Instituto Federal de Educação, Ciência e Tecnologia do Rio Grande do Norte
Campus Natal Central
Diretoria Acadêmica de Gestão e Tecnologia da Informação
Cálculo Diferencial e Integral

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A S S Í N T O T A S

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> restart :
> with(plots) : with(plottools) :
> with(Student[Calculus1]) : with(Student[Precalculus]) :
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Assíntotas: são curvas (retas horizontais, retas verticais, retas oblíquas e curvas em gerais), em que nos limites infinitos e/ou limites nos infinitos, a diferença entre o gráfico da expressão (ou função) e essas curvas (assíntotas) tendem a zero.

Assíntotas Horizontais (AH), Verticais (AV), Oblíquas (AO) e Curvilíneas (AC):

1. AH ==> reta $y = k$

$$\lim_{x\to\infty} f(x) = k \quad ou \quad \lim_{x\to-\infty} f(x) = k$$

2. AV ==> reta $x = c$

$$\lim_{x\to c^+} f(x) = \infty \; ou \quad \lim_{x\to c^+} f(x) = -\infty \quad ou \quad \lim_{x\to c^-} f(x) = \infty \; ou \quad \lim_{x\to c^-} f(x) = -\infty$$

Sendo $f(x) = \dfrac{p(x)}{q(x)}$, resolver a equação $q(x) = 0$.

3. AO ==> reta $y = m \cdot x + b$, onde:

$$m = \lim_{x\to\infty} \frac{f(x)}{x} \quad ou \quad m = \lim_{x\to-\infty} \frac{f(x)}{x}$$

$$b = \lim_{x\to\infty} (f(x) - m\,x) \quad ou \quad b = \lim_{x\to-\infty} (f(x) - m\,x)$$

4. AC ==> curva Q(x)

Sendo $f(x) = \dfrac{p(x)}{q(x)}$, *então podemos escrever* :

$$f(x) = \frac{p(x)}{q(x)} = Q(x) + \frac{R(x)}{q(x)} ,$$

$$onde : Q(x) = quociente\ da\ divisão$$
$$e \quad R(x) = resto\ da\ divisão$$

Exemplo 01:

> $f(x) = \frac{4}{x+3}$

Assíntota Horizontal: $y = 0$

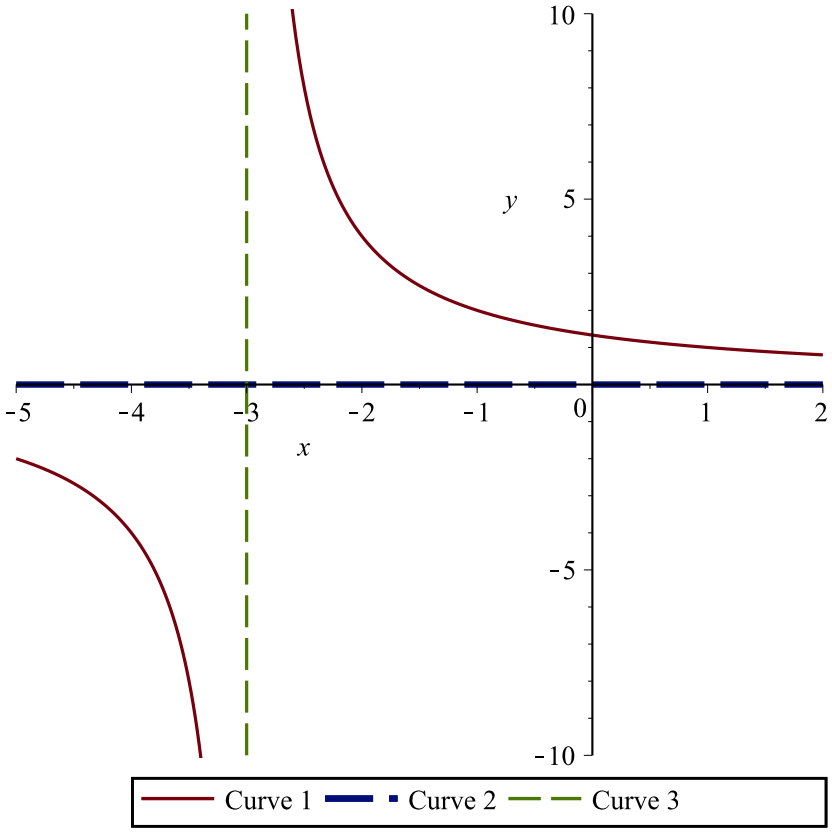
$$\lim_{x \to \infty} \frac{4}{x+3} = 0 \quad \textbf{(1)}$$

Assíntota Vertical: $x = -3$

> $x = solve(x+3=0, x);$

$$x = -3 \quad \textbf{(2)}$$

$$\lim_{x \to -3+} \frac{4}{x+3} = \infty \quad \textbf{(3)}$$



Exemplo 02:

> $f(x) = \frac{3 \cdot x}{x-2}$

Assíntota Horizontal: $y = 3$

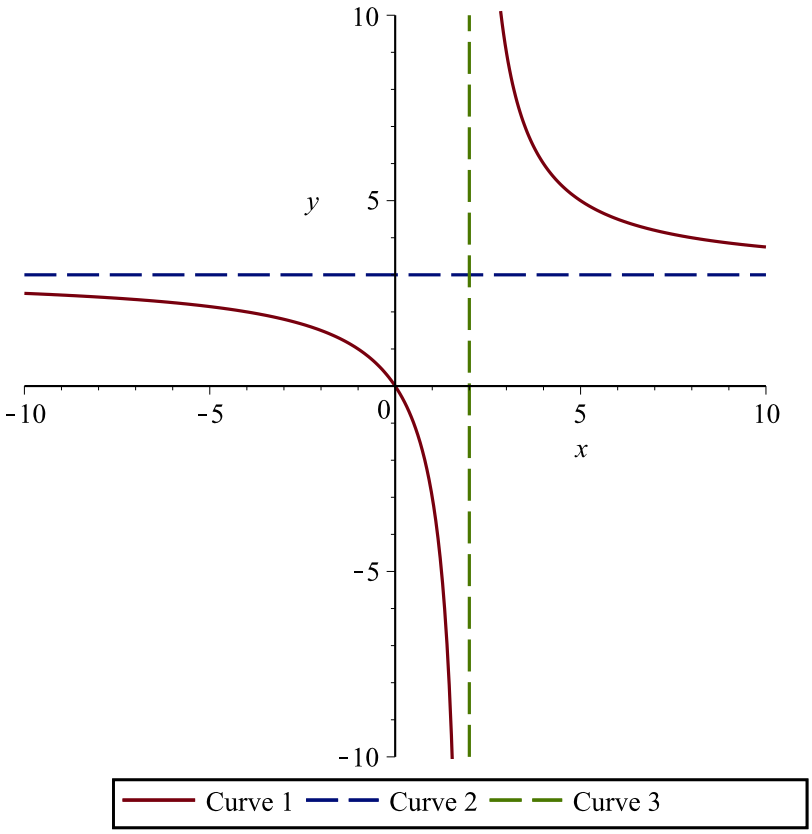
$$\lim_{x \to \infty} \frac{3x}{x-2} = 3 \quad \textbf{(4)}$$

Assíntota Vertical: $x = 2$

> $x = solve(x - 2 = 0, x);$

$$x = 2 \quad \textbf{(5)}$$

$$\lim_{x \to 2^+} \frac{3x}{x-2} = \infty \quad \textbf{(6)}$$



Exemplo 03:

> $f(x) = \frac{x-1}{x+1}$

Assíntota Horizontal: $y = 1$

$$\lim_{x \to \infty} \frac{x-1}{x+1} = 1 \quad \textbf{(7)}$$

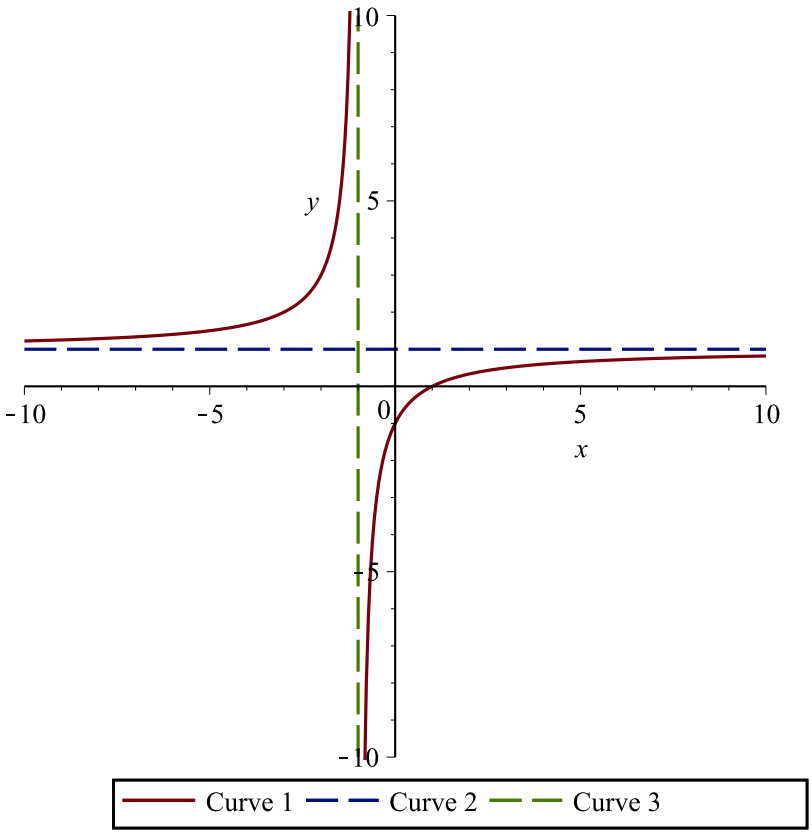
Assíntota Vertical: $x = -1$

> $x = solve(x + 1 = 0, x);$

$$x = -1 \quad \textbf{(8)}$$

$$\lim_{x \to -1^+} \frac{x-1}{x+1} = -\infty \quad \textbf{(9)}$$

$$\lim_{x \to -1^-} \frac{x-1}{x+1} = \infty \quad \textbf{(10)}$$



Exemplo 04:

> $f(x) = \frac{1}{x^2 - 4}$

Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{1}{x^2 - 4} = 0 \quad \textbf{(11)}$$

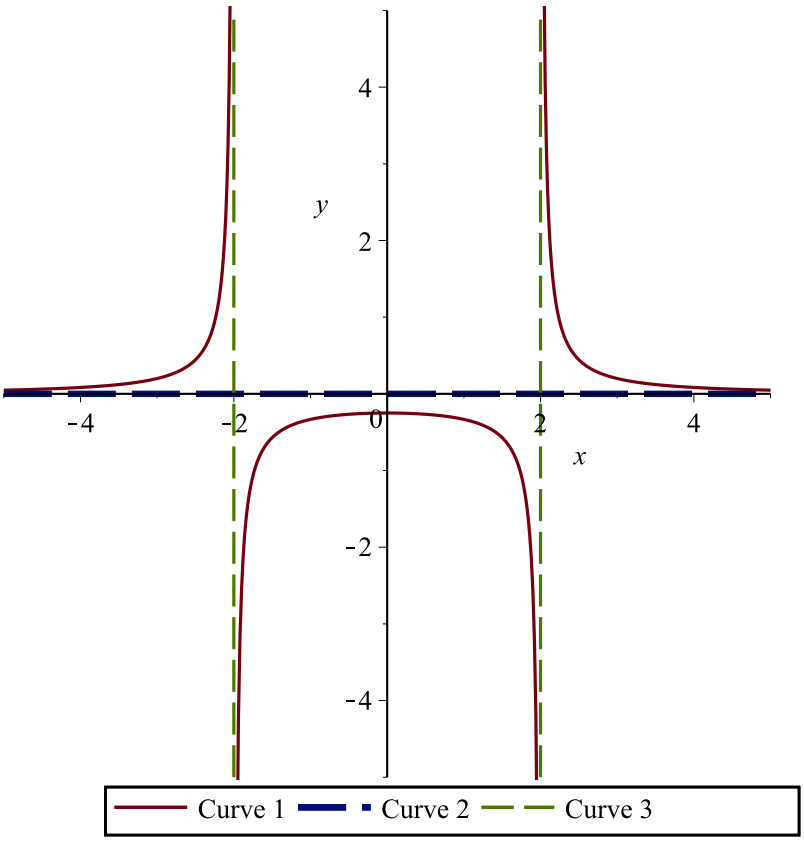
Assíntota Vertical: $x = 2$ ou $x = -2$

> $x = solve(x^2 - 4 = 0, x);$

$$x = (2, -2) \quad \textbf{(12)}$$

$$\lim_{x \to -2+} \frac{1}{x^2 - 4} = -\infty \quad \textbf{(13)}$$

$$\lim_{x \to 2+} \frac{1}{x^2 - 4} = \infty \quad \textbf{(14)}$$



Exemplo 05:

> $f(x) = \dfrac{x}{x^2 - 4}$

Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{x}{x^2 - 4} = 0 \qquad \textbf{(15)}$$

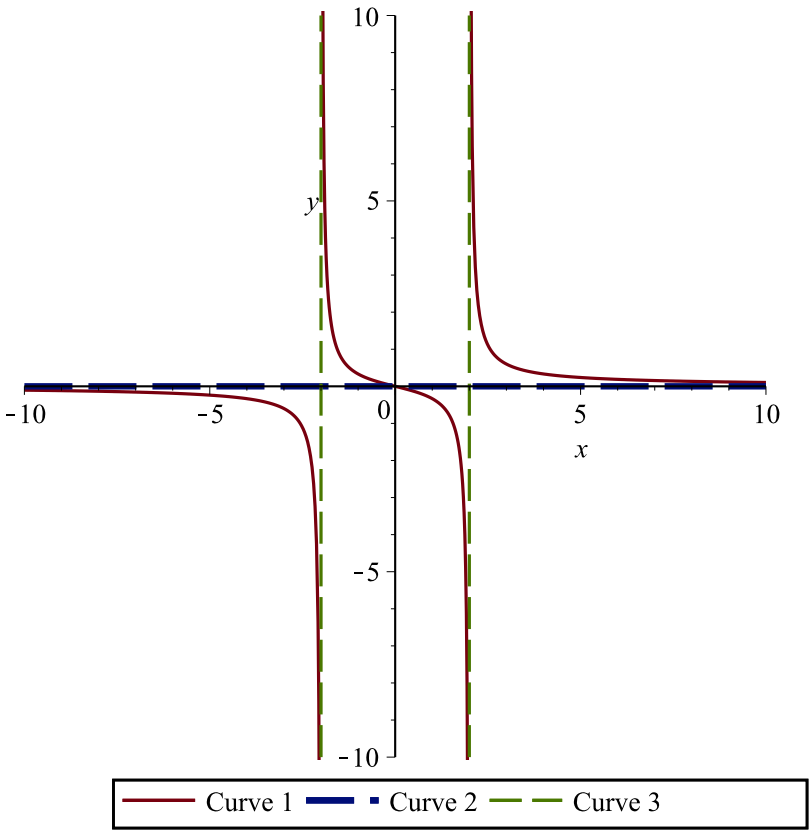
Assíntota Vertical: $x = 2$ ou $x = -2$

> $x = solve(x^2 - 4 = 0, x);$

$$x = (2, -2) \qquad \textbf{(16)}$$

$$\lim_{x \to -2+} \frac{1}{x^2 - 4} = -\infty \qquad \textbf{(17)}$$

$$\lim_{x \to 2+} \frac{1}{x^2 - 4} = \infty \qquad \textbf{(18)}$$



Exemplo 06:

> $f(x) = \dfrac{(x+2)}{(x^2 - 2\cdot x)}$

Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{x+2}{x^2 - 2x} = 0 \qquad \textbf{(19)}$$

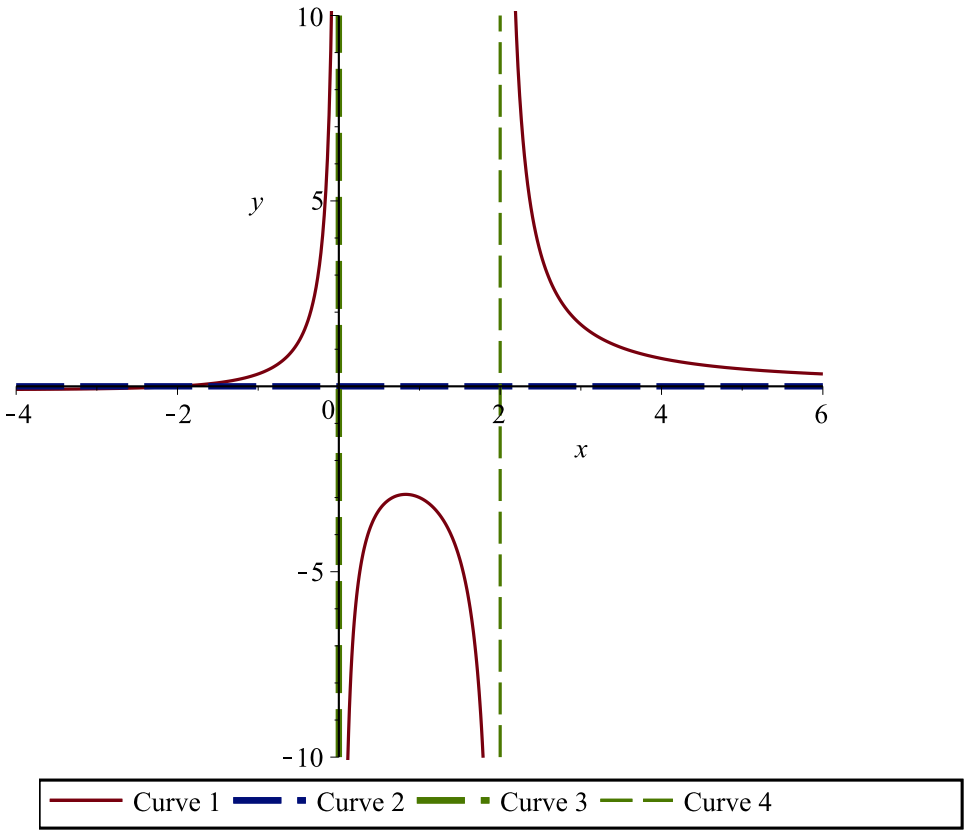
Assíntota Vertical: $x = 0$ ou $x = 2$

> $x = solve(x^2 - 2\cdot x = 0, x);$

$$x = (0, 2) \qquad \textbf{(20)}$$

$$\lim_{x \to 0+} \frac{1}{x^2 - 2x} = -\infty \qquad \textbf{(21)}$$

$$\lim_{x \to 2+} \frac{1}{x^2 - 2x} = \infty \qquad \textbf{(22)}$$



Exemplo 07:

> $f(x) = \dfrac{1}{x^2 + 1}$

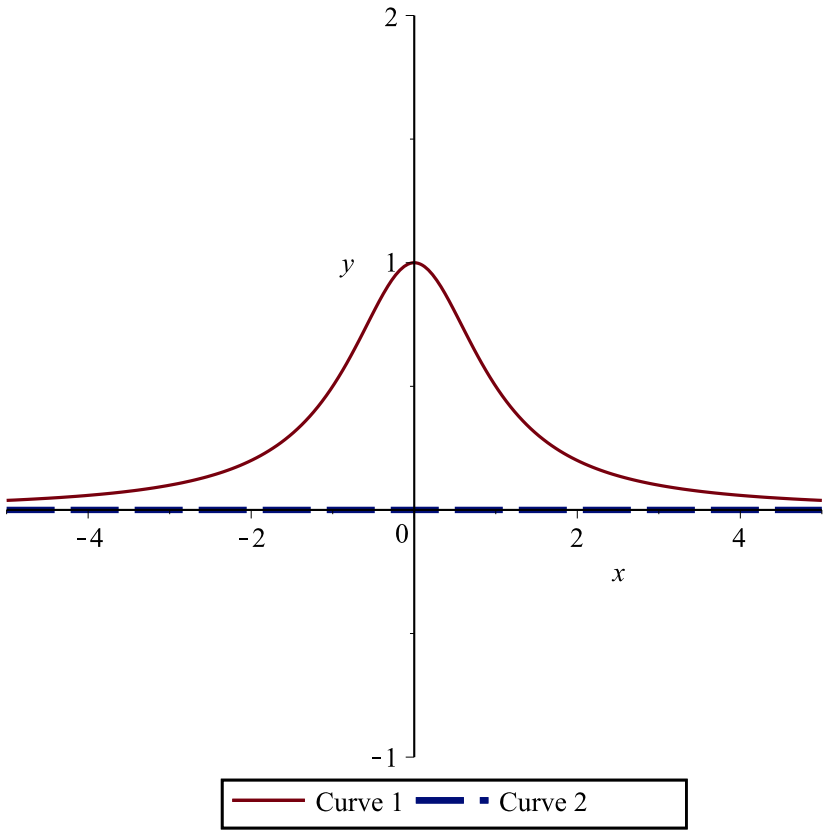
Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{1}{x^2 + 1} = 0 \qquad \textbf{(23)}$$

Assíntota Vertical: *não existem raizes reais, logo não existe AV*

> $x = solve(x^2 + 1 = 0, x);$

$$x = (I, -I) \qquad \textbf{(24)}$$



Exemplo 08:

> $f(x) = \frac{2 \cdot x}{x^2 + 1}$

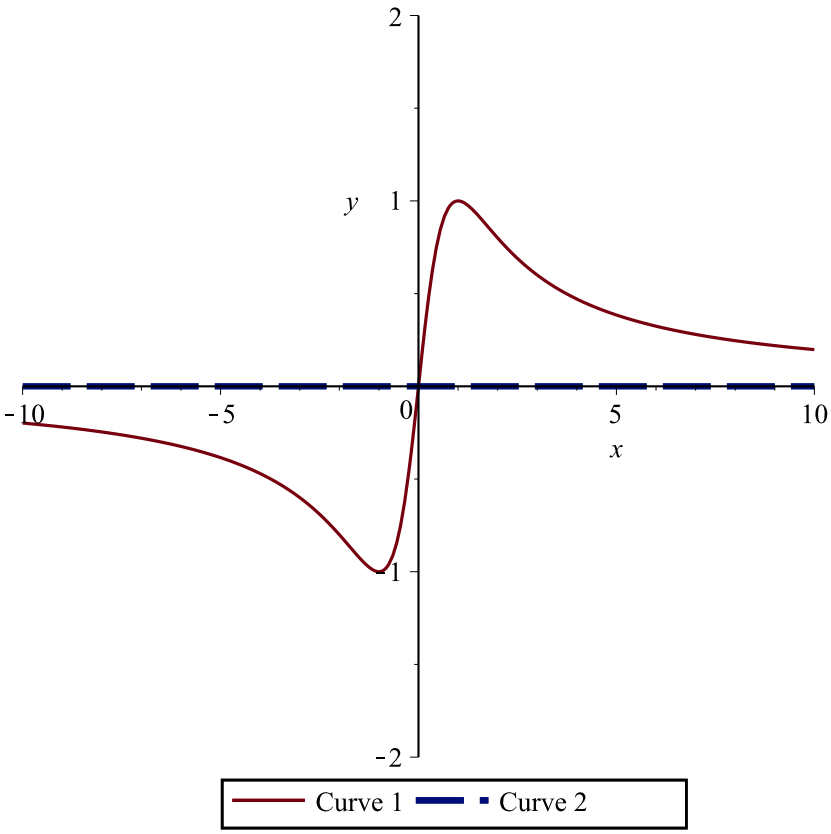
Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{2\,x}{x^2 + 1} = 0 \qquad \textbf{(25)}$$

Assíntota Vertical: *não existem raizes reais, logo não existe AV*

> $x = solve(x^2 + 1 = 0, x);$

$$x = (I, -I) \qquad \textbf{(26)}$$



Exemplo 09:

> $f(x)\ \dfrac{x^2-4}{x^2-9}$

Assíntota Horizontal: $y = 1$

$$\lim_{x\to\infty} \frac{x^2-4}{x^2-9} = 1 \qquad \textbf{(27)}$$

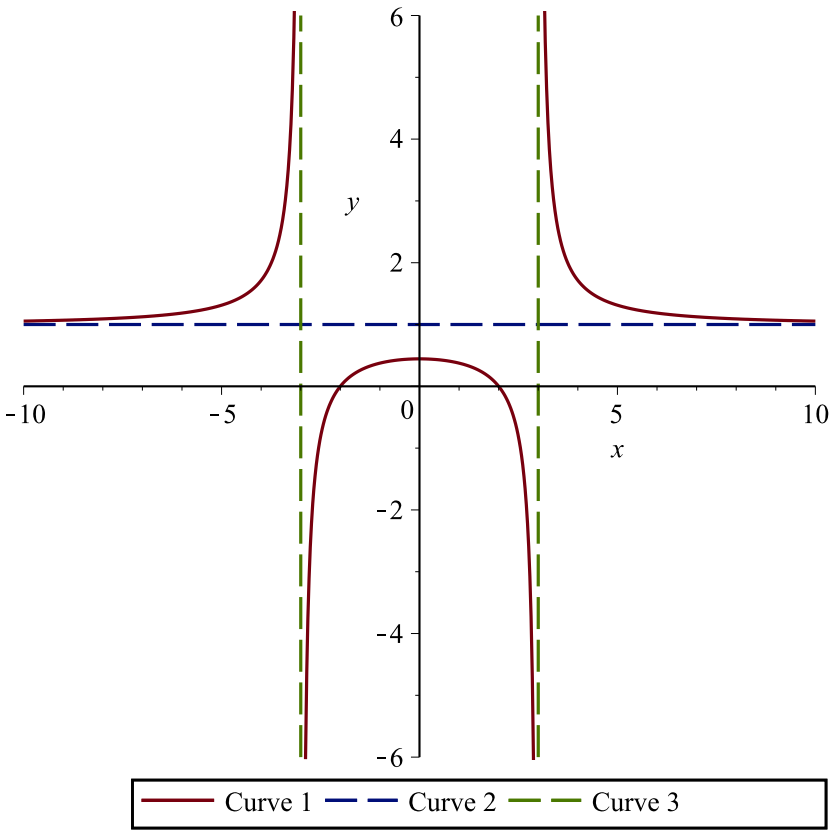
Assíntota Vertical: $x = 3$ ou $x = -3$

> $x = solve\left(x^2-9=0, x\right);$

$$x = (3,\ -3) \qquad \textbf{(28)}$$

$$\lim_{x\to -3+} \frac{x^2-4}{x^2-9} = -\infty \qquad \textbf{(29)}$$

$$\lim_{x\to 3+} \frac{x^2-4}{x^2-9} = \infty \qquad \textbf{(30)}$$



Exemplo 10:

> $f(x) = \frac{\sqrt{x^2+4}}{2x-4}$

Assíntota Horizontal: $y = \frac{1}{2}$ e $y = -\frac{1}{2}$

$$\lim_{x \to \infty} \frac{\sqrt{x^2+4}}{2x-4} = \frac{1}{2} \quad \textbf{(31)}$$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+4}}{2x-4} = -\frac{1}{2} \quad \textbf{(32)}$$

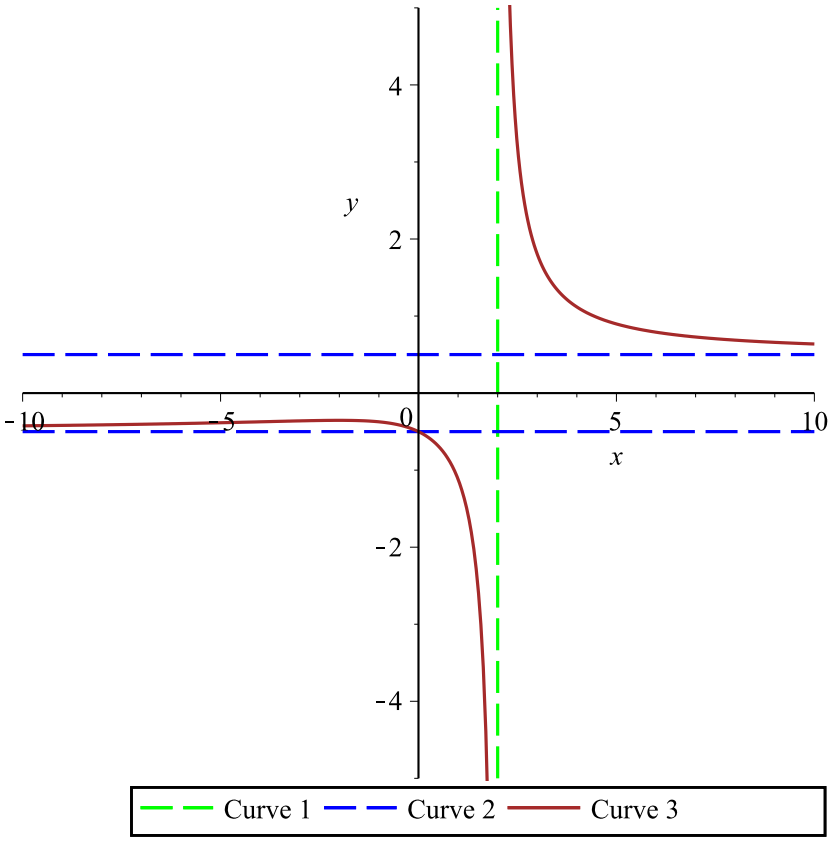
Assíntota Vertical: $x = 2$

> $x = solve(2 \cdot x - 4 = 0, x);$

$$x = 2 \quad \textbf{(33)}$$

$$\lim_{x \to 2+} \frac{\sqrt{x^2+4}}{2x-4} = \infty \quad \textbf{(34)}$$

$$\lim_{x \to 2-} \frac{\sqrt{x^2+4}}{2x-4} = -\infty \quad \textbf{(35)}$$



Exemplo 11:

> $f(x) = \dfrac{3x+4}{\sqrt{x^2-4}}$

Assíntota Horizontal: $y = 3$ *e* $y = -3$

$$\lim_{x \to \infty} \frac{3x+4}{\sqrt{x^2-4}} = 3 \qquad \textbf{(36)}$$

$$\lim_{x \to -\infty} \frac{3x+4}{\sqrt{x^2-4}} = -3 \qquad \textbf{(37)}$$

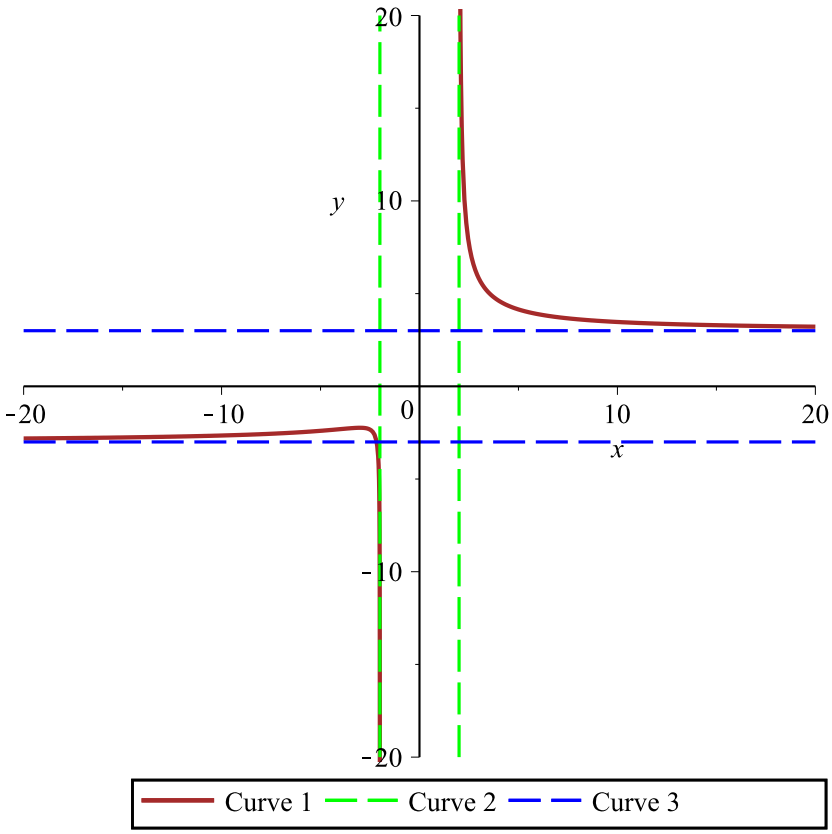
Assíntota Vertical: $x = 2$ *e* $x = -2$

> $x = solve\left(\sqrt{x^2-4} = 0, x\right);$

$$x = (2, -2) \qquad \textbf{(38)}$$

$$\lim_{x \to 2+} \frac{3x+4}{\sqrt{x^2-4}} = \infty \qquad \textbf{(39)}$$

$$\lim_{x \to -2-} \frac{3x+4}{\sqrt{x^2-4}} = -\infty \qquad \textbf{(40)}$$



Exemplo 12:

> $f(x) := \dfrac{3x+4}{\sqrt{x^2+4}}$

Assíntota Horizontal: $y = -3$ e $y = 3$

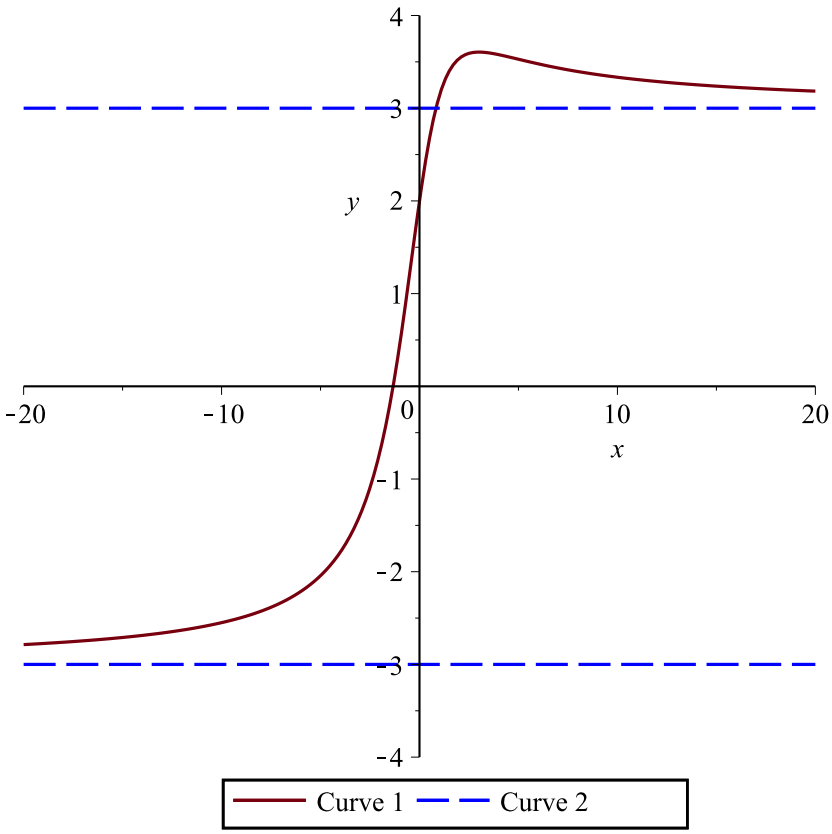
$$\lim_{x \to \infty} \frac{3x+4}{\sqrt{x^2+4}} = 3 \qquad \textbf{(41)}$$

$$\lim_{x \to -\infty} \frac{3x+4}{\sqrt{x^2+4}} = -3 \qquad \textbf{(42)}$$

Assíntota Vertical: *não existe AV*

> $x = solve\left(\sqrt{x^2+4} = 0, x\right);$

$$x = (2\,I,\ -2\,I) \qquad \textbf{(43)}$$



Exemplo 13:

> $f(x) = \frac{sen(x)}{x}$

Assíntota Horizontal: $y = 0$

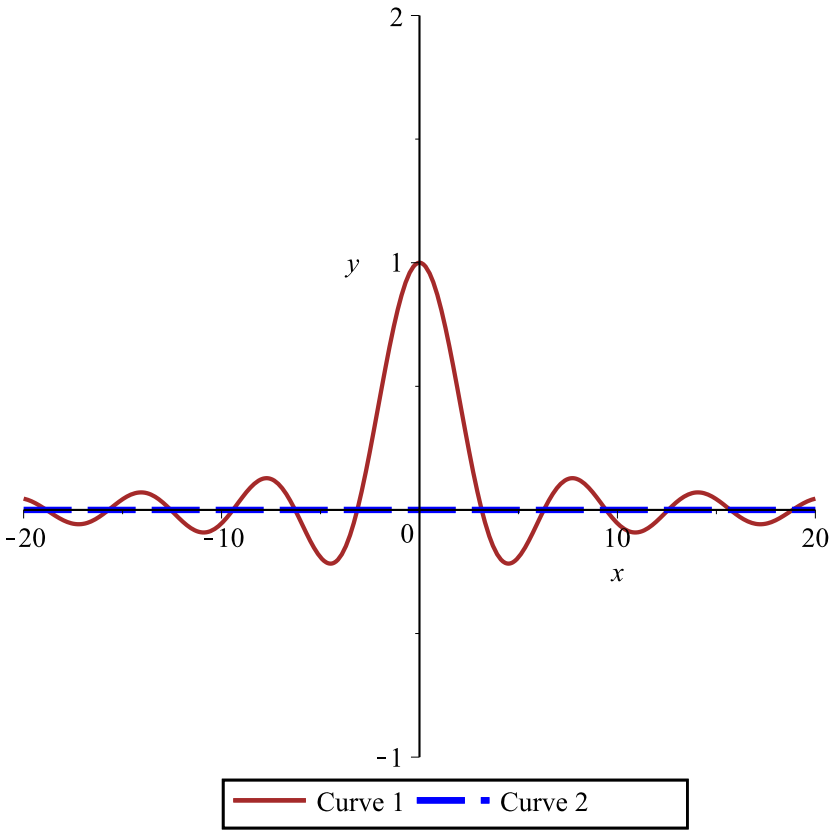
$$\lim_{x \to \infty} \frac{\sin(x)}{x} = 0 \quad \textbf{(44)}$$

$$\lim_{x \to -\infty} \frac{\sin(x)}{x} = 0 \quad \textbf{(45)}$$

Assíntota Vertical: *não existe AV*

$$\lim_{x \to 0^+} \frac{\sin(x)}{x} = 1 \quad \textbf{(46)}$$

$$\lim_{x \to 0^-} \frac{\sin(x)}{x} = 1 \quad \textbf{(47)}$$



Exemplo 14:

> $f(x) = \frac{\cos(x)}{x}$

Assíntota Horizontal: $y = 0$

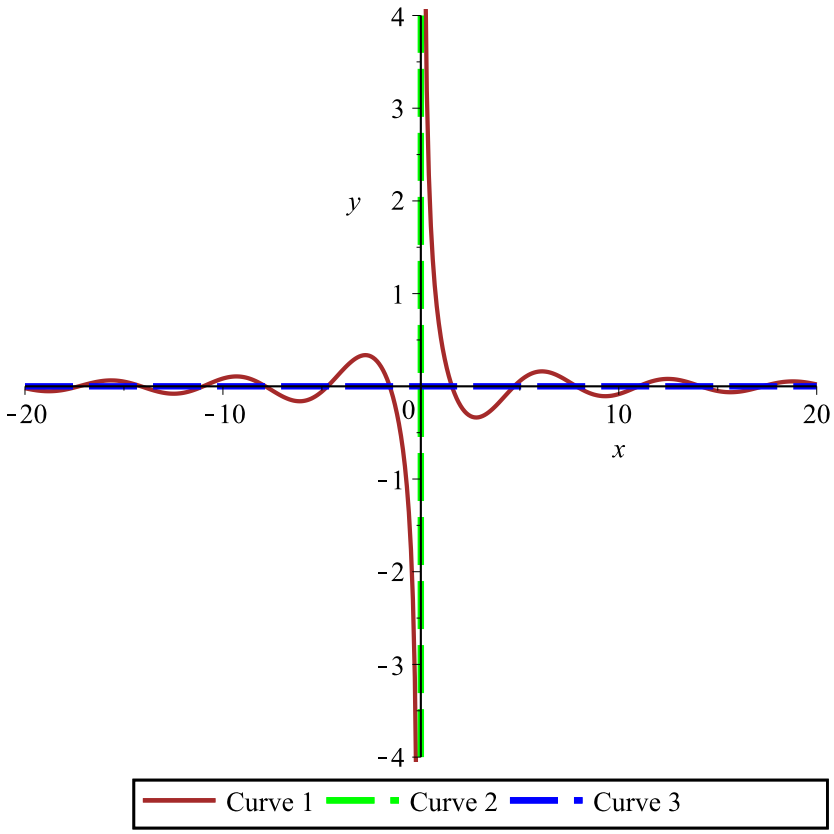
$$\lim_{x \to \infty} \frac{\cos(x)}{x} = 0 \quad \textbf{(48)}$$

$$\lim_{x \to -\infty} \frac{\cos(x)}{x} = 0 \quad \textbf{(49)}$$

Assíntota Vertical: $x = 0$

$$\lim_{x \to 0^+} \frac{\cos(x)}{x} = \infty \quad \textbf{(50)}$$

$$\lim_{x \to 0^-} \frac{\cos(x)}{x} = -\infty \quad \textbf{(51)}$$



Exemplo 15:

Síntese de Assíntotas em funções racionais polinomiais tipo: $\frac{p(x)}{q(x)}$

> $f(x) = \frac{3}{(3 \cdot x^2 + 2)}$

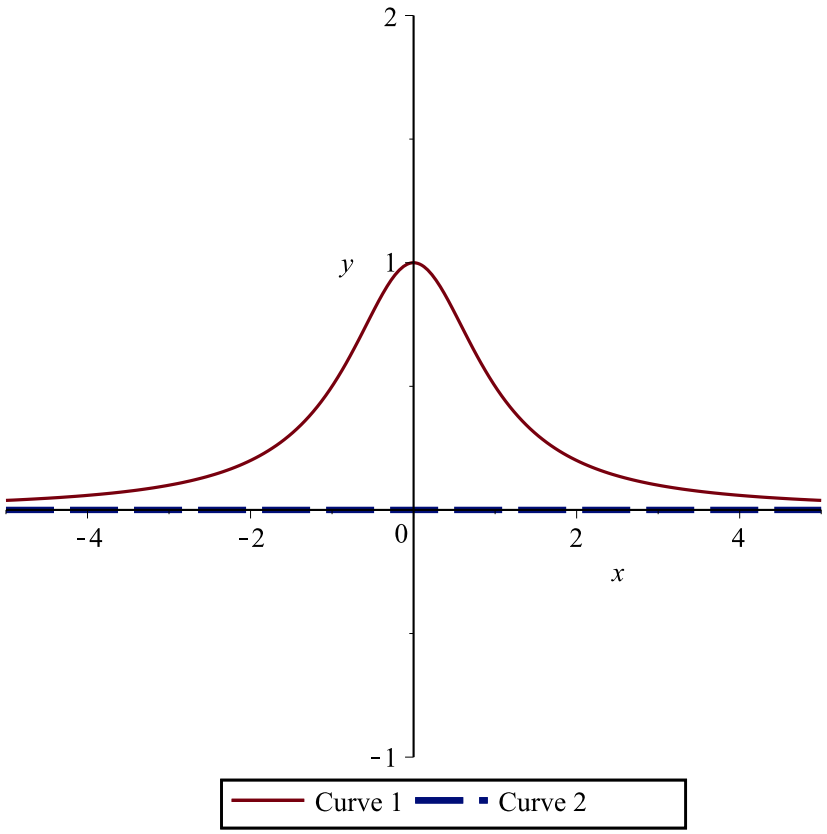
Assíntota Horizontal: $y = 0$

$$\lim_{x \to \infty} \frac{3}{3\,x^2 + 2} = 0 \qquad \textbf{(52)}$$

Assíntota Vertical: *não existem raizes reais, logo não existe AV*

> $x = solve(3 \cdot x^2 + 2 = 0, x)$;

$$x = \left(\frac{1}{3} \mathrm{I} \sqrt{6}, -\frac{1}{3} \mathrm{I} \sqrt{6} \right) \qquad \textbf{(53)}$$



Exemplo 16:

> $f(x) = \frac{(2 \cdot x + 3)}{(3 \cdot x^2 + 2)}$

Assíntota Horizontal: $y = 0$

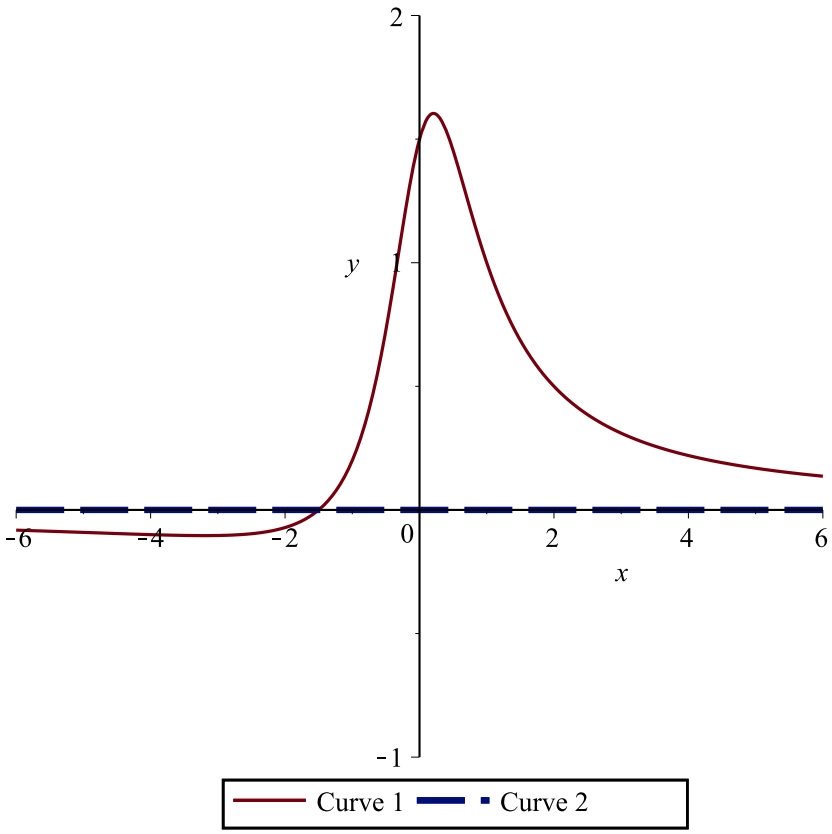
$$\lim_{x \to \infty} \frac{2x + 3}{3x^2 + 2} = 0 \quad \textbf{(54)}$$

$$\lim_{x \to -\infty} \frac{2x + 3}{3x^2 + 2} = 0 \quad \textbf{(55)}$$

Assíntota Vertical: *não existem raizes reais, logo não existe AV*

> $x = solve(3 \cdot x^2 + 2 = 0, x);$

$$x = \left(\frac{1}{3} I\sqrt{6}, -\frac{1}{3} I\sqrt{6} \right) \quad \textbf{(56)}$$



Exemplo 17:

> $f(x) = \frac{(2 \cdot x^2 + 3)}{(3 \cdot x^2 + 2)}$

Assíntota Horizontal: $y = \frac{2}{3}$

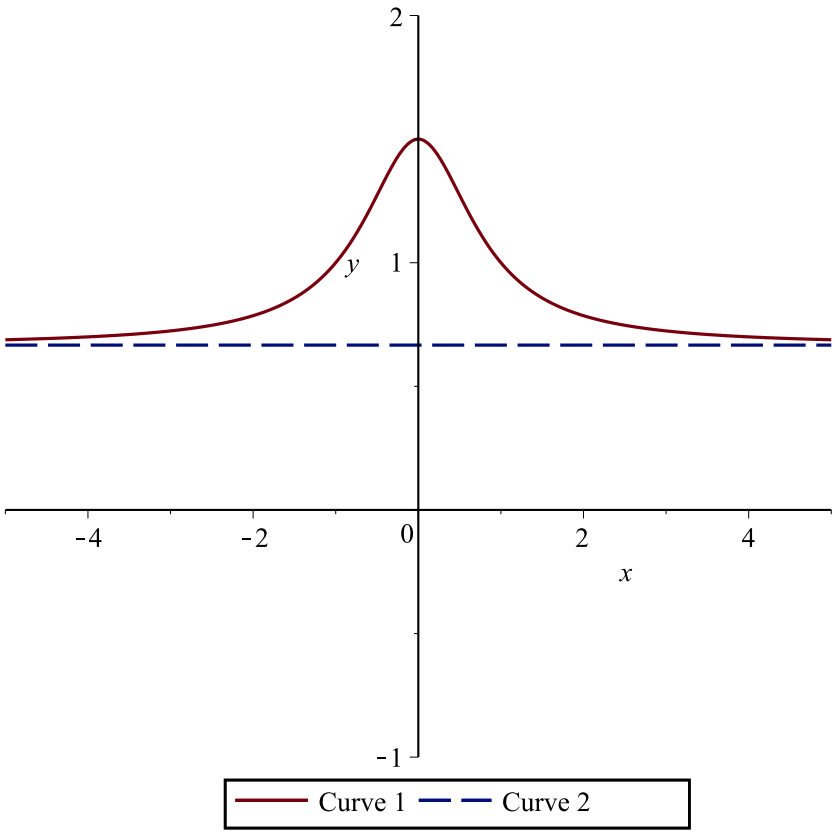
$$\lim_{x \to \infty} \frac{2x^2 + 3}{3x^2 + 2} = \frac{2}{3} \quad \textbf{(57)}$$

$$\lim_{x \to -\infty} \frac{2x^2 + 3}{3x^2 + 2} = \frac{2}{3} \quad \textbf{(58)}$$

Assíntota Vertical: *não existem raizes reais, logo não existe AV*

> $x = solve(3 \cdot x^2 + 2 = 0, x);$

$$x = \left(\frac{1}{3} I\sqrt{6}, -\frac{1}{3} I\sqrt{6} \right) \quad \textbf{(59)}$$



Exemplo 18:

> $f(x) = \tan(x)$

Assíntota Horizontal: *não existe AH*

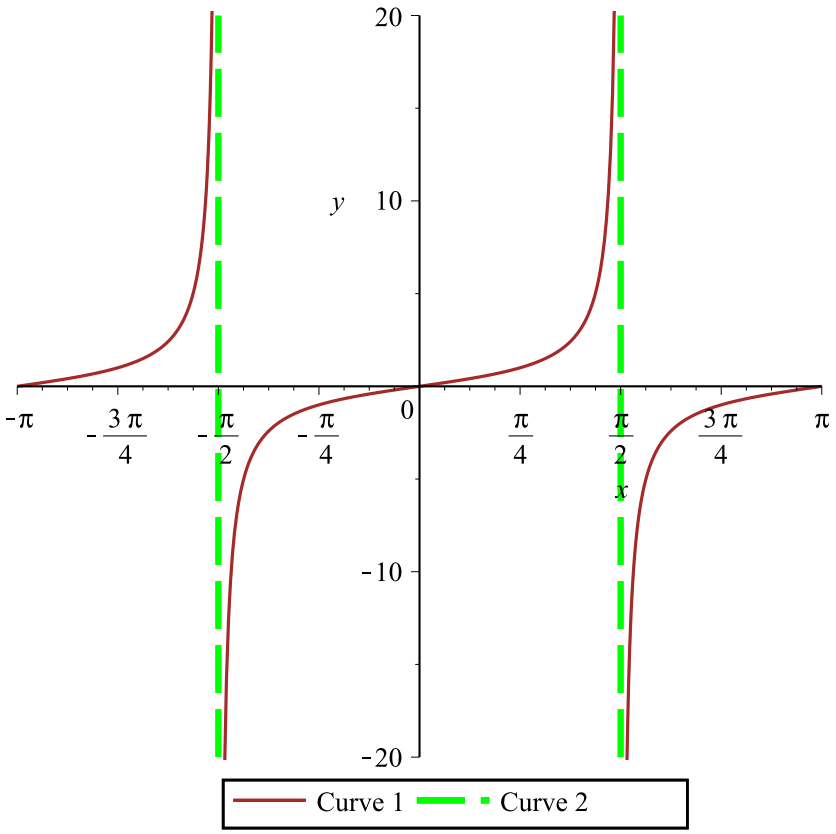
Assíntota Vertical: $x = -\frac{1}{2}\pi$ *e* $x = \frac{1}{2}\pi$

$$\lim_{x \to -\frac{1}{2}\pi^{+}} \tan(x) = -\infty \quad \textbf{(60)}$$

$$\lim_{x \to -\frac{1}{2}\pi^{-}} \tan(x) = \infty \quad \textbf{(61)}$$

$$\lim_{x \to \frac{1}{2}\pi^{+}} \tan(x) = -\infty \quad \textbf{(62)}$$

$$\lim_{x \to \frac{1}{2}\pi^{-}} \tan(x) = \infty \quad \textbf{(63)}$$



Exemplo 19:
Assintotas Oblíquas

> $f(x) = \dfrac{(2 \cdot x^2 + 3)}{(3 \cdot x + 2)}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = -\dfrac{2}{3}$

> $x = solve(3 \cdot x + 2 = 0, x);$

$$x = -\frac{2}{3} \tag{64}$$

$$\lim_{x \to -\frac{2}{3}+} \frac{2x^2+3}{3x+2} = \infty \tag{65}$$

$$\lim_{x \to -\frac{2}{3}-} \frac{2x^2+3}{3x+2} = -\infty \tag{66}$$

Assíntota Oblíqua - grau do polinômio do numerador maior do que o grau do polinômio do

denominador: $y = m\,x + b$

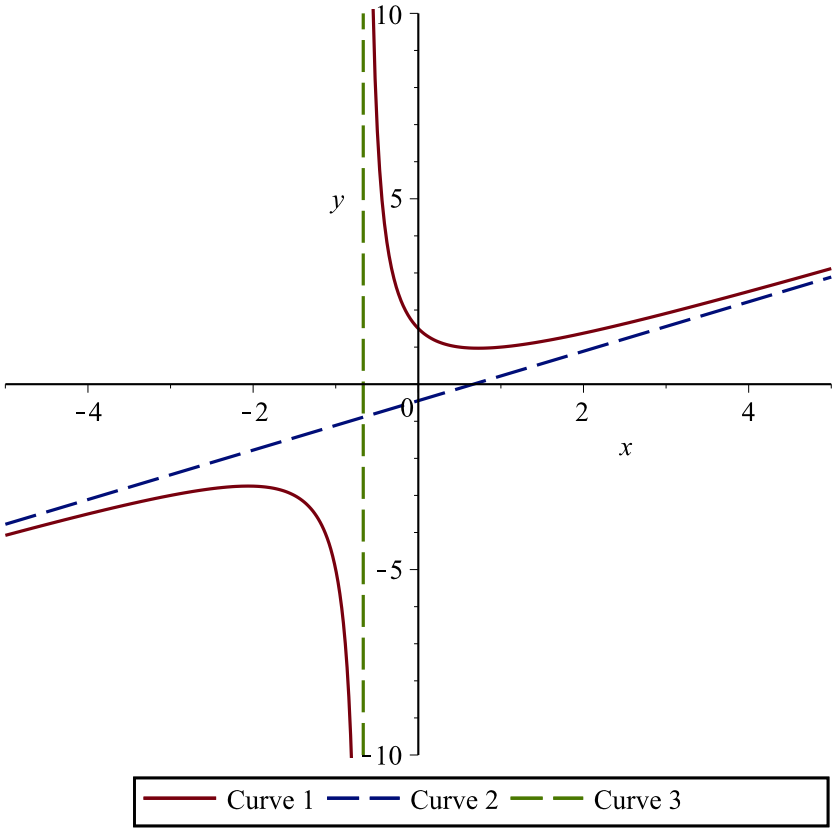
Reta Assíntota: $y = \frac{2}{3}\,x - \frac{4}{9}$

$$m = \lim_{x \to \infty} \frac{2\,x^2 + 3}{x\,(3\,x + 2)} \tag{67}$$

$$m := \frac{2}{3} \tag{68}$$

$$b = \lim_{x \to \infty} \left(\frac{2\,x^2 + 3}{3\,x + 2} - \frac{2}{3}\,x \right) \tag{69}$$

$$b := -\frac{4}{9} \tag{70}$$



Exemplo 20:

> $f(x) = \frac{x^2 + 1}{x} = x + \frac{1}{x}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = 0$

> $x = solve(x = 0, x)$;

$$x = 0 \tag{71}$$

$$\lim_{x \to 0^+} \frac{x^2+1}{x} = \infty \quad \textbf{(72)}$$

$$\lim_{x \to 0^-} \frac{x^2+1}{x} = -\infty \quad \textbf{(73)}$$

Assíntota Oblíqua - grau do polinômio do numerador maior do que o grau do polinômio do denominador: $y = m\,x + b$

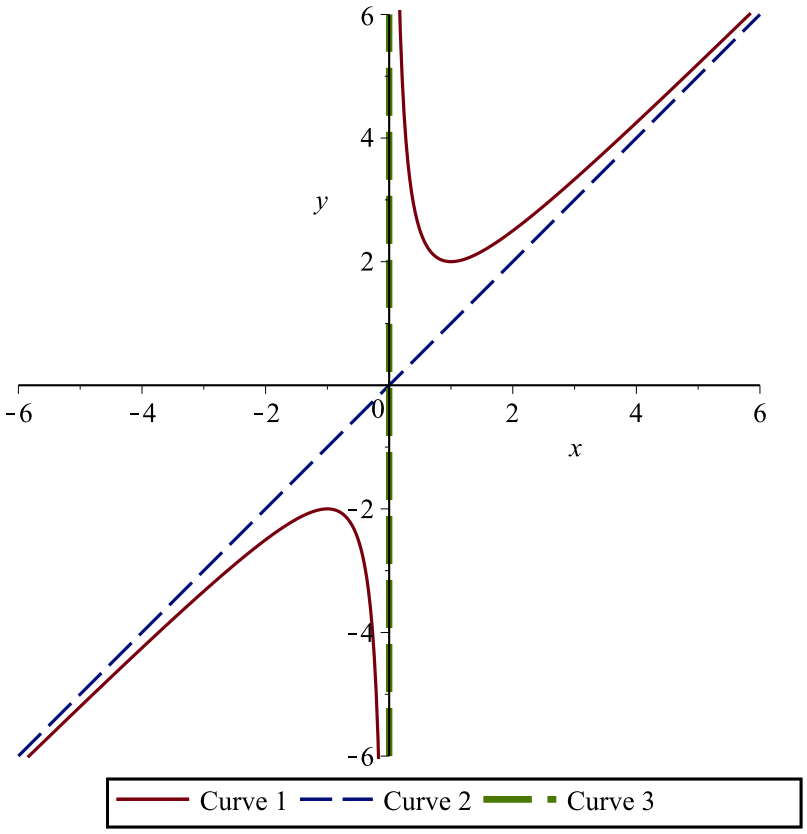
Reta Assíntota: $y = 1.\,x + 0 = x$

$$m := \lim_{x \to \infty} \frac{x^2+1}{x^2} \quad \textbf{(74)}$$

$$m := 1 \quad \textbf{(75)}$$

$$b := \lim_{x \to \infty} \left(\frac{x^2+1}{x} - x \right) \quad \textbf{(76)}$$

$$b := 0 \quad \textbf{(77)}$$



Exemplo 21:

> $f(x) = \dfrac{x^2+1}{x+2}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = -2$

```
> x = solve(x + 2 = 0, x);
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$$x = -2 \tag{78}$$

$$\lim_{x \to -2^+} \frac{x^2 + 1}{x + 2} = \infty \tag{79}$$

$$\lim_{x \to -2^-} \frac{x^2 + 1}{x + 2} = -\infty \tag{80}$$

Assíntota Oblíqua - grau do polinômio do numerador maior do que o grau do polinômio do denominador: $y = m\,x + b$

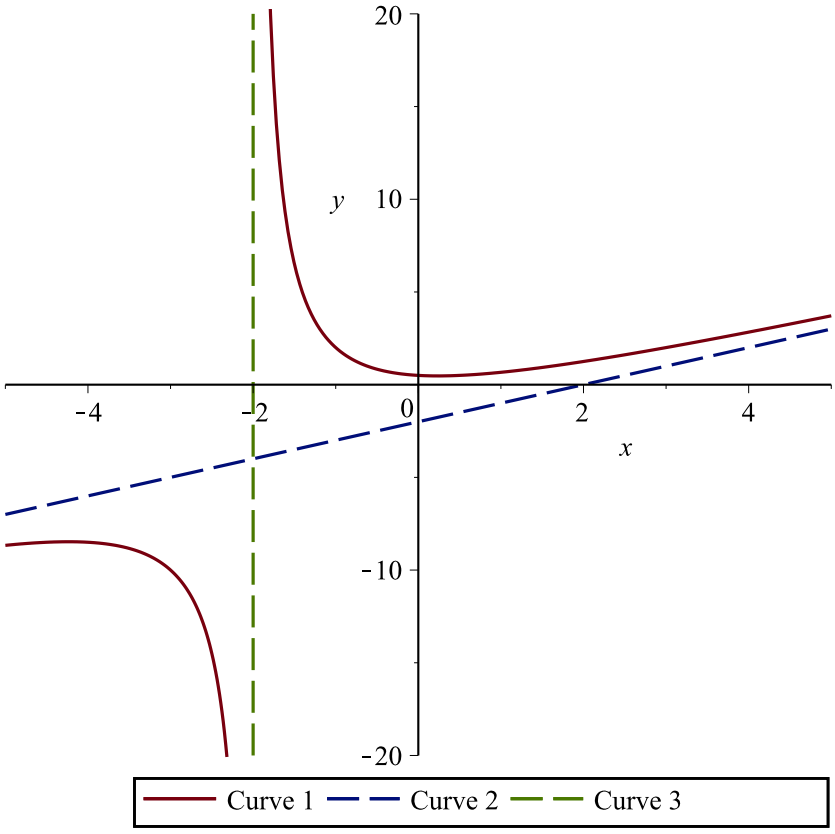
Reta Assíntota: $y = 1.\,x - 2 = x - 2$

$$m := \lim_{x \to \infty} \frac{x^2 + 1}{x\,(x + 2)} \tag{81}$$

$$m := 1 \tag{82}$$

$$b := \lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 2} - x \right) \tag{83}$$

$$b := -2 \tag{84}$$




```
> 'resto' = rem(x^2 + 1, x + 2, x);
```

(85)

$$resto = 5 \quad \textbf{(85)}$$

> $'quociente' = quo(x^2 + 1, x + 2, x);$

$$quociente = x - 2 \quad \textbf{(86)}$$

Exemplo 22:

> $f(x) = \frac{3 \cdot x^2 + 2 \cdot x - 1}{x - 2}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = -2$

> $x = solve(x - 2 = 0, x);$

$$x = 2 \quad \textbf{(87)}$$

$$\lim_{x \to 2+} \frac{3x^2 + 2x - 1}{x - 2} = \infty \quad \textbf{(88)}$$

$$\lim_{x \to 2-} \frac{3x^2 + 2x - 1}{x - 2} = -\infty \quad \textbf{(89)}$$

Assíntota Oblíqua - grau do polinômio do numerador maior do que o grau do polinômio do denominador: $y = mx + b$

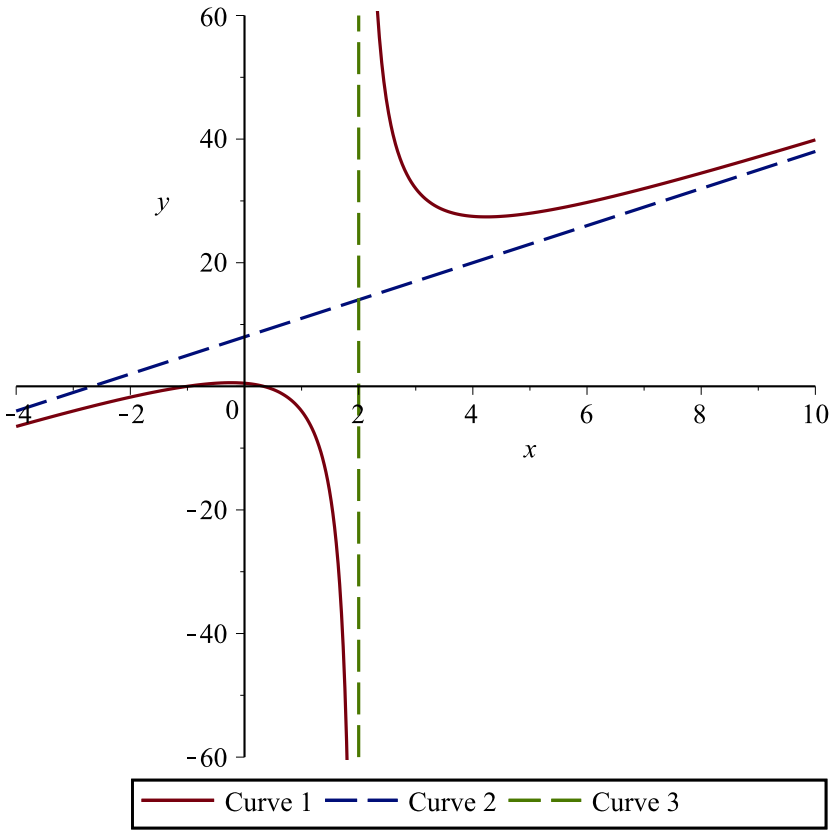
Reta Assíntota: $y = 3.x + 8$

$$m := \lim_{x \to \infty} \frac{3x^2 + 2x - 1}{x(x - 2)} \quad \textbf{(90)}$$

$$m := 3 \quad \textbf{(91)}$$

$$b := \lim_{x \to \infty} \left(\frac{3x^2 + 2x - 1}{x - 2} - 3x \right) \quad \textbf{(92)}$$

$$b := 8 \quad \textbf{(93)}$$



Exemplo 23:

> $f(x) = \frac{x^3}{x^2+1}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: *não existe AV*

> $x = solve(x^2 + 1 = 0, x)$;

$$x = (I, -I) \quad \textbf{(94)}$$

Assíntota Oblíqua - grau do polinômio do numerador maior do que o grau do polinômio do denominador: $y = m\,x + b$

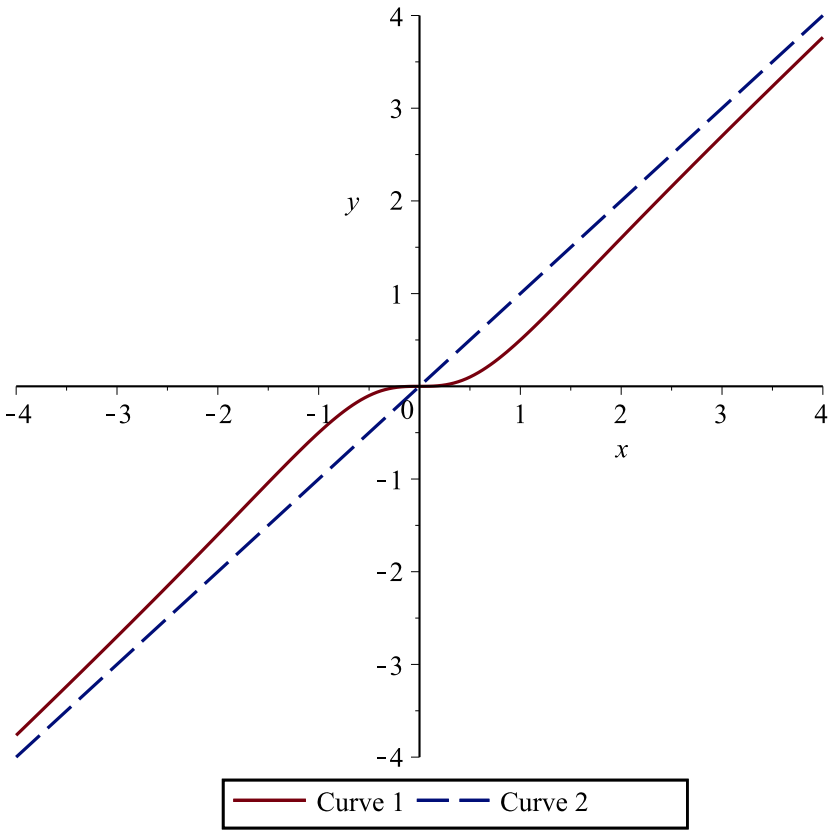
Reta Assíntota: $y = x$

$$m := \lim_{x \to \infty} \frac{x^2}{x^2+1} \quad \textbf{(95)}$$

$$m := 1 \quad \textbf{(96)}$$

$$b := \lim_{x \to \infty} \left(\frac{x^3}{x^2+1} - x \right) \quad \textbf{(97)}$$

$$b := 0 \quad \textbf{(98)}$$



Exemplo 24:
Assíntotas Curvilíneas

> $f(x) = \frac{x^3 - 1}{x + 1}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = -1$

> $x = solve(x + 1 = 0, x);$

$$x = -1 \quad \textbf{(99)}$$

$$\lim_{x \to -1+} \frac{x^3 - 1}{x + 1} = -\infty \quad \textbf{(100)}$$

$$\lim_{x \to -1-} \frac{x^3 - 1}{x + 1} = \infty \quad \textbf{(101)}$$

Assíntota Curvilínea: $y = x^2 - x + 1$

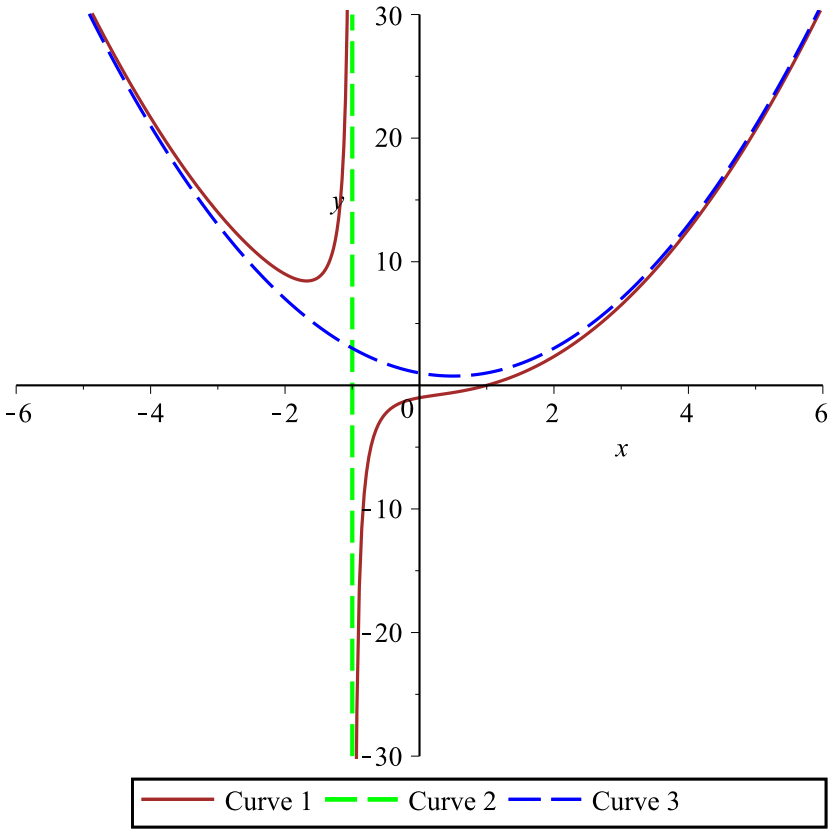
> $'quociente' = quo(x^3 - 1, x + 1, x);$

$$quociente = x^2 - x + 1 \quad \textbf{(102)}$$

> $'resto' = rem(x^3 - 1, x + 1, x);$

$$resto = -2 \quad \textbf{(103)}$$

(104)



Exemplo 25:

> $f(x) = \frac{x^5+1}{(x-1)^2}$

Assíntota Horizontal: *não existe AH*

Assíntota Vertical: $x = 1$

> $x = solve(x - 1 = 0, x)$;

$$x = 1 \quad \textbf{(105)}$$

$$\lim_{x \to 1+} \frac{x^5+1}{(x-1)^2} = \infty \quad \textbf{(106)}$$

$$\lim_{x \to 1-} \frac{x^5+1}{(x-1)^2} = \infty \quad \textbf{(107)}$$

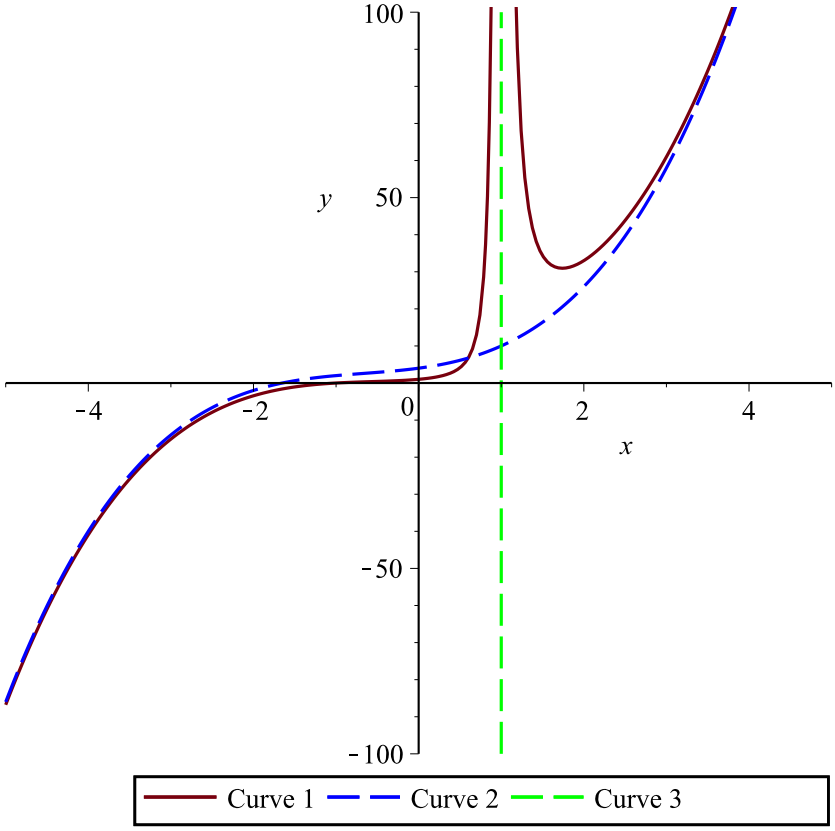
Assíntota Curvilínea: $y = x^3 + 2x^2 + 3x + 4$

> $'quociente' = quo(x^5+1, (x-1)^2, x)$;

$$quociente = x^3 + 2x^2 + 3x + 4 \quad \textbf{(108)}$$

> $'resto' = rem(x^5 + 1, (x-1)^2, x);$

$$resto = 5\,x - 3 \qquad \textbf{(109)}$$



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