
Topology Filters - Notes

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Carlos Caralps

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FILTER DEFINITION AND ALGEBRAIC STRUCTURE

In this chapter, we will define filters. Their basic propositions will be proved by the common way and by Lean. The final aim is to define an algebraic structure with filters, defining two operations.

1.1 Filter definition

Firstly, we will introduce the filter definition of a giving set.

Here goes the latex definition

The reader might have noticed we haven't included the empty axiom, normally used in the filter common definitions. Assuming this axiom would make it impossible to define the neutral element in one of the operations we will use later.

Having the conceptual definition of filters, we can define this structure in Lean. The following code lines were published in the mathlib repository by Johannes Hölzl in August of 2018.

```
structure filter (X : Type) :=
  (sets
    : set (set X))
  (univ_sets
    : set.univ ∈ sets)
  (sets_of_superset {x y} : x ∈ sets → x ⊆ y → y ∈ sets)
  (inter_sets {x y}      : x ∈ sets → y ∈ sets → x ∩ y ∈ sets)
```