Lógicas modales

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1er Cuatrimestre 2017
Córdoba, Argentina

Part I

Memory logics

Changing the model

- The Modal Logic book says

> A modal formula is a little automaton standing at some state in a relational structure, and only permitted to explore the structure by making journeys to neighbouring states.

- What about granting our automaton the additional power to **modify the model** during its exploratory trips?
- There may be many ways to modify a model (changing the domain, the edges, the valuation, ...)
- We want to restrict our atention to a specific way of modifying a model: **adding a memory** to the model, and **performing changes** on it

Changing the model

- We are going to add a storage structure to standard Kripke models:

$$\mathcal{M} = \langle W, (R_r)_{r \in \mathsf{rel}}, V \rangle \quad +$$

- There are many possible types of structures: a set, a list, a stack, ...
- We want to start with a very simple structure, so we are going to add a **set** S to the standard Kripke model:

Memory Kripke model

Given a set $S \subseteq W$, a memory Kripke model is

$$\mathcal{M} = \langle W, (R_r)_{r \in \mathsf{rel}}, V, S \rangle$$

Changing the model

We have to add suitable operators to manipulate the memory

- Since we are using a set S as the container, there are two "natural" operators to use:
  - An operator ⓡ to *remember* the current point, storing it in S.
  - An operator ⓚ to check membership of the current point, and find out whether it is *known*

Some notation

Given $\mathcal{M} = \langle W, (R_r)_{r\in\mathsf{rel}}, V, S\rangle$, $w \in W$, we define

$$\mathcal{M}[w] = \langle W, (R_r)_{r\in\mathsf{rel}}, V, S \cup \{w\}\rangle$$

Now, more formally

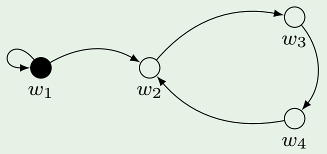
Semantics of ⓡ and ⓚ

$$\begin{array}{lll} \mathcal{M}, w \models ⓡ\varphi & \text{iff} & \mathcal{M}[w], w \models \varphi \\ \mathcal{M}, w \models ⓚ & \text{iff} & w \in S \end{array}$$

Changing the model

Let's see the use of ⓡ and ⓚ with an example. Suppose we start with the following model:

A model with an initially empty memory



- $V(p) = \emptyset$ for all $p \in \mathsf{prop}$
- $S = \emptyset$
- $S = \{w_1\}$

- How can we check whether w_1 has a successor different from itself?

$$\begin{array}{c} \mathcal{M}, w_1 \models ⓡ\Diamond\neg ⓚ \\ \Updownarrow \\ \mathcal{M}[w_1], w_1 \models \Diamond\neg ⓚ \\ \Updownarrow \\ \mathcal{M}[w_1], w_2 \models \neg ⓚ \quad \checkmark \end{array}$$

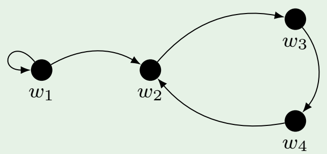
Memory logics

- The idea of using operators that **change** the model is not new
- The family of languages with these characteristics are sometimes called **dynamic logics**
- For example:
  - Dynamic epistemic logics
  - Real time logics
  - Dynamic predicate logic
- Memory logics can be seen as dynamic languages that
  - Do not add any domain-specific behaviour in the evolution of the model
  - Analyze dynamic behaviour from a very simple perspective
  - Can be thought of as a 'weak' version of the standard $\downarrow$ modal binder
- Can be combined with other modal and hybrid operators (A, nominals, @, etc.)

Other operators

- We can think in other operators, that *delete* elements from the memory.
- In the previous example, the memory was initially empty, which was quite convenient

A model where every point is memorized



- How can we check whether w_1 has a successor different from itself?
- There doesn't seem to be a way...

Other operators

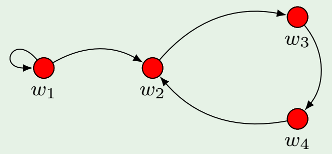
We can define an operator ⓔ (for 'erase') that completely wipes out the memory

Semantics of ⓔ

$$\langle M, (R_r)_{r\in\mathsf{rel}}, V, S\rangle, w \models \text{ⓔ}\varphi \quad \text{iff} \quad \langle M, (R_r)_{r\in\mathsf{rel}}, V, \emptyset\rangle, w \models \varphi$$

So now, in order to check in $\mathcal{M}$ whether w_1 has a successor different from itself

A model $\mathcal{M}$, where every point is memorized



we can evaluate

$$\mathcal{M}, w_1 \models \text{ⓔⓔⓡ}\Diamond\neg\text{ⓚⓡ}\Diamond\neg\text{ⓚ}$$

This formula works independently of the initial state of the memory

Other operators

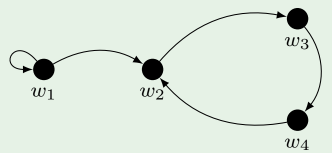
- We can also think in a 'local' version of ⓔ, that only deletes the current point of evaluation.
- Let's consider then the operator ⓕ (for 'forget')

Semantics of ⓕ

$$\langle M, (R_r)_{r\in\mathsf{rel}}, S\rangle, w \models \text{ⓕ}\varphi \quad \text{iff} \quad \langle M, (R_r)_{r\in\mathsf{rel}}, S\setminus\{w\}\rangle, w \models \varphi$$

Again, if we want to check in $\mathcal{M}$ whether w_1 has a successor different from itself

A model $\mathcal{M}$, where every point is memorized



we can evaluate

$$\mathcal{M}, w_1 \models \text{ⓕⓡ}\Diamond\text{ⓚ}$$

Other ingredients

There are other "dimensions" we can take into consideration:

- Class of models: for example, it is quite natural to consider the class of models whose memory is initially empty
- Memorizing policies: we can try to impose some restrictions on the interplay between memory and modal operators
  - These restrictions are going to help us find decidable fragments
- Other memory operators and containers: are there other memory operators? What happens if we change a set by other type of structure?
  - We can define ⓕ, a local version of ⓔ
  - We can try using a stack instead of a set as the memory container

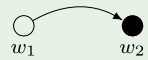
Other ingredients: classes of models

Observe that when the memory of $\mathcal{M}$ is initially empty,

$$\mathcal{M}, w \models \text{ⓡ}\langle r\rangle\text{ⓚ} \quad \text{iff} \quad wR_rw$$

But this formula is also true at

A model with a non-empty memory



Taking this into consideration, it is natural to consider memory logics restricted to

$$\mathcal{C}_\emptyset = \{\mathcal{M} \mid \mathcal{M} = \langle W, (R_r)_{r\in\mathsf{rel}}, V, \emptyset\rangle\}$$

the class of models with an empty memory.

Other ingredients: memorizing policies

- Until now memory and modal operators were working 'in parallel'
- Restricting expressivity sometimes can be helpful to reduce computational cost
- We can try to impose some restrictions in the interplay between memory and modal operators

Let's define an operator where $\langle r \rangle$ and ⓡ act **at the same time**

$\langle r \rangle$ and ⓡ working together

$$\mathcal{M}, w \models \langle\!\langle r \rangle\!\rangle \varphi \quad \text{iff} \quad \exists w' \in W, R_r(w, w') \text{ and } \mathcal{M}[w], w' \models \varphi.$$

We are going to see later that this operator helps us to find **decidable** memory fragments

Notation

We are going to work with several memory logic fragments

Notational convention

- We call $\mathcal{ML}$ the basic modal logic, and $\mathcal{HL}$ the extension of $\mathcal{ML}$ with nominals
- When we add a set S and the operators ⓡ and ⓚ we add m as a superscript, e.g. $\mathcal{ML}^m(\ldots$
- We add $\emptyset$ as a subscript when we work with $\mathcal{C}_\emptyset$ (otherwise is the class of all models), e.g. $\mathcal{ML}^m_\emptyset(\ldots$
- Then we list the additional operators

For example

- $\mathcal{ML}^m_\emptyset(\langle r \rangle, ⓔ)$: the modal memory logic with ⓡ, ⓚ, ⓔ and the usual diamond $\langle r \rangle$ over the class $\mathcal{C}_\emptyset$
- $\mathcal{HL}^m(@, \langle r \rangle)$: the hybrid memory logic with ⓡ, ⓚ, $\langle r \rangle$, @ over the class of all models

Getting to know a logic

This is a new family of logics, and there are characteristics that are worth investigating

- Expressivity: What can we say with memory logics? Which is the relation between them and other well-known logics?
- Decidability: Which is the computational complexity of the different fragments? How much are memory operators adding to the basic modal logic?
- Interpolation: How they behave in term of Craig interpolation and Beth definability?
- Axiomatization: Do they have sound and complete axiomatic systems?
- Tableau systems: Can we adapt known tableau techniques to produce sound and complete tableau systems? Can we find terminating tableaux for the decidable memory fragments?

Disclaimer: we are not going to see all these topics during this talk

Expressivity results

We compare the expressive power of the different fragments via the existence of *equivalence preserving translations*

$\mathcal{L}'$ is as least as expressive as $\mathcal{L}$ ($\mathcal{L} \leq \mathcal{L}'$) if there is a Tr such that

$$\mathcal{M}, w \models_{\mathcal{L}} \varphi \text{ iff } \mathcal{M}, w \models_{\mathcal{L}'} \mathrm{Tr}(\varphi)$$

Theorem

$\mathcal{ML}^m_\emptyset(\langle r \rangle) < \mathcal{HL}(\downarrow)$.

To see that $\mathcal{ML}^m_\emptyset(\langle r \rangle) \leq \mathcal{HL}(\downarrow)$ we define a translation Tr that maps formulas of $\mathcal{ML}^m_\emptyset(\langle r \rangle)$ into sentences of $\mathcal{HL}(\downarrow)$.

- We use $\downarrow$ to simulate ⓡ.
- We use a finite set N to simulate that ⓚ does not distinguish between different memorized states.

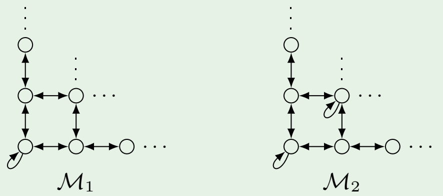
$$\begin{array}{rcl} \mathrm{Tr}_N(ⓡ\varphi) & = & \downarrow i.\mathrm{Tr}_{N \cup \{i\}}(\varphi) \quad (\text{for } i \text{ a new nominal}) \\ \mathrm{Tr}_N(ⓚ) & = & \bigvee_{i \in N} i \end{array}$$

Expressivity results

How can we see that $\mathcal{ML}^m_\emptyset(\langle r\rangle) \neq \mathcal{HL}(\downarrow)$? We need to show that there is *no possible* translation from $\mathcal{HL}(\downarrow)$ to $\mathcal{ML}^m_\emptyset(\langle r\rangle)$...

- We developed a notion of *bisimulation* for each fragment. Intuitively, two models are bisimilar for a logic $\mathcal{L}$ when they cannot be distinguished by $\mathcal{L}$-formulas

$\mathcal{M}_1$ and $\mathcal{M}_2$ are $\mathcal{ML}^m_\emptyset(\langle r\rangle)$-bisimilar



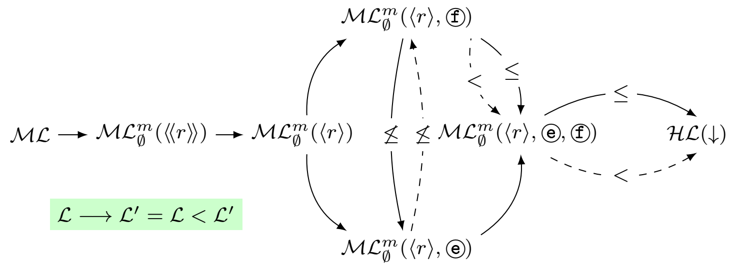
But there is a formula $\varphi \in \mathcal{HL}(\downarrow)$ such that

$$\mathcal{M}_1, w \models_{\mathcal{HL}(\downarrow)} \varphi \text{ and } \mathcal{M}_2, v \not\models_{\mathcal{HL}(\downarrow)} \varphi$$

So a translation from $\mathcal{HL}(\downarrow)$ to $\mathcal{ML}^m_\emptyset(\langle r\rangle)$ cannot exist

Expressivity results

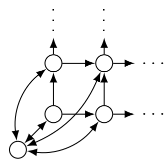
We establish in this way an "expressivity map" for many memory logic fragments:



- All the memory logic fragments are between the basic modal logic and the logic $\mathcal{HL}(\downarrow)$ (and therefore below first order logic)

Decidability results

- We have encoded the tiling problem for several memory fragments using a *spy point*: a point that sees every other point in the model



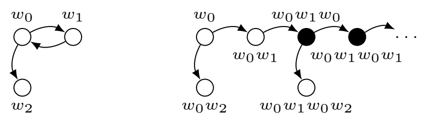
- Most of the memory logic fragments turned out to be undecidable
- We found decidable fragments restricting the interplay between $\langle r\rangle$ and ⓡ: we force them to act at the same time

$\langle r\rangle$ and ⓡ working together

$$\mathcal{M}, w \models \langle\!\langle r\rangle\!\rangle\varphi \quad \text{iff} \quad \exists w' \in W, R_r(w, w') \text{ and } \mathcal{M}[w], w' \models \varphi.$$

Decidability results

- We proved that some fragments are PSPACE-complete showing that they enjoy the bounded tree-model property: every satisfiable formula can be satisfied in a bounded tree
- We showed that there is a procedure to transform an arbitrary model into a tree-like model, preserving equivalence



- We also built a "decidability map" for the different memory fragments

PSPACE-complete	Undecidable
$\mathcal{ML}^m(\langle\!\langle r\rangle\!\rangle)$	$\mathcal{ML}^m_\emptyset(\langle\!\langle r\rangle\!\rangle)$, $\mathcal{ML}^m(\langle\!\langle r\rangle\!\rangle) + i$
$\mathcal{ML}^m(\langle\!\langle r\rangle\!\rangle, ⓕ)$	$\mathcal{ML}^m(\langle r\rangle)$, ...

Axiomatizations

- We characterized many memory logics fragments in terms of axiomatic systems *à la Hilbert*
- **Nominals** proved to be a very useful device to find sound and complete axiomatizations

Axiomatization for $\mathcal{HL}^m(@, \langle r \rangle)$

All axioms and rules for $\mathcal{HL}(@)$

$$+$$

$$\vdash @_i(ⓡ\varphi \leftrightarrow \varphi[ⓚ/(ⓚ \vee i)])$$

- We found sound and complete axiomatizations for all the *hybrid* memory fragments (and establish automatic completeness for pure extensions)
- We could provide axiomatizations for some cases even in the absence of nominals (i.e., $\mathcal{ML}^m(\langle\!\langle r \rangle\!\rangle)$ and $\mathcal{ML}^m(\langle\!\langle r \rangle\!\rangle, ⓕ)$)
- The tree-model property was a key feature to use when nominals were not present

Tableau systems

- We presented a sound and complete tableau system for $\mathcal{ML}^m(\langle r \rangle, ⓔ, ⓕ)$, $\mathcal{ML}^m_\emptyset(\langle r \rangle, ⓔ, ⓕ)$, and its sublanguages
- It is a *prefixed* tableau where we use prefixed formulas with the shape

$$\langle w, R, F \rangle^{\mathcal{C}} : \varphi$$

- w: point of evaluation
- R: set of memorized labels
- F: set of forgotten labels
- $\mathcal{C}$: either $\mathcal{C}_\emptyset$ or the class of all models
- φ: current formula

- The rules for propositional and modal operators are standard

Tableau systems

- For example, the rule for ⓡ is quite straightforward

$$(ⓡ) \quad \frac{\langle w, R, F \rangle^{C} : ⓡ\varphi}{\langle w, R \cup \{w\}, F - \{w\} \rangle^{C} : \varphi}$$

- The rule for ⓚ (and for ¬ⓚ) introduces an equivalence class

$$(ⓚ) \quad \frac{\langle w, \{v_1, \dots v_k\}, F \rangle^{C} : ⓚ}{w \approx v_1 \mid \cdots \mid w \approx v_k \mid \langle w, \emptyset, \emptyset \rangle^{C} : ⓚ}$$

$$(\text{repl}) \quad \frac{\begin{array}{c}\langle w, R, F \rangle^{C} : \varphi \\ w \approx^* w' \end{array}}{\langle w', R[w \mapsto w'], F[w \mapsto w'] \rangle^{C} : \varphi}$$

- Since this fragment in undecidable, the tableau is non-terminating
- We also provided a sound, complete and terminating tableau for the decidable fragments

Open questions

- We left some missing links in the expressivity map. We would like to complete it.
- The decidable fragments we found are strictly more expressive than $\mathcal{ML}$, but still really close to it. Can we find more expressive but still decidable fragments? We have some ideas
  - Concrete domains: storing values, not points
  - Restricted classes of models
  - Weaker containers (or syntactic restrictions)
- Beth definability needs further research, we would like some general result
- We want to explore the relation between memory logics and other dynamic logics (DEL is a good candidate). This could also lead to decidable fragments
- Can we find suitable axiomatizations in the absence of nominals. We still don't have one for $\mathcal{ML}^m(\langle r \rangle)$!

References

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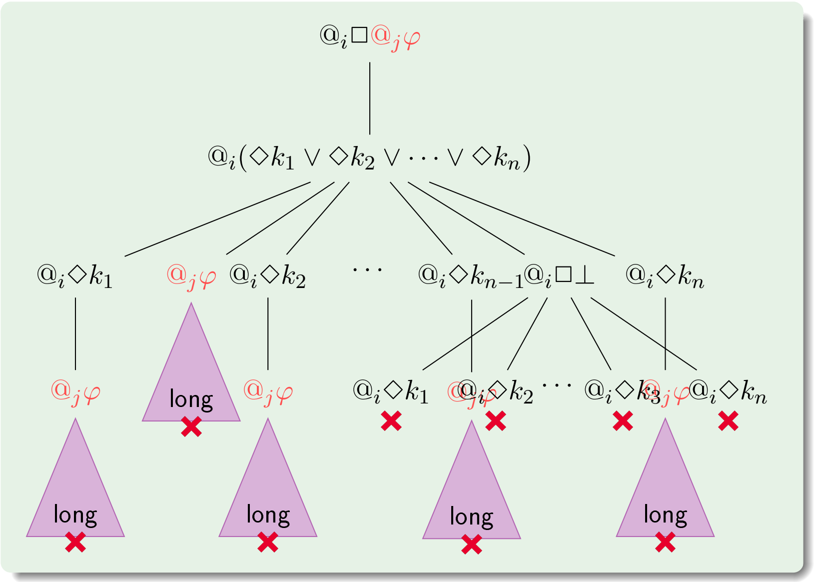
- Areces, C., Figueira, D., Gorin, D., and Mera, S.. Tableaux and Model Checking for Memory Logics. In Automated Reasoning with Analytic Tableaux and Related Methods, pp. 47–61, Springer Berling / Heidelberg, Oslo, Norway, 2009.

Part III

Coinduction, extractability, normal forms

Global modalities should be "extracted"



Globality $\sim$ extractability?

Global modalities are extractable from other modalities...

$$[r]@_i\varphi \equiv [r]\bot \vee @_i\varphi \qquad [r]\mathsf{A}\varphi \equiv [r]\bot \vee \mathsf{A}\varphi$$
$$@_j@_i\varphi \equiv @_j\bot \vee @_i\varphi \qquad @_j\mathsf{A}\varphi \equiv @_j\bot \vee \mathsf{A}\varphi$$
$$\mathsf{A}@_i\varphi \equiv \mathsf{A}\bot \vee @_i\varphi \qquad \mathsf{A}\mathsf{A}\varphi \equiv \mathsf{A}\bot \vee \mathsf{A}\varphi$$
$$\vdots \qquad \vdots$$

...but some modalities are more equal than others

$$\downarrow i.@_i\varphi \not\equiv \downarrow i.\bot \vee @_i\varphi$$
$$ⓡ\mathsf{A}\varphi \not\equiv ⓡ\bot \vee \mathsf{A}\varphi$$

Coinductive models – a unifying framework

The class of all (rooted) Kripke models with domain W

- $\text{Kripke}_W \stackrel{def}{=}$ all the tuples $\langle W, w_0, V, R \rangle$ such that
  - $w_0 \in W$
  - $V(p) \subseteq W$
  - $R(r, w) \subseteq W$
- $\langle W, w, V, R \rangle \models [r]\varphi$ iff $\langle W, v, V, R \rangle \models \varphi$, $\forall v \in R(r, w)$
- Many modal operators can be defined as classes of models

The class of all *coinductive models* with domain W

- $\text{Mods}_W \stackrel{def}{=}$ all the tuples $\langle W, w_0, V, R \rangle$ such that
  - $w_0 \in W$
  - $V(p) \subseteq W$
  - $R(r, w) \subseteq \text{Mods}_W \Longleftarrow$ **coinductive definition!**
- $\langle W, w, V, R \rangle \models [r]\varphi$ iff $\mathcal{M} \models \varphi$, $\forall \mathcal{M} \in R(r, w)$
- More modal operators can be defined as classes of models

Defining Conditions

Defining condition

$$\mathcal{P}_{\mathsf{A}}(\mathcal{M}) \iff R^{\mathcal{M}}(\mathsf{A}, w) = \{\langle v, |\mathcal{M}|, V^{\mathcal{M}}, R^{\mathcal{M}} \rangle \mid v \in |\mathcal{M}|\}$$

Defining condition

$$\mathcal{P}_{@_i}(\mathcal{M}) \iff R^{\mathcal{M}}(@_i, w) = \{\langle v, |\mathcal{M}|, V^{\mathcal{M}}, R^{\mathcal{M}} \rangle \mid v \in V(i)\}, i \in \mathsf{Nom}$$
$$\mathcal{P}_{\downarrow i}(\mathcal{M}) \iff R^{\mathcal{M}}(\downarrow i, w) = \{\langle w, |\mathcal{M}|, V^{\mathcal{M}}[i \mapsto \{w\}], R^{\mathcal{M}} \rangle\}, i \in \mathsf{Nom}$$
$$\mathcal{P}_{\mathsf{Nom}}(\mathcal{M}) \iff V^{\mathcal{M}}(i) \text{ is a singleton}, \forall i \in \mathsf{Nom}$$

Defining condition

$$\mathcal{P}_{\textcircled{r}}(\mathcal{M}) \iff R^{\mathcal{M}}(\textcircled{r}, w) = \{\langle w, |\mathcal{M}|, V^{\mathcal{M}}[\textcircled{k} \mapsto V^{\mathcal{M}}(\textcircled{k}) \cup \{w\}], R^{\mathcal{M}} \rangle\}$$
$$\mathcal{P}_{\textcircled{f}}(\mathcal{M}) \iff R^{\mathcal{M}}(\textcircled{f}, w) = \{\langle w, |\mathcal{M}|, V^{\mathcal{M}}[\textcircled{k} \mapsto V^{\mathcal{M}}(\textcircled{k}) \setminus \{w\}], R^{\mathcal{M}} \rangle\}$$
$$\mathcal{P}_{\textcircled{f}}(\mathcal{M}) \iff R^{\mathcal{M}}(\textcircled{e}, w) = \{\langle w, |\mathcal{M}|, V^{\mathcal{M}}[\textcircled{k} \mapsto \emptyset], R^{\mathcal{M}} \rangle\}$$

Some initial results using the coinductive framework

- The basic modal logic is complete wrt coinductive models
- *Bisimulations*: one size fits all
- General conditions that guarantee extractability
- Extractability is preserved when new operators are added

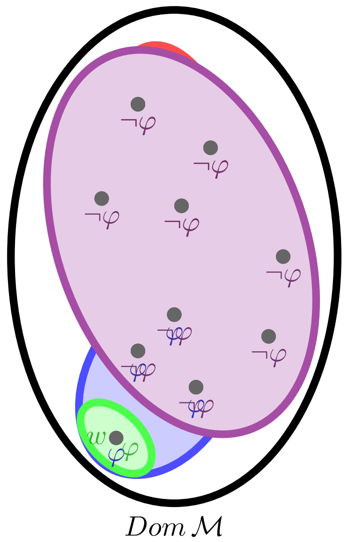
References

- Areces, C. and Gorín, D.. Coinductive models and normal forms for modal logics (or how we learned to stop worrying and love coinduction). Journal of Applied Logic, 8(4):305–318, Elsevier, 2010.

Part IV

Logical methods in the generation de referring expressions

Separations and descriptions



- Let $\mathcal{M}$ be a Kripke model
- And let $\emptyset \neq C, D \subset Dom\,\mathcal{M}$
- For any formula φ, we say that:
  - φ *separates* C and D in $\mathcal{M}$ iff

    $$\mathcal{M}, C \models \varphi \text{ and } \mathcal{M}, D \not\models \varphi$$

- Similarly, for $w \in Dom\,\mathcal{M}$ we say:
  - φ *describes* w in $\mathcal{M}$ iff

    $$\mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, v \not\models \varphi \forall v \neq w$$

- Description is a form of separation
- φ could be of any suitable logic

Separation and description problems

The separation problem

Given a finite model $\mathcal{M}$ and sets $C, D \subset Dom\,\mathcal{M}$, find a φ that separates C and D, if possible.

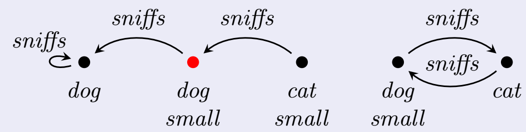
The description problem

Given a finite model $\mathcal{M}$ and a world $w \in Dom\,\mathcal{M}$, find a φ that describes w, if possible.

- They can be seen as another kind of inference task
- But they didn't receive much attention so far
- We are interested in their computational properties

Motivation: Generation of Referring Expressions

An application of logics in Natural Language Generation



content determination
(e.g., as a FOL formula)

- $dog(x) \wedge \exists y.(x \neq y \wedge dog(y) \wedge sniffs(x,y))$

surface realization

- "the dog that sniffs another dog"

(logical) content determination $\approx$ description problem

Motivation: Modal logics in the GRE

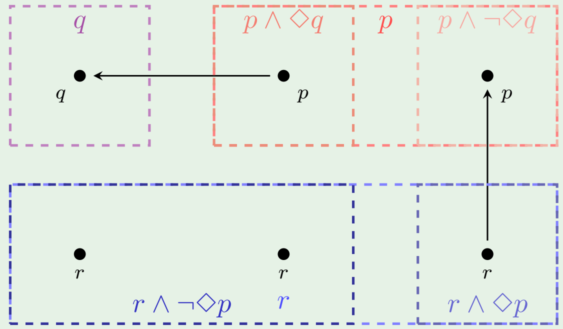
Areces, Koller & Striegnitz (2008)

- We propose modal logics for content determination:
  - $\mathcal{ML}$ – the basic modal language ($\neg$, $\wedge$, $\Diamond$)
  - $\mathcal{EL}$ – the existential positive fragment of $\mathcal{ML}$ ($\wedge$, $\Diamond$)
- *Rationale*:
  - Good expressive power
  - Simple surface realization algorithms
  - Relatively low computational complexity for inference tasks
- In particular, we show that:

  "The modal description problem needs polynomial time"

The modal description problem in polynomial time

A variation of Tarjan's bisimulation contraction algorithm



- Tarjan's algorithm runs in polynomial time
- Hence, the modal description problem is polynomial
- But this is assuming that $\wedge$ takes constant time!

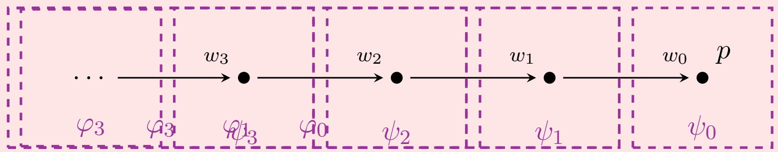
The modal description problem in polynomial time

for DAG representation!

- This algorithm produces a formula represented as a DAG
- The size of the DAG is polynomial in the size of the model
- Surface realization step doesn't exploit DAG representation
  - Most probably can't be done anyway
- Is the *tree* representation of this formula also polynomial?
- If not, "modal content determination" can't be said to take polynomial time

The modal description problem in polynomial time

also for tree representation?



$$\varphi_0 = \neg p \qquad \psi_0 = p$$
$$\varphi_1 = \varphi_0 \wedge \neg\Diamond\psi_0 \qquad \psi_1 = \varphi_0 \wedge \Diamond\psi_0$$
$$\varphi_2 = \varphi_1 \wedge \neg\Diamond\psi_1 \qquad \psi_2 = \varphi_1 \wedge \Diamond\psi_1$$
$$\vdots \qquad \vdots$$
$$\varphi_{i+1} = \varphi_i \wedge \neg\Diamond\psi_i \qquad \psi_{i+1} = \varphi_i \wedge \Diamond\psi_i$$

- Each ψ_i is description for w_i with size exponential in i
- Observe that w_i admits a linear description: $\underbrace{\Diamond\Diamond\ldots\Diamond}_{i \text{ times}} p$

Where do we go from here?

- The example shows that this algorithm is not polynomial
- Can we *fix* it?
- Can we find *another* one that is indeed polynomial?
- We show that no such algorithm exists!

Bounds for the separation / description problems

Basic modal language $\mathcal{ML}$

Theorem (Lower bound)

Any upper bound for the size of a solution for the separation or description problem for $\mathcal{ML}$ is at least exponential.

Corollary

No polynomial time algorithm exists that solves the description or separation problem returning the formula as a tree.

Theorem (Upper bound)

If $\varphi \in \mathcal{ML}$ is a minimum description for v in $\mathcal{M} = \langle W, R, V \rangle$, then $|\varphi| \in O(2^{\frac{1}{2}|W|^2} \cdot |V|)$.