University of Mannheim
School of Social Sciences
Mathematics for Political Scientists, Fall 2022
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Problem Set: Linear Algebra II

1. Let
$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -4 & 0 \\ 0 & 2 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -6 & 9 \\ 0 & 2 & 3 \\ 0 & 0 & -2 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix}$
Determine $x = \dfrac{\det((\mathbf{A}')^{-1}) \cdot \det(\mathbf{A}^3 \cdot \mathbf{B} \cdot \mathbf{C}) \cdot \det(((\mathbf{A}+\mathbf{B})^{-1})^4)}{\det(\mathbf{C}^{-1})}$

2. Calculate the determinant of $\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 \\ -2 & -2 & -1 \\ 0 & 1 & t \end{pmatrix}$, and show that it is never 0.

3. Determine the inverse of the following matrices.
$\mathbf{A} = (-\frac{1}{42}), \mathbf{B} = \begin{pmatrix} -1 & 8 \\ 2 & 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 4 & 3 \\ 0 & 2 & 0 \\ 2 & 4 & 7 \end{pmatrix}$

4. $\mathbf{D} = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 3 & -4 \\ 1 & 0 \end{pmatrix}, \mathbf{G} = \begin{pmatrix} 1 & 4 \\ -2 & -1 \end{pmatrix}$
Solve the matrix equation $-\mathbf{D}^3 \cdot \mathbf{X} + \mathbf{D} \cdot \mathbf{F} = \mathbf{D}^2 \cdot \mathbf{I} \cdot \mathbf{D} \cdot \mathbf{X} \cdot \mathbf{I} - \mathbf{D} \cdot \mathbf{F} \cdot \mathbf{G}$ for $\mathbf{X}$.

5. $\mathbf{A} = \begin{pmatrix} 4 & -7 \\ 1 & 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & -7 \\ 1 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 & 2 \\ 2 & -1 \end{pmatrix}$
Solve the matrix equation $3 \cdot \mathbf{X} - (\mathbf{A} - \mathbf{B})^3 \cdot \mathbf{X} - \mathbf{C} \cdot \mathbf{X} \cdot \mathbf{I} = \mathbf{I} - \mathbf{C}' \cdot \mathbf{X}$ for $\mathbf{X}$.

6. Consider the following system of equations.

$$\begin{aligned} x_1 - x_2 - x_3 &= 1 \\ -3x_1 + 4x_2 + 2x_3 &= -8 \\ -2x_1 + 3x_2 + x_3 &= -7 \end{aligned}$$

   (a) Rewrite the system of equations in matrix form.
   (b) How many solutions does the system have? Why?