# HW7: Portfolio Optimization

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Consider inverting in $N$ securities, in order to price their capital in the financial market, i.e giving their risk premium, one aims to predict their return with pricing models. Let us start by giving some previous definitions. Consider the returns of $N$ assets, $\{r_i\}_i$, $(\sigma_P, \mu_P)$ their feasible set of portfolios and their efficient frontier. Given a single free-risk return, i.e with 0 variance, we define the *tangency portfolio* as the most optimal efficient portfolio, and we denote by

$$\mu_\pi \text{ the return of the tangent portfolio}$$

$$\sigma_\pi \text{ the risk of the tangent portfolio}$$

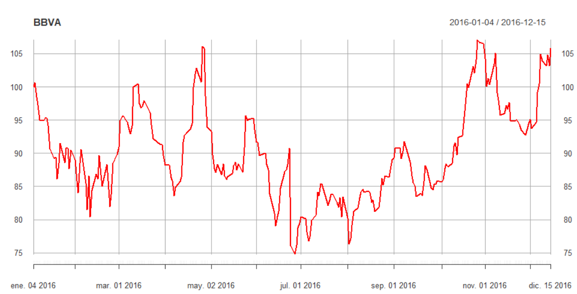
We may proceed by introducing the *Security Market Line*, which is a particular case of pricing model. The main function is to best predict the stock return using the returns of the tangent portfolio. Given the return of a risk-free asset $\{r_j\}_j$, it relates the excess of return with the slope of its regression on the tangent portfolio. We define then $sigma_{j,\pi}$ the covariance between the returns on the $j-$th asset and the tangent portfolio, then the $SML$ aims that the risk-premium (the price) for a security is the product of a parameter $\beta_j$ that measures how aggressive the asset is, and the risk-premium of the tangent portfolio. In particular, if $\{\mu_j\}_j$ is the return of the risk-free asset, then it satisfies

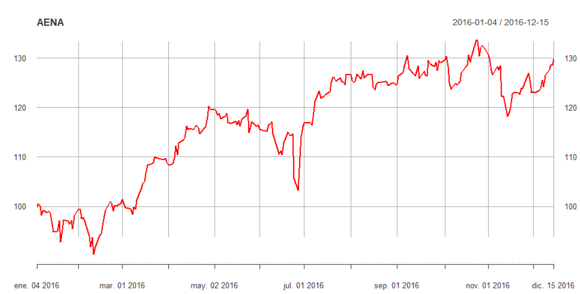
$$\mu_j - \mu_f = \beta_j(\mu_\pi - \mu_f)$$

where $\mu_f$ is the *risk free rate*, and $\beta_j = \dfrac{\sigma_{j,\pi}}{\sigma_\pi^2}$.
As mentioned, $\beta_j$ measures the aggressiveness of the asset with the financial market. While $\beta > 1$ are more aggressive and tend to overreact the market, $\beta < 1$ are more defensive. Let us show a particular case of using $SML$ in the stock index IBEX 35, and the performance of different $\beta_i$ assets, while the Brexit referendum took place. Assuming the risk-free is $\mu_f = 0$ then the $SML$ is simply modeled with

$$\mu_j = \frac{\sigma_{j,\pi}}{\sigma_\pi^2}\mu_\pi$$

which is a regression model of the asset returns, and here $\beta_j$ coefficients can be easily computed. Observe that, since the variation of the set index (IBEX 35) is defined as the variation of the tangent portfolio of the return, in our case we will use $\sigma_\pi^2 = var(r_{IBEX})$. Then among all the stocks we used one from BBVA.MC and AENA the ones that have respective $\beta_1 = 1.444175$ and $\beta_2 = 0.5655301$. We can display from each of them their performance




As we can see from the first plot, the behaviour of the first asset shows huge volatility clusters, as if the asset itself is easily disturbed from the information shocks, while in the second one, even if one can perceive some volatility left, it appears to have a constant tendency, no matter what information it receives. This is highly accurate from the nature of the $\beta_i$. Let us put the example of the case of the United Kingdom withdrawal, that took place the $23^{rd}$ of July of 2016. On one hand, since the first asset has $\beta > 1$, it appears to overreact to the information around as did the brexit to it. On the other hand, the conservative nature the second asset is also reflected in its performance. In this case $\beta < 1$, which this shocks seem to be less effective on the variation of its tendency, and it has a more conservative behaviour, despite that huge peak around July.