UNIVERSIDAD NACIONAL DE INGENIERÍA

FACULTAD DE CIENCIAS

Corregir

CALIFICACIÓN

Preg N°	Puntos
1	0
2	2.5
3	—
4	4
5	0.5
6	
Total	07

CURSO: Cálculo diferencial e integral avanzado COD. CURSO: CM-211

PRACTICA: Calificada N°4 SECCIÓN: D

APELLIDOS Y NOMBRES (Alumno) CODIGO FIRMA

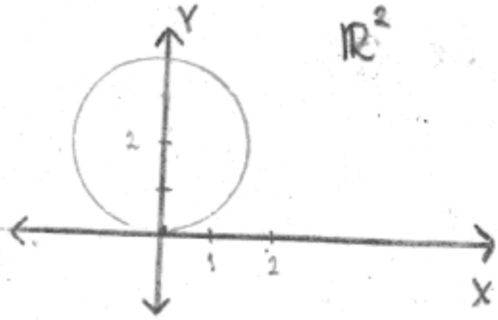
Lima, 15 de Mayo del 2018 N° Lista 3

NOTA

En números En letras Nombre del Profesor Firma del Profesor

2. Sea $f(x,y) = 25 - x^2 - y^2$.

$$\text{máx}/\text{mín} \quad f(x,y)$$



Sujeto a

$$g: x^2 + y^2 - 4y = 0 \quad \left(\mathcal{C}: x^2 + (y-2)^2 = 2^2 \right).$$

Por el método de los multiplicadores de Lagrange:

$$\nabla f = \lambda \nabla g$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (-2x, -2y).$$

$$\nabla g = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right) = (2x, 2y-4)$$

Con el siguiente sistema de ecuaciones:

$$-2x = \lambda \cdot 2x \quad \dots (\mathrm{I})$$

$$-2y = \lambda \cdot (2y-4) \quad \dots (\mathrm{II})$$

$$x^2 + y^2 - 4y = 0 \quad (\mathrm{III})$$

Casos

1^{er} caso: De (II):

$$-2y = \lambda(2y-4)$$
$$0 = 2y\lambda + 2y - 4\lambda$$
$$0 = 2y(\lambda+1) - 4\lambda$$
$$4\lambda = 2y(\lambda+1)$$
$$\frac{2\lambda}{\lambda+1} = y \quad , \quad \lambda+1 \neq 0 \Rightarrow \boxed{\lambda \neq -1}$$

2^{do} caso:

De (I):

$$-2x = \lambda \cdot 2x$$
$$0 = \lambda 2x + 2x$$
$$0 = 2x(\lambda+1)$$

Así, $x = 0 \quad \vee \quad \lambda+1=0$

$$\boxed{\lambda=-1}$$

Pero del 1^{er} caso, λ no puede ser -1. (Valor prohibido).

Con la información obtenida en el 1^{er} y 2^{do} caso.

De (III):

$$x^2 + y^2 - 4y = 0$$

$(\lambda \neq -1)$

$x=0 \quad , \quad y = \frac{2\lambda}{\lambda+1}$

Reemplazando:

$$0^2 + \left(\frac{2\lambda}{\lambda+1}\right)^2 - 4\left(\frac{2\lambda}{\lambda+1}\right) = 0$$

$$\frac{(2\lambda)^2 - 4(2\lambda)(\lambda+1)}{(\lambda+1)^2} = 0$$

Así:

$$4\lambda^2 - 8\lambda^2 - 8\lambda = 0$$
$$-4\lambda^2 - 8\lambda = 0 \iff \lambda^2 - 2\lambda = 0 \iff \lambda(\lambda-2) = 0$$

Los multiplicadores de Lagrange son: $\lambda_1 = 0$ y $\lambda_2 = 2$.

Para λ_1:

$$x = 0$$

$$y = \frac{2\lambda_1}{\lambda_1 + 1} = 0.$$

Punto $(0,0) \in \mathbb{R}^2$.

$$f(0,0) = 25 - 0^2 - 0^2$$

$$f(0,0) = 25.$$

<u>máximo</u>.

Para λ_2:

$$x = 0$$

$$y = \frac{2\lambda_2}{\lambda_2 + 1} = \frac{4}{3}.$$

Punto $(0, \frac{4}{3}) \in \mathbb{R}^2$

$$f(0, \tfrac{4}{3}) = 25 - 0^2 - \left(\tfrac{4}{3}\right)^2$$

$$f(0, \tfrac{4}{3}) = \frac{209}{9} \approx 23{,}2.$$

<u>mínimo</u>.

El punto es $(0,4)$ y $f(0,4) = 9$
mínimo

1. b) Verdadero. $Jf_{(1,0,2)} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix}$

5.

(a)

$$\mathbb{R}^3 \xrightarrow{g} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}^2$$
$$f \circ g$$

Con

$f: \mathbb{R}^2 \to \mathbb{R}^2$
$(x,y) \mapsto (e^{x+2y}, \operatorname{sen}(y+2x))$, $g: \mathbb{R}^3 \to \mathbb{R}^2$
$(u,v,w) \mapsto (u+2v^2+3w^3, 2v-u^2)$

$\operatorname{Dom} f = \{(x,y) \in \mathbb{R}^2\}$ 0,5 $\operatorname{Dom} g = \{(u,v,w) \in \mathbb{R}^3\}$

Así, $f \circ g_{(u,v,w)} = f(g(u,v,w)) = f(u+2v^2+3w^3, 2v-u^2)$

$f \circ g_{(u,v,w)} = (e^{u+2v^2+3w^3+2(2v-u^2)}, \operatorname{sen}(2v-u^2+2(u+2v^2+3w^3)))$

$f \circ g_{(u,v,w)} = (e^{u-2u^2+4v+2v^2+3w^3}, \operatorname{sen}(2u-u^2+2v+4v^2+\cancel{3w^3}))$.

Con $\operatorname{Dom} f \circ g = \{(u,v,w) \in \mathbb{R}^3 \wedge g(u,v,w) \in \operatorname{Dom} f\} = \{(u,v,w) \in \mathbb{R}^3\}$.

(b).

$$\operatorname{Jac} f \circ g_{(1,-1,1)} = \begin{bmatrix} \frac{\partial g_1}{\partial u} & \frac{\partial g_2}{\partial u} \\ \frac{\partial g_1}{\partial v} & \frac{\partial g_2}{\partial v} \\ \frac{\partial g_1}{\partial w} & \frac{\partial g_2}{\partial w} \end{bmatrix}_{(1,-1,1)} \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}_{(6,-3)}$$

$\mathbb{R}^2$

$g(1,-1,1) = (6,-3)$

$f(6,-3) = (1,0)$.

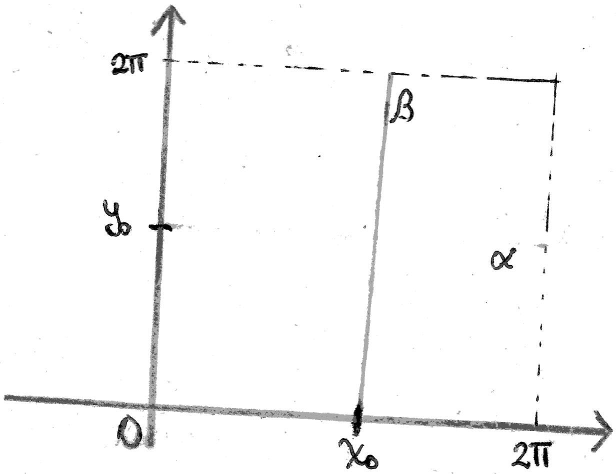
$$\operatorname{Jac} f \circ g_{(1,-1,1)} = \begin{bmatrix} 1 & -2u \\ 4v & 2 \\ 9w^2 & 0 \end{bmatrix}_{(1,-1,1)} \begin{bmatrix} e^{x+2y} & 2e^{x+2y} \\ 2\cos(y+2x) & \cos(y+2x) \end{bmatrix}_{(6,-3)}$$

$$\operatorname{Jac} f \circ g_{(1,-1,1)} = \begin{bmatrix} 1 & -2 \\ -4 & 2 \\ 9 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2\cos 9 & \cos 9 \end{bmatrix} = \begin{bmatrix} 1-4\cos 9 & 2-2\cos 9 \\ -4+4\cos 9 & -8+2\cos 9 \\ 9 & 18 \end{bmatrix}.$$

4. Sea $f: [0,2\pi]^2 \longrightarrow \mathbb{R}^3$ $\ulcorner\ 0<b<a\ \lrcorner$

$(x,y) \longmapsto ((a+b\cos x)\cos y,\ (a+b\cos x)\operatorname{sen} y,\ b\operatorname{sen} x)$

Si $x_0, y_0 \in [0,2\pi]$ fijos.



Se definen los conjuntos:

$\alpha_{y_0} = \{(x, y_0) \in [0,2\pi]^2 : 0 \le x \le 2\pi\}$.

$\beta_{x_0} = \{(x_0, y) \in [0,2\pi]^2 : 0 \le y \le 2\pi\}$.

y Sean $path_1 = \alpha_0$, $path_2 = \beta_{2\pi}$, $Path_3 = \alpha_{2\pi}$ y $Path_4 = \beta_0$.

Veamos

$f(\alpha_{2\pi}) = f(\alpha_0) = ((a+b\cos x)\cos 0,\ (a+b\cos x)\operatorname{sen} 0,\ b\operatorname{sen} x)$

$f(\alpha_{2\pi}) = f(\alpha_0) = (a+b\cos x,\ 0,\ b\operatorname{sen} x) = (a,0,0) + (b\cos x, 0, b\operatorname{sen} x)$,

Con $0 \le x \le 2\pi$.

circunferencia de radio b.

$f(\beta_0) = f(\beta_{2\pi}) = ((a+b\cos 2\pi)\cos y,\ (a+b\cos 2\pi)\operatorname{sen} y,\ b\operatorname{sen} 2\pi)$

$f(\beta_0) = f(\beta_{2\pi}) = ((a+b)\cos y,\ (a+b)\operatorname{sen} y,\ 0)$.

Con $0 \le y \le 2\pi$.

circunferencia de rado a+b

Así, para un

$f(x_0,y) = f(\beta_{x_0}) = ((a+b\cos x_0)\cos y,\ (a+b\cos x_0)\operatorname{sen} y,\ b\operatorname{sen} y)$

$f(x_0,y) = f(\beta_{x_0}) = (0,0,b\operatorname{sen} y) + ((a+b\cos x_0)\cos y,\ (a+b\cos x_0)\operatorname{sen} y,\ 0)$.

con $0 \le y \le 2\pi$.

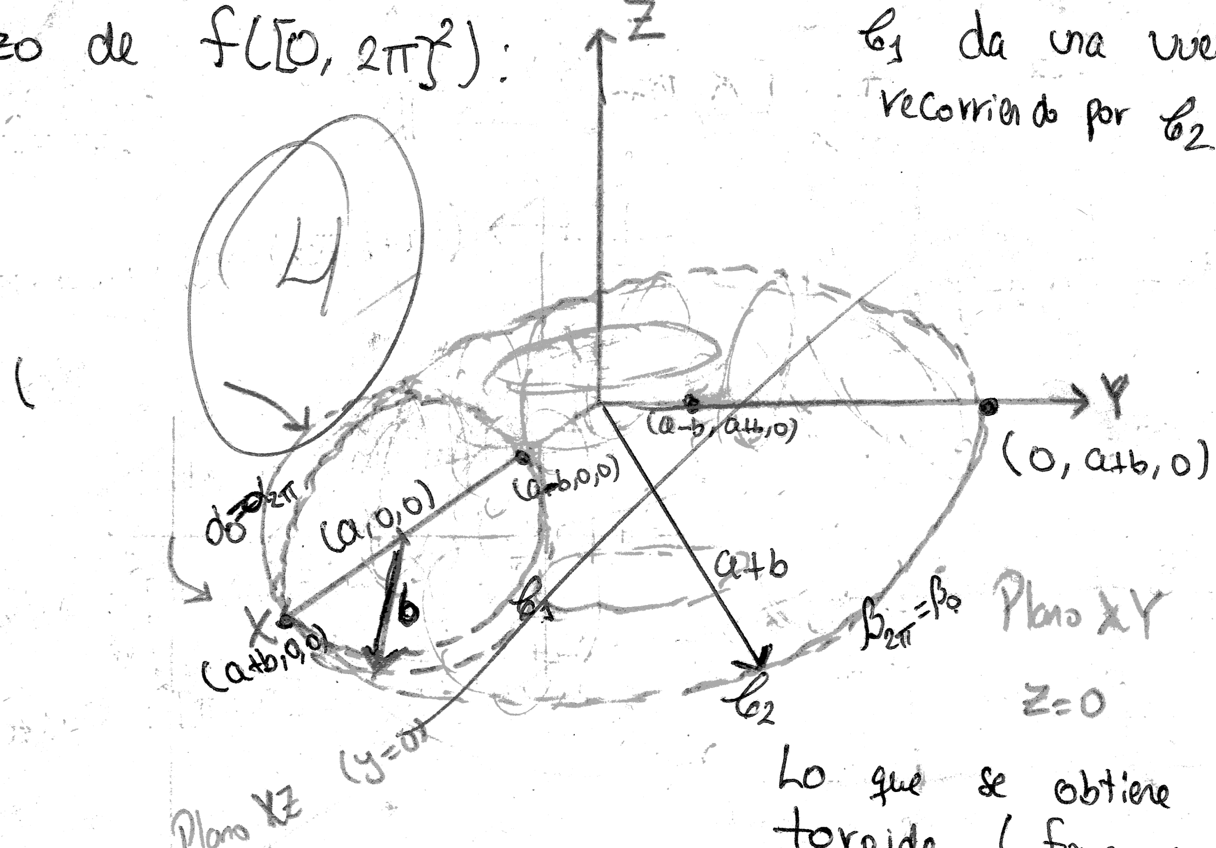
$f(x,y_0) = f(\alpha_{y_0}) = ((a+b\cos x)\cos y_0,\ (a+b\cos x)\operatorname{sen} y_0,\ b\operatorname{sen} x)$

$f(x,y_0) = f(\alpha_{y_0}) = (a\cos y_0,\ a\operatorname{sen} y_0,\ 0) + (b\cos y_0 \cdot \cos x,\ b\operatorname{sen} y_0 \cos x,\ b\operatorname{sen} x)$

Con $0 \le x \le 2\pi$.

Esbozo de $f([0, 2\pi]^2)$:

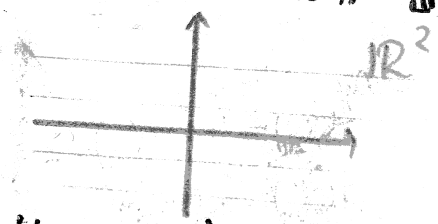
b_1 da una vuelta recorriendo por b_2



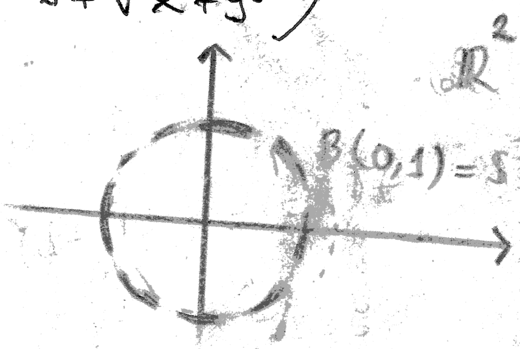
Lo que se obtiene es un toroide (forma de donut). ∎

3. (a) Sea

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$(x,y) \longmapsto \left(\frac{x}{1+\sqrt{x^2+y^2}}, \frac{y}{1+\sqrt{x^2+y^2}} \right)$$



y $B(0,1) = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 < 1\}$.



Viernes: 10:30 AM – 11:00 PM (1 vez x semana)
Curso de programación en C.