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Fall 2025

# Problem Set 1

## Mathematics for Social Scientists

**Exercise 1.** .

a. For each of the following pairs of sets $A, B$, compute the intersection $A \cap B$, the union $A \cup B$ and the difference $A \backslash B$.

   i. $A = \{1, 2, 3, 5, 6, 7, 8, 10\}$, $B = \{3, 4, 9, 10\}$

   ii. $A = \{n \in \mathbb{N} : n \text{ is a prime number}\}$, $B = \{n \in \mathbb{N} : n \text{ is odd}\}$

   iii. $A = [0, 1]$, $B = (0, 2)$

b. Evaluate the following:

   i. $A = \{a, b, 5\}$. What is $P(A)$, that is, the power set of A?

   ii. $A = \{a, b, c\}$, $B = \{1, 2, f\}$. What is $A$ x $B$, i.e. the Cartesian product of A and B?

   iii. $A = \{a, b\}$, $B = \{a, b, c\}$. What is $P(B) \backslash P(A)$?
   What is $|P(A)|$, i.e. the cardinality[1] of the power set of A?
   What is $|P(B)|$?

   iv. Let $|A| = n$, i.e. A has $n$ elements, where $n \in \mathbb{N}$. Can you come up with a rule to compute $|P(A)|$?

**Exercise 2.** Fill in the blanks.

$\boxed{\Rightarrow}$ $\boxed{\not\Rightarrow}$ $\boxed{\Leftrightarrow}$ $\boxed{\exists}$ $\boxed{\exists!}$ $\boxed{\in}$ $\boxed{\notin}$ $\boxed{\forall}$

(i) $n \in \mathbb{Z}$ is even $\boxed{\phantom{x}}$ $\frac{n}{2} \in \mathbb{Z}$.

(ii) $n \in \mathbb{N}$ is not prime $\Leftrightarrow$ $\boxed{\phantom{x}}$ $m \in \mathbb{N}$ such that $m$ $\boxed{\phantom{x}}$ $\{1, n\}$, and $\frac{n}{m}$ $\boxed{\phantom{x}}$ $\mathbb{N}$.

(iii) $\boxed{\phantom{x}}$ $q \in \mathbb{Q}$, $\boxed{\phantom{x}}$ $n \in \mathbb{Z}$ such that $n \leq q < n + 1$.

(iv) $x \in [0, 1]$ $\boxed{\phantom{x}}$ $x \in (0, 1)$.

(v) $y \in (0, 1)$ $\boxed{\phantom{x}}$ $y \in [0, 1]$.

**Exercise 3.** Let $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2$.

[1] Cardinality = Number of elements

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(i) Let $g : \mathbb{R} \to \mathbb{R} : x \mapsto x/2$. What are $g \circ f$ and $f \circ g$?

(ii) Suppose that $g \circ f : \mathbb{R} \to \mathbb{R} : x \mapsto x$. What is $g$?

(iii) Define a function $g$ such that $f \circ g(2) = 2$, $f \circ g(0) = 2$.

(iv) In which of the following cases is $f \circ g$ a well-defined function?

   (a) $g : \mathbb{N} \to \mathbb{R} : x \mapsto x + 1$

   (b) $g : \mathbb{R} \to \mathbb{R} : x \mapsto \sqrt{x}$

   (c) $g : \mathbb{R} \to [0, \infty) : x \to |x|$, the *modulus (absolute value)* function.

**Exercise 4.** Let $X$ and $Y$ be sets, $f : X \to Y$ a function. Define the **image** of $f$ as

$$\{y \in Y : \exists x \in X \text{ such that } f(x) = y\}.$$

Write down the images of the following functions.

(i) $X = Y = \mathbb{Q}$, $f(x) = x^3$.

(ii) $X = (0, 1]$, $Y = \mathbb{R}$, $f(x) = \frac{1}{x}$.

(iii) $X = Y = \mathbb{R}$, $f(x) = \sin(x)$.

(iv) $X = Y = \mathbb{R}$, $f(x) = 1$.