Automata and temporal logic

— a biased overview —

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Plan of this talk

- from tableaux to Büchi automata
- alternating automata for LTL
- branching-time logics

Plan of this talk

- from tableaux to Büchi automata

  a crash course in pre-history

- alternating automata for LTL

  some innovations since the 1990s

- branching-time logics

  nice theory, but does it work?

Tableaux for modal logic

- decomposition of formulas : top-down

- closure criteria : bottom-up, determine satisfiability

Tableaux for modal logic

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$$\frac{\Gamma, \varphi \wedge \psi}{\Gamma, \varphi, \psi} \qquad \frac{\Gamma, \varphi \vee \psi}{\Gamma, \varphi} \qquad \frac{l_1, ..., l_k, \Box\varphi_1, ..., \Box\varphi_m, \Diamond\psi_1, ..., \Diamond\psi_n}{\varphi_1, ..., \varphi_m, \psi_1 \quad \cdots \quad \varphi_1, ..., \varphi_m, \psi_n}$$

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- rules reflect properties of accessibility relation

$$\frac{\Gamma, \Box A}{\Gamma, A}\ \Box \text{ reflexive} \qquad \frac{\Gamma}{\Gamma, \Diamond\top}\ \Box \text{ serial}$$

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Tableaux for LTL

- decomposition based on recursive laws

$$\mathbf{G}\,\varphi \;\leftrightarrow\; \varphi \wedge \mathbf{X}\,\mathbf{G}\,\varphi \qquad\qquad \mathbf{F}\,\varphi \;\leftrightarrow\; \varphi \vee \mathbf{X}\,\mathbf{F}\,\varphi$$

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- distinguish "good" from "bad" loops

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ω-automata

- separate concerns: logic + combinatorics

translation	$\varphi \mapsto \mathcal{A}_\varphi$	$\mathcal{L}(\mathcal{A}_\varphi) =$ models of φ
emptiness checking	$\mathcal{L}(\mathcal{A}) \stackrel{?}{=} \emptyset$	(efficient, if possible)

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- useful for satisfiability *and* model checking

$$\mathcal{T} \models \varphi$$

(T transition system)

iff

every run of $\mathcal{T}$ satisfies φ

iff

$$\mathcal{L}(\mathcal{T} \times \mathcal{A}_{\neg\varphi}) = \emptyset$$

Büchi automata

- $\mathcal{A} = (\Sigma, Q, I, \delta, F)$

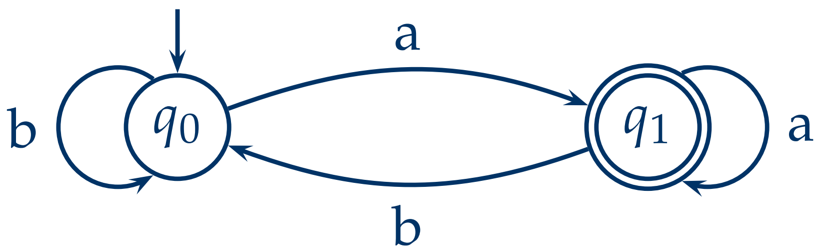
Σ	alphabet
Q	finite set of locations
$I \subseteq Q$	initial locations
$\delta \subseteq Q \times \Sigma \times Q$	transition relation
$F \subseteq Q$	accepting locations

- run of $\mathcal{A}$ over ω-word $\quad q_0 \xrightarrow{a_0} q_1 \xrightarrow{a_1} q_2 \xrightarrow{a_2} \ldots$

initialization	$q_0 \in I$
consecution	$(q_i, a_i, q_{i+1}) \in \delta$
acceptance	$q_i \in F$ for infinitely many i

Büchi automata: examples

- infinitely often ‘a’



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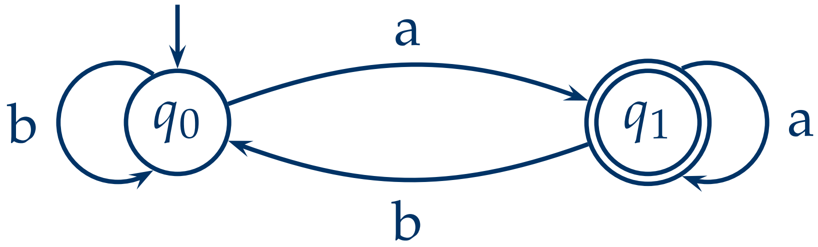

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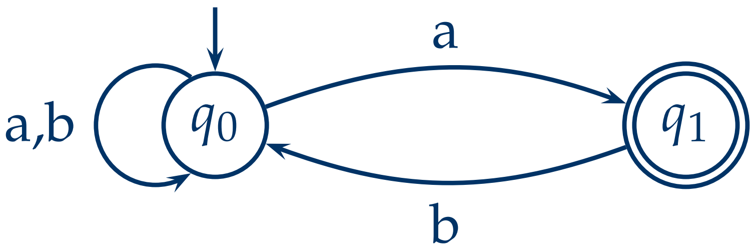

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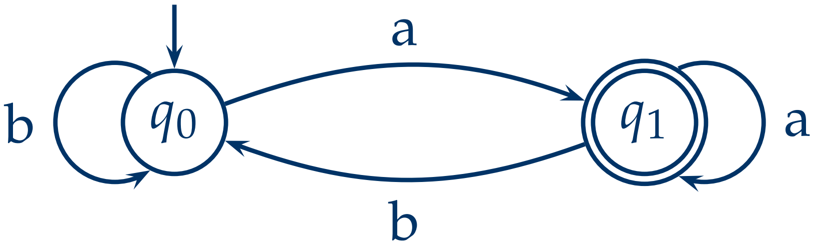

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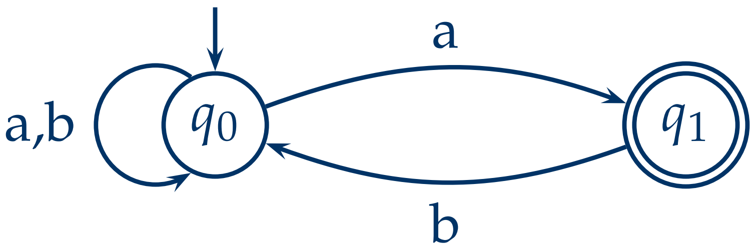

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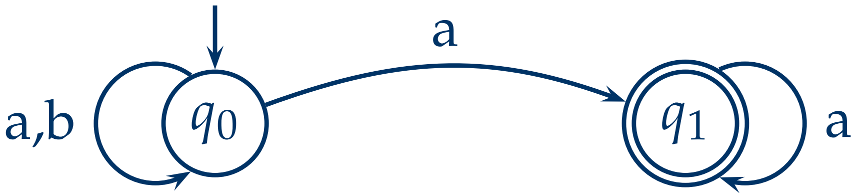

no equivalent deterministic Büchi automaton

Büchi automata: results

- emptiness checking

$$\mathcal{L}(\mathcal{A}) \neq \emptyset \quad \text{iff} \quad q_0 \stackrel{\Sigma^*}{\Longrightarrow} q \stackrel{\Sigma^+}{\Longrightarrow} q \quad \text{for some } q_0 \in I, q \in F$$

"$\Leftarrow$" obvious

"$\Rightarrow$" any accepting run contains repetition of some $q \in F$

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- closure properties

union	standard NFA construction
intersection	"marked" product
complement	difficult (Safra 1988: $O(2^{n \log n})$ locations)

Büchi automata: variants

- Generalized Büchi $\mathcal{A} = (\Sigma, Q, I, \delta, \{F_1, \ldots, F_n\})$

acceptance	infinitely many $q_i \in F_k$, for all k
intersection	simple product construction
translate	to Büchi automaton using counter $\pmod{n}$

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- Muller automaton $\mathcal{A} = (\Sigma, Q, I, \delta, \mathcal{F})$

  acceptance set of locations visited infinitely often $\in \mathcal{F}$

  Streett automata exponentially more succinct than Büchi

From LTL to GBA

- basic insight
  - let $\mathcal{V}$ be set of atoms containing those of formula φ
  - interpret temporal structure as ω-word over alphabet $2^{\mathcal{V}}$
  - models of φ define ω-language $\mathcal{L}(\varphi)$

From LTL to GBA

- basic insight
  - let $\mathcal{V}$ be set of atoms containing those of formula φ
  - interpret temporal structure as ω-word over alphabet $2^{\mathcal{V}}$
  - models of φ define ω-language $\mathcal{L}(\varphi)$
- construct $\mathcal{A}_\varphi$ such that $\mathcal{L}(\mathcal{A}_\varphi) = \mathcal{L}(\varphi)$

locations	sets of "subformulas" of φ promised to be true
initial locations	locations promising φ
transition relation	non-temporal formulas: ensure satisfaction
	temporal formulas: apply recursion laws
accepting locations	defined from "eventualities" $\mathbf{F}\,\psi$, $\psi_1\ \mathbf{U}\ \psi_2$

GBA for $\mathbf{G}\,\mathbf{F}\,p \wedge \mathbf{G}\,\mathbf{F}\,\neg p$

- construct the tableau for the formula

$$\boxed{\mathbf{G}\,\mathbf{F}\,p \wedge \mathbf{G}\,\mathbf{F}\,\neg p}$$

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$$
\begin{tikzcd}
 & & \boxed{\mathbf{G}\,\mathbf{F}\,p \wedge \mathbf{G}\,\mathbf{F}\,\neg p} \arrow[d] & \\
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$$

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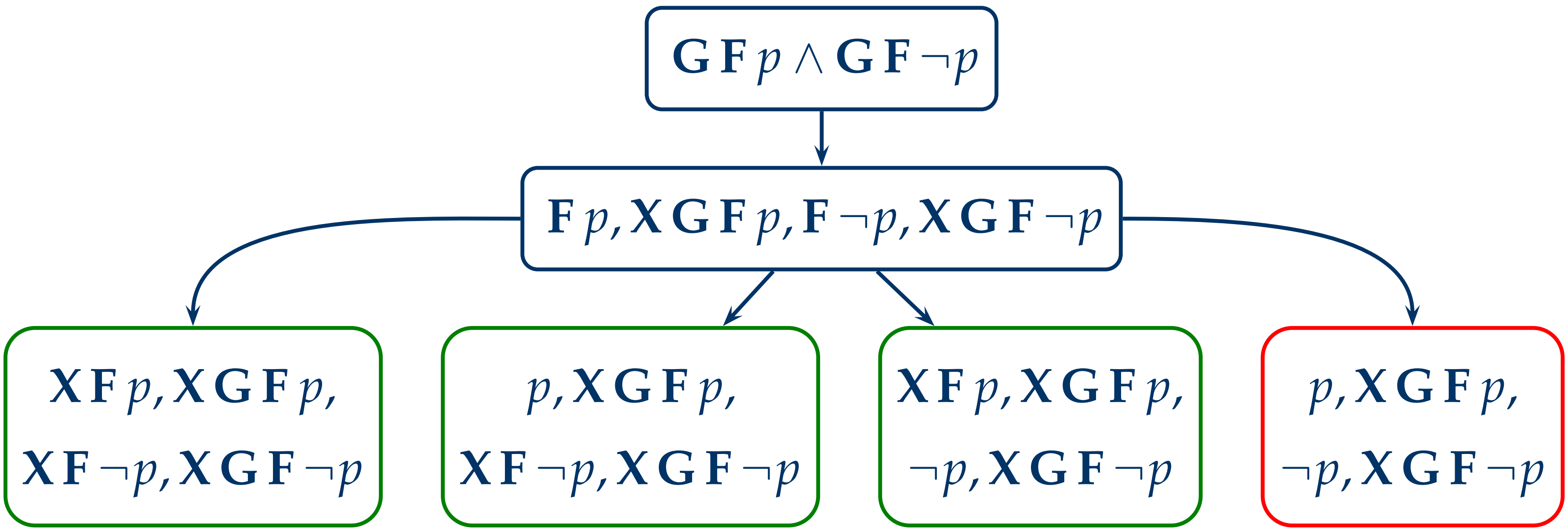

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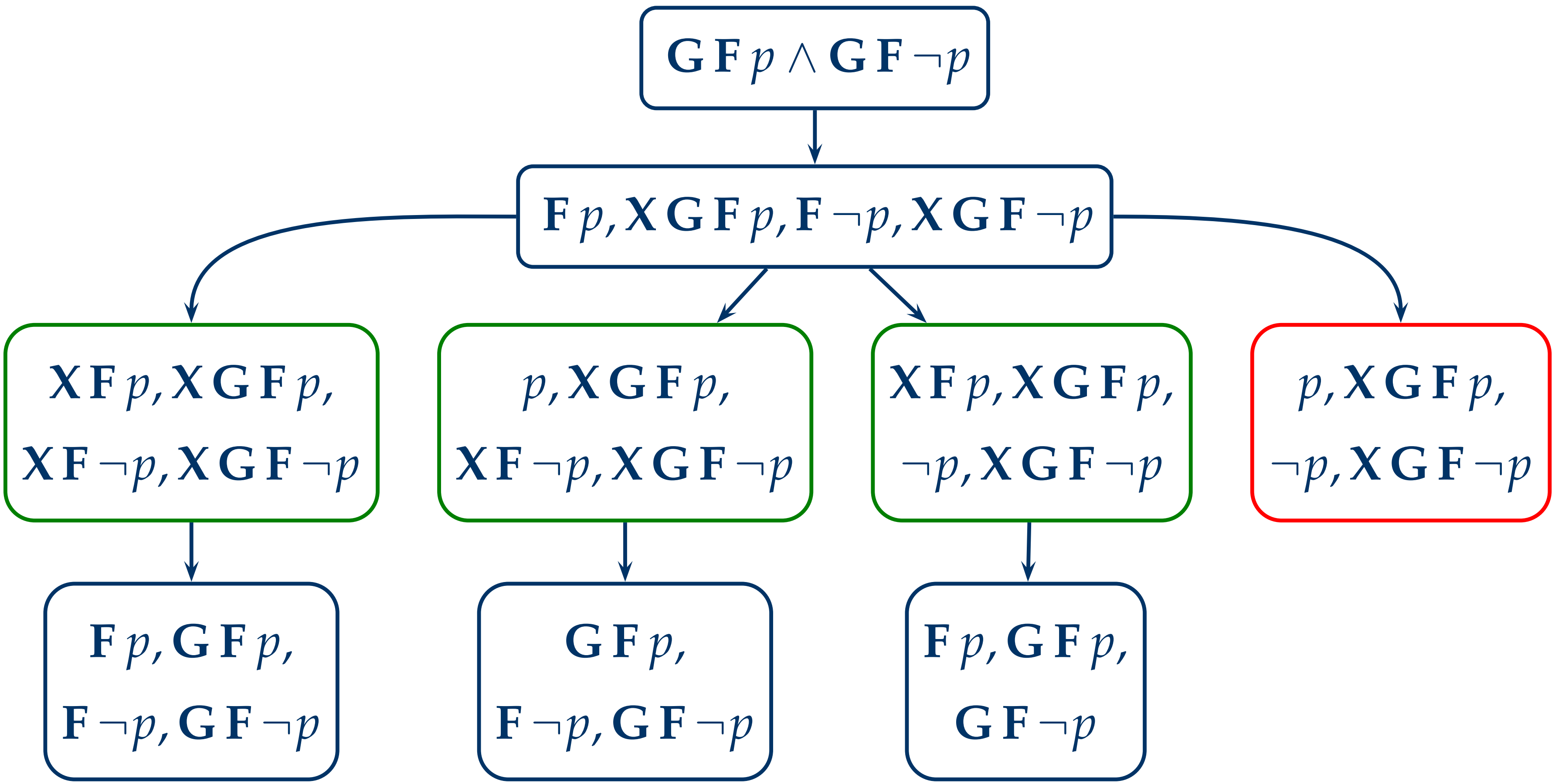

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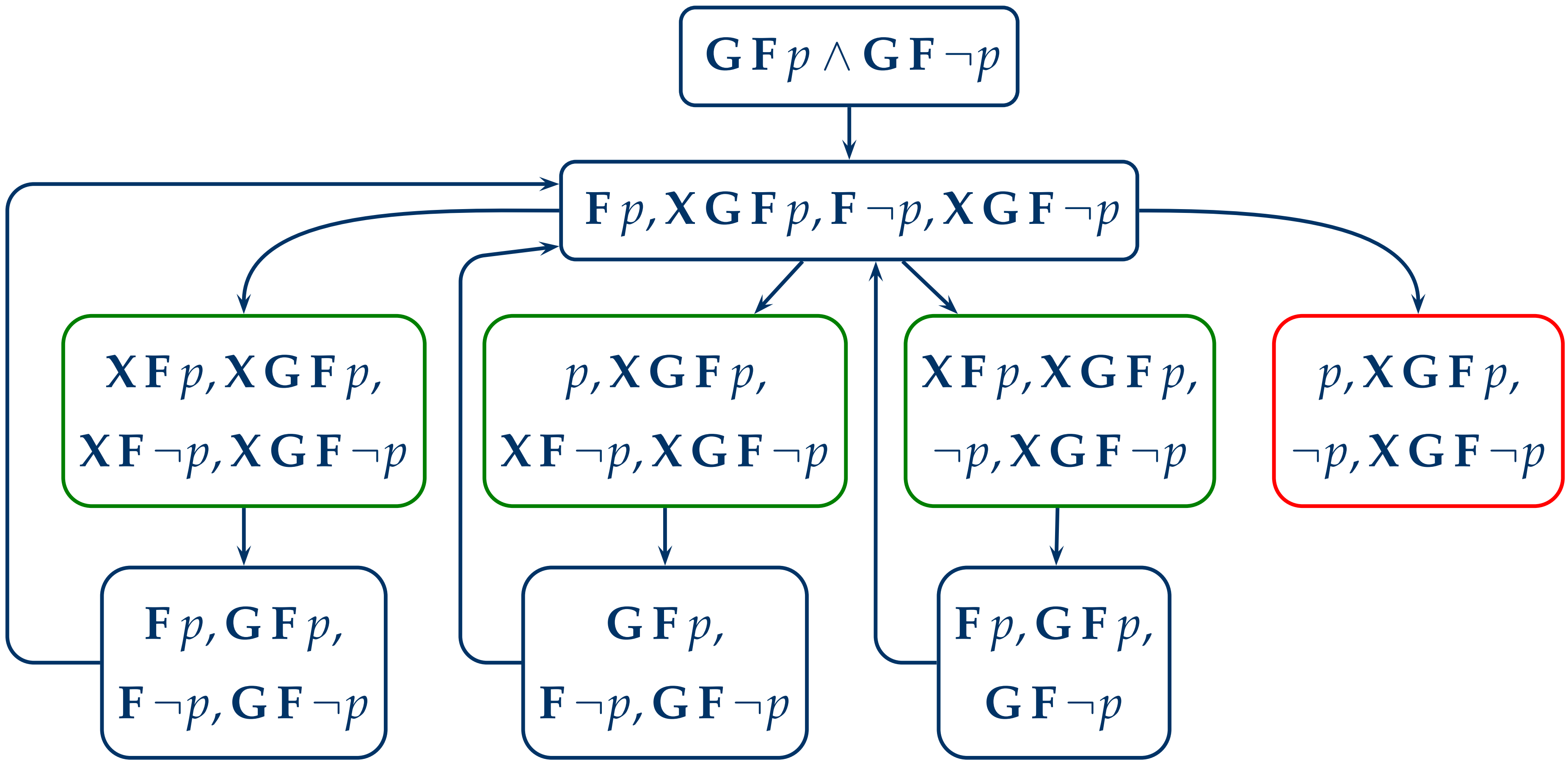

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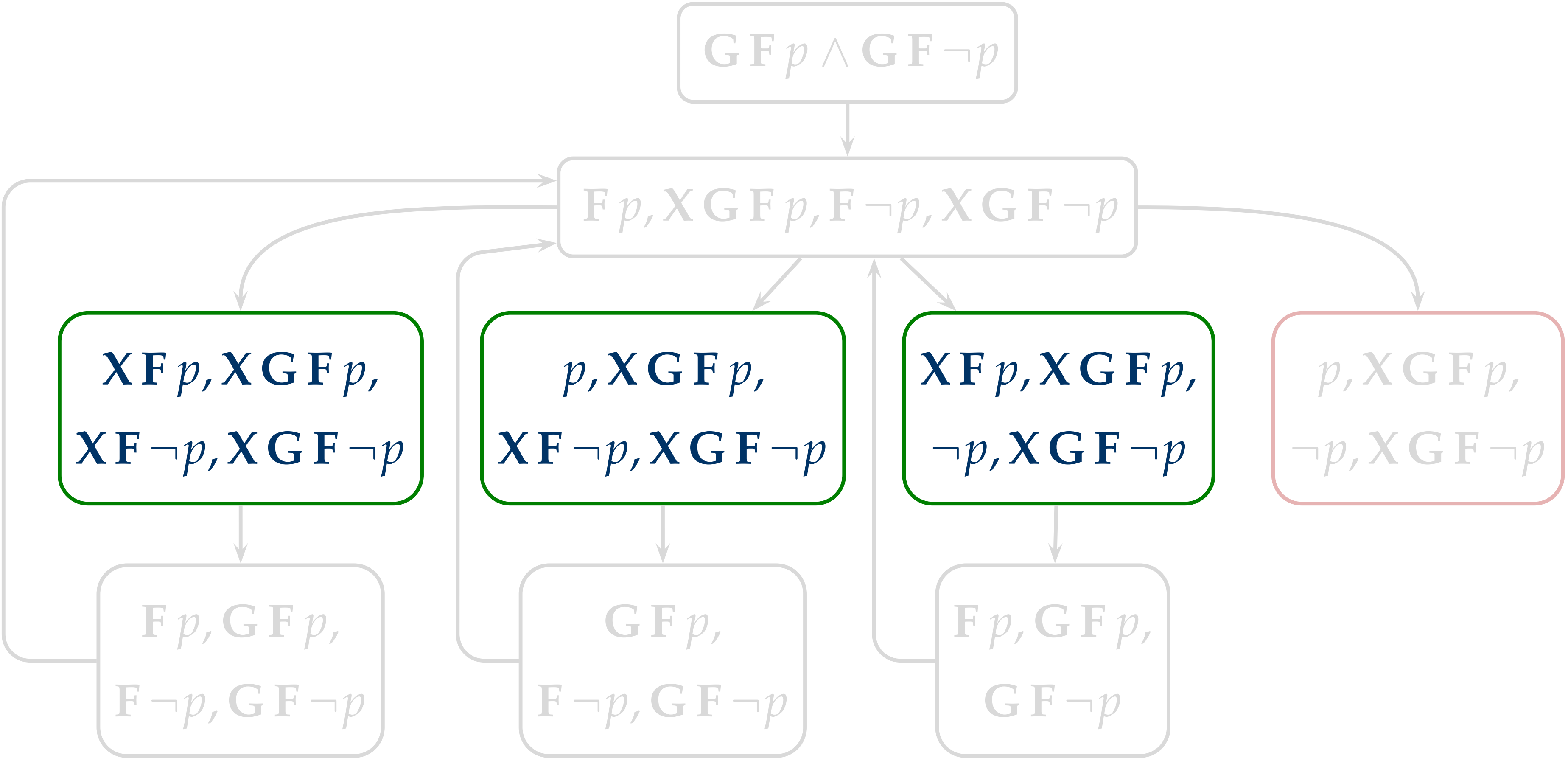
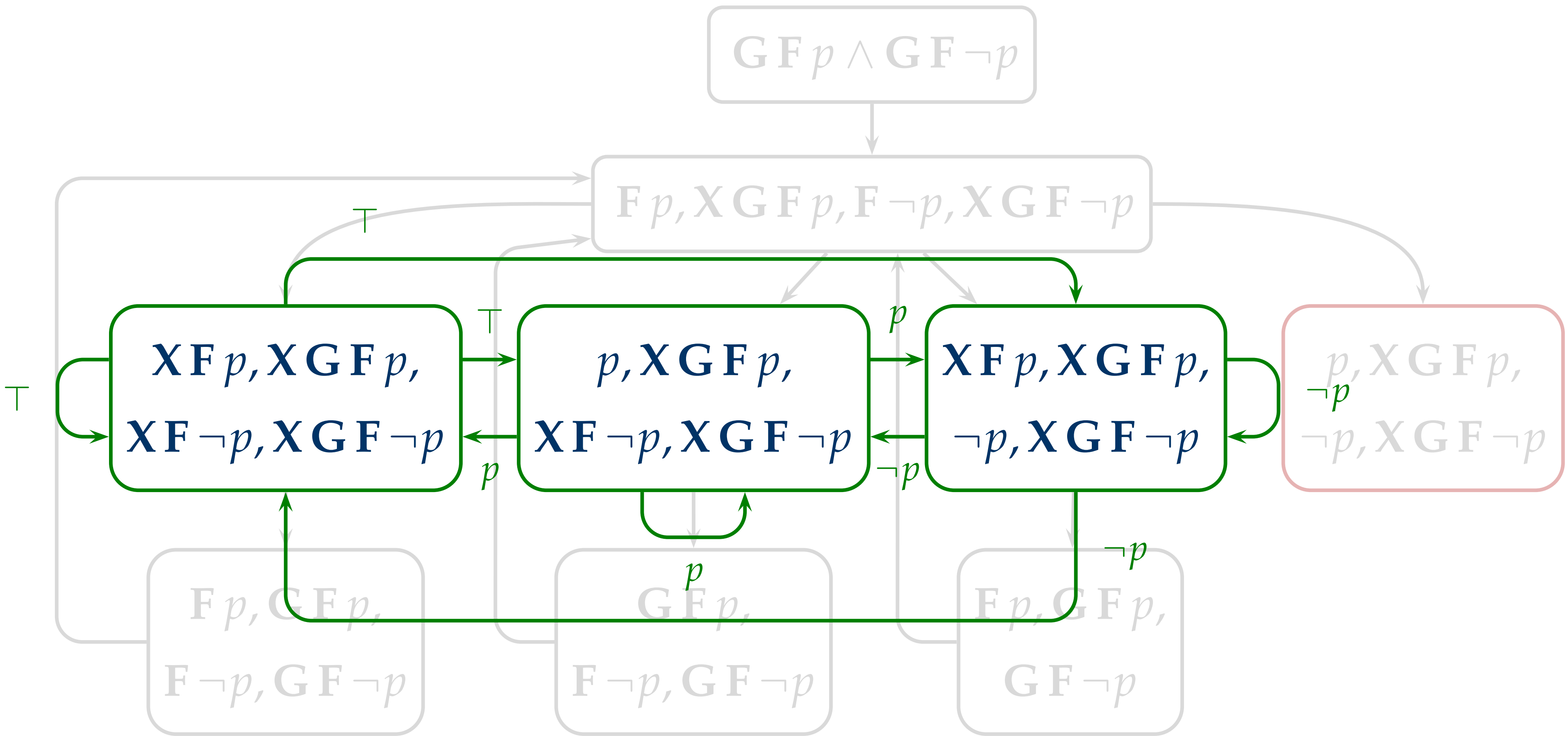
- tableau states yield locations of GBA



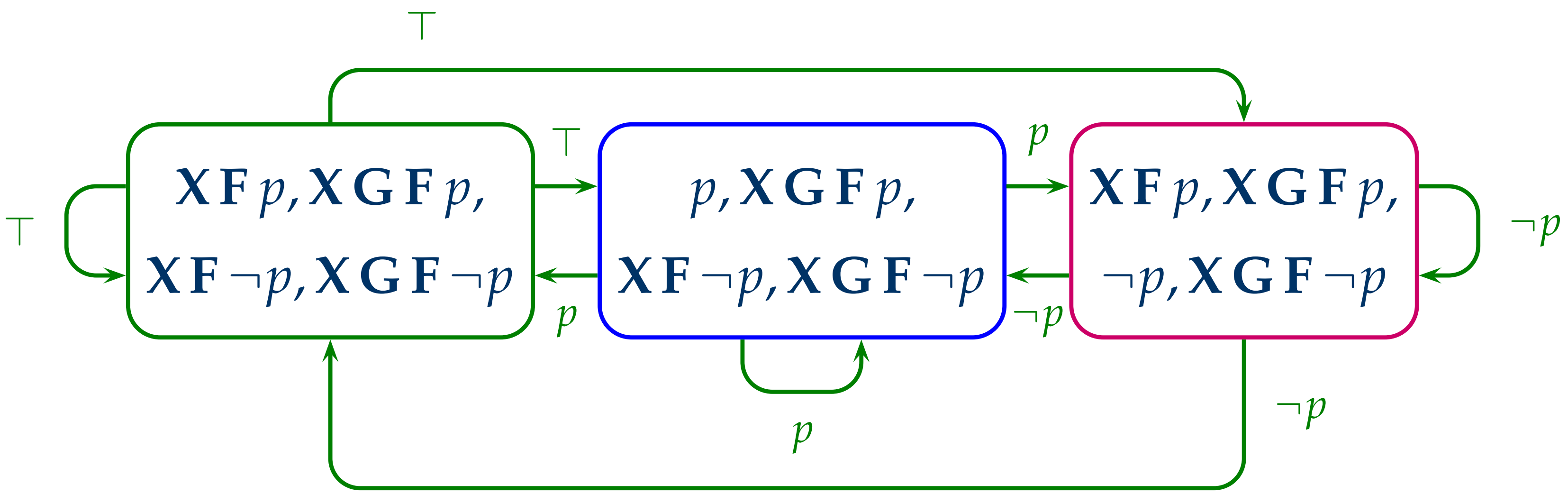
GBA for $\mathbf{G}\,\mathbf{F}\,p \wedge \mathbf{G}\,\mathbf{F}\,\neg p$

- follow paths in tableau to obtain GBA transitions



GBA for $\mathbf{G}\,\mathbf{F}\,p \wedge \mathbf{G}\,\mathbf{F}\,\neg p$

- acceptance conditions from formulas $\mathbf{F}\,p$ and $\mathbf{F}\,\neg p$



Practical considerations

- size of $\mathcal{A}_\varphi$ exponential in length of φ
  - minimizing Büchi automata is hard
  - analyze SCCs to reduce automaton size

    [Gastin & Oddoux, 2001]
  - apply simulation relations to reduce automaton size

    [Etessami et al, 2000, 2001]
- worst-case complexity of LTL model checking

$$O(|\mathcal{T}| \cdot |\mathcal{A}_{\neg\varphi}|) = O(|\mathcal{T}| \cdot 2^{|\varphi|})$$

State explosion problem

- $\mathcal{T} \times \mathcal{A}_{\neg\varphi}$ is too big to be computed effectively
- problems start around 10^6 – 10^7 states
- partial remedies (mainly concerning $\mathcal{T}$)

reduce	ignore irrelevant parts of $\mathcal{T} \times \mathcal{A}_{\neg\varphi}$	
	on-the-fly	interleave construction and verification
	partial-order	identify equivalent interleavings
compress	construct compact representations of $\mathcal{T} \times \mathcal{A}_{\neg\varphi}$	
abstract	"coarser" representation of $\mathcal{T}$	

Extensions

- Linear-time logics

PLTL	straightforward variant	[Lichtenstein&Pnueli, 1984]
NLTL	doubly exponential	[Laroussinie et al, 2002]
QPTL	non-elementary	[Sistla et al, 1987]

- Branching-time logics

satisfiability	similar theory around tree automata
model checking	tree automata introduce exponential blowup

Summing up

- mature theory and efficient implementations
- logic and combinatorics nicely decoupled
- disadvantages
  - somewhat clumsy construction
  - automaton generation can be bottleneck
  - branching-time model checking does not fit into framework

Alternating automata (for LTL)

- $\mathcal{A} = (\mathcal{V}, Q, q_0, \delta, \mathcal{F})$

$\mathcal{V}$	finite set of atomic propositions
Q	finite set of locations $\quad (\mathcal{V} \cap Q = \emptyset)$
$q_0 \in Q$	initial location
$\delta : Q \to \mathcal{B}^+(\overline{\mathcal{V}} \cup Q)$	transition function
$\mathcal{F}$	acceptance condition (*e.g.*, Büchi)

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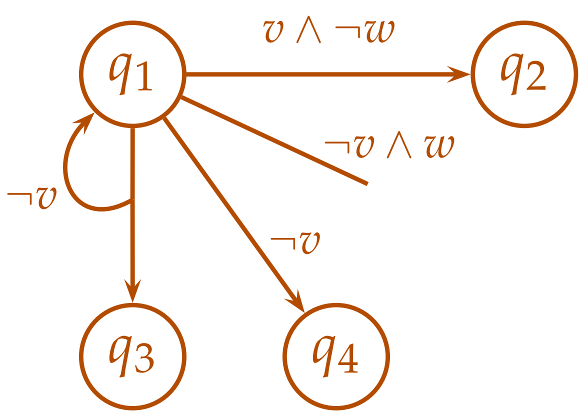
- interpretation of transitions

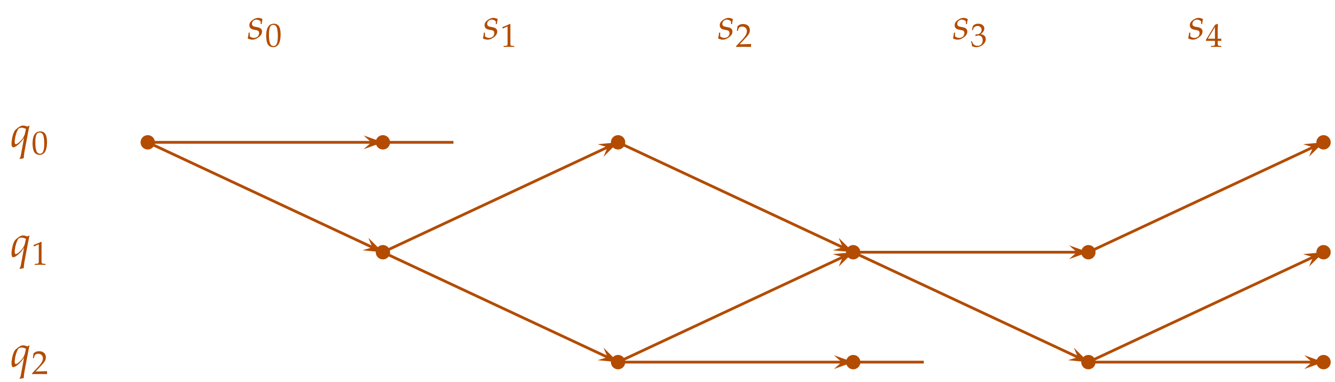
$$\begin{array}{rcl} \delta(q_1) & = & v \wedge \neg w \wedge q_2 \\ & \vee & \neg v \wedge (q_4 \vee (q_1 \wedge q_3)) \\ & \vee & \neg v \wedge w \end{array}$$



Alternating automata: runs

- run of $\mathcal{A}$ over temporal structure $s_0 s_1 \ldots$ is a dag

initialization	dag rooted at initial location q_0
consecution	current state and next dag slice satisfy transitions



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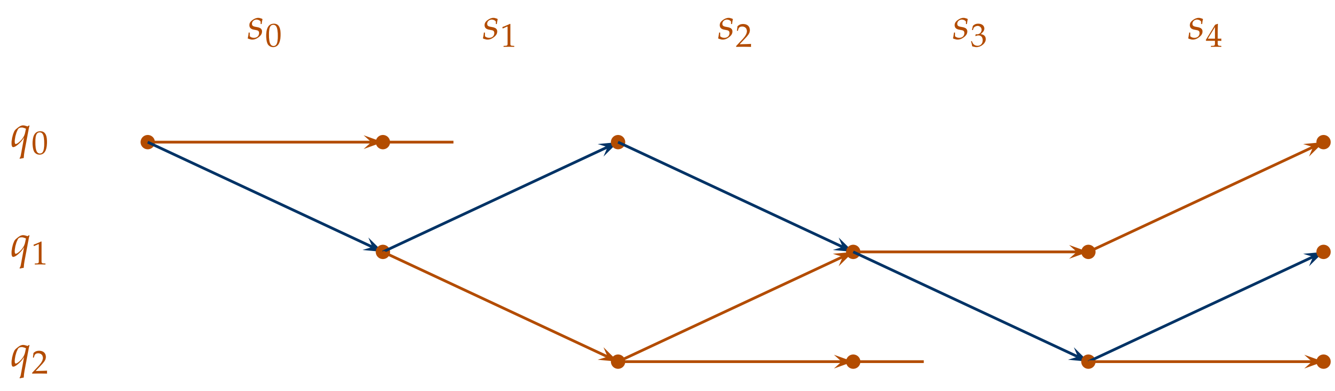
acceptance	must hold for all infinite paths through the dag

- language: ω-words for which there is accepting dag



Alternating automata: results

- no more powerful than Büchi automata
  - subset construction for translation [Miyano&Hayashi 1984]
  - exponentially more succinct
  - offer both non-determinism and parallelism
- closure properties
  - union and intersection: change initial state
  - complement also simple:
    dualize transition relation and modify acceptance condition

Weak alternating automata

- ranking function on states $\rho : Q \to \mathbb{N}$

$\rho(q') \leq \rho(q)$	whenever q' occurs in $\delta(q)$
"limit rank"	every infinite path eventually at constant rank
acceptance	limit rank is even

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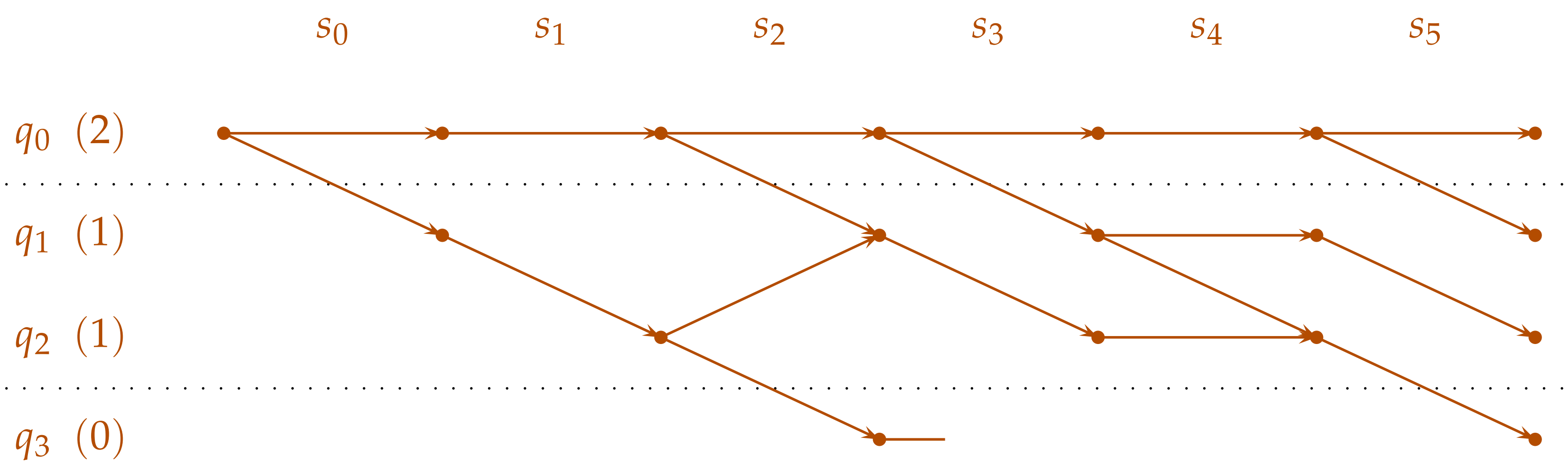

From LTL to WAA

- construct $\mathcal{A}_\varphi$ such that $\mathcal{L}(\mathcal{A}_\varphi) = \mathcal{L}(\varphi)$

locations	a location ψ for every subformula ψ
initial location	location φ corresponding to φ
transitions	non-temporal formulas: ensure satisfaction
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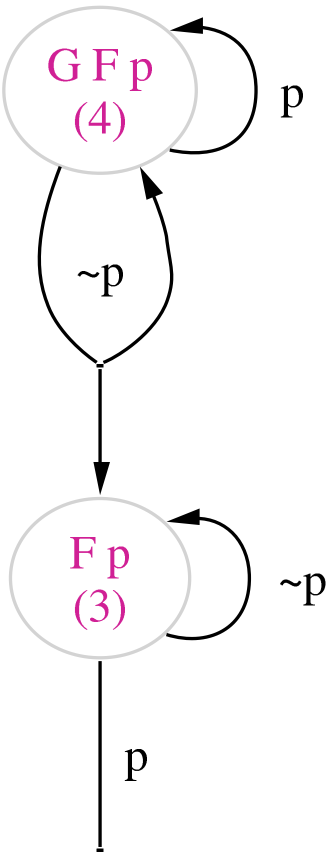
location	δ	ρ
ψ non-temporal	ψ	0
$\mathbf{X}\,\psi$	ψ	$\rho(\psi)$
$\mathbf{G}\,\psi$	$\delta(\psi) \wedge \mathbf{G}\,\psi$	$\geq \rho(\psi)$ even
$\mathbf{F}\,\psi$	$\delta(\psi) \vee \mathbf{F}\,\psi$	$\geq \rho(\psi)$ odd
$\psi_1 \overset{\wedge}{\vee} \psi_2$	$\delta(\psi_1) \overset{\wedge}{\vee} \delta(\psi_2)$	$\max(\rho(\psi_1), \rho(\psi_2))$

Examples

G F p



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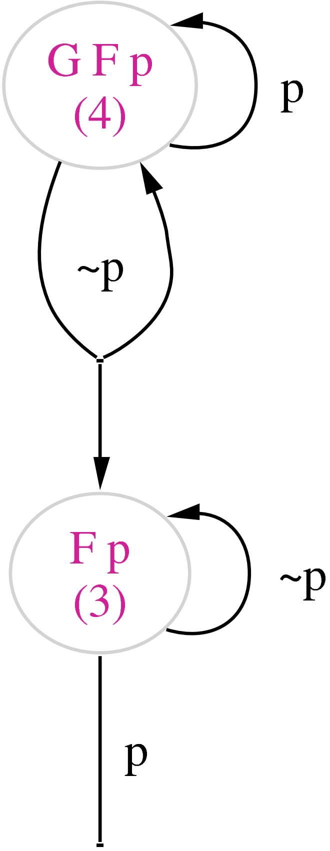
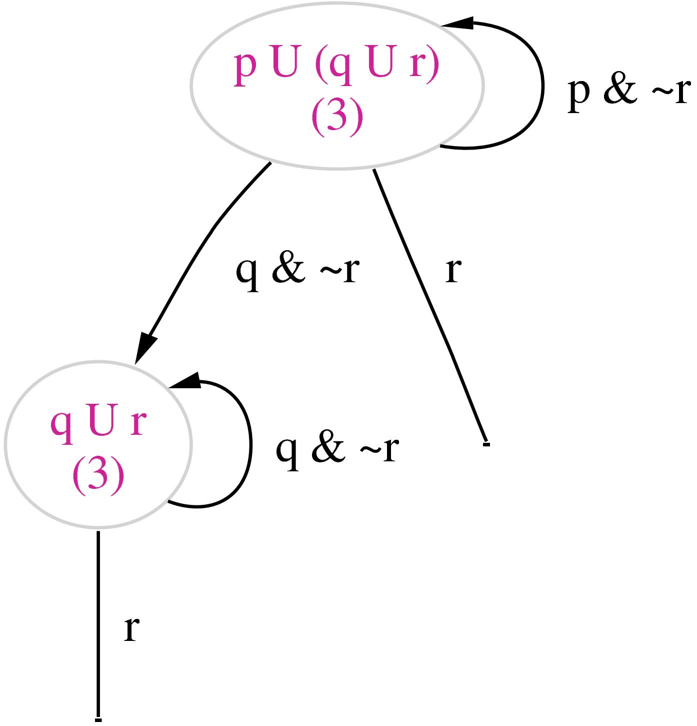


$p\ \mathbf{U}\ (q\ \mathbf{U}\ r)$



Linear WAA

- format of recursion laws for LTL $\quad \varphi \equiv \ldots$
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- linear WAA: graph has only trivial loops
  - $\rightsquigarrow$ $q = q'$ holds whenever $q \Longrightarrow^+ q'$ and $q' \Longrightarrow^+ q$
- run dag contains no “rising edges”
  - $\rightsquigarrow$ non-accepting infinite path must be trapped at odd state

Emptiness checking

- traditionally: transformation to Büchi automaton
  [Gastin&Oddoux 2001, Fritz&Wilke 2003]

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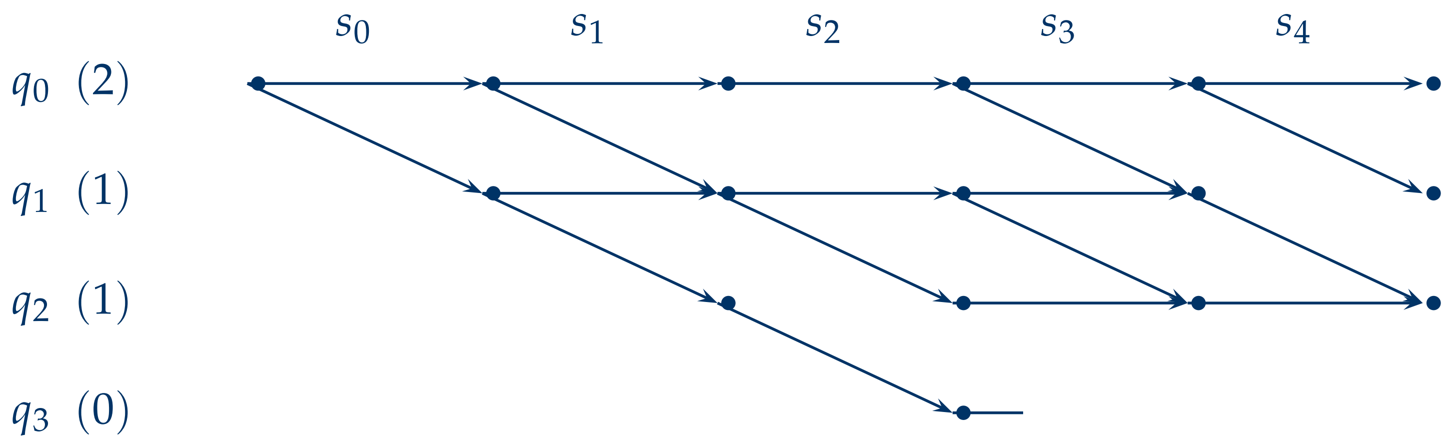
- direct non-emptiness criterion for linear WAA



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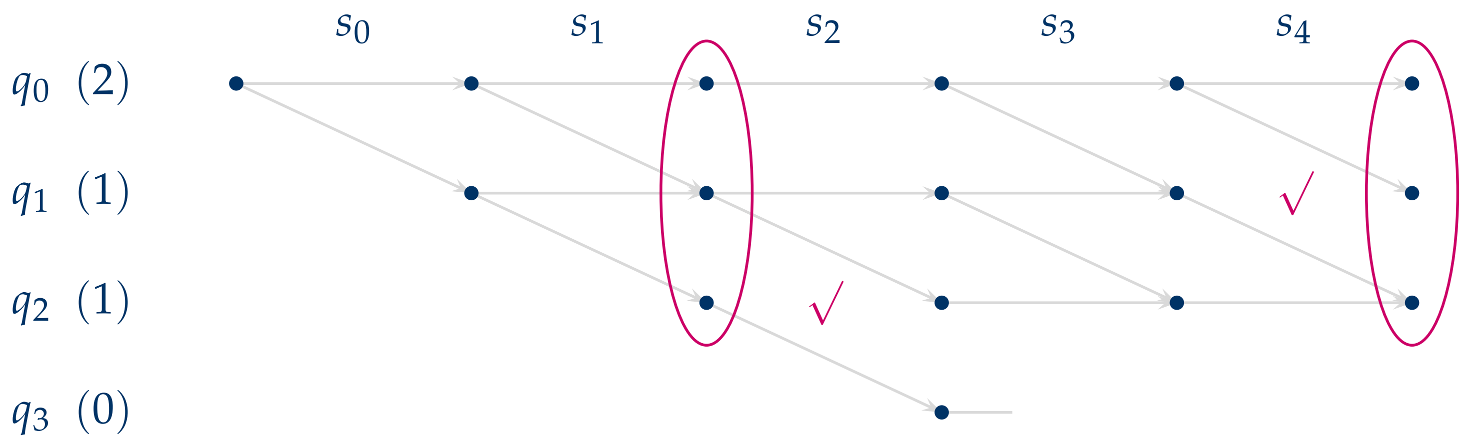

$\Rightarrow$ $s_0 s_1 (s_2 s_3 s_4)^\omega$ will be accepted by $\mathcal{A}$

Practical considerations

- prototype implementation
  - encoding of run dags for SAT solvers
  - direct search based on Tarjan's algorithm
- extension to model checking as for Büchi automata
- emptiness checking exponential in size of WAA
  - minimizing WAAs is hard (but automata are small)
  - simple ad-hoc optimizations
  - apply simulation relations [Fritz&Wilke 2003]

Branching time: principle

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- automata-theoretic approach [Kupferman et al, 1999]

  - alternating tree automaton $\mathcal{A}_{\mathcal{D},\varphi}$
    $\rightsquigarrow$ recognize the models of φ with out-degrees in $\mathcal{D}$

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    $\rightsquigarrow$ recognize the models of φ with out-degrees in $\mathcal{D}$

  - alternating 1-letter word automaton $\mathcal{A}_{\mathcal{T},\varphi} = \mathcal{T} \times \mathcal{A}_{\mathcal{D},\varphi}$
    $\rightsquigarrow$ simulate run of $\mathcal{T}$ over $\mathcal{A}_{\mathcal{D},\varphi}$

Branching time: principle

- model checking problem $\mathcal{T} \models \varphi$

  consider $\mathcal{T}$ as a whole, not its computations

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  - either $\mathcal{L}(\mathcal{A}_{\mathcal{T},\varphi}) = \{\mathcal{T}\}$ or $\mathcal{L}(\mathcal{A}_{\mathcal{T},\varphi}) = \emptyset$
    $\rightsquigarrow$ decide emptiness problem for $\mathcal{A}_{\mathcal{T},\varphi}$

Branching time: results

- CTL and alternation-free μ-calculus
  - $\mathcal{A}_{\mathcal{D},\varphi}$ and $\mathcal{A}_{\mathcal{T},\varphi}$ are WAAs
  - size linear in $|\varphi|$ and $|\mathcal{T}|$
  - emptiness problem for 1-letter WAAs linear

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- matches time complexity, tightens space complexity

Thank you — questions?