Sriram, Olympiad Combinatorics

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${\bf Abstract}$

Notes and selected solutions from Sriram, $Olypiad\ Combinatorics$. I will generally leave out tedious computations but will refer to the text whenever possible.

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1 Algorithms

1.1 Greedy Algorithms

Example 1.1

In a graph G with n vertices, no vertex has degree greater than Δ . Show that one can color the vertices using at most $\Delta + 1$ colors, such that no two neighboring vertices are the same color.

Proof. Use a greedy coloring algorithm with the vertices in any order: color the first vertex with 1. For the second vertex, use the smallest available color so that no two neighboring vertices are the same color. Since no vertex has degree greater than Δ , we will be able to color each vertex with one of $\{1, 2, \ldots, \Delta + 1\}$.

Example 1.2

In a graph G with V vertices and E edges, show that there exists an induced subgraph H with each vertex having degree at least E/V.

Proof.