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Lima, Oct. 30, 2019

Spring School on the Introduction on Numerical Modelling of Differential Equations – Programming Exercise 6

Exercise 6.1 [Poisson's equation]

Let $\alpha \in \mathbb{R}$. We are given the Poisson problem in 1D on the interval $\Omega = (0, 1)$:

$$\begin{aligned} -\alpha u''(x) &= f \quad \text{in } \Omega \\ u(0) = u(1) &= 0 \end{aligned}$$

with $\alpha = 1$ and the right hand side $f = -a$ with $a > 0$. The code of this example can be found on the cloud in `fem1d_linear.cc`.

Note: Please note that the above form is only correct when α is constant. The general formulation is

$$-\frac{d}{dx}(\alpha u')$$

which reduces to the above one, when α is constant.

(a) Run the code and observe the results using gnuplot, with $a = 1$ and $h = 0.1$.
Hint: Please work in the optimized compiling mode

(b) We play now with two parameters:

(i) Vary the *discretization parameter* h and use other parameters. What do you observe?

(ii) Vary now the *model parameter* α. What do you observe?

(iii) Choose now a different *right hand side* f. What do you observe?

(c) Check your solution by observing whether the maximum principle is fulfilled or not.

(d) We study in this final task the structure of the code. Go into the code and try to understand the different functions and methods that are implemented therein. Please have a specific look into the `assemble_system` method.