Example of the DP Algorithm

Set of clauses

$$\begin{aligned}\Sigma = \{ \quad & \{p, q\},\\ & \{r, \neg p, q\},\\ & \{\neg q, \neg p\},\\ & \{\neg r, q\}\\ & \}\end{aligned}$$

Σ is not empty, $\{\} \notin \Sigma$ and there is no unit clauses. We can only apply the split rule.

Case 1): Split Rule, Test for true We chose to make p **true**.

$$\begin{aligned}\Sigma' = \Sigma[p := T] = \{ \quad & \{T, q\},\\ & \{r, \neg T, q\},\\ & \{\neg q, \neg T\},\\ & \{\neg r, q\} \quad \}\end{aligned}$$

We simplify Σ'

$$\begin{aligned}\Sigma' = \{ \quad & \cancel{\{T, q\}},\\ & \{r, \cancel{\neg T}, q\},\\ & \{\neg q, \cancel{\neg T}\},\\ & \{\neg r, q\} \quad \}\end{aligned}$$

Obtaining:

$$\begin{aligned}\Sigma' = \{ \quad & \{r, q\},\\ & \{\neg q\},\\ & \{\neg r, q\} \quad \}\end{aligned}$$

As there is a unit clause, we apply unit propagation and set $\neg q$ **to true**, that is q **to false**.

$$\begin{aligned}\Sigma'' = \Sigma'[\neg q := T] = \{ \quad & \{r, F\},\\ & \{T\},\\ & \{\neg r, F\} \quad \}\end{aligned}$$

We simplify Σ''

$$\begin{aligned}\Sigma'' = \{ \quad & \{r, \cancel{F}\},\\ & \cancel{\{T\}},\\ & \{\neg r, \cancel{F}\} \quad \}\end{aligned}$$

Obtaining $\Sigma'' = \{\{r\}, \{\neg r\}\}$

As there is a unit clause, we apply unit propagation and st r **to true**.

$$\Sigma''' = \Sigma''[r := T] = \{ \quad \{T\}, \{\neg T\}\}$$

We simplify Σ'''

$$\Sigma''' = \{ \quad \cancel{\{T\}}, \{ \ \cancel{\neg T} \ \}\}$$

Obtaining $\Sigma''' = \{\{\}\}$.

As $\{\} \in \Sigma$ we return UNSAT for this case (but remember that we started with a split, so we have to check the other posibility)

Case 2): Split Rule, Test for false We chose to make p **false** (that is, we make $\neg p$ **true**).

$$\begin{aligned} \Sigma' = \Sigma[p := F] = \{ \quad & \{F, q\}, \\ & \{r, T, q\}, \\ & \{\neg q, T\}, \\ & \{\neg r, q\} \quad \} \end{aligned}$$

We simplify Σ'

$$\begin{aligned} \Sigma' = \{ \quad & \{\cancel{F}, q\}, \\ & \cancel{\{r, T, q\}}, \\ & \cancel{\{\neg q, T\}}, \\ & \{\neg r, q\} \quad \} \end{aligned}$$

Obtaining:

$$\begin{aligned} \Sigma' = \{ \quad & \{q\}, \\ & \{\neg r, q\} \quad \} \end{aligned}$$

As there is a unit clause, we apply unit propagation and set q **to true**

$$\begin{aligned} \Sigma'' = \Sigma'[q := T] = \{ \quad & \{T\}, \\ & \{\neg r, T\} \quad \} \end{aligned}$$

We simplify Σ''

$$\begin{aligned} \Sigma'' = \Sigma'[q := T] = \{ \quad & \cancel{\{T\}}, \\ & \cancel{\{\neg r, T\}} \quad \} \end{aligned}$$

Obtaining $\Sigma = \{\}$, and we return SAT.

Some Remarks:

- Notice that we **cannot stop the algorithm** in the first case of the Split. We should try the other option. Otherwise we would have ansered UNSAT when the clause set is actually SAT.
- Notice that if we would have chosen to explore first the second case of the Split **we could have stoped**. If we find a SAT answer then we are done.
- Can you list which is the valuation making the clause set true?
- Can you imagine a heuristic (a strategy) that help us decide whether we explore case 1 or case 2 of the Split first?
- Can can we do, if we want to get **all** the models that satisfy certain clause set?