Machine-learning Based Heuristics for Propositional SAT Solving

Ezequiel Orbe

Grupo de Procesamiento de Lenguaje Natural
Fa.M.A.F., Universidad Nacional de Córdoba

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Introducing Myself...

Info

- **Me:** Alejandro Ezequiel Orbe
- **Supervisors:** Gabriel Infante Lopez & Carlos Areces
- **Research Area:** Propositional Logic $\cap$ Machine Learning.

Research Topic

Machine-learning based Heuristics for Propositional SAT.

Outline of the Talk

1. Machine-learning based heuristics for Propositional SAT

2. Exploiting Symmetry in Propositional SAT Solving

Background

Facts about SAT Solving

- *Propositional satisfiability (SAT)* $\Rightarrow$ well-known $\mathcal{NP}$-complete problem.
- Relevance $\Rightarrow$ Many $\mathcal{NP}$-problems mapped into a SAT problem.
- Two types of solvers $\Rightarrow$ **Complete** & **Incomplete**.
- Complete Solvers $\Rightarrow$ **Conflict-driven (CDCL)** & **Look-ahead**.
- General purpose heuristics.

Background

Previous Work

- **Understanding Random SAT: Beyond the Clauses-to-Variables Ratio**. Nudelman et al. 2007.
- **Hierarchical hardness models for SAT**. Lin Xu et al. 2007.
- **Restart Strategy Selection using Machine Learning Techniques**. Toby Walsh et al. 2009.
- **Problem-Sensitive Restart Heuristics for the DPLL Procedure**. Carsten Sinz et al. 2009.

Our Research...

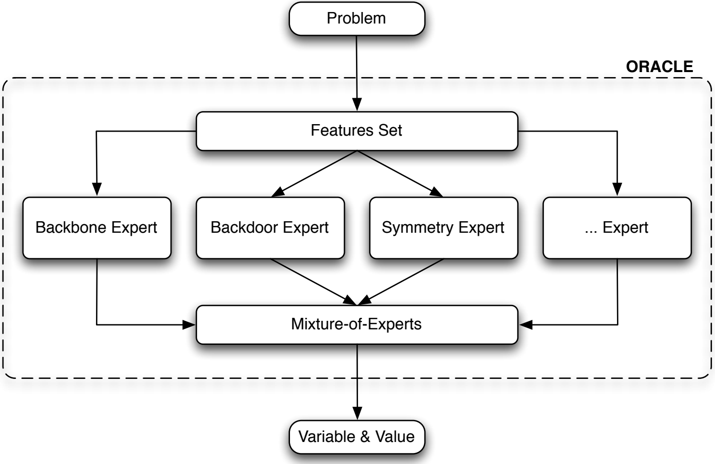
Goals

- Automatically find structure in problems from an specific domain.
- Exploit the structure to guide the search.

Setup

- Problem Generator $\Rightarrow$ Training Material.
- Features Set $\Rightarrow$ Captures structural properties.
- Oracle $\Rightarrow$ Branching Heuristic $\Rightarrow$ Decision & Direction
- Oracle $\Rightarrow$ Probability Distribution $\Rightarrow$ Mixture-of-experts.

Setup


Problem
ORACLE
Features Set
Backbone Expert
Backdoor Expert
Symmetry Expert
... Expert
Mixture-of-Experts
Variable & Value

What is Symmetry?

Informal Definition

- In general $\Rightarrow$ The symmetry of a discrete object is a permutation of its components that leaves the object intact.
- In SAT-solving $\Rightarrow$ mapping of a problem into itself that preserves its structure and solutions.

Example

- $\mathcal{T} = (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee C)$.
- $\theta_1 = \{A \mapsto B, B \mapsto A\}$

$$\theta_1(\mathcal{T}) = (\neg B \vee A \vee C) \wedge (B \vee \neg A \vee C)$$

- $\theta_2 = \{A \mapsto \neg A, B \mapsto \neg B\}$

$$\theta_2(\mathcal{T}) = (\neg\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg\neg B \vee C)$$

Example

How can we use it?

- Symmetries creates a partition in the assignment set.
- In search problems, if there are several points of the search space that are symmetric $\Rightarrow$ just explore one of them.

Example

- $\mathcal{T} = (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee C)$.
- $\theta_1 = \{A \mapsto B, B \mapsto A\}$

A	B	C	$\mathcal{T}$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned}
\theta_1(000) &= 000\\
\theta_1(001) &= 001\\
\theta_1(010) &= 100\\
\theta_1(011) &= 101\\
\theta_1(100) &= 010\\
\theta_1(101) &= 011\\
\theta_1(110) &= 110\\
\theta_1(111) &= 111
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_0 &= \{000\}\\
\mathcal{C}_1 &= \{001\}\\
\mathcal{C}_2 &= \{010, 100\}\\
\mathcal{C}_3 &= \{011, 101\}\\
\mathcal{C}_4 &= \{110\}\\
\mathcal{C}_5 &= \{111\}
\end{aligned}$$

What is Symmetry?

Notation & Definitions

- Let $V = \{x_1, x_2, \ldots, x_n\}$ be a set of boolean variables.
- Lets assume a total order of the variables in V:
  $x_1 < x_2 < \ldots < x_n$
- Let $L = \{x_1, \neg x_1, x_2, \neg x_2, \ldots, x_n, \neg x_n\}$ be the set of literals of variables in V.
- Let $Sym(L)$ be the symmetric group of the set L of literals.
- If $v \in L$ and $\theta \in Sym(L)$, the image of v under θ is denoted by v^θ.

What is Symmetry?

Definition

- Let T be a theory over L and let $\theta \in Sym(L)$.
- The permutation θ is a **symmetry**, or **automorphism**, of T iff $T^{\theta} = T$.
- The set of symmetries of T is a subgroup of $Sym(L)$ and is denoted by $Aut(T)$.

Definition

- A **truth assignment** for a set of variables V is a function $A : V \rightarrow \{0, 1\}$.
- A truth assignment A of V is called a **model** of theory T if $A(T) = 1$.
- The set of models of T is denoted $\mathcal{M}(T)$.

What is Symmetry?

Some facts...

- Symmetries can be viewed as acting on assignments.
- If $\theta \in Sym(L)$ then θ acts on the set of truth assignments by mapping $A \mapsto {}^{\theta}A$ where ${}^{\theta}A(v) = A(v^{\theta})$ for $v \in L$.
- Hence, if T is a theory over L, $A(T^{\theta}) = {}^{\theta}A(T)$.
- Any symmetry of T maps models of T to models of T, and non-models of T to non-models

Proposition

Let T be a theory over V, $\theta \in Aut(T)$, and A a truth assignment of V. Then $A \in \mathcal{M}(T)$ iff ${}^{\theta}A \in \mathcal{M}(T)$

What is Symmetry?

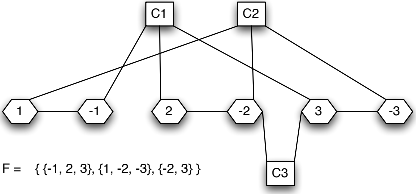
More facts...

- $Aut(T)$ induces an equivalences relation on the set of truth assignments of L, wherein $A \sim B$ if $B =^{\theta} A$ for some $\theta \in Aut(T)$
- The equivalence classes are the *orbits* of $Aut(T)$ in the set of assignments.
- Any equivalence class contains only models of T or contains no models of T.
- This indicates why symmetries can be used to reduce search: we can determine whether T has a model by visiting each equivalence class rather than visiting each truth assignment.

Symmetry Detection

How can we do it?

- The problem of extracting symmetries of a CNF formula is reduced to the **colored graph automorphism problem**.
- **Main idea:** Find a colored graph whose symmetry group is isomorphic to the symmetry group of the CNF formula.
- Available Tool $\Rightarrow$ Saucy
- Generates a graph with $2V + C$ vertices.



F = { {-1, 2, 3}, {1, -2, -3}, {-2, 3} }

Symmetry Breaking

Pre-processing Approach

- Complete Symmetry Breaking:
  - Augment the original formula with **Symmetry Breaking Predicates (SBP)**.
  - Pros:
    - Each predicate select exactly on representative assignment from each equivalence class.
  - Cons:
    - The size of the predicate can exceed significantly the size of the original formula.

$$
\begin{aligned}
\mathrm{PP}(\pi; X) &= \mathrm{leq}(X, X^{\pi}) \\
&= \bigwedge_{i \in [0, n-1]} \left[\left[\bigwedge_{j \in [i+1, n-1]} \left(x_j = x_j^{\pi} \right) \right] \rightarrow \left(x_i \leqslant x_i^{\pi} \right) \right]
\end{aligned}
$$

Symmetry Breaking

Pre-processing Approach

- Partial Symmetry Breaking
  - Full symmetry breaking is unfeasible.
  - Pros:
    - Only breaks symmetries for the set of irredundant generators of the symmetry group.
  - Cons:
    - A partial SBP selects the least assignment in each orbit of the symmetry group but may include other assignments as well.

Symmetry Breaking

In-processing Approach

- Dynamic Symmetry Breaking
  - Breaks symmetries during the search.
  - There is little mention to this approach in the literature.
  - Pros:
    - Identify local symmetries.
    - Avoid the creation of SBPs.
    - Boost the performance of conflict-driven learning.
  - Cons:
    - Difficult to integrate with current solvers.

Symmetry Breaking

Our approach to Symmetry-Breaking

- Dynamic Partial Symmetry Breaking.
- Use the set of irredundant generators.
- Integration of symmetry detection in the search process.
- Symmetry breaking as component of a complex heuristic (Symmetry Expert).

Results

Thank you!