

# Solutions Problem Set 7

## Probability Theory II

1. Consider the p.d.f.  $f(x) = 2x$  for  $0 \leq x \leq 1$ .

(a) Calculate the c.d.f. of  $f(x)$ .

$$F(x) = x^2$$

(b) Is  $f(x)$  a proper p.d.f.?

$$\begin{aligned}\int_0^1 f(x) &= 1 \\ 1^2 - 0^2 &= 1\end{aligned}$$

$f(x)$  is a proper p.d.f.

2. Consider the c.d.f.  $G(x) = \frac{1}{9}x^2$  for  $0 \leq x \leq 3$ .

(a) Calculate the p.d.f. of  $G(x)$ ,  $g(x)$ .

$$g(x) = \frac{2}{9}x$$

(b) Is  $g(x)$  a proper p.d.f.?

$$\begin{aligned}\int_0^3 g(x) &= 1 \\ \frac{1}{9} \times 3^2 - \frac{1}{9} \times 0^2 &= 1\end{aligned}$$

$g(x)$  is a proper p.d.f.

3. Consider the p.d.f.  $h(x) = \frac{4}{3}(1 - x^3)$  for  $0 < x < 1$ . Determine

(a)  $\Pr(X < \frac{1}{2})$ .

$$H(x) = -\frac{1}{3}x(x^3 - 4)$$

$$H\left(\frac{1}{2}\right) \approx 0.65$$

(b)  $\Pr(X > \frac{1}{3})$ .

$$1 - H\left(\frac{1}{3}\right) \approx 0.56$$

(c)  $\Pr(\frac{1}{4} < X < \frac{3}{4})$ .

$$H\left(\frac{3}{4}\right) - H\left(\frac{1}{4}\right) \approx 0.56$$

4. Consider the p.d.f.  $k(x) = cx^2$  for  $1 \leq x \leq 2$ . Determine

(a) Find the value of the constant  $c$ .

$$\int_1^2 k(x)dx = 1$$

$$\frac{1}{3}cx^3 \Big|_1^2 = 1$$

$$c = \frac{3}{7}$$

(b) Find  $\Pr(X > \frac{3}{2})$ .

$$K(x) = \frac{1}{7}x^3$$

$$1 - K\left(\frac{3}{2}\right) \approx 0.52$$