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20.12.2018

Exercise Nr. 3, *Summer School on Finite Elements*
Universidad Nacional Agraria La Molina
December, 2018

1. **Study the program:** Study the matlab program provided to you. Some files have changed:

   `assemblematrix2d.m` We have implemented 3 different matrix stencils. `Slap` is the matrix of the Laplace operator $-\Delta$, `St_x` and `St_y` are the stencils of the transport operators ∂_x and ∂_y. `SUPG` is the stencil of the streamline diffusion method. In line 22, the stencil `S` is combined. This stencil `S` is then used to assemble the system matrix.

2. **Solve a transport problem** On $\Omega = (0,1)^2$ we solve the transport-diffusion problem

$$-\epsilon\Delta u + \beta\cdot\nabla u = 0 \text{ and } u(x,y) = \begin{cases} 1 & x+y < \frac{1}{2} \\ 0 & x+y \geqslant \frac{1}{2} \end{cases} \text{ on } \partial\Omega,$$

   with the transport direction

$$\beta = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

   Modify the right hand side in `righthandside.m` and the dirichlet data in `dirichlet.m`. In `assemblematrix2d.m` set ϵ and the transport direction $\beta_x = 2$ and $\beta_y = 1$.

   Solve the problem for $\epsilon = 1$, $\epsilon = 0.01$ and $\epsilon = 0.0001$. Also, change the paramter M controlling the mesh size $h = 1/M$. What do you observe?

3. **Aritificial Diffusion:** You have seen, that the solution is not stable if $h < \epsilon$. Implement the *artificial diffusion method*

$$-\left(\epsilon + \frac{|\beta|}{2}h\right)\Delta u + \beta\cdot\nabla u = 0.$$

   Look at the file `assemblematrix2d.h` and change line 22, where the matrix stencil is assembled.

   Solve the problem for $\epsilon = 1$, $\epsilon = 0.01$ and $\epsilon = 0.0001$ on a coarse mesh with $M = 20$. What do you observe? How does the solution change?

   Now, for $\epsilon = 0.01$ solve the problem for $M = 20$, $M = 40, \ldots$. What do you observe?

4. **Streamline Diffusion:** Now we solve the Laplace problem with streamline diffusion

$$-\epsilon\Delta u + \beta \cdot \nabla u + \frac{h}{|\beta|}\frac{\partial^2}{\partial^2 \beta} u = 0 \text{ in } \Omega.$$

In line 19 of `assemblematrix2d.h` we have defined the stencil of the streamline diffusion method.

In line 22 of this file, combine the correct stencil.

Solve the problem for $\epsilon = 1$, $\epsilon = 0.01$ and $\epsilon = 0.0001$ and compare the result to the artificial diffusion method. What do you observe? Write a 2 page essay describing your findings and send it to Malte and Dandy.