HW0: Introduction to Financial Engineering

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Financial Engineering

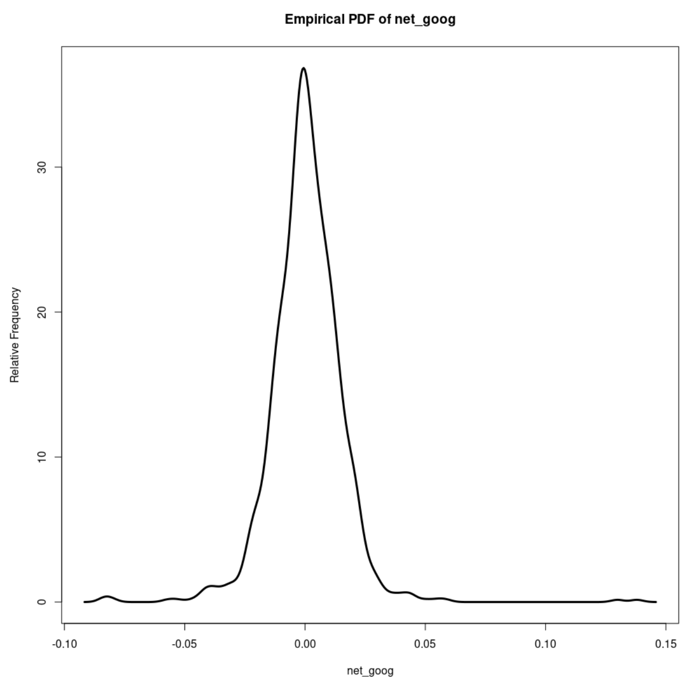
Exercise 1

It is well known that in order to make profit on investing, one needs to predict wisely the return fluctuations of the price assets. Therefore, it is logical to try to compare and expect the data to behave like a known distribution. For instance, the main goal of this exercise is to reproduce a normality test on the daily net returns and log returns from the Close price of Google stock and the S&P composite index.
Some basic statistics that can give us some useful predictive information are the sample mean, standard deviation, skewness, excess kurtosis, minimum and maximum value of the data. Particularly, we can expose these statistics from the data mentioned with the following table:

Return	Sample Mean	Std. Deviation	Skewness	Excess kurtosis	Minimum	Maximum
GOOG	0.0006759628	0.0151873329	0.8228599640	14.3999673167	-0.0837750389	0.1379628181
SP	0.0004323746	0.0191832255	0.0688880851	2.5776904851	-0.0824615385	0.0950775455

Table 1: Statistics of the daily net returns of Google stock and the S&P composite index

In addition, another useful predictive tool is the the empirical density function, which in our case the Google stock net returns is



Judging from the plot, one may think the data is normally distributed with mean 0, however that's not a valid argument to justify it. Hence, reproducing a normality test over the stock data is required to ensure it. R source code can provide this information with the orders

```
shapiro.test(net_sp)
 => p-value < 2.2e-16
```

Which is enough to reject the null hypothesis due to the low $p-$value obtained, therefore the Google net return stock is not normally distributed.

As a consequence of the flawless data result, it is reasonable to try to predict the behaviour of return transformations. For example studying the log returns, which have better continuous properties, is far more interesting. Considering R_t the net return of some price of an asset, the log return transformation under consideration can be easily obtained with

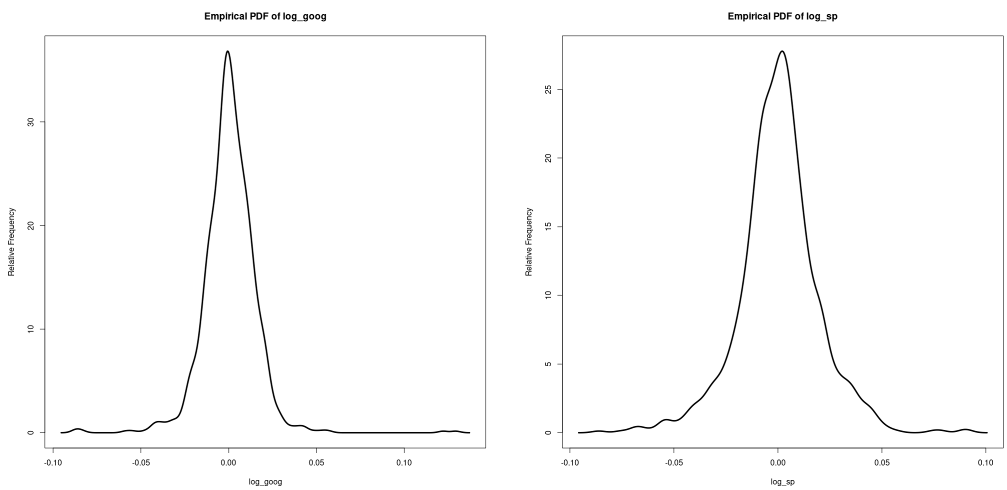
$$r_t = \log(1 + R_t)$$

which give us a new source of data to extract information from. This time, the data statistics can be displayed in the following table:

Log Return	Sample Mean	Std. Deviation	Skewness	Excess kurtosis	Minimum	Maximum
GOOG	0.0005614166	0.0151079312	0.4874215524	12.5439851585	-0.0874933536	0.1292396621
SP	0.0002486003	0.019179229	-0.060943009	2.5045189891	-0.08606078	0.0908251786

Table 2: Statistics of the daily log returns of Google stock and the S&P composite index

As well as providing the empirical density function of the daily returns, both from Google stock and S&P composite index, from the plots we can further expect the Google log returns to behave like a normal distribution



with mean zero, and even if testing the mean we obtain that

```
t.test(log_goog)
 => p-value = 0.2393
```

in which we accept the null hypothesis and the mean of the log return is zero, we still need to compute a test that provides us a sufficient $p-$value> 0.005 to accept that the log returns data distributes normally. Nevertheless we still reject the null hypothesis, in fact

```
shapiro.test(log_goog)
 => p-value = 4.963e-14
  shapiro.test(log_sp)
 => p-value < 2.2e-16
```

in both cases we have to reject the null hypotheses and none of the log return data is normally distributed.
However we can construct a 95% confidence interval for the expected daily log return of Google stock which is

$$(-0.0003731154\ ,\ 0.0014959487)$$

Exercise 2

Assuming the following situation, being let EUR D the loan one takes from the bank, that has to be amortized within T years with an interest rate of $R\%$ on a monthly installment. Then the constant monthly installment, given by the *French Amortization* method, is

$$D\frac{R}{12} + \frac{D\frac{R}{12}}{(1+\frac{R}{12})^{12T} - 1}$$

Where $D\frac{R}{12}$ is the interest component and $\frac{D\frac{R}{12}}{(1+\frac{R}{12})^{12T}-1}$ the amortized capital. Replacing the constants with the values $D = 240.000$, $T = 30$, $R = 1,99$ we have the following amount one has to pay to the bank on a monthly basis which is

$$\text{EUR } 885,89$$

As well as the monthly installment we can reproduce the amortization table that displays the interest portion, capital repaid and the debt during the the whole period of 360 months. However the values can be summarised by only showing the annual quantities, with the table below.

Year	Yearly Installment	Interest Portion	Capital Repayment	Debt
0				240.000,00
1	10.630,64	4.722,30	5.908,34	234.091,66
2	10.630,64	4.603,65	6.026,99	228.064,67
3	10.630,64	4.482,61	6.148,03	221.916,63
4	10.630,64	4.359,15	6.271,50	215.645,13
5	10.630,64	4.233,20	6.397,45	209.247,69
6	10.630,64	4.104,72	6.525,92	202.721,76
7	10.630,64	3.973,66	6.656,98	196.064,78
8	10.630,64	3.839,97	6.790,67	189.274,11
9	10.630,64	3.703,60	6.927,04	182.347,07
10	10.630,64	3.564,49	7.066,16	175.280,91
11	10.630,64	3.422,58	7.208,06	168.072,85
12	10.630,64	3.277,83	7.352,82	160.720,03
13	10.630,64	3.130,16	7.500,48	153.219,55
14	10.630,64	2.979,54	7.651,11	145.568,44
15	10.630,64	2.825,88	7.804,76	137.763,68
16	10.630,64	2.669,14	7.961,50	129.802,18
17	10.630,64	2.509,26	8.121,39	121.680,79
18	10.630,64	2.346,16	8.284,49	113.396,30
19	10.630,64	2.179,78	8.450,86	104.945,44
20	10.630,64	2.010,07	8.620,57	96.324,87
21	10.630,64	1.836,95	8.793,70	87.531,17
22	10.630,64	1.660,35	8.970,30	78.560,87
23	10.630,64	1.480,20	9.150,44	69.410,43
24	10.630,64	1.296,44	9.334,21	60.076,22
25	10.630,64	1.108,98	9.521,66	50.554,56
26	10.630,64	917,76	9.712,88	40.841,68
27	10.630,64	722,71	9.907,94	30.933,74
28	10.630,64	523,73	10.106,92	20.826,82
29	10.630,64	320,76	10.309,89	10.516,94
30	10.630,64	113,71	10.516,94	0,00

Therefore to compute how much we have paid back to the bank we just need to multiply the monthly installment by T, hence obtaining the value of EUR $318.919, 34$.

Let us assume a new situation where we work with a non constant interest rate anymore. To be precise, which is formed by a fixed market index m_i and a variable market spread m_s that increases $+0.10\%$ every year. Obtaining the amortization table in this case is quite similar to the previous one, however we must consider the fluctuating values the interest holds yearly.
Properly explained, assuming we start at year 0, we have that D_0, $R_0\%$, T_0 are respectively the initial debt, interest rate and years in which one must pay back the loan. Then, as done in the previous example, the table must be computed considering fixed the interest rate. That gave us a first monthly installment of EUR X_0. However, by the time the interest rate changes to $R_0 + 0.10\%$, which is at the end of 12 months, one needs to consider doing again the table, this time starting at year 1, with the values

$$\begin{aligned} D_1 =& D_0 - Y = \text{ EUR } 234.091, 66 \\ T_1 =& T_0 - 1 = 29 \text{ years} \\ R_1 =& R_0 + 0.10 = 2, 09\% \end{aligned}$$

where Y is the total amortized capital at year 0, leading us a new monthly installment of EUR $X_1 = 897, 57$. By iterating this process at every year, we obtain this new amortization table, where we can easily obtain the proper value of what one has actually paid back to the bank

$$\sum_{i=0}^{29} 12 \cdot X_i = \text{ EUR } 363655.30$$