The (Mis)use of Information in Decentralised Markets

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Abstract

A seller offers an asset in a decentralised market. Buyers have private signals about their common value. I study whether the market becomes allocatively more efficient with (i) more buyers, (ii) better-informed buyers. Both increase the information available about buyers' common value, but also the adverse selection each buyer faces. With more buyers, trade surplus eventually increases and converges to the full-information upper bound if and only if the likelihood ratio of buyers' signals are unbounded from above. Otherwise, it eventually decreases and converges to the no-information lower bound. With better information about trades buyers would have accepted, trade surplus increases. With better information about trades they would have rejected, trade surplus decreases—unless adverse selection is irrelevant. For binary signals, a sharper characterisation emerges: stronger good news increase total surplus, but stronger bad news eventually decrease it.

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1 Introduction

In this paper, I ask whether more information about buyers' common value for an asset improves or harms allocative efficiency in a decentralised market. The setting is parsimonious: the seller sequentially visits n buyers until one accepts to trade at the seller's commonly known reservation value[1]. Negotiations are private: no buyer knows how many others the seller visited already. Trade is efficient when buyers' common value for the asset—its *quality*—is High, but not when it is Low. Each buyer holds a private signal about the asset's quality; conditional on quality, these signals are IID. A buyer accepts trade when she expects it to yield positive surplus; otherwise she rejects. I ask:

1. Does the expected surplus from trade increase with more buyers, each with an additional signal?
2. Does the expected surplus from trade increase with better-informed buyers, i.e., each with a more informative signal?

Both more and better-informed buyers increase the amount of information available in the market about the asset's quality. However, more information in the market—through either channel—is a double-edged sword for allocative efficiency. On the one hand, it might push buyers to better trades by helping them screen the asset's quality better. On the other hand, it might push them to worse trades by exposing them to greater adverse selection: when there are more buyers in the market, more might have already rejected the seller; when each buyer is better-informed, each rejection might owe to a worse signal. This paper shows that the *kind* of information in the market determines how this trade-off is resolved.

My first main result, Theorem 1, answers how increasing the number of buyers in the market influences allocative efficiency. This hinges on whether the likelihood ratio of buyers' signals are unbounded from above. If it is, the expected surplus from trade eventually increases in the number of buyers, and converges to the full-information benchmark: a High quality seller almost surely trades, but a Low quality seller never does. This is the upper bound for equilibrium surplus in the market: all gains from trade are exhausted. If it is not, the expected surplus from trade eventually decreases in the number of buyers, and converges to the no-information benchmark: either the seller almost surely trades regardless of his quality, or expected surplus is zero when he trades. This is the lower bound for equilibrium surplus in the market (Proposition 3): buyers' ability to screen the asset's value generates no additional gains from trade.

That the outcome (whether trade occurs) in a large market reveals buyers' common value for the asset if and only if the likelihood ratio of their signals are unbounded from above is reminiscent of a large sealed bid common value auction à la Wilson (1977) and Milgrom (1979). There, too, the outcome (the winning bid) reveals the asset's quality if and only if the bidders' signal structure satisfies the same condition[2]. However, when this condition is violated, trade in a decentralised

[1] I relax this simplifying assumption in Section 7: I show that this emerges as the equilibrium outcome in a setting where buyers make ultimatum price offers.

[2] This is when bidders' common value for the item can assume two values. Milgrom (1979) identifies "distinguisha-

market either becomes completely uninformative about the asset's quality, or only reveals that the expected gains from trade are zero. In contrast, the winning bid in a large auction may still approximate buyers' common value well[3].

This also offers an interesting contrast with Lauermann and Wolinsky (2016). In a model like mine[4] but where the gains from trade are always positive, they find that (generically) the outcome in a large market either fully reveals or is completely independent of the asset's quality. Theorem 1 establishes that another possibility emerges when, ex-ante, the expected gains from trade are negative: trade might be *partially* informative about the asset's quality. However, trade only reveals the expected gains not to be negative but zero instead[5]—this information has no bearing on total surplus.

In Theorems 2 and 3, I answer how giving better information to each existing buyer influences allocative efficiency. Theorem 2 shows that, when buyers' signal structure is binary, stronger good news (higher likelihood ratio at the top) always increases surplus; but stronger bad news (lower likelihood ratio at the bottom) eventually decreases it. The former might prevent a seller from trading, but recovers surplus in doing so. The latter might help a seller trade, but this eventually destroys surplus due to adverse selection. Theorem 3 generalises this insight to arbitrary finite signal structures: additional information where a buyer would have accepted trade (a *negative override*) increases surplus; but additional information where she would have rejected trade (a *positive override*) decreases it—unless *adverse selection is irrelevant* in the appropriate sense (Definition 4).

To understand the main insight, consider buyers with a binary signal structure: each buyer receives either a good, or a bad signal. For simplicity, ignore equilibrium considerations; simply let buyers accept upon a good signal and reject upon a bad signal. Now consider revealing additional information to each buyer—another binary signal. This additional information could serve two purposes. If it is revealed after an initial good signal, it could lead the buyer to revise her initial decision to a rejection. I call information that serves this purpose a *negative override*[6]. If it is revealed after an initial bad signal, it could lead her to revise her initial decision to an acceptance. I call such information a *positive override*. In this simple binary-on-binary example, we can interpret a negative override as strengthening good news: a buyer can rely on two good signals to accept. A positive override strengthens bad news: a buyer can rely on two bad signals to reject.

A negative override increases surplus. It makes it harder for the seller to trade—a seller some buyer would have accepted before the negative override became available might now be rejected by every buyer. But when this happens, it reveals the expected surplus from trade to be negative: each buyer observed a bad signal and concluded that trading would reduce surplus, despite not

bility" as a condition that generalises "unbounded likelihood ratio from above" when buyers' common value for the item can assume any number of finite values.

[3]See, for instance, Section IV in Lauermann and Wolinsky (2017).

[4]See the Related Literature section for a more detailed discussion.

[5]Section 8 illustrates this with a numerical example.

[6]I follow the language used in credit markets: a negative (downgrade) override occurs when a human evaluator revises a prospective borrower's algorithmic credit score downwards, in light of overlooked information. A positive (upgrade) override occurs when she revises it upwards. See Section 2.5 in Van Gestel and Baesens (2008); as well as par. 110 in ECB (2024) and pg. 140 in Stellantis Financial Services Italia S.p.A. (2024).

knowing (but suspecting) that all buyers reached the same conclusion.

In contrast, a positive override might decrease surplus. It makes it easier for the seller to trade—a seller every buyer would have rejected might now trade with some buyer. However, due to adverse selection, the expected surplus from such a trade might be negative: the buyer who trades with the seller does not observe how many others rejected him previously. If too many did, those buyers' bad signals might reveal expected surplus from trade to be negative despite her good signal. The buyer might find that she traded when she should not have.

I show that adverse selection severely limits positive overrides from raising surplus: unless *adverse selection is irrelevant*, i.e., a buyer need not care about the number of previous refusals the seller received, a positive override decreases efficiency.

This insight underpins Theorem 2's sharp characterisation for binary signals. To extend it to arbitrary signal structures in Theorem 3, I formalise a positive (and, negative) override as a *local mean preserving spread*[7] of a signal upon which buyers reject (and, accept). Studying informativeness at the level of local spreads is essential to the tractability of my exercise but sacrifices no generality: any Blackwell improvement is a combination of finitely many local spreads.

Theorems 2 and 3 show that too much information can be detrimental for allocative efficiency. So, finally, I study how a regulator can coarsen buyers' information to maximise expected surplus. Proposition 9 shows that through this policy tool, the regulator aims to prevent a buyer from trading unless adverse selection is irrelevant, i.e., unless she should trade even if everyone else rejected the seller. The implication is striking: the regulator wants buyers to base their decisions on the highest number of rejections the seller may have received, not the expected number.

Contribution

I view the main contribution of my paper to be twofold. First, I study a question that has been largely overlooked by the literature on information aggregation in markets. Most of this literature[8] asks whether the outcome in a large market reflects all information its participants have. Instead, I ask whether a *finite* decentralised market can convert *more information* among its participants to more efficient outcomes. I study two channels which increase information in the market. The first is through an additional buyer, bringing an additional signal to the market. This paper is the first to explore this channel in a decentralised market[9]. The second is through better-informed buyers. To the best of my knowledge, this channel has not been explored by previous work[10].

Second, my findings have important policy implications for markets where trades are negotiated

[7]See Definitions 2 and 3.

[8]Prominent and related work in this literature includes Wilson (1977), Milgrom (1979), Riordan (1993) and Lauermann and Wolinsky (2017) for centralised exchanges (auctions) and Wolinsky (1990), Zhu (2012), and Lauermann and Wolinsky (2016) for decentralised markets.

[9]Riordan (1993) studies how allocative efficiency in a common value auction changes with an additional bidder.

[10]Notably, Glode and Opp (2019) show that a decentralised OTC market provides buyers with greater information acquisition incentives than a centralised limit-order market, so might be more efficient than the latter. I discuss their work in light of my contribution under Related Literature.

bilaterally and with little to no trade transparency: such as over-the-counter markets[11], credit markets[12], and the housing market[13]. Recent technological advances have allowed participants in these markets to enjoy increasingly greater access to information[14]. It is commonly presumed that the "more efficient processing of information, for example in credit markets, financial markets, [...] contribute to a more efficient financial system" (Financial Stability Board, 2017). My Theorems 2 and 3 show that this presumption—which ignores the adverse selection problem in these markets—is misleading: adverse selection may claw back on market participants' ability to screen for efficient trades, and *decrease* surplus when each can access better information. My Theorem 1 shows that adverse selection might cause increased competition to hurt efficiency, too. This validates a concern empirically recognised by regulators and industry leaders[15].

Existing regulation in credit markets already limits the information lenders can use to assess borrowers[16]. Directly resonating with Theorems 2 and 3, ECB guidelines (2024) state that "institutions should be more restrictive with positive overrides than with negative ones". I offer a novel justification[17] for such policies, rooted in the adverse selection problem inherent to the market.

Literature Review

The first question I ask is whether allocative efficiency in a decentralised market increases with more buyers. Riordan (1993) asks this question in a first price auction with common values[18]. There, the adverse selection problem is simply the winner's curse—the winner understands that she had the highest signal among all bidders. In contrast, here, a buyer who trades understands that she had the highest signal among those buyers the seller previously visited. Consequently, the sufficient condition Riordan (1993) identifies[19] for surplus to be increasing or decreasing in the number of bidders differs from the necessary and sufficient condition Theorem 1 supplies for a decentralised market.

[11]OTC markets are characterised by sequential contacts and little transparency (Duffie, 2012; Zhu, 2012). Liquidity providers typically make ultimatum offers that only last "as long as the breath is warm" (Bessembinder and Maxwell, 2008).

[12]In the US and the UK, credit scores mask borrowers' recent applications, and borrowers exercise little bargaining power against lenders (Agarwal et al., 2024 and Consumer Rights, 2024).

[13]In the housing market, "buyers and sellers must search for each other" (Han and Strange, 2015). Sellers frequently relist, making it difficult to infer how many viewings resulted in no trade: RE/MAX (2024) advises "if a property has been sitting on the market and going stale, there is no harm in relisting it so that it appears fresh and new".

[14]Hedge funds and broker-dealers use increasingly sophisticated data and algorithms to assess trades' profitability (Financial Stability Board, 2017); lenders use cutting-edge ML technology in credit scoring (Financial Stability Board, 2017); algorithmic traders in housing markets analyse and execute trades faster than traditional investors (Raymond, 2024).

[15]Regulators (partially) blamed adverse selection for the collapse of a British bank, HBOS: "the borrowers who came through its doors inevitably included many whom better established banks had turned away" (Kay (2024)).

[16]For instance, following the 2008 crash, the Basel III Accord severely limited the use of "advanced internal ratings systems" to determine credit risk exposure. This overturned the conventions set in Basel II. See BCBS (2017).

[17]Currently, these policies are mostly justified by a distrust in the "robustness and prudence" of lenders' abilities to screen borrowers (BCBS, 2017).

[18]Relatedly, Di Tillio et al. (2021) study whether the winning bid in a first price common value auction becomes more informative about the asset's value when there are more bidders. Efficiency is of no direct concern in their setting: trade is always efficient and always materialises.

[19]Where $F_\theta(s)$ is the CDF of the signal distribution for quality $\theta \in \{L, H\}$, he finds that the sign of the expression $\frac{f_H(.)/F_H(.)}{f_L(.)/F_L(.)} - \frac{F_H(.)}{F_L(.)}$ over the support is a sufficient condition to determine this.

My second question—whether efficiency increases when buyers are *better-informed*—is novel in this literature. The closest papers, to the best of my knowledge, are Levin (2001) and Glode and Opp (2019). Levin (2001) asks whether in a lemons market à la Akerlof (1970)—where the seller's private information is the root cause of inefficiency—a better-informed seller necessarily hurts efficiency. He finds that the right kind of information can increase efficiency. His environment, and therefore his characterisation, differs than mine. Glode and Opp (2019) show that buyers might have greater incentives to acquire information in an OTC market than in a limit-order market; so, the former can be more efficient than the latter. They investigate a particular information technology: buyers invest in their probability of getting a fully revealing signal. My Theorems 2 and 3 show that in general, adverse selection might cause a market where buyers are better-informed to be less efficient.

My model is closest to Zhu (2012) and Lauermann and Wolinsky (2016). Zhu (2012) assumes that trade is efficient regardless of the asset's quality[20]; otherwise his model is identical to mine. He shows that unless the likelihood ratio of buyers' signals are unbounded from above, a large market might fail to be efficient—a High quality seller might fail to trade. Where trading with a Low quality seller is inefficient, Theorem 1 offers a stronger conclusion: unless the same condition holds, surplus in a large market converges to the no-information lower bound.

Lauermann and Wolinsky, 2016, too, study a decentralised market for a common value asset; but they focus on a large market where the seller (there, the buyer) (i) pays a small cost for each buyer (there, seller) he visits and (ii) has bargaining power. There is no efficiency concern: trade is always efficient and executed. Generically, the transaction price carries either full or no information about the asset's common value. Furthermore, the seller's costly search further impedes the revelation of the asset's value: the condition necessary for the transaction price to be fully revealing is stronger than the unboundedness of the likelihood ratio.

My model also admits a social learning interpretation, in the tradition of Bikhchandani et al. (1992). It can be considered as a variant of the classic model where later decision makers (buyers) are called to decide only if those before them reject, and no one observes her position in the queue (as in Herrera and Hörner (2013)). Most work in this literature focuses on whether full learning attains with a large number of decision makers. Instead, my results speak to how more information, through more or better-informed decision makers, influences the welfare of finitely many decision makers.

The remainder is organised as follows. Section 2 presents the model. Section 3 presents preliminary analyses about equilibria and total surplus in the market. Section 4 presents Theorem 1. Section 5 presents Theorems 2 and 3 (in Subsections 5.1 and 5.2). Section 6 discusses how a regulator can optimally coarsen buyers' information to maximise total surplus. Section 7 presents an extension where buyers offer take-it-or-leave it prices to the seller. Section 8 presents a numerical example that supplements the discussion in Section 4. All proofs omitted in the main text appear in Section 9.

[20]Both buyers' common value and the seller's reservation value for a Low quality asset is 0.

2 Model

The seller (he) of an indivisible asset sequentially visits $n \in \mathbb{N}$ prospective buyers (she) in a uniformly random order. He sells to the first one who accepts to pay his reservation value $c \in [0,1]$. The asset's (seller's) quality θ is either High or Low, $\theta \in \{H, L\}$. When the buyer he visits rejects to trade, no transaction takes place and the seller visits the next buyer in line. If she accepts, she pays the seller his reservation value and enjoys a surplus of 1 if the asset's quality is High, but 0 if it is Low. The game ends when a buyer accepts the seller, or they all reject him.

At the outset of the game, the asset's quality is unknown[21]; all players share the common prior that it is High with probability ρ and Low otherwise. Each buyer obtains additional private information about the asset's quality through the outcome of a Blackwell experiment $\mathcal{E} = (\mathbf{S}, p_L, p_H)$. The outcome s of the experiment—the buyer's *signal*—is drawn from the finite set $\mathbf{S}$ with a distribution p_θ. Conditional on the asset's quality, buyers' signals are IID. The joint distribution of buyers' signals conditional on the asset's quality is common knowledge.

The buyer visited by the seller receives no information about how many others the seller previously visited. Nonetheless, she deduces that all those buyers rejected the seller. Through this, she extracts additional information about the asset's quality.

The buyer visited by the seller forms her posterior belief about the asset's quality with these two pieces of information. For clarity, I study her belief update in two stages. First, she uses the information conveyed by the seller's visit to revise her prior belief ρ to an interim belief ψ. Then, she uses her private signal to revise her interim belief to a posterior belief $\mathbb{P}_\psi(\theta = H \mid s)$.

A buyer's strategy $\sigma : \mathbf{S} \to [0,1]$ maps every signal $s \in \mathbf{S}$ she might observe to a probability $\sigma(s)$ with which she accepts to trade. Her strategy σ is *optimal against the interim belief* ψ if, given this interim belief and her signal, the buyer accepts (rejects) to trade whenever her expected payoff from trading with the seller is positive (negative):

$$\sigma(s) = \begin{cases} 0 & \mathbb{P}_\psi(\theta = H \mid s) < c \\ 1 & \mathbb{P}_\psi(\theta = H \mid s) > c \end{cases}$$

She may accept to trade with any probability when she expects zero surplus from trading.

I focus on *symmetric Bayesian Nash Equilibria* of this game. Hereafter, I reserve the term *equilibrium* for such equilibria unless I state otherwise. An *equilibrium* is a strategy and interim belief pair (σ^*, ψ^*) such that:

1. The interim belief ψ^* is *consistent* with the strategy σ^*; i.e., it is the interim belief of a buyer who believes all other buyers use the strategy σ^*.
2. The strategy σ^* is optimal given the interim belief ψ^*.

I call any strategy σ^* that constitutes part of an equilibrium an *equilibrium strategy.*

[21] Until Section 7, the seller's knowledge about the asset's quality is immaterial.

3 Buyers' Beliefs, Equilibria, and Total Surplus

This section lays the necessary groundwork to discuss my main results. First, I discuss how buyers form their interim beliefs, and the fundamental properties of the set of equilibria. Then, I discuss how the total surplus from trade varies across different equilibria.

3.1 Buyers' Beliefs and Equilibria

No buyer learns how many others the seller visited before her. But, she deduces that all those past visits resulted in rejections. How does she interpret this information?

When each buyer uses a strategy σ, a seller of quality θ faces a probability $r_\theta(\sigma;\mathcal{E})$ of getting rejected in any of his visits:

$$r_\theta(\sigma;\mathcal{E}) := 1 - \sum_{j=1}^{m} p_\theta\left(s_j\right) \times \sigma\left(s_j\right)$$

Every buyer understands that the seller is equally likely to decide to visit any number $k \in \{0, 1, 2, ..., n-1\}$ of other buyers before her. She will receive the seller's visit if and only if he is rejected by all those k buyers. Therefore, she assigns a probability $\nu_\theta\left(\sigma;\mathcal{E}\right)$ to being visited by the seller:

$$\nu_\theta\left(\sigma;\mathcal{E}\right) := \frac{1}{n} \times \sum_{k=0}^{n-1} r_\theta\left(\sigma;\mathcal{E}\right)^k$$

When the seller *does* visit her, the buyer uses this information to update her prior belief about the seller's quality to an interim belief ψ:

$$\begin{aligned}\psi = \mathbb{P}\left(\theta = H \mid \text{visit received}\right) &= \frac{\mathbb{P}\left(\text{visit received} \mid \theta = H\right) \times \mathbb{P}(\theta = H)}{\mathbb{P}\left(\text{visit received}\right)} \\ &= \frac{\rho \times \nu_H\left(\sigma;\mathcal{E}\right)}{\rho \times \nu_H\left(\sigma;\mathcal{E}\right) + (1-\rho) \times \nu_L\left(\sigma;\mathcal{E}\right)}\end{aligned}$$

This is the unique interim belief that is *consistent* with every buyer using the strategy σ. The buyer then uses her private signal $s \in \mathbf{S}$ about the seller's quality to update her interim belief to a posterior belief:

$$\mathbb{P}_\psi\left(\theta = H \mid s\right) = \frac{\psi \times p_H(s)}{\psi \times p_H(s) + (1-\psi) \times p_L(s)}$$

Note that the informational content of the buyer's signal $s \in \mathbf{S}$ is distilled by the ratio $\frac{p_H(s)}{p_H(s)+p_L(s)}$. For notational convenience, I will use the signal's label, s, to refer to this ratio:

$$s := \frac{p_H(s)}{p_H(s) + p_L(s)} \in [0,1] \qquad \text{for all } s \in \mathbf{S}$$

Under this notation, the ratio $\frac{s}{1-s}$ simply equals the signal's likelihood ratio, $\frac{p_H(s)}{p_L(s)}$. For further convenience, I also enumerate the signals $\mathbf{S}$ in order of increasing likelihood ratios; $\mathbf{S} := \{s_1, s_2, ..., s_m\}$ where $s_1 \leq s_2 \leq ... \leq s_m$. Note that, for the same interim belief, a buyer's posterior belief is increasing in her signal's index.

Whenever a strategy σ^* is optimal against the unique interim belief ψ^* consistent with it, the pair (σ^*, ψ^*) forms an equilibrium. In principle, there might be many such pairs, or none at all. Proposition 1 sets the ground by ruling the latter possibility out and characterising the set of equilibria.

Proposition 1. Let Σ be the set of equilibrium strategies. Then:

1. Σ is non-empty and compact.

2. Any equilibrium strategy σ^* is monotone: for any $\sigma^* \in \Sigma$, $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.

3. All equilibria exhibit adverse selection: $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

Proof outline: To establish the existence of an equilibrium, I construct a best response correspondence: Φ for buyers. Φ maps any strategy σ to the set of strategies that are optimal against the unique interim belief consistent with σ; i.e. those that maximise a buyer's expected payoff when her all her peers use the strategy σ. Note that σ^* is an equilibrium strategy if and only if it is a fixed point of this best response correspondence; i.e., $\sigma^* \in \Phi(\sigma^*)$. Through a routine application of Kakutani's Fixed Point Theorem, I show that Φ indeed has a fixed point. In the process, I prove that Φ is upper semicontinuous; this also establishes that the set of equilibrium strategies is compact.

Monotonicity is a straightforward necessity for a strategy to be optimal: higher signals induce higher posterior beliefs, so buyers (weakly) prefer to trade upon higher signals. A crucial consequence of monotonicity is that a Low quality seller is likelier to be rejected in any of his visits, as buyers are likelier to observe lower signals for him. Thus, the seller is adversely selected through his past rejections, and buyers' interim beliefs always lie below their prior beliefs. □

An equilibrium is guaranteed to exist, but it need not be unique. The following example, which I will modify and revisit on occasion, illustrates this. Let there be two buyers who share the prior belief $\rho = 0.5$, and the seller's reservation value be $c = 0.2$. Furthermore, let buyers' experiment $\mathcal{E}$ be binary, $\mathbf{S} = \{0.2, 0.8\}$, and its outcome have the conditional distribution:

$$p_L(s) = \begin{cases} 0.8 & s = 0.2 \\ 0.2 & s = 0.8 \end{cases} \qquad p_H(s) = \begin{cases} 0.2 & s = 0.2 \\ 0.8 & s = 0.8 \end{cases}$$

There are two equilibrium strategies in this example[22], which I denote as $\hat{\sigma}$ and $\check{\sigma}$. Table 1 summarises these strategies, and how buyers form their interim and posterior beliefs under them.

[22]There are no other equilibria. Under any monotone strategy that assigns a positive probability to trade, buyers' interim belief lies between 0.5 and 0.4—so buyers must always trade upon the high signal 0.8. Lemma 17 in Section 9.3 shows that in equilibrium, buyers either always or never trade upon the low signal.

	$\check{\sigma}$	$\hat{\sigma}$
Prob. the buyer accepts upon $\mathbf{s = 0.8}$	$\check{\sigma}(0.8) = 1$	$\hat{\sigma}(0.8) = 1$
Prob. the buyer accepts upon $\mathbf{s = 0.2}$	$\check{\sigma}(0.2) = 1$	$\hat{\sigma}(0.2) = 0$
Buyers' interim belief	$\psi = 0.5$	$\psi = 0.4$ [23]
Buyers' posterior belief upon $\mathbf{s = 0.8}$	0.8	0.7
Buyers' posterior belief upon $\mathbf{s = 0.2}$	0.2	≈ 0.14

Table 1: Running Example: Comparing Equilibrium Strategies $\check{\sigma}$ and $\hat{\sigma}$

Under the strategy $\check{\sigma}$, a buyer accepts trade regardless of her signal. This eliminates adverse selection: since the first buyer the seller visits accepts trade, whoever he visits is certain that she is the first one he visited; and so, buyers' interim belief ψ equals their prior ρ. In this equilibrium, a buyer finds trade optimal even if she receives the low signal 0.2; given the posterior belief this signal induces, she expects zero net surplus from trade.

Under the strategy $\hat{\sigma}$, a buyer accepts trade if only if she receives the high signal 0.8. Buyers' selectivity triggers adverse selection: each buyer understands that she need not be the first one the seller visited. So, buyers' interim belief ψ falls below their prior belief. A buyer no longer finds trade optimal when she receives the low signal 0.2: this signal induces a posterior belief ≈ 0.14, so she expects a loss. She does, however, find it optimal when she receives high signal $s = 0.8$: this signal induces a posterior belief of 0.7, so she expects positive net surplus from trade.

In the equilibrium where buyers use the strategy $\hat{\sigma}$, the seller is likelier to be rejected in any of his visits. The buyers are *more selective*—they are (weakly) likelier to reject trade at any signal they might observe.

Definition 1. Where σ' and σ are two strategies, σ' is *more selective than* σ (or, σ is *less selective than* σ') if $\sigma'(s) \le \sigma(s)$ for all $s \in \mathbf{S}$.

Selectivity offers a natural way to order buyers' equilibrium strategies. Proposition 2 shows that it is also a complete order over them.

Proposition 2. Selectivity is a complete order over the set of equilibrium strategies Σ. Moreover, Σ contains a *most* and *least* selective strategy, $\hat{\sigma} \in \Sigma$ and $\check{\sigma} \in \Sigma$ respectively:

$$\hat{\sigma}(s) \le \sigma^*(s) \le \check{\sigma}(s) \qquad \text{for all } s \in \mathbf{S} \text{ and } \sigma^* \in \Sigma$$

Proof. By Proposition 1, the set of equilibrium strategies Σ is a subset of the set of monotone strategies. The latter is a chain under the selectivity order: for any signal $s \in \mathbf{S}$ and two monotone

[23] The interim belief in this case is easily calculated as: $\psi = \frac{1+r_H(\sigma;\mathcal{E})}{(1+r_H(\sigma;\mathcal{E}))+(1+r_L(\sigma;\mathcal{E}))} = \frac{1.2}{1.2+1.8} = 0.4$.

strategies σ and σ', we have:

$$\sigma'(s) > \sigma(s) \implies \begin{array}{ll} 1 = \sigma'(s^{\cdot}) \geq \sigma(s^{\cdot}) & \text{for any } s^{\cdot} > s \in \mathbf{S} \\ \sigma'(s_{\cdot}) \geq \sigma(s_{\cdot}) = 0 & \text{for any } s_{\cdot} < s \in \mathbf{S} \end{array}$$

Since any subset of a chain is also a chain, Σ is a chain too.

By Proposition 1, Σ is a compact set. Since it is also a chain, by applying a suitably general Extreme Value Theorem (see Theorem 27.4 in Munkres (2000)) to the identity mapping on Σ, one verifies that Σ has a minimum and maximum element with respect to this order; i.e. there are two strategies $\hat{\sigma}, \check{\sigma} \in \Sigma$ such that for any other strategy $\sigma^* \in \Sigma$ we have $\hat{\sigma}(s) \leq \sigma^*(s) \leq \check{\sigma}(s)$ for all $s \in \mathbf{S}$. □

Likewise, I call an equilibrium "more selective than another" whenever buyers use a more selective strategy in the former.

3.2 Total Surplus

Trading generates a surplus of $1 - c$ when the asset's quality is High, but destroys a surplus of c when the asset's quality is Low. So, the expected surplus from trade in the market—total surplus, for short—depends on how well buyers can screen the asset's quality before they decide whether to trade. Given buyers' experiment $\mathcal{E}$ and strategy σ, total surplus equals:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) &:= (1-c) \times \mathbb{P}_{\sigma;\mathcal{E}}\left(\theta = H \cap \text{some buyer trades}\right) - c \times \mathbb{P}_{\sigma;\mathcal{E}}\left(\theta = L \cap \text{some buyer trades}\right) \\ &= (1-c) \times \rho \times [1 - r_H(\sigma;\mathcal{E})^n] - c \times (1-\rho) \times [1 - r_L(\sigma;\mathcal{E})^n]\end{aligned}$$

Buyers fully appropriate this surplus—the seller is only paid his reservation value when he trades.

Two benchmarks are natural to consider. If buyers had full information about the asset's quality, they would trade whenever the asset's quality is High, but never when it is Low—all gains from trade would be realised. Total surplus in this *full-information benchmark* would equal $\Pi^f := \rho \times (1 - c)$.

If instead, buyers had no information about the asset's quality, their decisions would be guided solely by their prior belief. There would be no adverse selection: previous rejections would convey no private information since no buyer has any. Buyers would trade if they expected positive surplus given their prior beliefs, and not otherwise. Total surplus in this *no-information benchmark* would equal $\Pi^n := \max\{0, \rho - c\}$.

Proposition 3 establishes that, in equilibrium, total surplus is bounded by these benchmarks; moreover, total surplus is always higher in more selective equilibria.

Proposition 3. Equilibrium total surplus is bounded above by the full-information benchmark Π^f and below by the no-information benchmark $\Pi^{\emptyset}$. Furthermore, it is higher under more selective equilibrium strategies:

$$\max\{0, \rho - c\} = \Pi^{\emptyset} \leq \Pi(\sigma^{**}; \mathcal{E}) \leq \Pi(\sigma^{*}; \mathcal{E}) \leq \Pi^{f} = \rho \times (1 - c)$$

where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*.

Proof. The second part of the Proposition follows as a corollary to Lemma 4, presented later in this section. For the first part, note that all gains from trade is realised when buyers have full information; hence Π^f bounds total surplus from above. Since total surplus equals buyers' surplus from trade, it is bounded below by 0 in any equilibrium—a buyer can always reject. Thus, when $\rho \leq c$, $\Pi^{\emptyset} = 0$ bounds total surplus from below. Now let $\rho > c$, and assume for contradiction that there is an equilibrium strategy σ^* such that $\Pi(\sigma^*; \mathcal{E}) < \Pi^{\emptyset}$. Then:

$$\begin{aligned}
&\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\text{some buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\theta = H \mid \text{some buyer trades}) - c] < \\
&\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\text{some buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\theta = H \mid \text{some buyer trades}) - c] + \\
&\qquad\qquad \mathbb{P}_{\sigma^*;\mathcal{E}}\,(\text{no buyer trades}) \times [\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\theta = H \mid \text{no buyer trades}) - c] = \Pi^{\emptyset}
\end{aligned}$$

So, $\mathbb{P}_{\sigma^*;\mathcal{E}}\,(\theta = H \mid \text{no buyer trades}) > c$. However, σ^* then cannot be an equilibrium strategy; each buyer has a profitable deviation to trade with the seller whenever he visits. $\square$

Their private information helps buyers raise surplus by avoiding trade when the asset's quality is Low, and executing it when it is High. Equilibrium multiplicity presents a trade-off: in a more selective equilibrium, trade is less likely—this conserves surplus when the asset's quality is Low. In a less selective equilibrium, trade is more likely—this raises surplus when the asset's quality is High. Proposition 3 establishes that this trade-off is always resolved in favour of more selective equilibria.

Our running example illustrates this. The equilibrium strategies $\hat{\sigma}$ and $\check{\sigma}$ we identified there are the most and least selective ones, as they are the only equilibrium strategies. Table 2 summarises how the probability that the seller trades and the total surplus vary across these equilibria, and compares them to the full- and no-information benchmarks.

	$\check{\sigma}$	$\hat{\sigma}$	**no-info.**	**full-info.**
Prob. seller trades when $\theta = \mathbf{H}$	1	0.96 [24]	1	1
Prob. seller trades when $\theta = \mathbf{L}$	1	0.36	1	0
Total surplus	0.3	0.348	0.6	0.3

Table 2: Running Example: Comparing Surplus Across Equilibria

In the least selective equilibrium where buyers use the strategies $\check{\sigma}$, the seller always trades; total surplus equals that in the no-information benchmark. In the most selective equilibrium where buyers use the strategies $\hat{\sigma}$, the seller does not trade unless some buyer receives the high signal. This decreases the probability of trade regardless of the asset's quality, but still increases total surplus.

[24] This is the probability that at least one buyer will receive the high signal 0.8 when the seller has High quality: $1 - p_H^2(0.2) = 0.96$. For a seller of Low quality, this probability is: $1 - p_L^2(0.2) = 0.36$.

That total surplus is higher in more selective equilibria follows from a starker fact established in Lemma 4.

Lemma 4. Let σ^* and σ be two monotone strategies such that (i) σ^* is more selective than σ, and (ii) σ^* is an equilibrium strategy. Then, total surplus is higher under σ^*: $\Pi(\sigma^*;\mathcal{E}) \geq \Pi(\sigma;\mathcal{E})$.

The intuition behind Proposition 3 and Lemma 4 is, once again, illustrated by our running example. There, the seller trades under both equilibria unless both buyers observe the low signal, 0.2. If both buyers observe the low signal, he does not trade under the most selective equilibrium: a buyer rejects him upon a low signal for fear that the other buyer may have already done so. But this reveals that the expected surplus from trading with this seller must be negative. Despite this, the seller trades in the least selective equilibrium as buyers hold different beliefs about each other's signals: a buyer now understands that she receives the seller's visit only if she is the first to do so, therefore she places a lower probability on the other having received a low signal.

4 Efficiency with More Buyers

In this section, I discuss how total surplus changes as the number of buyers in the market increases.

Theorem 1. Let $\Pi^n(\hat{\sigma};\mathcal{E})$ be total surplus under the most selective equilibrium in a market with n buyers. If $\mathcal{E}$ has an outcome that fully reveals High quality ($s_m = 1$), the sequence $\{\Pi^n(\hat{\sigma};\mathcal{E})\}_{n=1}^{\infty}$ is eventually increasing and converges to surplus in the full-information benchmark. Otherwise, it is eventually decreasing and converges to surplus in the no-information benchmark.

Each additional buyer brings an additional signal about the asset's quality to the market. As the market becomes arbitrarily large, buyers' collective information becomes sufficient to fully reveal the asset's quality—unless buyers' signals carry no information. Whether the market outcome incorporates this information and reaches full allocative efficiency depends on the *kind* of information each buyer has.

If buyers have a signal which fully reveals High quality, total surplus reaches its full-information upper bound as the number of buyers grows arbitrarily large. Beyond a threshold number of buyers, adverse selection forces each buyer to reject the seller unless she observes that fully-revealing high signal. Therefore, a Low quality seller never trades. A High quality seller, however, might. Moreover, a High quality seller is likelier to trade in a larger market, as it is likelier that at least one buyer will observe the fully-revealing high signal.

If, however, buyers do not have such a signal, the market experiences a surplus breakdown as the number of buyers grows large—total surplus reaches its no-information lower bound. How market outcomes evolve as the number of buyers grows depends on whether the expected gains from trade are positive, $\rho > c$, or weakly negative, $\rho \leq c$:

- When the expected gains from trade are positive, no matter how large the market, a buyer who observes the highest possible signal trades when the seller visits. The larger the market, the likelier that some buyer will observe this signal—regardless of the seller's quality. Beyond a threshold number of buyers, this hurts total surplus through the increased probability that a Low quality seller trades. As the number of buyers grows arbitrarily large, the seller almost surely trades. This yields the level of surplus in the no-information benchmark, $\rho - c$.

- When the expected gains from trade are negative, a buyer accepts the seller with an arbitrarily small probability in a large market, if at all. Adverse selection, however, ensures that she expects zero surplus from doing so. The seller may trade with positive probability, but total surplus is equal to that in the no-information benchmark: 0.

Notably, even when buyers have a signal that fully reveals High quality, total surplus need not converge to the full-information benchmark in *every* equilibrium. To see this, consider a slightly modified version of our running example. As before, the common prior is $\rho = 0.5$, and the seller's reservation value is $c = 0.2$. But we modify buyers' experiment; they now observe the outcome of $\mathcal{E}^g = (\mathbf{S}^g, p_L^g, p_H^g)$:

$$p_L^g(s) = \begin{cases} 1 & s = 0.2 \\ 0 & s = 1 \end{cases} \qquad\qquad p_H^g(s) = \begin{cases} 0.25 & s = 0.2 \\ 0.75 & s = 1 \end{cases}$$

The signal $s = 1$ fully reveals High quality. However, buyers trade regardless of the signal they receive in the least selective equilibrium—irrespective of the number of buyers in the market. Thus, the seller always trades. Total surplus always equals that in the no-information benchmark, $\Pi^{\emptyset} = 0.3$. On the other hand, buyers only trade upon the high signal $s = 1$ in the most selective equilibrium. Hence, a Low quality seller never trades. A High quality seller trades with probability $1 - (0.25)^n$ in a market with n buyers. In an arbitrarily large market, he trades almost surely; total surplus converges to the full-information benchmark, $\Pi^f = 0.5 \times [1 - 0.2]$.

Theorem 1 helps compare a decentralised market with a first price auction where the seller simultaneously solicits buyers' bids and sells to the highest bidder. There too, total surplus may decrease with an additional bidder (Riordan (1993)), due to an increased probability of trade when it is inefficient. Similar to a decentralised market, adverse selection is the culprit: the winner's curse is more severe in an auction with more bidders. However, adverse selection in a decentralised market differs from the winner's curse in an auction: a buyer who trades in a decentralised market understands that her signal was the highest among the *previous* buyers to be visited; a bidder who wins in an auction understands that her signal was the highest among *all* bidders. Thus, Riordan (1993)'s sufficient condition for an additional participant to decrease total surplus differs from the necessary and sufficient condition Theorem 1 recovers for total surplus to eventually increase with the number of buyers in a decentralised market.

This condition is also necessary and sufficient for buyers' common value to be revealed through trade in a decentralised market. Strikingly, the same condition is also necessary and sufficient for a first price common value auction to reveal bidders' common value through the winning bid[25]. However, when this condition is violated, a common value auction with an arbitrarily large number of bidders may nonetheless aggregate information "well"[26]. This is not the case in a decentralised market with an arbitrarily large number of buyers. Instead, three possibilities emerge:

1. The expected gains from trade are positive, $\rho > c$, and the seller almost surely trades. The fact that she trades is completely uninformative about the asset's quality.

2. The expected gains from trade are negative, $\rho < c$, and the seller never trades[27]. The fact that she does not trade is completely uninformative about the asset's quality.

3. The expected gains from trade are negative, $\rho < c$, and if a buyer trades, she expects zero surplus from doing so. Trade—when it happens—reveals that the asset has High quality with probability c, i.e., that the expected surplus from trade is zero[28].

So, whenever the likelihood ratio of buyers' signals is *not* unbounded at the top, trade is at most partially informative about the asset's quality, unlike in an auction. The information it reveals has no bearing on market participants' surplus: at most, trade is revealed to be to be no worse than no trade in expectation.

This offers an interesting contrast with Lauermann and Wolinsky, 2016, too[29]. They assume that trade is always efficient, and find that (generically) the outcome in a large market either fully reveals or is completely uninformative about buyers' common value for the asset. Theorem 1 echoes their finding where the expected gains from trade are positive, $\rho \geq c$. However, it also shows that another possibility arises when the expected gains from trade are negative, $\rho < c$: trade in a large market may be partially informative about the asset's quality.

5 Efficiency with Better-Informed Buyers

In this section, I discuss how giving each buyer better information—a Blackwell more informative experiment—affects total surplus. Throughout, I thus take the number of buyers n to be a primitive rather than a parameter. Equilibrium surplus attains its extremes in the most and least selective equilibria, so I focus on these equilibria. My main results describe the comparative statics of total surplus under both of these equilibria. For brevity, I write *equilibrium** wherever the reader may

[25]See Wilson (1977) and Milgrom (1979) for this classic result: in a setting where the item's common value may take countably many values, Milgrom (1979) recovers "distinguishability" as a necessary and sufficient condition. When the item's value is binary, "distinguishability" corresponds to an unbounded likelihood ratio from above.

[26]See, for instance, Section IV in Lauermann and Wolinsky (2017).

[27]For instance, if buyers' experiment is uninformative about the asset's value.

[28]Section 8 illustrates this scenario with a numerical example.

[29]Lauermann and Wolinsky (2017) find a similar result in a first price auction where the number of bidders varies with the common value of the auctioned item.

read *the most selective equilibrium* or *least selective equilibrium.* I denote the equilibrium* strategy for an experiment $\mathcal{E}$ as $\sigma^\star_{\mathcal{E}}$.

If there were a single buyer in the market—exposed to no adverse selection—a Blackwell improvement of her experiment would be necessary and sufficient for total surplus to rise regardless of the seller's reservation value[30]. The reason is simple: better information improves the buyer's ability to screen the asset and target efficient trades.

In a market with multiple buyers, buyers' ability to screen the seller is shaped both by the quality of their private signals and the extent of adverse selection each face in the market. So, better information becomes a double-edged sword. On the one hand, it allows each buyer to screen the seller more effectively. On the other hand, it might exacerbate adverse selection: previous refusals might become likelier, and each might carry worse news about the quality of the asset. This latter channel pushes the buyer to worse trades. If it overwhelms the buyers' increased ability to screen the asset's quality, it might lead to lower total surplus. The *kind* of improvement in buyers' information determines how this trade-off is resolved.

5.1 Binary Experiments

I start by restricting buyers to binary signals, where I obtain a sharper characterisation and offer the main insights which drive my result for general signal structures, in Theorem 3. A binary experiment $\mathcal{E}$ has two possible outcomes $s_1, s_2 \in \mathbf{S}$, which I relabel as $s_L, s_H \in \mathbf{S}$ for convenience[31]. The low outcome $s_L \in [0, 0.5]$ decreases a buyer's interim belief about the quality of the asset, while a high outcome $s_H \in [0.5, 1]$ increases it.

Ranking two binary experiments $\mathcal{E}'$ and $\mathcal{E}$ in their (Blackwell) informativeness is simple. Where the former has the possible outcomes $s'_L, s'_H \in \mathbf{S}'$, the experiment $\mathcal{E}'$ is Blackwell more informative than $\mathcal{E}$ if and only if[32]:

- it has a lower likelihood ratio at the bottom, $s'_L \leq s_L$; i.e., delivers *stronger bad news*, and
- it has a greater likelihood ratio at the top, $s'_H \geq s_H$; i.e. delivers *stronger good news.*

Theorem 2 answers how total surplus evolves when buyers' binary experiment becomes Blackwell more informative—delivers either stronger good news, or stronger bad news.

Theorem 2. Let buyers' experiment $\mathcal{E}$ be binary. Then, equilibrium* total surplus is increasing in the strength of good news (s_H) but is quasiconcave and eventually decreasing in the strength of bad news (s_L).

[30] In general, a Blackwell improvement is sufficient, but not necessary for a decision maker to extract higher value from a decision problem (see Blackwell (1953)). However, it is necessary for the decision maker to extract higher value from *any* decision problem where the unknown state is binary—such our buyers' screening problem. I present a self contained proof of this fact in Section 9.2, Lemma 12 for completeness.

[31] Recall from Section 3 that, to ease notation, I use labelling convention $s := \frac{p_H(s)}{p_H(s)+p_L(s)}$.

[32] See Section 12.5 in Blackwell and Girshick (1954) for a textbook exposition of this classic result.

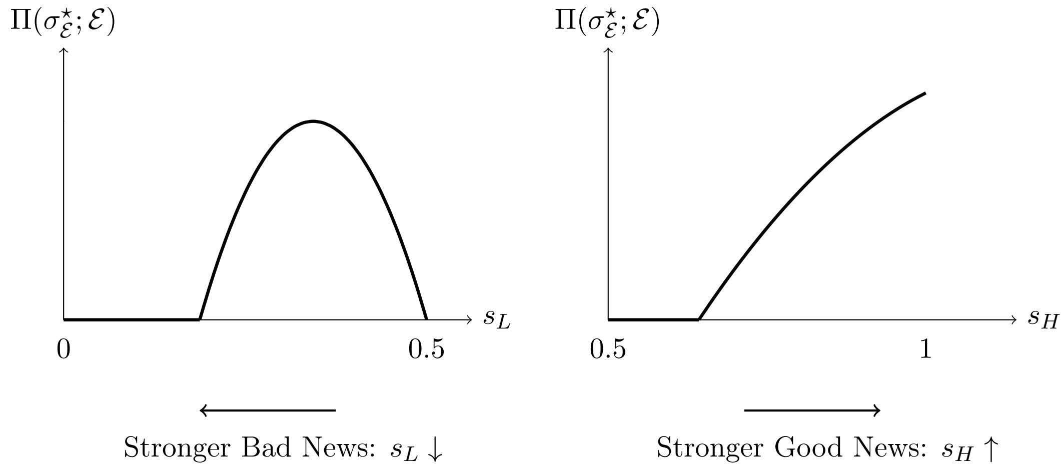


Figure 5.1: Theorem 2 illustrated

To understand the intuition behind Theorem 2, let us start from the case of stronger good news. Instead of the experiment $\mathcal{E}$, buyers now observe the outcome of $\mathcal{E}' = (\mathbf{S}', p'_L, p'_H)$ which delivers stronger good news than $\mathcal{E}$, $s'_H > s_H$, but the same strength of bad news as $\mathcal{E}$, $s_L = s'_L$. Brushing equilibrium considerations aside, simply assume that a buyer accepts upon the high signal, s_H or s'_H, and rejects upon the low signal, s_L or s'_L, under both experiments. How does, then, this improvement in buyers' information affect the seller's chances of trading?

The answer is clearest when we reinterpret the stronger good news buyers receive as an additional piece of evidence they might observe after an initial high signal, s_H, from the original experiment $\mathcal{E}$. After an initial low signal, s_L, a buyer observes no further evidence. However, after an initial high signal s_H, she observes the outcome of an *additional* binary experiment $\mathcal{E}^a = (\{s^a_L, s^a_H\}, p^a_L, p^a_H)$. Conditional on the asset's quality, $\mathcal{E}^a$ is independent from $\mathcal{E}$ and IID across buyers. We construc it carefully so that when appended to $\mathcal{E}$, it mimics the improvement in information that $\mathcal{E}'$ offers:

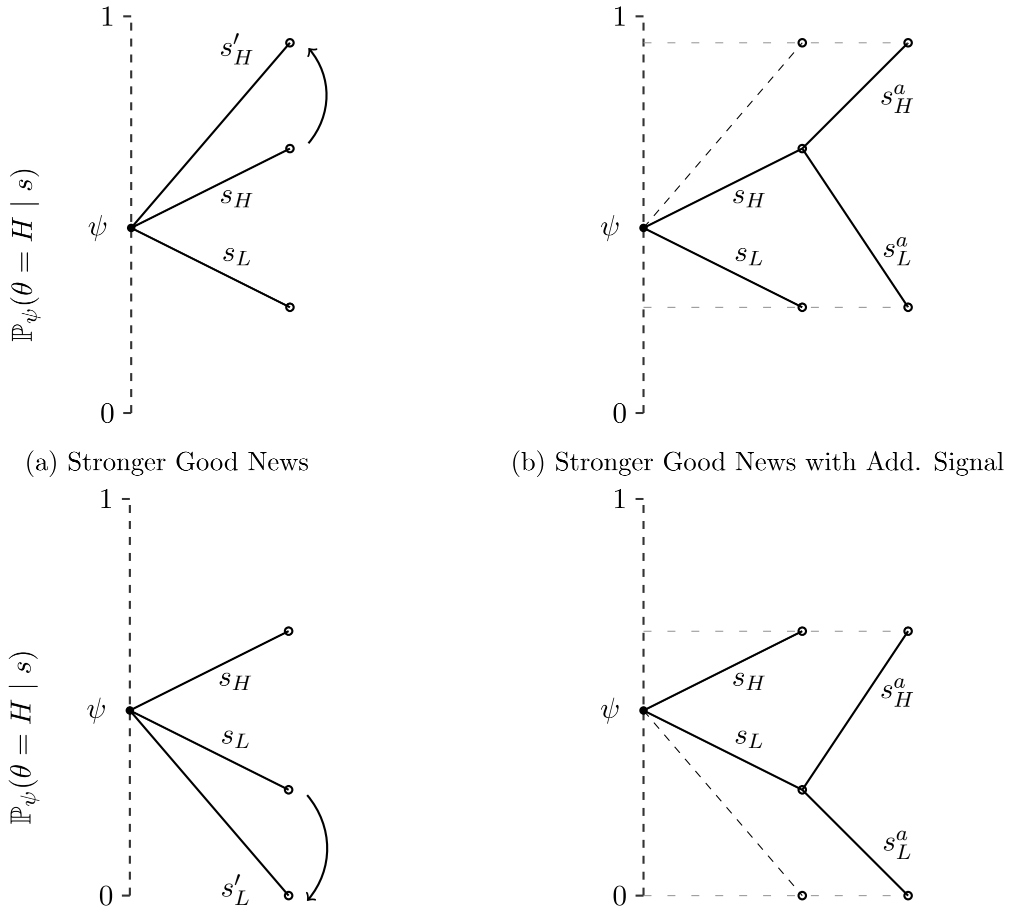
$$\frac{s_H}{1-s_H} \times \frac{s^a_H}{1-s^a_H} = \frac{s'_H}{1-s'_H} \qquad\qquad \frac{s_H}{1-s_H} \times \frac{s^a_L}{1-s^a_L} = \frac{s'_L}{1-s'_L}$$

Figure 5.2b illustrates this construction.

Thus, observing the sequence (s_H, s^a_H) conveys the same information as observing the signal s'_H from $\mathcal{E}'$. This information leads to an acceptance. The sequence (s_H, s^a_L) or simply s_L, instead, conveys the same information as observing the signal s'_L from $\mathcal{E}'$. This information leads to a rejection.

This reinterpretation reveals how additional information affects the seller's trading chances. A buyer who receives initial bad news through the signal s_L observes no additional evidence. She rejects the seller, as before. However, a buyer who receives initial good news through the signal s_H observes additional evidence through $\mathcal{E}^a$. Absent this additional evidence, she would have traded. But a low signal s^a_L from $\mathcal{E}^a$ *negatively overrides* that initial verdict: now, she rejects the seller.

So, stronger good news jeopardises trade: a seller who, before this improvement, would have



(a) Stronger Good News (b) Stronger Good News with Add. Signal

(c) Stronger Bad News (d) Stronger Bad News with Add. Signal

Figure 5.2: Blackwell Improvements of a Binary Signal

traded with some buyer, may now be rejected by every buyer. This raises total surplus: the seller is rejected by every buyer because each observe a low signal, either s_L or s^a_L. Upon this signal, each of those buyers—mindful that the seller *may* have received previous refusals—expect negative surplus from trade. If any of those buyers knew that the seller, in fact, was rejected by *every* buyer, her expectation would only deteriorate:

$$\underbrace{\mathbb{P}\left(\theta = H \mid s_L, n-1 \text{ rejections before visit}\right)}_{\text{belief given every other buyer rejected the seller}} < \underbrace{\sum_{k=0}^{n-1} \begin{bmatrix} \mathbb{P}\left(\theta = H \mid s_L, k \text{ rejections before visit}\right) \\ \times \ \mathbb{P}\left(k \text{ rejections before visit} \mid \text{visit}\right) \end{bmatrix}}_{\text{belief given some buyers rejected the seller}}$$

Now, let us turn to the case of stronger bad news. Theorem 2 establishes that there is a threshold level of bad news beyond which stronger bad news decreases total surplus. I characterise this threshold in Proposition 5, but to foreshadow it here, let us focus on a an experiment $\mathcal{E}'$ that delivers "marginally" stronger bad news than $\mathcal{E}$, i.e., $s'_L = s_L - \delta$ for a vanishingly small $\delta > 0$, but the same strength of good news as $\mathcal{E}$, $s'_H = s_H$.

As before, I reinterpret this improvement as an additional piece of evidence buyers observe; this time, after an initial low signal s_L. As illustrated in Figure 5.2d, we construct the additional binary experiment $\mathcal{E}^a$ buyers observe to mimic the improvement $\mathcal{E}'$ offers over $\mathcal{E}$:

$$\frac{s_L}{1-s_L} \times \frac{s_H^a}{1-s_H^a} = \frac{s_H}{1-s_H} \qquad\qquad \frac{s_L}{1-s_L} \times \frac{s_L^a}{1-s_L^a} = \frac{s_L'}{1-s_L'} = \frac{s_L-\delta}{1-(s_L-\delta)}$$

A buyer who receives initial good news through the signal s_H observes no additional evidence. She trades with the seller. However, a buyer who receives initial bad news through the signal s_L observes additional evidence through $\mathcal{E}^a$. Absent this additional evidence, she would have rejected the seller. But a high signal s_H^a from $\mathcal{E}^a$ *positively overrides* her initial verdict: now, she trades.

So, stronger bad news encourages trade: a seller who previously would be rejected by every buyer might now trade with one. The effect this has on total surplus, however, is less clear. What can we infer about the quality of the asset given stronger bad news allowed the seller to trade?

We are considering a seller who, before the improvement, was rejected by every buyer; so, initially all buyers observed low signals, s_L. After the additional experiment $\mathcal{E}^a$ was introduced, some buyer accepted to trade; so, at least one buyer observed the high signal, s_H^a. However, the rest observed the low signal s_L^a from this additional experiment. Whether this trade raises expected surplus depends on *how many* buyers observed the high signal s_H^a:

$$\mathbb{P}\left(\theta = H \mid \geq 1 \text{ buyer observed } s_H^a\right) = \sum_{k=1}^{n} \begin{bmatrix} \mathbb{P}\left(\theta = H \mid k \text{ buyers observed } s_H^a\right) \\ \times\, \mathbb{P}\left(k \text{ buyers observed } s_H^a \mid \geq 1 \text{ buyer obs. } s_H^a\right) \end{bmatrix}$$

However, we may deduce that almost surely only one buyer observed the high signal s_H^a. To see this, observe the likelihood ratios of the signals a buyer may observe from the experiment $\mathcal{E}^a$:

$$\frac{s_L^a}{1-s_L^a} = \frac{\frac{s_L-\delta}{1-(s_L-\delta)}}{\frac{s_L}{1-s_L}} \qquad\qquad \frac{s_H^a}{1-s_H^a} = \frac{\frac{s_H}{1-s_H}}{\frac{s_L}{1-s_L}}$$

While the likelihood ratio for the high signal s_H^a is constant, that for the low signal s_L^a converges to 1 as $\delta \downarrow 0$ and the improvement in buyers' information becomes "smaller". Due to the martingale property of likelihood ratios, these likelihood ratios must average to 1; so $\mathbb{P}_{\mathcal{E}^a}(s_H^a)$ must vanish as $\delta \downarrow 0$.

So, the expected surplus from this trade is non-negative if and only if:

$$\lim_{\delta \downarrow 0} \mathbb{P}\left(\theta = H \mid \geq 1 \text{ buyer observed } s_H^a\right) - c = \mathbb{P}\left(\theta = H \mid 1 \text{ buyer observed } s_H^a\right) - c \geq 0$$
$$\iff \frac{\rho}{1-\rho} \times \left[\frac{s_L}{1-s_L}\right]^{n-1} \times \frac{s_H}{1-s_H} \geq \frac{c}{1-c}$$

Marginally stronger bad news allows the *most adversely selected seller* to trade. Before buyers' information improved, he was rejected by every buyer. After, he is accepted by only one. The RHS of the expression above reflects this: trading with this seller increases surplus if and only if the high signal s_H^a observed by that one buyer overpowers the low signals s_L^a observed by the remaining

$n-1$ buyers[33].

This is a stark condition; when it holds, we may say that *adverse selection is irrelevant*: a buyer who observes the high signal need not be concerned about any previous refusals the seller may have received. It is also closely linked to the cutoff Theorem 2 identifies, described in Proposition 5.

Proposition 5. Where buyers' experiment is binary, total surplus in equilibrium* weakly decreases with stronger bad news (lower s_L) when:

$$\frac{\rho}{1-\rho} \times \max\left\{\frac{s_L}{1-s_L}, \left[\frac{s_L}{1-s_L}\right]^{n-1} \times \frac{s_H}{1-s_H}\right\} \leq \frac{c}{1-c}$$

This condition is also necessary in the least selective equilibrium.

Corollary 6. Where buyers' experiment is binary and $\rho \leq c$, equilibrium* total surplus weakly decreases with stronger bad news (lower s_L) when $\left(\frac{s_L}{1-s_L}\right)^{n-1} \times \frac{s_H}{1-s_H} \leq \frac{c}{1-c}$.

Total surplus falls with stronger bad news once bad news is strong enough to:

1. violate the "irrelevance of adverse selection" condition we informally identified, and
2. for buyers to reject the seller with positive probability in their equilibrium* strategies.

For some parameter constellations, the first condition is met before the second; bad news may need to get stronger before buyers reject the seller with positive probability. Before the strength of bad news hits this critical threshold, total surplus equals the no-information benchmark. Once it does, total surplus experiences a one time upward jump. Thereafter, stronger bad news decreases total surplus. When $\rho < c$ however, there is no parameter region where the seller always trades in equilibrium; the second condition is always satisfied. That leaves us with the "irrelevance of adverse selection" condition, as stated in Corollary 6.

5.2 Finite Experiments

In this section, I show how the ideas we developed for Theorem 2 generalise to Blackwell improvements of experiments with an arbitrary finite number of outcomes. We cannot deploy those ideas immediately: first, for non-binary experiments, the ideas of stronger good and bad news lose meaning; second, Blackwell improvements of such experiments are complex—they cannot be described by simple movements of likelihood ratios. Nonetheless, the core idea behind Theorem 2 supplies the answer: whether total surplus improves depends on whether an improvement is a *positive override*—information about a seller who would be rejected—or a *negative override*—information about a seller who would be approved.

Before I state Theorem 3, the main result of this section, I introduce Definition 2, which formalises the notion of a positive and negative override; and Definition 4, which formalises the notion of the "irrelevance of adverse selection" that I introduced informally in the previous section.

[33] Note that $s_L^a \to s_L$ as $\delta \downarrow 0$.

Definition 2. Enumerate the joint outcome set of the experiments $\mathcal{E}' = (\mathbf{S}', p'_L, p'_H)$ and $\mathcal{E} = (\mathbf{S}, p_L, p_H)$ as $\mathbf{S}' \cup \mathbf{S} = \{s_1, s_2, ..., s_M\}$. The experiment $\mathcal{E}'$ differs from $\mathcal{E}$ by a *local mean preserving spread* (or, *local spread*) *at* $s_j \in \mathbf{S}$ if:

$$p'_\theta(s_j) = 0 \qquad p_\theta(s_{j+1}) = p_\theta(s_{j-1}) = 0 \qquad p'_\theta(s_{j+1}) + p'_\theta(s_{j-1}) = p_\theta(s_j)$$

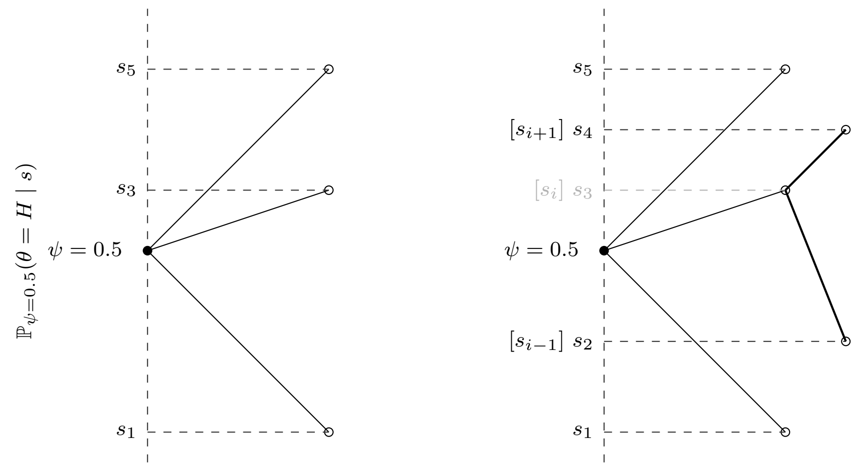
and $p'_\theta(s) = p_\theta(s)$ for any $s \in \mathbf{S}' \cup \mathbf{S} \setminus \{s_j\}$.

A local spread moves all the probability mass experiment $\mathcal{E}$ places on a particular signal to two new signals—one better news about the asset's quality, one worse. In this sense, we can think of it as providing additional information to a buyer who observes the original signal $s_j \in \mathbf{S}$.

Every local spread is an ordinary mean preserving spread[34]. The converse is not true; local spreads must move all the probability mass on a signal, and they must move it to its neighbouring signals: the mass on s_j is spread to s_{j+1} and s_{j-1}. I visualise the construction of a local spread in Figure 5.3, where I fix $\psi = 0.5$ for convenience.

Though local spreads cover a narrower set of improvements than ordinary mean preserving spreads, restricting to them is without loss for finite experiments[35]—every Blackwell improvement, and *a fortiori*, ordinary mean preserving spread can be constructed through a finite number of local spreads.

Remark 1. [Müller and Stoyan (2002), Theorem 1.5.29] An experiment $\mathcal{E}'$ is Blackwell more informative than another, $\mathcal{E}$, if and only if there is a finite sequence of experiments $\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_k$ such that $\mathcal{E}_1 = \mathcal{E}$, $\mathcal{E}_k = \mathcal{E}'$, and $\mathcal{E}_{i+1}$ differs from $\mathcal{E}_i$ by a local spread.



(a) Before local spread

(b) After local spread

Figure 5.3: A Local Mean Preserving Spread

[34] Specifically, a "3-part MPS", in the language of Rasmusen and Petrakis (1992). Mean preserving spreads are originally due to Muirhead (1900) and were popularised in economics by Rothschild and Stiglitz (1970).

[35] However, local mean preserving spreads are only defined for finite experiments; see Müller and Scarsini (2001) and Müller and Stoyan (2002).

Definition 3. Let experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at s_j and $\sigma^\star_{\mathcal{E}}$ denote the equilibrium* strategy under experiment $\mathcal{E}$. I say this local spread is a *negative override* under equilibrium* if $\sigma^\star_{\mathcal{E}}(s_j) = 1$, and a *positive override* if $\sigma^\star_{\mathcal{E}}(s_j) = 0$ instead.

Both positive and negative overrides are local spreads of a buyer's experiment, but they differ in *which* seller they inform the buyer about. A negative override is a local spread of a signal upon which the buyer would trade in the equilibrium* of her original experiment. A positive override is a local spread of a signal upon which the buyer would reject trade in the equilibrium* of her original experiment.

Lastly before I introduce Theorem 3, I formalise the "irrelevance of adverse selection" condition we identified in the previous section, and extend it to arbitrary finite experiments.

Definition 4. Where σ is a monotone strategy for a fixed experiment $\mathcal{E}$, I say *adverse selection is σ-irrelevant for signal $s \in \mathbf{S}$* if:

$$\frac{\rho}{1-\rho} \times \left[\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right]^{n-1} \times \frac{s}{1-s} \geq \frac{c}{1-c}$$

When adverse selection is σ-irrelevant for a signal $s \in \mathbf{S}$, a buyer finds it optimal to trade upon the signal $s \in \mathbf{S}$ even if every other buyer rejected the seller—provided those buyers use the strategies σ.

Theorem 3. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j \in \mathbf{S}$. Equilibrium* total surplus is:

1. weakly greater under $\mathcal{E}'$ if the local spread is a negative override under equilibrium*.

2. weakly less under $\mathcal{E}'$ if the local spread is a positive override under equilibrium*, unless adverse selection is $\sigma^\star_{\mathcal{E}}$-irrelevant for signal s_{j+1}.

Theorem 3 shows that the effect of a local spread on total surplus depends on the *kind* of the spread. Negative overrides always increase total surplus. Positive overrides decrease it—unless adverse selection is irrelevant for a buyer who receives the override.

To prove Theorem 3, I show that a negative override indeed pushes buyers to reject the seller more often in the new equilibrium*. A positive override pushes buyers to trade more often with him. This exercise is severely complicated by the fact that identifying how buyers' interim beliefs change with their experiments is infeasible beyond the simplest cases. Studying "local", not ordinary, mean preserving spreads is crucial for tractability; this allows us to identify how equilibria respond to improvements in information without needing to pinpoint changes in interim beliefs.

. They provide the necessary tractability by allowing equilibrium comparative statics without needing to pinpoint changes in interim beliefs. Furthermore, there is no cost to generality: "local" spreads, like ordinary spreads, form the basis of every Blackwell improvement.

Unlike Theorem 2, Theorem 3 requires knowledge of buyers' equilibrium* strategies to identify the effect of an improvement in information on efficiency. In practice, an analyst might want to remain agnostic about equilibrium* strategies. To alleviate this concern, Proposition 7 offers a sufficient condition for a positive override to decrease total surplus in the most selective equilibrium.

Proposition 7. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at s_j. Total surplus in the most selective equilibrium is lower under $\mathcal{E}'$ if the following conditions hold:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right) \leq \frac{c}{1-c} \qquad \text{and} \qquad \frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

The sufficient conditions in Proposition 7 weaken the necessary and sufficient conditions supplied by Theorem 3. The condition on the left ensures that the local spread is a negative override; a buyer rejects the seller upon s_j in any equilibrium since interim beliefs always lie below the prior belief, ρ. The condition on the right weakens the irrelevance condition for adverse selection: it requires that a rejection be optimal even if the $n-1$ rejections the seller received were due to the best signals below s_{j+1}, s_j.

6 Maximising Surplus by Coarsening Information

Section 5 showed that efficiency might be lower in a market where buyers are better informed; more information in the market accentuates the adverse selection problem each buyer faces, counteracting each buyer's improved ability to screen the asset's quality. This invites a question: if a regulator could coarsen buyers' information—perhaps through banning the use and dissemination of certain data—could this raise efficiency? How should a regulator who can use this tool to maximise efficiency in the market go about this exercise?

In this section, I consider the problem of a regulator who wishes to garble buyers' experiment $\mathcal{E}$ in order to maximise total surplus in (the most selective) equilibrium. As in the previous section, I take the number of buyers n to be a primitive not a parameter. The regulator can choose any finite garbling $\mathcal{E}^G = \left(\mathbf{S}^G, p_L^G, p_H^G\right)$ of buyers' experiment $\mathcal{E}$; i.e. any finite set of outcomes $\mathbf{S}^G = \left\{s_1^G, s_2^G, ..., s_R^G\right\}$ and probability mass functions $p_\theta^G(.)$ over it such that for some Markov matrix $\mathbf{T}_{m\times R}$:

$$\underbrace{\begin{bmatrix} p_L(s_1) & \cdots & p_L(s_m) \\ p_H(s_1) & \cdots & p_H(s_m) \end{bmatrix}}_{=\mathbf{P}} \times \mathbf{T} = \underbrace{\begin{bmatrix} p_L^G(s_1^G) & \cdots & p_L^G(s_R^G) \\ p_H^G(s_1^G) & \cdots & p_H^G(s_R^G) \end{bmatrix}}_{=\mathbf{P^G}}$$

We can interpret this as a coarsening of the original data available to each buyer, generated with the process that matrix $\mathbf{P}$ summarises. The regulator transforms this data, using the Markov matrix $\mathbf{T}$, into a set of summary statistics. The buyer only observes this set of summary statistics, whose data generating process is now described by the matrix $\mathbf{P^G}$.

Once the regulator chooses the garbled experiment $\mathcal{E}^G$, the game proceeds as before—only, buyers' experiment $\mathcal{E}$ is replaced by $\mathcal{E}^G$. An equilibrium, as before, is a pair $\left(\sigma^G, \psi^G\right)$ such that the strategy $\sigma^G : \mathbf{S}^G \to [0,1]$ is optimal given the interim belief ψ^G; and the interim belief ψ^G is consistent with the strategy σ^G. I call this the game induced by the garbling $\mathcal{E}^G$. I call a garbling $\mathcal{E}^G$ regulator-optimal if efficiency in the most selective equilibrium of the game it induces weakly exceeds equilibrium efficiency in the game induced by any other garbling of $\mathcal{E}$.

In Proposition 9, I show that the regulator coarsens buyers' experiment to a recommender signal which transforms the original signal the buyer would have observed to an "acceptance" or "rejection" recommendation. The regulator wishes to recommend a rejection following every signal a buyer could observe unless adverse selection is irrelevant at that outcome. Before I describe the regulator-optimal garbling in Proposition 9, I describe the special class of garblings in which it is contained—monotone binary garblings which provide optimal recommendations. I then show how we can use the idea of the "irrelevance of adverse selection"—which I used to describe equilibria previously—to describe garblings.

I call a garbling $\mathcal{E}^G$ *monotone binary* if the garbling has two possible outcomes, $|\mathbf{S}^G| = 2$, and there is a cutoff signal $s_{i^*} \in \mathbf{S}$ such that the entries $\{t_{ij}\}$ of matrix $\mathbf{T}_{m\times 2}$ for which $\mathbf{P} \times \mathbf{T} = \mathbf{P^G}$ are:

$$t_{i1} = \begin{cases} 1 & i < i^* \\ \in [0,1] & i = i^* \\ 0 & i > i^* \end{cases} \qquad\qquad t_{i2} = 1 - t_{i1}$$

When defining a monotone binary garbling for which "adverse selection is irrelevant", it will be convenient to refer to the signal $s_{i^*} \in \mathbf{S}$ as the *threshold signal* of the garbling.

A monotone binary garbling gives the buyer an "acceptance recommendation" s_2^G when her original signal realises above a threshold signal $s_{i^*} \in \mathbf{S}$, and a "rejection recommendation" s_1^G whenever it lies below it. Following these recommendations—accepting trade upon the signal s_2^G, and rejecting when it upon the signal s_1^G—need not be an equilibrium strategy in the game induced by the coarsened experiment $\mathcal{E}^G$. When it does, I say that the garbling $\mathcal{E}^G$ *produces incentive compatible recommendations.*

Definition 5. A monotone binary garbling $\mathcal{E}^G$ *produces incentive compatible (IC) recommendations* if the strategy σ^G, defined below, is an equilibrium strategy in the game induced by $\mathcal{E}^G$:

$$\sigma^G(s^G) := \begin{cases} 0 & s^G = s_1^G \\ 1 & s^G = s_2^G \end{cases}$$

Lemma 8 establishes that the regulator can restrict herself to monotone binary garblings that produce IC recommendations.

Lemma 8. Where it exists, the regulator-optimal garbling is monotone binary and produces IC recommendations.

The reason that the regulator can restrict herself to binary garblings is closely connected to a fundamental principle in information design. A buyer ultimately distils the information relayed by the garbled experiment into an action recommendation; she either accepts or rejects. The regulator can distil that information herself—supplying only recommendations to a buyer[36]. Monotone garblings align buyers' decisions better with the regulator's goal of maximising efficiency—it raises the chance that the seller is accepted when the asset has High, but not Low quality. The non-trivial conclusion Lemma 8 establishes that the regulator faces no trade-off between a monotone garbling and one whose recommendations are IC.

Since monotone garblings, like monotone strategies, divide the set of outcomes the experiment $\mathcal{E}$, $\mathbf{S}^G$, into an acceptance and rejection region, we can adopt the "selectivity" order for them as well.

Definition 6. A monotone garbling $\mathcal{E}^G$ of $\mathcal{E}$ is *more selective* than another, $\mathcal{E}^{G'}$, if $p_\theta^G\left(s_2^G\right) \leq p_\theta^{G'}\left(s_2^{G'}\right)$ for all $\theta \in \{L, H\}$.

Like monotone strategies, and for the same reason, selectivity is a complete order over the set of monotone binary garblings.

The last idea needed to describe the regulator-optimal garbling is the "irrelevance of adverse selection" for a garbling.

Definition 7. Let $\mathcal{E}^G$ be a monotone binary garbling of $\mathcal{E}$, with the threshold signal $s^* \in \mathbf{S}$. *Adverse selection is irrelevant* under $\mathcal{E}^G$ either if:

$$\frac{\rho}{1-\rho} \times \frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{p_H^G(s_1^G)}{p_L^G(s_1^G)}\right)^{n-1} \geq \frac{c}{1-c}$$

or either of the following two conditions hold:

1. $\mathcal{E}^G$ recommends no acceptances; i.e. $p_\theta(s_2^G) = 0$.
2. $\mathcal{E}^G$ recommends no rejections; i.e. $p_\theta(s_1^G) = 0$ and $\frac{\rho}{1-\rho} \times \left(\frac{s_1}{1-s_1}\right)^n \geq \frac{c}{1-c}$.

If this condition is violated, I say that adverse selection is *not* irrelevant under $\mathcal{E}^G$.

This condition translates the corresponding idea we developed to describe strategies to describe garblings—*adverse selection is irrelevant* under a garbling if a buyer who receives an "acceptance" recommendation need not be concerned about the number of buyers who received "rejection" recommendations. This property is also satisfied when the garbling never recommends an "acceptance",

[36]Note that coarsening buyers' experiment $\mathcal{E}$ may also create new equilibrium outcomes, some of which yield lower payoff than the previous least selective equilibrium. Our focus on the most selective equilibrium—besides being the appropriate focus for this exercise—frees us from the need to worry about this complication and utilise this fundamental principle.

or when accepting the seller would yield positive expected surplus regardless of the signals the n buyers could observe under their original experiment.

Proposition 9. If the least selective monotone binary garbling under which adverse selection is irrelevant produces IC recommendations, it is the regulator-optimal garbling. Otherwise, the regulator-optimal garbling is either:

- the least selective garbling under which adverse selection is irrelevant, or
- the most selective garbling under which adverse selection is not irrelevant

among monotone binary garblings which produce IC recommendations.

Corollary 10. When the seller's reservation value c weakly exceeds the prior belief ρ, the regulator-optimal garbling is the least selective monotone binary garbling under which adverse selection is irrelevant.

Proposition 9 reveals that the solution to the regulator's problem takes a striking form: although the regulator wishes to maximise a buyer's *expected* contribution to trade surplus, she focuses on the "worst case" where a buyer is the last to receive the seller.

Proposition 9 is intimately connected to the insight the previous section delivers: unless "adverse selection is irrelevant", information which pushes buyers to accept trade more often can harm efficiency. The regulator whishes to censor such information by coarsening buyers' experiment: if she were not bound by buyers' incentives to follow her recommendations, her optimal garbling would bundle every outcome of the original experiment $\mathcal{E}$ into a "rejection recommendation" unless adverse selection is irrelevant at that outcome. Corollary 10 establishes that in a substantial parameter region—when the seller's reservation value c weakly exceeds buyers' prior belief ρ—such recommendations are IC, and hence are adopted by the regulator.

7 Ultimatum Price Offers by Buyers

In this section, I relax the assumption that the seller trades with the first buyer willing to pay his reservation value c. I show that this behaviour emerges endogenously in the extended model here, where each buyer the seller visits makes him an ultimatum (take-it-or-leave-it) price offer. Specifically, I show that in the only equilibrium that is robust to the seller's information about the asset's quality, a buyer either offers to pay the seller his reservation value c or nothing—unless a seller of some quality can guarantee trade with the first buyer he visits[37]. To simplify notation, in this extended model I assume that the seller knows the quality of the asset[38]. Proposition 11

[37]In this latter circumstance, a multiplicity of equilibria with no relevance to our exercise might emerge. In all such equilibria, either some buyer almost surely trades with the seller, or a buyer trades with the seller unless she receives conclusive evidence that the quality is Low (at any signal a High quality seller might generate). In the latter case, buyers might coordinate on some fixed price c that they offer a seller who has not revealed himself Low quality. These have no bearing on trading patterns besides shifting surplus to the seller.

[38]Otherwise, the notation gets more involved since the seller will also consider what a buyer's offer reveals about the asset's quality and therefore what offers he should expect should he continue his visits.

establishes that in this extended model, the aforementioned equilibrium is unique. The argument it presents straightforwardly reveals that the proposed equilibrium survives regardless of what the seller knows about the asset's quality.

As before, the seller visits n prospective buyers in a random order. Each buyer he visits makes him a take-it-or-leave-it price offer; the strategy $\omega : \mathbf{S} \to \Delta(\{0\} \cup [c,1])$ maps every signal a buyer might observe to a distribution over possible price offers. Without loss of generality, I exclude offers in the interval $(0,c)$; since these prices are below the seller's reservation value c, they will always be rejected—the buyer might as well offer 0 instead. The seller either takes the offered price o, or leaves it and visits the next buyer. If he eventually takes an offer of o, he enjoys a payoff of $o-c$. If he trades with no buyer, he enjoys his reservation value for the asset, c. Given the quality θ of his asset and for each of the $k \in \{1,2,...,n\}$ visits he might make, the seller's strategy $\chi_{k,\theta} : [0,1] \to [0,1]$ is a measurable mapping from his k^{th} offer to a probability that he takes the offer. I assume that the seller accepts his current offer if and only if his payoff from doing so weakly exceeds his expected continuation payoff; so, each $\chi_{k,\theta} : [0,1] \to [0,1]$ will be a cutoff strategy in equilibrium[39].

Given the strategies ω^* for each buyer, we can recursively calculate the continuation value the seller expects right before he pays his k^{th} buyer, $V^*_{k;\theta}$. Denoting the seller's value of not selling the asset as $V^*_{n+1;\theta} := c$, we get:

$$V^*_{k;\theta} := \sum_{s\in\mathbf{S}} p_\theta(s) \times \int\limits_{\{0\}\cup[c,1]} \max\left\{V^*_{k+1;\theta}, m-c\right\} d\omega(s)(m)$$

Furthermore, given the cutoff strategies $\chi^*_{k,\theta} : [0,1] \to [0,1]$ for the seller and the strategy ω^* for each buyer, we can calculate the probability that a seller of quality θ does not trade in his k^{th} visit as:

$$r^e_{k,\theta}(\omega^*;\mathcal{E}) = \sum_{s\in\mathbf{S}} p_\theta(s) \times \int\limits_{\{0\}\cup[c,1]} \left(1-\chi^*_{k,\theta}(m)\right) d\omega(s)(m)$$

Through these probabilities and by using Bayes Rule, the probability that the seller is in his k^{th} visit by the time he visited a given buyer can be easily calculated; call it $\kappa_{\omega^*}(k)$. Likewise, denote the probability a buyer assigns to the asset having High quality given her observed the signal $s \in \mathbf{S}$ and that the seller is in his k^{th} visit and as $\iota_{\omega^*}(s)$.

The strategies ω^* and $\left\{\chi^*_{k,\theta}\right\}_{k=1}^{n}$ for each quality $\theta \in \{L,H\}$ form an equilibrium in this extended model if they maximise the buyers' and the seller's expected payoffs, respectively, given their equilibrium beliefs:

$$\chi^*_{k,\theta}(o) = \begin{cases} 1 & o-c \geq V^*_{k+1,\theta} \\ 0 & o-c < V^*_{k+1,\theta} \end{cases} \qquad \operatorname{supp} \omega^*(s) \subseteq \underset{o\in\{0\}\cup[c,1]}{\arg\max} \sum_{k=1}^{n} \kappa_{\omega^*}(k) \times \begin{bmatrix} \iota_{\omega^*}(s) \times (1-o) \times \chi_{k,H}(o) \\ -(1-\iota_{\omega^*}(s)) \times o \times \chi_{k,L}(o) \end{bmatrix}$$

[39]Hence, assuming measurability is without loss.

Proposition 11. Unless a seller of some quality almost surely trades with the first buyer he visits, in the unique equilibrium of the extended model:

- a buyer offers either the seller's reservation value c or nothing; supp $\omega^* \subseteq \{0, c\}$.
- the seller trades with the first buyer who offers his reservation value; $\chi_{k,\theta}(o) = \mathbb{1}\{o \geq c\}$.

Proof. The proposed tuple of strategies obviously form an equilibrium. If, almost surely, every buyer offers a price weakly below c, no buyer has an incentive to offer anything above it. The seller's continuation value always equals c, so he takes the first offer of c.

Now, I prove that this is the unique equilibrium unless a seller of some quality can guarantee with the first buyer he visits. Note that the continuation value $V^*_{k,\theta}$ must be weakly decreasing in k in every equilibrium. Furthermore, it must always lie weakly above c. Denote $\theta^* \in \arg\max_{\theta \in \{L,H\}} V^*_{1,\theta}$. If $V_{1,\theta^*} = c$, then the seller is almost surely offered the price 0 or c in any of his visits; we are done. If $V_{1,\theta^*} > c$, any offer $o > V_{1,\theta^*} + c$ is strictly dominated for a buyer: the seller's payoff from taking the offer o is strictly greater than V_{1,θ^*} so he will also take an offer slightly below o. So, the seller can expect to be offered at most $V_{1,\theta^*} + c$. Since his continuation value at the first visit is V_{1,θ^*}, the seller almost surely expects the offer $V_{1,\theta^*} + c$. But this reveals that a seller of quality θ^* almost surely trades with the first buyer he visits: for all $s \in$ supp P_{θ^*}, $\omega^*(s) = \delta_{V_{1,\theta^*}+c}$ and the seller immediately trades.

$\square$

8 Supplemental Appendix

Section 4 claimed that the outcome in a large market might be partially informative about the common value of the asset. This section presents a numerical example to demonstrate this claim.

Let buyers' experiment $\mathcal{E}$ be binary; $\mathbf{S} = \{0.2, 0.8\}$ and:

$$p_L(s) = \begin{cases} 0.8 & s = 0.2 \\ 0.2 & s = 0.8 \end{cases} \qquad\qquad p_H(s) = \begin{cases} 0.2 & s = 0.2 \\ 0.8 & s = 0.8 \end{cases}$$

Furthermore, let buyers' common prior be $\rho = 0.5$ and the seller's reservation value be $c = 0.6$. For any number of buyers, the equilibrium of this game is unique[40]: buyers always reject upon the low signal, $\sigma_n^*(0.2) = 0$, but accept with some probability upon the high signal, $\sigma_n^*(0.8) \in [0, 1]$.

Figures 8.b through 8.f plot (a) total surplus in the market, and the probabilities that (b) a buyer trades upon receiving the high signal, (c and d) some buyer trades with a seller, given his quality is High or Low (e and f) the probability that a seller has High quality given he trades with some buyer or no buyer, as the number of buyers rises from 1 to 50.

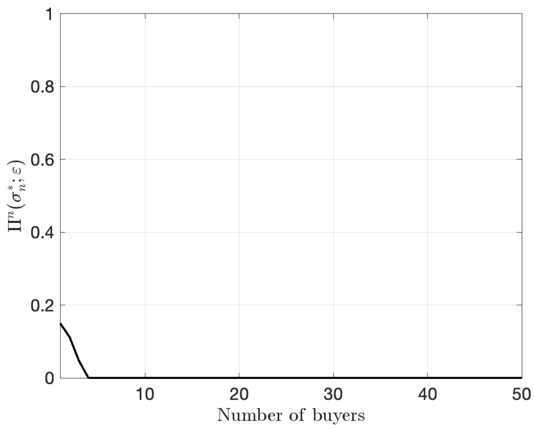


Figure 8.a: Total surplus

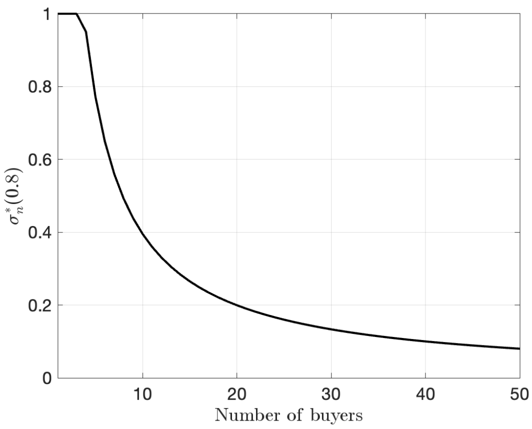


Figure 8.b: Probability that buyer accepts upon $s = 0.8$

[40]This follows a simple argument. No buyer can accept trade upon the low signal, since $\psi^* \leq 0.5$, so $\mathbb{P}_{\psi^*}(\theta = H \mid 0.2) \leq 0.2 < c$. As Lemma 15 shows, the interim belief is strictly decreasing in the probability that buyers approve upon the high signal. Thus, buyers either always accept trade at this signal, or there is a unique interior probability of acceptance that—given the interim belief consistent with those strategies—buyers are indifferent to accept.

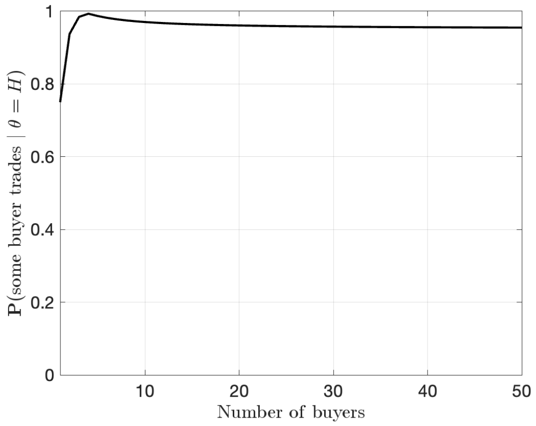


Figure 8.c: Probability that
some buyer trades when $\theta = H$

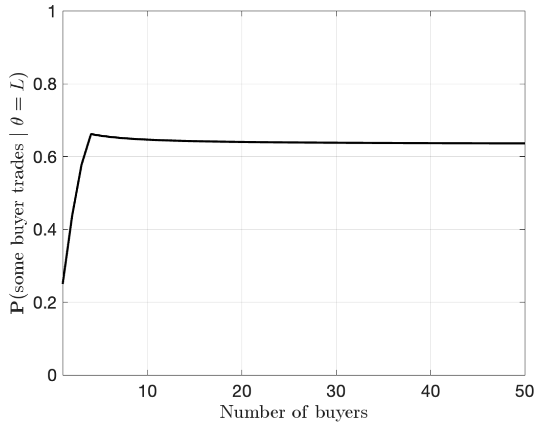


Figure 8.d: Probability that
some buyer trades when $\theta = L$

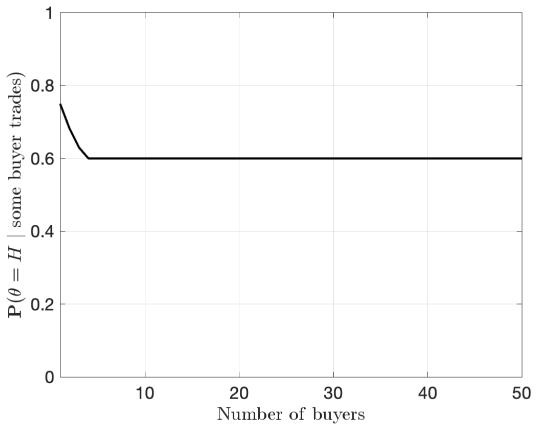


Figure 8.e: Probability that
$\theta = H$ given some buyer trades

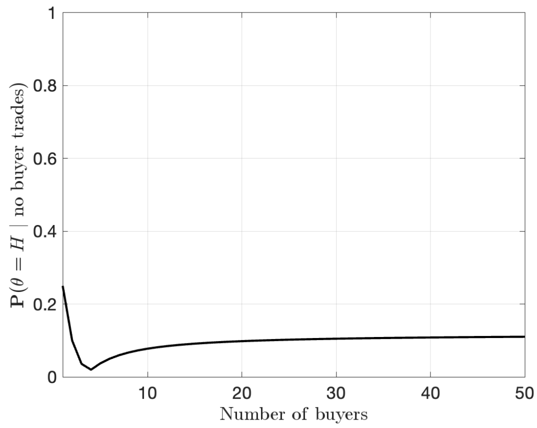


Figure 8.f: Probability that
$\theta = H$ given no buyer trades

Once the number of buyers in the market exceeds 3, total surplus in the market equals 0—the no-information benchmark. A buyer only trades if she receives the high signal. Even then, she rejects the seller with a strictly positive probability; this probability increases with the number of buyers in the market. However, the probability that some buyer trades with the seller reaches a constant level: 0.95 if he has High quality, and 0.63 if he has Low quality. Despite having no bearing on market participants' surplus, trade is informative about the seller's quality: the probability that the seller has High quality conditional on trading is 0.6 not $\rho = 0.5$; and the probability she has High quality conditional on not trading reaches 0.11[41].

[41]For clarity, the figures illustrate results as $n \to 50$; however, these asymptotic values remain valid as $n \to 1000$. As $n \to 1000$, the probability that a buyer accepts trade upon a high signal converges to 0.

9 Proof Appendix

9.1 Useful Definitions and Notation

In what follows, I occasionally operate with the likelihood ratios of beliefs for convenience. The reader can easily verify the identities:

$$\frac{\psi}{1-\psi} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})} \qquad\qquad \frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} = \frac{\psi}{1-\psi} \times \frac{s_i}{1-s_i}$$

Through similar reasoning, the reader can verify that it is optimal for a buyer to accept trade when:

$$\frac{\mathbb{P}_\psi(\theta = H \mid s_i)}{1-\mathbb{P}_\psi(\theta = H \mid s_i)} > \frac{c}{1-c}$$

Some strategies require buyers to randomise upon observing a particular outcome. To facilitate technical discussion, where it is warranted I assume that each buyer observes the realisation of a *tie-breaking signal* $u \sim U[0,1]$ alongside the outcome of her experiment. This signal is not informative about the asset's quality: it is distributed independently from it conditional on the experiment's outcome. I denote the outcome of buyer i's experiment as s^i and her tie-breaking signal as u^i. Without loss, buyer i accepts trade if and only if $\sigma(s^i) \leq u^i$; where σ is her strategy. I call the pair (s^i, u^i) the *score* buyer i observes for the seller.

Definition 8. The tuple $Z^i = (s^i, u^i)$, where $u^i \overset{IID}{\sim} U[0,1]$ is the *score* buyer i observes for the seller. The seller's *score profile* $\mathbf{z}$ is the set of scores each buyer observes; $\mathbf{z} = \{(s^i, u^i)\}_{i=1}^n$. Analogously, the seller's *signal profile* $\mathbf{s} = \{s^i\}_{i=1}^n$ is the set of outcomes of each buyer's experiment.

Some proofs in Section 9.3 require comparing interim beliefs across pairs of strategies and experiments; $(\sigma, \mathcal{E})$. For convenience, I define the mapping from such a pair to the interim belief consistent with them as $\Psi(.;\mathcal{E}) : [0,1]^n \to [0,1]$:

$$\Psi(\sigma;\mathcal{E}) := \frac{\rho \times \nu_H(\sigma;\mathcal{E})}{\rho \times \nu_H(\sigma;\mathcal{E}) + (1-\rho) \times \nu_L(\sigma;\mathcal{E})}$$

Wherever necessary, I treat each strategy $\sigma : \mathbf{S} \to [0,1]$ for an experiment $\mathcal{E}$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$. This is a finite dimensional vector space, so I endow it with the metric induced by the taxicab norm without loss of generality (see Kreyszig (1978) Theorem 2.4-5):

$$||\sigma' - \sigma|| = \sum_{j=1}^{m} |\sigma'(s_j) - \sigma(s_j)| \qquad\qquad \text{for any two strategies } \sigma' \text{ and } \sigma$$

Note that the interim belief function $\Psi(.;\mathcal{E})$ is thus a continuous function of buyers' strategies[42].

[42] r_θ is a continuous function of σ, thus both the nominator and denominator are strictly positive continuous functions of σ.

Definition 9. Where the experiment $\mathcal{E}$ is binary, s_L^{mute} is the *strongest* level of bad news for which there is an equilibrium where a buyer trades regardless of her signal:

$$\frac{\rho}{1-\rho} \times \frac{s_L^{\text{mute}}}{1-s_L^{\text{mute}}} = \frac{c}{1-c}$$

9.2 Omitted Results

Lemma 12. Suppose there is a single buyer, $n = 1$. Equilibrium total surplus under experiment $\mathcal{E}'$ exceeds that under $\mathcal{E}$ *regardless of the seller's reservation value* $c \in [0, 1]$ *and buyer's prior belief* $\rho \in [0, 1]$ if and only if $\mathcal{E}'$ is (Blackwell) more informative than $\mathcal{E}$.

Proof. The sufficiency part of this Lemma follows from Blackwell's Theorem (Blackwell and Girshick (1954), Theorem 12.2.2). To show necessity, I fix an arbitrary prior belief ρ for the evaluator.

Let q_j be the posterior belief the buyer forms about the asset's quality upon observing the outcome $s_j \in \mathbf{S}$:

$$q_j = \frac{\rho \times s_j}{\rho \times s_j + (1-\rho) \times (1-s_j)}$$

Furthermore, let $F(.)$ and $F'(.)$ be the CDFs of the posterior beliefs $\mathcal{E}$ and $\mathcal{E}'$ induce, respectively, for this prior belief ρ:

$$F(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p_H(s)$$
$$F'(q) = (1-\rho) \times \sum_{s \in \mathbf{S}: s \leq q} p'_L(s) + \rho \times \sum_{s \in \mathbf{S}: s \leq x} p'_H(s)$$

Equilibrium total surplus (and the buyer's expected payoff) under $\mathcal{E}$ is given by:

$$\int_c^1 (q-c) dF(q) = \int_c^1 q dF(q) - c \times (1 - F(c)) = (1-c) - \int_c^1 F(q) dq$$

An analogous expression gives equilibrium total surplus under $\mathcal{E}'$. For the former to exceed the latter for any $c \in [0, 1]$, we must have:

$$\int_c^1 \left(F(q) - F'(q)\right) dq \geq 0$$

which is equivalent to $\mathcal{E}'$ being Blackwell more informative than $\mathcal{E}$[43].

$\square$

Lemma 13 proves useful when proving Theorem 3, the main result of Section 5.2. This Lemma can also be used towards an alternative and direct proof for the equilibrium existence claim of

[43]See Müller and Stoyan (2002), Theorem 1.5.7. The Blackwell order between signal structures is equivalent to the convex order between the posterior belief distributions they induce; see Gentzkow and Kamenica (2016).

Proposition 1.

Lemma 13. For each $j \in \{1, 2, ..., m\}$, let σ_j be the strategy defined as:

$$\sigma_j(s) = \begin{cases} 0 & s < s_j \\ 1 & s \geq s_j \end{cases}$$

and ψ_j be the interim belief consistent with this strategy. Unless σ_j is itself an equilibrium strategy:

i There is an equilibrium strategy σ^* that is *more selective than* σ_j if $\mathbb{P}_{\psi_j}(\theta = H \mid s_j) < c$.

ii There is an equilibrium strategy σ^* that is *less selective than* σ_j if $\mathbb{P}_{\psi_j}(\theta = H \mid s_{j-1}) > c$.

Proof. Abusing notation slightly, I add two fully revealing outcomes s_0 and s_{m+1} to the set $\mathbf{S}$ (duplicating s_1 and s_m if either of them are already fully revealing), and denote the strategy which *never* accepts trade as σ_{m+1}:

$$\frac{s_{m+1}}{1 - s_{m+1}} = \infty \qquad \frac{s_0}{1 - s_0} = 0 \qquad \frac{\psi_{m+1}}{1 - \psi_{m+1}} = \frac{\rho}{1 - \rho}$$

Claim i.

The strategy σ_{m+1} is the most selective strategy buyers can adopt, and is an equilibrium strategy unless:

$$\frac{s_m}{1 - s_m} \times \frac{\psi_{m+1}}{1 - \psi_{m+1}} > \frac{c}{1 - c}$$

So, assume this condition is satisfied. Likewise, the strategy σ_k for $k \geq j$ is an equilibrium if the following inequality is satisfied:

$$\frac{s_k}{1 - s_k} \times \frac{\psi_k}{1 - \psi_k} \geq \frac{c}{1 - c} \geq \frac{s_{k-1}}{1 - s_{k-1}} \times \frac{\psi_k}{1 - \psi_k} \tag{9.1}$$

So, assume inequality 9.1 is violated for every $k \geq j$. This gives us:

$$\frac{s_{m+1}}{1 - s_{m+1}} \times \frac{\psi_{m+1}}{1 - \psi_{m+1}} > \frac{c}{1 - c} > \frac{s_j}{1 - s_j} \times \frac{\psi_j}{1 - \psi_j} \tag{9.2}$$

where the last part of this inequality is by hypothesis.

Now, let $k^* \in \{j, j + 1, ..., m\}$ be the first index for which the following inequality is satisfied:

$$\frac{s_{k^*+1}}{1 - s_{k^*+1}} \times \frac{\psi_{k^*+1}}{1 - \psi_{k^*+1}} \geq \frac{c}{1 - c} \geq \frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*}}{1 - \psi_{k^*}}$$

such a k^* must exist due to inequality 9.2. But since inequality 9.1 is violated, we must have:

$$\frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*+1}}{1 - \psi_{k^*+1}} > \frac{c}{1 - c} \geq \frac{s_{k^*}}{1 - s_{k^*}} \times \frac{\psi_{k^*}}{1 - \psi_{k^*}}$$

But since the function $\Psi(\sigma;\mathcal{E})$ is continuous in buyers' strategy σ, we can then find some strategy σ^*:

$$\sigma^*(s) = \begin{cases} 1 & s > s_{k^*} \\ \in [0,1] & s = s_{k^*} \\ 0 & s < s_{k^*} \end{cases}$$

such that:

$$\frac{s_{k^*+1}}{1-s_{k^*+1}} \times \frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})} \geq \frac{c}{1-c} = \frac{s_{k^*}}{1-s_{k^*}} \times \frac{\Psi(\sigma^*;\mathcal{E})}{1-\Psi(\sigma^*;\mathcal{E})}$$

The strategy σ^* is thus an equilibrium strategy. It is clearly more selective than σ_j; since it is more selective than σ_{k^*}, where $k^* \geq j$.

Claim ii.

For any $k \in \{1, 2, ..., j\}$, the strategy σ_k is an equilibrium if the inequality 9.1 is satisfied. So, as earlier, assume 9.1 is violated for every such k. Then, we get:

$$\frac{s_j}{1-s_j} \times \frac{\psi_j}{1-\psi_j} \geq \frac{s_{j-1}}{1-s_{j-1}} \times \frac{\psi_j}{1-\psi_j} > \frac{c}{1-c} > \frac{s_1}{1-s_1} \times \frac{\psi_1}{1-\psi_1}$$

where the second inequality in the chain follows by hypothesis and the last inequality follows from the violation of inequality 9.1 for $k = 1$. We can now repeat the argument we constructed after inequality 9.2 to prove Claim i, to prove the existence of an equilibrium strategy σ^* that is less selective than σ_j.

□

9.3 Omitted Proofs

Proposition 1. Let Σ be the set of equilibrium strategies. Then:

1. Σ is non-empty and compact.
2. Any equilibrium strategy σ^* is monotone: for any $\sigma^* \in \Sigma$, $\sigma^*(s) > 0$ for some $s \in \mathbf{S}$ implies that $\sigma^*(s') = 1$ for every $s' \in \mathbf{S}'$ such that $s' > s$.
3. All equilibria exhibit adverse selection: $\psi^* \leq \rho$ for any interim belief ψ^* consistent with an equilibrium strategy $\sigma^* \in \Sigma$.

Proof. In what follows, I treat each strategy $\sigma : \mathbf{S} \to [0,1]$ as a vector in the compact set $[0,1]^m \subset \mathbb{R}^n$, endowed with the taxicab metric (see the end of Section 9.1). I start by proving that any equilibrium strategy must be monotone and all equilibria exhibit adverse selection. Using these observations, I prove that the set of equilibrium strategies is non-empty and compact.

2. Any equilibrium strategy is monotone.

Any equilibrium strategy σ^* must be optimal against the interim belief ψ^* consistent with it. Whenever $\rho \in (0,1)$, $\psi^* = \Psi(\sigma^*;\mathcal{E}) \in (0,1)$, and so $\mathbb{P}_{\psi^*}(\theta = H \mid s') > \mathbb{P}_{\psi^*}(\theta = H \mid S = s)$ for $s', s \in \mathbf{S}$ such that $s' > s$.

3. All equilibria exhibit adverse selection.

A fortiori, $\Psi(\sigma;\mathcal{E}) \leq \rho$ for any monotone strategy σ. To see this, note that $p_H(.)$ first order stochastically dominates $p_L(.)$ since it likelihood ratio dominates it[44]. Therefore, $\nu_L(\sigma;\mathcal{E}) \geq \nu_H(\sigma;\mathcal{E})$. The result then follows since $\frac{\Psi(\sigma;\mathcal{E})}{1-\Psi(\sigma;\mathcal{E})} = \frac{\rho}{1-\rho} \times \frac{\nu_H(\sigma;\mathcal{E})}{\nu_L(\sigma;\mathcal{E})}$.

1. The set of equilibrium strategies is non-empty and compact.

i The set of equilibrium strategies is non-empty.

Define $\Phi(.) : [0,1]^m \to 2^{[0,1]^m}$ to be the buyers' *best response correspondence.* $\Phi(.)$ maps any strategy σ to the set of strategies that are optimal against the interim belief $\Psi(\sigma;\mathcal{E})$ it induces:

$$\Phi(\sigma) = \left\{\sigma' \in [0,1]^m : \sigma' \text{ is optimal against } \Psi(\sigma;\mathcal{E})\right\}$$

A strategy σ^* is an equilibrium strategy if and only if it is a fixed point of buyers' best response correspondence; $\sigma^* \in \Phi(\sigma^*)$. I establish that the correspondence Φ has at least such fixed point through Kakutani's Fixed Point Theorem.

Φ is trivially non-empty; every interim belief has some strategy optimal against it. It is also convex valued; if two distinct approval probabilities are optimal after some outcome $s \in \mathbf{S}$, *any* approval probability is optimal upon that outcome.

The only task that remains is to prove that Φ is upper-semi continuous. For this, take an arbitrary sequence of strategies $\{\sigma_n\}$ such that $\sigma_n \to \sigma_\infty$. Denote the interim beliefs consistent with these strategies as $\psi_n := \Psi(\sigma_n;\mathcal{E})$. Since $\Psi(.;\mathcal{E})$ is continuous in buyers' strategies, we also have $\psi_n \to \psi_\infty$ where $\psi_\infty = \Psi(\sigma_\infty;\mathcal{E})$. Now, take a sequence of strategies $\{\sigma_n^*\}$ where $\sigma_n^* \in \Phi(\sigma_n)$. Note that every σ_n^* is monotone since optimality against any interim belief $\psi \in (0,1)$ requires monotonicity. We want to show that Φ is upper semicontinuous; i.e.:

$$\sigma_n^* \to \sigma_\infty^* \implies \sigma_\infty^* \in \Phi(\sigma_\infty)$$

By the Monotone Subsequence Theorem, the sequence $\{\sigma_n^*\}$ has a subsequence $\sigma_{n_k}^* \to \sigma_\infty^*$ of strategies whose norms $||\sigma_{n_k}^*||$ are monotone in their indices n_k. Here, I take the case where these norms are increasing, the proof is analogous for the opposite case. Since σ_∞^* is the limit of a subsequence of monotone strategies, it must be a monotone strategy too. Assuming otherwise leads to a contradiction; for any $s, s' \in \mathbf{S}$ such that $s' > s$:

$$\sigma_\infty^*(s) > 0 \; \& \; \sigma_\infty^*(s') < 1 \implies \exists N \in \mathbb{N} \text{ s.t. } \forall\, n_k \geq N \quad \sigma_{n_k}^*(s) > 0 \; \& \; \sigma_{n_k}^*(s') < 1$$

[44] Theorem 1.C.1 in Shaked and Shanthikumar (2007).

Now let $\bar{s}$ be the highest outcome for which $\sigma^*_\infty(\bar{s}) > 0$. I show that:

- If $\sigma^*_\infty(\bar{s}) \in (0,1)$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

- If $\sigma^*_\infty(\bar{s}) = 1$, then:
$$\frac{\psi_\infty}{1-\psi_\infty} \times \frac{s}{1-s} \begin{cases} \leq \frac{c}{1-c} & s < \bar{s} \\ \geq \frac{c}{1-c} & s \geq \bar{s} \end{cases}$$

The first case easily follows by noting that:

$$\sigma^*_\infty(\bar{s}) \in (0,1) \implies \sigma^*_{n_k}(\bar{s}) \in (0,1) \implies \frac{\psi_{n_k}}{1-\psi_{n_k}} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c} \implies \frac{\psi_\infty}{1-\psi_\infty} \times \frac{\bar{s}}{1-\bar{s}} = \frac{c}{1-c}$$

for all $n_k \geq N' \in \mathbb{N}$. The second case follows similarly, by noting that $\sigma^*_\infty(\bar{s}) = 1$ and $\sigma^*_\infty(s') = 0$ for all $s' < \bar{s}$ implies $\sigma^*_{n_k}(\bar{s}) > 0$ and $\sigma^*_{n_k}(s') = 0$ for all $n_k \geq N'' \in \mathbb{N}$.

ii The set of equilibrium strategies is compact.

Σ is a subset of $[0,1]^m$ and therefore bounded, hence it suffices to show that is closed. Let $\{\sigma^{**}_n\}$ be a sequence of equilibrium strategies. Note that this means $\sigma^{**}_n \in \Phi(\sigma^{**}_n)$. Since $\Phi(.)$ is upper semicontinuous, $\sigma^{**}_n \to \sigma_\infty$ implies $\sigma_\infty \in \Phi(\sigma_\infty)$, and therefore σ_∞ is an equilibrium strategy itself. □

Proposition 3. Equilibrium total surplus is bounded above by the full-information benchmark Π^f and below by the no-information benchmark $\Pi^\emptyset$. Furthermore, it is higher under more selective equilibrium strategies:

$$\max\{0, \rho - c\} = \Pi^\emptyset \leq \Pi(\sigma^{**}; \mathcal{E}) \leq \Pi(\sigma^*; \mathcal{E}) \leq \Pi^f = \rho \times (1-c)$$

where σ^* and σ^{**} are two equilibrium strategies such that σ^{**} is more selective than σ^*.

Proof. This is an immediate corollary to Lemmas 4 and 14 below; both of independent interest. □

Lemma 14. Take three monotone strategies σ'', σ' and, σ, ordered from the least selective to the most. If $\Pi(\sigma'; \mathcal{E}) \leq \Pi(\sigma; \mathcal{E})$, then $\Pi(\sigma''; \mathcal{E}) \leq \Pi(\sigma'; \mathcal{E})$.

Proof. For the three strategies σ'', σ', and σ, consider three sets $Z, Z', Z'' \subset (S \times [0,1])^n$ where the seller's score profile $\mathbf{z}$ might lie:

$$\mathbf{z} \in \begin{cases} Z & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma'' \text{ but not } \sigma \\ Z' & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma' \text{ but not } \sigma \\ Z'' & \text{if } \mathbf{z} \text{ trades with some buyer under } \sigma'' \text{ but not } \sigma' \end{cases}$$

Notice that $Z' \cap Z'' = \emptyset$ and $Z' \cup Z'' = Z$. We can write the difference between total surplus under different strategies as:

$$\Pi(\sigma'; \mathcal{E}) - \Pi(\sigma; \mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z'\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) - c\right]$$

and:

$$\Pi(\sigma''; \mathcal{E}) - \Pi(\sigma'; \mathcal{E}) = \mathbb{P}\left(\mathbf{z} \in Z''\right) \times \left[\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) - c\right]$$

Therefore we want to prove that:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \le c \implies \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \le c$$

Now, note that $\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z\right)$ is a convex combination of $\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right)$ and $\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right)$. Furthermore:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z\right) \ge \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z \cap Z''\right) = \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right)$$

which then implies:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z''\right) \le \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z\right) \le \mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z'\right) \le c$$

$\square$

Lemma 4. Let σ^* and σ be two monotone strategies such that (i) σ^* is more selective than σ, and (ii) σ^* is an equilibrium strategy. Then, total surplus is higher under σ^*: $\Pi(\sigma^*; \mathcal{E}) \ge \Pi(\sigma; \mathcal{E})$.

Proof. Let $\mathbf{z}$ be the seller's *score profile*. Take an equilibrium strategy σ^* and a less selective strategy σ such that:

$$\sigma(s) - \sigma^*(s) = \begin{cases} \varepsilon & s = \underline{s} \\ 0 & \text{otherwise} \end{cases}$$

for some $\varepsilon > 0$, where $\underline{s} := \min\{s \in S : \sigma^*(s) < 1\}$. I show that:

$$\lim_{\varepsilon \to 0} \Pi\left(\sigma; \varepsilon\right) - \Pi\left(\sigma^*; \varepsilon\right) \le 0$$

By Lemma 14, this establishes the result.

Now, let $Z \subset (S \times [0,1])^n$ be the set of score profiles under which some buyer trades under σ, but all buyers reject the seller under σ^*:

$$\mathbf{z} \in Z \iff \begin{array}{rl} \sigma^*(s^i) > u^i & \text{for all } i \in \{1, 2, \ldots, n\}, \\ & \text{and} \\ \sigma(s^i) \le u^i & \text{for some } i \in \{1, 2, \ldots, n\}. \end{array}$$

Furthermore, for a given score profile $\mathbf{z}$, let $\#$ be the number of buyers whose observed scores are such that $\sigma(s^i) \geq u^i > \sigma^*(s^i)$. These buyers would accept trade under the strategy σ, but not under σ^*.

The seller's eventual outcome differs between the strategy profiles σ and σ^* if and only if his score profile $\mathbf{z}$ lies in Z. Furthermore, his eventual outcome can only change from a rejection by all buyers in σ^* to an approval by some buyer in σ. Thus:

$$\begin{aligned}\Pi(\sigma;\mathcal{E}) - \Pi(\sigma^*;\mathcal{E}) &= [\mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c] \times \mathbb{P}(\mathbf{z} \in Z) \\ &\propto \mathbb{P}(\theta = H \mid \mathbf{z} \in Z) - c\end{aligned}$$

Focus therefore, on the probability that $\theta = H$ given the seller's signal profile lies in Z:

$$\mathbb{P}\left(\theta = H \mid \mathbf{z} \in Z\right) = \sum_{i=1}^{n} \mathbb{P}\left(\theta = H \mid \# = i\right) \times \frac{\mathbb{P}\left(\# = i\right)}{\mathbb{P}(\mathbf{z} \in Z)}$$

Now note:

$$\mathbb{P}\left(\# = i \mid \theta\right) = \left(p_\theta(\underline{s})\right)^i \times \left(1 - p_\theta(\underline{s})\right)^{n-i} \times \varepsilon^i$$

and thus $\mathbb{P}\left(\# = i\right) \propto \varepsilon^i$. Since $\mathbb{P}(\mathbf{z} \in A) = \sum_{i=1}^{n} \mathbb{P}\left(\# = i\right)$, we have $\lim_{\varepsilon \to 0} \frac{\mathbb{P}(\# = i)}{\mathbb{P}(\mathbf{z} \in A)} = 0$ for any $i > 1$. Thus:

$$\lim_{\varepsilon \to 0} \mathbb{P}\left(\theta = H \mid \mathbf{z} \in A\right) - \mathbb{P}\left(\theta = H \mid \# = 1\right) = 0$$

I conclude the proof by showing that $\mathbb{P}\left(\theta = H \mid \# = 1\right) \leq c$ as $\varepsilon \to 0$:

$$\begin{aligned}\lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H \mid \# = 1\right)}{\mathbb{P}\left(\theta = L \mid \# = 1\right)} &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \frac{\mathbb{P}\left(\# = 1 \mid \theta = H\right)}{\mathbb{P}\left(\# = 1 \mid \theta = L\right)} \\ &= \lim_{\varepsilon \to 0} \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\ &= \frac{\mathbb{P}\left(\theta = H\right)}{\mathbb{P}\left(\theta = L\right)} \times \left(\frac{r_H(\sigma^*;\mathcal{E})}{r_L(\sigma^*;\mathcal{E})}\right)^{n-1} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \\ &\leq \frac{\psi^*}{1 - \psi^*} \times \frac{p_H(\underline{s})}{p_L(\underline{s})} \leq \frac{c}{1 - c}\end{aligned}$$

where $\psi^* = \Psi(\sigma^*;\mathcal{E})$ is the interim belief consistent with σ^*. The penultimate inequality holds due to the straightforward fact that:

$$\frac{\psi^*}{1 - \psi^*} = \frac{\rho}{1 - \rho} \times \frac{1 + r_H^* + ... + (r_H^*)^{n-1}}{1 + r_L^* + ... + (r_L^*)^{n-1}} \leq \frac{\rho}{1 - \rho} \times \left(\frac{r_H^*}{r_L^*}\right)^{n-1}$$

where $r_\theta^* := r_\theta(\sigma^*;\mathcal{E})$. The last inequality is due to the fact that $\underline{s} \in S$ is optimally rejected under σ^*.

$\square$

Theorem 1. Let $\Pi^n\left(\hat{\sigma};\mathcal{E}\right)$ be total surplus under the most selective equilibrium in a market with

n buyers. If $\mathcal{E}$ has an outcome that fully reveals High quality ($s_m = 1$), the sequence $\{\Pi^n(\hat{\sigma}; \mathcal{E})\}_{n=1}^{\infty}$ is eventually increasing and converges to surplus in the full-information benchmark. Otherwise, it is eventually decreasing and converges to surplus in the no-information benchmark.

Proof. For each $j \in \{1, 2, ..., m\}$, define σ_j to be the strategy:

$$\sigma_j(s) := \begin{cases} 0 & s < s_j \\ 1 & s \geq s_j \end{cases}$$

Moreover, let $F_\theta(x) = \sum_{s \in \mathbf{S}: s \leq x} p_\theta(x)$. In a market with n buyers, the interim belief $\psi_{j:n}$ consistent with buyers using the strategies σ_j is then implicitly given by:

$$\frac{\psi_{j:n}}{1 - \psi_{j:n}} = \frac{\sum_{k=0}^{n-1} F_H(s_{j-1})^k}{\sum_{k=0}^{n-1} F_L(s_{j-1})^k}$$

Note that for all $j > 1$, the RHS is bounded and strictly decreasing in n, so the sequence $\{\psi_{j:n}\}$ is convergent.

Case 1: $s_m = 1$.

To prove the Theorem's statement for this case, I first show that $\psi_{m:n} \xrightarrow{n} 0$. Let X_n be the random variable that is uniformly distributed over the set $\left\{F_H(s_{m-1})^k\right\}_{k=0}^{n-1}$. Then, note that:

$$\frac{\psi_{m:n}}{1 - \psi_{m:n}} = \frac{\sum_{k=0}^{n-1} F_H(s_{m-1})^k}{\sum_{k=0}^{n-1} F_L(s_{m-1})^k} = \mathbb{E}[X_n]$$

Now, fix any $x > 0$. Since $F_H(s_{m-1})^k$ is strictly decreasing in k, for any $\delta < 1$ of our choice, we can find some $N_{x;\delta} \in \mathbb{N}$ such that for all $n \geq N_{x;\delta}$ implies $\mathbb{P}(X_n \leq x) \geq \delta$ and $\mathbb{E}[X_n] \leq \delta x + (1 - \delta)$. Fixing $x = \frac{\varepsilon}{2\delta}$ for some $\varepsilon > 0$ small, we have $\mathbb{E}[X_n] \leq \frac{\varepsilon}{2} + 1 - \delta$. Since we can take δ arbitrarily close to 1, this shows that $\mathbb{E}[X_n] \to 0$, proving this first claim.

Since buyers must always accept to trade upon observing $s_m = 1$ in equilibrium, this implies that there is some $N \in \mathbb{N}$ for which σ_m is the most selective equilibrium strategy for all $n \geq N$. So, for $n \geq N$, a seller with a Low quality asset never trades. Moreover, every additional buyer increases the probability that a seller with a High quality asset trades. As $n \to \infty$, such a seller trades almost surely. We thus prove that $\Pi^n(\hat{\sigma}; \mathcal{E}) \to \Pi^f$.

Case 2: $s_m < 1$

The case where buyers' experiment $\mathcal{E}$ is uninformative is trivial; it always yields the no-information benchmark. So, I assume that $s_m \neq s_1$.

For any $j \in \{1, 2, ..., m\}$, the sequence $\left\{\frac{\psi_{j;n}}{1-\psi_{j;n}} \times \frac{s_j}{1-s_j}\right\}_{n=1}^{\infty}$ is bounded and monotone decreasing, thus convergent. Let $\mathcal{L}_j$ be the limit of this sequence. If $\mathcal{L}_m < \frac{c}{1-c}$, by Lemma 13, there is some $N' \in \mathbb{N}$ such that for all $n \geq N'$, the most selective equilibrium is more selective than σ_m. So, buyers must be indifferent when they trade—expected trade surplus must be 0. Since total surplus is bounded below by $\Pi^\emptyset$, we conclude that $\Pi^n(\hat{\sigma}; \mathcal{E}) = \Pi^\emptyset = 0$ for all $n \geq N'$.

Now consider the case $\mathcal{L}_m \geq \frac{c}{1-c}$. Since $\psi_{j:n}$ is decreasing in n, the most selective equilibrium with n buyers, $\hat{\sigma}_n$, must get weakly more selective with n. So, the sequence $\{r_\theta(\hat{\sigma}_n;\mathcal{E})\}_{n=1}^{\infty}$ is weakly increasing in n, convergent, and Cauchy. If for any $N \in \mathbb{N}$, $\hat{\sigma}_N$ is more selective than σ_m, we are done by the argument in the preceding paragraph. Otherwise, the sequence $\{r_\theta(\hat{\sigma}_n;\mathcal{E})\}_{n=1}^{\infty}$ converges to a number below 1. If are both constant at 0, we are done—total surplus is equal to that under the no-information benchmark along the sequence. Otherwise, $r_L(\hat{\sigma}_n;\mathcal{E}) > r_H(\hat{\sigma}_n;\mathcal{E})$ along the sequence. Since both sequences are Cauchy, there exists some $N \in \mathbb{N}$ and $M \geq N$ such that for all $m \geq M$, we have:

$$
\begin{aligned}
&\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_m;\mathcal{E})]^m - (1-\rho) \times c \times [1 - r_L(\hat{\sigma}_m;\mathcal{E})^m] \\
\approx &\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_N;\mathcal{E})]^m - (1-\rho) \times c \times [1 - r_L(\hat{\sigma}_N;\mathcal{E})^m] \\
> &\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_N;\mathcal{E})]^{m+1} - (1-\rho) \times c \times \left[1 - r_L(\hat{\sigma}_N;\mathcal{E})^{m+1}\right] \\
\approx &\rho \times (1-c) \times [1 - r_H(\hat{\sigma}_{m+1};\mathcal{E})]^{m+1} - (1-\rho) \times c \times \left[1 - r_L(\hat{\sigma}_{m+1};\mathcal{E})^{m+1}\right]
\end{aligned}
$$

This proves that total surplus is eventually decreasing.

Furthermore, since there is at least one outcome of the experiment $\mathcal{E}$ where a buyer surely trades with the seller, as $n \to \infty$, the seller trades almost surely regardless of quality. Hence, total surplus converges to $\rho - c$. Since total surplus can never be negative, it must be that $\Pi^{\emptyset} = \rho - c$ in this case.

□

Theorem 2. Let buyers' experiment $\mathcal{E}$ be binary. Then, equilibrium* total surplus is increasing in the strength of good news (s_H) but is quasiconcave and eventually decreasing in the strength of bad news (s_L).

I will use Lemmas 15, 17, and 18 below, possibly of independent interest, to prove Theorem 2. Throughout, I denote the most and least selective equilibrium strategies under the experiment $\mathcal{E}$ as $\hat{\sigma}^*_{\mathcal{E}}$ and $\check{\sigma}^*_{\mathcal{E}}$, respectively. I drop the subscript whenever the experiment in question is obvious.

Lemma 15. Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. $\Psi(\sigma;\mathcal{E})$ is:

i strictly increasing in $\sigma(s_L)$, whenever $\sigma(s_H) = 1$,

ii strictly decreasing in $\sigma(s_H)$ whenever $\sigma(s_L) = 0$.

Proof. Part i:

Let $\sigma(s_L) \in (0,1)$ and $\sigma(s_H) = 1$. The interim belief $\Psi(\sigma;\mathcal{E})$ is then given by:

$$\begin{aligned}
\Psi(\sigma;\mathcal{E}) &= \mathbb{P}\left(\theta = H \mid \text{visit received}\right) \\
&= \sum_{i=0}^{n-1} \mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection } \mid \text{visit received}) \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] \\
&= \sum_{i=0}^{n-1} \frac{\mathbb{P}(\text{visited after i}^{\text{th}} \text{ rejection })}{\mathbb{P}(\text{visit received})} \times \mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right]
\end{aligned}$$

Note that $\mathbb{E}\left[\theta = H \mid \text{i } s_L \text{ signals}\right] < \mathbb{E}\left[\theta = H \mid \text{i+1 } s_L \text{ signals}\right]$; since every s_L signal is further evidence for $\theta = L$. We have:

$$\begin{aligned}
\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection }) =& \mathbb{P}\left(\text{buyer was (i+1)}^{\text{st}} \text{ in order } \mid \text{seller got i rejections}\right) \\
&\times \mathbb{P}\left(\text{seller got i rejections}\right) \\
=& \frac{1}{n} \times \mathbb{P}(\text{i } s_L \text{ signals}) \times \left[1 - \sigma(s_L)\right]^i
\end{aligned}$$

The proof is completed by noting that:

$$\frac{\mathbb{P}(\text{ visited after (i+1)}^{\text{st}} \text{ rejection })}{\mathbb{P}(\text{ visited after i}^{\text{th}} \text{ rejection })} = \frac{\mathbb{P}(\text{i+1 } s_L \text{ signals})}{\mathbb{P}(\text{i } s_L \text{ signals})} \times \left[1 - \sigma(s_L)\right]$$

decreases, and thus $\Psi(\sigma;\mathcal{E})$ increases, in $\sigma(s_L)$.

Part ii:

Now take $\sigma(s_L) = 0$. We then have:

$$r_H(\sigma;\mathcal{E}) = 1 - p_H(s_H)\sigma(s_H) \qquad\qquad r_L(\sigma;\mathcal{E}) = 1 - p_L(s_H)\sigma(s_H)$$

and:

$$\begin{aligned}
\Psi(\sigma;\mathcal{E}) &\propto \frac{1 + r_H + ... + r_H^{n-1}}{1 + r_L + ... + r_L^{n-1}} \\
&= \frac{1 - r_H^n}{1 - r_L^n} \times \frac{1 - r_L}{1 - r_H} = \frac{1 - r_H^n}{1 - r_L^n} \times \frac{p_L(s_H)}{p_H(s_H)} \\
&\propto \frac{1 - r_H^n}{1 - r_L^n} = \frac{1 - \left(1 - p_H(s_H)\sigma(s_H)\right)^n}{1 - \left(1 - p_L(s_H)\sigma(s_H)\right)^n}
\end{aligned}$$

Differentiating the last expression with respect to $\sigma(s_H)$ and rearranging its terms reveals that this derivative is proportional to:

$$\frac{s_H}{1 - s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} - \frac{1 - (r_H)^n}{1 - (r_L)^n}$$

The positive term is the likelihood ratio of one s_H signal and $n-1$ rejections, and the negative term is the likelihood ratio from *at most* $n-1$ rejections. Since acceptances only happen with s_H

signals, the negative term strictly exceeds the positive term. This can be verified directly, too:

$$\frac{1-(r_H)^n}{1-(r_L)^n} > \frac{s_H}{1-s_H} \times \left(\frac{r_H}{r_L}\right)^{n-1} \iff \frac{1-(r_H)^n}{1-(r_L)^n} \times \frac{1-r_L}{1-r_H} > \left(\frac{r_H}{r_L}\right)^{n-1}$$
$$\iff \frac{1+...+(r_H)^{n-1}}{1+...+(r_L)^{n-1}} > \left(\frac{r_H}{r_L}\right)^{n-1}$$

The last inequality can be verified easily. Thus, $\Psi(\sigma;\mathcal{E})$ decreases in $\sigma(s_H)$.

□

The Corollary below follows from Lemma 15. Let both $\mathcal{E}'$ and $\mathcal{E}$ are binary experiments, where the former is Blackwell more informative than the latter. If, under both experiments, every buyer accepts upon "good news" and rejects upon "bad news", the interim belief under $\mathcal{E}'$ is lower.

Corollary 16. Let $\mathcal{E}'$ and $\mathcal{E}$ be two binary experiments, where the former is Blackwell more informative than the latter. Let the strategies σ' and σ for these respective experiments be defined as:

$$\sigma'(s') := \begin{cases} 0 & s' = s'_L \\ 1 & s' = s'_H \end{cases} \qquad \sigma(s) := \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

Then, $\Psi(\sigma';\mathcal{E}') \leq \Psi(\sigma;\mathcal{E})$.

Proof. Establishing that this holds for a pair $(\mathcal{E}',\mathcal{E})$ for which either (i) $s'_H > s_H$ and $s_L = s'_L$, or (ii) $s'_H = s_H$ and $s_L > s'_L$ suffices. I will only prove the first case, the second is analogous. Below I show that the outcome induced by σ under experiment $\mathcal{E}$ can be replicated by some strategy $\tilde{\sigma}$ under experiment $\mathcal{E}'$, where $\tilde{\sigma}(s_L) > 0$ and $\tilde{\sigma}(s_H) = 1$. Then, the desired conclusion follows from Lemma 15.

Take the pair $(\sigma,\mathcal{E})$. The probabilities that a buyer accepts or rejects trade, conditional on θ, is given by:

$$\frac{\mathbb{P}_\sigma(\text{rejected} \mid \theta = H)}{\mathbb{P}_\sigma(\text{rejected} \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}_\sigma(\text{accepted} \mid \theta = H)}{\mathbb{P}_\sigma(\text{accepted} \mid \theta = L)} = \frac{s_H}{1-s_H}$$

For the pair $(\tilde{\sigma},\mathcal{E}')$ where $\tilde{\sigma}(s'_H) = 1$, we have:

$$\frac{\mathbb{P}_{\tilde{\sigma}}(\text{rejected} \mid \theta = H)}{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = L)} = \frac{s_L}{1-s_L} \qquad \frac{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = H)}{\mathbb{P}_{\tilde{\sigma}}(\text{accepted} \mid \theta = L)} = \frac{p'_H(s_H) + \tilde{\sigma}(s_L)p'_H(s_L)}{p'_L(s_H) + \tilde{\sigma}(s_L)p'_L(s_L)}$$

where $\{p'_L, p'_H\}$ are the distributions for the experiment $\mathcal{E}'$. It is easy to verify that the expression on the right falls from $\frac{s'_H}{1-s'_H}$ to 1 monotonically and continuously as $\tilde{\sigma}(s_L)$ rises from 0 to 1. Thus, there is a unique interior value of $\tilde{\sigma}(s_L)$ that replicates the outcome of $(\sigma;\mathcal{E})$.

□

Lemma 17. Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. Let $\sigma^\star$ be buyers' equilibrium strategy, either in the most or least selective equilibrium. Then, $\sigma^\star(s_L) \in \{0, 1\}$.

Proof. I start by proving this for the least selective equilibrium; i.e. $\check{\sigma}^*(s_L) \in \{0, 1\}$. For s_L^{mute} defined in Definition 9, observe that when $s_L \geq s_L^{\text{mute}}$, $\sigma(s_L) = \sigma(s_H) = 1$ is an equilibrium; so we must have $\check{\sigma}^*(s_L) = 1$. The strategy σ defined by $\sigma(s_H) = \sigma(s_L) = 1$ gives rise to the interim belief $\Psi(\sigma; \mathcal{E}) = \rho$, which in turn renders approving upon the outcome s_L optimal. In turn, if $s_L < s_L^{\text{mute}}$, we must have $\sigma^*(s_L) = 1$ for any equilibrium strategy; since the equilibrium interim belief always lies below the prior belief (Proposition 1).

Now consider the most selective equilibrium strategy; $\hat{\sigma}^*$. For contradiction, let $1 > \hat{\sigma}^*(s_L) > 0$ and $\hat{\sigma}^*(s_H) = 1$. Lemma 15 establishes that the interim belief falls as $\sigma(s_L)$ falls; which implies there must be another, more selective equilibrium strategy σ^* such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) = 1$. $\square$

Lemma 17 establishes that when their experiment $\mathcal{E}$ is binary, buyers *never* mix upon seeing "bad news", $s = s_L$, neither in the most nor the least selective equilibrium. Following up, Lemma 18 establishes that a more informative binary experiment pushes buyers to reject upon bad news in both equilibria.

Lemma 18. Let $\mathcal{E}$ be a binary experiment, with outcomes in $\mathbf{S} = \{s_L, s_H\}$; $s_L \leq s_H$. Buyers' acceptance probabilities upon "bad news", $s = s_L$, in the least and most selective equilibrium strategies are given by:

$$\check{\sigma}^*(s_L) = \begin{cases} 1 & s_L \geq s_L^{\text{mute}} \\ 0 & s_L < s_L^{\text{mute}} \end{cases} \qquad \hat{\sigma}^*(s_L) = \begin{cases} 1 & s_L < s_L^\dagger(s_H) \\ 0 & s_L \geq s_L^\dagger(s_H) \end{cases}$$

where $s_L^\dagger(.)$ is an increasing function of s_H, and $s_L^\dagger(s_H) \geq s_L^{\text{safe}}$.

Proof. Note that there exists an equilibrium where $\sigma(s_L) = 1$ if and only if:

$$\frac{\rho}{1-\rho} \times \frac{s_L}{1-s_L} \geq \frac{c}{1-c}$$

which, combined with Lemma 17, proves the part of the Lemma for the selective equilibrium.

Now, define the strategies σ_0 as σ_1 as:

$$\sigma_0(s) = \begin{cases} 0 & s = s_L \\ 0 & s = s_H \end{cases} \qquad \sigma_1(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

A necessary and sufficient condition for an equilibrium σ^* where $\sigma^*(s_L) = 0$ to exist is:

$$\frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq \frac{c}{1-c}$$

Sufficiency follows since either:

$$\frac{\Psi(\sigma_0;\mathcal{E})}{1-\Psi(\sigma_0;\mathcal{E})} \times \frac{s_H}{1-s_H} \leq \frac{c}{1-c}$$

which implies σ_0 is an equilibrium, or there is an equilibrium strategy σ^* such that $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) > 0$ by Lemma 15. The condition is necessary, since any strategy that is less selective than σ_1 induces a higher interim belief, by Lemma 15.

By Corollary 16, whenever this necessary and sufficient condition holds for an experiment $\mathcal{E}$, it also holds for a (Blackwell) more informative experiment $\mathcal{E}'$. Moreover, whenever the low signals are rejected in the least selective equilibrium, they must be in the most selective equilibrium. This concludes the proof.

□

Proof, Theorem 2: By Lemma 18, Blackwell improving buyers' experiment shifts both their least selective and most selective equilibrium strategies once from *always* accepting trade to rejecting upon the low signal. By Lemma 14, this shift in buyers' strategy increases efficiency—and therefore each buyer's expected surplus.

Let $\{\sigma_\alpha\}_{\alpha\in[0,1]}$ be the family of strategies where buyers reject upon the low signal:

$$\sigma_\alpha(s) = \begin{cases} 0 & s = s_L \\ 1 & s = s_H \end{cases}$$

By Lemma 15, the interim belief ψ_α that the strategy σ_α induces is strictly decreasing in α. Thus, at most one of these can be an equilibrium strategy for a given experiment. Furthermore, whenever buyers' expected surplus from σ_1 is weakly positive, this must be the equilibrium strategy; decreasing α can only make approving upon the high signal *more* profitable. Hence, whenever buyers reject upon the low signal in equilibrium, efficiency is given by: $\Pi(\sigma^*;\mathcal{E}) = \max\{0, \Pi(\sigma_1;\mathcal{E})\}$. The Theorem then follows from the Claim below:

Claim. $\max\{0, \Pi(\sigma_1;\mathcal{E})\}$ is:

i weakly increasing in s_H whenever there is some equilibrium strategy σ^* s.t. $\sigma^*(s_L) = 0$.

ii hump-shaped in s_L. As s_L falls, it:

- weakly increases when $s_L \geq s_L^{as}$,
- weakly decreases when $s_L \leq s_L^{as}$

where s_L^{as} is defined implicitly as:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_L^{\text{as}}}{1-s_L^{\text{as}}}\right)^{n-1} \times \frac{s_H}{1-s_H} = \frac{c}{1-c}$$

Proof of the Claim.

Part i. Increasing the strength of good news; i.e. s_H.

Let $\mathcal{E}$ and $\mathcal{E}'$ be two binary experiments with outcome sets $\mathbf{S} = \{s_L, s_H\}$ and $\mathbf{S}' = \{s'_L, s'_H\}$. The experiment $\mathcal{E}'$ carries *marginally stronger good news* than experiment $\mathcal{E}$:

$$s'_L = s_L \qquad\qquad s'_H = s_H + \delta$$

for some small δ such that $1 - s_H \geq \delta > 0$. I show that $\Pi(\sigma'_1; \mathcal{E}') > \Pi(\sigma_1; \mathcal{E})$; where σ'_1 is defined analogously to σ_1 for experiment $\mathcal{E}'$.

Step 1. Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

Rather than observing the outcome of experiment $\mathcal{E}'$, say a buyer initially observes her original signal s, and then potentially an additional auxilliary signal $\hat{s}$. The first signal she receives, s, records the outcome of $\mathcal{E}$. If the low outcome s_L materialises, the buyer observes no more information. If, however, the high outcome s_H materialises, she then observes the additional auxiliary signal $\hat{s}$. This auxiliary signal records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. The outcome of $\hat{\mathcal{E}}$ is independent both from s and anything else any other buyer observes. Conditional on the asset's quality θ, the distribution over its outcomes is given by the pmf $p_\theta(.)$:

$$\hat{p}_H(\hat{s_H}) = 1 - \varepsilon \times \frac{s_L}{1 - s_L} \qquad\qquad \hat{p}_L(\hat{s_H}) = 1 - \varepsilon \times \frac{s_H}{1 - s_H}$$

The evolution of the buyer's beliefs when she observes this signal pair is determined by the two likelihood ratios:

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_H}) \mid \theta = L)} = \frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} \tag{9.3}$$

$$\frac{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = H)}{\mathbb{P}((s, \hat{s}) = (s_H, \hat{s_L}) \mid \theta = L)} = \frac{s_L}{1 - s_L} \tag{9.4}$$

Note that the likelihood ratio 9.3 increases continuously with ε.

The information from observing the pair $(s, \hat{s})$ as such is equivalent to observing the outcome of experiment $\mathcal{E}'$, when:

$$\frac{s_H}{1 - s_H} \times \frac{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}}{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}} = \frac{s_H + \delta}{1 - (s_H + \delta)} \tag{9.5}$$

for our chosen (δ, ε). I choose ε to satisfy this equality for our δ. As such, ε becomes a continuously increasing function of δ. Furthermore, note that by varying ε between 0 and $\frac{1-s_H}{s_H}$, we can replicate *any* experiment $\mathcal{E}'$ with $s'_L = s_L$ and $1 \geq s'_H \geq s_H$.

Step 2. $\pi(\sigma_1'; \mathcal{E}') \geq \pi(\sigma_1; \mathcal{E})$.

The buyer who observes the signal pair $(s, \hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma} : \{s_L, (s_H, \hat{s_H}), (s_H, \hat{s_L})\} \to [0, 1]$ for this signal pair that replicates the outcome of the strategy σ_1' for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_H, \hat{s_H}) = 1 \qquad\qquad \tilde{\sigma}(s_L) = \tilde{\sigma}(s_H, \hat{s_L}) = 0$$

and replicates the likelihood ratios of an acceptance and rejection signal under $\mathcal{E}'$.

Now, fix the seller's *signal profile* $\mathbf{s} = \{(s^i, \hat{s}^i)\}_{i=1}^n$ (defined in Section 9.1). I call a seller a *marginal admit* if his score profile is such that:

i for at least one $i \in \{1, 2, ..., n\}$, $s^i = s_H$, and

ii for *every* $i \in \{1, 2, ..., n\}$, either $s^i = s_L$, or $\hat{s}^i = \hat{s}_L$.

These marginal admits drive the wedge between efficiency under $\mathcal{E}'$ and $\mathcal{E}$: while some buyer trades under $\mathcal{E}$, they *all* reject him under $\hat{\mathcal{E}}$. So:

$$\Pi(\sigma_1'; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\,(\text{marginal admit}) \times \underbrace{[c - \mathbb{P}\,(\theta = H \mid \text{marginal admit})]}_{(1)}$$

A marginal admit only has signal realisations $(s, \hat{s}) = (s_H, \hat{s_L})$ or $s = s_L$. These carry equivalent information about θ. Thus, the expression (1) above equals:

$$c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right]$$

In the relevant region where there is an equilibrium strategy that leads to rejections after the low outcome s_L, the expression above must be weakly positive:

$$\begin{aligned}
c - \mathbb{P}\left[\theta = H \mid s^1 = ... = s^n = s_L\right] &\propto \frac{c}{1-c} \times \frac{\rho}{1-\rho} \times \left(\frac{s_L}{1-s_L}\right)^n \\
&\leq \frac{c}{1-c} - \frac{\rho}{1-\rho} \times \frac{\sum\limits_{k=0}^{n-1} p_H(s_L)^k}{\sum\limits_{k=0}^{n-1} p_L(s_L)^k} \times \frac{s_L}{1-s_L} \\
&= \frac{c}{1-c} - \frac{\Psi(\sigma_1; \mathcal{E})}{1 - \Psi(\sigma_1; \mathcal{E})} \times \frac{s_L}{1-s_L} \leq 0
\end{aligned}$$

where the last inequality follows from the necessary and sufficient condition the proof of Lemma 18 introduced for such an equilibrium to exist.

Part ii. Increasing the strength of bad news; i.e. decreasing s_L.

Now, let the experiment $\mathcal{E}'$ carry *marginally stronger bad news* than experiment $\mathcal{E}$ instead; for some arbitrarily small $\delta \in [0, s_L]$:

$$s'_L = s_L - \delta \qquad\qquad s'_H = s_H$$

Where σ'_1 and σ_1 are defined as before, I show that:

i $\Pi(\sigma'_1; \mathcal{E}) - \Pi(\sigma_1; \mathcal{E}) \geq 0$ when $s_L \geq s_L^{as}$, and

ii $\Pi(\sigma'_1; \mathcal{E}) - \Pi(\sigma_1; \mathcal{E}) \leq 0$ when $s_L \leq s_L^{as}$

Step 1. Replicating $\mathcal{E}'$ with a signal pair $(s, \hat{s})$.

As before, let each buyer observe *two* signals, potentially: s and $\hat{s}$. She first observes s, which records the outcome of $\mathcal{E}$. If the high outcome s_H materialises, she receives no further information. If, however, the low outcome s_L materialises, she then observes the additional auxiliary signal $\hat{s}$, which records the outcome of *another* binary experiment, $\hat{\mathcal{E}}$. As before, the outcome of this experiment is independent both from s and anything observed by any other buyer. Its distribution conditional on the asset's quality θ is given by the pmf $p_\theta(.)$:

$$\hat{p}_H\left(\hat{s}_H\right) = \varepsilon \times \frac{s_H}{1 - s_H} \qquad\qquad \hat{p}_L\left(\hat{s}_H\right) = \varepsilon \times \frac{s_L}{1 - s_L}$$

The evolution of the buyer's beliefs upon seeing the signal pair $(s, \hat{s})$ is then determined by the two likelihood ratios:

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_H}) \mid \theta = H\right)} = \frac{s_H}{1 - s_H} \tag{9.6}$$

$$\frac{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)}{\mathbb{P}\left((s, \hat{s}) = (s_L, \hat{s_L}) \mid \theta = H\right)} = \frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} \tag{9.7}$$

Note that 9.7 is continuously and strictly decreasing with ε, taking values between $\frac{s_L}{1-s_L}$ and 0 as ε varies between 0 and $\frac{s_H}{1-s_H}$. The signal pair $(s, \hat{s})$ is informationally equivalent to $\mathcal{E}'$ when:

$$\frac{s_L}{1 - s_L} \times \frac{1 - \varepsilon \times \dfrac{s_H}{1 - s_H}}{1 - \varepsilon \times \dfrac{s_L}{1 - s_L}} = \frac{s_L - \delta}{1 - (s_L - \delta)}$$

I choose ε to satisfy this equality. As before, ε then becomes a continuously increasing function of δ.

Step 2. $\Pi(\sigma'_1; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) \begin{cases} \geq 0 & s_L \geq s_L^{\text{as}} \\ \leq 0 & s_L \leq s_L^{\text{as}} \end{cases}$

The buyer who observes the signal pair $(s, \hat{s})$ obtains equivalent information to that from $\mathcal{E}'$. We now must identify the strategy $\tilde{\sigma} : \{(s_L, \hat{s_H}), (s_L, \hat{s_L}), s_H\} \to [0, 1]$ for this signal pair that replicates the outcome of the strategy σ_1' for experiment $\mathcal{E}'$. This strategy is defined as:

$$\tilde{\sigma}(s_L, \hat{s_H}) = \tilde{\sigma}(s_H) = 1 \qquad\qquad \tilde{\sigma}(s_L, \hat{s_L}) = 0$$

and replicates the likelihood ratios of an approval and rejection signal under $\mathcal{E}'$.

Now, fix the seller's *score profile*: $\mathbf{s} = \{(s^i, \hat{s}^i)\}_{i=1}^n$. I call a seller a *marginal reject* if:

i for every $i \in \{1, 2, ..., n\}$, $s^i = s_L$, and

ii for at least one $i \in \{1, 2, ..., n\}$, $\hat{s}^i = \hat{s}_H$.

Marginal rejects drive the wedge between efficiency under $\mathcal{E}'$ and $\mathcal{E}$: while *no* buyer trades under $\mathcal{E}$, *at least one* buyer does under $\mathcal{E}'$. So:

$$\Pi(\sigma_1'; \mathcal{E}') - \Pi(\sigma_1; \mathcal{E}) = \mathbb{P}\,(\text{marginal reject}) \times \underbrace{[\mathbb{P}\,(\theta = H \mid \text{marginal reject}) - c]}_{(2)}$$

For a marginal reject, buyers observe either $(s^i, \hat{s}^i) = (s_L, \hat{s}_L)$, or $(s^i, \hat{s}^i) = (s_L, \hat{s}_H)$. Denote the number of buyers who observed the latter as $\#$. Since the seller is a marginal reject, $\# \geq 1$. Then, (2) equals:

$$\sum_{i=1}^{n} \underbrace{\frac{\mathbb{P}\left(i\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\sum_{j=1}^{n} \mathbb{P}\left(j\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}}_{(3)} \times \mathbb{P}\,(\theta = H \mid \# = i) - c$$

where:

$$\begin{aligned}\mathbb{P}\left(i\ \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right) = k \times & \binom{n}{i} \times \left(\frac{s_H}{1 - s_H} \times \varepsilon\right)^i \times \left(1 - \frac{s_H}{1 - s_H} \times \varepsilon\right)^{n-i} \\ + (1-k) \times & \binom{n}{i} \times \left(\frac{s_L}{1 - s_L} \times \varepsilon\right)^i \times \left(1 - \frac{s_L}{1 - s_L} \times \varepsilon\right)^{n-i}\end{aligned}$$

and $k = \mathbb{P}\left(\theta = H \mid s^1 = ... = s^n = s_L\right)$. Thus, the limit of expression (3) as $\varepsilon \to 0$ (and therefore, $\delta \to 0$) for any $i > 1$ is:

$$\lim_{\varepsilon \to 0} \frac{\frac{1}{\varepsilon} \times \mathbb{P}\left(i\ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)}{\frac{1}{\varepsilon} \times \sum_{j=1}^{n} \mathbb{P}\left(j\ \hat{s} = \hat{s_H} \text{ signals } \mid s^1 = ... = s^n = s_L\right)} = 0 \tag{9.8}$$

Therefore, we get:

$$\lim_{\varepsilon\to 0}\sum_{i=1}^{n}\frac{\mathbb{P}\left(i\ \hat{s_H}\text{ signals }\mid s^1=...=s^n=s_L\right)}{\sum_{j=1}^{n}\mathbb{P}\left(j\ \hat{s_H}\text{ signals }\mid s^1=...=s^n=s_L\right)}\times\mathbb{P}\left(\theta=H\mid\#=i\right)-c$$

$$=\mathbb{P}\left(\theta=H\mid\#=1\right)-c$$

$$\propto\frac{\rho}{1-\rho}\times\left(\frac{s_L}{1-s_L}\right)^{n-1}\times\frac{s_H}{1-s_H}-\frac{c}{1-c}$$

proving the claim. □

□

Proposition 5. Where buyers' experiment is binary, total surplus in equilibrium* weakly decreases with stronger bad news (lower s_L) when:

$$\frac{\rho}{1-\rho}\times\max\left\{\frac{s_L}{1-s_L},\left[\frac{s_L}{1-s_L}\right]^{n-1}\times\frac{s_H}{1-s_H}\right\}\leq\frac{c}{1-c}$$

This condition is also necessary in the least selective equilibrium.

Proof.

i The least selective equilibrium:

By Lemma 17, the probability that the seller trades upon the low outcome in the least selective equilibrium is:

$$\check{\sigma}^*(s_L)=\begin{cases}1 & s_L\geq s_L^{\text{mute}}\\ 0 & s_L<s_L^{\text{mute}}\end{cases}$$

Thus, efficiency equals (i) the expected surplus from always approving the applicant when $s_L\geq s_L^{\text{mute}}$, and (ii) $\max\{0,\Pi(\sigma_1;\mathcal{E})\}$ when $s_L<s_L^{\text{mute}}$ (established in the proof of Theorem 2):

$$\Pi(\check{\sigma}^*;\mathcal{E})=\begin{cases}\rho-c & s_L\geq s_L^{\text{mute}}\\ \max\{0,\Pi(\sigma_1;\mathcal{E})\} & s_L<s_L^{\text{mute}}\end{cases}$$

Since always trading is always feasible, we have $\max\{0,\Pi(\sigma_1;\mathcal{E})\}\geq\rho-c$ when $s_L<s_L^{\text{mute}}$ by Lemma 14. Furthermore, the final Claim in Theorem 2's proof establishes that as s_L falls, the expression $\max\{0,\Pi(\sigma_1;\mathcal{E})\}$ weakly increases (decreases) when $s_L\geq s_L^{\text{as}}$ ($s_L\leq s_L^{\text{as}}$). Thus the desired conclusion is established.

ii The most selective equilibrium:

By Lemma 18, the most selective equilibrium shifts from one where a buyer always trades to one where she rejects upon the low signal when $s_H\geq s_H^{\dagger}(s_L)$, where $s_H^{\dagger}(.)$ is an increasing function of s_L. Following the arguments made for the least selective equilibrium then, efficiency:

- weakly increases as s_L decreases, when $s_L \geq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$
- weakly decreases as s_L decreases, when $s_L \leq \min\left\{s_L^{\text{as}}, s_L^{\dagger}(s_H)\right\}$.

The desired result follows by noting that $s_L^{\dagger}(s_H) \geq s_L^{\text{safe}}$, and therefore $\min\left\{s_L^{\dagger}, s_L^{\text{as}}(s_H)\right\} \geq \min\left\{s_L^{\text{safe}}, s_L^{\text{as}}(s_H)\right\}$.

□

Theorem 3. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at $s_j \in \mathbf{S}$. Equilibrium* total surplus is:

1. weakly greater under $\mathcal{E}'$ if the local spread is a negative override under equilibrium*.
2. weakly less under $\mathcal{E}'$ if the local spread is a positive override under equilibrium*, unless adverse selection is $\sigma^\star_{\mathcal{E}}$-irrelevant for signal s_{j+1}.

Proof. The Theorem focuses either on the least, or the most selective equilibrium strategies under both experiments. In the discussion below, I let σ^* and $\sigma^{*'}$ denote whichever equilibria we are focusing on under the respective experiments $\mathcal{E}$ and $\mathcal{E}'$. When I need to distinguish between the least and most selective equilibria, I denote them as $(\check{\sigma}, \check{\sigma}')$ and $(\hat{\sigma}, \hat{\sigma}')$, respectively. Following the notation introduced in Definition 2, let $\mathbf{S} \cup \mathbf{S}' = \{s_1, s_2, ..., s_M\}$ be the joint support of the experiments $\mathcal{E}$ and $\mathcal{E}'$, with elements increasing in their indices as usual. Since $\mathcal{E}'$ is obtained by a *local* mean preserving spread of $\mathcal{E}$, there is a monotone strategy $\sigma' : \mathbf{S}' \to [0,1]$ whose outcome under $\mathcal{E}'$ replicates the outcome of σ^* under $\mathcal{E}$:

$$\sigma'(s) = \begin{cases} \sigma^*(s_j) & s \in \{s_{j-1}, s_{j+1}\} \\ \sigma^*(s) & s \notin \{s_{j-1}, s_{j+1}\} \end{cases}$$

Claim 1. Efficiency under the most (least) selective equilibrium of $\mathcal{E}'$ weakly exceeds that under $\mathcal{E}$ when $\hat{\sigma}(s_j) = 1$ ($\check{\sigma}(s_j) = 1$).

Now suppose s_j leads to trade under σ^*; $\sigma^*(s_j) = 1$. Therefore, $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 1$. Below, I show that $\sigma^{*'}$ is *more selective than* σ'. By Lemma 4, it follows that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \geq \Pi\left(\sigma'; \mathcal{E}'\right) = \Pi\left(\sigma; \mathcal{E}\right)$.

If $s_{j-1} = \min \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j-2}) = 0$, $\sigma^{*'}$ must necessarily be more selective than σ'; and we are done. So, for contradiction, I assume the following:

- $s_{j-1} > \min \mathbf{S} \cup \mathbf{S}'$
- $\sigma^{*'}(s_{j-2}) > 0$
- $\sigma^{*'}$ is *less* selective than σ', where the two strategies differ.

Case i. σ^* and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$.

I will prove the contradiction by constructing a strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for experiment $\mathcal{E}$, such that:

i $\tilde{\sigma}$ replicates the outcome $\breve{\sigma}'$ induces in $\mathcal{E}'$,

ii That $\breve{\sigma}'$ is an eqm. strategy under $\mathcal{E}'$ implies that $\tilde{\sigma}$ is an eqm. strategy under $\mathcal{E}$,

iii But $\tilde{\sigma}$ is less selective than $\breve{\sigma}$, contradicting that $\breve{\sigma}$ is the least selective equilibrium strategy under $\mathcal{E}$.

I define the strategy $\tilde{\sigma} : \mathbf{S} \to [0,1]$ for $\mathcal{E}$ as:

$$\tilde{\sigma}(s) := \begin{cases} 1 & s = s_i \\ \sigma'(s) & s \neq s_i \end{cases}$$

it is seen easily that $\tilde{\sigma}$ replicates the outcome of $\breve{\sigma}'$. Furthermore, $\breve{\sigma}'$ is an equilibrium under $\mathcal{E}'$ if and only if $\tilde{\sigma}$ is an equilibrium under $\mathcal{E}$: they induce the same interim belief ψ, and share the following necessary and sufficient condition for optimality:

$$\mathbb{P}_\psi\left(\theta = H \mid s_{j-2}\right) \begin{cases} = c & \sigma'(s_{j-2}) < 1 \\ \geq c & \sigma'(s_{j-2}) = 1 \end{cases}$$

The strategy $\tilde{\sigma}$ under experiment $\mathcal{E}$ replicates the outcome of $\breve{\sigma}'$ under experiment $\mathcal{E}'$, and σ' under $\mathcal{E}'$ replicates the outcome of $\breve{\sigma}$ under experiment $\mathcal{E}$. Since we assumed that $\breve{\sigma}'$ is less selective than σ', it must be that $\tilde{\sigma}$ is less selective than $\breve{\sigma}$.

Case ii. σ^* and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

Since strategy σ' for experiment $\mathcal{E}'$ replicates the outcome of $\hat{\sigma}$ for experiment $\mathcal{E}$, the two strategies induce the same interim belief ψ. Therefore, if $\mathbb{P}_\psi\left(\theta = H \mid s_{j-1}\right) \geq c$, σ' is an equilibrium under $\mathcal{E}'$; meaning $\hat{\sigma}'$ must be more selective than σ'.

Otherwise, say $\mathbb{P}_\psi\left(\theta = H \mid s_{j-1}\right) < c$. Then, by Lemma 13, there must be an equilibrium strategy that is more selective than σ' under $\mathcal{E}'$.

Claim 2. Efficiency under the most (least) selective equilibrium of $\mathcal{E}'$ falls weakly below that under $\mathcal{E}$ if:

i. s_j leads to rejections under $\mathcal{E}$; i.e. $\hat{\sigma}(s_j) = 0$ ($\breve{\sigma}(s_j) = 0$), and

ii. the following condition holds:

$$\frac{\rho}{1-\rho} \times \left(\frac{r_H(\sigma;\mathcal{E})}{r_L(\sigma;\mathcal{E})}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

Now, suppose s_j leads to rejections under σ^*; $\sigma^*(s_j) = 0$. Consequently, we have $\sigma'(s_{j-1}) = \sigma'(s_{j+1}) = 0$. I establish Claim 2 in two steps:

Step 1. $\sigma^{*'}$ is less selective than σ'; trade is likelier when when s_j is locally spread.

Step 2. This efficiency when the condition in Claim 2 is met; $\Pi(\sigma^{*'}; \mathcal{E}') \leq \Pi(\sigma'; \mathcal{E}') = \Pi(\sigma^*; \mathcal{E})$.

Step 1.

If $s_{j+1} = \max \mathbf{S} \cup \mathbf{S}'$ or $\sigma^{*'}(s_{j+1}) > 0$, it must be the case that $\sigma^{*'}$ is less selective than σ', and we are done. So instead, I assume that $s_{j+1} < \max \mathbf{S} \cup \mathbf{S}'$ and $\sigma^{*'}(s_{j+1}) = 0$.

Case i. σ^* and $\sigma^{*'}$ are the least selective equilibrium strategies; i.e. $\sigma^* = \check{\sigma}$ and $\sigma^{*'} = \check{\sigma}'$

Since σ' replicates the outcome of $\check{\sigma}$, we have $\Psi(\check{\sigma}; \mathcal{E}) = \Psi(\sigma'; \mathcal{E}') = \psi$. Thus, σ' must be an equilibrium strategy under $\mathcal{E}'$ if $\mathbb{P}_\psi(\theta = H \mid s_{j+1}) \leq c$: the optimality conditions for all signals below s_{j+1} are satisfied *a fortiori*, and those for the signals above s_{j+1} are satisfied since $\check{\sigma}$ has the same optimality conditions under $\mathcal{E}$. So, $\check{\sigma}'$ must be less selective than σ', since the former is the least selective equilibrium. If on the other hand, $\mathbb{P}_\psi(\theta = H \mid s_{j+1}) > c$, there must be an equilibrium strategy under experiment $\mathcal{E}'$ that is *less* selective than σ', by Lemma 13.

Case ii. σ^* and $\sigma^{*'}$ are the most selective equilibrium strategies; i.e. $\sigma^* = \hat{\sigma}$ and $\sigma^{*'} = \hat{\sigma}'$.

$\hat{\sigma}'$ is the most selective equilibrium strategy under experiment $\mathcal{E}'$, and we assumed that $\hat{\sigma}'(s_{j+1}) = 0$. The strategy $\tilde{\sigma}$ defined below for experiment $\mathcal{E}$ replicates the outcome $\hat{\sigma}'$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \hat{\sigma}'(s) & s > s_j \end{cases}$$

Note that $\tilde{\sigma}$ must be an equilibrium under experiment $\mathcal{E}$, since the interim belief it induces is the same as the one $\hat{\sigma}'$ does, and its optimality constraints are a subset of the latter's. But since $\hat{\sigma}$ is the *most* selective equilibrium strategy under $\mathcal{E}$, $\tilde{\sigma}$ must be less selective than it.

Step 2.

The statement is trivially true when $\sigma' = \sigma^{*'}$, so I focus on the case where these two strategies differ. As Step 1 established, $\sigma^{*'}$ must be less selective than σ'. This implies that $\sigma^{*'}(s_{j+1}) > 0$. To see why, say we had $\sigma^{*'}(s_{j+1}) = 0$ instead. We can then construct a strategy $\tilde{\sigma}$ for experiment $\mathcal{E}$, which replicates the outcome $\sigma^{*'}$ generates under experiment $\mathcal{E}'$:

$$\tilde{\sigma}(s) = \begin{cases} 0 & s \leq s_j \\ \sigma^{*'}(s) & s > s_j \end{cases}$$

As they induce the same interim belief and the optimality constraints of the latter are a subset of the former's, $\tilde{\sigma}$ must be an equilibrium under $\mathcal{E}$. This contradicts with σ^* and $\sigma^{*'}$ being the least selective strategies; since $\sigma^{*'}$ being less selective than σ' implies that $\tilde{\sigma}$ must be less selective than σ^*. It also contradicts with σ^* and $\sigma^{*'}$ being the most selective strategies; since it would imply that σ', more selective than $\sigma^{*'}$, should be an equilibrium under $\mathcal{E}'$.

Given that $\sigma^{*'}(s_{j+1}) > 0$, I now take another strategy $\sigma^\delta_{\mathcal{E}'} : \mathbf{S}' \to [0,1]$ for experiment $\mathcal{E}'$:

$$\sigma^\delta_{\mathcal{E}'}(s) = \begin{cases} 1 & s > s_{j+1} \\ \delta & s = s_{j+1} \\ 0 & s < s_{j+1} \end{cases}$$

where $\delta > 0$ is small enough so that $\sigma^\delta_{\mathcal{E}'}$ is more selective than $\sigma^{*'}$, but less selective than σ'. I will show that, when the condition stated in Claim 2 holds, we have $\Pi\left(\sigma^\delta_{\mathcal{E}'}; \mathcal{E}'\right) \leq \Pi\left(\sigma'; \mathcal{E}'\right)$ for $\delta \to 0$. Lemma 14 then implies that $\Pi\left(\sigma^{*'}; \mathcal{E}'\right) \leq \Pi\left(\sigma^\delta_{\mathcal{E}'}; \mathcal{E}'\right)$, which coins the result.

To show this, I construct another experiment $\mathcal{E}^{\text{re}}$ under which I will use compare two strategies, σ_{re} and $\sigma^\delta_{\text{re}}$, that replicate the outcomes of the strategies σ' and $\sigma^\delta_{\mathcal{E}'}$, respectively. The experiment $\mathcal{E}^{\text{re}}$ has three possible outcomes, $\{s^{\text{re}}_L, s^{\text{re}}_\delta, s^{\text{re}}_H\}$. Conditional on the applicant's quality θ, its outcome distribution is independent from any other information any evaluator sees, and is given by the following pmf p^{re}_θ:

$$p_\theta(s^{\text{re}}) = \begin{cases} 1 - r_\theta(\sigma^*; \mathcal{E}) & s = s^{\text{re}}_H \\ \delta \times p'_\theta(s_{j+1}) & s = s^{\text{re}}_\delta \\ r_\theta\left(\sigma^*; \mathcal{E}\right) - \delta \times p'_\theta(s_{j+1}) & s = s^{\text{re}}_L \end{cases}$$

Define the strategies σ_{re} and $\sigma^\delta_{\text{re}}$ for this experiment as follows:

$$\sigma_{\text{re}}(s) = \begin{cases} 1 & s = s^{\text{re}}_H \\ 0 & s = s^{\text{re}}_\delta \\ 0 & s = s^{\text{re}}_L \end{cases} \qquad \sigma^\delta_{\text{re}} = \begin{cases} 1 & s = s^{\text{re}}_H \\ 1 & s = s^{\text{re}}_\delta \\ 0 & s = s^{\text{re}}_L \end{cases}$$

Now note that these two strategies replicate the outcomes of the strategies σ' and $\sigma^\delta_{\mathcal{E}'}$, respectively. Under $\sigma_{\text{re}}(s)$, the probability that a buyer trades, conditional on the seller's quality, is the same as it is under strategy σ' (or σ^*, which it replicates), and under $\sigma^\delta_{\text{re}}$, it is the same as it is under $\sigma^\delta_{\mathcal{E}'}$.

So, the difference between efficiency under these two strategies is determined by the *marginal reject* who:

- is rejected by *every* buyer under the strategy σ_{re}.

- is accepted by *at least one* buyer under the strategy $\sigma^{\delta}_{\text{re}}$.

Where $\mathbf{s^{re}} = \left\{s^1, ..., s^n\right\}$ is the seller's signal profile under the experiment $\mathcal{E}^{\text{re}}$, he has:

- *no* s^{re}_H signals; $s^i \neq s^{\text{re}}_H$ for all $i \in \{1, 2, ..., n\}$ and
- *at least one* s^{re}_δ signal; there exists some $i \in \{1, 2, ..., n\}$ such that $s^i = s^{\text{re}}_H$.

Thus we have:

$$\begin{aligned}\Pi(\sigma^{\delta}_{\mathcal{E}'}; \mathcal{E}') - \Pi(\sigma'; \mathcal{E}') &= \Pi(\sigma^{\delta}_{\text{re}}; \mathcal{E}^{\text{re}}) - \Pi(\sigma_{\text{re}}; \mathcal{E}^{\text{re}}) \\ &= \mathbb{P}\left(\text{marginal reject}\right) \times \underbrace{\left[\mathbb{P}\left(\theta = H \mid \text{marginal reject}\right) - c\right]}_{(2)}\end{aligned}$$

The expression labelled (2) above equals:

$$\sum_{i=1}^{n} \frac{\mathbb{P}\left(i \ s^{\text{re}}_\delta \text{ and } n-i \ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k \ s^{\text{re}}_\delta \text{ and } n-k \ s^{\text{re}}_L \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i \ s^{\text{re}}_\delta \text{ and } n-i \ s^{\text{re}}_L \text{ signals}\right) - c$$

Since the probability that a buyer observes the s^{re}_δ signal is proportional to δ, we have[45]:

$$\lim_{\delta \to 0} \frac{\mathbb{P}\left(i \ s^{\text{re}}_\delta \text{ and } n-i \ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k \ s^{\text{re}}_\delta \text{ and } n-k \ s^{\text{re}}_L \text{ signals}\right)} = 0$$

Therefore, we get:

$$\begin{aligned}&\lim_{\delta \to 0} \sum_{i=1}^{n} \frac{\mathbb{P}\left(i \ s^{\text{re}}_\delta \text{ and } n-i \ s^{\text{re}}_L \text{ signals}\right)}{\sum\limits_{k=1}^{n} \mathbb{P}\left(k \ s^{\text{re}}_\delta \text{ and } n-k \ s^{\text{re}}_L \text{ signals}\right)} \times \mathbb{P}\left(\theta = H \mid i \ s^{\text{re}}_\delta \text{ and } n-i \ s^{\text{re}}_L \text{ signals}\right) - c \\ &\lim_{\delta \to 0} \mathbb{P}\left(\theta = H \mid \text{one } s^{\text{re}}_\delta \text{ signal and } n-1 \ s^{\text{re}}_L \text{ signals}\right) - c \\ &\propto \lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \frac{p'_H(s_{j+1})}{p'_L(s_{j+1})} \times \left(\frac{r_H(\sigma^*; \mathcal{E}) - \delta \times p'_H(s_{j+1})}{r_L(\sigma^*; \mathcal{E}) - \delta \times p'_L(s_{j+1})}\right)^{n-1} - \frac{c}{1-c} \\ &= \frac{\rho}{1-\rho} \times \frac{s_{j+1}}{1-s_{j+1}} \left(\frac{r_H(\sigma^*; \mathcal{E})}{r_L(\sigma^*; \mathcal{E})}\right)^{n-1} - \frac{c}{1-c}\end{aligned}$$

□

Proposition 7. Let the experiment $\mathcal{E}'$ differ from $\mathcal{E}$ by a local spread at s_j. Total surplus in the most selective equilibrium is lower under $\mathcal{E}'$ if the following conditions hold:

$$\frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right) \leq \frac{c}{1-c} \quad \text{and} \quad \frac{\rho}{1-\rho} \times \left(\frac{s_j}{1-s_j}\right)^{n-1} \times \frac{s_{j+1}}{1-s_{j+1}} \leq \frac{c}{1-c}$$

[45] See expression 9.8 and the surrounding discussion in the proof of Theorem 2 for a more detailed explanation of this.

Proof. First, I let $\hat{\sigma}(s_{j+2}) < 1$. I show that this implies $\hat{\sigma}$ and $\hat{\sigma}'$ induce equivalent outcomes under their respective experiments. The strategy $\sigma' : \mathbf{S}' \to [0,1]$ which replicates the outcome of $\hat{\sigma}'$ under experiment $\mathcal{E}'$:

$$\sigma'(s) = \begin{cases} \hat{\sigma}(s) & s \geq s_{j+2} \\ 0 & s < s_{j+2} \end{cases}$$

must then be an equilibrium strategy under experiment $\mathcal{E}'$. This is because these strategies induce the same interim belief, that $\hat{\sigma}$ is an equilibrium strategy under $\mathcal{E}$ ensures that the optimality conditions of σ' for signals below s_{j+2} are satisfied, and for signals above s_{j+2}, the optimality conditions are the same as those for σ'. This means that $\hat{\sigma}'$ must be more selective than σ'. However, when proving Theorem 3, we established that σ' must be more selective than $\hat{\sigma}'$. So it must be that $\sigma' = \hat{\sigma}'$, and we are done.

So instead, let $\hat{\sigma}(s_{j+2}) = 1$. But then, it is easily established that:

$$\frac{r_H(\hat{\sigma}; \mathcal{E})}{r_L(\hat{\sigma}; \mathcal{E})} \leq \frac{s_j}{1 - s_j}$$

since $r_\theta(\hat{\sigma}; \mathcal{E}) = \sum_{k=1}^{j} p_\theta(s_k)$. So, the condition Proposition 7 supplies is sufficient for the one Theorem 3 does.

□

Lemma 8. Where it exists, the regulator-optimal garbling is monotone binary and produces IC recommendations.

Proof. To prove this statement, I take some garbling $\mathcal{E}^G$ and an equilibrium $\sigma^G : \mathbf{S}^G \to [0,1]$ it supports. I then construct a monotone binary garbling $\mathcal{E}^{G*}$ which produces IC recommendations, and show that efficiency under $\mathcal{E}^{G*}$ and the strategy which obeys its recommendations, σ^{G*}, are higher than those under $\mathcal{E}^G$ and σ^G.

For the monotone binary garbling $\mathcal{E}^{G*} = (\mathbf{S}^G, \mathbf{P}^{G*})$ and the garbling $\mathcal{E}^G = (\mathbf{S}^G, \mathbf{P}^G)$ in question:

$$\mathbf{P} \times \mathbf{T} = \mathbf{P}^G \qquad\qquad \mathbf{P} \times \mathbf{T}^* = \mathbf{P}^{G*}$$

define the expressions:

$$f^*(s) := p_L(s) \times t^*_{i1} \qquad\qquad f(s) := p_L(s) \times \sum_{s_j^G \in \mathbf{S}^G} t_{ij} \times \left(1 - \sigma^G(s_j^G)\right)$$

for each $s \in \mathbf{S}$. Given the asset has Low quality, $f^*(s)$ is the probability that (i) a buyer would have observed the signal $s \in \mathbf{S}$ in her original experiment, *and* (ii) the garbling $\mathcal{E}^{G*}$ issues her a "rejection recommendation". Similarly, $f(s)$ is the probability that (i) a buyer would observe the signal $s \in \mathbf{S}$ in her original experiment, *and* (ii) he would be rejected under the equilibrium strategies σ^G. For this Low quality seller, r_L^{G*} below is the probability that the buyer receives a rejection recommendation under $\mathcal{E}^{G*}$; and r_L^G is the probability that the buyer rejects him under $(\mathcal{E}^G, \sigma^G)$:

$$r_L^{G*} := \sum_{s \in \mathbf{S}} f^*(s) \qquad\qquad r_L^G := \sum_{s \in \mathbf{S}} f(s)$$

Now, take the least selective monotone binary garbling $\mathcal{E}^{G*}$ such that $r_L^{G*} = r_L^G$. Evidently, this garbling exists.

Clearly, one can treat f^* and f as probability density functions over $\mathbf{S}$ when normalised. Furthermore, the distribution the former describes is first order stochastically dominated by the one the latter does; $\frac{f^*(s_j)}{\sum\limits_{s \in \mathbf{S}} f^*(s)}$ crosses $\frac{f(s)}{\sum\limits_{s \in \mathbf{S}} f(s)}$ once from below. Therefore we get:

$$\begin{aligned} r_H^* := \sum_{s \in \mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f^*(s)}{\sum\limits_{s \in \mathbf{S}} f^*(s)} \\ \leq \sum_{s \in \mathbf{S}} \frac{p_H(s)}{p_L(s)} \times \frac{f(s)}{\sum\limits_{s \in \mathbf{S}} f(s)} =: r_H \end{aligned}$$

where r_H^* and r_H are the probabilities that a High quality seller is rejected from a visit under the strategies σ^{G*} and σ^G, respectively.

Since $r_H^* \geq r_H$ and $r_L^* = r_L$, efficiency is higher under σ^* than it is under σ. It only remains to show that the strategy σ^* is optimal against the interim belief ψ^* consistent with it.

The interim belief ψ^* consistent with $\mathcal{E}^{G*}$ and σ^{G*} lies below ψ—the interim belief consistent with $\mathcal{E}^G$ and σ^G:

$$\frac{\psi^*}{1-\psi^*} = \frac{\sum\limits_{k=0}^{n-1} \left(r_H^*\right)^k}{\sum\limits_{k=0}^{n-1} \left(r_L^*\right)^k} = \frac{\sum\limits_{k=0}^{n-1} (r_H)^k}{\sum\limits_{k=0}^{n-1} (r_L)^k} \leq \frac{\psi}{1-\psi}$$

Under the interim belief ψ^*, it is optimal for a buyer upon the signal s_L^{G*} if and only if:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{c}{1-c}$$

But this inequality must hold; since $\frac{r_H^*}{r_L^*} \leq \frac{r_H}{r_L}$, $\psi^* \leq \psi$, and σ^G is optimal against ψ:

$$\frac{\psi^*}{1-\psi^*} \times \frac{r_H^*}{r_L^*} \leq \frac{\psi}{1-\psi} \times \frac{r_H}{r_L} \leq \frac{c}{1-c}$$

Furthermore, that $\Pi(\sigma^{G*}; S^{G*}) \geq \Pi\left(\sigma; S^G\right) \geq 0$ suggests that the expected suprlus from accepting a seller upon the "approve" recommendation must be weakly positive; hence optimal. Thus, the strategy σ^{G*} is optimal against ψ^*.

$\square$

Proposition 9. If the least selective monotone binary garbling under which adverse selection is irrelevant produces IC recommendations, it is the regulator-optimal garbling. Otherwise, the regulator-optimal garbling is either:

- the least selective garbling under which adverse selection is irrelevant, or
- the most selective garbling under which adverse selection is not irrelevant

among monotone binary garblings which produce IC recommendations.

Proof. **Step 1:** The following are well-defined:

- the least selective monotone binary garbling under which adverse selection is irrelevant,
- the least (most) selective monotone binary garbling under which adverse selection is (not) irrelevant among those which produce IC recommendations.

I first show the least selective monotone binary garbling under which adverse selection is irrelevant is well defined. For any monotone binary garbling $\mathcal{E}^G = (\mathbf{S}^G, \mathbf{P}^G)$, let $\mathbf{P} \times \mathbf{T} = \mathbf{P}^G$ and define $d(\mathcal{E}^G) := \sum_{i=1}^{m} t_{i2}$. Evidently, $d(.)$ is a bijection between the space of monotone binary garblings of $\mathcal{E}$ and $[0, m]$. Also, where both are monotone binary garblings of $\mathcal{E}$, $\mathcal{E}^G$ is more selective than $\mathcal{E}^{G'}$ if and only if $d(\mathcal{E}^G) \leq d(\mathcal{E}^{G'})$. Thus, we seek the monotone binary garbling $d^{-1}(D^*)$ where $D^* := \max\{D \in [0, m] : \text{ a.s. is irrelevant under } d^{-1}(D)\}$. We must only show that D^* is well defined. To that end, define the Real valued function F over the space of monotone binary garblings, where:

$$F(S^G) = \begin{cases} \frac{\rho}{1-\rho} \frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{r_H^G}{r_L^G}\right)^{n-1} - \frac{c}{1-c} & d(\mathcal{E}^G) \in (0, m) \\ \lim_{D \downarrow 0} F \circ d^{-1}(D) & d(\mathcal{E}^G) = 0 \\ +\infty & d(\mathcal{E}^G) = m \end{cases} \qquad r_\theta^G := \sum_{s \in \mathbf{S}} p_s^G(s_L^G) \times p_\theta(s)$$

where s^* is the threshold signal of this garbling.

So, we seek $D^* := \max\{D \in [0, m] : F \circ d^{-1}(D) \geq 0\}$. But this maximiser exists because the function $F \circ d^{-1}$ is upper semicontinuous: $F \circ d^{-1}$ is a decreasing function, and for any $\bar{D} \in [0, m]$ and $\varepsilon > 0$, we can find some δ_ε such that $D \in (\bar{D} - \delta_\varepsilon, \bar{D}) \cap [0, m]$ implies $d^{-1}(\bar{D})$ and $d^{-1}(D)$ have the same threshold signal s^* and thus $F \circ d^{-1}(D) < F \circ d^{-1}(\bar{D}) + \varepsilon$ since r_H^G / r_L^G is continuous in $d(S^G)$.

Now say this garbling does not provide IC recommendations. Denote the interim belief that is consistent with evaluators following $\mathcal{E}^G$'s recommendations as ψ^G. The garbling $\mathcal{E}^G$ provides IC recommendations if:

$$\underbrace{\frac{r_H^G}{r_L^G} \times \frac{\psi^G}{1-\psi^G}}_{:=f_1(d(\mathcal{E}^G))} \leq \frac{c}{1-c} \qquad\qquad \underbrace{\frac{1-r_H^G}{1-r_L^G} \times \frac{\psi^G}{1-\psi^G}}_{:=f_2(d(\mathcal{E}^G))} \geq \frac{c}{1-c}$$

As defined above, both $f_1(.)$ and $f_2(.)$ are continuous. Therefore, the set of monotone binary garblings with optimal recommendations—$\left\{D \in [0,m] : f_1(D) \leq \frac{c}{1-c} \text{ and } f_2(D) \geq \frac{c}{1-c}\right\}$—is compact. Thus, both objects below are well-defined:

$$\max \left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has IC rec.s } : F \circ d^{-1}(D) \geq 0\right\}$$
$$\min \left\{D \in [0,m] \text{ and } d^{-1}(D) \text{ has IC rec.s } : F \circ d^{-1}(D) \leq 0\right\}$$

Among those with IC recommendations, the former gives us the least selective garbling under which adverse selection is irrelevant. The latter gives us the most selective garbling under which adverse selection is not irrelevant among such garblings, since the least-selective garbling under which adverse selection is irrelevant does *not* have IC recommendations (the minimiser of this set *must* have $F \circ d^{-1}(D) < 0$).

Step 2: Proving the statement of Proposition 9.

Efficiency under a monotone binary garbling S^G and strategies σ^G that obey its recommendations is given by:

$$\Pi\left(\sigma^G; S^G\right) = \rho - c - \rho \times \left(r_H^G\right)^n \times (1-c) + (1-\rho) \times \left(r_L^G\right)^n \times c$$

As a function of $d^{-1}(.)$, efficiency is continuous and therefore attains its maximum over the set $[0,m]$. I show that this maximum is attained with the least selective garbling under which adverse selection is irrelevant.

For the garbling $\mathcal{E}^G$, define $\mathcal{E}^G_{+\varepsilon} := d^{-1}(d(\mathcal{E}^G) + \varepsilon)$ and $S^G_{-\varepsilon} := d^{-1}(d(S^G) - \varepsilon)$. Likewise, let $s^*_{+\delta}$ and $s^*_{-\delta}$ be the threshold signals of these experiments, and $r^*_{\theta;+\delta}$, $r^*_{\theta;-\delta}$ be the probability that a seller of quality θ is rejected in a visit, under each garbling. From our earlier reasoning about the impact of making evaluators strategies marginally more (less) selective, we observe that:

$$\lim_{\delta \to 0} \Pi\left(\sigma^G_{+\delta}; \mathcal{E}^G_{+\delta}\right) - \Pi\left(\sigma^G; \mathcal{E}^G\right) \propto \lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \frac{p_H(s^*_{+\delta})}{p_L(s^*_{+\delta})} \times \left(\frac{r^G_{H;+\delta}}{r^G_{L;+\delta}}\right)^{n-1} - \frac{c}{1-c} \leq 0$$

where the last inequality follows since $\mathcal{E}^G$ is the least selective garbling under which adverse selection is irrelevant. We conclude that giving evaluators a marginally less selective garbling, and therefore (Lemma 14) any garbling that is less selective than S^G, cannot improve their payoffs. Likewise, for a marginally more selective garbling we have:

$$\lim_{\delta \to 0} \Pi\left(\sigma^G_{+\delta}; \mathcal{E}^G_{-\delta}\right) - \Pi\left(\sigma^G; \mathcal{E}^G\right) \propto -\lim_{\delta \to 0} \frac{\rho}{1-\rho} \times \frac{p_H(s^*)}{p_L(s^*)} \times \left(\frac{r^G_{H;+\delta}}{r^G_{L;+\delta}}\right)^{n-1} - \frac{c}{1-c} \geq 0$$

where the term on the RHS is now negative because the probability of trade *decreases* when strategies become more selective. By a reasoning similar to that behind Lemma 14, this reveals that *no* garbling that is more selective can improve efficiency either.

This also proves that among those with optimal recommendations, the least selective garbling under which adverse selection is irrelevant cannot be improved with a more selective garbling and the most selective garbling under which adverse selection is not irrelevant cannot be improved with a less selective garbling.

□

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