Silvia S. Pelozo
Becaria doctoral de CONICET (Tipo I)
Licenciada en Sistemas (UNNE)

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Grupo de Sistemas Dependibles

Tema Model checking probabilista para el análisis de propiedades del estado de régimen

Comisión de doctorado

- Pedro R. D'Argenio [Director]
- Ricardo J. Corin
- Laura Alonso i Alemany
- Héctor L. Gramaglia [Suplente]

Reasoning about security on distributed probabilistic systems through bounded-reachability

Silvia S. Pelozo

SUPERVISOR: PEDRO R. D'ARGENIO

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UNC Universidad Nacional de Córdoba

CONICET

Our problem

$$\left.\begin{array}{l}\text{Distributed systems}\\ +\ \text{random behavior}\\ +\ \text{privacy of components}\end{array}\right\} \xrightarrow{\text{model checking}} \left[\begin{array}{l} P\left(\Diamond\mathrm{GOOD}\right)\\ P\left(\Diamond\mathrm{BAD}\right)\end{array}\right.$$

Our problem

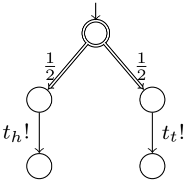
$$\left.\begin{array}{l}\text{Distributed systems}\\ +\ \text{random behavior}\\ +\ \text{privacy of components}\end{array}\right\} \xrightarrow{\text{model checking}} \left[\begin{array}{l}P\left(\Diamond\textsc{good}\right)\\ P\left(\Diamond\textsc{bad}\right)\end{array}\right.$$

Example

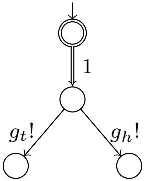
Player T tosses a coin

Player G tries to guess

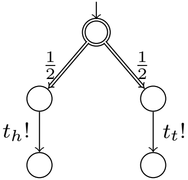
Player T



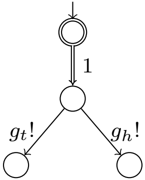
Player G

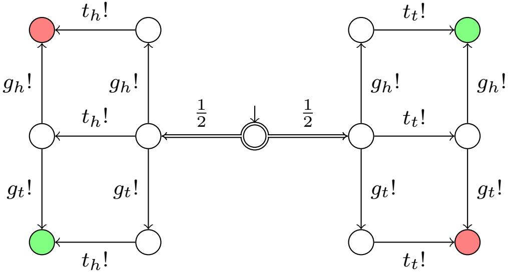


Player T



Player G





Formal model

$$\langle S, \quad \hat{s}, \ A, \ \rightarrow, \ \Rightarrow \rangle$$

Formal model

$$\langle S,\ \ \hat{s},\ \ A,\ \ \rightarrow,\ \ \Rightarrow \rangle$$

finite set of states

$\in S$ initial state

finite set of actions partitioned in inputs A_I and outputs A_O

$\subseteq S \times A \times S$ set of interactive transitions

Formal model

$$\langle S,\ \ \hat{s},\ \ A,\ \ \rightarrow,\ \ \Rightarrow \rangle$$

finite set
of states

$\in S$ initial
state

finite set of actions
partitioned in inputs
A_I and outputs A_O

$\subseteq S \times A \times S$ set
of interactive
transitions

$S \rightharpoonup Dist(S)$ set
of probabilistic
transitions

We require:

$$Dom\,(\rightarrow) \cap Dom\,(\Rightarrow) = \emptyset$$

Input enabledness:

$$\forall s \in S, a \in A_I : \exists s' \in S : s \xrightarrow{a} s'$$

Input determinism:

$$\forall s \in S, a \in A_I : \exists!\; s' \in S : s \xrightarrow{a} s'$$

Output isolation:

$$\forall s \in S, a', a'' \in A_O : s \xrightarrow{a'} s' \wedge s \xrightarrow{a''} s'' \\ \implies a' = a'' \wedge s' = s''$$

Parallel Composition

$$\mathcal{P} \text{ and } \mathcal{Q} \text{ are composable if } A_O^{\mathcal{P}} \cap A_O^{\mathcal{Q}} = \emptyset$$

$\mathcal{C} := \mathcal{P} \parallel \mathcal{Q}$ will be:

$$\left\langle S^{\mathcal{P}} \times S^{\mathcal{Q}}, (\hat{s}_{\mathcal{P}}, \hat{s}_{\mathcal{Q}}), A_I^{\mathcal{C}} \cup A_O^{\mathcal{C}}, \rightarrow_{\mathcal{C}}, \Rightarrow_{\mathcal{C}} \right\rangle$$

where:

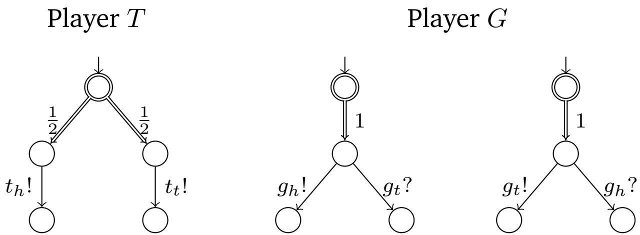
$$A_O^{\mathcal{C}} := A_O^{\mathcal{P}} \cup A_O^{\mathcal{Q}} \quad ; \quad A_I^{\mathcal{C}} := \left(A_I^{\mathcal{P}} \cup A_I^{\mathcal{Q}}\right) \setminus A_O^{\mathcal{C}}$$

and:

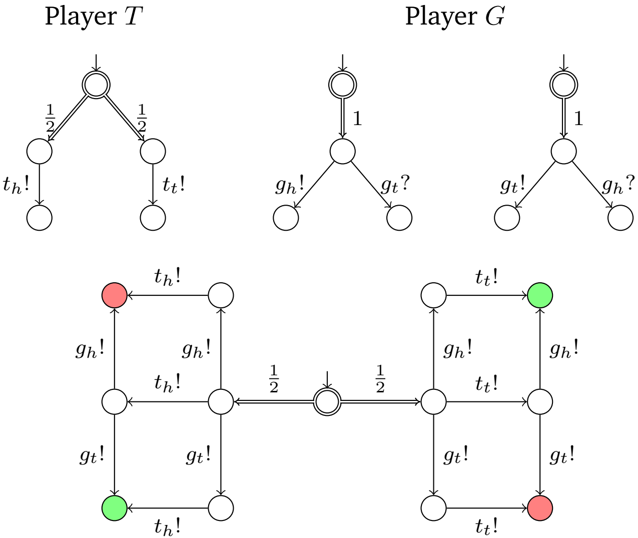
$$\frac{s \xrightarrow{a}_{\mathcal{P}} s'}{(s,t) \xrightarrow{a}_{\mathcal{C}} (s',t)} \quad a \notin A^{\mathcal{Q}} \qquad\qquad \frac{t \xrightarrow{a}_{\mathcal{Q}} t'}{(s,t) \xrightarrow{a}_{\mathcal{C}} (s,t')} \quad a \notin A^{\mathcal{P}}$$

$$\frac{s \xrightarrow{a}_{\mathcal{P}} s' \qquad t \xrightarrow{a}_{\mathcal{Q}} t'}{(s,t) \xrightarrow{a}_{\mathcal{C}} (s',t')} \qquad\qquad \frac{s \Rightarrow_{\mathcal{P}} \mu_s \qquad t \Rightarrow_{\mathcal{Q}} \mu_t}{(s,t) \Rightarrow_{\mathcal{C}} \mu_s \times \mu_t}$$

Can be extended to any finite set $\mathcal{C}$.



Player T
Player G
$\frac{1}{2}$
$\frac{1}{2}$
$t_h!$
$t_t!$
1
$g_h!$
$g_t?$
1
$g_t!$
$g_h?$



Player T
Player G
$\frac{1}{2}$
$\frac{1}{2}$
$t_h!$
$t_t!$
1
1
$g_h!$
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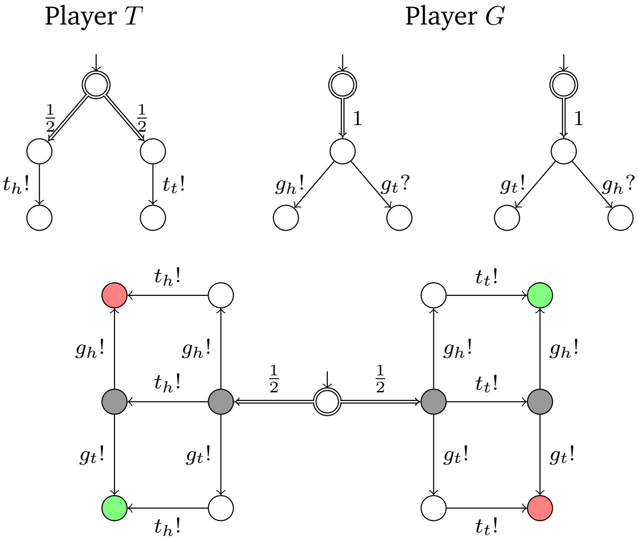
$t_t!$


Player T
Player G
1/2
1/2
t_h!
t_t!
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1
g_h!
g_t?
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1/2
t_t!
g_t!
g_t!
g_t!
g_t!
t_h!
t_t!

Resolution of non-determinism

A *finite path* of $\mathcal{C}$ is a sequence $s_0 a_0 s_1 a_1 \ldots a_{n-1} s_n$ where:

$$a_i \in A \text{ and } s_i \xrightarrow{a_i} s_{i+1}$$

or

$$a_i \in Dist\,(S)\,, s_i \Rightarrow a_i, \text{ and } a_i\,(s_{i+1}) > 0$$

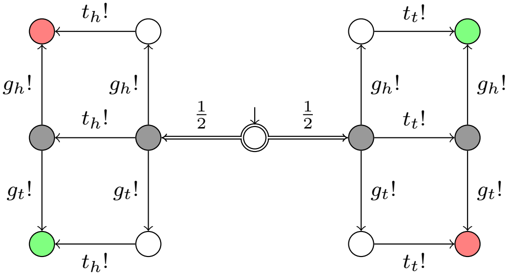
An *interleaving scheduler* is a function

$$\mathcal{I} : Paths\,(\mathcal{C}) \rightarrow Dist\,(\{\mathcal{P}_1, \ldots, \mathcal{P}_n\})$$

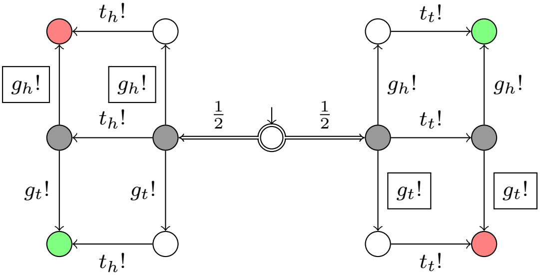
Defined for paths σ such that $last\,(\sigma)$ is vanishing:

$$\mathcal{I}\,(\sigma)\,(\mathcal{P}_i) > 0 \implies A^{en}_{last(\sigma),\mathcal{P}_i} \neq \emptyset$$

Why not all possible schedulers?



Why not all possible schedulers?



$$P\left(\Diamond \bigcirc\right) = 1$$

Distributed schedulers

[Giro and D'Argenio, 2009, Giro, 2010]

Projections

$\mathcal{C} = \mathcal{P}_1 \parallel \cdots \parallel \mathcal{P}_n$, $\sigma \in Paths\,(\mathcal{C})$, the projection $\sigma\,[\mathcal{P}_i]$ is:

$$(\hat{s}_{\mathcal{C}})\,[\mathcal{P}_i] = \pi_i\,(\hat{s}_{\mathcal{C}})$$

$$(\sigma a s)\,[\mathcal{P}_i] = \begin{cases} (\sigma)\,[\mathcal{P}_i] & \text{if } a \notin A^{\mathcal{P}} \\ (\sigma)\,[\mathcal{P}_i]\,a\,(\pi_i\,(s)) & \text{if } a \in A^{\mathcal{P}} \end{cases}$$

$$(\sigma\,(\mu_1 \times \cdots \times \mu_n)\,s)\,[\mathcal{P}_i] = (\sigma\,[\mathcal{P}_i])\,\mu_i\,(\pi_i\,(s))$$

where $\pi_i\,(s_1, \ldots, s_n) = s_i$.

Strongly distributed schedulers

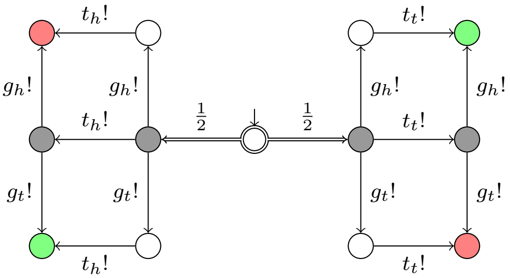
$\mathcal{I}$ of $\mathcal{C} = \mathcal{P}_1 \parallel \cdots \parallel \mathcal{P}_n$ is strongly distributed if:

$\forall \mathcal{P}_i, \mathcal{P}_j, \sigma, \sigma'$:

$$\left.\begin{array}{l} \sigma\left[\mathcal{P}_i\right] = \sigma'\left[\mathcal{P}_i\right] \\ \sigma\left[\mathcal{P}_j\right] = \sigma'\left[\mathcal{P}_j\right] \\ \mathcal{I}\left(\sigma\right)\left(\mathcal{P}_i\right) + \mathcal{I}\left(\sigma\right)\left(\mathcal{P}_j\right) \neq 0 \\ \mathcal{I}\left(\sigma'\right)\left(\mathcal{P}_i\right) + \mathcal{I}\left(\sigma'\right)\left(\mathcal{P}_j\right) \neq 0 \end{array}\right\} \implies$$

$$\implies \frac{\mathcal{I}\left(\sigma\right)\left(\mathcal{P}_i\right)}{\mathcal{I}\left(\sigma\right)\left(\mathcal{P}_i\right) + \mathcal{I}\left(\sigma\right)\left(\mathcal{P}_j\right)} = \frac{\mathcal{I}\left(\sigma'\right)\left(\mathcal{P}_i\right)}{\mathcal{I}\left(\sigma'\right)\left(\mathcal{P}_i\right) + \mathcal{I}\left(\sigma'\right)\left(\mathcal{P}_j\right)}$$

Is the problem solved?



$$\sigma\left[\mathcal{G}_h\right] = \sigma'\left[\mathcal{G}_h\right] \wedge \sigma\left[\mathcal{G}_t\right] = \sigma'\left[\mathcal{G}_t\right]$$

$$\Longrightarrow \frac{\mathcal{I}\left(\sigma\right)\left(\mathcal{G}_h\right)}{\mathcal{I}\left(\sigma\right)\left(\mathcal{G}_h\right) + \mathcal{I}\left(\sigma\right)\left(\mathcal{G}_t\right)} = \frac{\mathcal{I}\left(\sigma'\right)\left(\mathcal{G}_t\right)}{\mathcal{I}\left(\sigma'\right)\left(\mathcal{G}_h\right) + \mathcal{I}\left(\sigma'\right)\left(\mathcal{G}_t\right)}$$

But...

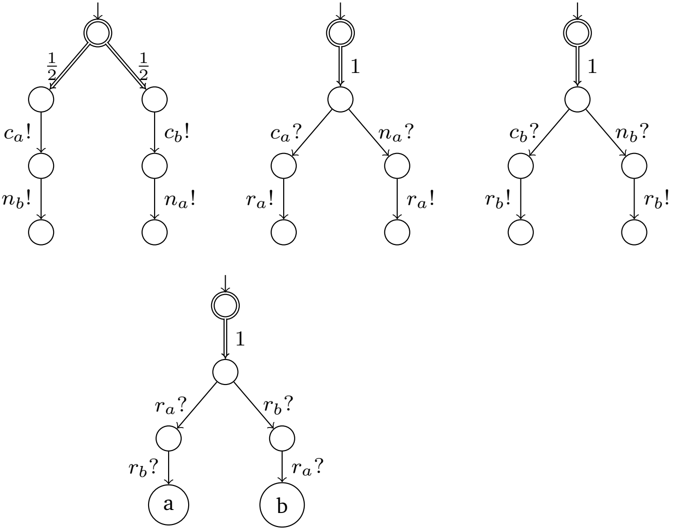
Unbounded reachability is undecidable [Giro and D'Argenio, 2007]

:-(

Bounded reachability IS decidable [Calin et al., 2010]:

Throught parametric interpretation, unfolding, and non-linear constrains
$\rightsquigarrow$ reduces to a non-linear programming problem

Anyway, they are sometimes still too powerful...



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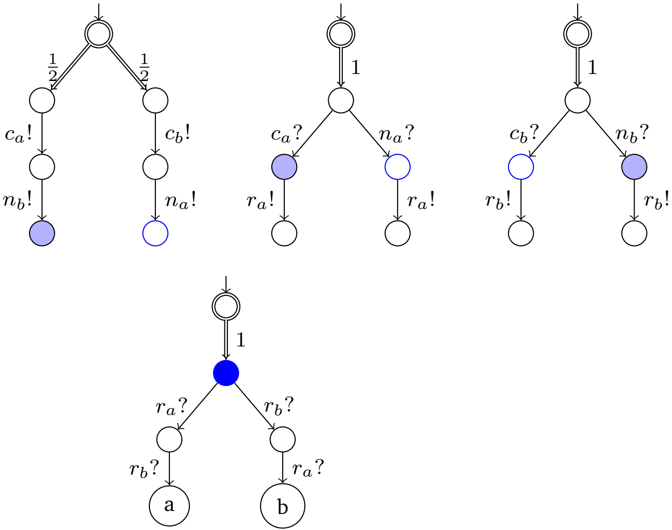

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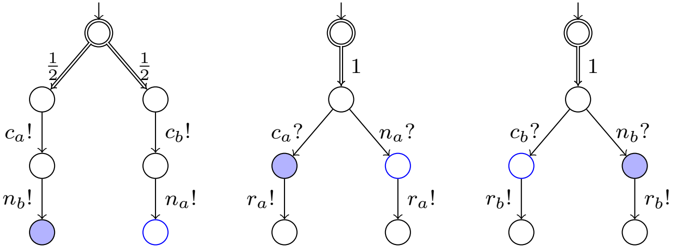
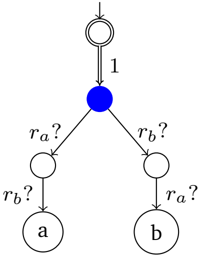




$$\sigma_{\bullet}(\mathcal{A}) \neq \sigma_{\circ}(\mathcal{A})\,;\sigma_{\bullet}(\mathcal{B}) \neq \sigma_{\circ}(\mathcal{B}) \implies$$

$$\mathcal{I}(\sigma_{\bullet})(\mathcal{A}) = 1, \mathcal{I}(\sigma_{\bullet})(\mathcal{B}) = 0$$

$$\mathcal{I}(\sigma_{\circ})(\mathcal{A}) = 0, \mathcal{I}(\sigma_{\circ})(\mathcal{B}) = 1$$

is a valid scheduler

Projection up to secrecy equivalence

Given:

$\mathcal{C} = \mathcal{P}_1 \parallel \cdots \parallel \mathcal{P}_n$

a path $\sigma \in Paths\left(\mathcal{C}\right)$

equivalence relations $\sim \subseteq S_i, \approx \subseteq A_i$ $(i = 1, \ldots, n)$

Projection $\left[\sigma\left[\mathcal{P}_i\right]\right]_\sim$ of σ is:

$$\left[\left(\hat{s}_\mathcal{C}\right)\left[\mathcal{P}_i\right]\right]_\sim = \left[\pi_i\left(\hat{s}_\mathcal{C}\right)\right]_\sim$$

$$\left[\left(\sigma a s\right)\left[\mathcal{P}_i\right]\right]_\sim = \begin{cases} \left[\left(\sigma\right)\left[\mathcal{P}_i\right]\right]_\sim & \text{if } a \notin A_{\mathcal{P}_i} \\ \left[\left(\sigma\right)\left[\mathcal{P}_i\right]\right]_\sim \left[a\right]_\approx \left[\pi_i\left(s\right)\right]_\sim & \text{if } a \in A_{\mathcal{P}_i} \end{cases}$$

$$\left[\left(\sigma\left(\mu_1 \times \cdots \times \mu_n\right) s\right)\left[\mathcal{P}_i\right]\right]_\sim = \left[\left(\sigma\left[\mathcal{P}_i\right]\right)\right]_\sim \mu_i\left(\left[\pi_i\left(s\right)\right]_\sim\right)$$

$$\mu_i\left(\left[s\right]_\sim\right) = \sum_{s' \in [s]_\sim} \mu_i\left(s'\right)$$

Distributed scheduler with secrecy

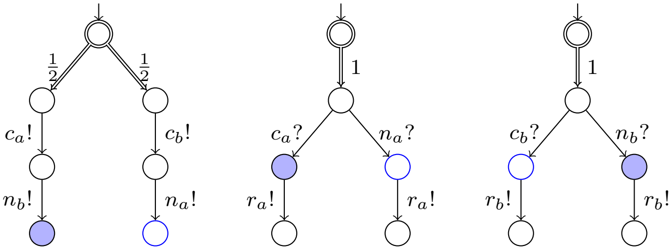
$\mathcal{I}$ of $\mathcal{C} = \mathcal{P}_1 \parallel \cdots \parallel \mathcal{P}_n$ such that

$\forall \mathcal{P}_i, \mathcal{P}_j, \sigma, \sigma'$:

$$\begin{array}{c} [\sigma\,[\mathcal{P}_i]]_\sim = [\sigma'\,[\mathcal{P}_i]]_\sim \\ [\sigma\,[\mathcal{P}_j]]_\sim = [\sigma'\,[\mathcal{P}_j]]_\sim \end{array} \qquad \left.\begin{array}{c} \mathcal{I}\,(\sigma)\,(\mathcal{P}_i) \\ \mathcal{I}\,(\sigma)\,(\mathcal{P}_j) \\ \mathcal{I}\,(\sigma')\,(\mathcal{P}_i) \\ \mathcal{I}\,(\sigma')\,(\mathcal{P}_j) \end{array}\right\} \neq 0$$

$$\Downarrow$$

$$\frac{\mathcal{I}\,(\sigma)\,(\mathcal{P}_i)}{\mathcal{I}\,(\sigma)\,(\mathcal{P}_i) + \mathcal{I}\,(\sigma)\,(\mathcal{P}_j)} = \frac{\mathcal{I}\,(\sigma')\,(\mathcal{P}_i)}{\mathcal{I}\,(\sigma')\,(\mathcal{P}_i) + \mathcal{I}\,(\sigma')\,(\mathcal{P}_j)}$$



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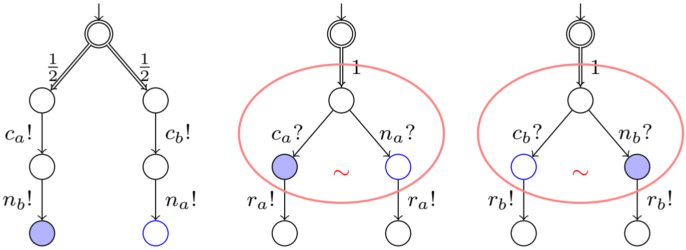

$$[\sigma_{\bullet}(\mathcal{A})]_{\sim} = [\sigma_{\circ}(\mathcal{A})]_{\sim} \wedge [\sigma_{\bullet}(\mathcal{B})]_{\sim} = [\sigma_{\circ}(\mathcal{B})]_{\sim}$$

$$\Downarrow$$

$$\mathcal{I}(\sigma_{\bullet})(\mathcal{A}) = 1, \mathcal{I}(\sigma_{\bullet})(\mathcal{B}) = 0$$

$$\mathcal{I}(\sigma_{\circ})(\mathcal{A}) = 0, \mathcal{I}(\sigma_{\circ})(\mathcal{B}) = 1$$

is NOT a valid scheduler

Closing remarks

In verification of distributed systems with *random behavior* + *privacy concerns*

$\rightarrow$ traditional probabilistic model-checking techniques are inadequate

Distributed schedulers work better

+ realistic bounds for probabilities
– undecidable in general
+ bounded reachability is decidable $\rightsquigarrow$ non-linear programming problem
– too powerful in some cases

We introduce *secrecy*

– some drawbacks of distributed schedulers
+ even more realistic results
+ also reducible to non-linear program
! some validation pending

References

[Calin et al., 2010] Calin, G., Crouzen, P., D'Argenio, P., Hahn, E., and Zhang, L. (2010). Time-bounded reachability in distributed input/output interactive probabilistic chains. In van de Pol, J. and Weber, M., editors, *Model Checking Software*, volume 6349 of *Lecture Notes in Computer Science*, pages 193–211. Springer Berlin / Heidelberg. 10.1007/978-3-642-16164-3-15.

[Giro, 2010] Giro, S. (2010). *On the automatic verification of distributed probabilistic automata with partial information*. PhD thesis, FaMAF, UNC.

[Giro and D'Argenio, 2009] Giro, S. and D'Argenio, P. (2009). On the expressive power of schedulers in distributed probabilistic systems. *Electronic Notes in Theoretical Computer Science*, 253(3):45 – 71. Proceedings of Seventh Workshop on Quantitative Aspects of Programming Languages (QAPL 2009).

[Giro and D'Argenio, 2007] Giro, S. and D'Argenio, P. R. (2007). Quantitative model checking revisited: Neither decidable nor approximable. In Raskin, J.-F. and Thiagarajan, P. S., editors, *FORMATS*, volume 4763 of *LNCS*, pages 179–194. Springer.