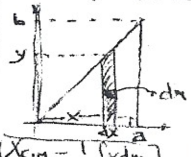
• Halla el C.M de la lámina triangular de masa M uniformemente distribuida.

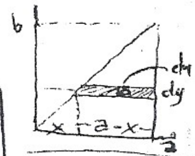


Pero

$\boxed{\frac{y}{x}=\frac{b}{a}}$

$\boxed{y=\frac{bx}{a}}$

$\boxed{dy=\frac{b}{a}dx}$



$\boxed{X_{c.m}=\frac{1}{M}\int x\,dm}$ $\boxed{y_{c.m}=\frac{1}{M}\int y\,dm}$

• $dA=y\,dx$ $dA=(a-x)\,dy$

• $\sigma=\frac{dm}{dA}=\frac{M}{\frac{ab}{2}}$ $\sigma=\frac{dm}{dA}=\frac{M}{\frac{ab}{2}}$

$\rightarrow dm=\frac{2M}{ab}y\,dx$ $\Rightarrow dm=\frac{2M}{ab}(a-x)\,dy$

$X_{c.m}=\frac{1}{\cancel{M}}\int x\frac{2\cancel{M}}{ab}y\,dx$ $y_{c.m}=\frac{1}{\cancel{M}}\int y\frac{2\cancel{M}}{ab}(a-x)\,dy$

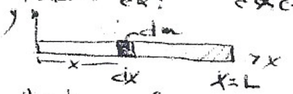
$X_{c.m}=\int_0^a x\frac{2}{ab}\frac{bx}{a}dx$ $y_{c.m}=\int_0^a \frac{bx}{a}\frac{2(a-x)}{ab}\frac{b}{a}dx$

$X_{c.m}=\frac{2}{a^2}\int_0^a x^2dx$ $y_{c.m}=\int_0^a\frac{2b}{a^3}(a-x)x\,dx$

$X_{c.m}=\frac{2}{a^2}\left[\frac{x^3}{3}\right]_0^a$ $y_{c.m}=\frac{2b}{a^3}\left[\frac{ax^2}{2}-\frac{x^3}{3}\right]_0^a$

$\boxed{X_{c.m}=\frac{2}{3}a}$ $\boxed{y_{c.m}=\frac{b}{3}}$

• La barra de longitud L y masa M tiene una densidad lineal de $\lambda=\alpha x^2$. Hallar: ¿α? ¿$X_{c.m}$?



$\lambda=\frac{dm}{dx}=\alpha x^2$ $dm=\frac{3M}{L^3}x^2dx$

$\int dm=\int_0^L\alpha x^2dx$ $X_{c.m}=\frac{1}{M}\int_0^L x\,dm$

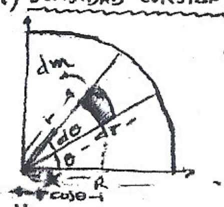
$M=\alpha\left[\frac{x^3}{3}\right]_0^L$ $X_{c.m}=\frac{1}{\cancel{M}}\int_0^L x\frac{3\cancel{M}}{L^3}x^2dx$

$\Rightarrow\boxed{\alpha=\frac{3M}{L^3}}$ $X_{c.m}=\frac{3}{L^3}\left[\frac{x^4}{4}\right]_0^L$

$\boxed{X_{c.m}=\frac{3}{4}L}$

• Halla el C.M de una lámina en forma $\frac{1}{4}$ circunferencia de masa M, radio R;

i) Densidad constante



• $\sigma=\frac{dm}{dA}=\frac{M}{\frac{\pi R^2}{4}}$

$dA=dr(r\,d\theta)$

$dm=\frac{4M}{\pi R^2}r\,d\theta\,dr$

$X_{c.m}=\frac{1}{M}\int x\,dm$

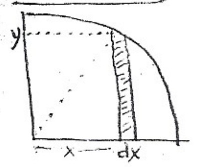
$=\frac{1}{\cancel{M}}\iint r\cos\theta\frac{4\cancel{M}}{\pi R^2}r\,d\theta\,dr$

$=\frac{4}{\pi R^2}\int_0^R r^2dr\int_0^{\frac{\pi}{2}}\cos\theta\,d\theta$

$=\frac{4}{\pi R^2}\left[\frac{r^3}{3}\right]_0^R\left[\operatorname{sen}\theta\right]_0^{\frac{\pi}{2}}$

$\boxed{X_{c.m}=\frac{4R}{3\pi}}$

Otro método



$dA=dx\,y$

$\sigma=\frac{dm}{dA}=\frac{M}{\frac{\pi R^2}{4}}$

$dm=\frac{4M}{\pi R^2}y\,dx$

Pero: $R^2=x^2+y^2$, $y=\sqrt{R^2-x^2}$

$X_{c.m}=\frac{1}{M}\int x\,dm$

$X_{c.m}=\frac{1}{\cancel{M}}\int x\frac{4\cancel{M}}{\pi R^2}y\,dx$

$X_{c.m}=\frac{4}{\pi R^2}\int_0^R x\sqrt{R^2-x^2}\,dx$

$u^2=R^2-x^2$
$2u\,du=-2x\,dx$
$u\,du=-x\,dx$
$x=R\Rightarrow u=0$
$x=0\Rightarrow u=R$

$X_{c.m}=\frac{4}{\pi R^2}\int_R^0 -u\,u\,du$

$X_{c.m}=\frac{4}{\pi R^2}\int_0^R u^2du$

$X_{c.m}=\frac{4}{\pi R^2}\left[\frac{u^3}{3}\right]_0^R$ $\boxed{X_{c.m}=\frac{4R}{3\pi}}$

ii) $\sigma=\alpha r\cos^2\theta$

$\sigma=\frac{dm}{dA}$ | $dA=r\,dr\,d\theta$

$dm=\alpha r\cos^2\theta\,dA$

$\int dm=\iint\alpha r\cos^2\theta\,r\,dr\,d\theta$

$M=\alpha\int_0^R r^2dr\int_0^{\pi/2}\cos^2\theta\,d\theta$

$M=\alpha\left[\frac{r^3}{3}\right]_0^R\int_0^{\pi/2}\frac{(\cos2\theta+1)}{2}d\theta$

$M=\alpha\frac{R^3}{3}\left[\frac{\operatorname{sen}2\theta}{2}+\theta\right]_0^{\pi/2}$

$M=\frac{\alpha R^3}{6}\left(\frac{\pi}{2}\right)$

$\Rightarrow\alpha=\frac{12M}{\pi R^3}$

$dm=\frac{12M}{\pi R^3}r\cos^2\theta\,r\,dr\,d\theta$

$X_{c.m}=\frac{1}{M}\int x\,dm$ $x=r\cos\theta$

$X_{c.m}=\frac{1}{\cancel{M}}\iint r\cos\theta\frac{12\cancel{M}}{\pi R^3}r\cos^2\theta\,r\,dr\,d\theta$

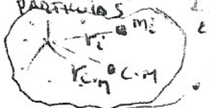
$X_{c.m}=\frac{12}{\pi R^3}\int_0^R r^3dr\int_0^{\pi/2}\cos^3\theta\,d\theta$

$X_{c.m}=\frac{12}{\pi R^3}\left[\frac{R^4}{4}\right]\int(\cos\theta-\operatorname{sen}^2\theta\cos\theta)\,d\theta$

$X_{c.m}=\frac{3R}{\pi}\left[\operatorname{sen}\theta-\frac{\operatorname{sen}^3\theta}{3}\right]_0^{\pi/2}$

$\boxed{X_{c.m}=\frac{2R}{\pi}}$

• Movimiento del C.M para un sist. de partículas



• $\bar r_{c.m}=\frac{\sum m_i\bar r_i}{M}$

• $\bar V_{c.m}=\frac{d\bar r_{c.m}}{dt}=\frac{1}{M}\sum m_i\bar v_i$

• $\bar a_{c.m}=\frac{d\bar V_{c.m}}{dt}=\frac{1}{M}\sum m_i\bar a_i$

$V_{c.m}=\frac{1}{M}\sum\bar P_i$

$V_{c.m}=\frac{\bar P_{sist}}{M}$

$\boxed{\bar P_{sist}=M\bar V_{c.m}}$

$\frac{d\bar P_s}{dt}=M\frac{d\bar V_{c.m}}{dt}=M\bar a_{c.m}=\bar F_R$

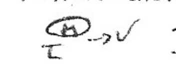
$\boxed{\frac{d\bar P_s}{dt}=\bar F_R}$

$\Rightarrow\frac{d\bar P_s}{dt}=\bar F_R$ Si $\bar F_R=0$
$\rightarrow\bar P_s=cte=M\bar V_{c.m}=cte$

El C.M se mueve con $V=cte$.

Sistema de masa variable

Para un cohete: $t+\Delta t$



$\bar P_i=M\bar v$

$\bar P_f=(M+\Delta m)(\bar v+\Delta\bar v)-\bar V_g(\Delta m)\approx 0$

$\bar P_f=M\bar v+M\Delta\bar v+\Delta m\bar v+\Delta m\Delta\bar v-\Delta m\bar V_g$

$\Delta\bar P=M\Delta\bar v+\Delta m\bar v-\Delta m\bar V_g$

$\Delta\bar P=M\Delta\bar v+(\bar V_g-\bar v)\Delta m$

$\lim_{\Delta t\to0}\frac{\Delta P}{\Delta t}=\frac{M\Delta v}{\Delta t}-\frac{V_r\Delta m}{\Delta t}$

$\boxed{\bar F_{ext}=M\frac{d\bar v}{dt}-\bar V_r\frac{dm}{dt}}$ $\frac{dm}{dt}$ = Rapidez con la cual se quema el comb.

$M\frac{d\bar v}{dt}=\bar F_{ext}+\bar F_{empuje}$

$\bar F_{empuje}=\bar V_r\frac{dm}{dt}$

$\bar V_r\equiv$ Velocidad relativa de expulsión de gases.

Rapidez con la cual debe quemarse el combustible para empezar a subir.

$\bar F_{ext}=-mg\hat j$

$\bar F_{ext}+\bar F_{emp}=\bar 0$

$-V_r\frac{dm}{dt}\hat j=mg\hat j$

$\frac{dm}{dt}=-\frac{mg}{V_r}$

$\int_{m_0}^{M}\frac{dm}{m}=\int_0^t-\frac{g}{V_r}dt$

$\ln\left(\frac{M(t)}{M_0}\right)=-\frac{g(t)}{V_r}$

$M(t)=M_0\,e^{\left(-\frac{gt}{V_r}\right)}$

• Si la masa empieza a subir con $\bar a=a\hat j$ desde el reposo.

$M\frac{d\bar v}{dt}=-Mg\hat j-\bar V_r\hat j\frac{dm}{dt}$

$M(a+g)=-V_r\frac{dm}{dt}$

$-\frac{(a+g)}{V_r}\int dt=\int\frac{dm}{M}$

$M(t)=M_0\,e^{-\frac{(a+g)}{V_r}t}$

$M_0=m_{nave}+m_{combustible}$.

$\bar r_{cm}=\frac{1}{M}\int\bar r\,dm$

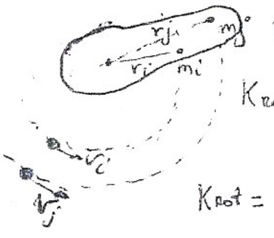
$\lambda=\frac{dm}{dl}$ $r_{cm}=\frac{1}{M}\int\bar r\,\lambda\,dl$

$\sigma=\frac{dm}{dA}$ $r_{cm}=\frac{1}{M}\iint\sigma\,dA$

$\rho=\frac{dm}{dV}$ $r_{cm}=\frac{1}{M}\iiint\rho\,dV$

$\boxed{\lambda=\frac{dm}{dl}=\frac{M}{L}}$

Rotación de cuerpos rígidos



$K_i = \frac{1}{2} m_i v_i^2$

$K_{rot} = \sum K_i = \sum \frac{1}{2} m_i v_i^2$

$K_{rot} = \frac{1}{2}\underbrace{\left(\sum m_i r_i^2\right)}_{I}\omega^2$

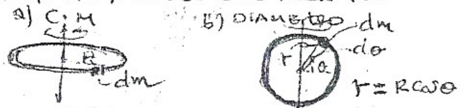
$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$ $\boxed{I = \sum m_i r_i^2}$

Cálculo del momento de inercia.

$\boxed{I = \sum m_i r_i^2 = \int r^2 dm}$

• Momento de inercia de un aro uniforme de masa M y radio R. Respecto a un eje que pasa por

a) C.M   b) Diámetro



$I = \int R^2 dm$

$\boxed{I = MR^2}$

$r = R\cos\theta$

$\lambda = \frac{M}{2\pi R} = \frac{dm}{R\,d\theta}$

$dm = \frac{M\,d\theta}{2\pi}$

$I = \int (R\cos\theta)^2 \frac{M}{2\pi} d\theta$

$I = \frac{MR^2}{2\pi}\int_0^{2\pi} \cos^2\theta\, d\theta$

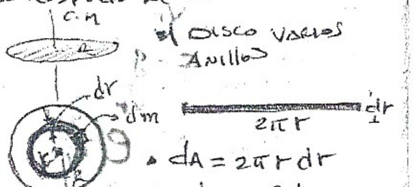
$I = \frac{MR^2}{2\pi}\int_0^{2\pi} \frac{(\cos 2\theta + 1)}{2}$

$I = \frac{MR^2}{4\pi}\left[\frac{\text{sen}\,2\theta}{2} + \theta\right]_0^{2\pi}$

$\boxed{I = \frac{MR^2}{2}}$

Momento de inercia de un disco respecto al eje que pasa por

a) C.M   b) Disco varios anillos



• $dA = 2\pi r\,dr$

• $dI = r^2 dm$

$\sigma = \frac{M}{\pi R^2} = \frac{dm}{2\pi r\,dr}$

$dm = \frac{2M r\,dr}{R^2}$

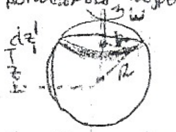
$I = \int r^2 dm$

$I = \int r^2 \frac{2M r\,dr}{R^2}$

$I = \frac{2M}{R^2}\int_0^R r^3 dr = \frac{2M}{R^2}\left[\frac{R^4}{4}\right]$

$I = \frac{MR^2}{2}$

• Momento de inercia de una esfera maciza homogénea respecto a su diámetro.



• $dI = \frac{1}{2} r^2 dm$

• $dV = \pi r^2 dz$

$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{dm}{\pi r^2 dz}$ • $dm = \left(\frac{3M}{4R^3}\right) r^2 dz$

$I = \frac{1}{2}\int r^2 dm = \frac{3M}{8R^3}\int r^4 dz$

Pero: $r^2 + z^2 = R^2$

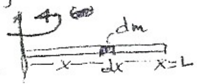
$r^2 = R^2 - z^2$

$I = \frac{3M}{8R^3}\int_{-R}^{R} (R^2 - z^2)^2 dz \Rightarrow \boxed{I = \frac{2}{5} MR^2}$

Momento de inercia de una barra respecto a uno de sus extremos. M, L

Si:

i) $\lambda = cte$



$\lambda = \frac{dm}{dx} = \frac{M}{L}$

$dm = \frac{M}{L} dx$

$I = \int x^2 dm$

$I = \frac{M}{L}\int_0^L x^2 dx$

$\boxed{I = \frac{ML^2}{3}}$

ii) $\lambda = cx^2$

$\lambda = \frac{dm}{dx} = cx^2$

$\int_0^M dm = c\int_0^L x^2 dx$

$M = c\left[\frac{x^3}{3}\right]_0^L$

$\boxed{C = \frac{3M}{L^3}}$

$\lambda = \left(\frac{3M}{L^3}\right)x^2$

• $I = \int x^2 dm$

• $\frac{dm}{dx} = \left(\frac{3M}{L^3}\right)x^2$

$dm = \left(\frac{3M}{L^3}\right)x^2 dx$

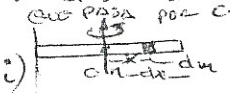
$I = \int_0^L x^2\left(\frac{3M}{L^3}\right)x^2 dx$

$I = \frac{3M}{L^3}\left[\frac{x^5}{5}\right]_0^L$

$\boxed{I = \frac{3}{5} ML^2}$ $\boxed{C.M = \frac{3}{4} L}$

• ¿Cuál será $I_{C.M}$ alrededor del eje que pasa por C.M.? M, L

i)



$I_{C.M} = \int x^2 dm$

$I_{C.M} = \int_{-L/2}^{L/2} x^2 \left(\frac{M}{L}\right) dx$

$I_{C.M} = \left(\frac{M}{L}\right)\int_{-L/2}^{L/2} x^2 dx$

$\lambda = \frac{dm}{dx} = \frac{M}{L}$

$dm = \left(\frac{M}{L}\right) dx$

$\boxed{I_{C.M} = \frac{ML^2}{12}}$

ii)

$I = \int r^2 dm$ } $r = x - \frac{3L}{4}$

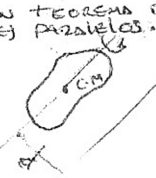
$I = \left(\frac{3M}{L^3}\right)\int_0^L \left(x - \frac{3L}{4}\right)^2 x^2 dx$

$I = \frac{3M}{L^3}\int_0^L \left(x^4 + \frac{9}{16}L^2 x^2 - \frac{3}{2}x^3 L\right) dx$

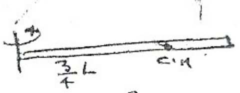
$I = \frac{3}{80} ML^2$

Otro método:

Con teorema de Steiner o de ejes paralelos.



$\boxed{I = I_{C.M} + Md^2}$



$I = \frac{3}{5} ML^2$

$I_{C.M} = I - Md^2$

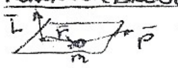
$I_{C.M} = \frac{3}{5} ML^2 - M\left(\frac{3}{4}L\right)^2$

$\boxed{I_{C.M} = \frac{3}{80} ML^2}$

Torque:

$\boxed{\bar{\tau}_{neto} = \bar{\tau}_{ext}}$ $\bar{L} = \bar{r} \times \bar{p}$

Momentum angular



$\frac{d\bar{L}}{dt} = \frac{d\bar{r}}{dt} \times \bar{p} + \bar{r} \times \frac{d\bar{p}}{dt}$

$\bar{v} \times \bar{p} = 0$

$\bar{L} = \bar{r} \times \bar{p}$

$\bar{L} = (r m v)\hat{k}$

$\bar{L} = (r m \omega r)\hat{k}$

$\bar{L} = (mr^2)\omega\hat{k}$

$\boxed{\bar{L} = I\bar{\omega}}$

$\frac{d\bar{L}}{dt} = \bar{r} \times \bar{F} = \bar{\tau}$

si $\bar{\tau} = 0 \Rightarrow \bar{L} = cte$

$\boxed{\sum \tau = I\alpha}$

$\bar{\tau} = r F_t$

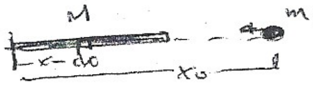
$\tau = r m a_T$

$\tau = r m \alpha r$

$\tau = (mr^2)\alpha$

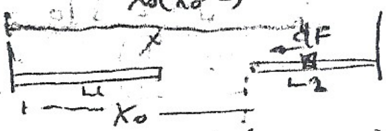
$\boxed{\bar{\tau} = I\alpha}$

i)



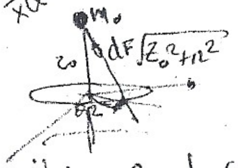
$dF = \frac{-G m_0 dm\,\hat{i}}{(x_0 - x)^2}$

$\bar{F} = \frac{G M m_0}{x_0(x_0 - L)}\hat{i}$



$F = -\frac{G M_1 M_2}{L_1 L_2} \ln\left(\frac{(x_0 + L_2 - L_1)(x_0)}{(x_0 + L_2)(x_0 - L_1)}\right)\hat{i}$

$dF = \frac{G M_1 dm}{x(x - L_1)}$



$F = -\frac{G m_0 M z_0 \hat{k}}{(z_0^2 + R^2)^{3/2}}$

i) $dF = \frac{-G m_0 dm(-r\hat{i} - y\hat{j} + z_0\hat{k})}{(z_0^2 + R^2)^{3/2}}$

MOMENTUM ANGULAR DE UNA PARTICULA.

$$\bar{L} = \bar{r} \times \bar{P}$$

$L_{máx} \Leftrightarrow r \perp P$

$L_{mín=0} \Leftrightarrow \bar{r} \parallel \bar{P}$

$$\frac{d\bar{L}}{dt} = \frac{d\bar{r}}{dt} \times \bar{P} + \bar{r} \times \frac{d\bar{P}}{dt}$$

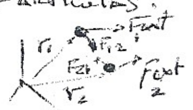
$$= \underbrace{\bar{v} \times \bar{P}}_{0} + \bar{r} \times \bar{F}$$

$$\frac{d\bar{L}}{dt} = \bar{r} \times \bar{F} = \bar{\tau}$$

si $\bar{\tau} = 0 \Rightarrow \boxed{\bar{L} = cte}$

PRINCIPIO DE CONSERVACIÓN DEL MOMENTUM ANGULAR DE UNA PARTICULA.

• SUPONGAMOS QUE SE TIENEN DOS PARTICULAS.



$$\boxed{\bar{L} = L_1 + L_2}$$

$$\left.\begin{aligned} \frac{d\bar{L}_1}{dt} &= \bar{\tau}_{ext_1} + \bar{\tau}_{12} \\ \frac{d\bar{L}_2}{dt} &= \bar{\tau}_{ext_2} + \bar{\tau}_{21} \end{aligned}\right\} (+)$$

$$\frac{d(\bar{L}_1 + \bar{L}_2)}{dt} = \bar{\tau}_{ext_1} + \bar{\tau}_{ext_2}$$

$$\frac{d\bar{L}}{dt} = \bar{\tau}_{ext}$$

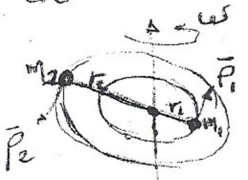
$\bar{\tau}_{ext} = \bar{0} \Rightarrow \bar{L} = \bar{L}_1 + \bar{L}_2 = cte.$

PRINCIPIO DE CONSERVACIÓN DEL MOMENTUM ANGULAR.

• EN GENERAL PARA UN SISTEMA DE PARTICULAS.

$$\bar{L} = \sum_{i=1}^{n} L_i \qquad \bar{\tau}_{ext} = \sum \bar{\tau}_i^{ext}$$

$$\frac{d\bar{L}}{dt} = \bar{\tau}_{ext} \Rightarrow \bar{\tau}_{ext} = \bar{0} \Rightarrow \bar{L} = \sum_{i=1}^{n} \bar{L}_i = cte.$$



$$\bar{L} = \bar{L}_1 + \bar{L}_2 = \bar{r}_1 \times \bar{P}_1 + \bar{r}_2 \times \bar{P}_2$$

$$\bar{L} = \bar{r}_1 \times \bar{P}_1 + \bar{r}_2 \times \bar{P}_2$$

$$\bar{L} = (r_1 P_1 + r_2 P_2) k$$

$$\bar{L} = (r_1 m_1 v_1 + r_2 m_2 v_2) k$$

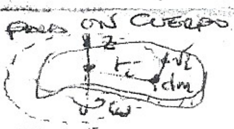
$$\bar{L} = (r_1 m_1 r_1 \omega + r_2 m_2 r_2 \omega) k$$

$$\bar{L} = (m_1 r_1^2 \omega + m_2 r_2^2 \omega) k$$

$$\bar{L} = (I_1 + I_2) \omega k$$

$$\boxed{\bar{L} = I\bar{\omega}} \Rightarrow \boxed{\bar{L} \parallel \bar{\omega}}$$

PARA UN CUERPO PLANO

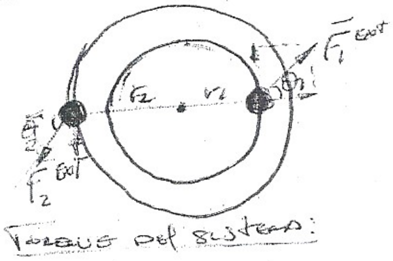


$$\bar{L} = \sum \bar{r}_i \times \bar{P}_i = \left(\sum r_i P_i\right)\hat{k}$$

$$\bar{L} = \left(\sum r_i m_i \omega r_i\right)\hat{k}$$

$$\bar{L} = \left(\sum m_i r_i^2\right)\omega\hat{k}$$

$$\boxed{\bar{L} = I\bar{\omega}} \Rightarrow \boxed{\bar{L} \parallel \bar{\omega}}$$



TORQUE DEL SISTEMA:

$$\bar{\tau}_{ext} = \bar{\tau}_1^{ext} + \bar{\tau}_2^{ext} = \bar{r}_1 \times \bar{F}_1^{ext} + \bar{r}_2 \times \bar{F}_2^{ext}$$

$$\bar{\tau}_{ext} = (r_1 F_1 \operatorname{sen}\theta_1 + r_2 F_2 \operatorname{sen}\theta_2)\hat{k}$$

OBS:

$$F_j^{ext} = \bar{F}_{Tj} = m_j \alpha r_j$$

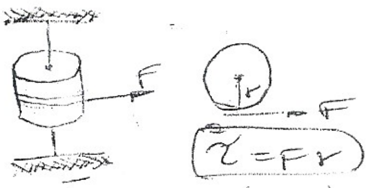
$$\Rightarrow \bar{\tau}_{ext} = (r_1 m_1 r_1 \alpha + r_2 m_2 r_2 \alpha)\hat{k}$$

$$\bar{\tau}_{ext} = (\underbrace{m_1 r_1^2 + m_2 r_2^2}_{I})\alpha\hat{k}$$

$$\boxed{\bar{\tau}_{ext} = I\bar{\alpha}}$$

$$\boxed{\bar{\tau}_{ext} = I\bar{\alpha}}$$

$$\boxed{\bar{\tau} \parallel \bar{\alpha}}$$



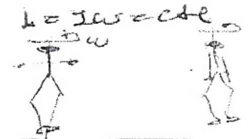
$$\boxed{\tau = Fr}$$

$$\frac{d\bar{L}}{dt} = \bar{\tau} = \frac{d(I\omega)}{dt}$$

$$\bar{\tau} = I\frac{d\omega}{dt} = I\bar{\alpha}$$

Si $\bar{\tau} = \bar{0} \rightarrow L = cte$

$L = I\omega = cte$



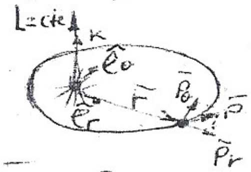
$$\boxed{I\omega = I\omega}$$

$\tau_{neto} = 0 \quad L = cte = I\omega$

SISTEMA SOL-TIERRA

$\bar{\tau}_T = \bar{r} \times \bar{F} = \bar{0} \Rightarrow \bar{L}_{Tierra} = cte$

$$\bar{L}_{Tierra} = \bar{r} \times \bar{P} = ct$$



$$\bar{P} = \bar{P}_\theta + \bar{P}_r$$

$$\left.\begin{aligned} \bar{P} &= m\bar{v}_\theta + m\bar{v}_r \\ \bar{P} &= m r\dot{\theta}\hat{e}_\theta + m\dot{r}\hat{e}_r \end{aligned}\right\} \bar{L} = \bar{r} \times \bar{P}$$

$$\bar{L} = \bar{r} \times \bar{P}$$

$$= (r\hat{e}_r) \times (m r\dot{\theta}\hat{e}_\theta + m\dot{r}\hat{e}_r)$$

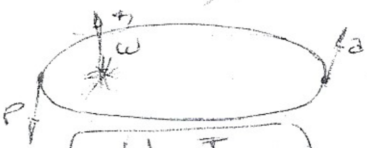
$$\bar{L} = m r^2 \dot{\theta}\hat{k} = cte$$

$$ds = \frac{1}{2} r^2 d\theta$$

$$\frac{ds}{dt} = \frac{1}{2} r^2 \dot{\theta} \cdot \frac{m}{m} = \frac{L}{2m} = cte$$

$$\boxed{\frac{ds}{dt} = \frac{L}{2m}}$$

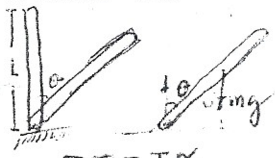
2da ley DE KEPLER.



$$\boxed{I_p \omega_p = I_a \omega_a}$$

PROBLEMAS

Hallo la α y ω de la barra en la posición horizontal (PARTE DEL REPOSO).



$$\sum \tau = I\alpha$$

$$\bar{\tau} = -mg\frac{L}{2}\operatorname{sen}\theta\,\hat{k}$$

$$-mg\frac{L}{2}\operatorname{sen}\theta\,\hat{k} = I\bar{\alpha}$$

$$-mg\frac{L}{2}\operatorname{sen}\theta\,\hat{k} = -\frac{mL^2}{3}\alpha\,\hat{k}$$

$$\boxed{\alpha = \frac{3}{2}\frac{g}{L}\operatorname{sen}\theta}$$

$\alpha = \frac{3}{2}\frac{g}{L} \quad a = \frac{3}{4}g \quad \frac{3g}{2} = a_x$

EN CADA PTO DE LA BARRA LA ACELERACIÓN ES DISTINTA.

$$\frac{d\omega}{dt} = \frac{3}{2}\frac{g}{L}\operatorname{sen}\theta$$

$$\frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{3}{2}\frac{g}{L}\operatorname{sen}\theta$$

$$\int_0^\omega \omega\, d\omega = \frac{3}{2}\frac{g}{L}\int_0^\theta \operatorname{sen}\theta\, d\theta$$

$$\frac{\omega^2}{2} = \frac{3}{2}\frac{g}{L}\left[-(\cos\theta - 1)\right]$$

$$\omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

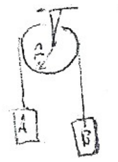
V del extremo.

$V = \omega L = \sqrt{3\frac{g}{L}(1-\cos\theta)}\sqrt{L^2}$

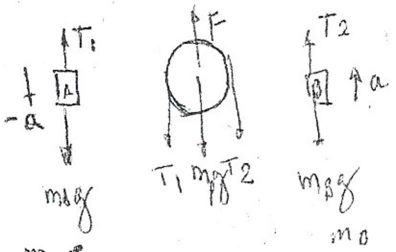
$V_A = \sqrt{3 g L(1-\cos\theta)}$ si $\theta = \pi/2$

$V_A = \sqrt{3gL} = 5{,}4\sqrt{L}$

Hallar la aceleración de los bloques. Y α de la polea.



$m_A = 10\,kg$
$m_B = 5\,kg$
$m_P = 2\,kg$
$R = 1/4$



m_A:

$T_1 - m_A g = 10(-a)$; $T_2 - m_B g = 5(a)$

$10g - T_1 = 10a$ (1) ; $T_2 - 5g = 5a$ (2)

$\Sigma \tau = I\alpha$

$T_1 R - T_2 R = \left(\frac{1}{2} m_P R^2\right)\alpha$

$T_1 - T_2 = \left(\frac{1}{2} m_P R\right)\alpha$

$T_1 - T_2 = a$ (3)

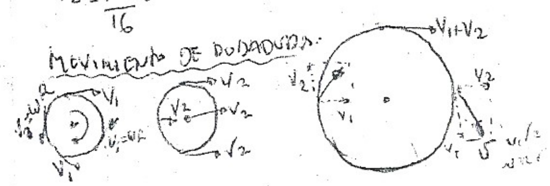
(1) + (2) + (3):

$5g = 16a \Rightarrow a = \frac{5}{16}g$

$T_1 = 10(g-a) = \frac{110}{16}g$

$T_2 = \frac{105}{16}g$

Movimiento de rodadura.



$V_{cm} = 0$
Rotación pura + Traslación pura $\Rightarrow$ R + T

Si $V_1 = V_2 = \omega R$

$\Rightarrow$

P: eje instantáneo de rotación

Si $V_{c.m} = \omega R \Rightarrow$ Rueda sin deslizarse

Energía cinética rotacional respecto al eje instantáneo de rotación.

$K = \frac{1}{2} I_P \omega^2$ ..... (α)

Del teorema de los ejes paralelos.

$I_P = I_{c.m} + MR^2$ ...(β)

(β) en (α)

$K = \frac{1}{2}(I_{c.m} + MR^2)\omega^2$

$K = \frac{1}{2} I_{c.m}\omega^2 + \frac{1}{2} M R^2 \omega^2$

$\boxed{K = \frac{1}{2} I_{c.m} \omega^2 + \frac{1}{2} M V_{cm}^2}$