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Exercise Nr. 1, ***Summer School on Finite Elements***
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1. **Study the program:** Study the matlab program provided to you. Some details on the different files:

   start.m This is the main program. It calls the necessary steps to set up the mesh, the matrix, and the right hand side. It solves the linear system and plots the solution

   createmesh.m This function creates a regular mesh of $N = (M + 1) * (M + 1)$ points of the domain $\Omega = (0, 1)^2$. The mesh is stored as a $N \times 2$ vector storing the coordinates of the nodes. By M we denode the number of cells in one spatial direction.

   assemblematrix2d.m This function assembles the finite element matrix. The matrix is given in stencil notation in the variable `S` describing the coupling of each node x_i to the surrounding nodes. Then, this stencil is added to the global matrix. Finally, we modify the matrix to include Dirichlet values by writing a 1 on the diagonals of the boundary nodes and setting the off-diagonals to 0

   assemblerhs2d.m This function integrates the right hand side vector $\mathbf{f}_i = (f, \phi_h^{(i)})$. As the function `f` is general, we use a numerical quadrature rule for integration

   righthandside.m Here we implement the function $f(x, y)$ that is integrated as right hand side

   plotsolution.m shows a plot of the solution vector as function over the mesh.

   To better understand the program you can

   - Print the mesh (for small values of M)
   - Print the matrix by `full(A)` (also for small values of M) or look at the *sparsity pattern* of the matrix by `spy(A)`.
   - Plot the right hand side `plotsolution('rhs',mesh,b)`

2. **Modify the solution:** The matlab problem solves the Laplace problem

$$-\Delta u = 1 \text{ in } \Omega = (0,1)^2 \text{ and } u = 0 \text{ on } \partial\Omega.$$

Change the right hand side in `righthandside.m` such that the solution to the Laplace problem is given by

$$u(x,y) = \sin(\pi x)\sin(\pi y).$$

The right hand side satisfies

$$f(x,y) = -\Delta u(x,y)$$

Compute the Laplacian and implement the result. Compare the plot of the solution with the result that you expect.

3. **Modify the equation:** We want to solve the *diffusion reaction problem*

$$-\epsilon\Delta u + u = f \text{ in } \Omega \text{ and } u = 0 \text{ on } \partial\Omega.$$

You have to modify the matrix.

- First, only add the constante ϵ. How does the solution to

$$-\epsilon\Delta u = 1$$

change if you choose $\epsilon = 1$, $\epsilon = 0.01$ and $\epsilon = 0.0001$.

- Compute the matrix stencil for the *reaction term*

$$M_{ij} = (\phi_h^{(j)}, \phi_h^{(i)})$$

and change the stencil `S` in the Matlab program. To keep it simple, you may use an approximate mass matrix $\widetilde{M}$ by numerical quadrature. Choosing trapezoidal rule, we get $\widetilde{M}_{ij} = 0$ for $i \neq j$, and $\widetilde{M}_{ii} = h^2$. This process is called *mass lumping*.