PÁGINA DE REFERÊNCIA 1

ÁLGEBRA

Operações Aritméticas

$a(b + c) = ab + ac$ $\qquad \dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + bc}{bd}$

$\dfrac{a + c}{b} = \dfrac{a}{b} + \dfrac{c}{b}$ $\qquad \dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \times \dfrac{d}{c} = \dfrac{ad}{bc}$

Expoentes e Radicais

$x^m x^n = x^{m+n}$ $\qquad \dfrac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$ $\qquad x^{-n} = \dfrac{1}{x^n}$

$(xy)^n = x^n y^n$ $\qquad \left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$ $\qquad \sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

Fatoração de Polinômios Especiais

$x^2 - y^2 = (x + y)(x - y)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Teorema Binomial

$(x + y)^2 = x^2 + 2xy + y^2$ $\qquad (x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

onde $\dbinom{n}{k} = \dfrac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot k}$

Fórmula Quadrática

Se $ax^2 + bx + c = 0$, então, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Desigualdades e Valor Absoluto

Se $a < b$ e $b < c$, então $a < c$.

Se $a < b$, então $a + c < b + c$.

Se $a < b$ e $c > 0$, então $ca < cb$.

Se $a < b$ e $c < 0$, então $ca > cb$.

Se $a > 0$, então

$|x| = a$ significa que $x = a$ ou $x = -a$

$|x| < a$ significa que $-a < x < a$

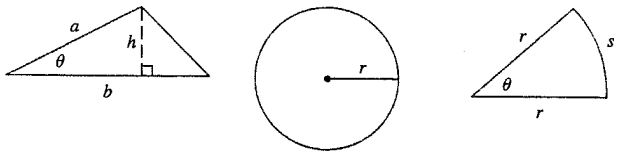
$|x| > a$ significa que $x > a$ ou $x < -a$

GEOMETRIA

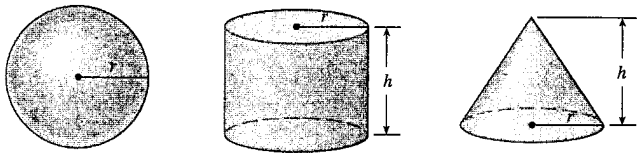
Fórmulas Geométricas

Fórmulas para área A, circunferência C e volume V:

Triângulo	Círculo	Setor do Círculo
$A = \frac{1}{2}bh$	$A = \pi r^2$	$A = \frac{1}{2}r^2\theta$
$= \frac{1}{2}ab \operatorname{sen}\theta$	$C = 2\pi r$	$s = r\theta$ (θ em radianos)



Esfera	Cilindro	Cone
$V = \frac{4}{3}\pi r^3$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$
$A = 4\pi r^2$		$A = \pi r\sqrt{r^2 + h^2}$



Fórmulas de Distância e Ponto Médio

Distância entre $P_1(x_1, y_1)$ e $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ponto Médio de $\overline{P_1P_2}$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Retas

Inclinação da reta através de $P_1(x_1, y_1)$ e $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Coeficiente angular da reta através de $P_1(x_1, y_1)$ com inclinação m:

$$y - y_1 = m(x - x_1)$$

Função afim da reta com inclinação m e interceptando o eixo y em b:

$$y = mx + b$$

Círculos

Equação do círculo com centro (h, k) e raio r:

$$(x - h)^2 + (y - k)^2 = r^2$$

PÁGINA DE REFERÊNCIA 2

TRIGONOMETRIA

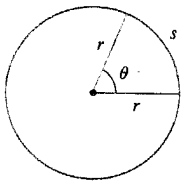
Medição do Ângulo

π radianos = 180°

$1° = \frac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \frac{180°}{\pi}$

$s = r\theta$

(θ em radianos)

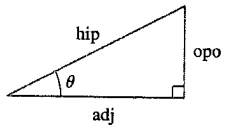


Trigonometria de Ângulo Reto

$\text{sen}\,\theta = \frac{\text{opo}}{\text{hip}}$ $\qquad \text{cossec}\,\theta = \frac{\text{hip}}{\text{opo}}$

$\cos\theta = \frac{\text{adj}}{\text{hip}}$ $\qquad \sec\theta = \frac{\text{hip}}{\text{adj}}$

$\text{tg}\,\theta = \frac{\text{opo}}{\text{adj}}$ $\qquad \text{cotg}\,\theta = \frac{\text{adj}}{\text{opo}}$

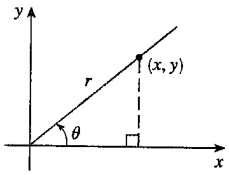


Funções Trigonométricas

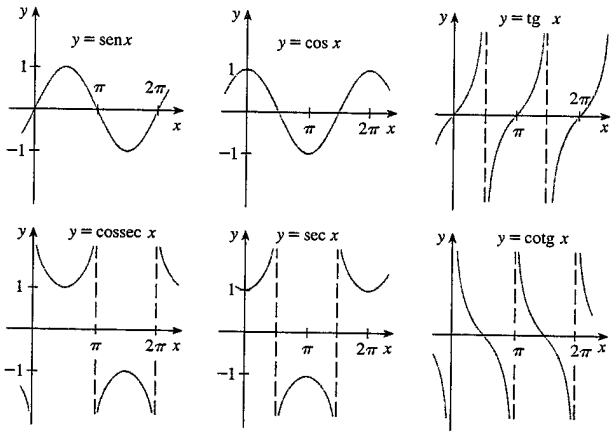
$\text{sen}\,\theta = \frac{y}{r}$ $\qquad \text{cossec}\,\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x}$

$\text{tg}\,\theta = \frac{y}{x}$ $\qquad \text{cotg}\,\theta = \frac{x}{y}$



Gráficos de Funções Trigonométricas



Funções Trigonométricas de Ângulos Importantes

θ	radianos	sen θ	cos θ	tg θ
0°	0	0	1	0
30°	$\pi/6$	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	$\pi/2$	1	0	—

Identidades Fundamentais

$\text{cossec}\,\theta = \frac{1}{\text{sen}\,\theta}$ $\qquad \sec\theta = \frac{1}{\cos\theta}$

$\text{tg}\,\theta = \frac{\text{sen}\,\theta}{\cos\theta}$ $\qquad \text{cotg}\,\theta = \frac{\cos\theta}{\text{sen}\,\theta}$

$\text{cotg}\,\theta = \frac{1}{\text{tg}\,\theta}$ $\qquad \text{sen}^2\theta + \cos^2\theta = 1$

$1 + \text{tg}^2\theta = \sec^2\theta$ $\qquad 1 + \text{cotg}^2\theta = \text{cossec}^2\theta$

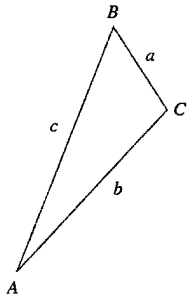
$\text{sen}(-\theta) = -\text{sen}\,\theta$ $\qquad \cos(-\theta) = \cos\theta$

$\text{tg}(-\theta) = -\text{tg}\,\theta$ $\qquad \text{sen}\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \text{sen}\,\theta$ $\qquad \text{tg}\left(\frac{\pi}{2} - \theta\right) = \text{cotg}\,\theta$

Lei dos Senos

$$\frac{\text{sen}\,A}{a} = \frac{\text{sen}\,B}{b} = \frac{\text{sen}\,C}{c}$$



Lei dos Cossenos

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Fórmulas de Adição e Subtração

$\text{sen}(x + y) = \text{sen}\,x\cos y + \cos x\,\text{sen}\,y$

$\text{sen}(x - y) = \text{sen}\,x\cos y - \cos x\,\text{sen}\,y$

$\cos(x + y) = \cos x\cos y - \text{sen}\,x\,\text{sen}\,y$

$\cos(x - y) = \cos x\cos y + \text{sen}\,x\,\text{sen}\,y$

$$\text{tg}(x + y) = \frac{\text{tg}\,x + \text{tg}\,y}{1 - \text{tg}\,x\,\text{tg}\,y}$$

$$\text{tg}(x - y) = \frac{\text{tg}\,x - \text{tg}\,y}{1 + \text{tg}\,x\,\text{tg}\,y}$$

Fórmulas de Ângulo Duplo

$\text{sen}\,2x = 2\,\text{sen}\,x\cos x$

$\cos 2x = \cos^2 x - \text{sen}^2 x = 2\cos^2 x - 1 = 1 - 2\,\text{sen}^2 x$

$$\text{tg}\,2x = \frac{2\,\text{tg}\,x}{1 - \text{tg}^2 x}$$

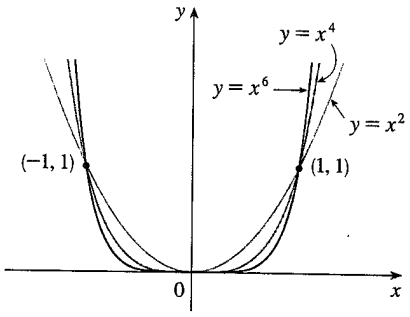
Fórmulas de Metade do Ângulo

$$\text{sen}^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

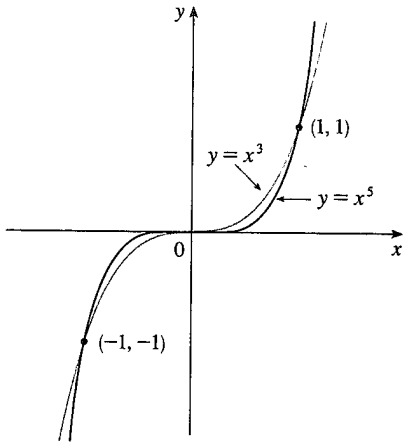
FUNÇÕES ESPECIAIS

Funções Potências $f(x) = x^n$

(i) $f(x) = x^n$, n um inteiro positivo

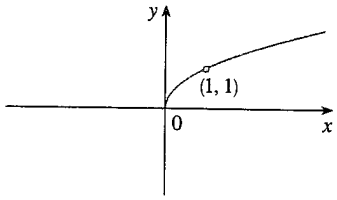


n par

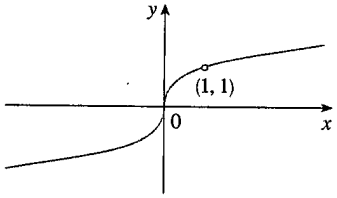


n ímpar

(ii) $f(x) = x^{1/n} = \sqrt[n]{x}$, n um inteiro positivo

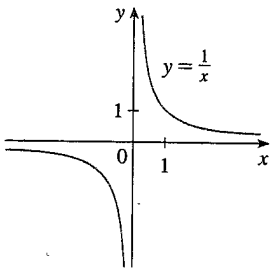


$f(x) = \sqrt{x}$



$f(x) = \sqrt[3]{x}$

(iii) $f(x) = x^{-1} = \dfrac{1}{x}$

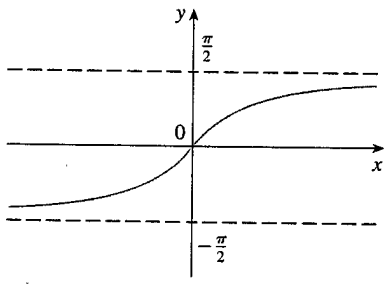


Funções Trigonométricas Inversas

$\arcsen x = \operatorname{sen}^{-1} x = y \iff \operatorname{sen} y = x \quad \text{e} \quad -\dfrac{\pi}{2} \leq y \leq \dfrac{\pi}{2}$

$\arccos x = \cos^{-1} x = y \iff \cos y = x \quad \text{e} \quad 0 \leq y \leq \pi$

$\operatorname{arctg} x = \operatorname{tg}^{-1} x = y \iff \operatorname{tg} y = x \quad \text{e} \quad -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$



$y = \operatorname{tg}^{-1} x = \operatorname{arctg} x$

$\lim_{x \to -\infty} \operatorname{tg}^{-1} x = -\dfrac{\pi}{2}$

$\lim_{x \to \infty} \operatorname{tg}^{-1} x = \dfrac{\pi}{2}$

FUNÇÕES ESPECIAIS

Funções Exponenciais e Logarítmicas

$\log_a x = y \iff a^y = x$

$\ln x = \log_e x$, onde $\ln e = 1$

$\ln x = y \iff e^y = x$

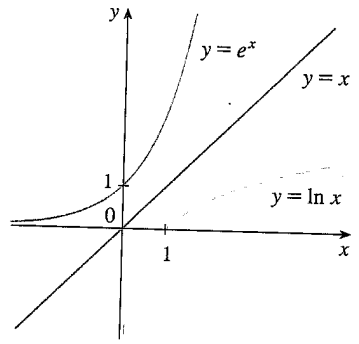
Equações de Cancelamento

$\log_a(a^x) = x \qquad a^{\log_a x} = x$

$\ln(e^x) = x \qquad e^{\ln x} = x$

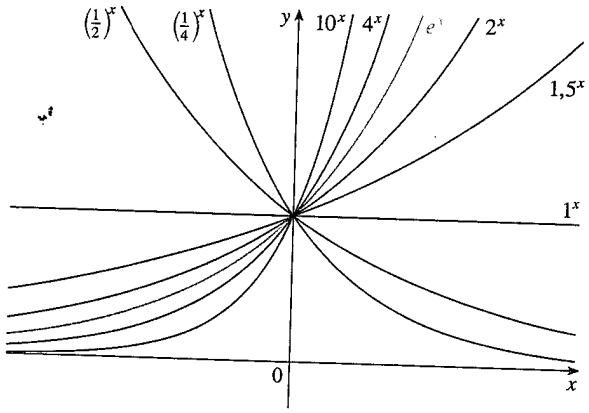
Leis de Logaritmos

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r \log_a x$

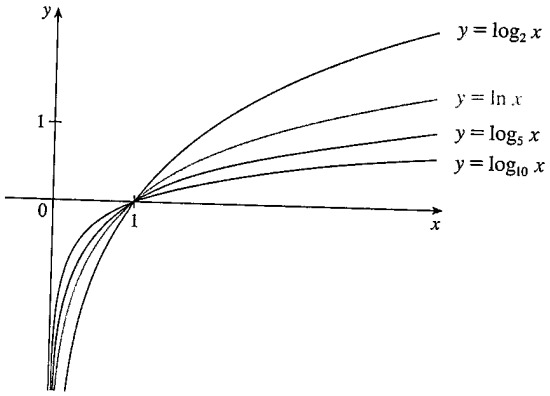


$\lim_{x \to -\infty} e^x = 0$ $\qquad$ $\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to 0^+} \ln x = -\infty$ $\qquad$ $\lim_{x \to \infty} \ln x = \infty$



Funções Exponenciais



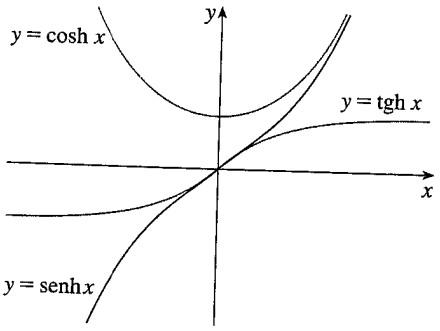
Funções Logarítmicas

Funções Hiperbólicas

$$\operatorname{senh} x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{cossech} x = \frac{1}{\operatorname{senh} x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{tgh} x = \frac{\operatorname{senh} x}{\cosh x} \qquad \operatorname{cotgh} x = \frac{\cosh x}{\operatorname{senh} x}$$



Funções Hiperbólicas Inversas

$y = \operatorname{senh}^{-1} x \iff \operatorname{senh} y = x$

$y = \cosh^{-1} x \iff \cosh y = x \quad \text{e} \quad y \geq 0$

$y = \operatorname{tgh}^{-1} x \iff \operatorname{tgh} y = x$

$$\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$\operatorname{tgh}^{-1} x = \tfrac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right)$$

PÁGINA DE REFERÊNCIA 5

REGRAS DE DIFERENCIAÇÃO

Fórmulas Gerais

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}[cf(x)] = cf'(x)$

3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Regra de Produto)

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Regra do Quociente)

7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Regra da Cadeia)

8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Regra da Potência)

Funções Exponenciais e Logarítmicas

9. $\frac{d}{dx}(e^x) = e^x$

10. $\frac{d}{dx}(a^x) = a^x \ln a$

11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$

12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Funções Trigonométricas

13. $\frac{d}{dx}(\operatorname{sen} x) = \cos x$

14. $\frac{d}{dx}(\cos x) = -\operatorname{sen} x$

15. $\frac{d}{dx}(\operatorname{tg} x) = \sec^2 x$

16. $\frac{d}{dx}(\operatorname{cossec} x) = -\operatorname{cossec} x \operatorname{cotg} x$

17. $\frac{d}{dx}(\sec x) = \sec x \operatorname{tg} x$

18. $\frac{d}{dx}(\operatorname{cotg} x) = -\operatorname{cossec}^2 x$

Funções Trigonométricas Inversas

19. $\frac{d}{dx}(\operatorname{sen}^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$

20. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$

21. $\frac{d}{dx}(\operatorname{tg}^{-1} x) = \frac{1}{1 + x^2}$

22. $\frac{d}{dx}(\operatorname{cossec}^{-1} x) = -\frac{1}{x\sqrt{x^2 - 1}}$

23. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2 - 1}}$

24. $\frac{d}{dx}(\operatorname{cotg}^{-1} x) = -\frac{1}{1 + x^2}$

Funções Hiperbólicas

25. $\frac{d}{dx}(\operatorname{senh} x) = \cosh x$

26. $\frac{d}{dx}(\cosh x) = \operatorname{senh} x$

27. $\frac{d}{dx}(\operatorname{tgh} x) = \operatorname{sech}^2 x$

28. $\frac{d}{dx}(\operatorname{cossech} x) = -\operatorname{cossech} x \operatorname{cotgh} x$

29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \operatorname{tgh} x$

30. $\frac{d}{dx}(\operatorname{cotgh} x) = -\operatorname{cossech}^2 x$

Funções Hiperbólicas Inversas

31. $\frac{d}{dx}(\operatorname{senh}^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$

32. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$

33. $\frac{d}{dx}(\operatorname{tgh}^{-1} x) = \frac{1}{1 - x^2}$

34. $\frac{d}{dx}(\operatorname{cossech}^{-1} x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$

35. $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1 - x^2}}$

36. $\frac{d}{dx}(\operatorname{cotgh}^{-1} x) = \frac{1}{1 - x^2}$

TABELA DE INTEGRAIS

Fórmulas Básicas

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$
6. $\int \operatorname{sen} u\,du = -\cos u + C$
7. $\int \cos u\,du = \operatorname{sen} u + C$
8. $\int \sec^2 u\,du = \operatorname{tg} u + C$
9. $\int \operatorname{cossec}^2 u\,du = -\operatorname{cotg} u + C$
10. $\int \sec u \operatorname{tg} u\,du = \sec u + C$
11. $\int \operatorname{cosec} u \operatorname{cotg} u\,du = -\operatorname{cossec} u + C$
12. $\int \operatorname{tg} u\,du = \ln|\sec u| + C$
13. $\int \operatorname{cotg} u\,du = \ln|\operatorname{sen} u| + C$
14. $\int \sec u\,du = \ln|\sec u + \operatorname{tg} u| + C$
15. $\int \operatorname{cossec} u\,du = \ln|\operatorname{cossec} u - \operatorname{cotg} u| + C$
16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1}\dfrac{u}{a} + C$
17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\operatorname{tg}^{-1}\dfrac{u}{a} + C$
18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u + a}{u - a}\right| + C$
20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u - a}{u + a}\right| + C$

Fórmulas Envolvendo $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2}\,du = \dfrac{u}{2}\sqrt{a^2 + u^2} + \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
22. $\int u^2\sqrt{a^2 + u^2}\,du = \dfrac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \dfrac{a^4}{8}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
23. $\int \dfrac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\dfrac{a + \sqrt{a^2 + u^2}}{u}\right| + C$
24. $\int \dfrac{\sqrt{a^2 + u^2}}{u^2}\,du = -\dfrac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
25. $\int \dfrac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
26. $\int \dfrac{u^2\,du}{\sqrt{a^2 + u^2}} = \dfrac{u}{2}\sqrt{a^2 + u^2} - \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
27. $\int \dfrac{du}{u\sqrt{a^2 + u^2}} = -\dfrac{1}{a}\ln\left|\dfrac{\sqrt{a^2 + u^2} + a}{u}\right| + C$
28. $\int \dfrac{du}{u^2\sqrt{a^2 + u^2}} = -\dfrac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \dfrac{du}{(a^2 + u^2)^{3/2}} = \dfrac{u}{a^2\sqrt{a^2 + u^2}} + C$

TABELA DE INTEGRAIS

Fórmulas Envolvendo $\sqrt{a^2 - u^2}$, $a > 0$

30. $\displaystyle\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

31. $\displaystyle\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

32. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\, du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2}\, du = -\frac{1}{u}\sqrt{a^2 - u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$

34. $\displaystyle\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

35. $\displaystyle\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

36. $\displaystyle\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$

37. $\displaystyle\int (a^2 - u^2)^{3/2}\, du = -\frac{u}{8}(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

38. $\displaystyle\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Fórmulas Envolvendo $\sqrt{u^2 - a^2}$, $a > 0$

39. $\displaystyle\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

40. $\displaystyle\int u^2\sqrt{u^2 - a^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

41. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a\cos^{-1}\frac{a}{|u|} + C$

42. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u^2}\, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

43. $\displaystyle\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

44. $\displaystyle\int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

45. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46. $\displaystyle\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$

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Fórmulas Envolvendo $a + bu$

47. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}\left(a + bu - a\ln|a+bu|\right) + C$

48. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C$

49. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

50. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

51. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$

52. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

53. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

54. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2} + C$

55. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a+bu} + C$

56. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a+bu} + C$

57. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad \text{se } a > 0$

$$= \frac{2}{\sqrt{-a}}\,\mathrm{tg}^{-1}\sqrt{\frac{a+bu}{-a}} + C, \quad \text{se } a < 0$$

58. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$

59. $\displaystyle\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$

60. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

61. $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$

62. $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$

TABELA DE INTEGRAIS

Fórmulas Trigonométricas

63. $\int \operatorname{sen}^2 u\,du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$

64. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$

65. $\int \operatorname{tg}^2 u\,du = \operatorname{tg} u - u + C$

66. $\int \operatorname{cotg}^2 u\,du = -\operatorname{cotg} u - u + C$

67. $\int \operatorname{sen}^3 u\,du = -\frac{1}{3}(2 + \operatorname{sen}^2 u)\cos u + C$

68. $\int \cos^3 u\,du = \frac{1}{3}(2 + \cos^2 u)\operatorname{sen} u + C$

69. $\int \operatorname{tg}^3 u\,du = \frac{1}{2}\operatorname{tg}^2 u + \ln|\cos u| + C$

70. $\int \operatorname{cotg}^3 u\,du = -\frac{1}{2}\operatorname{cotg}^2 u - \ln|\operatorname{sen} u| + C$

71. $\int \sec^3 u\,du = \frac{1}{2}\sec u \operatorname{tg} u + \frac{1}{2}\ln|\sec u + \operatorname{tg} u| + C$

72. $\int \operatorname{cossec}^3 u\,du = -\frac{1}{2}\operatorname{cossec} u \operatorname{cotg} u + \frac{1}{2}\ln|\operatorname{cossec} u - \operatorname{cotg} u| + C$

73. $\int \operatorname{sen}^n u\,du = -\frac{1}{n}\operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2} u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

75. $\int \operatorname{tg}^n u\,du = \frac{1}{n-1}\operatorname{tg}^{n-1} u - \int \operatorname{tg}^{n-2} u\,du$

76. $\int \operatorname{cotg}^n u\,du = \frac{-1}{n-1}\operatorname{cotg}^{n-1} u - \int \operatorname{cotg}^{n-2} u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\operatorname{tg} u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

78. $\int \operatorname{cossec}^n u\,du = \frac{-1}{n-1}\operatorname{cotg} u \operatorname{cossec}^{n-2} u + \frac{n-2}{n-1}\int \operatorname{cossec}^{n-2} u\,du$

79. $\int \operatorname{sen} au \operatorname{sen} bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

80. $\int \cos au \cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

81. $\int \operatorname{sen} au \cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u \operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$

83. $\int u \cos u\,du = \cos u + u \operatorname{sen} u + C$

TABELA DE INTEGRAIS

84. $\int u^n \operatorname{sen} u\, du = -u^n \cos u + n \int u^{n-1} \cos u\, du$

85. $\int u^n \cos u\, du = u^n \operatorname{sen} u - n \int u^{n-1} \operatorname{sen} u\, du$

86. $\int \operatorname{sen}^n u \cos^m u\, du = -\dfrac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \dfrac{n-1}{n+m} \int \operatorname{sen}^{n-2} u \cos^m u\, du$

$= \dfrac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \dfrac{m-1}{n+m} \int \operatorname{sen}^n u \cos^{m-2} u\, du$

Fórmulas Trigonométricas Inversas

87. $\int \operatorname{sen}^{-1} u\, du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$

88. $\int \cos^{-1} u\, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

89. $\int \operatorname{tg}^{-1} u\, du = u \operatorname{tg}^{-1} u - \frac{1}{2} \ln(1+u^2) + C$

90. $\int u \operatorname{sen}^{-1} u\, du = \dfrac{2u^2-1}{4} \operatorname{sen}^{-1} u + \dfrac{u\sqrt{1-u^2}}{4} + C$

91. $\int u \cos^{-1} u\, du = \dfrac{2u^2-1}{4} \cos^{-1} u - \dfrac{u\sqrt{1-u^2}}{4} + C$

92. $\int u \operatorname{tg}^{-1} u\, du = \dfrac{u^2+1}{2} \operatorname{tg}^{-1} u - \dfrac{u}{2} + C$

93. $\int u^n \operatorname{sen}^{-1} u\, du = \dfrac{1}{n+1}\left[u^{n+1} \operatorname{sen}^{-1} u - \int \dfrac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

94. $\int u^n \cos^{-1} u\, du = \dfrac{1}{n+1}\left[u^{n+1} \cos^{-1} u + \int \dfrac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

95. $\int u^n \operatorname{tg}^{-1} u\, du = \dfrac{1}{n+1}\left[u^{n+1} \operatorname{tg}^{-1} u - \int \dfrac{u^{n+1}\, du}{1+u^2}\right], \quad n \neq -1$

Fórmulas Exponenciais e Logarítmicas

96. $\int u e^{au}\, du = \dfrac{1}{a^2}(au-1)e^{au} + C$

97. $\int u^n e^{au}\, du = \dfrac{1}{a} u^n e^{au} - \dfrac{n}{a} \int u^{n-1} e^{au}\, du$

98. $\int e^{au} \operatorname{sen} bu\, du = \dfrac{e^{au}}{a^2+b^2}(a \operatorname{sen} bu - b \cos bu) + C$

99. $\int e^{au} \cos bu\, du = \dfrac{e^{au}}{a^2+b^2}(a \cos bu + b \operatorname{sen} bu) + C$

100. $\int \ln u\, du = u \ln u - u + C$

101. $\int u^n \ln u\, du = \dfrac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\int \dfrac{1}{u \ln u}\, du = \ln|\ln u| + C$

Fórmulas Hiperbólicas

103. $\int \operatorname{senh} u\, du = \cosh u + C$

104. $\int \cosh u\, du = \operatorname{senh} u + C$

105. $\int \operatorname{tgh} u\, du = \ln \cosh u + C$

106. $\int \operatorname{cotgh} u\, du = \ln|\operatorname{senh} u| + C$

107. $\int \operatorname{sech} u\, du = \operatorname{tg}^{-1}|\operatorname{senh} u| + C$

108. $\int \operatorname{cossech} u\, du = \ln\left|\operatorname{tgh} \tfrac{1}{2} u\right| + C$

109. $\int \operatorname{sech}^2 u\, du = \operatorname{tgh} u + C$

110. $\int \operatorname{cossech}^2 u\, du = -\operatorname{cotgh} u + C$

111. $\int \operatorname{sech} u \operatorname{tgh} u\, du = -\operatorname{sech} u + C$

112. $\int \operatorname{cossech} u \operatorname{cotgh} u\, du = -\operatorname{cossech} u + C$

TABELA DE INTEGRAIS

Fórmulas Envolvendo $\sqrt{2au - u^2}$, $a > 0$

113. $\displaystyle\int \sqrt{2au - u^2}\,du = \frac{u - a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

114. $\displaystyle\int u\sqrt{2au - u^2}\,du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

115. $\displaystyle\int \frac{\sqrt{2au - u^2}}{u}\,du = \sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

116. $\displaystyle\int \frac{\sqrt{2au - u^2}}{u^2}\,du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C$

117. $\displaystyle\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$

118. $\displaystyle\int \frac{u\,du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

119. $\displaystyle\int \frac{u^2\,du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2}\sqrt{2au - u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

120. $\displaystyle\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$