TECHNISCHE UNIVERSITÄT DRESDEN

ALGORITHMEN UND DATENSTRUKTUREN

ÜBUNG 4: EXTENDED BACKUS-NAUR-FORM

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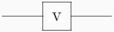
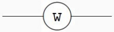
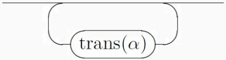
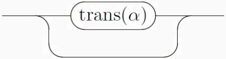
ÜBERSETZUNG SYNTAXDIAGRAMME $\leftrightarrow$ EBNF

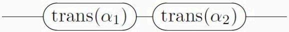
Die Funktion trans: $T(\Sigma, V) \to \mathrm{SynDia}(\Sigma, V)$ ist induktiv über den Aufbau von EBNF-Termen definiert:

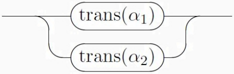
1. Sei $v \in V$; $\mathrm{trans}(v) =$ 

2. Sei $\mathtt{w} \in \Sigma$; $\mathrm{trans}(\mathtt{w}) =$ 

3. Sei $\alpha \in T(\Sigma, V)$;
   - $\mathrm{trans}(\hat{\{}\alpha\hat{\}}) =$ 

   - $\mathrm{trans}(\hat{[}\alpha\hat{]}) =$ 

   - $\mathrm{trans}(\hat{(}\alpha\hat{)}) = \mathrm{trans}(\alpha)$
4. Sei $\alpha_1, \alpha_2 \in T(\Sigma, V)$;
   - $\mathrm{trans}(\alpha_1\alpha_2) =$ 

   - $\mathrm{trans}(\hat{(}\alpha_1\hat{|}\alpha_2\hat{)}) =$ 

SEMANTIK VON EBNF-TERMEN

- Sei $\mathcal{E} = (V, \Sigma, S, R)$ eine EBNF-Definition.
- $v \in V \rightsquigarrow W(\mathcal{E}, v) = \rho(v)$ (syntaktische Kategorie)
- Semantik $[\![\cdot]\!] : \underbrace{T(\Sigma, V)}_{\alpha} \to ((\underbrace{V \to \mathcal{P}(\Sigma^*)}_{\rho}) \to \mathcal{P}(\Sigma^*))$

Definiere $[\![\alpha]\!](\rho)$ wie folgt:

- Wenn $\alpha = v \in V$, dann gilt $[\![\alpha]\!](\rho) = \rho(v)$.
- Wenn $\alpha \in \Sigma$, dann gilt $[\![\alpha]\!](\rho) = \{\alpha\}$.
- Wenn $\alpha = \alpha_1\alpha_2$, dann gilt $[\![\alpha]\!](\rho) = [\![\alpha_1]\!](\rho) \cdot [\![\alpha_2]\!](\rho)$.
- Wenn $\alpha = \hat{(}\alpha_1\hat{|}\alpha_2\hat{)}$, dann gilt $[\![\alpha]\!](\rho) = [\![\alpha_1]\!](\rho) \cup [\![\alpha_2]\!](\rho)$.
- Wenn $\alpha = \hat{\{}\alpha_1\hat{\}}$, dann gilt $[\![\alpha]\!](\rho) = ([\![\alpha_1]\!](\rho))^*$.
- Wenn $\alpha = \hat{[}\alpha_1\hat{]}$, dann gilt $[\![\alpha]\!](\rho) = [\![\alpha_1]\!](\rho) \cup \{\varepsilon\}$.
- Wenn $\alpha = \hat{(}\alpha_1\hat{)}$, dann gilt $[\![\alpha]\!](\rho) = [\![\alpha_1]\!](\rho)$.

Übungsblatt 3

AUFGABE 4 — TEIL (A)

$$W(\mathcal{E}, S) = \left\{ a^{n+m} w b^{\ell} a^{n} \mid n, m \geq 0, 0 \leq \ell \leq m, w \in \Sigma^{*} \right\}$$

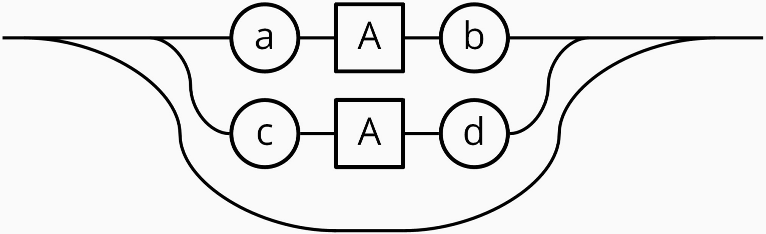
$$V = \{S, A, B\} \quad \text{und} \quad R = \left\{ \begin{aligned} S &::= \hat{(}\ aSa\ \hat{|}\ A\ \hat{)}, \\ A &::= \hat{(}\ aA\hat{[}\ b\ \hat{]}\ \hat{|}\ B\ \hat{)}, \\ B &::= \hat{\{}\ \hat{(}\ a\ \hat{|}\ b\ \hat{)}\ \hat{\}} \end{aligned} \right\}$$

AUFGABE 4 — TEIL (B)

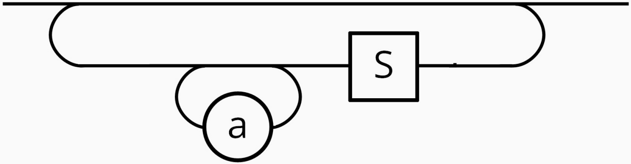
$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(A) \end{pmatrix} = \begin{pmatrix} \llbracket \hat{[}\ aAb\ \hat{]} \rrbracket\,(\rho) \\ \llbracket \hat{(}\ Sc\ \hat{|}\ cS\ \hat{)} \rrbracket\,(\rho) \end{pmatrix} = \begin{pmatrix} \{a\} \cdot \rho(A) \cdot \{b\} \\ \rho(S) \cdot \{c\} \cup \{c\} \cdot \rho(S) \end{pmatrix}$$

$$\begin{aligned} \begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{\varepsilon\} \\ \emptyset \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{\varepsilon\} \\ \{c\} \end{pmatrix} &\overset{f}{\mapsto} \begin{pmatrix} \{\varepsilon, acb\} \\ \{c\} \end{pmatrix} \\ &\overset{f}{\mapsto} \begin{pmatrix} \{\varepsilon, acb\} \\ \{c, acbc, cacb\} \end{pmatrix} \\ &\overset{f}{\mapsto} \begin{pmatrix} \{\varepsilon, acb, aacbcb, acacbb\} \\ \{c, acbc, cacb\} \end{pmatrix} \end{aligned}$$

S



A



Übungsblatt 4

AUFGABE 1 — TEIL (A)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(B) \end{pmatrix} = \begin{pmatrix} \left\llbracket \hat{[}\, B\, \hat{]}\, b \right\rrbracket (\rho) \\ \llbracket Sb \rrbracket (\rho) \end{pmatrix} = \begin{pmatrix} (\rho(B) \cup \{\varepsilon\}) \cdot \{b\} \\ \rho(S) \cdot \{b\} \end{pmatrix}$$

$$= \begin{pmatrix} \rho(B) \cdot \{b\} \cup \{b\} \\ \rho(S) \cdot \{b\} \end{pmatrix}$$

Fünf Iterationen:

$$\begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{b\} \\ \emptyset \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{b\} \\ \{b^2\} \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{b, b^3\} \\ \{b^2\} \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{b, b^3\} \\ \{b^2, b^4\} \end{pmatrix}$$

$$\overset{f}{\mapsto} \begin{pmatrix} \{b, b^3, b^5\} \\ \{b^2, b^4\} \end{pmatrix}$$

AUFGABE 1 — TEIL (B)

$$\ldots \overset{f}{\mapsto} \begin{pmatrix} \{b, b^3, b^5\} \\ \{b^2, b^4\} \end{pmatrix}$$

$$\implies W(\mathcal{E}, S) = \left\{ b^{2n+1} \mid n \geq 0 \right\}$$

$$W(\mathcal{E}, B) = \left\{ b^{2n+2} \mid n \geq 0 \right\} = \left\{ b^{2n} \mid n \geq 1 \right\}$$

AUFGABE 1 — TEIL (C)

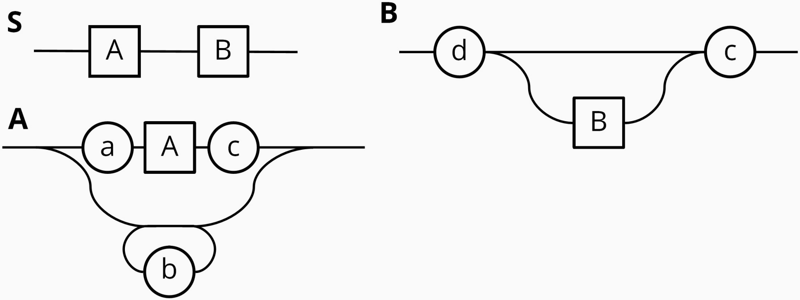
Vermutung:

$$W(\mathcal{E}, S) = \left\{ b^{2n+1} \mid n \geq 0 \right\} \quad \text{und} \quad W(\mathcal{E}, B) = \left\{ b^{2n} \mid n \geq 1 \right\}$$

$$\begin{aligned}
[\![S]\!]\,(\rho) &= \left[\!\!\left[\hat{[}\, B \,\hat{]}\, b \right]\!\!\right](\rho) \\
&= ([\![B]\!]\,(\rho) \cup \{\varepsilon\}) \cdot [\![b]\!]\,(\rho) \\
&= (\rho(B) \cup \{\varepsilon\}) \cdot \{b\} \\
&= \rho(B) \cdot \{b\} \cup \{b\} \\
&= \left\{ b^{2n} \mid n \geq 1 \right\} \cdot \{b\} \cup \{b\} \\
&= \left\{ b^{2n+1} \mid n \geq 1 \right\} \cup \left\{ b^{2 \cdot 0 + 1} \right\} \\
&= \left\{ b^{2n+1} \mid n \geq 0 \right\} \\
&= W(\mathcal{E}, S)
\end{aligned}$$

$$\begin{aligned}
[\![B]\!]\,(\rho) &= [\![S\,b]\!]\,(\rho) \\
&= [\![S]\!]\,(\rho) \cdot [\![b]\!]\,(\rho) \\
&= \rho(S) \cdot \{b\} \\
&= \left\{ b^{2n+1} \mid n \geq 0 \right\} \cdot \{b\} \\
&= \left\{ b^{2n+2} \mid n \geq 0 \right\} \\
&= \left\{ b^{2n} \mid n \geq 1 \right\} \\
&= W(\mathcal{E}, B)
\end{aligned}$$

AUFGABE 2 — TEIL (A)



AUFGABE 2 — TEIL (B)

$$f(\rho) = \begin{pmatrix} f(\rho)(S) \\ f(\rho)(A) \\ f(\rho)(B) \end{pmatrix} = \begin{pmatrix} [\![AB]\!]\,(\rho) \\ \left[\!\!\left[\hat{(}\ aAc\ \hat{|}\ \hat{\{}\ b\ \hat{\}}\ \hat{)}\right]\!\!\right](\rho) \\ \left[\!\!\left[d\ \hat{[}\ B\ \hat{]}\ c\right]\!\!\right](\rho) \end{pmatrix}$$

$$= \begin{pmatrix} \rho(A)\cdot\rho(B) \\ \{a\}\cdot\rho(A)\cdot\{c\}\cup\{b\}^* \\ \{d\}\cdot(\rho(B)\cup\{\varepsilon\})\cdot\{c\} \end{pmatrix}$$

$$= \begin{pmatrix} \rho(A)\cdot\rho(B) \\ \{a\}\cdot\rho(A)\cdot\{c\}\cup\{b\}^* \\ \{d\}\cdot\rho(B)\cdot\{c\}\cup\{dc\} \end{pmatrix}$$

$$\begin{pmatrix} \emptyset \\ \emptyset \\ \emptyset \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \emptyset \\ \{b\}^* \\ \{dc\} \end{pmatrix} \overset{f}{\mapsto} \begin{pmatrix} \{b^m dc \mid m\in\mathbb{N}\} \\ \{a^\ell b^m c^\ell \mid \ell\in\{0,1\}, m\in\mathbb{N}\} \\ \{d^n c^n \mid n\in\{1,2\}\} \end{pmatrix}$$

AUFGABE 2 — TEIL (C)

Wir wollen eine EBNF-Definition $\mathcal{E}' = (V, \Sigma, S, R)$ finden, sodass

$$W(\mathcal{E}') = \left\{ a^{n+\ell} c b^n (cd)^\ell \mid n, \ell \in \mathbb{N}, n \geq 1 \right\}$$

gilt. Wir zerlegen wie üblich die Sprache in unabhängige Teile:

$$L = \left\{ a^\ell \;\; a^n \; c \; b^n \;\; (cd)^\ell \mid n, \ell \in \mathbb{N}, n \geq 1 \right\}$$

Dann ergibt sich also nach dem Grundschema

$$\begin{aligned} V &= \{S, A\} \\ R &= \left\{ S ::= \hat{(}\; aScd \;\hat{|}\; A \;\hat{)}, A ::= \hat{(}\; aAb \;\hat{|}\; acb \;\hat{)} \right\} \end{aligned}$$