Committees

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COMMENTS ON!

Abstract

Orange notes: for Ian & Dan.

Green notes: for me (but Ian & Dan should feel free to chime in!).

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1 Introduction

2 The Model

A committee of N *members*, $m_1, ..., m_N$, wish to decide whether to *enact* or *block* a policy. The benefit a member m_i obtains from enacting the policy depends on an unknown state of the world, $\omega \in \{\omega_0, \omega_1\}$. When the state is favourable, $\omega = \omega_1$, enacting the policy yields her a payoff of u_i. When the state is unfavourable, $\omega = \omega_0$, it yields her a payoff loss of 1 instead. In contrast, blocking the policy yields her a payoff of 0 in either state. There is also a *lobbyist* outside the committee. He prefers the policy to be enacted regardless of the state.

At the outset of the game, the lobbyist and committee members share the prior belief that $\omega = \omega_1$ with probability $p \in [0, 1]$. The lobbyist can then provide additional information about the state. He does so by committing to perform and publicly reveal the outcome of a Blackwell experiment of his choice. Formally, a Blackwell experiment is a measurable space $(X, \mathcal{X})$, and, for each possible state ω_k, a probability measure P_k over this space. Once players observe the outcome $x \in X$ of this experiment, they update their prior belief to an *interim belief* $r \in [0, 1]$ that $\omega = \omega_1$.

Following a *belief-based approach*[1] I work directly with the distribution of interim beliefs the lobbyist's chosen Blackwell experiment induces. Thus, I model the lobbyist as choosing the distribution $\mu_l \in \Delta_p([0, 1])$ his experiment will induce over interim beliefs, where $\Delta_p([0, 1])$ denotes the set of probability measures over the interval $[0, 1]$ with a barycenter p.:

$$\Delta_p([0,1]) := \left\{ \mu \in \Delta([0,1]) : \int_0^1 r d\mu(r) = p \right\}$$

Once the outcome of the lobbyist's chosen experiment is revealed, committee members can acquire further information about the state. To this end, each member can privately choose another Blackwell experiment whose outcome she will observe. Following the outcome of her experiment, member m_i updates her interim belief r to a posterior belief $q_i \in [0, 1]$ that $\omega = \omega_1$. Again, I model m_i as choosing the distribution $\mu_{i;r} \in \Delta_r([0, 1])$ her Blackwell experiment will induce over her posterior beliefs, given her interim belief r. For simplicity, I assume that the outcomes of members' chosen experiments are independent conditional on the true state ω^2.

discuss in last section

Providing information is costless for the lobbyist. In contrast, acquiring additional information is costly for committee members: m_i pays a cost $C_i(\mu)$ to observe the outcome of an experiment which induces the distribution μ over her possible posterior beliefs. The cost function $C_i : \Delta([0, 1]) \to \mathbb{R}^+$

[1] See Kamenica and Gentzkow 2011 and Caplin and Dean 2013.
[2] This assumption has no bearing on the results of this paper; see Section 5

is *uniformly posterior separable*; i.e., there is a convex function $c_i : [0,1] \to \mathbb{R}^+$ such that:

$$C_i(\mu) := \int_0^1 c_i(q)d\mu(q) - c_i\left(\mathbb{E}_\mu[q]\right)$$

Following the lobbyist's provision and committee's acquisition information, each committee member casts a private vote into a voting mechanism: a tuple $\left(D, \{V_i\}_{i=1}^N, \{\mathcal{V}_i\}_{i=1}^N\right)$ comprised of a set $V_i \ni v_i$ of possible votes each member m_i may cast, a σ-algebra $\mathcal{V}_i$ over each of these sets, and a *decision rule* $D : \bigtimes_{i=1}^N V_i \to [0,1]$—a measurable mapping from the space of possible vote profiles $\left(\bigtimes_{i=1}^N V_i, \bigtimes_{i=1}^N \mathcal{V}_i\right)$ to a probability that the policy is enacted. For a given interim belief r and posterior belief q, I model member m_i's voting behaviour as a distribution $\tau_{i;q,r} \in \Delta(V_i)$ over the space of votes she may cast.

For any given voting mechanism, I adopt the *lobbyist-preferred perfect Bayesian equilibrium* (*equilibrium*, henceforth) as my solution concept. An equilibrium is a strategy μ_l^* for the lobbyist, and an information strategy $\left\{\mu_{i;r}^*\right\}_{r\in[0,1]}$ and voting strategy $\left\{\tau_{i;q,r}^*\right\}_{(q,r)\in[0,1]^2}$ for each member m_i, such that:

1. Member m_i's voting strategy maximises her expected payoff at every pair (r,q) of beliefs she may hold, given other members' information and voting strategies: i don't think i am missing off-path considerations..?

$$\begin{aligned} & v_i \in \operatorname{supp} \tau_{i;q,r} \\ \implies & v_i \in \underset{v \in V_i}{\operatorname{argmax}} \int\limits_{[0,1]^{N-1}} \left[\mathbb{P}_{r,q_1,\ldots,q_N}(\omega_1) \times (u_i+1) - 1\right] \\ & \qquad \times \left\{ \int\limits_{V_{-i}} D(v, \mathbf{v_{-i}}) d\tau^*_{-i;r,\mathbf{q}_{-i}}(\mathbf{v_{-i}}) \right\} \frac{d\mu_r^*(q, \mathbf{q_{-i}})}{\int_{Q_{-i}} \mu_r^*(q, \mathbf{q_{-i}}) d\mathbf{q_{-i}}} \end{aligned}$$

where:

- $(V_{-i}, \mathcal{V}_{-i}) := \left(\bigtimes_{j\neq i} V_j, \bigtimes_{j\neq i} \mathcal{V}_j\right)$ is the space of vote profiles members except m_i may submit, with $\mathbf{v}_{-i} \in V_{-i}$.
- $\tau^*_{-i;r,\mathbf{q}_{-i}} := \bigtimes_{j\neq i} \tau^*_{j;r,q_j}$ is the product measure over this space induced by those members' voting behaviour, given the vector $\mathbf{q}_{-i} \in V_{-i}$ of their posterior beliefs.
- $\mu_r^* := \bigtimes_{i=1}^N \mu_{i;r}^*$ is the probability measure over the space of committee members' joint posterior beliefs the information those members acquire induces[3].

[3]It is straightforward to construct this measure given the conditional independence of members' Blackwell experiments, see Section add Technical Appendix, add this there.

- $\mathbb{P}_{r,q_1,...,q_N}(\omega_1)$ is the probability that $\omega = \omega_1$ given members' interim and posterior beliefs:

$$\mathbb{P}_{q_1,...,q_N}(\omega_1) := \left(\mathcal{L}(r) \times \prod_{j=1}^{N} \frac{\mathcal{L}(q_j)}{\mathcal{L}(r)}\right) \times \left(1 + \mathcal{L}(r) \times \prod_{j=1}^{N} \frac{\mathcal{L}(q_j)}{\mathcal{L}(r)}\right)^{-1} \quad \text{where } \mathcal{L}(q) := \frac{q}{1-q}$$

This is extremely long..?

2. Member m_i's information strategy maximises her expected payoff at any interim belief $r \in [0, 1]$, given others' information and voting strategies:

$$\mu^*_{i;r} \in \underset{\mu \in \Delta_r[0,1]}{\operatorname{argmax}} \int [\mathbb{P}_{q_1,...,q_N}(\omega_1) \times (u_i + 1) - 1]$$
$$\times \left\{\int_{V_{-i}} D(v_1, ..., v_N)\tau^*_{-i;r,q}(d\mathbf{v}_{-i})\tau^*_{i;r,q}(dv_i)\right\} \frac{\mu_r(q_i, d\mathbf{q_{-i}})}{\int_{Q_{-i}} \mu_r(q_i, \mathbf{q_{-i}})d\mathbf{q_{-i}}} \mu_{i;r}(dq_i) - C_i(\mu)$$

where $\mu_r := \bigtimes_{j \neq i} \mu^*_{r;j} \times \mu$ for m_i's chosen distribution μ.

3. The lobbyist's information strategy μ^*_l tmaximises the equilibrium probability that the policy is enacted given committee members' information and voting strategies.

4. The equilibrium strategies maximise the probability that the policy is enacted among all strategy profiles which satisfy the three conditions above.

can we ensure this exists? does it matter for me?

if multiple exist there, the one with lowest info costs

Dominant Voting Mechanisms

This paper investigates whether there exists a *dominant* voting mechanism—one whose equilibrium achieves:

1. a (weakly) higher probability that the policy is blocked when $\omega = \omega_0$,
2. a (weakly) higher probability that the policy is enacted when $\omega = \omega_1$,
3. a (weakly) lower expected cost of information acquisition for each committee member.

than any alternative. Any committee member—regardless of the payoff she enjoys when the policy is enacted in the favourable state and her information acquisition costs—would prefer such a voting mechanism to its alternatives.

3 Equilibria Under Dictatorships

A *dictatorship*–appointing a single member whose vote fully determines whether the policy is enacted–is among the simplest voting mechanisms the committee may adopt.

Definition 1. A voting mechanism $\left(\overset{\text{♛}}{D}_i, \{V_j\}_{j=1}^{N}, \{\mathcal{V}_j\}_{j=1}^{N}\right)$ is an *m_i-dictatorship*, if:

1. $V_i = \{0, 1\}$, and V_j is a singleton for all $j \neq i$.

2. $\overset{\text{♛}}{D}_i(\mathbf{v}) = v_i$ for any profile of votes $\mathbf{v} = (v_1, ..., v_N)$.

"up to isomorphism", but how to say this nicer?

Despite dictatorships' simplicity, the main result of this paper (Theorem 1) establishes that there exists a member whose dictatorship is a dominant voting mechanism. This section sets the ground for this result by characterising the equilibrium a dictatorship induces.

Lemma 1. In the equilibrium of an m_i-dictatorship:

1. $\tau^*_{i;r,q} = \delta_{\mathbb{1}\{q \geq (1+u_i)^{-1}\}}$ for all interim beliefs $r \in [0, 1]$.

2. $\mu^*_{i;r}$ is the unique distribution for which supp $\mu^*_{i;r} \subseteq \left\{\min\{r, \underline{q}^*_i\}, \max\{r, \bar{q}_i{}^*\}\right\}$, for two threshold beliefs $\underline{q}^*_i$ and $\bar{q}_i{}^*$, $0 \leq \underline{q}^*_i \leq \bar{q}_i{}^* \leq 1$.

3. μ^*_l is the unique distribution for which supp $\mu^*_l \subseteq \{0, \max\{p, \bar{q}_i{}^*\}\}$.

Proof.

1. In an m_i-dictatorship, no vote except m_i's of has any bearing on the outcome. So, the outcome may only reflect m_i's posterior belief. Therefore, $v_i \in \tau^*_{i;r,q}$ implies that $v_i \in \arg\max_{v \in \{0,1\}} v \times [q \times (u_i + 1) - 1]$. This establishes that $\tau^*_i(r, q_i) = \delta_{\mathbb{1}\{q_i \geq (1+u_i)^{-1}\}}$.

2. Member m_i uses her information to decide between two possible votes; so, we can without loss of generality assume $|$supp $\mu^*_{i;r}| \leq 2$ for all $r \in [0, 1]$. Furthermore, since information acquisition is costly, any additional information she acquires must have decision-making value; i.e., $(1 + u_i)^{-1} \in$ conv $\left(\text{supp } \mu^*_{i;r}\right)$ whenever $|$supp $\mu^*_{i;r}| = 2$. Lastly, note that Theorem 1 in Caplin, Dean, and Leahy 2022 implies that m_i's strategy must be *Locally Posterior Invariant* since C_i is uniformly posterior separable; i.e., for any $\tilde{r} \in$ conv $\left(\text{supp } \mu^*_{i;r}\right)$, $\mu^*_{i;\tilde{r}}$ is the unique distribution such that supp $\mu^*_{i;\tilde{r}} =$ supp $\mu^*_{i;r}$.

   add here that these two are outcome equivalent for the lobbyist, so also satisfy lobbyist-preferred

   If m_i never acquires any information under the m_i-dictatorship; i.e., $\mu^*_{i;r} = \delta_r$ for all $r \in [0, 1]$, we are done. Otherwise, for any r such that $|$supp $\mu^*_{i;r}| = 2$, $(1 + u_i)^{-1} \in$ conv $\left(\mu^*_{i;r}\right)$ implies that supp $\mu^*_{i;r} =$ supp $\mu^*_{i;(1+u_i)^{-1}}$. Letting $\left\{\underline{q}^*_i, \bar{q}^*_i\right\} :=$ supp $\mu^*_{i;(1+u_i)^{-1}}$ where $\underline{q}^*_i \leq \bar{q}^*_i$, we thus conclude:

   (a) for all $r \in \left[\underline{q}^*_i, \bar{q}^*_i\right]$, $\mu^*_{i;r}$ is the unique distribution such that supp$\mu^*_{i;r} = \left\{\underline{q}^*_i, \bar{q}^*_i\right\}$.

   (b) for all $r \notin \left[\underline{q}^*_i, \bar{q}^*_i\right]$, $\mu^*_{i;r} = \delta_r$.

which proves our claim.

3. Given m_i's equilibrium voting and information strategies, let $\pi^*_{l;i}(r)$ be the equilibrium probability that the policy is enacted as a function of committee members' interim belief r. Further, let $\hat{\pi_{l;i}}^*(r)$ be the concave envelope of this function. These two functions are then given by:

$$\pi^*_{l;i}(r) := \begin{cases} 0 & r \leq \underline{q}_i^* \\ \frac{r-\underline{q}_i^*}{\bar{q}_i^*-\underline{q}_i^*} & r \in \left[\underline{q}_i^*, \bar{q}_i^*\right] \\ 1 & r \geq \bar{q}_i^* \end{cases} \qquad \hat{\pi_{l;i}}^*(r) := \begin{cases} \frac{r}{\bar{q}_i^*} & r \leq \bar{q}_i^* \\ 1 & r \geq \bar{q}_i^* \end{cases}$$

That $\hat{\pi_{l;i}}^*(r)$ is indeed the concave envelope of $\pi^*_{l;i}(r)$ can be easily verified by noting that any concave function which coincides with $\pi^*_{l;i}$ at $r = 0$ and $r = \bar{q}_i^*$ must weakly lie above the function $\hat{\pi_{l;i}}^*(r)$ on the interval $[0, \bar{q}_i^*]$.

By Kamenica and Gentzkow 2011, the condition $\mathbb{E}_\mu \, \pi^*_{l;i}(.) = \hat{\pi_{l;i}}^*(p)$ is then sufficient for the optimality of μ for the lobbyist. It is easily seen that the distribution μ_l^* defined in Lemma 1 satisfies this condition.

□

Discuss this Lemma.

4 The Dominance of Dictatorships

Definition 2. Member m_i is the *most-demanding member* of the committee if $\bar{q}_i^* = \max\{\bar{q}_1^*, \bar{q}_2^*, ..., \bar{q}_N^*\}$.

Discuss what makes a member most-demanding. Low u_i and "appropriately low" costs of acquiring information must interact.

Theorem 1. The dictatorship of the most demanding member is a dominant voting mechanism.

add the assumption that D must attain its min/max.

Proof. Where m_i is the most-demanding member, I use $\overset{\text{♛}}{Q} := \bar{q}_i^*$ to simplify notation in the ensuing. I fix $\left(\{V_j\}_{j=1}^N, \{\mathcal{V}_j\}_{j=1}^N, D\right)$ to be an arbitrary voting mechanism, and let $\overset{\text{♛}}{D}$ denote the voting rule for the dictatorship of the most-demanding member.

I will use Claim 2 to establish the Theorem:

Claim 2. Any voting mechanism induces an equilibrium where committee members:

(a) acquire no information for interim beliefs $r \in \{0\} \cup [\check{Q}, 1]$; i.e., $\mu^*_{1;r} = ... = \mu^*_{N;r} = \delta_r$ for such interim beliefs.

(b) vote to maximise the probability that the policy is executed for interim beliefs $r \geq \check{Q}$, but to minimise it for $r = 0$; i.e.:

$$\mathbb{E}_{\tau^*_{1;r,r},...,\tau^*_{N;r,r}} D(\mathbf{v}) = \begin{cases} \max\limits_{\mathbf{v}\in V} D(\mathbf{v}) := \bar{D} & r \geq \check{Q} \\ \min\limits_{\mathbf{v}\in V} D(\mathbf{v}) := \underline{D} & r = 0 \end{cases}$$

Proof. Assume that all members except m_i employ the asserted strategies. I will first show that the asserted strategies are a best response for m_i, in that case. I then show that, in the relevant subgames, there is no strategy profile that is (a) a mutual best response for all committee members, and (b) preferred by the lobbyist to those asserted above.

Now let $\pi_i(q_i; r)$ be the payoff m_i expects under this voting mechanism given (i) the interim belief r, (ii) her posterior belief q_i, (iii) other members' strategies, and (iv) that she votes optimally at each posterior belief. Since other committee members acquire no private information, we have:

$$\pi_i(q_i; r) = \max_{\tau\in\Delta(V_i)} [q_i \times (1 + u_i) - 1] \times \int\limits_{V_{-i}} \int\limits_{V_i} D(v, \mathbf{v}_{-i})\tau(dv)\tau^*_{-i;r,r}(d\mathbf{v}_{-i})$$

For $q > (1 + u_i)^{-1}$, $\pi_i(q; r)$ is strictly increasing in the probability that the policy is enacted. For $q < (1 + u_i)^{-1}$, it is strictly decreasing in the same probability. Thus, when $q > (1 + u_i)^{-1}$, m_i's optimal voting strategy must maximise the probability that the policy is enacted given others' voting strategies, but when $q < (1+u_i)^{-1}$, it must minimise it. When $q = (1+u_i)^{-1}$, $\pi_i(q_i; r) = 0$; so, any voting strategy is a best response. m_i's equilibrium strategy must then be lobbyist-preferred one, and maximise the probability that the policy is enacted. This verifies Claim 2's assertion on optimal voting strategies, and gives:

$$\pi_i(q; r) = \begin{cases} \bar{D} \times [q \times (1 + u_i) - 1] & q \geq (1 + u_i)^{-1} \\ \underline{D} \times [q \times (1 + u_i) - 1] & q < (1 + u_i)^{-1} \end{cases}$$

whenever $r \in \{0\} \cup [\check{Q}, 1]$.

When $r = 0$, $\Delta_0([0, 1]) = \delta_0$; so $\mu^*_{i;0} = \delta_0$. When $r \geq \check{Q}$, the strategy $\mu^*_{i;r} = \delta_r$ delivers the highest probability that the policy is executed given members' ensuing voting strategies; so, if $\mu^*_{i;r} = \delta_r$ is a best response for m_i, it is also part of her equilibrium information strategy. So to conclude our proof, we wish to show:

$$\int_0^1 \left[\pi_i(q;r) - \pi_i(r;r)\right] d\mu(q) - C_i(\mu) \leq 0 \qquad \text{for all } \mu \in \Delta_r([0,1]) \tag{4.1}$$

whenever $r \geq \overset{\text{♛}}{Q}$.

To this end, define $\overset{\text{♛}}{\pi}_i(q_i)$ to be the payoff m_i expects under an m_i-dictatorship given her posterior belief q_i; i.e., $\overset{\text{♛}}{\pi}_i(q_i) := \max\{0, q_i \times (1+u_i) - 1\}$. Since $\bar{q}_i \geq \overset{\text{♛}}{Q}$, Lemma 1 implies that:

did i show this in the proof of the lemma?

$$\int_0^1 \left[\overset{\text{♛}}{\pi}_i(q) - \overset{\text{♛}}{\pi}_i(r)\right] d\mu(q) - C_i(\mu) \leq 0 \qquad \text{for all } \mu \in \Delta_r([0,1]) \tag{4.2}$$

whenever $r \geq \overset{\text{♛}}{Q}$. So, we are done once we show:

$$\int_0^1 \tilde{\pi}_i(q;r) d\mu(q) \leq 0 \qquad \text{for all } \mu \in \Delta_r([0,1]) \tag{4.3}$$

where $\tilde{\pi}_i(q,r) := [\pi_i(q,r) - \pi_i(r,r)] - [\overset{\text{♛}}{\pi}_i(q) - \overset{\text{♛}}{\pi}_i(r)]$.

To see this, first note that $\pi_i(q,r) - \overset{\text{♛}}{\pi}_i(q) \leq \pi_i\left((1+u_i)^{-1}, r\right) - \overset{\text{♛}}{\pi}_i\left((1+u_i)^{-1}\right) = 0$, so $\tilde{\pi}_i(q,r)$ is piecewise linear, decreasing below $q = (1+u_i)^{-1}$, and increasing above $q = (1+u_i)^{-1}$. Therefore, $\tilde{\pi}_i(.,r)$ is a concave function for any $r \geq \overset{\text{♛}}{Q}$. Combined with the fact that $\tilde{\pi}_i(r,r) = 0$, this gives:

$$\int_0^1 \tilde{\pi}_i(q;r) d\mu(q) \leq \int_0^1 \tilde{\pi}_i(q;r) d\delta_r(q) = 0 \qquad \text{for all } \mu \in \Delta_r([0,1])$$

by Jensen's Inequality.

□

Claim 3. The dictatorship of the most-demanding member is a dominant voting mechanism.

Proof. In the equilibrium the dictatorship of the most-demanding member induces, no committee member acquires private information about the state. Any other voting mechanism must thus result in a weakly higher expected cost of information acquisition for each member.

To conclude the proof, it suffices to show that any fixed voting mechanism achieves:

(a) a lower probability that the policy is enacted when $\omega = \omega_1$, and

(b) a lower probability that the policy is blocked when $\omega = \omega_0$

than the dictatorship of the most-demanding member.

Recall (from Lemma 1) that, under this dictatorship, the lobbyist's equilibrium information strategy is the unique distribution $\overset{\star}{\mu}_l$ such that supp $\overset{\star}{\mu}_l = \left\{0, \min\{p, \overset{\star}{Q}\}\right\}$. Therefore, the dictator never blocks the policy unless her interim belief is $r = 0$; i.e. $\omega = \omega_0$ is revealed. The policy is implemented whenever $\omega = \omega_1$.

Note that, due to Claim 2, the probability that the policy is executed following interim beliefs $r = 0$ and $r = \overset{\star}{Q}$ are $\underline{D}$ and $\bar{D}$, respectively. Therefore, if the lobbyist employs the strategy $\overset{\star}{\mu}_l$ described above, we get:

$$\mathbb{P}_{D,\overset{\star}{\mu}_l}(\text{enact} \mid \omega = \omega_1) = \bar{D} \qquad \mathbb{P}_{D,\overset{\star}{\mu}_l}(\text{enact} \mid \omega = \omega_0) = 1 - \mathbb{P}_{\overset{\star}{\mu}_l}(r = 0 \mid \omega_0) \times (1 - \underline{D})$$

The lobbyist's equilibrium information strategy, μ_l^*, must weakly increase at least one of these probabilities. Since the probability that the policy is enacted cannot exceed $\bar{D}$ in either state, we conclude that:

$$\begin{aligned}
&\mathbb{P}_{D,\mu_l^*}(\text{enact} \mid \omega = \omega_1) \leq \bar{D} \leq 1 \\
&\mathbb{P}_{D,\mu_l^*}(\text{enact} \mid \omega = \omega_0) \geq 1 - \mathbb{P}_{\overset{\star}{\mu}_l}(r = 0 \mid \omega_0) \times (1 - \underline{D}) \geq 1 - \mathbb{P}_{\overset{\star}{\mu}_l}(r = 0 \mid \omega_0)
\end{aligned}$$

$\square$

$\square$

better to do this in two "steps" rather than "claim"s.

5 Discussion