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**Problem Set 5 – Gascoigne Workshop**
**Summer Term 2013**

## Systems of Partial Differential Equations

In this problem set, we consider systems of PDE's.

To solve a system of PDE's, we have to modify several classes of the usual GASCOIGNE code. The classes `Equation` and `RightHandSide` have the method `GetNcomp()`, which returns the number of components. The equations for the different components are to be generated in the class `Equation`, e.g.,

```
b[0] += s * ( U[0].x() * N.x() + U[0].y() * N.y() );
b[1] += s * ( U[1].x() * N.x() + U[1].y() * N.y() );
...
```

The classes `RightHandSide` and `DirichletData` must be changed accordingly. The boundary data in GASCOIGNE can be specified, e.g., by

```
b[2] = 5. * v.x();
```

Here, we do not multiply with a test function as we do in the class `RightHandSide`,

```
b[2] += v.y() * (v.y()-1.) * N.m();
```

In the parameter file `run.param`, we must set the Dirichlet data. Therefore, we consider the following example for an equation with three components:

```
dirichlet     2  2 4
dirichletcomp 2  1 0
dirichletcomp 4  2 1 2
```

The first line denotes the number of the different boundary colors for the Dirichlet boundary conditions (here 2) and the color values defined in the `inp`-file (here 2 and 4). The next two lines of the above code describe the two different boundaries. The second line means: the boundary with color 2 is active for 1 solution component, the component number 0. Other solution components on the part of the boundary marked by color 2 have Neumann boundary conditions. The third line of the code fragment specifies the boundary color 4.

For more detailed information compare Chapter *"Systems of partial differential equations"*.

**Problem 5.1:**

Solve the Poisson problem with two components $u := (u_1, u_2) : \Omega \to \mathbb{R}^2$:

$$-\Delta u = f, \quad f = \begin{pmatrix} 1 \\ 10 \end{pmatrix},$$

where $\Delta u := (\Delta u_1, \Delta u_2)$ with the boundary data

$$\begin{aligned} &u_1 = 0 \text{ and } \partial_n u_2 = 0 \text{ on } \Gamma_{\text{left}} \cup \Gamma_{\text{right}} \\ &u_2 = 0 \text{ and } \partial_n u_1 = 0 \text{ on } \Gamma_{\text{top}} \cup \Gamma_{\text{bottom}} \end{aligned}$$

**Problem 5.2:**

Solve the system of PDE's:

$$\begin{aligned} -\Delta u_1 + u_1 u_2 &= 8\sin(7\pi x)\cos(5\pi y), \\ -\frac{1}{100}\Delta u_2 + \frac{3}{2}\partial_x u_1 \partial_x u_2 + \partial_y u_1 \partial_y u_2 &= 5, \end{aligned}$$

with the boundary data

$$\begin{aligned} \partial_n u_1 &= 0 \text{ on } \Gamma, \\ u_2 &= 0 \text{ on } \Gamma_{\text{left}} \cup \Gamma_{\text{right}}, \\ \partial_n u_2 &= 0 \text{ on } \Gamma_{\text{top}} \cup \Gamma_{\text{bottom}}. \end{aligned}$$