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Problem Set 1

Mathematics for Social Scientists

Exercise 1. .

a. For each of the following pairs of sets A, B, compute the intersection $A \cap B$, the union $A \cup B$ and the difference $A \backslash B$.

   i. $A = \{1, 2, 3, 5, 6, 7, 8, 10\}$, $B = \{3, 4, 9, 10\}$

   ii. $A = \{n \in \mathbb{N} : n \text{ is a prime number}\}$, $B = \{n \in \mathbb{N} : n \text{ is odd}\}$

   iii. $A = [0, 1]$, $B = (0, 2)$

b. Evaluate the following:

   i. $A = \{a, b, 5\}$. What is $P(A)$, that is, the power set of A?

   ii. $A = \{a, b, c\}$, $B = \{1, 2, f\}$. What is A x B, i.e. the Cartesian product of A and B?

   iii. $A = \{a, b\}$, $B = \{a, b, c\}$. What is $P(B) \backslash P(A)$?
   What is $|P(A)|$, i.e. the cardinality[1] of the power set of A?
   What is $|P(B)|$?

   iv. Let $|A| = n$, i.e. A has n elements, where $n \in \mathbb{N}$. Can you come up with a rule to compute $|P(A)|$?

Exercise 2. Fill in the blanks.

$\boxed{\Rightarrow}$ $\boxed{\not\Rightarrow}$ $\boxed{\Leftrightarrow}$ $\boxed{\exists}$ $\boxed{\exists!}$ $\boxed{\in}$ $\boxed{\notin}$ $\boxed{\forall}$

(i) $n \in \mathbb{Z}$ is even $\boxed{}$ $\frac{n}{2} \in \mathbb{Z}$.

(ii) $n \in \mathbb{N}$ is not prime $\Leftrightarrow$ $\boxed{}$ $m \in \mathbb{N}$ such that m $\boxed{}$ $\{1, n\}$, and $\frac{n}{m}$ $\boxed{}$ $\mathbb{N}$.

(iii) $\boxed{}$ $q \in \mathbb{Q}$, $\boxed{}$ $n \in \mathbb{Z}$ such that $n \leq q < n + 1$.

(iv) $x \in [0, 1]$ $\boxed{}$ $x \in (0, 1)$.

(v) $y \in (0, 1)$ $\boxed{}$ $y \in [0, 1]$.

Exercise 3. Let $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2$.

[1] Cardinality = Number of elements

(i) Let $g : \mathbb{R} \to \mathbb{R} : x \mapsto x/2$. What are $g \circ f$ and $f \circ g$?

(ii) Suppose that $g \circ f : \mathbb{R} \to \mathbb{R} : x \mapsto x$. What is g?

(iii) Define a function g such that $f \circ g(2) = 2$, $f \circ g(0) = 2$.

(iv) In which of the following cases is $f \circ g$ a well-defined function?

  (a) $g : \mathbb{N} \to \mathbb{R} : x \mapsto x + 1$

  (b) $g : \mathbb{R} \to \mathbb{R} : x \mapsto \sqrt{x}$

  (c) $g : \mathbb{R} \to [0, \infty) : x \to |x|$, the *modulus (absolute value)* function.

Exercise 4. Let X and Y be sets, $f : X \to Y$ a function. Define the **image** of f as

$$\{y \in Y : \exists x \in X \text{ such that } f(x) = y\}.$$

Write down the images of the following functions.

(i) $X = Y = \mathbb{Q}$, $f(x) = x^3$.

(ii) $X = (0, 1]$, $Y = \mathbb{R}$, $f(x) = \frac{1}{x}$.

(iii) $X = Y = \mathbb{R}$, $f(x) = \sin(x)$.

(iv) $X = Y = \mathbb{R}$, $f(x) = 1$.

Exercise 5. Sketch the following functions:

(i) $f(x) = 2$

(ii) $f(x) = 3x - 1$

(iii) $f(x) = x^2 + 2x + 1$

(iv) $f(x) = (x - 3)^{-1}$

(v) $f(x) = |2x - 2|$

(vi) $f(x) = e^{2x}$

(vii) $f(x) = -\sqrt{x}$

Exercise 6. Which of the following functions is injective, bijective, or surjective?

(i) $a(x) = 2x + 1$

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(ii) $b(x) = x^2$

(iii) $c(x) = \ln x$

(iv) $d(x) = e^x$