

Now we want to calculate the range R for all the values of T (as a function of k) that we have just found. To do this, we need to solve Equation 2.43 for each of the values of k and $t = T$ that we have just found.

$$x := 100$$

$$f(k, T) := \text{root} \left[x - \frac{u}{k} \cdot (1 - \exp(-k \cdot T)), x \right]$$

$$R_j := f(K_j, T_{rj})$$

$$R_1 = 3.182 \cdot 10^4$$

This is the guess for the first value of x . The actual value of the guess does not matter. This is the Equation 2.43 that we need to solve to find the range R .

Now calculate the range R for all the values.

We just list the first value and plot the remainder. This is the range for no air resistance, that is $k = 0$.

Now let's calculate and plot the range determined from the approximate calculation. Calculate Equation 2.55.

$$R_{pj} := R_1 \cdot \left(1 - \frac{4 \cdot K_j \cdot v}{3 \cdot g} \right)$$

[Now plot R_j and R_{pj} versus K_j to produce Figure 2-9.]

Plot approximate and numerical solutions. Figure 2-9.

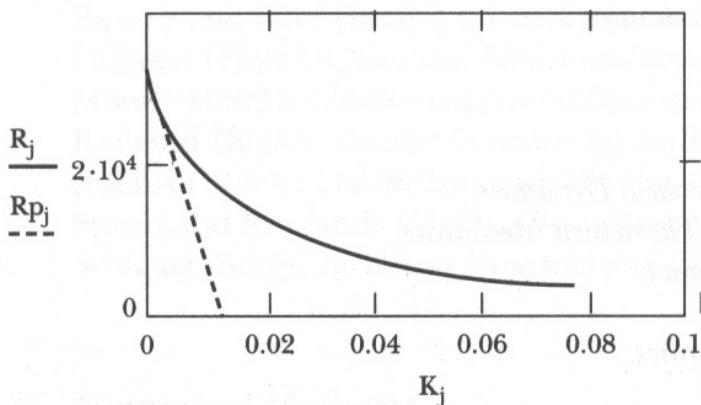


FIGURE 2-9

Selected References

The following texts are particularly recommended as general sources of collateral reading material.

A. General Theoretical Physics

- Blass (B162), *Theoretical Physics*.
Lindsay and Margenau (Li36), *Foundations of Physics*.
Wangsness (Wa63), *Introduction to Theoretical Physics*.

B. Elementary Mechanics

- Baierlein (Ba83), *Newtonian Dynamics*.
Barger and Olsson (Ba73), *Classical Mechanics*.
Davis (Da86), *Classical Mechanics*.
Fowles and Cassiday (Fo99), *Analytical Mechanics*.
French (Fr71), *Newtonian Mechanics*.
Knudsen and Hjorth (Kn00), *Elements of Newtonian Mechanics*.
McCall (Mc01), *Classical Mechanics*.
Rossberg (Ro83), *Analytical Mechanics*.

C. Intermediate Mechanics

- Arya (Ar98), *Introduction to Classical Dynamics*.
Becker (Be54), *Introduction to Theoretical Mechanics*.
Lindsay (Li61), *Physical Mechanics*.
Scheck(Sc99), *Mechanics*.
Slater and Frank (Sl47), *Mechanics*.
Symon (Sy71), *Mechanics*.

D. Advanced Mechanics

- Baruh (Ba99), *Analytical Dynamics*.
Goldstein (Go80), *Classical Mechanics*.

Landau and Lifshitz (La76), *Mechanics*.
McCuskey (Mc59), *An Introduction to Advanced Dynamics*.

E. Mathematical Methods

Abramowitz and Stegun (Ab65), *Handbook of Mathematical Functions*.
Arfken (Ar85), *Mathematical Methods for Physicists*.
Byron and Fuller (By69), *Mathematics of Classical and Quantum Physics*.
Churchill (Ch78), *Fourier Series and Boundary Value Problems*.
Davis (Da61), *Introduction to Vector Analysis*.
Dennery and Krzywicki (De67), *Mathematics for Physicists*.
Dwight (Dw61), *Tables of Integrals and Other Mathematical Data*.
Kaplan (Ka84), *Advanced Calculus*.
Mathews and Walker (Ma70), *Mathematical Methods of Physics*.
Pipes and Harvill (Pi70), *Applied Mathematics for Engineers and Physicists*.

F. Special Relativity

Einstein (Ei61), *Relativity*.
French (Fr68), *Special Relativity*.
Resnick (Re72), *Basic Concepts in Relativity and Early Quantum Theory*.
Rindler (Ri82), *Introduction to Special Relativity*.
Taylor and Wheeler (Ta66), *Spacetime Physics*.

G. Chaos

Baker and Gollub (Ba90), *Chaotic Dynamics*.
Besshoir and Wolf (Be91), *Chaos Simulations*.
Hilborn (Hi94), *Chaos and Nonlinear Dynamics*.
Moon (Mo92), *Chaotic and Fractal Dynamics*.
Rasband (Ra90), *Chaotic Dynamics of Nonlinear Systems*.
Rollins (Ro90), *Chaotic Dynamics Workbench*.
Sprott and Rowlands (Sp92), *Chaos Demonstrations*.
Strogatz (St94), *Nonlinear Dynamics and Chaos*.

H. Numerical Methods

DeJong (De91), *Introduction to Computational Physics*.
Johnson and Reiss (Jo82), *Numerical Analysis*.
Press, Teukolsky, Vetterling, and Flannery (Pr92), *Numerical Recipes*.

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Answers to Even-Numbered Problems

Chapter 1

10. (a) $\mathbf{v} = 2b\omega \cos \omega t \mathbf{i} - b\omega \sin \omega t \mathbf{j}$ (b) 90°

$$\mathbf{a} = -\omega^2 \mathbf{r}$$

$$|\mathbf{v}| = b\omega[3 \cos^2 \omega t + 1]^{\frac{1}{2}}$$

12. $h = \frac{|\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}|}{|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|}$

$$A = \frac{1}{2} |(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{b})| = \frac{1}{2} |(\mathbf{a} - \mathbf{c}) \times (\mathbf{b} - \mathbf{a})|$$

$$= \frac{1}{2} |(\mathbf{c} - \mathbf{b}) \times (\mathbf{a} - \mathbf{c})|$$

14. (a) -104 (b) $\begin{pmatrix} 9 & 7 \\ 13 & 9 \\ 5 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} -5 & -5 \\ 3 & -5 \\ 25 & 14 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -3 & -4 \\ 3 & 0 & 6 \\ 4 & -6 & 0 \end{pmatrix}$

26. $\mathbf{a} \cdot \mathbf{e}_r = -\frac{3}{4} \frac{v^2}{k}; |\mathbf{a}| = \frac{3}{4} \frac{v^2}{k} \sqrt{\frac{2}{1 + \cos \theta}}; \dot{\theta} = \frac{v}{\sqrt{2kr}}$

34. $\int (\mathbf{A} \times \ddot{\mathbf{A}}) dt = (\mathbf{A} \times \dot{\mathbf{A}}) + \mathbf{C}$, where \mathbf{C} is a constant vector

36. $\pi c^2 d$

38. $-\pi$

40. (a) $x = -2$ m, $y = 3$ m, $z_{\max} = 72$ m; (c) SE

Chapter 2

2. $F_\theta = mR(\ddot{\theta} - \dot{\phi}^2 \sin \theta \cos \theta)$

$F_\phi = mR(2\dot{\theta}\dot{\phi} \cos \theta + \ddot{\phi} \sin \theta)$

4. $13.2 \text{ m} \cdot \text{s}^{-1}$

6. (a) 210 m behind (b) can be no more than 0.68 s late

14. (a) $d = \frac{2v_0^2 \cos \alpha \sin(\alpha - \beta)}{g \cos^2 \beta}$ (b) $\frac{\pi}{4} + \frac{\beta}{2}$ (c) $d_{\max} = \frac{v_0^2}{g(1 + \sin \beta)}$

16. $\frac{2v_0}{g \sin \alpha}$

18. (a) $35.2 \text{ m} \cdot \text{s}^{-1}$ (b) 40.7° ; 1.1 m

20. 17.4°

22. (c) $\dot{x}(t) = C_1 \cos \omega_c t + C_2 \sin \omega_c t + \frac{E_y}{B}$
 $\dot{y}(t) = -C_1 \sin \omega_c t + C_2 \cos \omega_c t$

24. $\mu_k = 0.18$; $v_B = 15.6 \text{ m/s}$

26. 2.3 m; 1.1 m

28. $h_{\text{marble}} = h \left(\frac{3-a}{1+a} \right)^2$; $h_{\text{superball}} = h \left(\frac{1-3a}{1+a} \right)^2$ where $a = m/M$

30. 71 m

32. $\sin \theta_0 = \frac{1 \pm \mu_k \sqrt{3 + 4\mu_k^2}}{2(1 + \mu_k^2)}$

34. (a) $y = -\frac{m}{\alpha} \left[v + \frac{mg}{\alpha} \ln \left(1 - \frac{\alpha v}{mg} \right) \right]$ (b) $y = -\frac{m}{2\beta} \ln \left(1 - \frac{\beta v^2}{mg} \right)$

36. $R = \frac{v_0^2}{g} \cos \theta \left(\sin \theta + \sqrt{\sin^2 \theta + \frac{2gh}{v_0^2}} \right)$

38. (a) $F(x) = -mn\alpha^2 x^{-(2n+1)}$

(b) $x(t) = [(n+1)at]^{\frac{1}{n+1}}$

(c) $F(t) = -mn\alpha^2 [(n+1)at]^{-(2n+1)/(n+1)}$

40. (a) $a_t = \frac{2A\alpha^2 \sin \alpha t}{\sqrt{5 - 4 \cos \alpha t}}$; $a_n = \frac{A\alpha^2 |2 \cos \alpha t - 1|}{\sqrt{5 - 4 \cos \alpha t}}$

(b) $\frac{n\pi}{\alpha}$ where $n = \text{integer}$

42. Stable if $R > b/2$; unstable if $R \leq b/2$

48. $\tau = \pi d^{3/2} \sqrt{\frac{2}{mG}}$

50. (a) $x(t) = \frac{c^2}{F} \left(\sqrt{m_0^2 + \frac{F^2 t^2}{c^2}} - m_0 \right)$, $v(t) = -\frac{Ft}{\sqrt{m_0^2 + \frac{F^2 t^2}{c^2}}}$

(c) $t(v = c/2) = 0.55$ yr; $t(v = 0.99c) = 6.67$ yr.

52. (a) $F(x) = -\frac{4U_0x}{a^2} \left(1 - \frac{x^2}{a^2} \right)$, (c) $\omega = \sqrt{\frac{4U_0}{ma^2}}$, (d) $v_{\min} = \sqrt{\frac{2U_0}{m}}$

(e) $x(t) = \frac{a[\exp(t\sqrt{8U_0/ma^2}) - 1]}{[\exp(t\sqrt{8U_0/ma^2}) + 1]}$

54. (a) $v = g/k = 1000$ m/s, (b) $height = \frac{v_0}{k} + \frac{g}{k^2} \ln \left(\frac{g}{g + kv_0} \right) = 680$ m

Chapter 3

2. (a) 6.9×10^{-2} s⁻¹ (b) $\frac{10}{2\pi}(1 - 2.40 \times 10^{-5})$ s⁻¹ (c) 1.0445

4. $\langle T \rangle = \langle U \rangle = \frac{mA^2\omega_0^2}{4} \bar{U} = \frac{1}{2} \bar{T} = \frac{mA^2\omega_0^2}{6}$

6. 2.74 rad · s⁻¹

12. $\ddot{\theta} = -\frac{g}{l} \sin \theta$

14. $x(t) = (\cosh \beta t - \sinh \beta t)[(A_1 + A_2)\cosh \omega_2 t + (A_1 - A_2)\sinh \omega_2 t]$

$\dot{x}(t) = (\cosh \beta t - \sinh \beta t)[(A_1\omega_2 - A_1\beta)(\cosh \omega_2 t + \sinh \omega_2 t) - (A_2\beta + A_2\omega_2)(\cosh \omega_2 t - \sinh \omega_2 t)]$

26. $\frac{R_1[R_2(R_2 + R_1) + \omega^2 L_2^2] + i[R_1\omega L_2 + (\omega L_1 - 1/\omega C)((R_1 + R_2)^2 + \omega^2 L_2^2)]}{(R_1 + R_2)^2 + \omega^2 L_2^2}$

28. $F(t) = \frac{4}{\pi} \sin t + \frac{4}{3\pi} \sin 3t + \frac{4}{5\pi} \sin 5t + \dots$

30. $F(t) = \frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t - \dots$

32. (a) $x(t) = \frac{H(0)}{\omega_0^2} \left(1 - e^{-\beta t} \cosh \omega_2 t - \frac{\beta e^{-\beta t}}{\omega_2} \sinh \omega_2 t \right)$

(b) $x(t) = \frac{b}{\omega_2} e^{-\beta t} \sinh \omega_2 t; t > 0$

$$34. x(t) = \begin{cases} 0 & t < 0 \\ 4[1 - \cos(0.5t)] \text{ m} & 0 < t < 4\pi \\ 0 & t > 4\pi \end{cases}$$

$$36. x(t) = e^{-\beta(t-t_0)} \left[x_0 \cos \omega_1(t-t_0) + \left(\frac{\dot{x}_0}{\omega_1} + \frac{\beta x_0}{\omega_1} + \frac{b}{\omega_1} \right) \sin \omega_1(t-t_0) \right]; \quad t > t_0$$

$$38. x(t) = \frac{F_0}{m} \frac{\omega}{[(\beta - \gamma)^2 + (\omega + \omega_1)^2][(\beta - \gamma)^2 + (\omega - \omega_1)^2]} \\ \times \left[e^{-\gamma t} \left[2(\gamma - \beta) \cos \omega t + ([\beta - \gamma]^2 + \omega_1^2 - \omega^2) \frac{\sin \omega t}{\omega} \right] \right. \\ \left. + e^{-\beta t} \left[2(\beta - \gamma) \cos \omega_1 t + ([\beta - \gamma]^2 + \omega^2 - \omega_1^2) \frac{\sin \omega_1 t}{\omega_1} \right] \right]$$

40. Amplitude = -0.16 mm, minus sign indicates spring is compressed

$$42. (a) x(t) = \frac{F}{m\omega_0} \frac{1}{(\omega_0 + \omega)(\omega_0 - \omega)} (\omega_0 \sin \omega t - \omega \sin \omega_0 t). \quad (b) x(t) = \frac{F_0 t^3 \omega_0}{6m}$$

$$44. \frac{\omega_1}{\omega_0} = \frac{8\pi}{\sqrt{64\pi^2 + 1}}$$

Chapter 4

$$6. \dot{\theta} = \sqrt{\frac{2}{ml^2}} [E - mgl(1 - \cos \theta)]^{1/2}$$

$$8. \tau = 4\sqrt{\frac{2mA}{F_0}}$$

10. Only 0.6 and 0.7 are chaotic

14. $n = 30$

22. Transitions at $B_1 = 9.8 - 9.9$, $B_2 = 11.6 - 11.7$, and $B_3 = 13.3 - 13.4$. Behavior: (i) one period per three drive cycles when $B < B_1$, (ii) chaotic when $B_1 < B < B_2$, (iii) mixed chaotic/one period per drive cycle (depending on initial conditions) when $B_2 < B < B_3$, and (iv) one period per drive cycle when $B > B_3$

Chapter 5

$$2. \rho = \frac{C}{2\pi Gr} \text{ where } C = \frac{\partial \phi}{\partial r} = \text{const.}$$

$$6. \mathbf{g} = -\frac{GM}{r^2} \mathbf{e}_r$$

8. $g_z = -2\pi G\rho \left(\sqrt{a^2 + (z_0 - l)^2} - \sqrt{a^2 + z_0^2} + l \right)$

10. $\phi(R) \cong -\frac{GM}{R} \left[1 - \frac{1}{2} \frac{a^2}{R^2} \left(1 - \frac{3}{2} \sin^2 \theta \right) \right]$

16. $F_z = 2\pi\rho_s GM$

20. $\Phi(z) = -\frac{2GM}{R^2} (\sqrt{z^2 + R^2} - z), g(z) = -k \frac{2GM}{R^2} \left(\frac{\sqrt{z^2 + R^2} - z}{\sqrt{z^2 + R^2}} \right)$

Chapter 6

8. (a) $a_1 = b_1 = c_1 = \frac{2}{\sqrt{3}} R$ (b) $a_1 = a \frac{2}{\sqrt{3}}, b_1 = b \frac{2}{\sqrt{3}}, c_1 = c \frac{2}{\sqrt{3}}$

10. $R = \frac{1}{2}H$

14. length $= 2\sqrt{2} \sin \frac{\pi}{2\sqrt{2}}$

16. $y(x) = \frac{8}{13^{3/2} - 8} \left[\left(1 + \frac{9x}{4} \right)^{3/2} - 1 \right]$ and $z = x^{3/2}$

18. $x = -y = \sqrt{-z}$ where $x > 0, y < 0, z < 0$. Parabolic line.

Chapter 7

4. $m\ddot{r} - mr\dot{\theta}^2 + Ar^{\alpha-1} = 0; \frac{d}{dt}(mr^2\dot{\theta}) = 0; \text{ yes; yes}$

6. $2m\ddot{S} + m\ddot{\xi} \cos \alpha - mg \sin \alpha = 0$

$(m + M)\ddot{\xi} + m\ddot{S} \cos \alpha = 0$

10. (a) $y(t) = -\frac{g}{4}t^2$ (b) $y(t) = \frac{Ml}{m}(1 - \cosh \gamma t)$

12. $r(t) = r_0 \cosh \alpha t + \frac{g}{2\alpha^2}(\sin \alpha t - \sinh \alpha t)$

14. (a) $\ddot{\theta} + \frac{a+g}{b} \sin \theta = 0$ (b) $2\pi \sqrt{\frac{b}{a+g}}$

16. $\ddot{\theta} + \frac{g}{b} \sin \theta - \frac{a}{b} \omega^2 \sin \omega t \cos \theta = 0$

18. $\omega = \sqrt{\frac{g \sin \theta_0}{l - R\theta_0}}, \theta_0 = \frac{\pi}{2}$

22. $L = \frac{1}{2}m\dot{x}^2 - \frac{k}{x}e^{-t/\tau}; H = \frac{p_x^2}{2m} + \frac{k}{x}e^{-t/\tau}$

24. $L = \frac{1}{2}m(\alpha^2 + l^2\dot{\theta}^2) + mgl \cos \theta$

$$H = \frac{p_\theta^2}{2ml^2} - \frac{1}{2}m\alpha^2 - mgl \cos \theta$$

26. (a) $H = \frac{p_\theta^2}{2ml^2} - mgl \cos \theta; \dot{\theta} = \frac{p_\theta}{ml^2}; \dot{p}_\theta = -mgl \sin \theta$

(b) $H = \frac{p_x^2}{2(m_1 + m_2 + I/a^2)} - m_1gx - m_2g(l - x)$

$$\dot{p}_x = \left(m_1 + m_2 + \frac{l}{a^2} \right) \dot{x}$$

$$\dot{p}_x = g(m_1 - m_2)$$

28. $p_r = mr; \dot{p}_r = \frac{p_\theta^2}{mr^3} - \frac{k}{r^2}; p_\theta = mr^2\dot{\theta}; \dot{p}_\theta = 0$

32. $H = \frac{1}{2m} \left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 \sin^2 \theta} \right) - \frac{k}{r}; \dot{p}_r = -\frac{k}{r^2} + \frac{p_\theta^2}{mr^3} + \frac{p_\phi^2}{mr^3 \sin^2 \theta};$

$$\dot{p}_\theta = \frac{p_\phi^2 \cot \theta}{mr^2 \sin^2 \theta}; \dot{p}_\phi = 0$$

34. (a) $\ddot{x} = aR(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta); \ddot{\theta} = \frac{\ddot{x} \sin \theta + g \cos \theta}{R}; \text{ where } a \equiv \frac{m}{M+m}$

(b) $\lambda = -\frac{mMg(3 \sin \theta - a \sin^3 \theta - 2 \sin \theta_0)}{(M+m)(1 - a \sin^2 \theta)^2}$

38. $\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{p}{m}, \frac{dp}{dt} = -\frac{\partial H}{\partial x} = -(kx + bx^3)$

40. $0 = 4 \frac{d^2x}{dt^2} + b \left(2 \frac{d^2\theta_1}{dt^2} \cos \theta_1 + \frac{d^2\theta_2}{dt^2} \cos \theta_2 \right) - b \left(2 \left(\frac{d\theta_1}{dt} \right)^2 \sin \theta_1 + \left(\frac{d\theta_2}{dt} \right)^2 \sin \theta_2 \right)$

$$-2g \sin \theta_1 = 2b \frac{d^2\theta_1}{dt^2} + 2 \frac{d^2x}{dt^2} \cos \theta_1 + b \frac{d^2\theta_2}{dt^2} \cos(\theta_1 - \theta_2)$$

$$+ b \left(\frac{d\theta_2}{dt} \right)^2 \sin(\theta_1 - \theta_2)$$

$$-g \sin \theta_2 = b \frac{d^2\theta_2}{dt^2} + \frac{d^2x}{dt^2} \cos \theta_2 + b \frac{d^2\theta_1}{dt^2} \cos(\theta_1 - \theta_2)$$

$$- b \left(\frac{d\theta_1}{dt} \right)^2 \sin(\theta_1 - \theta_2)$$

Chapter 8

4. $\langle U \rangle = -\frac{k}{a}; \quad \langle T \rangle = \frac{k}{2a}$

10. Parabola; yes

12. 76 days

14. $F(r) = -\frac{l^2}{\mu} \left(\frac{6k}{r^4} + \frac{1}{r^3} \right)$

22. No

24. (a) 1590 km (b) 1900 km

28. 2380 m/s

30. $\Delta v = 3.23$ km/s; parabola

32. Stable if $r < a$

38. $\Delta v = 5275$ m/s (opposite to direction of motion); 146 days

40. Carrying the waste out of the solar system requires less energy than crashing it into the sun

42. $2.57 \times 10^{11} \text{ J}$

44. $\frac{\alpha}{r} = 1 + \varepsilon \cos \theta,$ where $\alpha = \frac{\ell^2}{\mu k}, \quad \varepsilon = \sqrt{1 + \frac{2\ell^2}{\mu k^2} \left(E - \frac{k}{a} \right)}$

If $0 < \varepsilon < 1$, the orbit is ellipsoid. If $\varepsilon = 0$, the orbit is circular

46. $T = 9 \times 10^7 \text{ yr.}$

Chapter 9

2. On the axis; $\frac{3}{4}h$ from vertex

4. $\bar{x} = \frac{2a}{\theta} \sin \frac{\theta}{2}; \quad \bar{y} = 0$

6. $\mathbf{r}_{\text{cm}} = \frac{F_0}{4m} t^2 \mathbf{i}; \quad \mathbf{v}_{\text{cm}} = \frac{F_0}{2m} t \mathbf{i}; \quad \mathbf{a}_{\text{cm}} = \frac{F_0}{2m} \mathbf{i}$

8. $\bar{x} = 0; \quad \bar{y} = \frac{a}{3\sqrt{2}}$

10. $\frac{v_0}{g} \sin \theta \sqrt{\frac{2E}{m_1 + m_2}} \left(\sqrt{\frac{m_1}{m_2}} + \sqrt{\frac{m_2}{m_1}} \right)$

12. (a) yes (b) 11 m/s

14. No

20. \sqrt{ga}

22. (a) two sets of solutions: $v_n = 5.18 \text{ km/s}, v_d = 14.44 \text{ km/s}$ and $v_n = 19.79 \text{ km/s}, v_d = 5.12 \text{ km/s.}$ (b) 74.8° and $5.2^\circ.$ (c) 30°

24. $\omega = \frac{\omega_0}{1 - \frac{a}{b}\theta}; T = mb\omega_0\omega$

26. $\mathbf{N} = \frac{kr}{v_0} (\mathbf{r}_1 - \mathbf{r}_2) \times (\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)$

28. $\frac{4m_1 m_2}{(m_1 + m_2)^2}$

30. (a) $(-0.09\mathbf{i} + 1.27\mathbf{j})\text{N} \cdot \mathbf{s}$ (b) $(-9\mathbf{i} + 127\mathbf{j})\text{N}$

32. $v_1 = v_2 = \frac{u_1}{3}$

34. $v_1 = v_2 = \frac{u_1}{\sqrt{2}}$; $\theta = 45^\circ$

36. $\frac{m_1}{m_2} = 3 \pm 2\sqrt{2}$; $\frac{u_2}{u_1} = -(1 \pm \sqrt{2})$ with $\begin{cases} +: \alpha < 0 \\ -: \alpha > 0 \end{cases}$

40. $v_1 = \frac{u_1(m_1 \sin^2 \alpha - \epsilon m_2)}{m_1 \sin^2 \alpha + m_2}$; along \mathbf{u}_1

$v_2 = \frac{(\epsilon + 1)m_1 u_1 \sin \alpha}{m_1 \sin^2 \alpha + m_2}$; straight up

42. 4.3 m/s, 36° from normal

44. $\mu ag \left(1 + \frac{u_1^2}{ag}\right)$

46. $\sigma(\theta) = \frac{a^2}{4}; \sigma_t = \pi a^2$

48. $\sigma_{\text{LAB}}(\psi) \cong \frac{\left(\frac{m_1^2 k}{2m_2^2 T_0}\right)^2}{\left[1 - \sqrt{1 - \left(\frac{m_1}{m_2}\psi\right)^2}\right]^2 \sqrt{1 - \left(\frac{m_1}{m_2}\psi\right)^2}}$

54. e^{-1}

58. $\frac{v_B^2}{2g}$

60. 25 s

62. 273 s

64. (a) 3700 km (b) 890 km (c) 950 km (d) 8900 km

66. (a) 131 m/s. (b) 108 m

Chapter 10

2. The location is given by $\tan \theta = \frac{ar_0}{v^2}$, where θ is the angle between the radius

and the horizontal; $|a_f| = a + \sqrt{a^2 + \frac{v^4}{r_0^2}}$

4. $v_0 = 0.5 \omega R$, in y direction; a circle

6. paraboloid $\left(z = \frac{\omega^2}{2g} r^2 + \text{const.} \right)$

12. $0.0018 \text{ rad} = 6 \text{ min}$

16. (a) 77 km (b) 8.9 km (c) 10 km (d) 160 km (all to the west)

18. 260 m to the left

20. $g(\text{poles}) = 9.832 \text{ m/s}^2$, $g(\text{equator}) = 9.780 \text{ m/s}^2$

22. 2.26 mm to the right

Chapter 11

2. $I_1 = I_2 = \frac{3}{20}M(R^2 + 4h^2)$; $I_3 = \frac{3}{10}MR^2$;

$I'_1 = I'_2 = \frac{3}{20}M\left(R^2 + \frac{1}{4}h^2\right)$; $I'_3 = I_3$

4. $I = \frac{1}{3}ml^2$; $a = \frac{\ell}{\sqrt{3}}$

14. $I_1 = I_2 = \frac{83}{320}Mb^2$; $I_3 = \frac{2}{5}Mb^2$

20. $\sqrt{\frac{3g}{b}}$

24. (a) $\sqrt{\sqrt{3}\frac{g}{a}}$ (b) $\sqrt{\frac{12}{5\sqrt{3}}\frac{g}{a}}$

32. 53.7 rad/s

34. $\omega_x = \omega_{x_0} \exp(-bt/I_x)$

Chapter 12

8. $\omega_1 = \sqrt{2 + \sqrt{2}} \sqrt{\frac{g}{l}}$; $\omega_2 = \sqrt{2 - \sqrt{2}} \sqrt{\frac{g}{l}}$

10. $m\ddot{x}_1 + b\dot{x}_1 + (\kappa + \kappa_{12})x_1 - \kappa_{12}x_2 = F_0 \cos \omega t$

$m\ddot{x}_2 + b\dot{x}_2 + (\kappa + \kappa_{12})x_2 - \kappa_{12}x_1 = 0$

16. $\theta_0 = -\frac{1}{2}\phi_0$, Mode 1; $\theta_0 = \phi_0$, Mode 2

18. $\omega_1 = 0$; $\omega_2 = \sqrt{\frac{g}{Mb}(M + m)}$

20. $\mathbf{a}_1 = \left(\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}} \right)$; $\mathbf{a}_2 = \left(\frac{4}{\sqrt{42}}, -\frac{5}{\sqrt{42}}, \frac{1}{\sqrt{42}} \right)$

22. $\omega_1 = 2\sqrt{\frac{\kappa}{M}}$; $\omega_2 = 2\sqrt{\frac{3\kappa}{M+m}}$; $\omega_3 = 2\sqrt{\frac{3\kappa}{M}}$

26. 4.57, 4.64, 4.81 rad/s

28. $\theta_{2,\max} = 0.96$ rad (but at this angle, small angle approximation is not completely valid, so this is a rough estimate)

Chapter 13

4. $\omega_n = \frac{n\pi}{L} \sqrt{\frac{\tau}{\rho}}$

The amplitude of the n th mode is given by $\mu_n = \begin{cases} 0, & n \text{ even} \\ \frac{32}{n^3 \pi^3}, & n \text{ odd} \end{cases}$

6. The second harmonic is down 4.4 dB; the third, 13.3 dB

12. $\eta_s(t) = e^{-Dt/2\rho} \left[A_1 \exp\left(\sqrt{\frac{D^2}{4\rho^2} - \frac{s^2\pi^2\tau}{\rho b}} t\right) + A_2 \exp\left(-\sqrt{\frac{D^2}{4\rho^2} - \frac{s^2\pi^2\tau}{\rho b}} t\right) \right]$

20. $\phi_{B_1} - \phi_{A_1} = \tan^{-1}(\cot \theta)$ $\phi_{A_2} - \phi_{A_1} = -\theta$

Chapter 14

12. 55.3 m; 0.22 μ s; 2.5×10^8 m/s; 2.5×10^8 m/s

16. $\cos \theta = \frac{\cos \theta' - \beta}{1 - \beta \cos \theta'}$

20. The astronaut ages 25.4 years; those on Earth age 26.7 years.

22. 4.4×10^9 kg/s; 1.4×10^{13} years

24. $7m_p c^2$, including the rest mass of the proton (kinetic energy is 6 $m_p c^2$)

28. $v \leq 0.115c$

30. 0.8 MeV

32. $T_{\text{electron}} = 999.5$ MeV
 $T_{\text{proton}} = 433$ MeV

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