A THEORY FOR COMMUNICATING SEQUENTIAL PROCESSES IN COQ

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INTRODUCTION

- Concurrent systems
  - Parallel execution of components
  - Deadlock, nondeterminism and other issues
  - Testing cannot guarantee properties such as determinism
- CSP: a theory for Communicating Sequential Processes
  - Clear and accurate description of concurrent systems
  - Designs can be proven correct with respect to desired properties
- Refinement (model) checkers
  - Analysis and verification of systems via state exploration
  - FDR: most popular refinement checker for CSP
  - State explosion problem
- Verifying properties by proof development

OBJECTIVES

“Provide an initial formalisation of the CSP language in Coq.”

- Simplified syntax for CSP in Coq
  - Machine-readable version of CSP
- Support for the Labelled Transition System representation
  - Operational semantics via the SOS approach
- Verification of traces refinement relations
  - Property-based random testing

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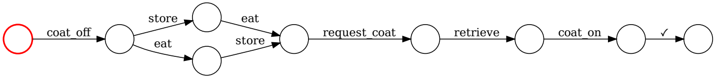
CSP

Example: the cloakroom attendant

- CSP$_M$

```
channel coat_on, coat_off, store, retrieve, request_coat, eat
SYSTEM =
   coat_off -> store -> request_coat -> retrieve -> coat_on -> SKIP
   [| {coat_off, request_coat, coat_on} |]
   coat_off -> eat -> request_coat -> coat_on -> SKIP
```

- LTS



- Traces
  - $\langle coat\_off,\ store,\ eat\rangle$, $\langle coat\_off,\ eat,\ store,\ request\_coat\rangle$ ...

COQ

- Functional and inductive definitions

```
Fixpoint evenb (n:nat) : bool :=
  match n with
    | O ⇒ true
    | S O ⇒ false
    | S (S n') ⇒ evenb n'
  end.
```

```
Inductive ev : nat → Prop :=
  | ev_0 : ev 0
  | ev_SS (n : nat) (H : ev n) : ev (S (S n)).
```

- Proof development and the tactics language Ltac

```
Lemma negb_involutive : ∀ (b : bool),
  negb (negb b) = b.
Proof.
  destruct b.
  - simpl. reflexivity.
  - simpl. reflexivity.
Qed.
```

```
Ltac solve_negb_inv b :=
  destruct b; simpl; reflexivity.
```

QUICKCHICK

- Randomised property-based testing tool
  - Example

```
Fixpoint remove (x : nat) (l : list nat) : list nat :=
  match l with
    | [] ⇒ []
    | h::t ⇒ if h =? x then t else h :: remove x t
  end.
```

```
Conjecture removeP : ∀ x l, ¬ (In x (remove x l)).
```

```
QuickChick removeP.
```

  - Output

```
0
[0, 0]
Failed!  After 17 tests and 12 shrinks
```

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ABSTRACT AND CONCRETE SYNTAXES

- Event prefix

Abstract	Concrete
`ProcPrefix (Event "e") STOP`	`"e" --> STOP`

- Example: process "PRINTER"
  - CSP_{Coq}

Abstract

```
Proc "PRINTER"
 (ProcPrefix (Event "accept")
  (ProcPrefix (Event "print")
   STOP))
```

Concrete

```
"PRINTER" ::= "accept" -->
      "print" --> STOP
```

  - CSP_{M}

```
PRINTER = accept -> print -> STOP
```

ABSTRACT AND CONCRETE SYNTAXES

Constructor	CSP_M	CSP_{Coq}
Stop	STOP	STOP
Skip	SKIP	SKIP
Event prefix	e -> P	e --> P
External choice	P [] Q	P [] Q
Internal choice	P |~| Q	P |~| Q
Alphabetised parallel	P [A || B] Q	P [[A \\ B]] Q
Generalised parallel	P [| A |] Q	P [| A |] Q
Interleave	P ||| Q	P ||| Q
Sequential composition	P ; Q	P ;; Q
Event hiding	P \ A	P \ A
Process definition	P := Q	P ::= Q
Process name	P	ProcRef “P”

STRUCTURED OPERATIONAL SEMANTICS

- Inference rule

Event prefix

$$\frac{}{(a \rightarrow P) \xrightarrow{a} P}$$

External choice

$$\frac{P \xrightarrow{a} P'}{P \,\Box\, Q \xrightarrow{a} P'} \quad (a \neq \tau)$$

- Inductive definition *sosR*

```
Inductive sosR : specification → proc_body → event_tau_tick →
proc_body → Prop :=
  | prefix_rule (S : specification) (P : proc_body) (a : event) :
    S # (a --> P) // Event a ==> P
  | ext_choice_left_rule (S : specification) (P Q : proc_body) :
    ∀ (P' : proc_body) (a : event_tau_tick),
      ¬ eq a Tau →
      (S # P // a ==> P') →
      (S # P [] Q // a ==> P')
```

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LABELLED TRANSITION SYSTEMS

- Inductive definition *ltsR'* (1/2)

```
Inductive ltsR' :
  specification →
  set transition →
  set proc_body →
  set proc_body →
  Prop :=
  | lts_empty_rule (S : specification) (visited : set proc_body) :
      ltsR' S nil nil visited
```

LABELLED TRANSITION SYSTEMS

- Inductive definition *ltsR'* (2/2)

```
| lts_inductive_rule
        (S : specification)
        (T : set transition)
        (P : proc_body)
        (tl visited : set proc_body) :
    let T' := transitions_from P T in
    let T'' := set_diff transition_eq_dec T T' in
    let visited' := set_add proc_body_eq_dec P visited in
    let to_visit := set_diff proc_body_eq_dec
        (set_union proc_body_eq_dec tl (target_proc_bodies T'))
        visited' in
    (∀ (a : event_tau_tick) (P' : proc_body),
        (S # P // a ==> P') ↔ In (P,a,P') T') →
    ltsR' S T'' to_visit visited' →
    ltsR' S T (P :: tl) visited.
```

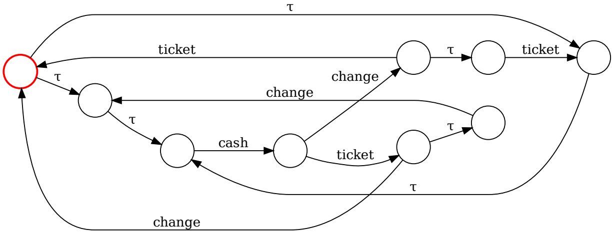
LABELLED TRANSITION SYSTEMS

- Functional definitions: *compute_ltsR* and *generate_dot*

  `Definition` *compute_ltsR*
  (*S* : *specification*) (*name* : *string*) (*limit* : *nat*) : *option* (`set` *transition*).

  `Definition` *generate_dot* (*lts* : *option* (`set` *transition*)) : *string*.

- Graph visualisation (GraphViz)



TRACES REFINEMENT

- Inductive definition *traceR'*

```
Inductive traceR' : specification → proc_body → trace → Prop :=
  | empty_trace_rule (S : specification) (P : proc_body) :
      traceR' S P nil
  | event_trace_rule (S : specification) (P P' : proc_body) (h :
event_tau_tick) (tl : trace) :
      ¬ eq h Tau →
      (S # P // h ==> P') →
      traceR' S P' tl →
      traceR' S P (h::tl)
  | tau_trace_rule (S : specification) (P P' : proc_body) (t : trace) :
      (S # P // Tau ==> P') →
      traceR' S P' t →
      traceR' S P t.
```

TRACES REFINEMENT

- Proof automation for the is-a-trace relation: *solve_trace*

  Example *MACHINE_TRACE* :
  traceR PARKING_PERMIT_MCH "MACHINE" ["cash" ; "ticket" ; "change"].
  Proof. *solve_trace*. Qed.

- Traces refinement formalisation

  Definition *trace_refinement* (*S* : *specification*) (*Spec Imp* : *string*) : Prop :=
  ∀ (*t* : *trace*), *traceR S Imp t* → *traceR S Spec t*.

  Notation "S '#' P '[T=' Q" := (*trace_refinement S P Q*)
  (at level 150, left associativity).

TRACES REFINEMENT

- Traces generator: *gen_valid_trace'*

```
Fixpoint gen_valid_trace'
  (S : specification) (P : proc_body) (size : nat)
  : G (option semantics_trace.trace) :=
  match size with
  | O ⇒ ret nil
  | S size' ⇒
    freq_ (ret nil) [
      (1, ret nil) ;
      (size,
        bind (gen_valid_trans S P) (
          fun t ⇒ (
            match t with
            | nil ⇒ ret nil
            | (Event e, Q) :: _ ⇒
              bind (gen_valid_trace' S Q
size') (
                fun ts ⇒ ret (Event e :: ts)
              )
            | (Tick, Q) :: _ ⇒
              bind (gen_valid_trace' S Q
size') (
                fun ts ⇒ ret (Tick :: ts)
              )
            | (Tau, Q) :: _ ⇒
              bind (gen_valid_trace' S Q
size') (
                fun ts ⇒ ret ts
              )
            end
          )
        )
      )
    ]
  end.
```

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TRACES REFINEMENT

- Demonstrating the generator

  Sample (*gen_valid_trace PARKING_PERMIT_MCH* “MACHINE” 10).

  Output:

```
[Some []; Some [“cash”; “change”; “ticket”; “cash”;
“change”]; Some [“cash”]; Some [“cash”; “ticket”;
“change”; “cash”]; Some []; Some [“cash”; “change”;
“ticket”; “cash”]; Some [“cash”; “change”; “ticket”];
...]
```

TRACES REFINEMENT

- Executable property *traceP*

```
Definition traceP
  (S : specification)
  (proc_id : string)
  (fuel : nat)
  (t : option semantics_trace.trace) : bool.
```

- Refinement checker *trace_refinement_checker*

```
Definition trace_refinement_checker
  (S : specification)
  (Imp Spec : string)
  (trace_max_size : nat)
  (fuel : nat) : Checker :=
    forAll (gen_valid_trace S Imp trace_max_size) (traceP S Spec fuel).
```

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TRACES REFINEMENT

- Searching for counterexamples with QuickChick

```
Definition EXAMPLE : specification.
Proof.
  solve_spec_ctx_rules (
    Build_Spec
      [ Channel {{"a", "b", "c"}} ]
      [ "P" ::= "a" --> "b" --> ProcRef "P" ;
        "Q" ::= ("a" --> "b" --> ProcRef "Q") [] ("c" --> STOP) ]
  ).
Defined.
```

```
QuickChick (trace_refinement_checker EXAMPLE "Q" "P" 5 1000).
```

Output:

```
Some ["c"]
*** Failed after 3 tests
and 0 shrinks.  (0 discards)
```

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SUMMARY

- Abstract and concrete syntaxes for a subset of CSP operators
- Operational semantics via the SOS approach
- Inductive and functional definitions of labelled transition systems
- Third-party visualisation support for LTS representation
- Inductive and functional definitions of traces
- Proof automation for checking the is-a-trace relation
- Random testing for refinement relations using QuickChick

RELATED WORK

- CSP-Prover
  - Interactive theorem prover based on Isabelle
  - Stable failures model as the underlying denotational semantics
  - Semi-automated proof tactics for verification of refinement relations
- Isabelle/UTP
  - Implementation of the *Unifying Theories of Programming* framework
  - Support for construction of denotational semantic meta-models
  - Useful to construct program verification tools
- Distinguishable features of CSP_{Coq}
  - Graphical representation of LTSs
  - Property-based testing for checking traces refinement relations

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FUTURE WORK

- Extend CSP_{Coq} dialect to include remaining CSP operators
- Check for invalid recursions (hiding and parallelism operations)
- Define a tactic to automate proofs involving the relation *ltsR*
- Prove correctness of definition *compute_ltsR*
- Prove correctness of generator *gen_valid_trace*
- Define traces refinement in terms of bi-simulation

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