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Problem Set 7 – Gascoigne Workshop
Summer Term 2013

Navier-Stokes equations

In this problem set, we consider the Navier-Stokes equations

$$\begin{aligned} -\nu\Delta u + u\cdot\nabla u + \nabla p &= f \\ \nabla\cdot u &= 0 \end{aligned} \quad \text{in } \Omega.$$

The boundary conditions are already specified for the computational domain Ω given in the file `channel.inp`. This system describes the flow of an incompressible fluid. Because this problem is nonlinear, we first consider the linear Stokes equations

$$\begin{aligned} -\nu\Delta u + \nabla p &= f \\ \nabla\cdot u &= 0 \end{aligned} \quad \text{in } \Omega,$$

which is the appropriate physical model for viscous fluids like honey. The parameter $\nu \in \mathbb{R}^+$ describes the viscosity of the fluid.

Gascoigne uses bilinear or biquadratic finite elements for the calculations. Usually, we discretize the equations with the Q_1 elementy for both pressure p and for the velocity vector u. Without stabilization this can lead to instabilities since the discrete *“inf-sup”*-condition is not fulfilled.

Problem 7.1:

Copy the problem set 7 as usual. Use the last six problem sets and implement the Stokes-equations in the weak formulation (modify the class `MyEquation`)..

For the stabilization use at first the additional bilinear form

$$a_1^s(\boldsymbol{u},p)(\varphi,\psi) = \alpha h^2(\nabla p, \nabla\psi),$$

in the divergence equation. Try different values for α. Save the corresponding results.

Hint: In the file `ref_equation` the correct `Form` and `Matrix` of the Stokes system are already implemented. Do not cheat yourself by copying this lines! Use these files only as guideline if you have troubles with your own implementation.

Problem 7.2:

Now, we want to use the *"local projection stabilization"* (LPS). Detailed informations about the implementation can be found in Chapter *"Flow problems and stabilization"*.

In this case, we use a derivation of the class `LpsEquation`. Use the class `LocalEquation`, which is already implemented. Disable the naive stabilization from **Problem 7.1** in your file `localequation.cc` and implement the `StabForm` and `StabMatrix` for the LPS method with the help of the script. Make sure to change the `discname` in `param`-file from `Q1` to `Q1Lps` and the pointer for the equation in the file `local.h`.

Save the results of the computations in a folder with the name `LPS`. Now compare the results to the previous stabilization technique.

Problem 7.3:

Implement the Navier-Stokes equations. Therefore, add the convection term $(u \cdot \nabla u, \varphi)$ into `Form` and `Matrix` of your `localequation.cc`. The convective term leads to further instabilities (see *Problem Set 6*). Use the LPS method for the Navier-Stokes system. Change the computational domain by using the file `nsbench4.inp` in the `param`-file. Make sure to use the correct boundary colors. In the file `local.h` change the value `double high` describing the vertical height of the channel to a value of 4.1.