# Handout
# APPM 4720/5720 Fall 2018
# Advanced Convex Optimization

**Date**: Wed, Sep 5 2018 **Lecturer**: Stephen Becker

**Instructions** Solve the following exercises using CVX (MATLAB) or similar. Use the following matrices:

$$A = \begin{bmatrix} 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 & 8 & 2 \\ 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 \\ 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 \\ 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 \\ 5 & 10 & 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 \end{bmatrix}$$

$$y = [1, 2, 3, 4, 5]^T$$

## Exercises

1. Solve $\min_x \|x\|_2$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.294216.
2. Solve $\min_x \|x\|_2^2$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.0865633.
3. Solve $\min_x \|x\|_1$ subject to $\|Ax - y\|_2 \leq 0.1$. The minimum should be 0.787669.
4. Re-solve the equation in Exer. 1 and request the dual variable $\lambda$, and use this value so that your solution of $\min_x \|x\|_2 + \lambda\|Ax - y\|_2$ coincides with that of Exer. 1.
5. Matrices in objective: Solve $\min_{x\in\mathbb{R}^5} g(A - x\mathbb{1}^T)$ where $g(A) = \sum_{j=1}^n \sqrt{\sum_{i=1}^m A_{ij}^2}$ is the sum of column $\ell_2$ norms. The minimum should be 63.9551.
6. Matrices in objective: Solve $\min_{x\in\mathbb{R}^5} h(A - x\mathbb{1}^T)$ where $h(A) = \|A\|$ is the spectral norm of $A$ (largest singular value). The minimum should be 14.3922.
7. Matrix variables: Solve $\min_{X\in\mathbb{R}^{5\times 10}} \|X - A\|_F$ subject to $\mathbb{1}_5^T X \mathbb{1}_{10} = 1$. The minimum should be 40.1637.
8. Matrix variables: Let $B$ be the first 5 columns of $A$. Solve $min_{X\in\mathbb{R}^{5\times 5}} \|X - B\|_F$ subject to $X \succeq 0$ (i.e., $X$ is positive semi-definite). The minimum should be 14.436.

**Remarks** With CVX and Matlab, note that if $X$ is a matrix, then `norm(X,1)` and `norm(X(:),1)` are different (same for the 2 norm), as the former gives you the operator norm. The CVX command `norms(X,1)` is also different — try all three versions with the $A$ matrix given above.