Tableaux Example

Exercise: Given the following definitions

$$\begin{array}{ll} Def \ 1 & D \sqsubseteq \neg A \\ Def \ 2 & \neg A \sqsubseteq \neg B. \end{array}$$

prove that the following assertion is unsatisfiable using tableaux

$$a: \forall R. (A \lor B) \land \exists R. D$$

Solution:

1.
$$a: \forall R.(A \lor B) \land \exists R.D$$

2. $a: \forall R.(A \lor B) \leftarrow (\land) \text{ in } 1$
3. $a: \exists R.D \leftarrow (\land) \text{ in } 1$
4. $(a,b):R \leftarrow (\exists) \text{ in } 3$
5. $b:D \leftarrow (\exists) \text{ in } 3$
6. $b: (A \lor B) \leftarrow (\forall) \text{ in } 2 \text{ and } 4$
7. $b: (\neg D \lor \neg A) \leftarrow (Def 1) \text{ in } b$
8. $b: (\neg \neg A \lor \neg B) \leftarrow (Def 2) \text{ in } b$
(\lor) in $7 \rightarrow 9a$. $b: \neg D$ 9b. $b: \neg A \leftarrow (\lor) \text{ in } 7$
 \otimes
(\lor) in $8 \rightarrow 11a$. $b: \neg \neg A \rightarrow 11b$. $b: \neg B \rightarrow (\lor) \text{ in } 8$
(\lor) in $11a \rightarrow 12$. $b: A \rightarrow \otimes$