Optimisation Problems

PROBLEM1

Creams & Lotions wants to schedule the production of three new lotions that will be launched to the market. The lotions are produced with the super-secret ingredient SSS and this ingredient must be compounded by very experienced staff. The requirements in experienced labour and SSS for each lotion are depicted in the next table.

Lotion	**A**	**B**	**C**
Labor (hours per kg of lotion produced)	7	3	6
Raw material (kg per kg of lotion produced)	4	4	5
Profit (€/kg of lotion produced)	4	2	3

The raw material, SSS, is limited to 200 kg/day. There is also a limit of 150 labour hours per day. The objective is to maximize the total profit. Write the linear programming model for this problem. Consider now that Creams & Lotion knows the maximum demand for each one of the products. The demand is 10kg for lotions A and C and 15kg for lotion B. Add these constraints to the previous model. Consider now additionally that the quantity to produce of each one of the lotions must always be greater than or equal to 25% of the total quantity of lotions produced. Add these constraints to the previous model.

PROBLEM2

The Diaz Coffee Company blends three types of coffee beans (Brazilian, Colombian, and Peruvian) into ground coffee to be sold to retailers. Suppose that each kind of bean has a distinctive aroma and strength, and the company has a chief taster who can rate these features on a scale of 1 to 100. The features of the beans are tabulated below:

Beans	**Aroma (Level)**	**Strength (Level)**	**Cost (MU/kg)**	**Supply (kg)**
Brazil	75	12	0.5	1,500
Colombia	60	20	0.6	1,200
Peru	85	18	0.7	2,000

The company would like to create a blend that has an aroma rating of at least 78 and a strength rating of at least 16. Its supplies of the various beans are limited, however. The available quantities are specified above and all beans are delivered under a previously arranged purchase agreement. Diaz wants to make 4 000 kg of the blend at the lowest possible cost.

PROBLEM3

One oil refinery can mix three types of crude to produce normal and super gasoline. The oil refinery has two mixing units, an older unit and a more recent one.

The older unit uses for each production cycle 5 barrels of crude A, 7 barrels of crude B and 2 barrels of crude C to produce 9 tanks of normal gasoline and 7 tanks of super. The more recent unit uses for each production cycle 3 barrels of crude A, 9 barrels of crude B and 4 barrels of crude C to produce 5 tanks of normal gasoline and 9 tanks of super.

The refinery has already signed contracts with some customers that impose a production of at least 500 tanks of normal gasoline and 300 tanks of super. For the total production they have in stock 1500 barrels of crude A, 1900 of crude B and 1000 of crude C. The profit of the oil refinery is 6 monetary units for each tank of normal gasoline and 9 monetary units for each tank of super.

Write the linear programming model for that problem.

PROBLEM4

A company needs to rent space in a warehouse. The needs for the next 3 months are represented in the following table.

Month	Space Needed (m2)
1	1,500
2	500
3	5,000

The renting price per square meter is dependent on the number of months rented and is represented in the following table.

Renting Period (months)	Price (€/m2)
1	28
2	40
3	50

Write a linear programming model for this problem.

PROBLEM5

The GoodFlight aviation company will buy several jet planes for long, medium and short lights that were named P_l , Pm and P_s. The unit costs, in thousand euros, are 5,000, 3,800 and 2,000. The board of the company has a maximum budget of 112,000 to buy the planes.

The expected annual profits for each plane are respectively 310, 230 and 200 thousand euros. The company has pilots to y with a maximum of 30 new planes.

If they only buy planes of type P_s, the maintenance department could assure the maintenance of 40 new planes. However, in what concerns maintenance, each plane P_m is equivalent to 4/3 of a plane P_s and each plane P_l to 5/3 of a plane P_s.

The technical department requires that for each plane P_s that is bought at least one plane P_l or P_m is bought. They require also that for each plane P_l that is bought, at least 8 planes P_s or P_m are bought. The company must decide how many planes of each type to buy in order to maximize its profit.

Build a linear programming model for this problem.

PROBLEM6

In the biggest Paper Mill in the World, the paper is produced in jumbo rolls (characterized by a width and a diameter), that are divided in smaller rolls that can be sent directly to clients or that can be cut.

Consider the following example:

The paper is produced in rolls with a width of 6 meters. Out of these rolls the paper mill needs to produce 30 rolls of width 280cm, 60 rolls of width 200cm and 48 rolls of width 150cm. One roll of 6 meter might be divided in 2 rolls of 280cm with a waste of 40cm. Assume that they have enough rolls in stock to produce the quantities that are needed.

Write the linear programming model for this example.

PROBLEM6.1

After seeing the solution, the production manager of the paper mill recalled a very important additional constraint. The di_culty of placing the blades cutting position requires a minimum production, ie whenever you use a standard cut it is necessary to cut at least 15 rolls.

PROBLEM7

The city of Metropolis is in the process of designing a new public emergency system, and their design calls for locating emergency vehicles around the city. The city is divided into nine districts, and seven potential sites have been identi_ed as possible locations for emergency vehicles. Equipment located at each potential site can reach some (but not all) of the districts within the 3-minute time requirement specified by the city. In the following table, an entry of 1 means that the district can be serviced from the corresponding site within the time requirement.

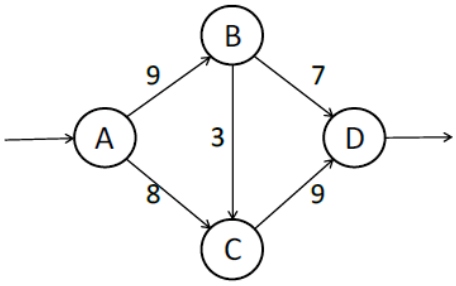
	S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**
District 1	0	1	0	1	0	0	1
District 2	1	0	0	0	0	1	1
District 3	0	1	0	0	0	1	1
District 4	0	1	1	0	1	1	0
District 5	1	0	1	0	1	0	0
District 6	1	0	0	1	0	1	0
District 7	1	0	0	0	0	0	1
District 8	0	0	1	1	1	0	0
District 9	1	0	0	0	1	0	0

The city wants to provide coverage to all nine districts within the specified time, using the minimum number of sites. Write the linear programming model for the problem and then add the following constraints to the model:

1. At least two of the three sites S1, S3 and S4 must be chosen.
2. From the three sites S5, S6 and S7 exactly two sites must be chosen.
3. Sites S3 and S4 cannot be simultaneously chosen.
4. If site S2 is chosen then site S7 must also be chosen.

PROBLEM8

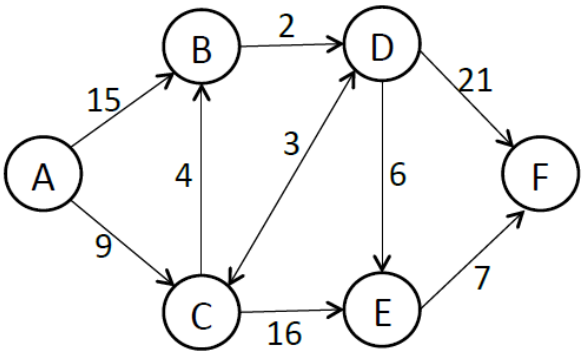
The maximum ow problem is the problem of finding a maximum feasible flow through a capacitated directed graph from a single-source to a single-sink. Consider the network that is represented in the figure where the numbers near each arc correspond to the maximum capacity of each arc.



The objective is to determine the maximum ow through the network from node A to node D. Write the linear programming model for the maximum ow problem represented in the figure.

PROBLEM9

The shortest path problem is the problem of finding a path between two vertices (or nodes) of a graph such that the sum of the weights of its constituent edges is minimized. Consider the graph represented in the following figure:



Write the linear programming model for the shortest path problem represented in the figure. The source node is node A and the sink node is node F. The numbers near the arcs represent their lengths.