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Spring School on the Introduction on Numerical Modelling of Differential Equations – Programming Exercise 5

Exercise 5.1 [Condition for explicit Euler]

Consider the linear, scalar model problem

$$u'(t) = \lambda u(t), \quad u(0) = 1, \quad \mathbb{R} \ni \lambda < 0.$$

Derive the explicit Euler scheme. What is the condition on Δt such that the explicit Euler scheme produces bounded approximations for all $t > 0$? Confirm your result with the implementation in file `eemodelproblem.cc` provided in the exercise yesterday.

Exercise 5.2 [Object oriented ODE solver]

Download the file `linearoscillator.cc` available on the cloud. It solves the problem

$$u'(t) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} u(t) \ \text{ in } (0, 20\pi], \qquad u(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

using the methods

#	Scheme	#	Scheme
0	Explicit Euler	4	Implicit Euler
1	Heun 2nd order	5	Implicit midpoint
2	Heun 3rd order	6	Alexander
3	Runge-Kutta 4th order	7	Crouzieux
		8	Gauß 6th order

and provides errors $e(T)$ and convergence rates for all schemes. What conclusions can you draw from the tables?

Exercise 5.3 [Van der Pol oscillator]

In this exercise we explore the nonlinear Van der Pol oscillator

$$\begin{aligned} u_0'(t) &= -u_1(t) & u_0(0) &= 1 \\ u_1'(t) &= 1000 \cdot (u_0(t) - u_1^3(t)) & u_1(0) &= 2 \end{aligned}$$

which is an example for a stiff ODE system.
Download an updated version of the file `linearoscillator.cc` from the cloud. Compile and run the following four combinations of methods and timesteps:

- `RKF45` method is an adaptive embedded Runge Kutta method of 5th order. Run it with tolerances $TOL_1 = 0.2$ and $TOL_2 = 0.001$ using an initial time step $\Delta t = 1/16$.
- The implicit Euler method. Run it with $\Delta t_1 = 1/16$ and $\Delta t_2 = 1/512$.

The output file contains in each line

$$t_i \quad u_0(t_i) \quad u_1(t_i) \quad \Delta t_i$$

Compare the solutions, especially $u_1(t)$ as well as the time step sizes Δt_i for all four runs. What do you observe?