

**Exercise Nr. 3, Summer School on Finite Elements**  
**Universidad Nacional Agraria La Molina**  
**December, 2018**

1. **Study the program:** Study the matlab program provided to you. Some files have changed:

**assembledmatrix2d.m** We have implemented 3 different matrix stencils. `Slap` is the matrix of the Laplace operator  $-\Delta$ , `St_x` and `St_y` are the stencils of the transport operators  $\partial_x$  and  $\partial_y$ . `SUPG` is the stencil of the streamline diffusion method. In line 22, the stencil `S` is combined. This stencil `S` is then used to assemble the system matrix.

2. **Solve a transport problem** On  $\Omega = (0, 1)^2$  we solve the transport-diffusion problem

$$-\epsilon \Delta u + \beta \cdot \nabla u = 0 \text{ and } u(x, y) = \begin{cases} 1 & x + y < \frac{1}{2} \\ 0 & x + y \geq \frac{1}{2} \end{cases} \text{ on } \partial\Omega,$$

with the transport direction

$$\beta = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

Modify the right hand side in `righthandside.m` and the dirichlet data in `dirichlet.m`. In `assembledmatrix2d.m` set  $\epsilon$  and the transport direction  $\beta_x = 2$  and  $\beta_y = 1$ .

Solve the problem for  $\epsilon = 1$ ,  $\epsilon = 0.01$  and  $\epsilon = 0.0001$ . Also, change the parameter  $M$  controlling the mesh size  $h = 1/M$ . What do you observe?

3. **Artificial Diffusion:** You have seen, that the solution is not stable if  $h < \epsilon$ . Implement the *artificial diffusion method*

$$-\left(\epsilon + \frac{|\beta|}{2}h\right)\Delta u + \beta \cdot \nabla u = 0.$$

Look at the file `assembledmatrix2d.h` and change line 22, where the matrix stencil is assembled.

Solve the problem for  $\epsilon = 1$ ,  $\epsilon = 0.01$  and  $\epsilon = 0.0001$  on a coarse mesh with  $M = 20$ . What do you observe? How does the solution change?

Now, for  $\epsilon = 0.01$  solve the problem for  $M = 20, M = 40, \dots$ . What do you observe?

4. **Streamline Diffusion:** Now we solve the Laplace problem with streamline diffusion

$$-\epsilon \Delta u + \beta \cdot \nabla u + \frac{h}{|\beta|} \frac{\partial^2}{\partial^2 \beta} u = 0 \text{ in } \Omega.$$

In line 19 of `assemblematrix2d.h` we have defined the stencil of the streamline diffusion method.

In line 22 of this file, combine the correct stencil.

Solve the problem for  $\epsilon = 1$ ,  $\epsilon = 0.01$  and  $\epsilon = 0.0001$  and compare the result to the artificial diffusion method. What do you observe? Write a 2 page essay describing your findings and send it to Malte and Dandy.