\right)}{\mathbb{P}\left(x = s_L, \hat{x} = \hat{s_L} \mid \theta = L\right)} = \underbrace{\frac{s_L}{1 - s_L} \times \frac{1 - \frac{s_H}{1 - s_H} \times \varepsilon}{1 - \frac{s_L}{1 - s_L} \times \varepsilon}}_{(L, \hat{L})} < \frac{s_L}{1 - s_L}$$

This likelihood ratio labeled $(L, \hat{L})$ above continuously and monotonically decreases as ε varies from 0 to $\frac{1 - s_H}{s_H}$. We can thus choose ε to equate $(L, \hat{L})$ with the likelihood ratio for the signal $x' = s'_L$, labeled (L') below:

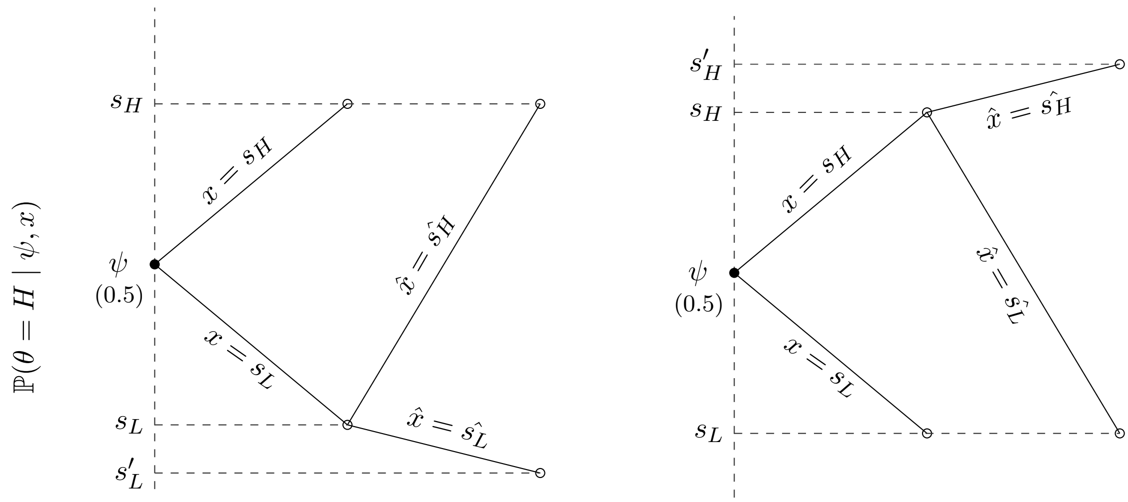
$$\frac{\mathbb{P}\left(x' = s'_L \mid \theta = H\right)}{\mathbb{P}\left(x' = s'_L \mid \theta = L\right)} = \underbrace{\frac{s_L - \delta}{1 - (s_L - \delta)}}_{(L')}$$

Note that the value of ε which equates the likelihood ratios $(L, \hat{L})$ and (L') is then continuous and strictly increasing as a function of δ.

When the likelihood ratios $(L, \hat{L})$ and (L') are equal, the information an evaluator can extract about the applicant's quality from the signal pair $(x, \hat{x})$ is equivalent to the information $\mathcal{X}'$ would provide. Eventually observing a high signal in the former – either $x = s_H$ or $(x, \hat{x}) = (s_L, \hat{s_H})$ – carries the same evidence for *High* quality as observing $x' = s'_H$ does. Similarly, observing only low signals – the pair $(x, \hat{x}) = (s_L, \hat{s_L})$ – carries the same evidence for *Low* quality as $x' = s'_L$. Thus for a given interim belief, the distribution of an evaluator's posterior beliefs about θ are the same whether she observes the signal tuple $(x, \hat{x})$ or just x'. This construction is illustrated in the left panel of Figure 3 .

We wish to compare applicants' eventual outcomes when evaluators approve upon high signals and reject upon low ones under both signal structures. Receiving the information contained in $\mathcal{X}'$ through this procedure is no obstacle to implementing an equivalent decision rule. Whenever they observe an equivalent of $x' = s'_H$ – either $x = s_H$ or $(x, \hat{x}) = (s_L, \hat{s_H})$, evaluators simply approve. Upon an equivalent of $x' = s'_L$ though – $(x, \hat{x}) = (s_L, \hat{s_L})$ – they reject.

One can interpret this reformulation of evaluators' improved information as a "second inspection" of their initial "rejection" pile. Now, an evaluator re-assesses applicants initially placed in



(a) improving $\theta = L$ evidence (b) improving $\theta = H$ evidence

Figure 3: Improving Binary Signals

the “rejection” pile upon a low signal. Upon this second assessment, she either concludes that she erred and should have placed him in the “approval” pile, or reinforces her verdict that the applicant must have *Low* quality.

This reformulation exposes whose eventual outcomes vary between $\mathcal{X}$ and $\mathcal{X}'$, or when evaluators observe the auxiliary signal $\hat{x}$ after x. Applicants approved by some evaluator under $\mathcal{X}$ are also approved under the pair $(x, \hat{x})$, after the same $x = s_H$ signal. Applicants rejected by all n evaluators under $\mathcal{X}$ however, get n *second chances.* These *marginal admits* reverse some of their evaluators' initial rejection verdicts by receiving a $\hat{x} = \hat{s_H}$ signal in their second assessment, saving themselves of a rejection.

How approving these *marginal admits* under $(x, \hat{x})$ affects evaluators' payoffs depends on how likely such an applicant is to have *High* quality. Any and all information evaluators could hope to scour about this is contained in the collection of signal pairs $\{(x_i, \hat{x}_i)\}_{i=1}^n$ they would observe if he visited them all. He was initially rejected in unison; so all evaluators observed $x = s_L$ initially. As he later reversed some of these rejection verdicts, at least one of them later observed a high signal, $\hat{x} = \hat{s_H}$. How many?

For small δ, and therefore ε, the answer is *almost surely, just the one.* The probability that any k evaluators observe $\hat{x} = \hat{s_H}$ is proportional to ε^k. So, as ε shrinks to 0, the probability that any $k > 1$ evaluators observed $\hat{x} = \hat{s_H}$ in their second assessment vanishes in favour of the probability that only one did. He is the *most adversely selected admit*; approved only by his *last* evaluator.

Whether approving this marginal admit raises or hurts evaluators' payoffs then depends on whether this single $\hat{x} = \hat{s_H}$ signal is strong enough to justify his approval despite the remaining

$n-1$ $\hat{x}=\hat{s_L}$ signals:

$$\begin{aligned}
&\frac{\mathbb{P}\left(\theta=H \mid \; n-1 \; \hat{x}=\hat{s_L} \text{ signals and one } \hat{x}=\hat{s_H}\right)}{\mathbb{P}\left(\theta=L \mid \; n-1 \; \hat{x}=\hat{s_L} \text{ signals and one } \hat{x}=\hat{s_H}\right)} \\
&= \lim_{\delta\to 0} \frac{\rho}{1-\rho} \times \left(\frac{\hat{s_L}}{1-\hat{s_L}}\right)^{n-1} \times \frac{s_H}{1-s_H} \\
&= \frac{\rho}{1-\rho} \times \underbrace{\left(\frac{s_L}{1-s_L}\right)^{n-1}}_{n-1 \text{ low signals}} \times \underbrace{\frac{s_H}{1-s_H}}_{\text{a single high signal}} \lesseqgtr \frac{c}{1-c}
\end{aligned}$$

When the LHS above exceeds the RHS, this marginal admit is sufficiently likely to have *High* quality, despite his $n-1$ low signals. *Adverse selection poses no threat* in this case: an evaluator who observes a high signal is happy to approve the applicant with no interest in his predecessors' evaluations. Strengthening evidence for *Low* quality – decreasing s_L – however, eventually sinks the LHS below the RHS. The evidence $n-1$ low signals carry for *Low* quality becomes too daunting to justify an approval at cost c thereafter, despite the single high signal an evaluator observed. I denote this tipping point for s_L below which adverse selection starts posing a threat, and such marginal admits start hurting evaluators, as s_L^{as}.

Definition 1. For a binary signal structure $\mathcal{X}$ with given strength of evidence for *High* quality s_H, s_L^{as} is the *strongest* possible evidence for *Low* quality where adverse selection poses no threat:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

Note that increasing the approval cost c or the number of evaluators n brings this tipping point forward; adverse selection becomes a threat earlier. Increasing evaluators' prior ρ and s_H pushes it back.

As the reader might be anticipating, s_L^{as} plays an important role in characterising the threshold Theorem 1 pointed at. Indeed, it is no coincidence that Figure 2 depicts evaluators' payoffs as decreasing precisely when s_L falls below s_L^{as}. Recall that in the unique equilibria whose payoffs it tracks, evaluators approve upon high signals, and reject upon low ones granted this yields positive payoffs. Hence, precisely these marginal admits determine how their payoffs respond to stronger evidence for *Low* quality.

Equilibrium dynamics might differ in general, distinguishing this threshold from s_L^{as}. If, for instance, evaluators approve all applicants anyway in the original equilibrium, stronger evidence for *Low* quality cannot bring about marginal admits. In contrast, it might push evaluators to a more selective equilibrium, even. Such equilibria where evaluators approve anyone exist whenever evidence for *Low* quality is too weak: absent adverse selection, evaluators have no reason to reject an applicant upon a low signal. I denote this second tipping point for s_L as s_L^{mute}.

Definition 2. For a binary signal structure $\mathcal{X}$, s_L^{mute} is the strongest possible evidence for *Low* quality where all applicants are approved in the most embracive equilibrium:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

s_L^{mute} rises with the approval cost c and falls with evaluators' prior ρ. Note that it need not be below 0.5; there might be no feasible value for s_L where the most embracive equilibrium sees all applicants approved. In fact, it never is when $c \geq \rho$.

Once this equilibrium ceases to be the most or least selective one, the effect of decreasing s_L further on the payoffs at these equilibria is indeed determined by the *marginal admit*. The most selective equilibrium switches earlier than the least selective equilibrium. The least selective equilibrium remains to see all applicants approved until s_L finally falls below s_L^{mute}.

Proposition 4. Let $\mathcal{X}$ be a binary signal structure with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. Evaluators' payoffs across the most embracive equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < \min\{s_L^{\text{mute}}, s_L^{as}\}$.

Similarly, there exists a threshold $s_L^{\dagger} \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$, such that their payoffs across the most selective equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq s_L^{\dagger}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < s_L^{\dagger}$.

Can you pin this threshold down? If not, why?

We can interpret a marginal strengthening of evidence for *High* quality analogously. This time, consider the signal structure $\mathcal{X}'$ with normalised posterior beliefs:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small $\delta > 0$. This time, evaluators will observe the realisation of an auxiliary signal $\hat{x}$ *only if* they first observe $x = s_H$. The complementary signal $\hat{x}$, conditionally independent from x as before, has the distribution:

$$\mathbb{P}\left(\hat{x} = \hat{s_L} \mid \theta = H\right) = \varepsilon \times \frac{s_L}{1-s_L} \qquad\qquad \mathbb{P}\left(\hat{x} = \hat{s_L} \mid \theta = L\right) = \varepsilon \times \frac{s_H}{1-s_H}$$

We choose ε so that the pair $(x, \hat{x})$ provides the same information about quality as $\mathcal{X}'$:

$$\frac{\mathbb{P}\left(x = s_H, \hat{x} = \hat{s_H} \mid \theta = H\right)}{\mathbb{P}\left(x = s_H, \hat{x} = \hat{s_H} \mid \theta = L\right)} = \frac{s_H}{1-s_H} \times \frac{1 - \varepsilon \times \frac{s_L}{1-s_L}}{1 - \varepsilon \times \frac{s_H}{1-s_H}} = \frac{s_H + \delta}{1 - s_H - \delta}$$

As before, ε is a continuous and increasing function of δ. The right panel of Figure 3 illustrates.

Under this signal pair $(x, \hat{x})$, observing consecutive high signals $(s_H, \hat{s_H})$ elevates the evaluator's belief about the applicant's quality as observing $x' = s'_H$ would. The evaluator then approves. If he sees any low signal however, be it s_L or $\hat{s_L}$, his belief sinks as it would had he observed $x' = s'_L$. He rejects.

As before, we can interpret evaluators' observations of the auxiliary signal as a "second inspection", this time of their initial "approval" pile. Each evaluator re-assesses applicants she initially placed in her approval pile upon a high signal. Consecutively, she either concludes that she erred and that her applicant should have been in the "rejection" pile instead, or reinforces her conviction that the applicant likely has *High* quality.

This reformulation exposes the new group of applicants whose eventual outcomes switch when we strengthen evidence for *High* quality. An applicant who would be rejected by all evaluators under $\mathcal{X}$ faces the same fate under $(x, \hat{x})$: his initial $x = s_L$ signals still lead to rejections. But an applicant who previously would be approved by some evaluator faces a renewed threat of rejection by all. Any initial $x = s_H$ signal he had can be overturned by a $\hat{x} = \hat{s_L}$ signal now. Unlucky enough, and he might overturn all his initial high signals, being left with nothing but rejections.

How pushing this *marginal reject* out affects evaluators' payoffs depends again on how likely he is to have *High* quality. As before, all and any information about this is contained in the signals his evaluators would observe for him if he visited them all. Inferring what these signals must be is now easier. As all evaluators eventually rejected him, they all must have observed low signals; either $x = s_L$ or $\hat{x} = \hat{s_L}$.

If low signals indeed lead to a rejection in equilibrium, pushing this marginal reject out is sure to raise evaluators' payoffs. Intuitively, equilibrium behaviour shows that the *fear* of adverse selection suffices to keep an evaluator from an applicant upon a low signal. So it can certainly not be optimal to approve an applicant who, in fact, *is* the most adversely selected one, upon a low signal.

If a low signal *already* leads to a rejection in equilibrium, this marginal reject is *sure* to raise evaluators' expected payoffs. When evaluators find rejecting upon a low signal optimal, learning that *all* evaluators saw low signals can only strengthen this conviction.

3.2 General Discrete Signals

The previous section uncovered how evaluators' equilibrium payoffs vary across binary signal structures. Such a signal structure is *more informative* whenever it provides *stronger evidence* either for *Low* or *High* quality. Theorem 1 showed that while stronger evidence for *High* quality always benefits evaluators, stronger evidence for *Low* quality *eventually* harms them.

In many settings of interest, however, the evaluators in concern have richer signal structures.

Traders of financial assets and derivatives, for instance, might get recommendations of varying levels of strength, such as "Strong Sell", "Sell", "Buy", and "Strong Buy". Similarly, a bank's credit scoring algorithm might output varying probabilities that the loan seeker will default, rather than simply summarising this information as "Good" or "Bad". These highlight the importance of extending our characterisation in Theorem 1 to improvements of such richer signal structures. ~~This is precisely the present section's aim. Its main result, Theorem TWO, characterises the effect of more information on evaluators' equilibrium payoffs, for any *arbitrary* discrete signal structure they might hold.~~

Evidence for this

When studying improvements of binary signals, I introduced the idea of auxiliary signals engineered to replicate any given improvement. This construction helped pin down the applicants whose eventual outcomes a given improvement affects. I now formalise and generalise this idea, using *local mean preserving spreads* of a signal structure. Local mean preserving spreads are the key to identify who the applicants affected by an *arbitrary* Blackwell improvement are.

Definition 3 (Local Mean Preserving Spread)**.** Let p and p' be the normalised posterior densities for the signal structures $\mathcal{X}$ and $\mathcal{X}'$. Additionally, let $s_1 < s_2 < ... < s_M$ be the elements of $S \cup S'$; the joint support of $\mathcal{X}$ and $\mathcal{X}'$. If there exists some $i \in \{2, ..., M-1\}$ such that:

$$p'(s_{i-1}) \geq p(s_{i-1}) \qquad 0 = p'(s_i) \leq p(s_i) \qquad p'(s_{i+1}) \geq p(s_{i+1})$$

$$p'(s_j) = p(s_j) \quad \text{for all } j \notin \{i-1, i, i+1\}$$
$$\sum_{i=1}^{M} s_i \times p'(s_i) = \sum_{i=1}^{M} s_i \times p(s_i)$$

I say $\mathcal{X}'$ *differs from* $\mathcal{X}$ *by a local mean preserving spread* (*at* s_i).

the original definition has F not $\mathcal{X}$, is that a problem?

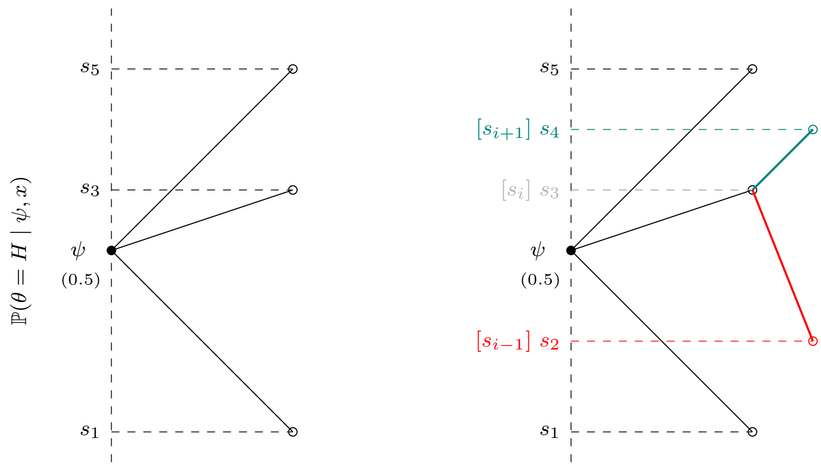


Figure 4: A Local Mean Preserving Spread

Much like an ordinary mean preserving spread (Rothschild and Stiglitz, 1970[8]), a *local mean preserving spread* distributes probability away from an *origin* point to two *destination* points, one above and one below it. It does so while preserving the mean of the original distribution. Crucially however, a mean preserving spread is *local* if and only if the destination points are the immediate neighbours of the origin point[9]. In other words, neither the original nor the resulting distribution assign positive probability to any other point between the origin and the two destination points[10].

The auxiliary signals I introduced in the previous section create such local mean preserving spreads. To strengthen evidence for *Low* quality, for instance, the auxiliary signal spreads all the probability mass assigned to the origin point s_L to the neighbouring destination points s_H and s'_L, where $s_H > s_L > s'_L$.

Local mean preserving spreads are simple ways to Blackwell improve signal structures. Nonetheless, they are powerful enough to characterise *any* Blackwell improvement of a signal structure, too. Remark 1, slightly refining the classic result in Rothschild and Stiglitz, 1970[11], states that we can decompose *any* Blackwell improvement of a discrete signal structure into a sequence of finitely many *local* mean preserving spreads. So, I focus on the effect of such local mean preserving spreads on evaluators' equilibrium payoffs.

Remark 1. [Müller and Stoyan, 2002, Theorem 1.5.29] $\mathcal{X}'$ is Blackwell more informative than $\mathcal{X}$ if and only if there is a finite sequence $\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_k$ such that $\mathcal{X}_1 = \mathcal{X}$, $\mathcal{X}_k = \mathcal{X}'$, and $\mathcal{X}_{i+1}$ differs from $\mathcal{X}_i$ by a local mean preserving spread.

As before, we can interpret a local MPS of s_i as a re-evaluation of the applicant upon that signal realisation; I illustrate this in Figure **??**. This re-evaluation might change an evaluator's initial verdict upon s_i. As a result, whether applicants face a renewed rejection threat or get a second approval chance hinges on what this initial verdict would have been.

If the signal s_i originally led to an approval, applicants who initially owed an approval to this signal might get rejected upon this re-evaluation. Consequently, an applicant who would initially be approved by some evaluator might now get rejected by all. As Theorem 2 states, pushing such applicants out always benefits evaluators. Intuitively, upon his re-evaluation, this applicant falls into the rejection region of *every* evaluator. Given her signal, *no* evaluator can

[8]Rothschild and Stiglitz, 1970 describe mean preserving spreads through *four* points in the support of the distribution. Here, I describe them through *three*. This is without loss of generality. In fact, mean preserving spreads were first characterised by Muirhead, 1900 in the context of majorisation (transformations T), with *three* points. Rasmusen and Petrakis, 1992 show formally that these the three or four point characterisations of MPS are in fact equivalent.

[9]The reader will notice that this statement is ill-defined unless the signal structure is discrete. To the best of the author's knowledge, no counterpart for *local mean preserving spreads* exist for, say, atomless signal structures.

[10]The attentive reader will also realise that this definition also requires that *all* probability mass be spread away from the origin point. This difference is insignificant in our current setting.

[11]This result appeared in earlier work related to majorisation. See Muirhead, 1900, whose textbook exposition appears in Hardy et al., 1952.

justify approving him under the *threat* of adverse selection. The evaluator's fear of adverse selection and the rejection it motivates are, in fact, valid. Her approval would indeed stand alone among *all* her peers' rejections.

If the signal s_i originally led to a rejection, however, this re-evaluation presents an applicant who were rejected with it a *second chance.* Some of his initial rejections owing to s_i might be overturned upon a positive revision of his signal to s_{i+1}. This positive re-evaluation is good news about his quality, but it still comes against the potential backdrop of some – unchanged – poor evaluations. The evaluator who approves upon her re-assessment benefits if the signal s_{i+1} is strong enough to counter this adverse selection threat.

If the signal s_{i+1} is strong enough to overwhelm rejections even by *all* $n-1$ evaluators, our evaluator faces no such threat. She would find it beneficial to approve the applicant upon the signal s_{i+1}, regardless of the number of his past rejections. For a fixed strategy σ all evaluators use, I say *adverse selection poses no threat at signal* s_{i+1} when so.

Definition 4. Fix the signal structure $\mathcal{X}$ and a monotone strategy σ. I say *adverse selection poses no threat at signal s* if:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{X})}{r_L(\sigma;\mathcal{X})}\right)^{n-1} \times \frac{s}{1-s} > \frac{c}{1-c}$$

~~If adverse selection poses no threat at signal s_{i+1}, the applicant approved upon a revision of s_i to s_{i+1} increases evaluators' payoffs. How many rejections other evaluators would issue is irrelevant; these are insufficient to overwhelm the good news s_{i+1} carries. Strikingly however, this is also *necessary* for this admit to improve evaluators' payoffs. *Whenever* adverse selection poses a threat at signal s_{i+1}, evaluators are worse off due to these admits brought about with the local spread of s_i.~~

Strikingly, *any* threat of adverse selection at s_{i+1} suffices for evaluators to be worse off with the admit s_i's spread brings about. The argument parallels the one describing how *marginal admits* affect evaluators' payoffs in the binary case. Consider, again, a "small" spread which revises the signal s_i to s_{i+1} with a vanishingly small probability. The probability that our applicant, previously rejected by all evaluators, overturned *multiple* s_i signals positively is vanishingly small. Hence, the evaluator revising her rejection to an approval *does* in fact suffer from the most severe form of adverse selection.

Theorem 2. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $x = s_i$ leads to approvals under σ.

2. *weakly lower* than under σ:

i if $x = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{X}$ and σ.

Whether adverse selection poses a threat at s_{i+1} depends on evaluators' precise equilibrium strategy. This might concern an analyst with no knowledge of this strategy when she wants to judge whether a given spread guarantees to harm evaluators. This is less alarming than it first appears; the analyst can locate both the most and least selective equilibrium strategies precisely with the algorithm I present to prove equilibrium existence in Proposition 1. Nonetheless, Proposition 5 offers a stronger sufficient condition for the most selective equilibrium. It strengthens the notion of adverse selection threat at s_{i+1} Theorem 2 uses to one which depends only on the signals s_i and s_{i+1}.

Proposition 5. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $x = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proposition 5 still requires knowing that s_i is a rejection signal in the most selective equilibrium. This too, can be strengthened to a sufficient condition that relies only on the signal realisation. Recall from the previous subsection that signal realisations below s_L^{mute} ought to be rejected in any equilibrium. Corollary 6 uses this fact:

Corollary 6. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

4 Evaluators with Regulated Strategies

This section will be about a social planner who can dictate evaluators' strategies to maximise their total payoffs can achieve.

1. First result: a planner who wishes to dictate the same strategy to every evaluator can *never benefits* from evaluators having information that would be harmful in equilibrium.

   Reason: dictating the same strategy to every evaluator $\equiv$ giving each a Blackwell inferior experiment. So at best, you can do as well as you did with a Blackwell inferior experiment.

   In the binary case, you can do only as well as the equilibrium outcome of some inferior experiment. I don't yet know if this is true in general.

2. Result 1.5: with binary signals, an evaluator can simply raise the approval cost to implement this symmetric optimum.

3. Second result: a planner who can dictate *different* strategies to evaluators can break this. Example: binary signal with $s_L < s_L^{\text{mute}}$. Optimal solution: k evaluators approve upon s_H and reject upon s_L, remaining $n-k$ reject always. Evaluators' payoffs are then monotone with any improvement in information.

4. Third result: the above is equivalent to giving evaluators' full information about history. In general, not optimal (I can maybe cite Makarov and Plantin, 2023 on this, or give my own example).

5. Fourth result (follow up to two): conjectured. In general, evaluators' payoffs are not monotone in information even when they have full history information. I will cook an example for this.

5 Information with Arbitrary History Signals

This Section is about the generalisation to arbitrary history signals that I wish to make. I conjecture that Theorem 1 is going to generalise in some form to *any* history signal. I do not wish to get a fully general result in the spirit of Theorem 2; i.e. will restrict myself to binary in this section. This is because with partially (or fully) observed past decisions, Blackwell improvements might behave weirdly simply because decisions are censored data à la classic social learning anyway.

6 Proof Appendix

6.1 Useful Definitions and Notation

In what follows, I occasionally express beliefs in *likelihood form* for convenience. The reader can verify the following with ease: where $\frac{\psi}{1-\psi}$ is the likelihood ratio of an evaluator's belief about quality before he observes x_i, $\frac{s}{1-s} \times \frac{\psi}{1-\psi}$ is the likelihood ratio of her belief *after* observing $x_i = s$. It is (strictly) optimal for her to approve the applicant when this ratio (strictly) exceeds $\frac{c}{1-c}$.

Some strategies require evaluators to randomise when approving their applicant upon observing a particular signal realisation. To facilitate the discussion when so, I let evaluators formulate a *score* for the applicant they receive, based on the signal realisation x_i they observe.

Definition 5. An applicant's score from evaluator i is the pair (x_i, u_i), where $u_i \overset{IID}{\sim} U[0,1]$. His *score profile* $Z := \{(x_1, u_1), ..., (x_n, u_n)\}$ is the set of scores he would receive from each evaluator, were he to visit all.

Evaluator i approves the applicant upon his visit if and only if $\sigma_i(x_i) \leq u_i$.

Following the terminology of the applicant's *score profile*, I call the analogous set $\mathcal{X} := \{x_1, ..., x_n\}$ his *signal profile.*

6.2 Omitted Results

Lemma 7. Let there be a *single* evaluator who approves the applicant if and only if $\mathbb{P}(\theta = H \mid x) \geq c$. Her expected payoff under signal structure $\mathcal{X}'$ is greater than under $\mathcal{X}$ *regardless of her approval cost* $c \in (0,1)$ and prior belief $\rho \in (0,1)$ if and only if $\mathcal{X}' \underset{B}{\succeq} \mathcal{X}$.

Proof. Sufficiency is due to Blackwell's Theorem (Blackwell and Girshick, 1954, Theorem 12.2.2). I show necessity by fixing a prior belief ρ for the evaluator.

Let $F(.)$ and $F'(.)$ be the posterior distributions $\mathcal{X}$ and $\mathcal{X}'$ induce, respectively, for this prior belief ρ:

$$F(s_i) = (1-\rho) \times \sum_{j=1}^{i} p_L(s_j) + \rho \times \sum_{j=1}^{i} p_H(s_j)$$
$$F'(s_i) = (1-\rho) \times \sum_{j=1}^{i} p'_L(s_j) + \rho \times \sum_{j=1}^{i} p'_H(s_j)$$

The evaluator's expected payoff under $\mathcal{X}$ is then:

$$\int_c^1 (\omega - c) dF(\omega) = \int_c^1 \omega dF(\omega) - c \times (1 - F(c)) = (1-c) - \int_c^1 F(\omega) d\omega$$

Of course, an analogous expression gives her expected payoff under $\mathcal{X}'$. Therefore, for her expected payoffs under $\mathcal{X}'$ to exceed those under $\mathcal{X}$ for any $c \in (0,1)$, we must have:

$$\int_c^1 \big(F(\omega) - F'(\omega)\big)\, d\omega \geq 0$$

which is equivalent to $\mathcal{X}'$ being Blackwell more informative than $\mathcal{X}$[12]

□

6.3 Omitted Proofs

Proposition 1. Let $p_H \neq p_L$. Where Σ is the set of equilibrium strategies:

1. *an equilibrium exists;* $\Sigma \neq \emptyset$,
2. *all equilibrium strategies are monotone;* for any $\sigma^* \in \Sigma$ and $s' > s$, $\sigma^*(s) > 0$ implies $\sigma^*(s') = 1$,
3. *all equilibria exhibit adverse selection;* $\psi^* \leq \rho$ for any ψ^* induced by an equilibrium strategy,
4. Σ is compact. Moreover, elements of Σ are pointwise totally ordered.

Proof. Existence: I provide an algorithm which locates an equilibrium. The same algorithm can be used to locate *every* equilibrium. Without loss of generality, add the signals $s_0 = 0$ and $s_{m+1} = 1$ to the set $S = \{s_1, s_2, ..., s_m\}$, where $p_\theta(s_{m+1}) = 0$ for $\theta \in \{L, H\}$. Define σ_j to be the strategy which approves the applicant if and only if $x \geq s_j$:

$$\sigma_j(x) = \begin{cases} 0 & x < s_j \\ 1 & x \geq s_j \end{cases}$$

Note that the strategy σ_{m+1} approves no applicant, and σ_1 approves all applicants. Lastly, denote the interim belief σ_j induces as $\psi_j := \Psi(\sigma_j, \mathcal{X})$.

If the inequality:

$$\frac{s_j}{1 - s_j} \times \frac{\psi_j}{1 - \psi_j} \geq \frac{c}{1 - c} \geq \frac{s_{j-1}}{1 - s_{j-1}} \times \frac{\psi_j}{1 - \psi_j} \tag{6.1}$$

is satisfied for some $j \in \{1, 2, ..., m+1\}$, (σ_j, ψ_j) is an equilibrium.

[12] See Müller and Stoyan, 2002, Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica, 2016.

Say this holds for no index $j \in \{1, 2, ..., m+1\}$. Then, we must have:

$$\frac{s_m}{1-s_m} \times \frac{\psi_{m+1}}{1-\psi_{m+1}} \geq \frac{s_m}{1-s_m} \times \frac{\psi_m}{1-\psi_m} > \frac{c}{1-c} > \frac{s_1}{1-s_1} \times \frac{\psi_1}{1-\psi_1}$$

The first inequality holds as $\psi_{m+1} = \rho \geq \psi_m$. The remaining part of the inequality follows from inequality 6.1 being violated both for $j = 1$ and $j = m + 1$. Now take the lowest index j^* for which:

$$\frac{s_{j^*}}{1-s_{j^*}} \times \frac{\psi_{j^*}}{1-\psi_{j^*}} \geq \frac{c}{1-c} > \frac{s_{j^*-1}}{1-s_{j^*-1}} \times \frac{\psi_{j^*-1}}{1-\psi_{j^*-1}}$$

Since Ψ is continuous in $\sigma(s_{j^*})$, we can find a strategy σ^* such that $\sigma_{j^*} \leq \sigma^* \leq \sigma_{j^*-1}$ which induces a interim belief ψ^* such that:

$$\frac{s_{j^*}}{1-s_{j^*}} \times \frac{\psi^*}{1-\psi^*} = \frac{c}{1-c}$$

and thus, (σ^*, ψ^*) constitutes an equilibrium.

Finally, finding all equilibria σ^* such that $\sigma_j \leq \sigma^* \leq \sigma_{j-1}$ for all $j \in \{1, 2, ..., m+1\}$ for which the inequality:

$$\frac{s_j}{1-s_j} \times \frac{\psi_j}{1-\psi_j} \geq \frac{c}{1-c} > \frac{s_{j-1}}{1-s_{j-1}} \times \frac{\psi_{j-1}}{1-\psi_{j-1}}$$

holds exhausts the set of all equilibria.

Monotonicity: Take some strategy σ that's optimal against the interim belief it induces $\psi = \Psi(\sigma; \mathcal{X})$. Optimality demands that $\sigma(s') \geq \sigma(s)$ for any $s', s \in S$ s.t. $s' > s$. Additionally, if $\sigma(s) \in (0, 1)$, it must be that:

$$\frac{s_H}{1-s_H} \times \frac{\psi}{1-\psi} > \frac{s}{1-s} \times \frac{\psi}{1-\psi} = \frac{c}{1-c} > \frac{s_L}{1-s_L} \times \frac{\psi}{1-\psi}$$

for any $s_H > s > s_L$, thus $\sigma(s_H) = 1$ and $\sigma(s_L) = 0$.

Mention once where this update comes from.

Adverse Selection: Take a *monotone* strategy σ. Since $p_H(.)$ likelihood ratio dominates $p_L(.)$, it also first order stochastically dominates it[13]. Therefore, $r_L(\sigma; \mathcal{X}) \geq r_H(\sigma; \mathcal{X})$ and $\Psi(\sigma; \mathcal{X}) \leq \rho$.

Totally Orderedness and Compactness of the Equilibrium Set: Let Σ be the set of equilibrium strategies. Totally orderedness follows automatically since every element of Σ is a monotone strategy.

Since the set of *all* strategies is a bounded subset of $\mathbb{R}^m$, we only need to show that Σ is *closed* to establish compactness. So take a sequence $\{\sigma^n\} \in \Sigma$ s.t. $\sigma_n \to \sigma^*$. Note that all σ^n must be monotone strategies. Denote the respective interim beliefs as $\psi^n := \Psi(\sigma^n; \mathcal{X})$ and $\psi^* = \Psi(\sigma^*; \mathcal{X})$.

[13] Theorem 1.C.1 in Shaked and Shanthikumar, 2007.

We would like to prove $\sigma \in \Sigma$. Wlog, I restrict attention to cases where $\{\sigma^n\}$ is either an increasing or a decreasing sequence; otherwise one can simply take a monotone subsequence of $\{\sigma^n\}$ which converges to the same limit. I take the case where $\{\sigma^n\}$ is a decreasing sequence here, the complementary case is analogous.

Let first $\sigma = \sigma_j$ for some $j \in \{1, 2, ..., m\}$. Then, there is some $N \in \mathbb{N}$ such that for all $n \geq N$, $1 > \sigma^n(s_{j-1}) \to 0$ and thus:

$$\frac{\psi^n}{1-\psi^n} \times \frac{s_{j-1}}{1-s_{j-1}} = \frac{c}{1-c}$$

and so by the continuity of Ψ in $\sigma(s_{j-1})$:

$$\frac{\psi^*}{1-\psi^*} \times \frac{s_j}{1-s_j} > \frac{c}{1-c} = \frac{\psi^*}{1-\psi^*} \times \frac{s_{j-1}}{1-s_{j-1}}$$

thereby establishing that σ^* is an equilibrium strategy. The proof when $\sigma^*(s_j) \in (0,1)$ for some $j \in \{1, 2, ..., m\}$ is similar.

$\square$

The following Lemma, of independent interest itself, will be useful when proving Proposition 2.

Lemma 8. Take three monotone strategies $\sigma'' > \sigma' > \sigma$. If $\Pi(\sigma'; \mathcal{X}) \leq \Pi(\sigma; \mathcal{X})$, then $\Pi(\sigma''; \mathcal{X}) \leq \Pi(\sigma'; \mathcal{X})$.

Proof. For the three strategies $\sigma'' > \sigma' > \sigma$, consider three sets $A, A', A'' \subset (S \times [0,1])^n$ where the applicant's score profile might lie:

$$\begin{aligned} &Z \in A && \text{if } Z \text{ is eventually approved under } \sigma'' \text{ but not } \sigma \\ &Z \in A' && \text{if } Z \text{ is eventually approved under } \sigma' \text{ but not } \sigma \\ &Z \in A'' && \text{if } Z \text{ is eventually approved under } \sigma'' \text{ but not } \sigma' \end{aligned}$$

Notice that $A' \cap A'' = \emptyset$ and $A' \cup A'' = A$. We can write the difference between the sum of evaluators' payoffs under different strategies as:

$$\Pi(\sigma'; \mathcal{X}) - \Pi(\sigma; \mathcal{X}) = \mathbb{P}\left(Z \in A'\right) \times \left[\mathbb{P}\left(\theta = H \mid Z \in A'\right) - c\right]$$

and:

$$\Pi(\sigma''; \mathcal{X}) - \Pi(\sigma'; \mathcal{X}) = \mathbb{P}\left(Z \in A''\right) \times \left[\mathbb{P}\left(\theta = H \mid Z \in A''\right) - c\right]$$

$\Pi(\sigma'; \mathcal{X}) \leq \Pi(\sigma; \mathcal{X})$ implies $\mathbb{P}(\theta = H \mid \mathbf{Z} \in A') \leq c$. But then we must have $\mathbb{P}(\theta = H \mid \mathbf{Z} \in A'') \leq c$, since $\mathbb{P}(\theta = H \mid Z \in A)$ is a convex combination of $\mathbb{P}(\theta = H \mid Z \in A')$ and $\mathbb{P}(\theta = H \mid Z \in A'')$, and:

$$\mathbb{P}(\theta = H \mid Z \in A) \geq \mathbb{P}\left(\theta = H \mid Z \in A \cap A''\right) = \mathbb{P}\left(\theta = H \mid Z \in A''\right)$$

Therefore, we have $\mathbb{P}(\theta = H \mid Z \in A'') \leq \mathbb{P}(\theta = H \mid Z \in A) \leq \mathbb{P}(\theta = H \mid Z \in A') \leq c$.

□

Proposition 2. Let σ^* be an equilibrium strategy, and σ be any other monotone strategy more embracive than σ^*. Evaluators' expected payoffs under σ^* exceed those under σ; $\Pi(\sigma^*; \mathcal{X}) \geq \Pi(\sigma; \mathcal{X})$.

Proof. Let Z be the applicant's *score profile*, as in Definition 5. Take an equilibrium strategy σ^* and a more embracive strategy σ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & \underline{s} \\ 0 & s \neq \underline{s} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. Now let $A \subset (S \times [0,1])^n$ be the set of score profiles which lead to rejections by all evaluators under σ^*, but an eventual approval under σ. Thus, $Z \in A$ if $x_i = \underline{s}$ and $\sigma(\underline{s}) \geq u_i > \sigma^*(\underline{s})$ for at least one evaluator $i \in \{1, ..., n\}$. Let $\#$ be the number of evaluators holding such scores for the applicant.

Only applicants' whose score profiles are in A change their eventual outcome from a rejection in σ^* to an approval in σ, thus:

$$\begin{aligned} \Pi(\sigma; \mathcal{X}) - \Pi(\sigma^*; \mathcal{X}) &= [\mathbb{P}(\theta = H \mid Z \in A) - c] \times \mathbb{P}(Z \in A) \\ &\propto \mathbb{P}(\theta = H \mid Z \in A) - c \end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the applicant's signal profile is in Z:

$$\mathbb{P}(\theta \mid Z \in A) = \sum_{i=1}^{n} \mathbb{P}(\theta = H \mid \# = i) \times \frac{\mathbb{P}(\# = i)}{\mathbb{P}(Z \in A)}$$

Now note:

$$\mathbb{P}(\# = i \mid \theta) = (p_\theta(\underline{s}))^i \times (1 - p_\theta(\underline{s}))^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}(\# = i) \propto \varepsilon^i$. Since $\mathbb{P}(Z \in A) = \sum_{i=1}^{n} \mathbb{P}(\# = i)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\# = i)}{\mathbb{P}(Z \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}(\theta = H \mid Z \in A) - \mathbb{P}(\theta = H \mid \# = 1) = 0$$

I conclude the proof by showing that $\mathbb{P}(\theta = H \mid \# = 1) < c$, and invoking Lemma 8.

$$\begin{aligned}
\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H \mid \# = 1)}{\mathbb{P}(\theta = L \mid \# = 1)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \frac{\mathbb{P}(\# = 1 \mid \theta = H)}{\mathbb{P}(\# = 1 \mid \theta = L)} \\
&= \lim_{\varepsilon \to 0} \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma; \mathcal{X})}{r_L(\sigma; \mathcal{X})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&= \frac{\mathbb{P}(\theta = H)}{\mathbb{P}(\theta = L)} \times \left(\frac{r_H(\sigma^*; \mathcal{X})}{r_L(\sigma^*; \mathcal{X})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\
&\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \leq \frac{c}{1 - c}
\end{aligned}$$

where $\psi^* = \Psi(\sigma^*; \mathcal{X})$ is the interim belief of the evaluators induced by σ^*. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{1 + r_H^* + \dots + (r_H^*)^{n-1}}{1 + r_L^* + \dots + (r_L^*)^{n-1}} \leq \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*; \mathcal{X})$. The last inequality is due to the fact that $\underline{s} \in S$ is optimally rejected under σ^*.

□

Theorem 1. Let $\pi(\sigma^*; \mathcal{X})$ be an evaluator's payoff in an extreme equilibrium under the binary signal structure $\mathcal{X}$ with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. $\pi(\sigma^*; \mathcal{X})$ is weakly:

a) increasing with the strength of evidence for $\theta = H$ (s_H),

b) increasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is above a threshold,

c) decreasing with the strength of evidence for $\theta = L$ (s_L^{-1}) when s_L is below that threshold.

I will use the five lemmata below, possibly of independent interest, to prove Theorem 1. Throughout, I denote the most selective equilibrium under the signal structure $\mathcal{X}$ as $\sigma_{\mathcal{X}}^{\text{sel}*}$. Similarly, $\sigma_{\mathcal{X}}^{\text{em}*}$ is the most embracive equilibrium. The subscript is dropped when the signal structure in question is obvious.

insert numbers!

Lemma 9. Let $\mathcal{X}$ be binary. $\Psi(\sigma; \mathcal{X})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

is strictly true here?

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

Proof. Part i:

Let $\sigma(F) \in (0,1)$ and $\sigma(S) = 1$. The interim belief ψ is then given by:

$$\begin{aligned}
\Psi(\sigma;\mathcal{X}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right) \\
&= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] \\
&= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]
\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every s_L signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}
\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) =& \mathbb{P}\left(\text{ev. was (i+1)}^{\text{th}} \text{ in order } \mid \text{applicant got i rejections}\right) \\
&\times \mathbb{P}\left(\text{applicant got i rejections}\right) \\
=& \frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i
\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus ψ increases, in $\sigma(s_L)$.

Part ii:

Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma;\mathcal{X}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma;\mathcal{X}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}
\Psi(\sigma;\mathcal{X}) &\propto \frac{1 + r_H + \ldots + r_H^{n-1}}{1 + r_L + \ldots + r_L^{n-1}} \\
&= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_H}{1 - r_L} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)} \\
&\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - \left(1 - p_H(s_H)\sigma(s_H)\right)^n}{1 - \left(1 - p_L(s_H)\sigma(s_H)\right)^n}
\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1 - s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1 - (r_H)^n}{1 - (r_L)^n}$$

The positive term is the likelihood ratio of one s_H signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since approvals only happen with s_H signals, the negative term strictly exceeds the positive term. Thus, $\Psi(\sigma;\mathcal{X})$ decreases in $\sigma(s_H)$.

$\square$

The Corollary below follows from Lemma 9: if $\mathcal{X}' \underset{B}{\succeq} \mathcal{X}$ where both signal structures are binary, adverse selection is stronger under $\mathcal{X}'$, if evaluators always (i) approve upon the high signal, and (ii) reject upon the low signal, under both signal structures.

Corollary 10. Let $\mathcal{X}'$ be more informative than $\mathcal{X}$, the strategies $\sigma'_{(0,1)}$ and σ_1 be $\sigma'_{(0,1)}(s'_L) = \sigma_1(s_L) = 0$ and $\sigma'_{(0,1)}(s'_H) = \sigma_1(s_H) = 1$. Then, $\Psi(\sigma';\mathcal{X}') \leq \Psi(\sigma;\mathcal{X})$.

Proof. I will only prove that the assertion holds when $s_L = s'_L$ but $s'_H > s_H$. The mirror case, which establishes the second part of the corollary, is analogous.

The proof will show that the outcome induced by σ under signal structure $\mathcal{X}$ can be replicated by $\tilde{\sigma}$ under signal structure $\mathcal{X}'$, where $\tilde{\mathcal{X}}(s_L) > 0$ and $\tilde{\mathcal{X}}(s_H) = 1$. Then, the conclusion follows from Lemma 9.

Take the pair $(\sigma,\mathcal{X})$. The probabilities that the applicant is rejected or approved upon a visit, conditional on θ, is given by:

$$\frac{\mathbb{P}(\sigma \text{ rejects } \mid \theta = H)}{\mathbb{P}(\sigma \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\sigma \text{ approves } \mid \theta = H)}{\mathbb{P}(\sigma \text{ approves } \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma},\mathcal{X}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}(\tilde{\sigma} \text{ rejects} \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ rejects } \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = H)}{\mathbb{P}(\tilde{\sigma} \text{ approves } \mid \theta = L)} = \frac{p'_H(s_H) + \tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H) + \tilde{\sigma}(s_L)p'_L(s_L)}$$

where the family of distributions $\{p'_\theta\}$ belong to $\mathcal{X}'$. It is easy to verify that this last fraction on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma;\mathcal{X})$.

This proves the corollary.

$\square$

Lemma 11. Let $\mathcal{X}$ be binary. There is no mixing at $x = s_L$ neither in $\sigma^{\text{sel}*}$ nor in $\sigma^{\text{emb}*}$; i.e. $\sigma^{\text{sel}*}(s_L), \sigma^{\text{em}*}(s_L) \in \{0,1\}$.

Proof. I start by showing $\sigma^{\text{em}*}(s_L) \in \{0,1\}$. Where s_L^{mute} is as it was defined in Definition **??**, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium. This is because $\psi = \rho$ at this induced equilibrium, thus approving upon the low signal is optimal. This is the most embracive equilibrium, since there is no strategy that's more embracive. When $s_L < s_L^{\text{mute}}$, any equilibrium σ must feature $\sigma(s_L) = 0$, since $\psi \leq \rho$.

Now consider $\sigma^{\text{sel}*}$. For contradiction, let $\sigma^{\text{sel}*}(s_L) > 0$. By Lemma 9, and an argument used while proving equilibrium existence in Proposition 1, there is then another equilibrium σ clarify!

where $\sigma(s_L) = 0$.

$\square$

Lemma 12 characterises evaluators' decisions upon seeing the low signal in the extreme equilibria, given how informative $\mathcal{X}$ is. Broadly, more informative signal structures push evaluators to reject upon the low signal under both equilibria.

Lemma 12. Let $\mathcal{X}$ be binary, with signal realisations s_L and s_H. Then:

$$\sigma^{\text{em}*}(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \sigma^{\text{sel}*}(s_L) = \begin{cases} 1 & s_H < s_H^{\dagger}(s_L) \\ 0 & s_H \geq s_H^{\dagger}(s_L) \end{cases}$$

where $s_H^{\dagger}(.)$ is an increasing function, and $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

Proof. Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1-s_L} \geq \frac{c}{1-c}$$

which proves the part for the most embracive equilibrium, combined with Lemma 11.

Let the strategy σ_1 be such that $\sigma_1(s_L) = 0$ and $\sigma_1(s_H) = 1$. The following is a necessary and sufficient condition for an equilibrium σ^* where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1; \mathcal{X})}{1 - \Psi(\sigma_1; \mathcal{X})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

Necessity follows from $\Psi(\sigma_1; \mathcal{X}) \geq \Psi(\sigma^*; \mathcal{X})$ due to Lemma 9. Sufficiency follows from the fact that an equilibrium always exists, and the condition above implies s_L must always be rejected in it. Due to Corollary 10, we know that this condition holds when s_H is weakly above some threshold $s_H^{\dagger}(s_L)$, increasing with s_L. The necessary and sufficient condition holds whenever s_L^{mute}, therefore $s_H^{\dagger}(s_L^{\text{mute}}) = 0.5$.

$\square$

Proof, Theorem 1: I prove Theorem 1 by establishing four facts:

1. The expected payoff in an equilibrium σ^* where $\sigma^*(s_L) = 0$ is higher than the expected payoff of approving all applicants.

   This follows directly from Proposition 2.

2. There is at most one equilibrium where $\sigma^*(s_L) = 0$.

   Let $\{\sigma_\alpha\}_{\alpha \in [0,1]}$ be the family of strategies where the low signal is rejected: $\sigma_\alpha(s_L) := 0$ and $\sigma_\alpha(s_H) := \alpha$. If:

$$\frac{\Psi(\sigma_1; \mathcal{X})}{1 - \Psi(\sigma_1; \mathcal{X})} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c}$$

σ_1 is the only equilibrium candidate among this family; the interim belief is higher under any lower α by Lemma 9. Otherwise, again by Lemma 9, there is at most one $\alpha \in [0,1]$ for which:

$$\frac{\Psi(\sigma_\alpha;\mathcal{X})}{1-\Psi(\sigma_\alpha;\mathcal{X})} \times \frac{s_H}{1-s_H} - \frac{c}{1-c} = 0$$

When such an α exists, σ_α is the only equilibrium candidate in this family. Under higher α, approving upon $x = s_H$ is not optimal. Under lower α, rejecting upon $x = s_L$ is not optimal. If the expression above is strictly negative for *any* α, then the only equilibrium candidate where the low signal is rejected is σ_0.

3. When an equilibrium $\sigma^* \in \{\sigma_\alpha\}_{\alpha\in[0,1]}$ where all low signals are rejected exists, the expected payoff in this equilibrium is given by $\pi_i(\sigma^*;\mathcal{X}) = \max\{0, \pi_i(\sigma_1;\mathcal{X})\}$.

   Above we showed that evaluators expect positive expected payoff (necessarily from approving an applicant) only when $\alpha = 1$. Otherwise, they either approve no applicant or are indifferent to rejecting those they do.

Theorem 1 then follows from our fourth claim:

Below, I use s_L^{as} but do not reiterate what it means.

4. $\max\{0, \Pi(\sigma_1;\mathcal{X})\}$ is:

   i weakly increasing in s_H whenever σ_α is an equilibrium strategy for some $\alpha \in [0,1]$,

   make sure the notation is either Π or π

   ii hump-shaped in s_L. As s_L falls, it is:

   - weakly increasing when $s_L \geq s_L^{as}$,
   - weakly decreasing when $s_L \leq s_L^{as}$

   where s_L^{as} is implicitly defined as:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

   for the signal structure $\mathcal{X}$.

Due to Lemma 12, both the most embracive and most selective equilibria shift once from the equilibrium where *all* s_L signals are approved to the one where *none* are approved, as the binary signal structure $\mathcal{X}$ becomes more informative. Due to the first fact laid out in the proof of this Theorem, this induces an increase in evaluators' expected payoff. Therefore, this last assertion about the shape of evaluators' payoffs in the equilibrium where the low signal is rejected concludes the proof.

Proof for the fourth claim:

Part i: Increasing s_H; i.e. the strength of evidence for $\theta = H$.

Let $\mathcal{X}$ and $\mathcal{X}'$ be two binary signal structures with respective signal realisations $\{s_L, s_H\}$ and $\{s'_L, s'_H\}$. Let $s'_L = s_L$ and $s'_H = s_H + \delta$ for $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma_1;\mathcal{X}') > \Pi(\sigma_1;\mathcal{X})$.

Step 1: Replicating $\mathcal{X}'$ with a signal pair $(x, \hat{x})$.

Rather than having evaluators observe one draw from the signal structure $\mathcal{X}'$, say an evaluator potentially observes *two* signal realisations; x and $\hat{x}$. She first observes x, a single draw from $\mathcal{X}$. If this signal realises as $x = s_L$, she observes no further information. If instead $x = s_H$, she observes another signal $\hat{x} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from the signal structure $\hat{\mathcal{X}}$. $\hat{x}$ has the following distribution, and is independent from x, conditional on θ:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(x, \hat{x})$ is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((x, \hat{x}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((x, \hat{x}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{6.2}$$

$$\frac{\mathbb{P}((x, \hat{x}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((x, \hat{x}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{6.3}$$

Note that the likelihood ratio 6.2 increases continuously with ε. The signal pair $(x, \hat{x})$ is informationally equivalent to $\mathcal{X}'$ when:

$$\frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - (s_H + \delta)} \tag{6.4}$$

for our chosen (δ, ε). Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ. Furthermore, by varying ε between 0 and $\frac{1-s_H}{s_H}$, the equivalent of *any* signal structure $\mathcal{X}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$ can be obtained.

Step 2: $\pi(\sigma_1; \mathcal{X}') > \pi(\sigma_1; \mathcal{X}')$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(x, \hat{x})$ instead of x'. To do so, the evaluator approves if and only if the pair $(x, \hat{x}) = (s_H, \hat{s_H})$ is observed. Note that, conditional on the visiting applicant's quality, the probability that the evaluator approves him is the same under these two policies. This is due to the identical informational content of these signals, as laid out in equations 6.3 and 6.4. Thus, evaluators' payoffs are also identical under these policies.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(x_i, \hat{x}_i)\}_{i=1}^{n}$. An applicant is a *marginal reject* if he has no $(x_i, \hat{x}_i) = (s_H, \hat{s_H})$ signals. The difference between evaluators' payoffs under $(\mathcal{X}, \hat{\mathcal{X}})$ and $\mathcal{X}$ is determined by these *marginal* *rejects*: they are *eventually rejected* under $(\mathcal{X}, \hat{\mathcal{X}})$ but *eventually approved* under $\mathcal{X}$. So:

$$\Pi(\sigma_1; \mathcal{X}') - \Pi(\sigma_1; \mathcal{X}) = \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[c - \mathbb{P}\left(\theta = H \mid \text{marginal reject}\right)\right]}_{(1)}$$

A marginal reject only has signal realisations $(x, \hat{x}) = (s_H, \hat{s_L})$ or $x = s_L$. These carry equivalent information about θ. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid x_1 = ... = x_n = s_L\right]$$

In the relevant region where $x = s_L$ leads to a rejection, the expression above must be weakly positive. Therefore, $\Pi(\sigma_1; \mathcal{X}') - \Pi(\sigma_1; \mathcal{X}) \geq 0$.

Maybe a brief explainer.

This concludes the first part of the claim that $\max\{0, \pi(\sigma_1; \mathcal{X})\}$ is weakly increasing in s_H.

I should probably just focus on the π not the whole thing, correct!

Part ii: Decreasing s_L; i.e. increasing the strength of evidence for $\theta = L$.

Now I show that replacing $\mathcal{X}$ with $\mathcal{X}'$ when $s'_L = s_L - \delta$ and $s'_H = s_H$:

i increases $\pi(\sigma_1; \mathcal{X})$ when $s_L \leq s_L^{as}$,

ii decreases $\pi(\sigma_1; \mathcal{X})$ when $s_L > s_L^{as}$

for $\delta > 0$ arbitrarily small. The desired assertion follows.

I should probably use the sum of evaluators' payoffs here.

Step 1: Replicating $\mathcal{X}'$ in two signals.

As before, let the evaluator potentially observe *two* signal realisations, x and $\hat{x}$. She first observes x, a single draw from $\mathcal{X}$. If this signal realises as $x = s_H$, she receives no further information. If it realises as $x = s_L$, she observes another signal $\hat{x} \in \{\hat{s_L}, \hat{s_H}\}$, a draw from a signal structure we construct now, $\hat{\mathcal{X}}$. $\hat{x}$ is distributed independently from x conditional on θ, as follows:

$$\mathbb{P}\left(\hat{x} = s_H \mid \theta = H\right) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad \mathbb{P}\left(\hat{x} = s_H \mid \theta = L\right) = \varepsilon \times \frac{s_L}{1 - s_L}$$

The evolution of the evaluator's beliefs upon seeing the signal pair $(x, \hat{x})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((x, \hat{x}) = (s_L, \hat{s_H}) \mid \theta = H\right)}{\mathbb{P}\left((x, \hat{x}) = (s_L, \hat{s_H}) \mid \theta = H\right)} = \frac{s_H}{1 - s_H} \tag{6.5}$$

$$\frac{\mathbb{P}\left((x, \hat{x}) = (s_L, \hat{s_L}) \mid \theta = H\right)}{\mathbb{P}\left((x, \hat{x}) = (s_L, \hat{s_L}) \mid \theta = H\right)} = \frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} \tag{6.6}$$

Note that 6.6 is continuously and strictly decreasing with ε, taking values between $\frac{s_L}{1-s_L}$ and 0 as ε varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(x, \hat{x})$ is informationally equivalent to $\mathcal{X}'$ when:

$$\frac{s_L}{1-s_L} \times \frac{1-\varepsilon \times \dfrac{s_H}{1-s_H}}{1-\varepsilon \times \dfrac{s_L}{1-s_L}} = \frac{s_L-\delta}{1-(s_L-\delta)}$$

Choose ε to satisfy this equality; note that ε becomes a continuously increasing function of δ.

Step 2: $\pi(\sigma_1; \mathcal{X})$ increases (decreases) with a marginal decrease in s_L, whenever $s_L \geq s_L^{as}$.

The strategy σ_1 can be replicated by an evaluator who receives the signal pair $(x, \hat{x})$ instead of x'. To do so, the evaluator rejects if and only if the pair $(x, \hat{x}) = (s_L, \hat{s_L})$ is observed.

Fix the collection of signal draws evaluators will see for the applicant if he visits them all: $\{(x_i, \hat{x}_i)\}_{i=1}^n$. An applicant is a *marginal admit* if: (i) he has *no* $x = s_H$ signals, and (ii) he has *at least one* $\hat{x} = \hat{s_L}$ signal. The difference between evaluators' payoffs under $(\mathcal{X}, \hat{\mathcal{X}})$ and $\mathcal{X}$ is determined by these *marginal admits*, who are *eventually rejected* under $\mathcal{X}$, but *eventually approved* under $(\mathcal{X}, \hat{\mathcal{X}})$. So:

$$\Pi(\sigma_1; \mathcal{X}') - \Pi(\sigma_1; \mathcal{X}) = \mathbb{P}\left(\text{marginal admit}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

For a marginal admit, $(x_i, \hat{x}_i) \in \{(s_L, \hat{s_H}), (s_L, \hat{s_L})\}$, and $(x_j, \hat{x}_j) = (s_L, \hat{s_H})$ for at least one evaluator j. Denote the number of evaluators who observe $(s_L, \hat{s_H})$ as $\#$. Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots = x_n = s_L\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots = x_n = s_L\right)}}_{(3)} \times \mathbb{P}\left(\theta = H \mid \# = i\right) - c$$

where:

$$\mathbb{P}\left(i \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots x_n = s_L\right) = \binom{n}{i} \times (k \times \varepsilon)^i \times (1 - k \times \varepsilon)^{n-1}$$

for $k = \mathbb{P}\left(\theta = H \mid x_1 = \ldots = x_n = s_L\right)$. The limit of expression (3) as $\varepsilon \to 0$ (thus $\delta \to 0$) is:

$$\lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(i \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots = x_n = s_L\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots = x_n = s_L\right)} = \mathbb{P}\left(1 \ \hat{x} = \hat{s_H} \text{ signals } \mid x_1 = \ldots = x_n = s_L\right)$$

and thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c = \lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \# = 1\right) - c$$

This expression is strictly positive (negative) when the expression below is strictly positive (negative):

there is a limit below as well, add.

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} - \frac{c}{1-c}$$

proving the claim.

$\square$

Proposition 4. Let $\mathcal{X}$ be a binary signal structure with normalised posterior beliefs $0 \leq s_L \leq 0.5$ and $0.5 \leq s_H \leq 1$. Evaluators' payoffs across the most embracive equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < \min\{s_L^{\text{mute}}, s_L^{as}\}$.

Similarly, there exists a threshold $s_L^\dagger \geq \min\{s_L^{\text{mute}}, s_L^{as}\}$, such that their payoffs across the most selective equilibria are:

i weakly increasing with the strength of evidence for $\theta = L$ when $s_L \geq s_L^\dagger$,

ii weakly decreasing with the strength of evidence for $\theta = L$ when $s_L < s_L^\dagger$.

Proof. I start with the most embracive equilibrium. When $s_L \geq s_L^{\text{safe}}$, the strategy $\sigma_{(1,1)}$ which approves everyone; i.e. $\sigma_{(1,1)}(s_L) = \sigma_{(1,1)}(s_H) = 1$, is an equilibrium. This owes to $\Psi(\sigma_{(1,1)}; \mathcal{X}) = \rho$ as it can be easily checked, and to the definition of s_L^{safe}. Since no strategy is more embracive, $\sigma^{\text{em}*} = \sigma_{(1,1)}$. In this parameter region, $\pi(\sigma^{\text{em}*}; \mathcal{X})$ does not vary as every applicant is approved. When $s_L < s_L^{\text{safe}}$, this equilibrium is no longer possible, and evaluators' equilibrium payoffs are thus given by $\pi(\sigma^{\text{em}*}; \mathcal{X}) = \max\{0, \pi(\sigma_1; \mathcal{X})\}$; as it was explained in the second fact under Theorem 1's proof. As s_L decreases, this increases (decreases) when $s_L \geq s_L^{as}$ ($s_L < s_L^{as}$). This establishes the first part of Proposition 4.

For the most selective equilibrium to have $\sigma^*(\text{sel}*) = 0$, a necessary and sufficient condition is:

$$\frac{\Psi(\sigma_1; \mathcal{X})}{1 - \Psi(\sigma_1; \mathcal{X})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

This owes to Lemma 9, which establishes that the interim belief *increases* in $\sigma(s_H)$.

Clearly, this condition is satisfied when $s_L \leq s_L^{\text{safe}}$. Thus, the most selective equilibrium becomes one where s_L leads to a rejection once s_L falls below some threshold $s_L^{\text{thr}} \geq s_L^{\text{safe}}$. Evaluators' equilibrium payoffs then start falling with stronger evidence for $\theta = L$ once $s_L \leq \min\{s_L^{\text{thr}}, s_L^{as}\}$.

$\square$

Theorem 2. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ both are either the most or least selective equilibrium strategies under the respective signal structures, evaluators' expected payoffs under σ' are:

1. *weakly higher* than under σ if $x = s_i$ leads to approvals under σ.

2. *weakly lower* than under σ:

   i if $x = s_i$ leads to rejections under σ; $\sigma(s_i) = 0$, and

   ii unless adverse selection poses a threat at signal s_{i+1} for $\mathcal{X}$ and σ.

Proof. Following the notation introduced in Definition 3, let $S \cup S'$ be the joint support of the signal structures $\mathcal{X}$ and $\mathcal{X}'$, and $s_1 < s_2 < ... < s_M$ be its elements. I begin by noting that the outcome the monotone strategy $\sigma : S \to [0,1]$ generates under $\mathcal{X}$ can be replicated under $\mathcal{X}'$ by another monotone strategy $\tilde{\sigma}' : S' \to [0,1]$ provided $\sigma(s_i) \in \{0,1\}$[14]:

$$\tilde{\sigma}'(s_j) = \begin{cases} \sigma(s_i) & j \in \{i-1, i+1\} \\ \sigma(s_j) & j \notin \{i-1, i+1\} \end{cases}$$

Part 1:
Now suppose s_i leads to approvals under σ; $\sigma(s_i) = 1$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 1$. I argue below that $\tilde{\sigma} \geq \sigma'$; evaluators *reject* more when s_i is spread. From Proposition 2, it follows that $\pi(\sigma; \mathcal{X}) = \pi(\tilde{\sigma}'; \mathcal{X}') \leq \pi(\sigma'; \mathcal{X}')$.

If $s_{i-1} = \min S \cup S'$ or $\sigma'(s_{i-2}) = 0$, we necessarily have $\tilde{\sigma} \geq \sigma'$ and are done. So, for contradiction, let $\sigma'(s_{i-2}) > 0$, and $\sigma' > \tilde{\sigma}'$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{X}$ and $\mathcal{X}'$, respectively.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{X}$ such that:

i $\tilde{\sigma}$ replicates the outcome σ' induces in $\mathcal{X}'$,

ii $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{X}$ if and only if σ' is an equilibrium strategy under $\mathcal{X}'$,

iii $\tilde{\sigma} > \sigma$, so σ cannot be the most embracive equilibrium under $\mathcal{X}$.

So, define the strategy $\tilde{\sigma} : S \to [0,1]$ for $\mathcal{X}$ as simply:

$$\tilde{\sigma}(s_j) := \begin{cases} 1 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of σ'. Furthermore, σ' is an equilibrium under $\mathcal{X}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{X}$: they induce the same interim belief ψ as the latter replicates the former, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}(\theta = H \mid \psi, x = s_{i-2}) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

[14]The characterisation of $\tilde{\sigma}$ is otherwise the same, but it ceases to be *monotone* by our definition.

Lastly, since $\sigma' > \tilde{\sigma}'$, it must be that $\tilde{\sigma} > \sigma$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{X}$ and $\mathcal{X}'$, respectively.

Recall that $\tilde{\sigma}$ and σ induce the same interim belief ψ under their respective signal structures; $\Psi(\tilde{\sigma}; \mathcal{X}') = \Psi(\sigma; \mathcal{X}) = \psi$. Therefore, if $\mathbb{P}(\theta = H \mid x = s_{i-1}, \psi) \geq c$, $\tilde{\sigma}$ is an equilibrium under $\mathcal{X}'$. This implies that $\sigma' \leq \tilde{\sigma}$, since σ' is the most selective equilibrium under $\mathcal{X}'$. If $\mathbb{P}(\theta = H \mid x = s_{i-1}, \psi) < c$ otherwise, there is an equilibrium $\sigma' < \tilde{\sigma}$ under $\mathcal{X}'$ due to the intermediate value argument presented when equilibrium existence was established in Proposition 1.

make this reference clear, we are using it a lot.

Part 2:

Now suppose s_i leads to rejections under σ; $\sigma(s_i) = 0$. Consequently, $\tilde{\sigma}'(s_{i-1}) = \tilde{\sigma}'(s_{i+1}) = 0$. I will establish Theorem 2's claim in two steps:

1. $\sigma' \geq \tilde{\sigma}'$; evaluators *approve* more when s_i is spread,

2. $\pi(\sigma'; \mathcal{X}') \leq \pi(\tilde{\sigma}'; \mathcal{X}') = \pi(\sigma; \mathcal{X})$ when adverse selection poses a threat at signal s_{i+1} for signal structure $\mathcal{X}$ and strategy σ.

Step 1:

If $s_{i+1} = \max S \cup S'$ or $\sigma'(s_{i+1}) > 0$, we necessarily have $\sigma' \geq \tilde{\sigma}$. So, let $s_{i+1} < \max S \cup S'$ and $\sigma'(s_{i+1}) = 0$.

Case 1: σ and σ' are the most embracive equilibria under $\mathcal{X}$ and $\mathcal{X}'$, respectively.

Recall that $\Psi(\sigma; \mathcal{X}) = \Psi(\tilde{\sigma}'; \mathcal{X}') = \psi$ since $\tilde{\sigma}'$ replicates the outcome of σ. Thus, if:

$$\mathbb{P}(\theta = H \mid x = s_{i+1}, \psi) \leq c$$

$\tilde{\sigma}'$ must be an equilibrium strategy under $\mathcal{X}'$; the optimality conditions for all signals below s_{i+1} are satisfied *a fortiori*, and those for the signals above s_{i+1} are satisfied since σ is an equilibrium strategy in $\mathcal{X}$. Then, $\sigma' \geq \tilde{\sigma}'$, since σ' is the most embracive equilibrium. If

$$\mathbb{P}(\theta = H \mid x = s_{i+1}, \psi) > c$$

on the other hand, by the intermediate value argument we used to establish equilibrium existence in Proposition 1, there is an equilibrium strategy $\sigma' > \tilde{\sigma}$ under $\mathcal{X}'$.

Case 2: σ and σ' are the most selective equilibria under $\mathcal{X}$ and $\mathcal{X}'$, respectively.

Since $\sigma'(s_{i+1})$, its outcome under $\mathcal{X}'$ can be replicated $\mathcal{X}$ with a strategy $\tilde{\sigma}$, defined as:

$$\tilde{\sigma}(s_j) = \begin{cases} 0 & j = i \\ \sigma'(s_j) & j \neq i \end{cases}$$

I will show that this necessarily implies that $\sigma' \geq \tilde{\sigma}'$, in two steps:

i $\tilde{\sigma}$ is an equilibrium strategy under $\mathcal{X}$ if and only if σ' is an equilibrium strategy under $\mathcal{X}'$,

ii $\tilde{\sigma} \leq \sigma$, and therefore $\tilde{\sigma} = \sigma$ since σ is the most selective equilibrium under $\mathcal{X}$.

(i) follows trivially, since both strategies have the same optimality condition for every signal realisation above s_{i+1}. Now, since σ is the most selective equilibrium under $\mathcal{X}$, we must have $\sigma \leq \tilde{\sigma}$; as $\tilde{\sigma}$ is an equilibrium strategy by (i). However, this means $\tilde{\sigma}' \leq \sigma'$. Since $\tilde{\sigma}'$ must also be an equilibrium in $\mathcal{X}'$, we must have $\tilde{\sigma}' = \sigma'$ and therefore $\sigma \leq \tilde{\sigma}$.

Step 2:

Now I establish the second part. The case where $\tilde{\sigma}' = \sigma'$ is trivial, so I focus on the case $\sigma' > \tilde{\sigma}'$. As we showed when establishing Case 2 in the first step, we must then have $\sigma'(s_{i+1}) > 0$.

Now take a strategy σ^{ε} for $\mathcal{X}'$, defined as $\sigma^{\varepsilon}(s_{i+1}) := \varepsilon$. We take ε small enough so that $\sigma' > \sigma^{\varepsilon} > \tilde{\sigma}'$. I will now show that when adverse selection poses a threat at signal s_{i+1} for $(\sigma; \mathcal{X})$, we have:

$$\pi(\sigma^{\varepsilon}; \mathcal{X}') \leq \pi(\tilde{\sigma}'; \mathcal{X}') = \pi(\sigma; \mathcal{X})$$

Proposition 2 then coins the result.

I show this slightly circuitously. Construct a ternary signal $\mathcal{X}^{\mathrm{re}}$ which we will use to replicate the outcomes σ^{ε} and $\tilde{\sigma}'$ generate. This signal admits the realisations $x^{\mathrm{re}} \in \{s_L^{\mathrm{re}}, s_{\varepsilon}^{\mathrm{re}}, s_H^{\mathrm{re}}\}$ and has distribution:

$$\mathbb{P}\left(x^{\mathrm{re}} = s \mid \theta\right) = \begin{cases} 1 - r_{\theta}(\sigma; \mathcal{X}) & s = s_H^{\mathrm{re}} \\ \varepsilon \times p'_{\theta}(s_{i+1}) & s = s_{\varepsilon}^{\mathrm{re}} \\ r_{\theta}(\sigma; \mathcal{X}) - \varepsilon \times p'_{\theta}(s_{i+1}) & s = s_L^{\mathrm{re}} \end{cases}$$

Clearly, as defined below, the strategies σ^{re} and $\sigma^{\mathrm{re}\text{-}\varepsilon}$ for $\mathcal{X}^{\mathrm{re}}$ replicate the outcomes of $\tilde{\sigma}$ and σ^{ε} under $\mathcal{X}'$:

$$\sigma^{\mathrm{re}}(s) = \begin{cases} 1 & s = s_H^{\mathrm{re}} \\ 0 & s = s_{\varepsilon}^{\mathrm{re}} \\ 0 & s = s_L^{\mathrm{re}} \end{cases} \qquad\qquad \sigma^{\mathrm{re}}(s) = \begin{cases} 1 & s = s_H^{\mathrm{re}} \\ 1 & s = s_{\varepsilon}^{\mathrm{re}} \\ 0 & s = s_L^{\mathrm{re}} \end{cases}$$

This makes it clear that the difference in evaluators' payoffs between $\tilde{\sigma}$ and σ^{ε} will be the *marginal admits* whose evaluators will observe:

i no s_H^{re} signal realisation,

ii at least one $s_{\varepsilon}^{\mathrm{re}}$ signal realisation.

if they visit all evaluators. Thus, we have:

$$\Pi(\sigma^{\varepsilon}; \mathcal{X}') - \Pi(\tilde{\sigma}; \mathcal{X}') = \mathbb{P}\left(\text{marginal admits}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal admit}\right) - c\right]}_{(2)}$$

where (2) then equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_{L}^{\text{re}} \text{ signals}\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ s_{\varepsilon}^{\text{re}} \text{ and } n-j\ s_{L}^{\text{re}} \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i\ s_{\varepsilon}^{\text{re}} \text{ and } n-i\ s_{L}^{\text{re}} \text{ signals}\right)$$

COMPLETE! □

Proposition 5. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $x = s_i$ leads to rejections under σ; i.e. $\sigma(s_i) = 0$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proof. □

Corollary 6. Let $\mathcal{X}'$ differ from $\mathcal{X}$ by a local MPS at s_i. Where σ' and σ are the most selective equilibria under the respective signal structures, evaluators' payoffs are lower in the former if $s_i < s_L^{\text{mute}}$, and:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_i}{1-s_i}\right)^{n-1} \times \frac{s_{i+1}}{1-s_{i+1}} \leq \frac{c}{1-c}$$

Proof. □

why does this appear with the incorrect number?

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