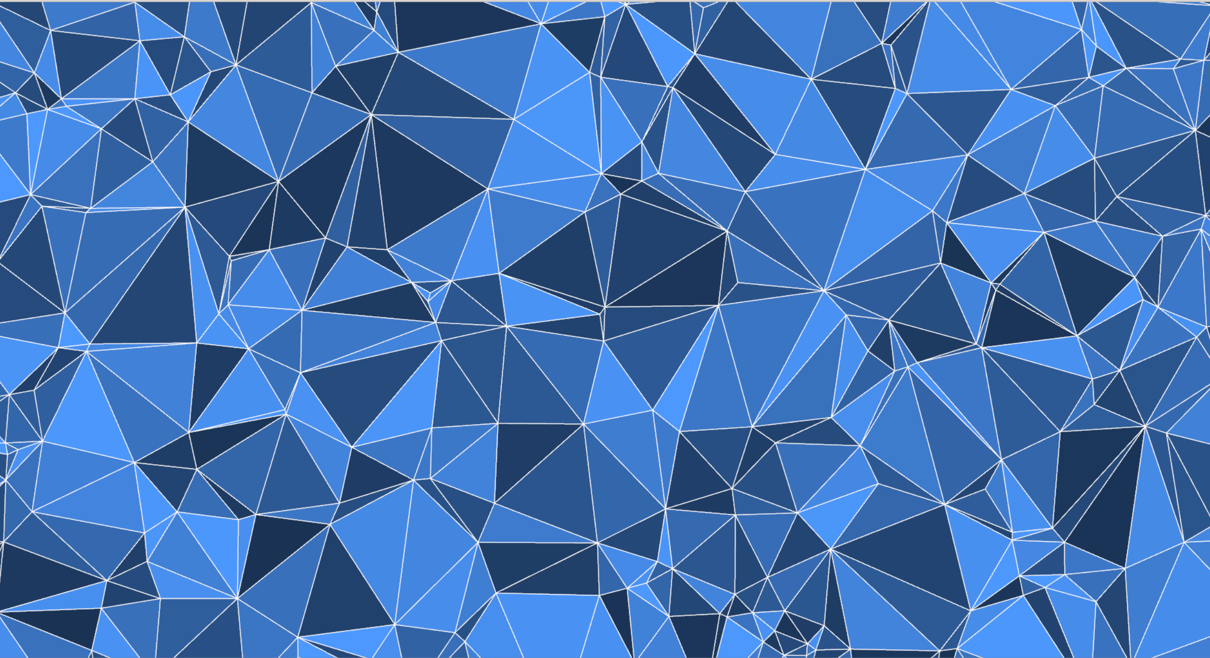
Delaunay Triangulation Mosaic



Algorithmical Geometry: Delaunay Triangulation

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December 28, 2021

Outline

Introduction

Introduction: Previous Work and Hands-On Approach

(1) Shewchuk, “Triangle: Engineering a 2D Quality Mesh Generator and Delaunay Triangulator”, 1996

(2) Guibas and Stolfi, “Primitives for the Manipulation of General Subdivisions and the Computation of Voronoi Diagrams”, 1985

(3) Dwyer, “A Faster Divide-and-Conquer Algorithm for Constructing Delaunay Triangulations”, 1987

Introduction: Overview

Educational Problems:

- Duality to Voronoi Diagrams, Dirichlet
- Incremental, Sweepline, Divide-and-Conquer Algorithms
- Varying Data Structures

Here: Triangular Data Structure and Divide-and-Conquer Algorithm

- Smallest Data Structure
- Fastest Algorithm
- Robust when using tweaks

Background

Background: Triangle

Definition: (Triangle)

We say that three points $A, B, C \in \mathbb{R}^2$ are building a triangle if they are affine independent.

Definition: (Circumcircle)

If $A, B, C \in \mathbb{R}^2$ are building a triangle, we define the circumcircle of the built triangle to be the circle that intersects with A, B, and C. We call its center the circumcenter of the triangle.

Background: Triangulation

Definition: (Triangulation)

- $n \in \mathbb{N}, n \geq 3$
- $\forall i \in \mathbb{N}, i \leq n : \quad x_i \in \mathbb{R}^2$ affine independent
- $\mathcal{V} := \{x_i \mid i \in \mathbb{N}, i \leq n\}$
- $\mathcal{T}(\mathcal{V})$ is a planar graph such that all faces are triangles when vertices are drawn at their given positions.
- $\mathcal{DT}(\mathcal{V})$ is a Delaunay triangulation if it is a triangulation such that for all triangle faces the interior of the circumcircle contains no other points of $\mathcal{V}$.

Background: Properties of Delaunay Triangulation

- Duality to Voronoi Diagram
- always exists
- If no points are cocircular, unique
- optimality:
- boundary is convex hull
- Delaunay condition implies triangulation

Background: Existence and Uniqueness of Delaunay Triangulation

Geometric Primitives

Geometric Primitives: Counter-Clockwise

$$\begin{vmatrix} A_x & A_y & 1 \\ B_x & B_y & 1 \\ C_x & C_y & 1 \end{vmatrix} > 0$$

Geometric Primitives: Inside Circumcircle

$$\begin{vmatrix} A_x & A_y & A_x^2 + A_y^2 & 1 \\ B_x & B_y & B_x^2 + B_y^2 & 1 \\ C_x & C_y & C_x^2 + C_y^2 & 1 \\ D_x & D_y & D_x^2 + D_y^2 & 1 \end{vmatrix} > 0$$

Data Structures

Algorithm

Implementation Aspects

Applications

Conclusions

Thank you for Your Attention!

References

(1) Jonathan Richard Shewchuk. "Triangle: Engineering a 2D Quality Mesh Generator and Delaunay Triangulator". In: *Applied Computational Geometry: Towards Geometric Engineering*. Ed. by Ming C. Lin and Dinesh Manocha. Vol. 1148. Lecture Notes in Computer Science. From the First ACM Workshop on Applied Computational Geometry. Springer-Verlag, May 1996, pp. 203–222. URL: https://people.eecs.berkeley.edu/~jrs/papers/triangle.pdf (visited on 11/07/2020).

(2) Leonidas Guibas and Jorge Stolfi. "Primitives for the Manipulation of General Subdivisions and the Computation of Voronoi Diagrams". In: *ACM Transactions on Graphics* 4 (April 1985), pp. 74–123. DOI: 10.1145/282918.282923. URL: http://sccg.sk/~samuelcik/dgs/quad_edge.pdf (visited on 11/07/2020).

(3) Rex A. Dwyer. "A Faster Divide-and-Conquer Algorithm for Constructing Delaunay Triangulations". In: *Algorithmica* 2 (November 1987), pp. 137–151. DOI: 10.1007/BF01840356.

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(8) V. Fuetterling, C. Lojewski, and F.-J. Pfreundt. "High-Performance Delaunay Triangulation for Many-Core Computers". In: *High Performance Graphics 2014* (2014), pp. 97–104. DOI: 10.2312/hpg.20141098.

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