§1 April 1st, 2021

§1.1 Principal Bundle

Definition 1.1. A principal bundle $P \to B$ is a fiber bundle with fiber G, a lie group, and cocycles $g_{\alpha beta} : U_\alpha \cap U_\beta \to G$ where G acts on itself by multiplication: $\varphi : G \to \mathrm{Aut}(G)$ is given by $g \mapsto (h \mapsto hg)$.

P admits an action G such that the action respects the fibration; i. e given $p \in \pi^{-1}(b)$, $pg \in \pi^{-1}(b)$. This action on one fiber is transitive and free.

§1.2 Examples: $G = \mathbb{Z}_2$