Committees

D. Carlos Akkar*

May 24, 2025

COMMENTS ON!

Abstract

Orange notes: for Ian & Dan.

Green notes: for me (but Ian & Dan should feel free to chime in!).

*Nuffield College and Department of Economics, Oxford. akkarcarlos@gmail.com

I thank Ian Jewitt, Margaret Meyer, Daniel Quigley, and Manos Perdikakis for long discussions and generous guidance.

1 Introduction

2 The Model

A committee of N *members*, $m_1, ..., m_N$, wish to decide whether to *enact* or *block* a policy. The benefit a member m_i obtains from enacting the policy depends on an unknown state of the world, $\omega \in \{\omega_0, \omega_1\}$. When the state is favourable, $\omega = \omega_1$, she obtains a payoff of $u_i \geq 0$ from enacting the policy. When the state is unfavourable, $\omega = \omega_0$, she incurs a payoff loss of 1 instead. Blocking the policy, in contrast, yields her a payoff of 0 in either state. There is also a *lobbyist* outside the committee. He prefers the policy to be enacted regardless of the state.

At the outset of the game, the lobbyist and committee members share the prior belief that $\omega = \omega_1$ with probability $p \in [0, 1]$. The lobbyist can then provide additional *public* information about the state. He does so by committing to perform and publicly disclose the outcome of an (Blackwell) experiment of his choosing. Formally, an experiment is comprised of a measurable space $(X, \mathcal{X})$ and, for each possible state ω_k, a probability measure P_k over it. Upon observing the outcome of the lobbyist's experiment, players use Bayes Rule to update their prior belief p that $\omega = \omega_1$ to an *interim belief* $r \in [0, 1]$.

I follow a *belief-based approach*[1] and model the lobbyist as directly choosing the distribution μ_l his experiment will induce over interim beliefs. The distribution μ_l must be *Bayes plausible*; i.e., $\mu_l \in \Delta_p([0, 1])$, where:

$$\Delta_p([0,1]) := \left\{\mu \in \Delta([0,1]) : \int_0^1 r d\mu(r) = p\right\} \qquad \Delta([0,1]) := \underset{p \in [0,1]}{\cup} \Delta_p([0,1])$$

After the lobbyist reveals the outcome of his experiment, each committee member acquires additional information about the state. She does so by choosing an additional experiment whose outcome she observes. Committee members acquire this additional information *privately*: neither the experiment m_i chooses to perform nor its outcome are disclosed to anyone.

Following the outcome of her experiment, m_i updates her interim belief r to a *posterior belief* $q_i \in [0, 1]$ that $\omega = \omega_1$. Again, I model each member m_i as choosing the distribution $\mu_i(. \mid r) \in \Delta_r([0, 1])$ her experiment will induce over her posterior beliefs, given her interim belief r. Conditional on the true state ω, the outcomes of members' chosen experiments are independent.

meaningless to wonder about corr. between exp.s—info. acq. is private? introducing correlation only strengthens result, though.

Producing and providing public information is costless for the lobbyist. In contrast, acquiring private information is costly for committee members: m_i must pay a cost $C_i(\mu)$ to observe the outcome of an experiment which induces the distribution μ over her possible posterior beliefs. The

[1] See Kamenica and Gentzkow 2011 and Caplin and Dean 2013.

cost function $C_i : \Delta([0,1]) \to \mathbb{R}^+$ is *uniformly posterior separable*[2]; i.e., there is a convex function $c_i : [0,1] \to \mathbb{R}^+$ whose effective domain contains $(0,1)$, such that[3]:

is it ok that i assume $\infty - \infty = \infty$?

check if this truly squares with CDL

$$C_i(\mu) := \begin{cases} 0 & \text{for } \mu = \{\delta_0, \delta_1\} \\ \int c_i(q) d\mu(q) - c_i\left(\mathbb{E}_\mu[q]\right) & \text{otherwise} \end{cases}$$

Finally, each committee member casts a private vote into a *voting mechanism.* A voting mechanism is a tuple $\left(D, \{V_i\}_{i=1}^N, \{\mathcal{V}_i\}_{i=1}^N\right)$ where $V_i \ni v_i$ is a set of possible votes each member m_i may cast, $\mathcal{V}_i$ is the σ-algebra over V_i, and the *decision rule* $D : \bigtimes_{i=1}^N V_i \to [0,1]$ is a measurable mapping from the space of possible vote profiles to the probability that the policy is enacted—the *decision.* I further require that the committee's decision rule attain its maximum and minimum over its domain; i.e:

$$\exists\ \underline{\mathbf{v}}, \bar{\mathbf{v}} \in \bigtimes_{i=1}^N V_i \quad \text{such that for all } \mathbf{v} \in \bigtimes_{i=1}^N V_i \quad D(\underline{\mathbf{v}}) \leq D(\mathbf{v}) \leq D(\bar{\mathbf{v}})$$

For any given voting mechanism, I focus on the lobbyist-preferred perfect Bayesian equilibrium of this game. A perfect Bayesian equilibrium of this game consists of:

1. a voting strategy $\{\tau_i^*(. \mid q, r)\}_{(q,r)\in[0,1]^2}$ where $\tau_i^*(. \mid q, r) \in \Delta(V_i)$ for each member m_i,
2. an information strategy $\{\mu_i^*(. \mid r)\}_{r\in[0,1]}$ where $\mu_i^*(. \mid r) \in \Delta_r([0,1])$ for each member m_i, and
3. a strategy $\mu_l^* \in \Delta_p([0,1])$ for the lobbyist

such that under this fixed voting mechanism:

1. Given other members' strategies, member m_i's voting strategy maximises her expected payoff at any pair of interim and posterior beliefs $(r, q) \in [0,1]^2$; i.e.

$$\begin{aligned} & v_i \in \operatorname{supp} \tau_i^*(. \mid q, r) \\ \implies & v_i \in \underset{v \in V_i}{\operatorname{argmax}} \int_{[0,1]^{N-1}} U_i(r, q_i, \mathbf{q_{-i}}) \times \left\{ \int_{V_{-i}} D(v, \mathbf{v}_{-i}) \tau_{-i}^*(d\mathbf{v}_{-i} \mid r, \mathbf{q_{-i}}) \right\} \mu_{-\mathbf{i}}^*\left(d\mathbf{q_{-i}} \mid r, q_i\right) \end{aligned}$$

where:

worry: measurability; votes needn't be measurable in beliefs, eg. but: i am not claiming eqm shd ex-

i. the product measure $\mu_{-\mathbf{i}}^*(\cdot \mid r, q_i)$ over the posterior beliefs of members $m_{j\neq i}$ is induced

[2]See, for instance, Caplin, Dean, and Leahy 2022.
[3]For notational convenience, I assume that $\infty - \infty = \infty$.

by those members information strategies, and is conditional on their interim belief r and m_i's posterior belief q_i[4],

ii. $(V_{-i}, \mathcal{V}_{-i}) := \left(\bigtimes_{j\neq i} V_j, \bigtimes_{j\neq i} \mathcal{V}_j \right)$ is the product space of votes $\mathbf{v}_{-i} \in V_{-i}$ members $m_{j\neq i}$ may submit,

iii. the product measure $\tau^*_{-i}(\cdot \mid r, \mathbf{q}_{-i}) := \bigtimes_{j\neq i} \tau^*_j(. \mid; r, q_j)$ over the space $(V_{-i}, \mathcal{V}_{-i})$ is induced by the voting strategies of members $m_{j\neq i}$, and is conditional on their interim belief r and posterior beliefs $\mathbf{q_{-i}} \in V_{-i}$, and

iv. $U_i(r, q_i, \mathbf{q_{-i}}) := \mathbb{P}_{r,q_1,\ldots q_N}(\omega_1) \times (u_i + 1) - 1$ is the payoff m_i expects from enactment given all players' information about the state, where:

$$\mathbb{P}_{r,q_1,\ldots,q_N}(\omega_1) := \left(\frac{r}{1-r} \times \prod_{j=1}^{N} \frac{q_j/(1-q_j)}{r/(1-r)} \right) \times \left(1 + \frac{r}{1-r} \times \prod_{j=1}^{N} \frac{q_j/(1-q_j)}{r/(1-r)} \right)^{-1}$$

denotes the probability that $\omega = \omega_1$ given the same collection of information.

This is extremely long..?

2. Given others' strategies, member m_i's information strategy maximises her expected payoff at any interim belief r she may hold; i.e.

$$\mu^*_i(\cdot \mid r) \in \arg\max_{\mu \in \Delta_r[0,1]} \int \vartheta^*_i(r,q)\mu(dq) - C_i(\mu)$$

where $\vartheta^*_i(r,q)$ is the payoff m_i expects in the subgame where she holds the interim belief r and posterior belief q, given her ensuing voting strategy,

3. The lobbyist's information strategy μ^*_l maximises the probability that the policy is enacted given others' information and voting strategies.

can we ensure this exists? does it matter for me?

if multiple exist there, the one with lowest info costs

The lobbyist-preferred perfect Bayesian equilibrium (hereon, simply "equilibrium") is the perfect Bayesian equilibrium with the highest probability of enactment.

Dominant Voting Mechanisms

I call a voting mechanism *dominant* if it gives rise to an equilibrium that yields:

i. a (weakly) higher probability that the policy is blocked when $\omega = \omega_0$,

ii. a (weakly) higher probability that the policy is enacted when $\omega = \omega_1$, and

iii. a (weakly) lower expected cost of information acquisition for each committee member

[4]This measure is easily derived using Bayes' Rule, see Technical Appendix.

compared to any equilibrium under any other voting mechanism. A dominant voting mechanism—if it exists—yields (weakly) higher equilibrium payoffs for all committee members than any other mechanism.

This paper asks whether such a mechanism indeed exists.

3 Dictatorships

One of the simplest voting mechanisms the committee may adopt is a *dictatorship*, in which one member is delegated *exclusive* and *complete* control over the outcome.

Definition 1. A voting mechanism $(\overset{\text{♛}}{D}_i, \{V_j\}_{j=1}^N, \{\mathcal{V}_j\}_{j=1}^N)$ is an m_i-*dictatorship* if:

$$\textbf{(exclusive control)} \quad \overset{\text{♛}}{D}_i(v_i, \mathbf{v}_{-i}) = \overset{\text{♛}}{D}_i(v_i, \mathbf{v}'_{-i}) \quad \text{for all } v_i \in V_i \text{ and } \mathbf{v}_{-i}, \mathbf{v}'_{-i} \in V_{-i}$$

$$\textbf{(complete control)} \quad \min \overset{\text{♛}}{D}_i = 0 \text{ and } \max \overset{\text{♛}}{D}_i = 1$$

i hope to make the point that none can be relaxed– relaxing exclusivity (override mechanisms) destroys result

Theorem 1 establishes that there exists a particular committee member, denoted $\overset{\text{♛}}{m}$ and called the *most-demanding member*, such that every $\overset{\text{♛}}{m}$-dictatorship is a dominant voting mechanism. The rest of this section lays the groundwork for this result by characterising the equilibria dictatorships induce, and introducing the most-demanding member of the committee.

Lemma 1. In the equilibrium of an m_i-dictatorship:

1. m_i's voting strategy satisfies:

$$\mathbb{E}_{\tau_i^*(.|r,q)} \overset{\text{♛}}{D}_i = \begin{cases} 0 & \text{for } q < (1+u_i)^{-1} \\ 1 & \text{otherwise} \end{cases}$$

for any interim belief $r \in [0, 1]$.

2. m_i's information acquisition strategy $\{\mu_i^*(. \mid r)\}_{r \in [0,1]}$ is the unique one which satisfies:

$$\text{supp } \mu_i^*(. \mid r) \subseteq \left\{\min\{r, \underline{q}_i^*\}, \max\{r, \bar{q}_i{}^*\}\right\}$$

where $\underline{q}_i^*$ and $\bar{q}_i{}^*$ are two fixed threshold beliefs such that $\underline{q}_i^* \leq (1+u_i)^{-1} \leq \bar{q}_i{}^*$.

3. The lobbyist's information strategy μ_l^* is the unique distribution satisfying:

$$\text{supp } \mu_l^* \subseteq \{0, \max\{p, \bar{q}_i{}^*\}\}$$

4. No member except m_i expends a positive cost to acquire information:

$$C_j\left(\mu_j^*(. \mid r)\right) = 0 \quad \text{for each } r \in [0,1]$$

In an m_i-dictatorship, m_i has exclusive control over the outcome, and thus exclusive influence over the lobbyist's incentives. She votes to ensure that the outcome maximises her expected payoff: the policy is enacted whenever she expects (weakly) positive payoff from enacting it, and it is blocked whenever she does not.

Owing to her uniformly posterior separable cost of acquiring information, m_i adopts a simple information strategy. Whenever her interim belief falls between the two threshold beliefs $\underline{q}_i^*$ and $\bar{q}_i^*$, the additional information m_i acquires ensures that her posterior belief lands on one of these thresholds. When her interim belief lies outside this interval, m_i acquires no further information: almost surely, her posterior belief is the same as her interim belief. Importantly, the thresholds $\underline{q}_i^*$ and $\bar{q}_i^*$ bracket m_i's "threshold of indifference"—the posterior belief $(1+u_i)^{-1}$ at which she would be indifferent between enacting and blocking the policy—so that $\underline{q}_i^* \leq (1+u_i)^{-1} \leq \bar{q}_i^*$. This simply reflects that m_i never acquires costly but "useless" information—an experiment whose outcome would always lead to the same vote. Her peers, having no influence over the outcome, never acquire any costly information.

explain why posterior separability helps later.

The lobbyist wishes to maximise the probability that the committee enacts the policy. In an m_i-dictatorship, this amounts to maximising the probability that m_i's posterior belief weakly exceeds her threshold of indifference, $(1+u_i)^{-1}$, so that she votes for enactment. Owing to m_i's simple information strategy, the lobbyist can restrict himself to inducing interim beliefs outside the interval $\left(\underline{q}_i^*, \bar{q}_i^*\right)$: interim beliefs inside this interval merely prompt m_i to distribute her posterior beliefs between the thresholds $\underline{q}_i^*$ or $\bar{q}_i^*$, while those outside the interval prompt her to vote without acquiring further information [5]. The lobbyist's problem thus reduces to a simple Bayesian Persuasion exercise a là Kamenica and Gentzkow 2011. The distribution of interim beliefs he induces is binary, and is supported over $\bar{q}_i^*$ and 0. The former is the weakest interim belief that prompts m_i to vote for enactment. The latter is induced only when the state is unfavourable, $\omega = \omega_0$, ensuring that m_i votes to block the policy only when any doubt that $\omega = \omega_1$ is exhausted.

Proof, Lemma 1:

[5] Matysková and Montes 2023 establish a more general version of this argument in a setting where there is a single "Sender" (replacing the lobbyist here) and a single "Receiver" (replacing the committee member with exclusive and complete control here), showing that the Sender can restrict himself to inducing interim beliefs where the Receiver acquires no further information, provided that the latter has a uniformly posterior separable cost of acquiring information.

1. Since m_i has exclusive control, we can define the measurable function $\overset{♛}{f}_i : V_i \to [0,1]$ to be $\overset{♛}{f}_i(v_i) := \overset{♛}{D}_i(v_i, \mathbf{v_{-i}})$ for any arbitrary $\mathbf{v_{-i}} \in V_{-i}$. Then, $v_i \in \operatorname{supp}$ only if:

$$
\begin{aligned}
& v_i \in \arg\max_{v \in V_i} \int\limits_{[0,1]^{N-1}} U_i(r, q_i, \mathbf{q_{-i}}) \times \left\{ \int\limits_{V_{-i}} \overset{♛}{f}_i(v_i) \tau^*_{-i}(d\mathbf{v_{-i}} \mid r, \mathbf{q_{-i}}) \right\} \mu^*_{\mathbf{-i}}\left(d\mathbf{q_{-i}} \mid r, q_i\right) \\
\implies & v_i \in \arg\max_{v \in V_i} \overset{♛}{f}_i(v_i) \times \int\limits_{[0,1]^{N-1}} U_i(r, q_i, \mathbf{q_{-i}}) \mu^*_{\mathbf{-i}}\left(d\mathbf{q_{-i}} \mid r, q_i\right) \\
\implies & v_i \in \arg\max_{v \in V_i} \overset{♛}{f}_i(v_i) \times [q_i \times (1 + u_i) - 1]
\end{aligned}
$$

where the last implication follows from the martingale property of posterior beliefs; i.e., $\mathbb{E}_{\mathbf{q_{-i}}} \mathbb{P}_{r, q_1, \ldots, q_N} = q_i$. Thus, $\tau^*_i(\cdot \mid r, q)$ is an equilibrium only if:

$$
\mathbb{E}_{\tau^*_i(\cdot \mid r, q)} \overset{♛}{f}_i(.) = \begin{cases} 0 & q < (1 + u_i)^{-1} \\ 1 & q > (1 + u_i)^{-1} \end{cases}
$$

When $q = (1 + u_i)^{-1}$, m_i is indifferent between enacting and blocking the policy, hence any vote. He must then submit the lobbyist-preferred vote; i.e., $\mathbb{E}_{\tau^*_i(\cdot \mid r, (1+u_i)^{-1})} \overset{♛}{f}_i(.) = 1$.

2. m_i only needs to decide whether to maximise or minimise the enactment probability of the policy. So, she can without loss of generality restrict to information strategies where $|\operatorname{supp} \mu^*_i(\cdot \mid r)| \leq 2$ for all $r \in [0,1]$. Furthermore, for any interim belief, m_i can restrict herself to either acquiring no information, or acquiring information that has decision-making value; i.e., $(1 + u_i)^{-1} \in \operatorname{conv}(\operatorname{supp} \mu^*_i(\cdot \mid r))$ for any $|\operatorname{supp} \mu^*_i(\cdot \mid r)| = 2$. Lastly, note that Theorem 1 in Caplin, Dean, and Leahy 2022 implies that m_i's strategy must be *Locally Posterior Invariant* since C_i is uniformly posterior separable; i.e., for any $\tilde{r} \in \operatorname{conv}(\operatorname{supp} \mu^*_i(\cdot \mid r))$, $\mu^*_i(\cdot \mid \tilde{r})$ is the unique distribution satisfying $\operatorname{supp} \mu^*_i(\cdot \mid \tilde{r}) = \operatorname{supp} \mu^*_i(\cdot \mid r)$.

   add here that these two are outcome equivalent for the lobbyist, so also satisfy lobbyist-preferred

   Letting $\left\{\underline{q}^*_i, \bar{q}^*_i\right\} := \operatorname{supp} \mu^*_i(\cdot \mid (1 + u_i)^{-1})$ where $\underline{q}^*_i \leq \bar{q}^*_i$, we thus conclude:

   (a) for all $r \in \left[\underline{q}^*_i, \bar{q}^*_i\right]$, $\mu^*_i(\cdot \mid r)$ is the unique distribution such that $\operatorname{supp}\mu^*_i(\cdot \mid r) = \left\{\underline{q}^*_i, \bar{q}^*_i\right\}$.

   (b) for all $r \notin \left[\underline{q}^*_i, \bar{q}^*_i\right]$, $\mu^*_{i;r} = \delta_r$.

   which proves our claim.

3. Given members' voting and information strategies, let $\zeta^*_i(r)$ be the equilibrium enactment probability under the m_i-dictatorship, given the interim belief r. Denote the concave envelope

of this function as $\hat{\zeta}_i{}^*(r)$. These two functions are given by:

$$\zeta_i^*(r) := \begin{cases} 0 & r \leq \underline{q}_i^* \\ \frac{r-\underline{q}_i^*}{\bar{q}_i^*-\underline{q}_i^*} & r \in \left[\underline{q}_i^*, \bar{q}_i^*\right] \\ 1 & r \geq \bar{q}_i^* \end{cases} \qquad\qquad \hat{\zeta}_i{}^*(r) := \begin{cases} \frac{r}{\bar{q}_i^*} & r \leq \bar{q}_i^* \\ 1 & r \geq \bar{q}_i^* \end{cases}$$

That $\hat{\zeta}_i{}^*(r)$ is indeed the concave envelope of $\zeta_i^*(r)$ can be easily verified: any concave function that coincides with ζ_i^* at $r = 0$ and $r = \bar{q}_i^*$ must weakly lie above $\hat{\zeta}_i{}^*(r)$ on the interval $[0, \bar{q}_i^*]$. The condition $\mathbb{E}_\mu \zeta_i^*(.) = \hat{\zeta}_i{}^*(p)$ is then sufficient for the optimality of μ for the lobbyist (Kamenica and Gentzkow 2011). It is easily seen that the distribution μ_l^* defined in Lemma 1 satisfies this condition.

verify that KG is correct citation.

□

4 A Dominant Dictatorship

Definition 2. Member m_i is the *most-demanding member* of the committee if $\bar{q}_i^* = \max\{\bar{q}_1^*, \bar{q}_2^*, ..., \bar{q}_N^*\}$.

Discuss what makes a member most-demanding. Low u_i and "appropriately low" costs of acquiring information must interact.

Theorem 1. The dictatorship of the most demanding member is a dominant voting mechanism.

add the assumption that D must attain its min/max.

Proof. Where m_i is the most-demanding member, I use $\overset{\text{♛}}{Q} := \bar{q}_i^*$ to simplify notation in the ensuing. I fix $\left(\{V_j\}_{j=1}^N, \{\mathcal{V}_j\}_{j=1}^N, D\right)$ to be an arbitrary voting mechanism, and let $\overset{\text{♛}}{D}$ denote the voting rule for the dictatorship of the most-demanding member.

I will use Claim 2 to establish the Theorem:

Claim 2. Any voting mechanism induces an equilibrium where committee members:

(a) acquire no information for interim beliefs $r \in \{0\} \cup [\overset{\text{♛}}{Q}, 1]$; i.e., $\mu_{1;r}^* = ... = \mu_{N;r}^* = \delta_r$ for such interim beliefs.

(b) vote to maximise the probability that the policy is executed for interim beliefs $r \geq \overset{\text{♛}}{Q}$, but to minimise it for $r = 0$; i.e.:

$$\mathbb{E}_{\tau^*_{1;r,r},\ldots,\tau^*_{N;r,r}} D(\mathbf{v}) = \begin{cases} \max_{\mathbf{v}\in V} D(\mathbf{v}) := \bar{D} & r \geq \check{Q} \\ \min_{\mathbf{v}\in V} D(\mathbf{v}) := \underline{D} & r = 0 \end{cases}$$

Proof. Assume that all members except m_i employ the asserted strategies. I will first show that the asserted strategies are a best response for m_i, in that case. I then show that, in the relevant subgames, there is no strategy profile that is (a) a mutual best response for all committee members, and (b) preferred by the lobbyist to those asserted above.

Now let $\pi_i(q_i; r)$ be the payoff m_i expects under this voting mechanism given (i) the interim belief r, (ii) her posterior belief q_i, (iii) other members' strategies, and (iv) that she votes optimally at each posterior belief. Since other committee members acquire no private information, we have:

$$\pi_i(q_i; r) = \max_{\tau\in\Delta(V_i)} [q_i \times (1 + u_i) - 1] \times \int_{V_{-i}} \int_{V_i} D(v, \mathbf{v}_{-i})\tau(dv)\tau^*_{-i;r,r}(d\mathbf{v}_{-i})$$

For $q > (1 + u_i)^{-1}$, $\pi_i(q; r)$ is strictly increasing in the probability that the policy is enacted. For $q < (1 + u_i)^{-1}$, it is strictly decreasing in the same probability. Thus, when $q > (1 + u_i)^{-1}$, m_i's optimal voting strategy must maximise the probability that the policy is enacted given others' voting strategies, but when $q < (1+u_i)^{-1}$, it must minimise it. When $q = (1+u_i)^{-1}$, $\pi_i(q_i; r) = 0$; so, any voting strategy is a best response. m_i's equilibrium strategy must then be lobbyist-preferred one, and maximise the probability that the policy is enacted. This verifies Claim 2's assertion on optimal voting strategies, and gives:

$$\pi_i(q; r) = \begin{cases} \bar{D} \times [q \times (1 + u_i) - 1] & q \geq (1 + u_i)^{-1} \\ \underline{D} \times [q \times (1 + u_i) - 1] & q < (1 + u_i)^{-1} \end{cases}$$

whenever $r \in \{0\} \cup [\check{Q}, 1]$.

When $r = 0$, $\Delta_0 ([0, 1]) = \delta_0$; so $\mu^*_{i;0} = \delta_0$. When $r \geq \check{Q}$, the strategy $\mu^*_{i;r} = \delta_r$ delivers the highest probability that the policy is executed given members' ensuing voting strategies; so, if $\mu^*_{i;r} = \delta_r$ is a best response for m_i, it is also part of her equilibrium information strategy. So to conclude our proof, we wish to show:

$$\int_0^1 [\pi_i(q; r) - \pi_i(r; r)] \, d\mu(q) - C_i (\mu) \leq 0 \qquad \text{for all } \mu \in \Delta_r ([0, 1]) \tag{4.1}$$

whenever $r \geq \overset{\star}{Q}$.

To this end, define $\overset{\star}{\pi}_i(q_i)$ to be the payoff m_i expects under an m_i-dictatorship given her posterior belief q_i; i.e., $\overset{\star}{\pi}_i(q_i) := \max\{0, q_i \times (1+u_i) - 1\}$. Since $\bar{q}_i \geq \overset{\star}{Q}$, Lemma 1 implies that:

did i show this in the proof of the lemma?

$$\int_0^1 \left[\overset{\star}{\pi}_i(q) - \overset{\star}{\pi}_i(r)\right] d\mu(q) - C_i(\mu) \leq 0 \qquad \text{for all } \mu \in \Delta_r([0,1]) \tag{4.2}$$

whenever $r \geq \overset{\star}{Q}$. So, we are done once we show:

$$\int_0^1 \tilde{\pi}_i(q;r) d\mu(q) \leq 0 \qquad \text{for all } \mu \in \Delta_r([0,1]) \tag{4.3}$$

where $\tilde{\pi}_i(q,r) := [\pi_i(q,r) - \pi_i(r,r)] - [\overset{\star}{\pi}_i(q) - \overset{\star}{\pi}_i(r)]$.

To see this, first note that $\pi_i(q,r) - \overset{\star}{\pi}_i(q) \leq \pi_i\left((1+u_i)^{-1}, r\right) - \overset{\star}{\pi}_i\left((1+u_i)^{-1}\right) = 0$, so $\tilde{\pi}_i(q,r)$ is piecewise linear, decreasing below $q = (1+u_i)^{-1}$, and increasing above $q = (1+u_i)^{-1}$. Therefore, $\tilde{\pi}_i(.,r)$ is a concave function for any $r \geq \overset{\star}{Q}$. Combined with the fact that $\tilde{\pi}_i(r,r) = 0$, this gives:

$$\int_0^1 \tilde{\pi}_i(q;r) d\mu(q) \leq \int_0^1 \tilde{\pi}_i(q;r) d\delta_r(q) = 0 \qquad \text{for all } \mu \in \Delta_r([0,1])$$

by Jensen's Inequality.

□

Claim 3. The dictatorship of the most-demanding member is a dominant voting mechanism.

Proof. In the equilibrium the dictatorship of the most-demanding member induces, no committee member acquires private information about the state. Any other voting mechanism must thus result in a weakly higher expected cost of information acquisition for each member.

To conclude the proof, it suffices to show that any fixed voting mechanism achieves:

(a) a lower probability that the policy is enacted when $\omega = \omega_1$, and

(b) a lower probability that the policy is blocked when $\omega = \omega_0$

than the dictatorship of the most-demanding member.

Recall (from Lemma 1) that, under this dictatorship, the lobbyist's equilibrium information strategy is the unique distribution $\overset{\star}{\mu}_l$ such that supp $\overset{\star}{\mu}_l = \left\{0, \min\{p, \overset{\star}{Q}\}\right\}$. Therefore, the dictator never blocks the policy unless her interim belief is $r = 0$; i.e. $\omega = \omega_0$ is revealed. The policy is implemented whenever $\omega = \omega_1$.

Note that, due to Claim 2, the probability that the policy is executed following interim beliefs $r = 0$ and $r = \overset{\star}{Q}$ are $\underline{D}$ and $\bar{D}$, respectively. Therefore, if the lobbyist employs the strategy $\overset{\star}{\mu}_l$ described above, we get:

$$\mathbb{P}_{D,\overset{\star}{\mu}_l}\left(\text{enact} \mid \omega = \omega_1\right) = \bar{D} \qquad \mathbb{P}_{D,\overset{\star}{\mu}_l}\left(\text{enact} \mid \omega = \omega_0\right) = 1 - \mathbb{P}_{\overset{\star}{\mu}_l}\left(r = 0 \mid \omega_0\right) \times \left(1 - \underline{D}\right)$$

The lobbyist's equilibrium information strategy, μ_l^*, must weakly increase at least one of these probabilities. Since the probability that the policy is enacted cannot exceed $\bar{D}$ in either state, we conclude that:

$$\mathbb{P}_{D,\mu_l^*}\left(\text{enact} \mid \omega = \omega_1\right) \leq \bar{D} \leq 1$$
$$\mathbb{P}_{D,\mu_l^*}\left(\text{enact} \mid \omega = \omega_0\right) \geq 1 - \mathbb{P}_{\overset{\star}{\mu}_l}\left(r = 0 \mid \omega_0\right) \times \left(1 - \underline{D}\right) \geq 1 - \mathbb{P}_{\overset{\star}{\mu}_l}\left(r = 0 \mid \omega_0\right)$$

$\square$

better to do this in two “steps” rather than “claim”s.

$\square$

5 Discussion

Scrapped Sections

6 Overrides and Dictatorships

Overrides are dominated by dictatorships. See the note in my compass lexecon notebook.

In general, how one member's vote influences the outcome will depend on the votes her peers submit. Certain mechanisms, however, may privilege a particular member to *override* the rest: allow her a vote that (almost surely) guarantees that the policy is enacted or blocked, regardless of her peers' votes. Such mechanisms will be central to Theorem 1, the main result of this paper. I call them *overrides*.

Definition 3. A voting mechanism $(\overset{\scriptscriptstyle\Psi}{D}_i, \{V_j\}_{j=1}^N, \{\mathcal{V}_j\}_{j=1}^N)$ is an m_i*-override* if for some $\underline{v}_i, \bar{v}_i \in V_i$:

$$\forall \mathbf{v_{-i}} \in V_{-i}: \qquad \overset{\scriptscriptstyle\Psi}{D}_i(\underline{v}_i, \mathbf{v_{-i}}) = 0 \quad \text{and} \quad \overset{\scriptscriptstyle\Psi}{D}_i(\bar{v}_i, \mathbf{v_{-i}}) = 1$$

A particularly simple override—indeed, one of the simplest voting mechanisms—is a *dictatorship*, which bestows the privileged member exclusive authority over the outcome.

Definition 4. An m_i-override $(\overset{\scriptscriptstyle\Psi}{D}_i, \{V_j\}_{j=1}^N, \{\mathcal{V}_j\}_{j=1}^N)$ is an m_i*-dictatorship* if:

$$\overset{\scriptscriptstyle\Psi}{D}_i\left(v_i, \mathbf{v_{-i}}\right) = \overset{\scriptscriptstyle\Psi}{D}_i\left(v_i, \mathbf{v'_{-i}}\right) \quad \text{for any } \mathbf{v_{-i}}, \mathbf{v'_{-i}} \in V_{-i} \text{ and } v_i \in V_i$$

Theorem 1 establishes that there exists a particular committee member—denoted $\overset{\scriptscriptstyle\Psi}{m}$ and called the *most-demanding member*—such that every $\overset{\scriptscriptstyle\Psi}{m}$-override (and hence every $\overset{\scriptscriptstyle\Psi}{m}$-dictatorship) is a dominant voting mechanism. In the rest of this section, I lay the groundwork for Theorem 1: I characterise the equilibria override mechanisms induce, and introduce the most-demanding member of the committee.