University of Mannheim
School of Social Sciences
Math Refresher for Political Science, Fall 2025
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Problem Set: Set Theory I

1. Identify whether each of the following indicators is measured at a nominal, ordinal, interval, or ratio level. Note also whether each is a discrete or a continuous measure:
   (a) Highest level of education as (1) some high school, (2) high school graduate, (3) some college, (4) college graduate, (5) postgraduate.
   (b) Annual income.
   (c) State welfare expenditures, measured in millions of dollar.
   (d) Vote choice among Bush, Clinton, and Perot.
   (e) Absence or presence of a militarized interstate dispute.
   (f) Military personnel, measured in 1,000s of persons.
   (g) The number of wars in which countries have participated.
2. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{6, 8\}$. Find following:
   (a) $A \cup B$
   (b) $A \cap B$
   (c) $A \cap B^C$
   (d) $B - A$
   (e) $C - B$
   (f) $A \cap C$
3. Let $A = \{1, 5, 10\}$, $B = \{1, 2, ..., 10\}$.
   (a) Is $A \subset B$, $B \subset A$, both or neither?
   (b) What is $A \cup B$?
   (c) What is $A \cap B$?
   (d) Partition B into two sets, A and everything else. Call everything else C. What is C?
   (e) What is $A \cup C$?
   (f) What is $A \cap C$?

4. Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$ and $C = \{a, b, 1, 2\}$. Show that:

   (a) Distributivity: $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$

   (b) Associativity: $(A \cap B) \cap C = A \cap (B \cap C)$

   (c) De Morgan Laws: $C - (A \cup B) = (C - A) \cap (C - B)$

5. Determine which of the following formulas are true. If any formula is false, find a counterexample to demonstrate this using a Venn diagram.

   (a) $A \setminus B = B \setminus A$

   (b) $A \subseteq B \iff A \cap B = A$

   (c) $A \cup B = A \cup C \implies B = C$

   (d) $A \subseteq B \iff A \cup B = B$

   (e) $A \cap B = A \cap C \implies B = C$

   (f) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$

6. Explain in words why it is true that for any sets A, B, C:

   (a) $(A \cup B) \cup C = A \cup (B \cup C)$

   (b) $(A \cap B) \cap C = A \cap (B \cap C)$

   (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   (d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

7. Find the interior point(s) and the boundary points(s) of the set $\{x : 1 \leq x \leq 5\}$.

8. Why does every set in $\mathbb{R}$ that is nonempty, closed, and bounded have a greatest member?

9. Which of the following sets are open, closed, or neither?

   (a) $D = \{x \in \mathbb{R}^1 : x = 2 \text{ or } 3 < x < 4\}$

   (b) $A = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y \leq 1\}$

   (c) $B = \{(x, y) \in \mathbb{R}^2 : x^2 < y < 1\}$

   (d) $C = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y < 1\}$

   (e) universal set

10. Sketch the following functions:

    (a) $f(x) = 2$

    (b) $f(x) = 3x - 1$

    (c) $f(x) = x^2 + 2x + 1$

    (d) $f(x) = (x - 3)^{-1}$

    (e) $f(x) = |2x - 2|$

(f) $f(x) = e^{2x}$

(g) $f(x) = -\sqrt{x}$

11. Which of the following functions is injective, bijective, or surjective?

(a) $a(x) = 2x + 1$

(b) $b(x) = x^2$

(c) $c(x) = \ln x$

(d) $d(x) = e^x$

12. Simplify the following expressions as much as possible:

(a) $(-x^4y^2)^2$

(b) $9(3^0)$

(c) $(2a^2)(4a^2)$

(d) x^4/x^3

(e) $(-2)^(7 - 4)$

(f) $(1/(27b^3)^(1/3)$

(g) $y^7y^6y^5y^4$

(h) $((2a/7b)/(11b/5a))$

(i) $(z^2)^4$

13. Solve:

(a) $\sqrt[3]{2^3}$

(b) $\sqrt[3]{27}$

(c) $\sqrt[4]{625}$

14. Solve the following inequalities so that the variable is the only term on the left-hand side:

(a) $x - 3 < 2x + 15$

(b) $11 - (4/3)t > 3$

(c) $(5/6)y + 3(y - 1) \leq (11/6)(1 - y) + 2y$

15. Find the roots (solutions) to the following quadratic equations:

(a) $4x^2 - 1 = 17$

(b) $9x^2 - 3x - 12 = 0$

(c) $x^2 - 2x - 16 = 0$

(d) $6x^2 - 6x - 6 = 0$

(e) $5 + 11x = -3x^2$