

# ALGORITHMEN UND DATENSTRUKTUREN

## ÜBUNG 13: DAS PROZESSPROBLEM

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# Algebraisches Pfadproblem

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# ABSTRAKTION: ALGEBRAISCHES PFADPROBLEM

- ▶ **bisher:** kürzeste Wege
  - ▷ Summation + entlang der Pfade
  - ▷ Minimum min über alle Pfade
- ▶ **jetzt:** Verallgemeinerung
  - ▷ Pfadoperation  $\odot$  entlang der Pfade
  - ▷ Akkumulationsoperation  $\oplus$
- ▶ **Ergebnis:** allgemeine algebraische Struktur — Semiring  
( $S, \oplus, \odot, 0, 1$ )

	Werte $S$	$\oplus$	$\odot$	<b>0</b>	<b>1</b>
kürzeste Wegeproblem	$\mathbb{R}_{\geq 0}^{\infty}$	min	+	$\infty$	0
Kapazitätsproblem	$\mathbb{N}_{\infty}$	max	min	0	$\infty$
Erreichbarkeitsproblem	$\{\text{true}, \text{false}\}$	$\vee$	$\wedge$	false	true
Zuverlässigkeitsproblem	$[0, 1]$	max	$\cdot$	0	1
Prozessproblem	$\mathcal{P}(\Sigma^*)$	$\cup$	$\circ$	$\emptyset$	$\{\varepsilon\}$

# FLOYD-WARSHALL → AHO-HOPCRAFT-ULLMANN

- ▶ modifizierte Adjazenzmatrix

$$mA_G = \begin{cases} A_G(u, v) & \text{wenn } u \neq v \\ A_G(u, v) \oplus \mathbf{1} & \text{wenn } u = v \end{cases}$$

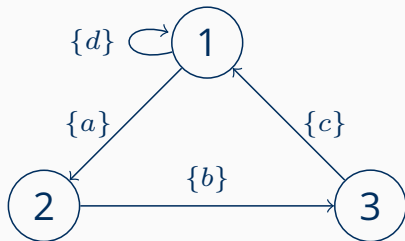
- ▶ **Initialisierung:**  $D_G^{(0)} = mA_G$
- ▶ **Rekursion:**

$$\begin{aligned} & D_G^{(k+1)}(u, v) \\ &= D_G^{(k)}(u, v) \oplus \left( D_G^{(k)}(u, k+1) \odot (D_G^{(k)}(k+1, k+1))^* \odot D_G^{(k)}(k+1, v) \right) \end{aligned}$$

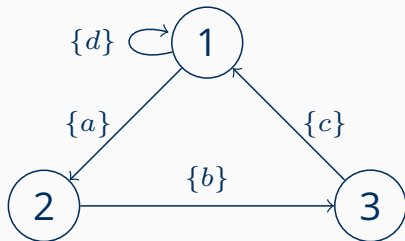
- ▶ vgl. dazu Floyd-Warshall:

$$\begin{aligned} & D_G^{(k+1)}(u, v) \\ &= \min \left\{ D_G^{(k)}(u, v), D_G^{(k)}(u, k+1) + D_G^{(k)}(k+1, v) \right\} \end{aligned}$$

## AUFGABE 12.3



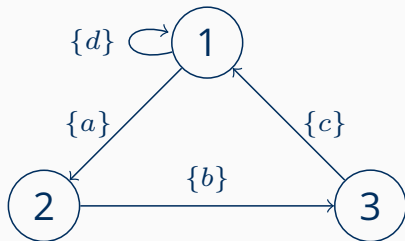
## AUFGABE 12.3



**Teil (a) :**

$$(S, \oplus, \odot, \mathbf{0}, \mathbf{1}) =$$
$$(\mathcal{P}(\Sigma^*), \cup, \circ, \emptyset, \{\varepsilon\})$$

## AUFGABE 12.3



**Teil (a) :**

$$(S, \oplus, \odot, \mathbf{0}, \mathbf{1}) = \\ (\mathcal{P}(\Sigma^*), \cup, \circ, \emptyset, \{\varepsilon\})$$

Update-Formel:  $D_G^{(k+1)}(u, v)$

$$\begin{aligned} &= D_G^{(k)}(u, v) \oplus \left( D_G^{(k)}(u, k+1) \odot (D_G^{(k)}(k+1, k+1))^* \odot D_G^{(k)}(k+1, v) \right) \\ &= D_G^{(k)}(u, v) \cup \left( D_G^{(k)}(u, k+1) \circ (D_G^{(k)}(k+1, k+1))^* \circ D_G^{(k)}(k+1, v) \right) \\ &= \text{alt} \quad \cup \left( \text{Zeile} \quad \circ \quad (\text{Diagonale})^* \quad \circ \quad \text{Spalte} \right) \end{aligned}$$

## AUFGABE 12.3

**Teil (b) :**  $mA_G = D_G^{(0)} = \begin{pmatrix} \{\varepsilon, d\} & \{a\} & \emptyset \\ \emptyset & \{\varepsilon\} & \{b\} \\ \{c\} & \emptyset & \{\varepsilon\} \end{pmatrix}$



## AUFGABE 12.3

**Teil (b) :**  $mA_G = D_G^{(0)} = \begin{pmatrix} \{\varepsilon, d\} & \{a\} & \emptyset \\ \emptyset & \{\varepsilon\} & \{b\} \\ \{c\} & \emptyset & \{\varepsilon\} \end{pmatrix}$

**Teil (c) :**  $D_G^{(1)} = \begin{pmatrix} \{d\}^* & \{d\}^* \{a\} & \emptyset \\ \emptyset & \{\varepsilon\} & \{b\} \\ \{c\} \{d\}^* & \{c\} \{d\}^* \{a\} & \{\varepsilon\} \end{pmatrix}$

**Teil (d) :**

$$\begin{aligned}D_G^{(2)}(3,3) &= D_G^{(1)}(3,3) \cup \left( D_G^{(1)}(3,2) \circ (D_G^{(1)}(2,2))^* \circ D_G^{(1)}(2,3) \right) \\&= \{\varepsilon\} \cup \left( \{c\} \{d\}^* \{a\} \circ \{\varepsilon\}^* \circ \{b\} \right) \\&= \{\varepsilon\} \cup \left( \{c\} \{d\}^* \{ab\} \right)\end{aligned}$$

## AUFGABE 12.3

Teil (d) :

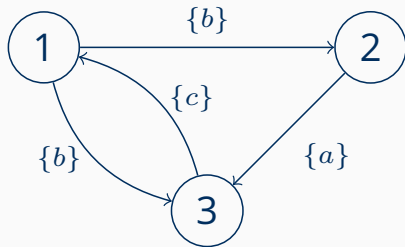
$$\begin{aligned}D_G^{(2)}(3,3) &= D_G^{(1)}(3,3) \cup \left( D_G^{(1)}(3,2) \circ (D_G^{(1)}(2,2))^* \circ D_G^{(1)}(2,3) \right) \\&= \{\varepsilon\} \cup \left( \{c\} \{d\}^* \{a\} \circ \{\varepsilon\}^* \circ \{b\} \right) \\&= \{\varepsilon\} \cup \left( \{c\} \{d\}^* \{ab\} \right)\end{aligned}$$

$$\begin{aligned}D_G^{(3)}(3,3) &= D_G^{(2)}(3,3) \cup \left( D_G^{(2)}(3,3) \circ (D_G^{(2)}(3,3))^* \circ D_G^{(2)}(3,3) \right) \\&= D_G^{(2)}(3,3) \cup \left( D_G^{(2)}(3,3) \right)^* \\&= \left( D_G^{(2)}(3,3) \right)^* \\&= \left( \{\varepsilon\} \cup \{c\} \{d\}^* \{ab\} \right)^* \\&= \left( \{c\} \{d\}^* \{ab\} \right)^*\end{aligned}$$

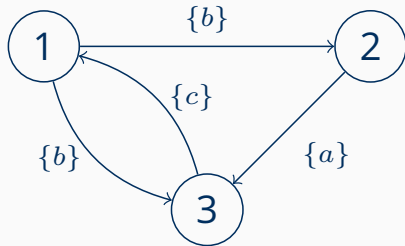
# Übungsblatt 13

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# AUFGABE 1



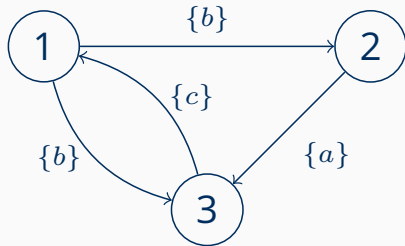
# AUFGABE 1



**Teil (a) :**

$$mA_G = \begin{pmatrix} \{\varepsilon\} & \{b\} & \{b\} \\ \emptyset & \{\varepsilon\} & \{a\} \\ \{c\} & \emptyset & \{\varepsilon\} \end{pmatrix}$$

# AUFGABE 1



**Teil (a) :**

$$mA_G = \begin{pmatrix} \{\varepsilon\} & \{b\} & \{b\} \\ \emptyset & \{\varepsilon\} & \{a\} \\ \{c\} & \emptyset & \{\varepsilon\} \end{pmatrix}$$

**Teil (b):**

$$D_G^{(1)} = \begin{pmatrix} \{\varepsilon\} & \{b\} & \{b\} \\ \emptyset & \{\varepsilon\} & \{a\} \\ \{c\} & \{cb\} & \{cb, \varepsilon\} \end{pmatrix} \quad D_G^{(2)} = \begin{pmatrix} \{\varepsilon\} & \{b\} & \{b, ba\} \\ \emptyset & \{\varepsilon\} & \{a\} \\ \{c\} & \{cb\} & \{cb, cba, \varepsilon\} \end{pmatrix}$$

# AUFGABE 1

Teil (c):

$$\begin{aligned} D_G(3,1) &= D_G^{(3)}(3,1) \\ &= D_G^{(2)}(3,1) \cup \left\{ D_G^{(2)}(3,3) \cdot (D_G^{(2)}(3,3))^* \cdot D_G^{(2)}(3,1) \right\} \\ &= \{c\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{c\} \right\} \\ &= \{cb, cba\}^* \cdot \{c\} \end{aligned}$$



# AUFGABE 1

## Teil (c):

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$$\begin{aligned} D_G(3,2) &= D_G^{(2)}(3,2) \cup \left\{ D_G^{(2)}(3,3) \cdot (D_G^{(2)}(3,3))^* \cdot D_G^{(2)}(3,2) \right\} \\ &= \{cb\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{cb\} \right\} \\ &= \{cb, cba\}^* \cdot \{cb\} \end{aligned}$$

# AUFGABE 1

## Teil (c):

$$\begin{aligned} D_G(3, 1) &= D_G^{(3)}(3, 1) \\ &= D_G^{(2)}(3, 1) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 1) \right\} \\ &= \{c\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{c\} \right\} \\ &= \{cb, cba\}^* \cdot \{c\} \end{aligned}$$

$$\begin{aligned} D_G(3, 2) &= D_G^{(2)}(3, 2) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 2) \right\} \\ &= \{cb\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{cb\} \right\} \\ &= \{cb, cba\}^* \cdot \{cb\} \end{aligned}$$

$$\begin{aligned} D_G(3, 3) &= D_G^{(2)}(3, 3) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 3) \right\} \\ &= \{cb, cba, \varepsilon\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{cb, cba, \varepsilon\} \right\} \\ &= \{cb, cba\}^* \end{aligned}$$

# AUFGABE 1

**Teil (c):**

$$\begin{aligned} D_G(3, 1) &= D_G^{(3)}(3, 1) \\ &= D_G^{(2)}(3, 1) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 1) \right\} \\ &= \{c\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{c\} \right\} \\ &= \{cb, cba\}^* \cdot \{c\} \end{aligned}$$

$$\begin{aligned} D_G(3, 2) &= D_G^{(2)}(3, 2) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 2) \right\} \\ &= \{cb\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{cb\} \right\} \\ &= \{cb, cba\}^* \cdot \{cb\} \end{aligned}$$

$$\begin{aligned} D_G(3, 3) &= D_G^{(2)}(3, 3) \cup \left\{ D_G^{(2)}(3, 3) \cdot (D_G^{(2)}(3, 3))^* \cdot D_G^{(2)}(3, 3) \right\} \\ &= \{cb, cba, \varepsilon\} \cup \left\{ \{cb, cba, \varepsilon\} \cdot \{cb, cba, \varepsilon\}^* \cdot \{cb, cba, \varepsilon\} \right\} \\ &= \{cb, cba\}^* \end{aligned}$$

**Teil (d):**  $D_G(3, 3) = \{cb, cba, ba\}^* \rightsquigarrow (3, 3, \{cb, cba, ba\}^*)$