Now we want to calculate the range R for all the values of T (as a function of k) that we have just found. To do this, we need to solve Equation 2.43 for each of the values of k and t = T that we have just found.

$x := 100$

This is the guess for the first value of x. The actual value of the guess does not matter.

$$f(k, T) := \mathrm{root}\left[x - \frac{u}{k}\cdot(1 - \exp(-k\cdot T)), x\right]$$

This is the Equation 2.43 that we need to solve to find the range R.

$R_j := f(K_j, Tr_j)$

Now calculate the range R for all the values.

$R_1 = 3.182\cdot 10^4$

We just list the first value and plot the remainder. This is the range for no air resistance, that is k = 0.

Now let's calculate and plot the range determined from the approximate calculation. Calculate Equation 2.55.

$$Rp_j := R_1\cdot\left(1 - \frac{4\cdot K_j\cdot v}{3\cdot g}\right)$$

[Now plot R_j and R_{pj} versus K_j to produce Figure 2-9.]

Plot approximate and numerical solutions. Figure 2-9.

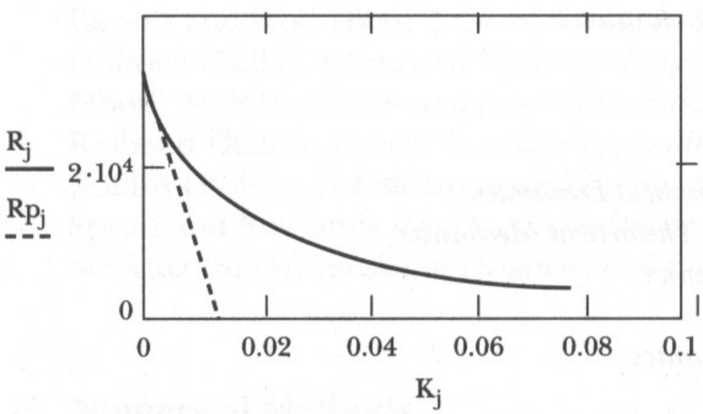

FIGURE 2-9

Selected References

The following texts are particularly recommended as general sources of collateral reading material.

A. General Theoretical Physics

Blass (B162), *Theoretical Physics.*
Lindsay and Margenau (Li36), *Foundations of Physics.*
Wangsness (Wa63), *Introduction to Theoretical Physics.*

B. Elementary Mechanics

Baierlein (Ba83), *Newtonian Dynamics.*
Barger and Olsson (Ba73), *Classical Mechanics.*
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Fowles and Cassiday (Fo99), *Analytical Mechanics.*
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Knudsen and Hjorth (Kn00), *Elements of Newtonian Mechanics.*
McCall (Mc01), *Classical Mechanics.*
Rossberg (Ro83), *Analytical Mechanics.*

C. Intermediate Mechanics

Arya (Ar98), *Introduction to Classical Dynamics.*
Becker (Be54), *Introduction to Theoretical Mechanics.*
Lindsay (Li61), *Physical Mechanics.*
Scheck(Sc99), *Mechanics.*
Slater and Frank (Sl47), *Mechanics.*
Symon (Sy71), *Mechanics.*

D. Advanced Mechanics

Baruh (Ba99), *Analytical Dynamics.*
Goldstein (Go80), *Classical Mechanics.*

Landau and Lifshitz (La76), *Mechanics.*
McCuskey (Mc59), *An Introduction to Advanced Dynamics.*

E. Mathematical Methods

Abramowitz and Stegun (Ab65), *Handbook of Mathematical Functions.*
Arfken (Ar85), *Mathematical Methods for Physicists.*
Byron and Fuller (By69), *Mathematics of Classical and Quantum Physics.*
Churchill (Ch78), *Fourier Series and Boundary Value Problems.*
Davis (Da61), *Introduction to Vector Analysis.*
Dennery and Krzywicki (De67), *Mathematics for Physicists.*
Dwight (Dw61), *Tables of Integrals and Other Mathematical Data.*
Kaplan (Ka84), *Advanced Calculus.*
Mathews and Walker (Ma70), *Mathematical Methods of Physics.*
Pipes and Harvill (Pi70), *Applied Mathematics for Engineers and Physicists.*

F. Special Relativity

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French (Fr68), *Special Relativity.*
Resnick (Re72), *Basic Concepts in Relativity and Early Quantum Theory.*
Rindler (Ri82), *Introduction to Special Relativity.*
Taylor and Wheeler (Ta66), *Spacetime Physics.*

G. Chaos

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Bessoir and Wolf (Be91), *Chaos Simulations.*
Hilborn (Hi94), *Chaos and Nonlinear Dynamics.*
Moon (Mo92), *Chaotic and Fractal Dynamics.*
Rasband (Ra90), *Chaotic Dynamics of Nonlinear Systems.*
Rollins (Ro90), *Chaotic Dynamics Workbench.*
Sprott and Rowlands (Sp92), *Chaos Demonstrations.*
Strogatz (St94), *Nonlinear Dynamics and Chaos.*

H. Numerical Methods

DeJong (De91), *Introduction to Computational Physics.*
Johnson and Reiss (Jo82), *Numerical Analysis.*
Press, Teukolsky, Vetterling, and Flannery (Pr92), *Numerical Recipes.*

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Answers to Even-Numbered Problems

Chapter 1

10. (a) $\mathbf{v} = 2b\omega \cos \omega t\,\mathbf{i} - b\omega \sin \omega t\,\mathbf{j}$ (b) 90°

$$\mathbf{a} = -\omega^2\mathbf{r}$$

$$|\mathbf{v}| = b\omega[3 \cos^2 \omega t + 1]^{\frac{1}{2}}$$

12. $h = \dfrac{|\mathbf{a}\cdot\mathbf{b} \times \mathbf{c}|}{|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|}$

$$A = \frac{1}{2}|(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{b})| = \frac{1}{2}|(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{a})|$$

$$= \frac{1}{2}|(\mathbf{c} - \mathbf{b}) \times (\mathbf{a} - \mathbf{c})|$$

14. (a) -104 (b) $\begin{pmatrix} 9 & 7 \\ 13 & 9 \\ 5 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} -5 & -5 \\ 3 & -5 \\ 25 & 14 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -3 & -4 \\ 3 & 0 & 6 \\ 4 & -6 & 0 \end{pmatrix}$

26. $\mathbf{a}\cdot\mathbf{e}_r = -\dfrac{3}{4}\dfrac{v^2}{k}$; $|\mathbf{a}| = \dfrac{3}{4}\dfrac{v^2}{k}\cdot\sqrt{\dfrac{2}{1 + \cos\theta}}$; $\dot{\theta} = \dfrac{v}{\sqrt{2kr}}$

34. $\displaystyle\int(\mathbf{A} \times \ddot{\mathbf{A}})\,dt = (\mathbf{A} \times \dot{\mathbf{A}}) + \mathbf{C}$, where $\mathbf{C}$ is a constant vector

36. $\pi c^2 d$

38. $-\pi$

40. (a) $x = -2$ m, $y = 3$ m, $z_{max} = 72$ m; (c) SE

Chapter 2

2. $F_\theta = mR(\ddot{\theta} - \dot{\phi}^2 \sin\theta\cos\theta)$

$F_\phi = mR(2\dot{\theta}\,\dot{\phi}\cos\theta + \ddot{\phi}\,\sin\theta)$

4. $13.2\ \mathrm{m\cdot s^{-1}}$

6. (a) 210 m behind (b) can be no more than 0.68 s late

14. (a) $d = \dfrac{2v_0^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$ (b) $\dfrac{\pi}{4} + \dfrac{\beta}{2}$ (c) $d_{\max} = \dfrac{v_0^2}{g(1+\sin\beta)}$

16. $\dfrac{2v_0}{g\sin\alpha}$

18. (a) $35.2\ \mathrm{m\cdot s^{-1}}$ (b) 40.7°; 1.1 m

20. 17.4°

22. (c) $\dot{x}(t) = C_1\cos\omega_c t + C_2\sin\omega_c t + \dfrac{E_y}{B}$

$\dot{y}(t) = -C_1\sin\omega_c t + C_2\cos\omega_c t$

24. $\mu_k = 0.18$; $v_B = 15.6$ m/s

26. 2.3 m; 1.1 m

28. $h_{\text{marble}} = h\left(\dfrac{3-a}{1+a}\right)^2$; $h_{\text{superball}} = h\left(\dfrac{1-3a}{1+a}\right)^2$ where $a = m/M$

30. 71 m

32. $\sin\theta_0 = \dfrac{1 \pm \mu_k\sqrt{3+4\mu_k^2}}{2(1+\mu_k^2)}$

34. (a) $y = -\dfrac{m}{\alpha}\left[v + \dfrac{mg}{\alpha}\ln\left(1 - \dfrac{\alpha v}{mg}\right)\right]$ (b) $y = -\dfrac{m}{2\beta}\ln\left(1 - \dfrac{\beta v^2}{mg}\right)$

36. $R = \dfrac{v_0^2}{g}\cos\theta\left(\sin\theta + \sqrt{\sin^2\theta + \dfrac{2gh}{v_0^2}}\right)$

38. (a) $F(x) = -mna^2x^{-(2n+1)}$

(b) $x(t) = [(n+1)at]^{\frac{1}{n+1}}$

(c) $F(t) = -mna^2[(n+1)at]^{-(2n+1)/(n+1)}$

40. (a) $a_t = \dfrac{2A\alpha^2\sin\alpha t}{\sqrt{5-4\cos\alpha t}}$; $a_n = \dfrac{A\alpha^2|2\cos\alpha t - 1|}{\sqrt{5-4\cos\alpha t}}$

(b) $\dfrac{n\pi}{\alpha}$ where n = integer

42. Stable if $R > b/2$; unstable if $R \leq b/2$

48. $\tau = \pi d^{3/2}\sqrt{\dfrac{2}{mG}}$

50. (a) $x(t) = \dfrac{c^2}{F}\left(\sqrt{m_0^2 + \dfrac{F^2t^2}{c^2}} - m_0\right)$, $v(t) = \dfrac{Ft}{\sqrt{m_0^2 + \dfrac{F^2t^2}{c^2}}}$

(c) $t(v = c/2) = 0.55$ yr; $t(v = 0.99c) = 6.67$ yr.

52. (a) $F(x) = -\dfrac{4U_0x}{a^2}\left(1 - \dfrac{x^2}{a^2}\right)$, (c) $\omega = \sqrt{\dfrac{4U_0}{ma^2}}$, (d) $v_{\min} = \sqrt{\dfrac{2U_0}{m}}$

(e) $x(t) = \dfrac{a[\exp(t\sqrt{8U_0/ma^2}) - 1]}{[\exp(t\sqrt{8U_0/ma^2}) + 1]}$

54. (a) $v = g/k = 1000$ m/s, (b) $height = \dfrac{v_0}{k} + \dfrac{g}{k^2}\ln\left(\dfrac{g}{g + kv_0}\right) = 680$ m

Chapter 3

2. (a) $6.9 \times 10^{-2}\,\mathrm{s}^{-1}$ (b) $\dfrac{10}{2\pi}(1 - 2.40 \times 10^{-5})\ \mathrm{s}^{-1}$ (c) 1.0445

4. $\langle T\rangle = \langle U\rangle = \dfrac{mA^2\omega_0^2}{4}$ $\overline{U} = \dfrac{1}{2}\ \overline{T} = \dfrac{mA^2\omega_0^2}{6}$

6. $2.74\ \mathrm{rad\cdot s^{-1}}$

12. $\ddot{\theta} = -\dfrac{g}{l}\sin\theta$

14. $x(t) = (\cosh\beta t - \sinh\beta t)[(A_1 + A_2)\cosh\omega_2 t + (A_1 - A_2)\sinh\omega_2 t]$

$\dot{x}(t) = (\cosh\beta t - \sinh\beta t)[(A_1\omega_2 - A_1\beta)(\cosh\omega_2 t + \sinh\omega_2 t)$
$\quad -(A_2\beta + A_2\omega_2)(\cosh\omega_2 t - \sinh\omega_2 t)]$

26. $\dfrac{R_1[R_2(R_2 + R_1) + \omega^2L_2^2] + i[R_1\omega L_2 + (\omega L_1 - 1/\omega C)((R_1 + R_2)^2 + \omega^2L_2^2)]}{(R_1 + R_2)^2 + \omega^2L_2^2}$

28. $F(t) = \dfrac{4}{\pi}\sin t + \dfrac{4}{3\pi}\sin 3t + \dfrac{4}{5\pi}\sin 5t + \cdots$

30. $F(t) = \dfrac{2}{\pi} - \dfrac{4}{3\pi}\cos 2\omega t - \dfrac{4}{15\pi}\cos 4\omega t - \cdots$

32. (a) $x(t) = \dfrac{H(0)}{\omega_0^2}\left(1 - e^{-\beta t}\cosh\omega_2 t - \dfrac{\beta e^{-\beta t}}{\omega_2}\sinh\omega_2 t\right)$

(b) $x(t) = \dfrac{b}{\omega_2}e^{-\beta t}\sinh\omega_2 t;\ t > 0$

34. $x(t) = \begin{cases} 0 & t < 0 \\ 4[1 - \cos(0.5t)] \text{ m} & 0 < t < 4\pi \\ 0 & t > 4\pi \end{cases}$

36. $x(t) = e^{-\beta(t-t_0)}\left[x_0 \cos \omega_1(t - t_0) + \left(\frac{\dot{x}_0}{\omega_1} + \frac{\beta x_0}{\omega_1} + \frac{b}{\omega_1}\right)\sin \omega_1(t - t_0)\right]; \quad t > t_0$

38. $$x(t) = \frac{F_0}{m}\frac{\omega}{[(\beta - \gamma)^2 + (\omega + \omega_1)^2][(\beta - \gamma)^2 + (\omega - \omega_1)^2]}$$
$$\times\left[e^{-\gamma t}\left[2(\gamma - \beta)\cos \omega t + \left([\beta - \gamma]^2 + \omega_1^2 - \omega^2\right)\frac{\sin \omega t}{\omega}\right]\right.$$
$$\left.+\, e^{-\beta t}\left[2(\beta - \gamma)\cos \omega_1 t + \left([\beta - \gamma]^2 + \omega^2 - \omega_1^2\right)\frac{\sin \omega_1 t}{\omega_1}\right]\right]$$

40. Amplitude = −0.16 mm, minus sign indicates spring is compressed

42. (a) $x(t) = \frac{F}{m\omega_0}\frac{1}{(\omega_0 + \omega)(\omega_0 - \omega)}(\omega_0 \sin \omega t - \omega \sin \omega_0 t)$. (b) $x(t) = \frac{F_0 t^3 \omega_0}{6m}$

44. $\frac{\omega_1}{\omega_0} = \frac{8\pi}{\sqrt{64\pi^2 + 1}}$

Chapter 4

6. $\dot{\theta} = \sqrt{\frac{2}{ml^2}}[E - mgl(1 - \cos\theta)]^{1/2}$

8. $\tau = 4\sqrt{\frac{2mA}{F_0}}$

10. Only 0.6 and 0.7 are chaotic

14. $n = 30$

22. Transitions at $B_1 = 9.8 - 9.9$, $B_2 = 11.6 - 11.7$, and $B_3 = 13.3 - 13.4$. Behavior: (i) one period per three drive cycles when $B < B_1$, (ii) chaotic when $B_1 < B < B_2$, (iii) mixed chaotic/one period per drive cycle (depending on initial conditions) when $B_2 < B < B_3$, and (iv) one period per drive cycle when $B > B_3$

Chapter 5

2. $\rho = \frac{C}{2\pi G r}$ where $C = \frac{\partial\phi}{\partial r} = $ const.

6. $\mathbf{g} = -\frac{GM}{r^2}\mathbf{e}_r$

8. $g_z = -2\pi G\rho\left(\sqrt{a^2 + (z_0 - l)^2} - \sqrt{a^2 + z_0^2} + l\right)$

10. $\phi(R) \cong -\dfrac{GM}{R}\left[1 - \dfrac{1}{2}\dfrac{a^2}{R^2}\left(1 - \dfrac{3}{2}\sin^2\theta\right)\right]$

16. $F_z = 2\pi\rho_s GM$

20. $\Phi(z) = -\dfrac{2GM}{R^2}(\sqrt{z^2 + R^2} - z),\ g(z) = -\mathbf{k}\dfrac{2GM}{R^2}\left(\dfrac{\sqrt{z^2 + R^2} - z}{\sqrt{z^2 + R^2}}\right)$

Chapter 6

8. (a) $a_1 = b_1 = c_1 = \dfrac{2}{\sqrt{3}}R$ (b) $a_1 = a\dfrac{2}{\sqrt{3}};\ b_1 = b\dfrac{2}{\sqrt{3}};\ c_1 = c\dfrac{2}{\sqrt{3}}$

10. $R = \dfrac{1}{2}H$

14. length $= 2\sqrt{2}\sin\dfrac{\pi}{2\sqrt{2}}$

16. $y(x) = \dfrac{8}{13^{3/2} - 8}\left[\left(1 + \dfrac{9x}{4}\right)^{3/2} - 1\right]$ and $z = x^{3/2}$

18. $x = -y = \sqrt{-z}$ where $x > 0, y < 0, z < 0$. Parabolic line.

Chapter 7

4. $m\ddot{r} - mr\dot{\theta}^2 + Ar^{\alpha-1} = 0;\ \dfrac{d}{dt}(mr^2\dot{\theta}) = 0;$ yes; yes

6. $2m\ddot{S} + m\ddot{\xi}\cos\alpha - mg\sin\alpha = 0$

$(m + M)\ddot{\xi} + m\ddot{S}\cos\alpha = 0$

10. (a) $y(t) = -\dfrac{g}{4}t^2$ (b) $y(t) = \dfrac{Ml}{m}(1 - \cosh\gamma t)$

12. $r(t) = r_0\cosh\alpha t + \dfrac{g}{2\alpha^2}(\sin\alpha t - \sinh\alpha t)$

14. (a) $\ddot{\theta} + \dfrac{a + g}{b}\sin\theta = 0$ (b) $2\pi\sqrt{\dfrac{b}{a + g}}$

16. $\ddot{\theta} + \dfrac{g}{b}\sin\theta - \dfrac{a}{b}\omega^2\sin\omega t\cos\theta = 0$

18. $\omega = \sqrt{\dfrac{g\sin\theta_0}{l - R\theta_0}};\quad \theta_0 = \dfrac{\pi}{2}$

22. $L = \frac{1}{2}m\dot{x}^2 - \frac{k}{x}e^{-t/\tau}$; $H = \frac{p_x^2}{2m} + \frac{k}{x}e^{-t/\tau}$

24. $L = \frac{1}{2}m(\alpha^2 + l^2\dot{\theta}^2) + mgl\cos\theta$

$$H = \frac{p_\theta^2}{2ml^2} - \frac{1}{2}m\alpha^2 - mgl\cos\theta$$

26. (a) $H = \frac{p_\theta^2}{2ml^2} - mgl\cos\theta$; $\dot{\theta} = \frac{p_\theta}{ml^2}$; $\dot{p}_\theta = -mgl\sin\theta$

(b) $H = \dfrac{p_x^2}{2(m_1 + m_2 + I/a^2)} - m_1 gx - m_2 g(l - x)$

$$p_x = \left(m_1 + m_2 + \frac{l}{a^2}\right)\dot{x}$$

$$\dot{p}_x = g(m_1 - m_2)$$

28. $p_r = m\dot{r}$; $\dot{p}_r = \frac{p_\theta^2}{mr^3} - \frac{k}{r^2}$; $p_\theta = mr^2\dot{\theta}$; $\dot{p}_\theta = 0$

32. $H = \frac{1}{2m}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2\sin^2\theta}\right) - \frac{k}{r}$; $\dot{p}_r = -\frac{k}{r^2} + \frac{p_\theta^2}{mr^3} + \frac{p_\phi^2}{mr^3\sin^2\theta}$;

$$\dot{p}_\theta = \frac{p_\phi^2\cot\theta}{mr^2\sin^2\theta};\quad \dot{p}_\phi = 0$$

34. (a) $\ddot{x} = aR(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$; $\ddot{\theta} = \frac{\ddot{x}\sin\theta + g\cos\theta}{R}$; where $a \equiv \frac{m}{M + m}$

(b) $\lambda = -\dfrac{mMg(3\sin\theta - a\sin^3\theta - 2\sin\theta_0)}{(M + m)(1 - a\sin^2\theta)^2}$

38. $\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{p}{m}$, $\frac{dp}{dt} = -\frac{\partial H}{\partial x} = -(kx + bx^3)$

40. $0 = 4\frac{d^2x}{dt^2} + b\left(2\frac{d^2\theta_1}{dt^2}\cos\theta_1 + \frac{d^2\theta_2}{dt^2}\cos\theta_2\right) - b\left(2\left(\frac{d\theta_1}{dt}\right)^2\sin\theta_1 + \left(\frac{d\theta_2}{dt}\right)^2\sin\theta_2\right)$

$$-2g\sin\theta_1 = 2b\frac{d^2\theta_1}{dt^2} + 2\frac{d^2x}{dt^2}\cos\theta_1 + b\frac{d^2\theta_2}{dt^2}\cos(\theta_1 - \theta_2) + b\left(\frac{d\theta_2}{dt}\right)^2\sin(\theta_1 - \theta_2)$$

$$-g\sin\theta_2 = b\frac{d^2\theta_2}{dt^2} + \frac{d^2x}{dt^2}\cos\theta_2 + b\frac{d^2\theta_1}{dt^2}\cos(\theta_1 - \theta_2) - b\left(\frac{d\theta_1}{dt}\right)^2\sin(\theta_1 - \theta_2)$$

Chapter 8

4. $\langle U \rangle = -\dfrac{k}{a}$; $\langle T \rangle = \dfrac{k}{2a}$

10. Parabola; yes

12. 76 days

14. $F(r) = -\dfrac{l^2}{\mu}\left(\dfrac{6k}{r^4} + \dfrac{1}{r^3}\right)$

22. No

24. (a) 1590 km (b) 1900 km

28. 2380 m/s

30. $\Delta v = 3.23$ km/s; parabola

32. Stable if $r < a$

38. $\Delta v = 5275$ m/s (opposite to direction of motion); 146 days

40. Carrying the waste out of the solar system requires less energy than crashing it into the sun

42. 2.57×10^{11} J

44. $\dfrac{\alpha}{r} = 1 + \varepsilon \cos\theta$, where $\alpha = \dfrac{\ell^2}{\mu k}$, $\varepsilon = \sqrt{1 + \dfrac{2\ell^2}{\mu k^2}\left(E - \dfrac{k}{a}\right)}$

If $0 < \varepsilon < 1$, the orbit is ellipsoid. If $\varepsilon = 0$, the orbit is circular

46. $T = 9 \times 10^7$ yr.

Chapter 9

2. On the axis; $\dfrac{3}{4}h$ from vertex

4. $\bar{x} = \dfrac{2a}{\theta}\sin\dfrac{\theta}{2}$; $\bar{y} = 0$

6. $\mathbf{r}_{\text{cm}} = \dfrac{F_0}{4m}t^2\mathbf{i}$; $\mathbf{v}_{\text{cm}} = \dfrac{F_0}{2m}t\mathbf{i}$; $\mathbf{a}_{\text{cm}} = \dfrac{F_0}{2m}\mathbf{i}$

8. $\bar{x} = 0$; $\bar{y} = \dfrac{a}{3\sqrt{2}}$

10. $\dfrac{v_0}{g}\sin\theta\sqrt{\dfrac{2E}{m_1 + m_2}}\left(\sqrt{\dfrac{m_1}{m_2}} + \sqrt{\dfrac{m_2}{m_1}}\right)$

12. (a) yes (b) 11 m/s

14. No

20. $\sqrt{ga}$

22. (a) two sets of solutions: $v_n = 5.18$ km/s, $v_d = 14.44$ km/s and $v_n = 19.79$ km/s, $v_d = 5.12$ km/s. (b) 74.8° and 5.2°. (c) 30°

24. $\omega = \dfrac{\omega_0}{1 - \dfrac{a}{b}\theta}$; $T = mb\omega_0\omega$

26. $\mathbf{N} = \dfrac{kr}{v_0}(\mathbf{r}_1 - \mathbf{r}_2) \times (\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)$

28. $\dfrac{4m_1m_2}{(m_1 + m_2)^2}$

30. (a) $(-0.09\,\mathbf{i} + 1.27\,\mathbf{j})\mathrm{N\cdot s}$ (b) $(-9\mathbf{i} + 127\mathbf{j})\mathrm{N}$

32. $v_1 = v_2 = \dfrac{u_1}{3}$

34. $v_1 = v_2 = \dfrac{u_1}{\sqrt{2}}$; $\theta = 45°$

36. $\dfrac{m_1}{m_2} = 3 \pm 2\sqrt{2}$; $\dfrac{u_2}{u_1} = -(1 \pm \sqrt{2})$ with $\begin{cases} +: \alpha < 0 \\ -: \alpha > 0 \end{cases}$

40. $v_1 = \dfrac{u_1(m_1 \sin^2\alpha - \varepsilon m_2)}{m_1 \sin^2\alpha + m_2}$; along $\mathbf{u}_1$

$v_2 = \dfrac{(\varepsilon + 1)m_1u_1 \sin\alpha}{m_1 \sin^2\alpha + m_2}$; straight up

42. 4.3 m/s, 36° from normal

44. $\mu ag\left(1 + \dfrac{u_1^2}{ag}\right)$

46. $\sigma(\theta) = \dfrac{a^2}{4}$; $\sigma_t = \pi a^2$

48. $$\sigma_{\mathrm{LAB}}(\psi) \cong \frac{\left(\dfrac{m_1^2 k}{2m_2^2 T_0}\right)^2}{\left[1 - \sqrt{1 - \left(\dfrac{m_1}{m_2}\psi\right)^2}\right]^2 \sqrt{1 - \left(\dfrac{m_1}{m_2}\psi\right)^2}}$$

54. e^{-1}

58. $\dfrac{v_B^2}{2g}$

60. 25 s

62. 273 s

64. (a) 3700 km (b) 890 km (c) 950 km (d) 8900 km

66. (a) 131 m/s. (b) 108 m

Chapter 10

2. The location is given by $\tan\theta = \dfrac{ar_0}{v^2}$, where θ is the angle between the radius and the horizontal; $|\mathbf{a}_f| = a + \sqrt{a^2 + \dfrac{v^4}{r_0^2}}$

4. $v_0 = 0.5\,\omega R$, in y direction; a circle

6. paraboloid $\left(z = \dfrac{\omega^2}{2g}r^2 + \text{const.}\right)$

12. 0.0018 rad = 6 min

16. (a) 77 km (b) 8.9 km (c) 10 km (d) 160 km (all to the west)

18. 260 m to the left

20. $g(\text{poles}) = 9.832\ \text{m/s}^2$, $g(\text{equator}) = 9.780\ \text{m/s}^2$

22. 2.26 mm to the right

Chapter 11

2. $I_1 = I_2 = \dfrac{3}{20}M(R^2 + 4h^2);\ I_3 = \dfrac{3}{10}MR^2;$

$$I_1' = I_2' = \frac{3}{20}M\left(R^2 + \frac{1}{4}h^2\right);\ I_3' = I_3$$

4. $I = \dfrac{1}{3}ml^2;\ a = \dfrac{\ell}{\sqrt{3}}$

14. $I_1 = I_2 = \dfrac{83}{320}Mb^2;\ I_3 = \dfrac{2}{5}Mb^2$

20. $\sqrt{\dfrac{3g}{b}}$

24. (a) $\sqrt{\sqrt{3}\,\dfrac{g}{a}}$ (b) $\sqrt{\dfrac{12}{5\sqrt{3}}\dfrac{g}{a}}$

32. 53.7 rad/s

34. $\omega_x = \omega_{x_0}\exp(-bt/I_x)$

Chapter 12

8. $\omega_1 = \sqrt{2 + \sqrt{2}}\sqrt{\dfrac{g}{l}};\ \omega_2 = \sqrt{2 - \sqrt{2}}\ \sqrt{\dfrac{g}{l}}$

10. $m\ddot{x}_1 + b\dot{x}_1 + (\kappa + \kappa_{12})x_1 - \kappa_{12}x_2 = F_0\cos\omega t$

$m\ddot{x}_2 + b\dot{x}_2 + (\kappa + \kappa_{12})x_2 - \kappa_{12}x_1 = 0$

16. $\theta_0 = -\dfrac{1}{2}\phi_0$, Mode 1; $\theta_0 = \phi_0$, Mode 2

18. $\omega_1 = 0;\ \omega_2 = \sqrt{\dfrac{g}{Mb}(M+m)}$

20. $\mathbf{a}_1 = \left(\dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}, -\dfrac{3}{\sqrt{14}}\right);\qquad \mathbf{a}_2 = \left(\dfrac{4}{\sqrt{42}}, -\dfrac{5}{\sqrt{42}}, \dfrac{1}{\sqrt{42}}\right)$

22. $\omega_1 = 2\sqrt{\dfrac{\kappa}{M}};\ \omega_2 = 2\sqrt{\dfrac{3\kappa}{M+m}};\ \omega_3 = 2\sqrt{\dfrac{3\kappa}{M}}$

26. 4.57, 4.64, 4.81 rad/s

28. $\theta_{2,\max} = 0.96$ rad (but at this angle, small angle approximation is not completely valid, so this is a rough estimate)

Chapter 13

4. $\omega_n = \dfrac{n\pi}{L}\sqrt{\dfrac{\tau}{\rho}}$

The amplitude of the nth mode is given by $\mu_n = \begin{cases} 0, & n \text{ even} \\ \dfrac{32}{n^3\pi^3}, & n \text{ odd} \end{cases}$

6. The second harmonic is down 4.4 dB; the third, 13.3 dB

12. $\eta_s(t) = e^{-Dt/2\rho}\left[A_1 \exp\left(\sqrt{\dfrac{D^2}{4\rho^2} - \dfrac{s^2\pi^2\tau}{\rho b}}\,t\right) + A_2 \exp\left(-\sqrt{\dfrac{D^2}{4\rho^2} - \dfrac{s^2\pi^2\tau}{\rho b}}\,t\right)\right]$

20. $\phi_{B_1} - \phi_{A_1} = \tan^{-1}(\cot\theta) \qquad \phi_{A_2} - \phi_{A_1} = -\theta$

Chapter 14

12. 55.3 m; 0.22 μs; 2.5×10^8 m/s; 2.5×10^8 m/s

16. $\cos\theta = \dfrac{\cos\theta' - \beta}{1 - \beta\cos\theta'}$

20. The astronaut ages 25.4 years; those on Earth age 26.7 years.

22. 4.4×10^9 kg/s; 1.4×10^{13} years

24. $7m_pc^2$, including the rest mass of the proton (kinetic energy is $6\ m_pc^2$)

28. $v \le 0.115c$

30. 0.8 MeV

32. $T_{\text{electron}} = 999.5$ MeV
$T_{\text{proton}} = 433$ MeV

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