High Performance Computing

Cerritos Lira, Carlos

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Work-Span Model

We represent a computation using a DAG (Directed Acyclic Graph)

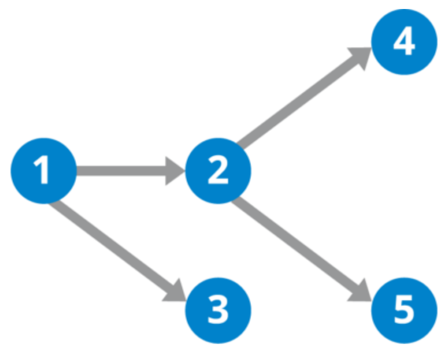

Figure 1: DAG

Each node represents an operation and the edges represent what operations should be done before. Suppose:

1. Each operation is made in the same time.
2. Each operation takes a unit of time.
3. No edge cost.

Define:

$$\begin{aligned} W(n) &= \text{ Total number of nodes.} \\ D(n) &= \text{ Vertices on the longest path.} \\ T_p(n) &= \text{ Time to execute given that we have pram procesors.} \end{aligned}$$

We want to know an upper bound for $T_p(n)$, we already know the following:

$$\begin{aligned} T_1(n) &= W(n) \\ T_\infty(n) &= D(n) \\ T_p(n) &\geq max\{D(n), \left\lceil \frac{W(n)}{p} \right\rceil\} \end{aligned}$$

Brent's Theorem

Let's suppose given a DAG we break it into phases, where:

1. Each phase has 1 c.p. vertex.
2. All vertices in a given phase are independent.
3. Every vertex has to appear in some phase (phase k has W_k vertices and takes t_k time to complete).

then:

$$\begin{aligned}
T_p &= \sum_{k=1}^{D} t_k \\
&= \sum_{k=1}^{D} \left\lceil \frac{W_k}{p} \right\rceil \\
&= \sum_{k=1}^{D} \left\lfloor \frac{W_k - 1}{p} \right\rfloor + 1 \\
&= \sum_{k=1}^{D} \frac{W_k - 1}{p} + 1 \\
&\leq \tfrac{W-D}{p} + D
\end{aligned}$$

Given a DAG we can see his performance with:

$$S_p(n) = \frac{T_*(n)}{T_p(n)} \text{ where } T_*(n) \text{ is best sequential time.}$$