The Anatomy of Contagion and Macroprudential Policies*

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Abstract

I propose a model to highlight the relevance of basic characteristics of the structural makeup of contagion for the design of macroprudential policies. Besides characterizing optimal policies under a variety of environments, I show that failing to incorporate said characteristics can lead to inappropriate strategies for crisis prevention.

Since the 2007-2009 crisis, policymakers around the world have adopted a more systemic approach to the regulation and supervision of financial institutions. As the crisis painfully demonstrated, the interconnected nature of modern economies can make the failure of a single institution threaten many others, potentially leading to cascades of failures with economy-wide implications. To avoid neglecting such connections, this new approach—often referred to as macroprudential—is designed with a view toward safeguarding the health of the entire financial system rather than the health of individual institutions—no matter how large they may be.

Unfortunately, policymakers face an important challenge when designing these policies. Because of the opacity and multifaceted nature of linkages among institutions, it is hard to determine how exactly contagion might materialize when economic conditions deteriorate. A natural question then arises: How can policymakers formulate macroprudential policies when they are uncertain about the precise structural makeup of contagion? Despite its importance, this question has been overlooked by the literature. This paper partly fills this gap by developing a model to help design these policies in such uncertain environments.

The main idea behind my paper can be illustrated with the help of Lazear (2011)'s analogy between contagion and domino toppling: "If one domino falls, it will topple the others, and conversely, if the first domino remains upright, the others will not fall." My paper's key insight is that aggregate characteristics of the sequence of standing dominoes—often referred to as a domino run—can alter the size of the chain reaction that occurs when the first domino falls. Consequently, when attempting to limit this reaction, policymakers should consider such characteristics. Importantly, knowing the exact pattern of the domino run might not be needed, as these characteristics can provide sufficient information to curtail the chain reaction.

I develop this idea in the context of a simple model in which financial institutions (banks, for short) are interconnected through an exogenous network of exposures that alters their payoffs. In times of economic stress, cascades of failures may occur because of contagion. As in domino toppling, the failure of a single bank could cause others to fail. This happens when such banks hold illiquid portfolios and are connected to the first failing bank via a sequence of contagious exposures. To capture uncertainty about the precise anatomy of contagion, contagious exposures are randomly determined. Yet, for tractability, the distribution of these exposures across banks is assumed to be known. Because banks fail to internalize the system-wide consequences of their individual portfolio choices, there is room for regulation. By imposing portfolio restrictions on certain banks, a policymaker seeks to maximize expected aggregate output. Because these restrictions limit banks' ability to allocate funds toward more productive investments, such restrictions are not costless. As a result, when designing interventions, the policymaker aims to strike a balance between forestalling contagion and decreasing resource misallocation.

With this model in hand, I show that optimal policies depend on the interaction between (1) simple moments of the distribution of contagious exposures—such as mean and variance—and (2) losses related to resource misallocation. Intuitively, the optimal intervention is designed to ensure that the expected benefits of restricting the portfolio of the last bank are equal to the anticipated losses resulting from these restrictions. Benefits arise because restricting the last bank prevents not only its potential failure, but also the failures of (direct and indirect) neighboring banks—which, in the domino toppling example, correspond to all dominoes placed later in the run. Because, for any given intervention, said moments alter the expected number of neighboring banks that could be saved from contagion, these moments are important for policymaking.

I also show that, as the economy grows large, heterogeneity in the number of contagious exposures across banks plays a central role in policymaking. When there is considerable heterogeneity, a small number of banks can play a disproportionately important role in contagion. Unless these banks are restricted, large cascades are unlikely to be eradicated. As a result, learning the identity of such banks becomes crucial to cost effectively prevent the emergence of large cascades. However, when contagious exposures are distributed somewhat equally across banks, most banks play a similar role from the perspective of failure propagation. Therefore, as the average number of contagious exposures increases, the more banks need to be restricted. A non-interventional policy becomes optimal when the expected losses resulting from regulation exceed the expected benefits associated with keeping cascades of failures locally confined.

This analysis contributes to the debate on the design of macroprudential policies in interconnected economies. Although the exact anatomy of contagion is not known, policymakers are likely to have information that broadly describes how contagion might possibly materialize in times of stress. My results show that failing to incorporate such information can lead to the wrong policies. By providing a mapping between this type of information and policy design, the proposed framework generates new predictions of how the anatomy of contagion can change the scope and design of macroprudential interventions.

Related literature.— My paper contributes to two strands of the literature. First, it adds to a body of work that explores how connections among institutions affect the likelihood of contagion.[1] Unlike those papers, my paper explicitly focuses on the design of macropudential policies in the presence of spillovers and contagion uncertainty. Second, my paper adds to recent research that explores how interventions affect the mechanism through which shocks might propagate.[2] While my paper also focuses on how contagion varies with different interventions, it provides a tractable framework—with which other models can be compared—in which optimal policies can be characterized under contagion uncertainty.

The rest of the paper is organized as follows. Section I introduces the baseline model. Section II explores how regulation alters the way failures propagate in times of stress, and, in doing so, reshapes the distribution of consumption. Section III characterizes the optimal

[1] An incomplete list includes Rochet and Tirole (1996), Allen and Gale (2000), Freixas et al. (2000), Eisenberg and Noe (2001), Lagunoff and Schreft (2001), Dasgupta (2004), Leitner (2005), Nier et al. (2007), Allen and Babus (2009), Haldane and May (2011), Allen et al. (2012), Amini et al. (2013), Cont et al. (2013), Georg (2013), Cabrales et al. (2014), Elliott et al. (2014), Acemoglu et al. (2015), Glasserman and Young (2015, 2016), and Castiglionesi et al. (2019).

[2] See, for example, Beale et al. (2011), Gai et al. (2011), Battiston et al. (2012), Goyal and Vigier (2014), Alvarez and Barlevy (2015), Adrian et al. (2015), Aldasoro et al. (2017), Erol and Ordoñez (2017), Gofman (2017), and Galeotti et al. (2020).

policy. Section IV describes how such a policy changes when policymakers can identify differences among banks before designing their interventions. Besides discussing modeling assumptions, Section V describes the results of an extended model wherein policymakers are also uncertain about the exact distribution of contagious exposures across banks. Section VI concludes. Derivations of propositions and examples, as well as supplemental analysis, appear in the Online Appendix.

I. Baseline Model

Consider a three-period economy consisting of n risk-neutral banks—each endowed with one dollar—whose payoffs are linked through an exogenous network of exposures. Although banks may differ in their number of exposures, they are ex-ante identical in other respects. There is a risk-neutral representative investor—who owns all assets in the economy—and a policymaker who imposes preemptive restrictions on certain banks to maximize the expected consumption of the representative investor. Time is indexed by $t \in \{0, 1, 2\}$ and banks are indexed by $i \in \{1, 2, \cdots, n\}$, with n being potentially large.

Assets.—There are two assets, cash and an illiquid asset, whose payoffs are realized at $t = 2$. In the absence of bank failures, cash pays one dollar, while the illiquid asset pays $(1 + \delta)$ dollars, with $\delta > 0$. Cash can be interpreted as liquid securities, while the illiquid asset may represent securities that yield higher returns in normal times but decrease in value in stressful times.

Timing.—At $t = 0$, the policymaker imposes portfolio restrictions on certain banks. These restrictions take a simple form in which the policymaker forces banks to hold a higher fraction of their portfolio in cash. At $t = 1$, banks react to regulation. Subject to their constraints, banks choose their portfolio to maximize expected profits. At $t = 2$, and shortly before asset payoffs are realized, economic conditions deteriorate. When conditions deteriorate, banks become more vulnerable to failures affecting related banks, as exposures can spread such failures. As a result, cascades of failures might occur at $t = 2$. These cascades can change the consumption of the representative investor, as failing banks generate a zero-dollar payoff. Consequently, expectations about the exact size of these cascades can reshape the design of policy interventions.

Banks' Portfolio.—To simplify the analysis, assume that banks invest either a fraction ω or $(\omega + \Delta)$ of their portfolio in cash, with $\omega \in (0, 1)$, $\Delta > 0$, and $(\omega + \Delta) \leq 1$. In addition, I deliberately assume that δ is sufficiently large that no bank chooses to hold $(\omega + \Delta)$ of its portfolio in cash unless forced by the policymaker. That is, from banks' perspective, investing in illiquid assets is more profitable than keeping funds in cash.[3]

Propagation of Bank Failures.—To gain tractability, the following process determines the spread of bank failures:

(A1) A bank (selected uniformly at random) is affected by an adverse shock $\varepsilon \sim U(\omega, \omega+\Delta)$. If this bank's cash holdings are strictly smaller than ε, it fails.

(A2) Although bank i may not be directly hit by ε, i can still fail because of contagion. In particular, i fails if (1) there is a sequence of contagious exposures between i and the first bank hit and (2) each bank in this sequence, including i, holds a fraction ω of its portfolio in cash.

For tractability, contagious exposures are randomly determined, and their distribution across banks follows $\{p_k\}_{k=0}^{n-1}$, where p_k denotes the probability that a randomly chosen bank exhibits k contagious exposures at $t = 2$. Although the exact number of contagious exposures in each bank is unknown at $t = 0$, the policymaker knows distribution $\{p_k\}_{k=0}^{n-1}$. The random selection of contagious exposures is a metaphor for regulators having difficulty assessing how exactly contagion manifests when economic conditions deteriorate. This difficulty makes the policymaker uncertain about which banks are more likely to spread failures. Notably, as

[3] This assumption effectively removes banks' strategic considerations from the analysis and makes the overall model tractable. Importantly, this assumption is consistent with the "underestimated risks" factor highlighted by the IGM Forum (2017) as one of the most prominent factors contributing to the 2007–2009 financial crisis as well as banks' lack of appreciation of downside risks (underscored by Gennaioli and Shleifer (2018)).

the architecture of the network of exposures is arbitrary, distribution $\{p_k\}_{k=0}^{n-1}$ may take any (well-defined) functional form.

II. Regulation and Consumption

This section examines how regulation can change the consumption of the representative investor. Because banks' individual choices do not internalize the consequences of cascades of failures, there is room for regulation. By construction, restricted banks absorb rather than amplify failures—as they neither are affected by nor propagate failures. From a contagion perspective, then, imposing restrictions on any given bank can be represented by the removal of such a bank and its exposures from any realized network of contagious exposures. Consequently, regulation reshapes the way contagion manifests itself in times of stress and, as a result, the distribution of consumption.

Let π_i denote the payoff of bank i at $t = 2$. Suppose the policymaker restricts a fraction x of banks. One can think of restricted banks as being selected uniformly at random as they are ex-ante identical from the policymaker's perspective. The (normalized) consumption of the representative investor at $t = 2$, denoted by $\mathcal{C}(x)$, can then be written as

$$\begin{aligned} \mathcal{C}(x) &= \left(\sum_{i \in \mathcal{R}} \pi_i\right) + \left(\sum_{j \notin \mathcal{R}} \pi_j\right), \\ &= x(1 + \delta(1 - (\omega + \Delta))) + \left(\sum_{j \notin \mathcal{R}} \pi_j\right), \end{aligned} \tag{1}$$

where $\mathcal{R}$ denotes the set of restricted banks. The first term on the right-hand side of equation (1) follows from the fact that restricted banks are forced to hold $(\omega+\Delta)$ in cash. Because restricted banks do not fail, they collectively generate $x(1 + \delta(1 - (\omega + \Delta)))$. However, unrestricted banks may fail. Because the exact number of contagious exposures per bank is random, $\left(\sum_{j \notin \mathcal{R}} \pi_j\right)$ is random, and, thus, $\mathcal{C}(x)$ is also random.

To determine the distribution of $\left(\sum_{j\notin\mathcal{R}}\pi_j\right)$—and, in doing so, the distribution of $\mathcal{C}(x)$—it is illustrative to consider how bank failures spread. Because the bank hit by ε is selected uniformly at random, the probability that said bank was restricted at $t=0$ is x. When this happens, contagion is prevented from its onset and the collective payoff of unrestricted banks equals $(1-x)[1+\delta(1-\omega)]$. However, when the bank hit by ε was not restricted, cascades of failures might arise. Let ϕ^x_m denote the probability that m (unrestricted) banks fail once a fraction x of banks were restricted. If m banks fail, which occurs with probability $(1-x)\phi^x_m$, the collective payoff of unrestricted banks equals $\left(1-x-\frac{m}{n}\right)[1+\delta(1-\omega)]$. Consequently, the distribution of $\left(\sum_{j\notin\mathcal{R}}\pi_j\right)$ can be written as

$$\sum_{j\notin\mathcal{R}}\pi_j \;=\; \begin{cases} (1-x)[1+\delta(1-\omega)] & \text{with probability } x, \\ \left(1-x-\frac{m}{n}\right)[1+\delta(1-\omega)] & \text{with probability } (1-x)\phi^x_m \text{ with } m=1,\cdots,n(1-x). \end{cases}$$

Leveraging the fact that contagious exposures are randomly determined, the first proposition characterizes probabilities $\{\phi^x_m\}_{m=1}^{n(1-x)}$ as a function of the underlying distribution of contagious exposures, $\{p_k\}_{k=0}^{n-1}$.

PROPOSITION 1 (Probabilities ϕ^x_m): *Suppose a fraction x of banks are restricted at $t=0$. The probability that an unrestricted bank shares k contagious exposures with other unrestricted banks at $t=2$, denoted by θ^x_k, is then given by*

$$\theta^x_k \;=\; \begin{cases} \sum_{j=k}^{n-1} p_j\binom{j}{k}(1-x)^k x^{j-k} & \textit{if } k=\{0,\cdots,n(1-x)-1\}, \\ 0 & \textit{otherwise.} \end{cases}$$

With the aforementioned probabilities in hand, probabilities $\{\phi^x_m\}_{m=1}^{n(1-x)}$ can be written as

$$\phi^x_m \;=\; \begin{cases} \frac{\langle\theta^x\rangle}{(m-1)!}\left(\frac{d^{m-2}}{dz^{m-2}}\left[g\left(z,\{\theta^x_k\}_k\right)^m\right]\right)\Bigg|_{z=0} & \textit{with } m=\{2,\cdots,n(1-x)\}, \\ \theta^x_0 & \textit{with } m=1, \end{cases}$$

where $g(z,\{\theta_k^x\}_k) \equiv \sum_{k=0}^{n(1-x)-2}\left(\frac{(k+1)\theta_{k+1}^x}{\langle\theta^x\rangle}\right)z^k$, $\langle\theta^x\rangle \equiv \sum_{k=0}^{n(1-x)-1} k\theta_k^x$, *and* $\left(\frac{d^{m-2}}{dz^{m-2}}\left[g\left(z,\{\theta_k^x\}_k\right)^m\right]\right)\Big|_{z=0}$ *denotes the* $(m-2)$ *derivative of the* m^{th} *power of function* $g(\cdot,\cdot)$ *evaluated at* $z=0$*.*

Although bank failures can spread in intricate ways, Proposition 1 shows that probabilities $\{\phi_m^x\}_{m=1}^{n(1-x)}$ can be determined in economies with arbitrary sizes and connectivity structures. It also emphasizes the importance of (1) the anatomy of contagion—encoded in distribution $\{p_k\}_{k=0}^{n-1}$—and (2) the fraction of restricted banks, x, for the emergence of cascades of failures of arbitrary size. In doing so, Proposition 1 helps characterize the entire distribution of $\mathcal{C}(x)$.

With the previous expressions in hand, expected consumption can be written as

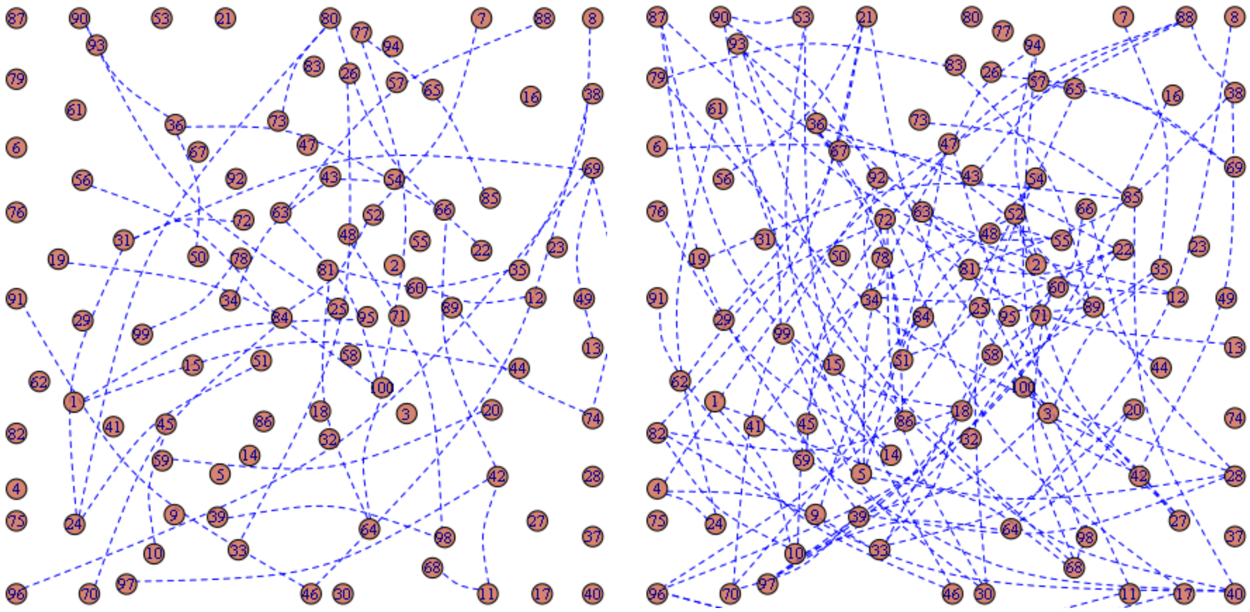
$$\mathbb{E}[\mathcal{C}(x)] \quad = \quad \underbrace{\eta(1-(1-x)e(x))}_{\text{expected benefits}} - \underbrace{x\delta\Delta,}_{\text{expected losses}} \tag{2}$$

where $\eta \equiv 1+\delta(1-\omega)$ and $e(x) \equiv \frac{1}{n}\left(\sum_{m=1}^{n(1-x)} m\phi_m^x\right)$ denotes the expected fraction of banks that fail once a fraction x of them have been restricted. Therefore, the term $(1-(1-x)e(x))$ represents the expected fraction of banks that would otherwise fail at $t=2$ if a fraction x of them had not been restricted at $t=0$. Simply put, this term refers to the expected fraction of banks saved from failure, as regulation helps keep contagion locally confined and, thus, captures the expected benefits of regulation. The term $x\delta\Delta$ represents the anticipated losses resulting from regulation, as restricted banks must forgo profitable investments. The higher the difference in asset payoffs, δ, or the difference in cash between restricted and unrestricted banks, Δ, the greater these losses.

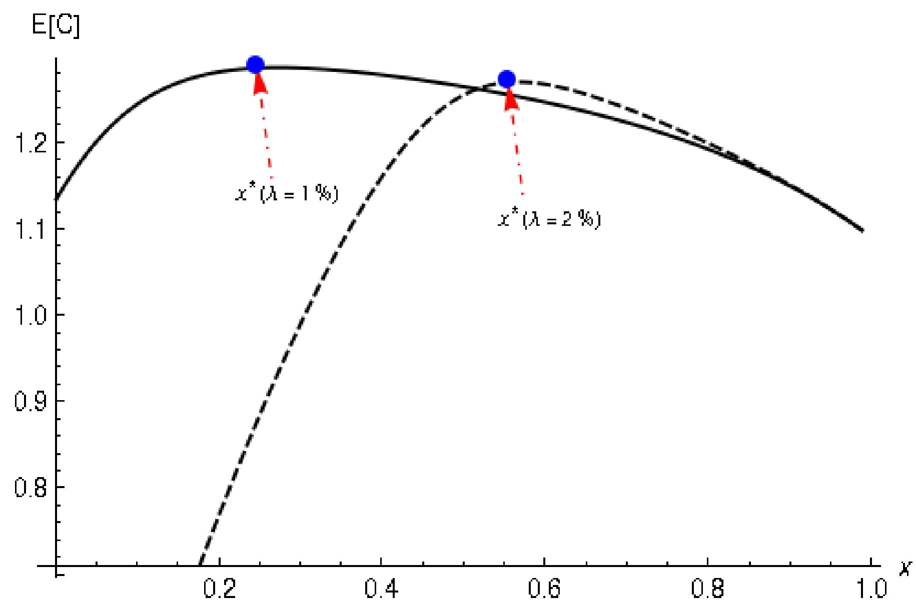
To better appreciate how regulation alters expected consumption, it is useful to examine how $\mathbb{E}[\mathcal{C}(x)]$ varies with x in equation (2). Although an increase in x increases the expected losses of regulation linearly, such an increase does not necessarily increase the expected benefits of regulation in the same fashion. While an increase in x decreases $(1-x)$, increasing x might increase or decrease $e(x)$. The reason is that changes in the expected fraction of failing banks resulting from changes in x—which can be approximately captured by the derivative of $e(x)$, $\frac{\partial e(\cdot)}{\partial x}$—hinge on distribution $\{p_k\}_{k=0}^{n-1}$.

The analogy between domino toppling and contagion can help illustrate the previous observation. In simple words, aggregate characteristics of the domino run—encoded in $\{p_k\}_{k=0}^{n-1}$—continue to determine the expected size of the chain reaction that occurs when the first domino falls. This is because restricted banks can be thought of as being selected uniformly at random. Consequently, the way the function $e(\cdot)$ changes with x depends on the precise functional form of $\{p_k\}_{k=0}^{n-1}$. In other words, the anatomy of contagion has first-order effects on how regulation alters expected consumption.

Figure 1 illustrates the previous observation with a simple example. Consider an economy with $n = 100$ banks wherein each bank is connected to every other bank. Assume $\omega = 0.1$, $\delta = 0.9$, and $\Delta = 0.8$. Suppose each exposure, independently of others, can be contagious with probability $\lambda > 0$. Then, $\{p_k\}_{k=0}^{n-1}$ can be approximated by a Poisson distribution with mean $\lambda \times n$. Figures 1(a) and 1(b) depict how the network of contagious exposures (represented by blue dotted lines) could look when $\lambda = 1\%$ or $\lambda = 2\%$, respectively. The juxtaposition of Figures 1(a) and 1(b) highlights that even small changes in λ can have a consequential effect on how contagion manifests within the model. With these parameterizations in hand, Figure 1(c) depicts expected consumption as a function of the policy choice, x. As Figure 1(c) shows, how expected consumption varies with x depends on the precise value of λ. The higher the λ, the higher the average number of contagious exposures per bank and, thus, the higher the susceptibility of the economy to contagion. Consequently, the higher the fraction of banks that must be restricted to keep cascades locally confined, $x^*(\lambda)$.



(a) $\lambda = 1\%$ (b) $\lambda = 2\%$



(c) Expected Consumption

Figure 1. Potential anatomy of contagion, expected consumption, and optimal policies, $x^*(\lambda)$, when $\{p_k\}_{k=0}^{n-1}$ follows a Poisson distribution. Parameterization: $n = 100$, $\omega = 0.1$, $\delta = 0.9$, and $\Delta = 0.8$. Nodes represent banks. Dotted lines represent contagious exposures.

III. Optimal Regulation

This section characterizes the fraction of restricted banks selected to maximize expected consumption, x^*, and how such a policy varies with the primitives of the model. Section III.A shows that, within the model, this policy always exists and provides an intuitive characterization. By focusing on the limiting case when n grows large, Section III.B offers a simpler characterization.

A. *Optimal policies in finite economies*

The next proposition shows that, irrespective of how large or interconnected the economy might be, an optimal policy always exists within the model.

PROPOSITION 2: *The solution of the optimization problem*

$$\max_{x\in[0,1]} \quad \mathbb{E}[\mathcal{C}(x)] \tag{3}$$

always exists for any (well-defined) distribution $\{p_k\}_{k=0}^{n-1}$.

Although Proposition 2 shows that an optimal policy exists, it is not clear how this policy can be found. By imposing more structure on distribution $\{p_k\}_{k=0}^{n-1}$, the next proposition characterizes the optimal policy as the solution of an optimality condition.

PROPOSITION 3: *Given parameters ω, δ, and Δ, suppose $\{p_k\}_{k=0}^{n-1}$ is such that $\mathbb{E}[\mathcal{C}(x)]$ is a strictly concave function of x, $\forall x \in (0,1)$. Then, the optimal policy, x^*, satisfies:*

$$\eta\left(e(x^*) - (1-x^*)\frac{\partial e(x)}{\partial x}\bigg|_{x=x^*}\right) = \delta\Delta. \tag{4}$$

Equation (4) helps illustrate the main trade-off of the policymaker. Intuitively, x^* is deliberately selected in such a way that the benefits of restricting the last bank are equal to the losses resulting from those restrictions. These benefits arise from the fact that restricting the last bank prevents not only its potential failure, but also the potential failure of its (direct and indirect) neighbors, thereby reducing the expected number of failures. Losses arise because the last restricted bank is forced to forgo profitable investments by holding a larger fraction of its portfolio in cash. In sum, x^* is chosen to limit the spread of failures while avoiding excessive losses resulting from such a regulation.

B. Optimal policies when n grows large

Although the previous characterization is useful, it relies on computing probabilities $\{\phi_m^x\}_{m=1}^{n(1-x)}$, as function $e(x)$ depends on them. Because these probabilities require the calculation of sums and derivatives that might not have closed-form expressions, one is likely to resort to numerical methods when solving for x^*. One way of simplifying the above calculations is to focus on economies of infinite size—which can be broadly thought of as economies composed of a finite but large number of banks. In these economies, any cascade of finite size becomes negligible as the economy grows large. Consequently, only cascades proportional to the size of the economy matter. Because only sufficiently large cascades are consequential, it is not necessary to keep track of the whole sequence of probabilities $\{\phi_m^x\}_{m=1}^{n(1-x)}$.

The next proposition shows that, as the economy grows large, only the first two moments of distribution $\{p_k\}_{k=0}^{n-1}$ are needed to characterize the optimal policy.

PROPOSITION 4: *Define the threshold*

$$x_r \equiv 1 - \lim_{n\to\infty}\left(\frac{\sum_{k=0}^{n-1} kp_k}{\left(\sum_{k=0}^{n-1} k^2 p_k\right) - \left(\sum_{k=0}^{n-1} kp_k\right)}\right). \tag{5}$$

Then,

$$x^* = \begin{cases} x_r & \textit{if } \delta\Delta \leq \frac{\eta}{x_r}, \\ 0 & \textit{otherwise.} \end{cases} \tag{6}$$

The basic idea behind Proposition 4 is simple. Once x exceeds a critical threshold, x_r, large cascades are almost surely prevented, as regulation effectively disintegrates the network of contagious exposures into smaller and disconnected parts, keeping cascades locally confined. Notably, as shown by equation (5), the value of this critical threshold depends only on the first two moments of distribution $\{p_k\}_{k=0}^{n-1}$. And the variance-to-mean ratio, $\frac{\left(\sum_{k=0}^{n-1} k^2 p_k\right)}{\left(\sum_{k=0}^{n-1} k p_k\right)}$, becomes key. The reason is that, at the limit, high variation in the number of contagious exposures makes the economy more susceptible to contagion, as banks with a disproportionately large number of exposures can reach a large number of banks if they fail. Finally, equation (6) shows that it is optimal to prevent large cascades only when losses resulting from regulation are sufficiently small. However, if these losses are significant, i.e., $\frac{\eta}{x_r} < \delta\Delta$, a non-interventional policy becomes optimal.

B.1. Relevance of the variance-to-mean ratio

This section illustrates the relevance of the variance-to-mean ratio for policymaking. By characterizing the threshold x_r when $\{p_k\}_{k=0}^{n-1}$ follows either a Poisson or a Power-law distribution, the following examples underscore that a long tail in $\{p_k\}_{k=0}^{n-1}$ can make non-interventional policies optimal even if the anticipated losses resulting from regulation are arbitrarily small.

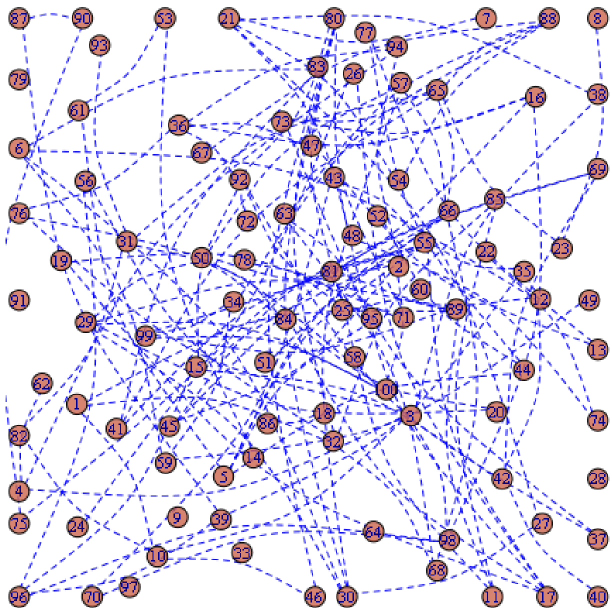
EXAMPLE 1 (Poisson): *Suppose* $p_k = \frac{\alpha^k e^{-\alpha}}{k!}$, *with* $\alpha \geq 1$. *Then,*

$$x_r = 1 - \frac{1}{\alpha}.$$

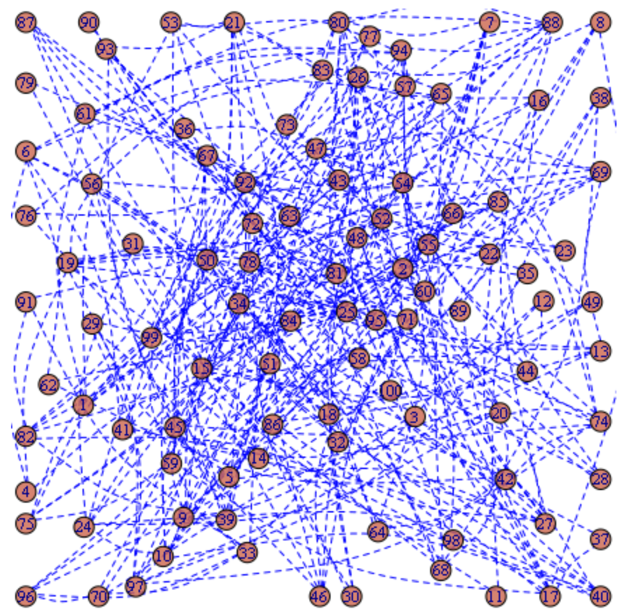
EXAMPLE 2 (Power-law): *Suppose* $p_k \propto k^{-\alpha}$, *with* $\alpha > 1$. *Then,*

$$x_r = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} & \textit{if } \alpha > 3 \\ 1 & \textit{if } 1 < \alpha \leq 3. \end{cases}$$

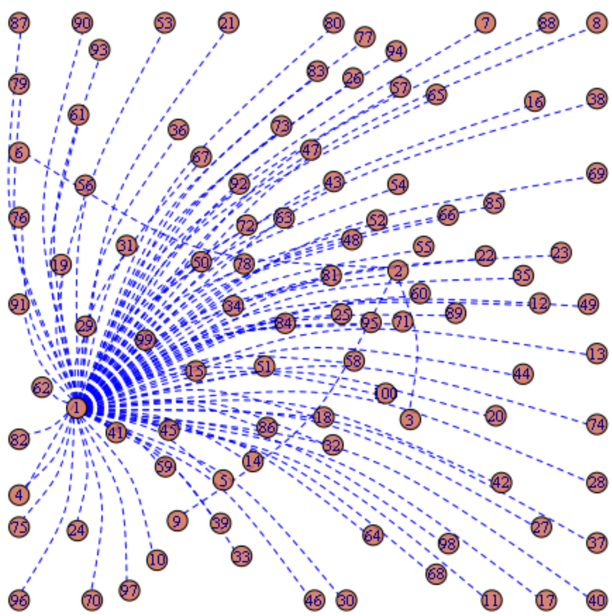
Figure 2 helps emphasize the critical differences in the anatomy of contagion if $\{p_k\}_{k=0}^{n-1}$ follows either a Poisson or a Power-law distribution. If $\{p_k\}_{k=0}^{n-1}$ follows a Poisson distribution, contagious exposures are somewhat equally spread across banks—see Figures 2(a) and 2(b). Hence, banks tend to act in a similar way from the perspective of contagion. As shown by Example 1, when the average number of contagious exposures—captured by parameter α—increases, x_r increases because a higher fraction of banks must be restricted to preclude the rise of large cascades. If $\{p_k\}_{k=0}^{n-1}$ follows a Power-law distribution, however, the situation is fundamentally different, as variation in the number of contagious exposures across banks can be significant—see Figures 2(c) and 2(d). As Example 2 shows, if $\alpha \leq 3$, large cascades can be prevented only by restricting every single bank. The reason is that the policymaker is unlikely to restrict the few banks driving contagion, as restrictions can be thought of as being implemented uniformly at random. Here, a non-interventional policy is optimal even if the anticipated losses resulting from regulation might be extremely small.



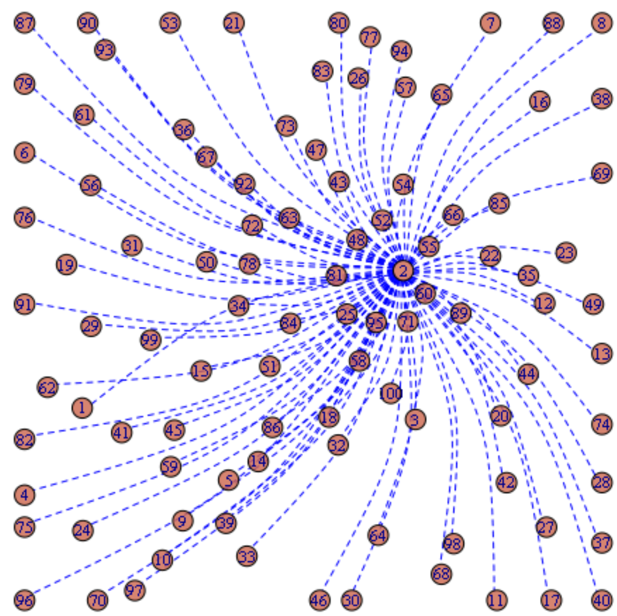
(a) Poisson: $\alpha = 3$



(b) Poisson: $\alpha = 6$



(c) Power-law: $\alpha = 3$



(d) Power-law: $\alpha = 6$

Figure 2. Potential anatomy of contagion under Poisson and Power-law distributions; $n = 100$. Nodes represent banks. Dotted lines represent contagious exposures.

IV. Targeted Regulation

Based on the assumption that banks are ex ante identical from policymakers' perspective, the previous section highlights that large variation in the number of contagious exposures can alter the ability of policymakers to keep contagion locally confined. However, this assumption misses a critical observation. Today policymakers are likely to have tools to determine whether certain banks can play a more relevant role in spreading failures in times of stress. This section investigates how these tools can reshape the design of optimal interventions.

To fix ideas, assume that before designing interventions the policymaker must decide whether to acquire an information technology at a cost κ. Although this technology allows the policymaker to rank banks based on their (future) number of contagious exposures, the precise number of contagious exposures per bank remains unknown. Parameter κ can be broadly interpreted as the cost of improving information about the role banks may play in contagion during times of stress.

The existence of this technology fundamentally changes the formulation of the optimal policy. If the policymaker decides not to pay κ, then banks are ex-ante identical from her point of view. However, by paying κ, the policymaker can target banks when implementing interventions. Although investing in this technology is costly, not doing so is also costly as it results in losses from regulating an excessively large number of banks. The optimal policy is then jointly determined by the choice of whether to pay κ and which banks (or how many) must be restricted.

To facilitate computations, the following analysis focuses on economies when n grows large. For tractability, I assume that after acquiring the information technology, the policymaker restricts all banks that are likely to exhibit more than K^* contagious exposures. Leveraging the fact that contagious exposures are randomly determined, the next proposition characterizes the critical threshold K^* as a function of distribution $\{p_k\}_{k=0}^{n-1}$.

PROPOSITION 5: *Suppose the policymaker has acquired the information technology. The* *optimal threshold K^* then solves*

$$\lim_{n\to\infty}\left(\sum_{k=0}^{n-1} kp_k\right) = \sum_{k=0}^{K^*} k(k-1)p_k. \tag{7}$$

The idea behind Proposition 5 is simple. Restricting every bank that is likely to exhibit more than K^* contagious exposures is equivalent to removing probability mass at the end of the right tail of distribution $\{p_k\}_{k=0}^{n-1}$. Importantly, removing such probability mass can have a disproportionate effect on how contagion manifests, as well-connected banks and their neighbors (and their neighbors' neighbors, and so on) no longer propagate failures when economic conditions deteriorate. The critical threshold K^* is selected to ensure that the probability mass removed from said tail is sufficiently large to almost surely prevent the emergence of large cascades of failures.

Naturally, the policymaker's decision to acquire this information technology depends on the extent to which this technology helps mitigate contagion more effectively. It is then illustrative to define the social value of acquiring such technology. Let $\tilde{x}_t$ and $\tilde{x}_r$ denote the optimal fraction of restricted banks selected after deciding whether or not to pay κ, respectively. Intuitively, the value of acquiring the information technology, $\mathcal{V}$, corresponds to the expected gains in consumption resulting from targeted interventions. Therefore,

$$\mathcal{V} \equiv \mathbb{E}[\mathcal{C}(\tilde{x}_t)] - \mathbb{E}[\mathcal{C}(\tilde{x}_r)] = (\tilde{x}_r - \tilde{x}_t)\delta\Delta + \eta\left((1-\tilde{x}_r)e(\tilde{x}_r) - (1-\tilde{x}_t)e(\tilde{x}_t)\right). \tag{8}$$

Equation (8) highlights the two components of the value of this information. The first term, $(\tilde{x}_r - \tilde{x}_t)\delta\Delta$, captures the idea that targeted interventions are expected to allow the policymaker to restrict fewer banks, decreasing losses resulting from excessive regulation. The second term, $\eta\left((1-\tilde{x}_r)e(\tilde{x}_r) - (1-\tilde{x}_t)e(\tilde{x}_t)\right)$, captures the idea that targeted interventions are expected to decrease the spread of failures more effectively, as the policymaker restricts banks that are likely to exhibit the highest number of contagious exposures. With the previous definition in hand, the next proposition characterizes the conditions under which paying

κ is optimal.

PROPOSITION 6: *Define the threshold* $x_t \equiv \left(1 - \sum_{k=0}^{K^*} p_k\right)$. *And let* x_r *denote the threshold defined in equation (5). The value of acquiring the information technology,* $\mathcal{V}$*, then equals*

$$\mathcal{V} \quad = \quad \begin{cases} \Delta x \delta\Delta & \text{if } \delta\Delta \leq \frac{\eta}{x_r} \\ \eta - x_t \delta\Delta & \text{if } \frac{\eta}{x_r} < \delta\Delta \leq \frac{\eta}{x_t} \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

where $\Delta x \equiv x_r - x_t$. *If* $\mathcal{V} \geq \kappa$*, then acquiring the information technology is optimal.*

With the above proposition in hand, the next proposition characterizes the optimal fraction of restricted banks, x^*, as a function of critical thresholds x_r, x_t, their difference, Δx, and the cost of acquiring the information technology, κ.

PROPOSITION 7: *The optimal fraction of restricted banks,* x^**, equals*

$$x^* \quad = \quad \begin{cases} x_r & \text{if } \delta\Delta \leq \min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\} \\ x_t & \text{if } \min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\} < \delta\Delta \leq \frac{\eta-\kappa}{x_t} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Intuitively, when $\delta\Delta \leq \min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\}$, losses resulting from nontargeted policies are small. Acquiring the information technology is then not optimal, and, thus, $x^* = x_r$. When $\min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\} < \delta\Delta \leq \frac{\eta-\kappa}{x_t}$, however, nontargeted policies become sufficiently costly as they restrict an excessively large number of banks. It is then optimal to acquire the information technology and implement targeted interventions, and, thus, $x^* = x_t$. Finally, when $\delta\Delta > \frac{\eta-\kappa}{x_t}$, losses associated with any type of regulation (targeted and nontargeted) are considerably large, making a non-interventional policy optimal.

A. *Relevance of cross-sectional heterogeneity*

To illustrate how the previously described policy can vary with the anatomy of contagion—encoded in $\{p_k\}_{k=0}^{n-1}$—this section characterizes such a policy when $\{p_k\}_{k=0}^{n-1}$ follows either a Poisson or a Power-law distribution. I show that heterogeneity in the number of contagious exposures continues to play a critical role in policymaking because it affects whether targeted or nontargeted interventions are optimal. If banks are expected to behave in a similar fashion when economic conditions deteriorate, there is no significant difference between targeted and nontargeted interventions from the perspective of forestalling contagion. Consequently, nontargeted or non-interventional policies tend to be optimal. However, if a small fraction of banks are likely to drive the spread of failures, implementing targeted interventions is optimal as long as κ is sufficiently small. In sum, the scope for welfare-improving interventions within the model is closely linked to variation in the number of contagious exposures across banks.

A.1. Poisson distribution

As in Example 1, suppose $\{p_k\}_{k=0}^{n-1}$ follows a Poisson distribution with parameter $\alpha > 1$. Then,

$$x_r = 1 - \frac{1}{\alpha}, \qquad x_t = x_r - \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}, \qquad \text{and} \qquad \Delta x = \frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}, \tag{11}$$

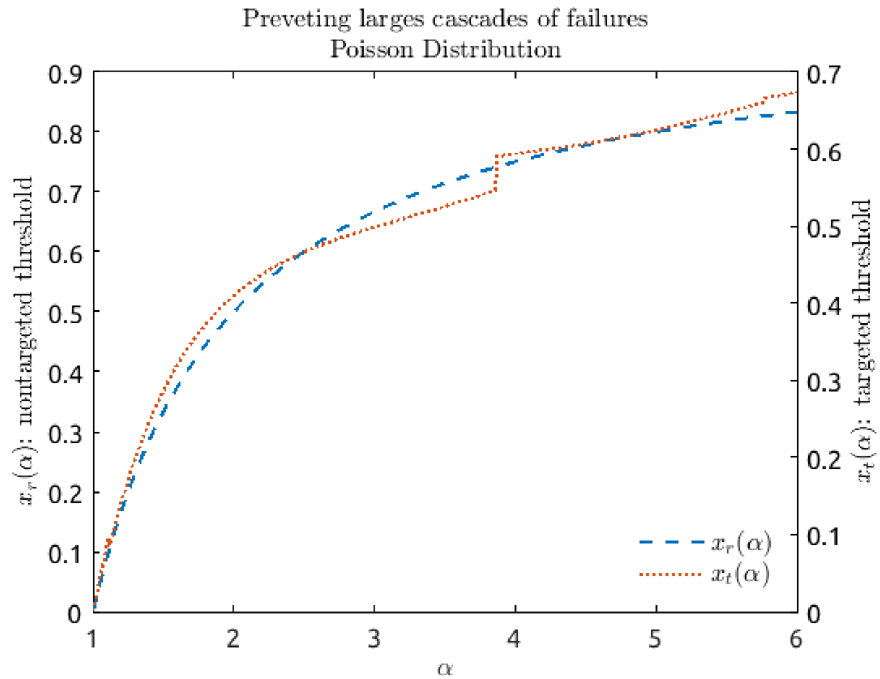
where K_α solves $\frac{1}{\alpha} = \sum_{j=0}^{(K_\alpha-2)} \frac{e^{-\alpha}\alpha^j}{j!}$. To better appreciate the differences between the aforementioned critical thresholds, Figure 3(a) depicts x_r and x_t as a function of α. As α grows, the average number of contagious exposures increases. Hence, a larger fraction of banks must be restricted to prevent large cascades, irrespective of whether policies are targeted or not.

Substituting x_r and x_t into expression (10) yields

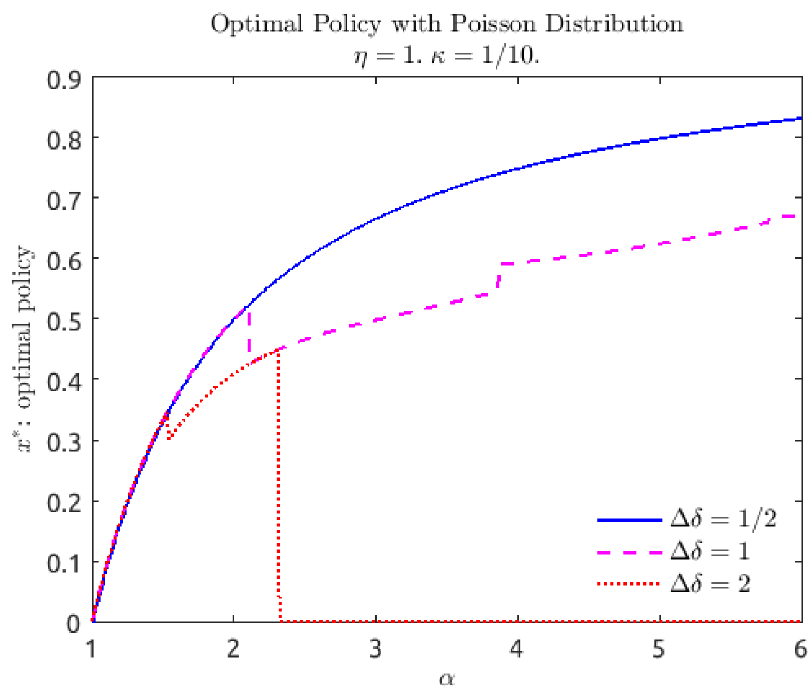
$$x^* = \begin{cases} \left(1 - \frac{1}{\alpha}\right) & \text{if } \delta\Delta \leq \min\left\{\frac{\eta\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} \\ \left(1 - \frac{1}{\alpha}\right) - \left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right) & \text{if } \min\left\{\frac{\eta\alpha}{\alpha-1}, \frac{\kappa K_\alpha!}{e^{-\alpha}\alpha^{K_\alpha}}\right\} < \delta\Delta \leq \frac{\eta-\kappa}{\left(1-\frac{1}{\alpha}\right)-\left(\frac{e^{-\alpha}\alpha^{K_\alpha}}{K_\alpha!}\right)} \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Figure 3(b) depicts x^* as a function of α for different values of $\delta\Delta$, under the assumption that $\eta = 1$ and $\kappa = 1/10$. When α is sufficiently small, contagious exposures are less frequent. Because banks behave in a similar fashion at $t = 2$, the policymaker has little incentive to implement targeted interventions when $\delta\Delta$ is small, and, thus, $x^* = x_r$. However, as α increases, more banks exhibit a higher number of contagious exposures. This increment, in turn, increases the policymaker's incentives to identify the set of most contagious banks. For moderate values of $\delta\Delta$, then, targeted interventions become optimal; that is, $x^* = x_t$. Finally, if $\delta\Delta$ is sufficiently large, losses resulting from excessive regulation are considerable, and, thus, either targeted or non-interventional policies become optimal.

Notably, when $\{p_k\}_{k=0}^{n-1}$ follows a Poisson distribution, Δx approaches zero as α grows large. Intuitively, targeted policies become less effective as contagious exposures become more and more frequent. Therefore, as α grows large, there is less room for policy improvement from targeted interventions.



(a) Optimal thresholds x_r and x_t



(b) Optimal fraction of restricted banks, x^*.

Figure 3. Critical thresholds x_r, x_t, and optimal policy, x^*, in Poisson distributions

A.2. Power-law distribution

As in Example 2, suppose that $\{p_k\}_{k=0}^{n-1}$ follows a Power-law distribution with parameter $\alpha > 1$ and the minimum number of contagious exposures per bank equals one. Then,

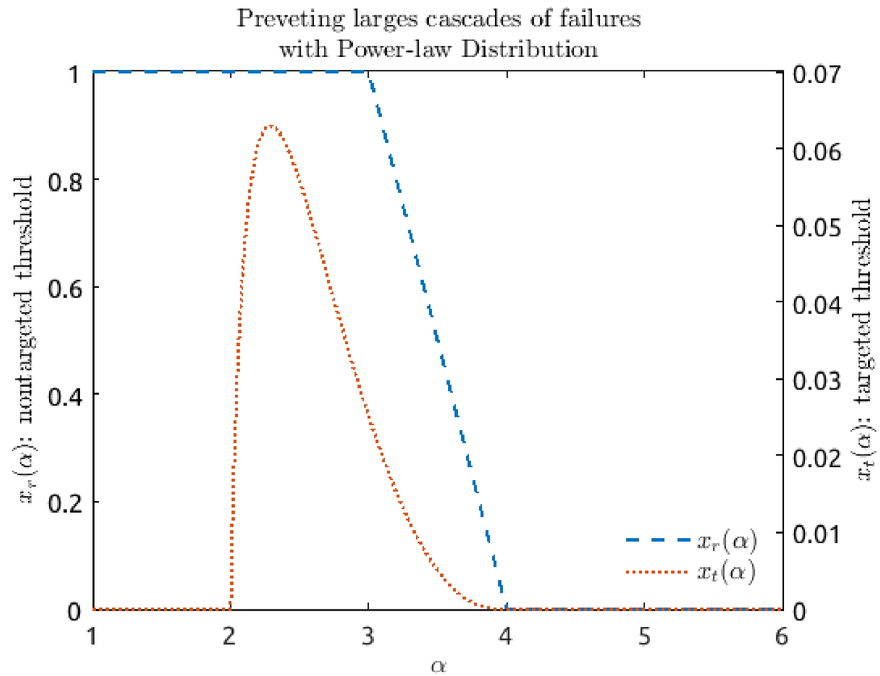
$$\begin{aligned} x_r &= \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 < \alpha \leq 3. \end{cases} \\ x_t &= K_\alpha^{(1-\alpha)} \end{aligned} \tag{13}$$

where K_α satisfies $K_\alpha^{2-\alpha} - 2 = \left(\frac{2-\alpha}{3-\alpha}\right)\left(K_\alpha^{3-\alpha} - 1\right)$. As a result,

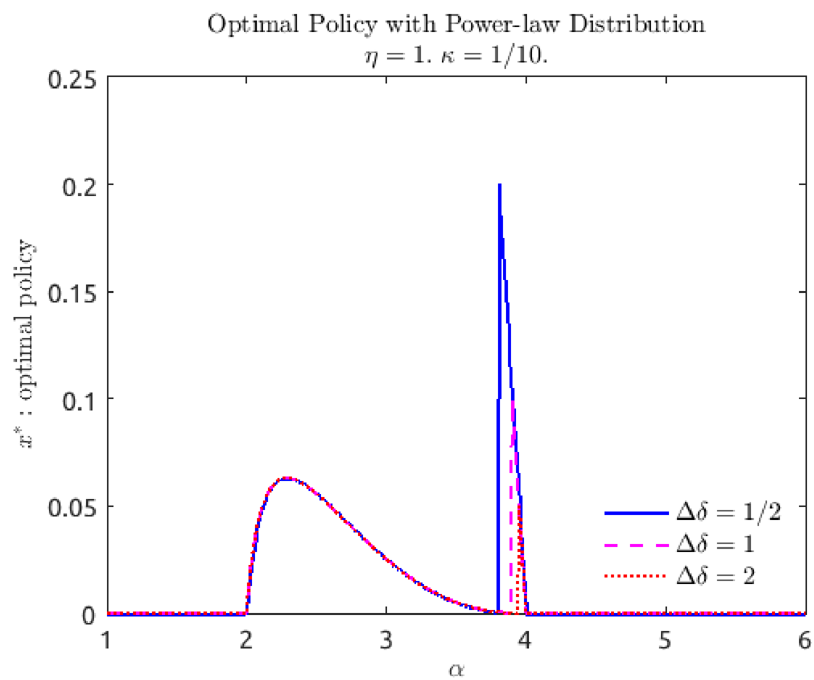
$$\Delta x = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} - K_\alpha^{(1-\alpha)} & \text{if } \alpha > 3 \\ 1 - K_\alpha^{(1-\alpha)} & \text{if } 1 < \alpha \leq 3. \end{cases} \tag{14}$$

Figure 4(a) depicts x_r and x_t as a function of α. The juxtaposition of Figures 3(a) and 4(a) shows that Δx is visibly larger in the Power-law than in the Poisson case. Notably, in the Power-law case if $\alpha \leq 3$ nontargeted policies require restricting each individual bank. Variation in the number of contagious exposures is so large that a failure affecting any single bank will almost surely affect a non-negligible fraction of them. Because only a small fraction of banks exhibit an excessively large number of contagious exposures, the policymaker is likely to miss such banks when implementing nontargeted policies. As a result, acquiring the information technology is more helpful in the Power-law than in the Poisson case.

As before, substituting x_r and x_t into expressions (10) and (9) yields the optimal fraction of restricted banks, x^*, as a function of α—which I omit here for conciseness. Figure 4(b) depicts x^* as a function of α for different values of $\delta\Delta$. Consistent with the previous analysis, when $\alpha \leq 3$ the policymaker implements targeted interventions as nontargeted policies cannot prevent large cascades; hence, $x^* = x_t$. For larger values of α, the size of large cascades decreases with α, and so does the policymaker's incentive to acquire the information technology. Consequently, when losses resulting from regulation are sufficiently small, $x^* = x_r$. Otherwise, $x^* = x_t$. Yet when losses associated with regulation are sufficiently large, a non-interventional policy is optimal, and, thus, $x^* = 0$. Importantly, because of the significant differences in their cross-sectional heterogeneity, the parameter space wherein a non-interventional policy is optimal is much smaller in the Power-law than in the Poisson case.



(a) Optimal thresholds x_r and x_t



(b) Optimal fraction of restricted banks, x^*.

Figure 4. Critical thresholds x_r, x_t, and optimal policy, x^*, in Power-law distributions

V. Discussion

This section discusses key assumptions behind the baseline model and how these assumptions relate to the existing literature and some of today's current policies. It also discusses the results of a model wherein policymakers may be uncertain about the distribution of contagious exposures across banks.

A. What do cascades of failures fundamentally capture?

In practice, cascades may capture liquidity-driven crises (as in Diamond and Rajan (2011), Caballero and Simsek (2013), and Stein (2013)) in which liquidity shocks affecting a small group of banks induce negative shocks for some of their neighbors. With high uncertainty, these neighbors may face a run due to solvency concerns, which could, in turn, cause solvency concerns about some of the neighbors' neighbors. Consequently, cascades may capture crises of confidence. Another example of cascades refers to situations in which adverse shocks affecting certain banks lead to losses in the balance sheets of their neighbors. If resulting losses exceed the capital of such neighbors, these neighbors might fail, which, in turn, may cause other banks to fail as well.

B. Why are contagious exposures randomly determined?

The random selection serves as a metaphor for regulators having difficulty assessing how exposures react when economic conditions deteriorate. This difficulty makes policymakers fundamentally uncertain about which banks are more prone to propagate failures in times of stress. Although this model does not reflect the economic incentives underlying the formation of exposures or the reasons certain institutions are more likely to propagate adverse shocks than others, it provides a simple approximation of the problem faced by policymakers nowadays—where the lack of detailed information and the high complexity of interactions among institutions besets their regulation and supervision. In doing so, this model provides a benchmark with which other models can be compared.

C. *What would happen if contagious exposures were known?*

To keep parsimony and tractability, I deliberately set aside banks' strategic considerations from the analysis. This is done by assuming that δ is sufficiently large so that no bank decides to hold a high fraction of its portfolio in cash unless forced by the policymaker. The idea is to develop a simple model to better understand how regulators formulate macroprudential policies in the face of contagion uncertainty. That said, if contagious exposures were known, banks' actions would be strategic substitutes: An increase in bank i's liquidity reduces the incentives of its neighbors to increase their liquidity. Intuitively, when bank i becomes more resilient to failures, i's neighbors also become more resilient, and, thus, have fewer incentives to increase their liquidity. It follows from Galeotti et al. (2020) that optimal interventions would target banks that do not necessarily share exposures so as to move neighbors' incentives in opposite directions.

D. *Mapping portfolio restrictions to actual policies*

In a broad sense, portfolio restrictions within the model aim to capture a diverse set of regulatory tools implemented after the 2007–2009 crisis. In particular, $(\omega + \Delta)$ is assumed to be sufficiently large that banks with more liquid portfolios are not vulnerable to failures. Intuitively, portfolio restrictions give banks incentives for prudent risk-taking, generating greater buffers to support their operations when a crisis manifests.

Although different forms of regulation, such as the risk-weighted capital ratio (RWC) and the liquidity coverage ratio (LCR), seek to address different distortions, arguments in favor of them share a common ground with the benefits of portfolio restrictions within the model. For example, the RWC aims to increase banks' skin in the game, effectively reducing banks' incentives to engage in excessive risk-taking and therefore decreasing their insolvency risk. Likewise, the LCR seeks to curb banks' incentives to engage in risky funding activities, thereby decreasing the likelihood of runs due to solvency concerns. Comparably, the net stable funding ratio (NSFR) seeks to address significant maturity mismatches between assets and liabilities, providing banks with better buffers to absorb losses when affected by adverse shocks.[4]

E. Incorporating ambiguity about contagion

Section II of the Online Appendix presents a generalization of the baseline model wherein policymakers are also uncertain about the exact functional form of $\{p_k\}_{k=0}^{n-1}$.[5] In such economies, I show that optimal policies are affected by policymakers' attitudes towards ambiguity and their beliefs about the susceptibility of the economy to contagion. Under certain conditions, small changes in beliefs generate significant changes in the optimal policy. In the face of ambiguity, policymakers are worried that their intervention may not be stringent enough to forestall large cascades when a crisis manifests. Consequently, as ambiguity increases, more banks need to be restricted. Here, the lack of certainty about $\{p_k\}_{k=0}^{n-1}$ is not a justification for inaction, but rather the opposite, as policymakers recognize the potentially large negative consequences of contagion.

VI. Conclusion

Leveraging on results from random graphs, this paper shows that basic aggregate characteristics of the anatomy of contagion can have first order implications for the design of macroprudential policies. Through the lens of a parsimonious and tractable model, I characterize optimal policies under a variety of economic environments. I show that said policies are designed to ensure that the expected benefits of forestalling contagion are equal to the anticipated losses associated with the implementation of these policies. This analysis high-

[4]See Tarullo (2019) for a broad description of the post-crisis approach to prudential regulation.

[5]See Routledge and Zin (2009) and Easley and O'Hara (2010) for models that connect liquidity and model uncertainty. See Ruffino (2014) for a discussion of some implications of model uncertainty for regulation.

lights that different connectivity structures among institutions can pose distinct challenges for policymakers who design preemptive policies that seek to mitigate contagion in times of stress.

REFERENCES

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, 2015, Systemic risk and stability in financial networks, *American Economic Review* 105, 564–608.

Adrian, Tobias, Daniel Covitz, and Nellie Liang, 2015, Financial stability monitoring, *Annual Review of Financial Economics* 7, 357–395.

Aldasoro, Iñaki, Domenico Delli Gatti, and Ester Faia, 2017, Bank networks: Contagion, systemic risk and prudential policy, *Journal of Economic Behavior and Organization* 142, 164 – 188.

Allen, Franklin, and Ana Babus, 2009, Networks in finance, in Paul R. Kleindorfer, and Yoram Wind, eds., *In The Network Challenge: Strategy, Profit, and Risk in an Interlinked World* (Wharton School Publishing).

Allen, Franklin, Ana Babus, and Elena Carletti, 2012, Asset commonality, debt maturity and systemic risk, *Journal of Financial Economics* 104, 519–534.

Allen, Franklin, and Douglas Gale, 2000, Financial contagion, *Journal of Political Economy* 108, 1–33.

Alvarez, Fernando, and Gadi Barlevy, 2015, Mandatory disclosure and financial contagion, Working Paper 21328, National Bureau of Economic Research.

Amini, Hamed, Rama Cont, and Andreea Minca, 2013, Resilience to contagion in financial networks, *Mathematical Finance* .

Battiston, Stefano, Domenico Delli Gatti, Mauro Gallegati, Bruce Greenwald, and Joseph Stiglitz, 2012, Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk, *Journal of Economic Dynamics and Control* 36, 1121–1141.

Beale, Nicholas, David G. Rand, Heather Battey, Karen Croxson, Robert M. May, and Martin A. Nowak, 2011, Individual versus systemic risk and the regulator's dilemma, *Proceedings of the National Academy of Sciences* 108, 12647–12652.

Caballero, Ricardo J., and Alp Simsek, 2013, Fire sales in a model of complexity, *Journal of Finance* 68, 2549–2587.

Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo, 2014, Risk sharing and contagion in networks, *CESifo Working Paper* .

Castiglionesi, Fabio, Fabio Feriozzi, and Guido Lorenzoni, 2019, Financial integration and liquidity crises, *Management Science* 65, 955–975.

Cohen, Reuven, Keren Erez, Daniel ben Avraham, and Shlomo Havlin, 2000, Resilience of the internet to random breakdowns, *Physical Review Letters* 85, 4626–4628.

Cohen, Reuven, Keren Erez, Daniel ben Avraham, and Shlomo Havlin, 2001, Breakdown of the internet under intentional attack, *Physical Review Letters* 86, 3682–3685.

Cont, Rama, Amal Moussa, and Edson Santos, 2013, Network structure and systemic risk in banking systems, in Jean-Pierre Fouque, and Joe Langsam, eds., *Handbook of Systemic Risk* (Cambridge University Press).

Dasgupta, Amil, 2004, Financial contagion through capital connections: A model of the origin and spread of bank panics, *Journal of the European Economics Association* 2, 1049–1084.

Diamond, Douglas W., and Raghuram G. Rajan, 2011, Fear of fire sales, illiquidity seeking, and credit freezes, *The Quarterly Journal of Economics* 126, 557–591.

Easley, David, and Maureen O'Hara, 2010, Liquidity and valuation in an uncertain world, *Journal of Financial Economics* 97, 1 – 11.

Eisenberg, Larry, and Thomas Noe, 2001, Systemic risk in financial systems, *Management Science* 47, 236–249.

Elliott, Matthew, Benjamin Golub, and Matthew Jackson, 2014, Financial networks and contagion, *American Economic Review* 104, 3115–3153.

Erol, Selman, and Guillermo Ordoñez, 2017, Network reactions to banking regulations, *Journal of Monetary Economics* 89, 51 – 67, Carnegie-Rochester-NYU Conference Series on the Macroeconomics of Liquidity in Capital Markets and the Corporate Sector.

Freixas, Xavier, Bruno M. Parigi, and Jean-Charles Rochet, 2000, Systemic risk, interbank relations, and liquidity provision by the central bank, *Journal of Money, Credit and Banking* 32, 611–638.

Gai, Prasanna, Andrew Haldane, and Sujit Kapadia, 2011, Complexity, concentration and contagion, *Journal of Monetary Economics* 58, 453–470.

Galeotti, Andrea, Benjamin Golub, and Sanjeev Goyal, 2020, Targeting interventions in networks, *Econometrica* 88, 2445–2471.

Gennaioli, Nicola, and Andrei Shleifer, 2018, *A Crisis of Beliefs Investor Psychology and Financial Fragility* (Princeton University Press).

Georg, Co-Pierre, 2013, The effect of the interbank network structure on contagion and common shocks, *Journal of Banking and Finance* 37, 2216–2228.

Glasserman, Paul, and H. Peyton Young, 2015, How likely is contagion in financial networks?, *Journal of Banking and Finance* 50, 383–399.

Glasserman, Paul, and H. Peyton Young, 2016, Contagion in financial networks, *Journal of Economic Literature* 54, 779–831.

Gofman, Michael, 2017, Efficiency and stability of a financial architecture with too-interconnected-to-fail institutions, *Journal of Financial Economics* 124, 113–146.

Goyal, Sanjeev, and Adrien Vigier, 2014, Attack, defense, and contagion in networks, *Review of Economic Studies* 81, 1518–1542.

Haldane, Andrew G., and Robert M. May, 2011, Systemic risk in banking ecosystems, *Nature* 469.

IGM Forum, 2017, Factors contributing to the 2008 global financial crisis.

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, 2005, A smooth model of decision making under ambiguity, *Econometrica* 73, 1849–1892.

Lagunoff, Roger, and Stacey L. Schreft, 2001, A model of financial fragility, *Journal of Economic Theory* 99, 220 − 264.

Lazear, Edward P., 2011, The euro crisis: Doubting the 'domino' effect.

Leitner, Yaron, 2005, Financial networks: Contagion, commitment, and private sector bailouts, *Journal of Finance* 60, 2925–2953.

Molloy, Michael, and Bruce Reed, 1995, A critical point for random graphs with a given degree sequence, *Random Structures and Algorithms* 6, 161–180.

Molloy, Michael, and Bruce Reed, 1998, The size of the giant component of a random graph with a given degree sequence, *Combinatorics, Probability and Computing* 7, 295–305.

Newman, M. E. J., 2007, Component sizes in networks with arbitrary degree distributions, *Phys. Rev. E* 76, 045101.

Nier, Erlend, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn, 2007, Network models and financial stability, *Journal of Economic Dynamics and Control* 31, 2033–2060.

Ok, Efe, 2007, *Real Analysis with Economic Applications* (Princeton University Press).

Rochet, Jean-Charles, and Jean Tirole, 1996, Interbank lending and systemic risk, *Journal of Money, Credit, and Banking* 28, 733–762.

Routledge, Bryan R., and Stanley E. Zin, 2009, Model uncertainty and liquidity, *Review of Economic Dynamics* 12, 543 – 566.

Ruffino, Doriana, 2014, Some implications of knightian uncertainty for finance and regulation, *FEDS Notes* .

Stein, Jeremy C., 2013, Liquidity regulation and central banking, Remarks at the "Finding the Right Balance" 2013 Credit Markets Symposium sponsored by the Federal Reserve Bank of Richmond, Charlotte, North Carolina.

Tarullo, Daniel K., 2019, Financial regulation: Still unsettled a decade after the crisis, *Journal of Economic Perspectives* 33, 61–80.

Online Appendix for The Anatomy of Contagion and Macroprudential Policies

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This appendix contains material to supplement the analysis in "The Anatomy of Contagion and Macroprudential Policies." Section I reports proofs of propositions and examples in the paper. Section II presents a generalization of the baseline model in which policymakers are uncertain about the precise functional form of distribution $\{p_k\}_{k=0}^{n-1}$.

I. Mathematical Derivations

This section contains derivations of propositions and examples in the paper. Most of these derivations are built from a simple observation. Given how failures propagate across banks and that contagious exposures are randomly determined, $\{p_k\}_{k=0}^{n-1}$ can be thought of as the degree distribution of random networks among n banks. As a result, one can leverage results from random graphs to solve for optimal policies.

Proof of Proposition 1. Let θ_k^x denote the probability that an unrestricted bank shares k contagious exposures with other unrestricted banks at $t = 2$. To compute such a probability, it is illustrative to explore the distribution of contagious exposures among unrestricted banks after a fraction x of them were restricted at $t = 0$. To fix ideas, consider a bank with k_0 contagious exposures. After restrictions have been imposed, that bank may have k contagious exposures, with $k \leq k_0$, as some of its neighbors may be restricted at $t = 0$. Because banks are ex-ante identical from the perspective of the policymaker, the probability that a subset of k neighbors is not restricted is $(1-x)^k$, whereas the probability that the remaining neighbors are restricted is $(x)^{k_0-k}$. Because there are $\binom{k_0}{k}$ different subsets of k neighbors, the distribution of contagious exposures among unrestricted banks is then

$$\theta_k^x = \begin{cases} \sum_{j=k}^{n-1} p_j \binom{j}{k} (1-x)^k (x)^{j-k} & \text{if } k = \{0, \cdots, n(1-x)-1\} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Given how failures spread across banks and that restricted banks can be thought of as being selected uniformly at random, the sequence of probabilities $\{\theta_k^x\}_{k=0}^{n(1-x)-1}$ is equivalent to the degree distribution of a randomly generated network among $n(1-x)$ unrestricted banks. Consequently, ϕ_m^x represents the probability that a randomly chosen unrestricted bank belongs to a connected subgraph of size m. Therefore, one can directly apply results in Newman (2007) and show that

$$\phi_m^x = \begin{cases} \frac{\langle \theta^x \rangle}{(m-1)!} \left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta_k^x\}_k\right)^m \right] \right) \Big|_{z=0}, & \text{with } m = \{2, \cdots, n(1-x)\} \\ \theta_0^x, & \text{with } m = 1. \end{cases}$$

where $\langle \theta^x \rangle$ denotes the average number of contagious exposures among unrestricted banks and $\left(\frac{d^{m-2}}{dz^{m-2}} \left[g\left(z, \{\theta_k^x\}_k\right)^m \right] \right) \Big|_{z=0}$ denotes the $(m-2)$ derivative of $g\left(z, \{\theta_k^x\}_k\right)^m$ evaluated at $z = 0$, where $g(z, \{\theta_k^x\}_k)$ represents the excess degree distribution function of $\{\theta_k^x\}_k$, defined as

$$g(z, \{\theta_k^x\}_k) \equiv \sum_{k=0}^{n(1-x)-2} \left(\frac{(k+1)\theta_{k+1}^x}{\langle \theta^x \rangle} \right) z^k.$$

$\square$

REMARK 1 (Numerical solutions): *When the model is solved numerically, the $(m-2)$ derivative of $g\left(z, \{\theta_k^x\}_k\right)^m$ evaluated at $z = 0$, can be approximated by*

$$\frac{1}{\epsilon^{m-2}} \left[\sum_{j=0}^{m-2} (-1)^{m-2-j} \binom{m-2}{j} g\left(j\epsilon, \{\theta_k^x\}_k\right)^m \right]$$

with $\epsilon > 0$ sufficiently small.

Proof of Proposition 2. Note that $\mathbb{E}[\mathcal{C}(x)]$ is bounded because $0 \leq e(x) \leq (1-x)$. Moreover, for any well-defined distribution $\{p_k\}_{k=0}^{n-1}$, function $e(x)$ is a semi-continuous function of x. Consequently, $\mathbb{E}[\mathcal{C}(x)]$ is also semi-continuous. Baire's generalization of the Weierstrass' theorem ensures that $\mathbb{E}[\mathcal{C}(x)]$ always attains its maximum over a compact set; see (Ok, 2007, Chapter 4). $\square$

Proof of Proposition 3. Because $\mathbb{E}[\mathcal{C}(x)]$ is a strictly concave function of x, the solution of Problem 3 in the paper must satisfy the first order condition[1]

$$\frac{\partial}{\partial x} \left(\mathbb{E}[\mathcal{C}(x)] \right) \Big|_{x=x^*} = 0, \tag{2}$$

which can be rewritten as

$$\eta \left(e(x^*) - (1-x^*) \frac{\partial e(x)}{\partial x} \Big|_{x=x^*} \right) = \delta \Delta.$$

$\square$

The rest of the analysis focuses on the case when n grows large. In this case, any cascade of finite size becomes negligible as $n \to \infty$. The following discussion helps better understand how large a cascade must be to matter and under what conditions such large cascades emerge.

The rise of large cascades.— To fix notation, let $\mathcal{G}_n$ denote a network of contagious exposures among n banks and $\{\mathcal{G}_n\}_{n \in \mathbb{N}}$ denote a sequence of such networks, indexed by the number of banks n. Let $\mathcal{S}\left(\mathcal{G}_n\right)$ denote the largest subset of connected banks in $\mathcal{G}_n$, and let $|\mathcal{S}\left(\mathcal{G}_n\right)|$ denote the cardinality of such a set. To determine the condition under which large

[1]The fraction of restricted banks must be a rational number because n is a natural number. However, the solution of equation (2) could be an irrational number. Nonetheless, the optimal intervention x^* gets arbitrarily close to the solution of equation (2) as n grows large.

cascades arise, one can use the following idea proposed by Molloy and Reed (1995) and Cohen et al. (2000). Let n_0 denote a large natural number. Suppose there are two banks that belong to each element in the subsequence $\{\mathcal{S}(\mathcal{G}_n)\}_{n\geq n_0}$—say, i and j, which are directly connected. If bank i (or j) is also directly connected to another bank—and loops of contagious exposures are ignored—then the size of the largest sequence of connected banks is proportional to the size of the system—i.e., $\lim_{n\to\infty}\frac{\mathbb{E}|\mathcal{S}(\mathcal{G}_n)|}{n}>0$—and, thus, large cascades occur; otherwise, the largest sequence of connected banks is fragmented, and, thus, $\lim_{n\to\infty}\frac{\mathbb{E}|\mathcal{S}(\mathcal{G}_n)|}{n}=0$.[2] Therefore, the condition that determines the emergence of large cascades is given by

$$\lim_{n\to\infty}\mathbb{E}_n\left[k_i|i\leftrightarrow j\right] \;=\; \lim_{n\to\infty}\sum_{k_i}k_i\mathbb{P}_n\left[k_i|i\leftrightarrow j\right] \;\leq\; 2, \tag{3}$$

where $\mathbb{P}_n\left[k_i|i\leftrightarrow j\right]$ denotes the probability that bank i has k_i contagious exposures, given that i and j are connected via one contagious exposure. It follows from Bayes' rule that

$$\mathbb{P}_n\left[k_i|i\leftrightarrow j\right] \;=\; \frac{\mathbb{P}_n\left[i\leftrightarrow j|k_i\right]\mathbb{P}_n\left[k_i\right]}{\mathbb{P}_n\left[i\leftrightarrow j\right]}.$$

Because contagious exposures are randomly determined,

$$\mathbb{P}_n\left[i\leftrightarrow j\right]=\frac{\mathbb{E}_n[k]}{n-1} \quad\text{and}\quad \mathbb{P}_n\left[i\leftrightarrow j|k_i\right]=\frac{k_i}{n-1}.$$

Thus, equation (3) is equivalent to

$$\lim_{n\to\infty}\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]} \;\leq\; 2, \tag{4}$$

That is, if the variance-to-mean ratio at the limit, $\lim_{n\to\infty}\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]}$, is smaller than or equal to 2, large cascades can be prevented. It is important to note that the derivation of equation (4) does not rely on the functional form of $\mathbb{P}_n[k]$ and applies to any distribution of exposures in which banks are randomly connected to each other. Intuitively, equation (4) establishes that if, at the limit, there is enough variation in the number of contagious exposures across banks, then the failure of a single bank almost surely affects a non-negligible fraction of them. High variation in the number of contagious exposures makes the economy more susceptible to contagion as banks with a large number of contagious exposures can effectively reach a large fraction of banks when failing.

Proof of Proposition 4. Because restricted banks neither fail nor propagate failures, restricting a sufficiently large fraction of banks can potentially prevent the emergence of large cascades. When x exceeds a critical threshold, x_r, large cascades can be prevented as the network of contagious exposures among unrestricted banks disintegrates into smaller and disconnected parts, keeping contagion locally confined. To determine such threshold, suppose a fraction x of bank are restricted uniformly at random. After restrictions have been

[2] As n grows large, loops of contagious exposures can be ignored for $\frac{\mathbb{E}_n(k^2)}{\mathbb{E}_n(k)}<2$. For more details, see Cohen et al. (2000).

imposed, a bank with k_0 contagious exposures may have only k contagious exposures, with $k \leq k_0$, as some of its neighbors may be restricted. In addition, the probability that a subset of k neighbors is not restricted is $(1-x)^k$, whereas the probability that the remaining neighbors are restricted is x^{k_0-k}. Because there are $\binom{k_0}{k}$ different subsets of k neighbors, the distribution of contagious exposures among unrestricted banks is

$$\mathbb{P}'_n(k) = \sum_{k \geq k_0} p_{k_0} \binom{k_0}{k} (1-x)^k x^{k_0-k},$$

and, thus,

$$\mathbb{E}'_n[k] = \langle k \rangle (1-x) \quad \text{and} \quad \mathbb{E}'_n[k^2] = \langle k^2 \rangle (1-x)^2 + \langle k \rangle x(1-x), \tag{5}$$

where $\langle k \rangle \equiv \sum_{k=0}^{n-1} k p_k$, $\langle k^2 \rangle \equiv \sum_{k=0}^{n-1} k^2 p_k$, and expectations with superscript prime denote expectations after restrictions have been implemented. As previously shown, large cascades arise if and only if

$$\lim_{n \to \infty} \frac{\mathbb{E}'_n[k^2]}{\mathbb{E}'_n[k]} \leq 2. \tag{6}$$

It then directly follows from substituting equation (5) into equation (6) that x_r must satisfy

$$x_r = 1 - \lim_{n \to \infty} \left(\frac{\sum_{k=0}^{n-1} k p_k}{\left(\sum_{k=0}^{n-1} k^2 p_k\right) - \left(\sum_{k=0}^{n-1} k p_k\right)} \right). \tag{7}$$

With the previous computations in hand, it is now illustrative to analyze how $\mathbb{E}[\mathcal{C}(x)]$ changes with x at the limit $n \to \infty$. To fix ideas, let x^* denote the smallest fraction of banks that must be restricted to prevent the emergence of large cascades. If $x \geq x^*$, then the size of the largest connected component in any realized network of contagious exposures among unrestricted banks is almost surely proportional to $\log(n)$. However, if $x < x^*$ the size of the largest connected component in said network is of order n—and the size of the second largest connected component is of order $\log(n)$; for more details, see Molloy and Reed (1998). As a result,

$$\lim_{n \to \infty} \mathbb{E}[\mathcal{C}(x)] = \begin{cases} x((1+\delta) - (\omega + \Delta)\delta) + \eta(1-x), & \text{if } x \geq x^* \\ x((1+\delta) - (\omega + \Delta)\delta) + \eta x(1-x), & \text{if } x < x^*. \end{cases}$$

Define $\Delta x = (x^* - x)$. To determine the optimal policy, it is worth noting that function $H(x) \equiv \lim_{n \to \infty} (\mathbb{E}[\mathcal{C}(x^*)] - \mathbb{E}[\mathcal{C}(x)])$ equals

$$H(x) = \begin{cases} -\Delta x \Delta \delta, & \text{if } x \geq x^* \\ \eta(1-x)^2 - \Delta x \Delta \delta, & \text{if } x < x^*. \end{cases}$$

If $\left(\frac{\eta}{\Delta\delta}\right) \geq x^*$, then $H(x) \geq 0$, $\forall\, x$ in $[0, x^*]$. Thus, x^* generates higher expected consumption than any other fraction $0 \leq x \leq 1$, as $H(x)$ is strictly positive when $x > x^*$. However, if

$\left(\frac{\eta}{\Delta\delta}\right) < x^*$, then $H(0) < 0$. Consequently, $H(x) < 0$, when $0 \leq x < x^*$, as $H(x)$ is an increasing function of x when $x < x^*$ and $H(x^*) = 0$. Therefore, $x = 0$ maximizes expected consumption. Consequently, as n grows large, the optimal policy converges to the following intervention:

$$x^{\text{optimal}} = \begin{cases} x^*, & \text{if } \left(\frac{\eta}{\Delta\delta}\right) \geq x^* \\ 0, & \text{otherwise.} \end{cases}$$

Importantly, the value of x^* does not depend on the exact values of ω, δ, and Δ as it depends only on the first two moments of distribution $\{p_k\}_{k=1}^{n-1}$ as equation (7) shows. $\square$

Proof of Example 1. For a Poisson distribution with parameter α, the first two moments are given by $\langle k \rangle = \alpha$ and $\langle k^2 \rangle = \alpha^2 + \alpha$. Thus, when restrictions are implemented at random, the critical threshold is given by $x_r(\alpha) = 1 - \frac{1}{\alpha}$. $\square$

Proof of Example 2. A continuous Power-law distribution with parameter α, minimal value k_0, and maximum value K, satisfies

$$\begin{aligned} \langle k \rangle = k_0^{\alpha-1} K^{2-\alpha} \left(\frac{\alpha-1}{\alpha-2}\right) \quad \text{and} \quad \langle k^2 \rangle &= k_0^{\alpha-1} K^{3-\alpha} \left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 1 < \alpha < 2 \\ \langle k \rangle = k_0 \left(\frac{\alpha-1}{\alpha-2}\right) \quad \text{and} \quad \langle k^2 \rangle &= k_0^{\alpha-1} K^{3-\alpha} \left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 2 < \alpha < 3 \\ \langle k \rangle = k_0 \left(\frac{\alpha-1}{\alpha-2}\right) \quad \text{and} \quad \langle k^2 \rangle &= k_0^2 \left(\frac{\alpha-1}{\alpha-3}\right) \quad \text{if } 3 < \alpha \end{aligned}$$

As a consequence, when K grows large,

$$\langle k \rangle = k_0 \left(\frac{\alpha-1}{\alpha-2}\right) \text{ if } \alpha > 2 \quad \text{and} \quad \langle k^2 \rangle = k_0^2 \left(\frac{\alpha-1}{\alpha-2}\right) \text{ if } \alpha > 3$$

and they diverge in all other cases.

Using the above equations and the condition that determines the emergence of large cascades implies that the critical threshold is then given by

$$x_r(\alpha) = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) k_0 - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$

as n grows large. $\square$

Proof of Proposition 5. Because the information technology has been acquired, the policy-maker can restrict banks with the highest number of contagious exposures. As noted by Cohen et al. (2001), restricting banks with more than K^* contagious exposures is approximately equivalent to restricting a fraction x_t of banks, where x_t satisfies

$$x_t = 1 - \sum_{k=0}^{K^*} p_k.$$

Take a bank with k contagious exposures. The fraction of contagious exposures attached to all banks with k contagious exposures equals $\frac{kp_k}{\langle k \rangle}$. Consequently, the fraction of contagious exposures attached to restricted banks is

$$s(x_t) \quad = \quad \frac{1}{\langle k \rangle}\left(\sum_{k=K^*+1}^{n-1} kp_k\right) \quad = \quad 1 - \frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K^*} kp_k\right)$$

Because imposing restrictions on a set of banks can be represented by the removal of such banks and their exposures from any realized network of contagious exposures, if x_t is equivalent to the critical threshold that prevents the rise of large cascades, x_t must satisfy

$$s(x_t) \quad = \quad x_t \quad = \quad \lim_{n \to \infty}\left(1 - \frac{\langle k(x_t) \rangle}{\langle k(x_t)^2 \rangle - \langle k(x_t) \rangle}\right).$$

Therefore,

$$\begin{aligned}
1 - s(x_t) \quad &= \quad \lim_{n \to \infty}\left(\frac{\langle k(x_t) \rangle}{\langle k(x_t)^2 \rangle - \langle k(x_t) \rangle}\right) \\
\lim_{n \to \infty}\left(\frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K^*} kp_k\right)\right) \quad &= \quad \frac{\sum_{k=0}^{K^*} kp_k}{\sum_{k=0}^{K^*} k^2 p_k - \sum_{k=0}^{K^*} kp_k} \\
\lim_{n \to \infty}\left(\frac{1}{\langle k \rangle}\left(\sum_{k=0}^{K^*} kp_k\right)\right) \quad &= \quad \frac{\sum_{k=0}^{K^*} kp_k}{\sum_{k=0}^{K^*} k(k-1)p_k} \\
\lim_{n \to \infty}\left(\langle k \rangle\right) \quad &= \quad \sum_{k=0}^{K^*} k(k-1)p_k. \qquad (8)
\end{aligned}$$

is the equation that the critical threshold K^* must solve. □

Proof of Propositions 6 and 7. To determine the value of information and optimal policy, it is illustrative to analyze how $\left(\frac{\eta}{\Delta\delta}\right)$ compares with x_t and x_r. First, suppose $x_r \leq \left(\frac{\eta}{\Delta\delta}\right)$. Then, $\mathcal{V} = \Delta x \Delta\delta$, which represents the value of information. Consequently, if $\Delta x \Delta\delta \geq \kappa$, then acquiring information is optimal and $x^{\text{optimal}} = x_t$. Otherwise, it is optimal not to acquire information and $x^{\text{optimal}} = x_r$. Second, suppose $x_t \leq \left(\frac{\eta}{\Delta\delta}\right) < x_r$. Then $\mathcal{V} = \eta - x_t\Delta\delta$ represents the value of information. Thus, if $\eta - x_t\Delta\delta \geq \kappa$, then acquiring information is optimal and $x^{\text{optimal}} = x_t$. Otherwise, acquiring information is not optimal and $x^{\text{optimal}} = 0$. Finally, suppose $\left(\frac{\eta}{\Delta\delta}\right) < x_t$. Then, $x^{\text{optimal}} = 0$, and, thus, the value of information is 0. As a result, acquiring information is not optimal.

As a consequence, the optimal intervention is given by

$$x^{\text{optimal}} \quad = \quad \begin{cases} x_r, & \text{if } x_r \leq \min\left\{\left(\frac{\eta}{\Delta\delta}\right), x_t + \frac{\kappa}{\Delta\delta}\right\} \\ x_t, & \text{if } \frac{\kappa}{\Delta\delta} + x_t \leq x_r \leq \left(\frac{\eta}{\Delta\delta}\right) \text{ or} \\ & \quad x_t \leq \left(\frac{\eta-\kappa}{\Delta\delta}\right) \text{ and } \left(\frac{\eta}{\Delta\delta}\right) < x_r \\ 0, & \quad \text{otherwise,} \end{cases} \qquad (9)$$

which can be rewritten as

$$x^{\text{optimal}} = \begin{cases} x_r, & \text{if } \Delta\delta \leq \min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\} \\ x_t, & \text{if } \min\left\{\frac{\eta}{x_r}, \frac{\kappa}{\Delta x}\right\} < \Delta\delta \leq \frac{\eta-\kappa}{x_t} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

And the value of information is

$$\mathcal{V} = \begin{cases} \Delta x \Delta\delta, & \text{if } x_r \leq \left(\frac{\eta}{\Delta\delta}\right) \\ \eta - x_t\Delta\delta, & \text{if } x_t \leq \left(\frac{\eta}{\Delta\delta}\right) \leq x_r \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

which is equivalent to

$$\mathcal{V} = \begin{cases} \Delta x \Delta\delta, & \text{if } \Delta\delta \leq \frac{\eta}{x_r} \\ \eta - x_t\Delta\delta, & \text{if } \frac{\eta}{x_r} < \Delta\delta \leq \frac{\eta}{x_t} \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

□

Critical thresholds for Poisson distributions. If information is not acquired, then $x_r = 1 - \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$. The result follows directly from the fact that $\langle k \rangle = \alpha$ and $\langle k^2 \rangle = \alpha^2 + \alpha$ for the Poisson distribution. If information is acquired, then $x_t = 1 - \sum_{k=k_{\min}}^{K^*} p_k = \sum_{k=K^*+1}^{\infty} p_k$— where K^* is the solution of equation $\langle k \rangle = \sum_{k=k_{\min}}^{K^*} k(k-1)p_k$, with $p_k = \frac{e^{-\alpha}\alpha^k}{k!}$. Notably, the fraction of exposures attached to restricted banks, p, equals $p = \frac{1}{\langle k \rangle}\sum_{k=K^*+1}^{\infty} kp_k$. Importantly, if $p = x_r$, then large cascades are prevented. Because $p = \frac{1}{\langle k \rangle}\sum_{k=K^*+1}^{\infty} kp_k = \sum_{k=K^*}^{\infty} \frac{e^{-\alpha}\alpha^k}{k!} = (\sum_{k=K^*+1}^{\infty} \frac{e^{-\alpha}\alpha^k}{k!}) + \frac{e^{-\alpha}\alpha^{K^*}}{K^*!} = x_t + \frac{e^{-\alpha}\alpha^{K^*}}{K^*!}$, then $x_t = x_r - \frac{e^{-\alpha}\alpha^{K^*}}{K^*!}$. □

Critical thresholds for Power-law distributions. If the policymaker implements targeted interventions, the following equation

$$\sum_{k=k_0}^{k_x} k(k-1)p_k = \langle k \rangle$$

determines the emergence of large cascades. Because the network follows a Power-law distribution with parameter α, the above equation is equivalent to

$$(\alpha-1)k_0^{\alpha-1}\left(\frac{k_x^{3-\alpha} - k_0^{3-\alpha}}{3-\alpha} - \frac{k_x^{2-\alpha} - k_0^{2-\alpha}}{2-\alpha}\right) = k_0\left(\frac{\alpha-1}{\alpha-2}\right)$$

which is equivalent to

$$\left(\frac{k_0}{3-\alpha}\right)\left(\left(\frac{k_x}{k_0}\right)^{3-\alpha} - 1\right) - \left(\frac{1}{2-\alpha}\right)\left(\left(\frac{k_x}{k_0}\right)^{2-\alpha} - 2\right) = 0,$$

and, thus, k_x can be derived from α—k_x is equivalent to K_α in the paper. It then follows directly from Example 2 that if the network exhibits a Power-law degree distribution of parameter α and $k_{\min} = 1$, then

$$\begin{aligned} x_r &= \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} & \text{if } \alpha > 3 \\ 1 & \text{if } 1 \leq \alpha \leq 3. \end{cases} \\ x_t &= K_\alpha^{(1-\alpha)}. \end{aligned}$$

Consequently, K_α satisfies

$$K_\alpha^{2-\alpha} - 2 = \left(\frac{2-\alpha}{3-\alpha}\right)\left(K_\alpha^{3-\alpha} - 1\right).$$

As a result,

$$\Delta x = \begin{cases} 1 - \left(\left(\frac{2-\alpha}{3-\alpha}\right) - 1\right)^{-1} - K_\alpha^{(1-\alpha)} & \text{if } \alpha > 3 \\ 1 - K_\alpha^{(1-\alpha)} & \text{if } 1 \leq \alpha \leq 3. \end{cases}$$

□

II. Extended Model

This section extends the baseline model to economies wherein the policymaker is uncertain about the precise functional form of distribution $\{p_k\}_k$. This type of uncertainty fundamentally differs from the uncertainty captured by the baseline model. In the baseline model, the policymaker knows the precise functional form of $\{p_k\}_k$. Here, however, the policymaker is unable to pin down such a distribution. Consequently, she faces model uncertainty as she is unsure about the rules that dictate how the economy behaves when economic conditions deteriorate.

To better appreciate the implications of model uncertainty, suppose distribution $\{p_k\}_k$ belongs to a known distribution family. And each distribution within that family can be pin down by a single parameter α. I capture model uncertainty by assuming that α is unknown and random. For a given value of α, the previous analysis shows that

$$\mathbb{E}_\alpha[\mathcal{C}(x)] = \eta - \eta(1-x)\left(\sum_{m=1}^{n(1-x)} \frac{m}{n}\phi_m^x(\alpha)\right) - x\Delta\delta$$

where probabilities ϕ_m^x are written as $\phi_m^x(\alpha)$ to emphasize their dependence on the exact value of α. Importantly, probabilities $\phi_m^x(\alpha)$ are now random variables as α is random, which, in turn, makes expected consumption a random variable. As a result, the previous framework—in which x is selected to maximize expected consumption—is incapable of dealing with this type of uncertainty.

To capture model uncertainty in a way consistent with the previous analysis, I extend the baseline model along two simple dimensions. First, I consider that the representative investor has preferences characterized by the smooth ambiguity model of Klibanoff et al. (2005).[3] Second, I assume that α only takes values within a known set $\mathcal{A}$. Although the policymaker does not know the exact value of α, she has prior beliefs over $\mathcal{A}$. These beliefs are captured by distribution f, which denotes a Borel probability measure on $\mathcal{A}$ with barycenter $\bar{\alpha} \equiv \int_{\alpha \in \mathcal{A}} \alpha df$.

A. *Optimal Intervention*

Optimal selection of restricted banks. With the previous modifications in hand, I now reformulate the policymaker's problem. Given a choice of whether or not to acquire the information technology, the policymaker chooses a set of banks to regulate, denoted by $\mathcal{R}$, to solve

$$\begin{aligned} \max_{\mathcal{R}} \quad & \mathbb{E}_{\bar{\alpha}}\left(\mathcal{C}_\alpha \middle| \mathcal{R}\right) - \left(\frac{\theta}{2}\right) \times \mathbb{V}_f\left(\mathbb{E}_\alpha(\mathcal{C}_\alpha \middle| \mathcal{R})\right) - \kappa \times \mathbb{1}_\kappa \\ \text{s.t.} \quad & 0 \leq |\mathcal{R}| \leq n, \end{aligned} \tag{13}$$

where operator $\mathbb{E}_{\bar{\alpha}}(\cdot)$ denotes the expectation operator when $\alpha = \bar{\alpha}$. Operator $\mathbb{V}_f(\cdot)$ denotes the variance of expected consumption based on distribution f. Parameter θ is a non-negative coefficient capturing the representative investor's attitude toward ambiguity. Notably, the extended model is equivalent to the baseline model when $\mathcal{A}$ is singleton or $\theta = 0$. When $\mathcal{A}$ is singleton, there is no model uncertainty, as the exact value of α is known. When $\theta = 0$, the representative investor is ambiguity neutral and, thus, does not mind not knowing α and behaves as if $\alpha = \bar{\alpha}$. And $\mathbb{1}_\kappa$ is an indicator function that equals 1 if the policymaker decides to acquire the information technology; otherwise, it is zero.

To illustrate the policymaker's trade-off in this new environment, I now rewrite her first order condition as

$$\begin{aligned} \eta\left(e_{\bar{\alpha}}(x^*) - (1-x^*)\left.\frac{\partial e_{\bar{\alpha}}(x)}{\partial x}\right|_{x=x^*}\right) &= \Delta\delta \\ &+ \left(\frac{\theta}{2}\right)\eta^2 \frac{\partial}{\partial x}\left.\left((1-x)^2 \int_{\alpha \in \mathcal{A}} \left(e_\alpha(x) - e_{\bar{\alpha}}(x)\right)^2 df(\alpha)\right)\right|_{x=x^*}. \end{aligned}$$

where $e_{\bar{\alpha}}(x^*) \equiv \sum_{m=1}^{n(1-x^*)} \frac{m}{n}\phi_m^x(\bar{\alpha})$. Proposition 9 shows that, under certain conditions, the optimal fraction of restricted banks gets arbitrarily close to x^*—which denotes the solution of the above equation—as the economy grows large.[4]

[3] In a broad sense, these preferences capture circumstances in which investors are uncertain about the "true model" that determines the behavior of the economy. Given the uncertainty about the model, investors may exhibit aversion to (or preference for) that uncertainty. For example, if investors are averse to such uncertainty, they worry about making non-optimal decisions ex ante because they do not know the "true model." Importantly, with these preferences, investors' tastes for risk and model uncertainty (henceforth referred to as ambiguity) can be separated in a simple form that makes it tractable to nest the baseline model into the new framework.

[4] The above equation characterizes the optimal fraction of restricted banks when the solution of the planner's problem is interior; for details about the general case, see Proposition 10 in Appendix II.B.

To appreciate the importance of model uncertainty, it is illustrative to emphasize the similarities between the above equation and equation (4) in the paper. Although the marginal benefit in both equations arises from the fact that interventions help limit the spread of cascades, the marginal cost now has an extra component. And this component is unrelated to the losses arising from portfolio restrictions. This cost arises solely from the fact that the policymaker (a) does not know the exact value of α and (b) exhibits aversion to ambiguity when $\theta > 0$. Consequently, the optimal fraction of restricted banks, x^*, now also hinges on the policymaker's prior beliefs, captured by distribution f, and her attitudes toward ambiguity, captured by parameter θ.

Value of information. Let x_t denote the optimal fraction of restricted banks chosen after acquiring the information technology, and let x_r denote the optimal fraction of restricted banks chosen when such technology is not acquired. The social value of information, $\mathcal{V}$, is then

$$
\begin{aligned}
\mathcal{V} &\equiv (\mathbb{E}_{\bar{\alpha}}[\mathcal{C}|x_t] - \mathbb{E}_{\bar{\alpha}}[\mathcal{C}|x_r]) - \left(\frac{\theta}{2}\right) \left(\mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}|x_t\right)\right) - \mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}|x_r\right)\right)\right) \\
&= (x_r - x_t)\Delta\delta + \eta\left((1-x_r)e_{\bar{\alpha}}(x_r) - (1-x_t)e_{\bar{\alpha}}(x_t)\right) \\
&+ \left(\frac{\theta}{2}\right)\eta^2\left((1-x_r)^2\left[\int_{\alpha\in\mathcal{A}}\left(e_\alpha(x_r) - e_{\bar{\alpha}}(x_r)\right)^2 df(\alpha)\right] - (1-x_t)^2\left[\int_{\alpha\in\mathcal{A}}\left(e_\alpha(x_t) - e_{\bar{\alpha}}(x_t)\right)^2 df(\alpha)\right]\right).
\end{aligned}
$$

Thus, the value of information now has three components. The first two terms in the right-hand side (RHS) of the above equation capture ideas similar to the two components described in the RHS of equation (8) in the paper. The first two terms arise from the fact that, on average, information is expected to help decrease losses generated from excessive regulation and limit the spread of cascades more effectively. But, the third term is new and captures that α is unknown and the representative investor is ambiguity averse. Intuitively, it represents the extent to which information allows the policymaker to hedge the risk of making non-optimal decisions ex-ante as a result of not knowing the precise value of α. Importantly, the perception of such risk is intimately linked to the underlying family of distributions $\{\{p_k^\alpha\}_k\}_{\alpha\in\mathcal{A}}$ as well as the policymaker prior beliefs over $\mathcal{A}$, f.

Optimal choice of information. As before, the choice of whether to acquire the information technology follows a simple rule. If $\mathcal{V} \geq \kappa$, the social benefit of information outweighs its cost, and, thus, it is optimal to acquire it.

B. Optimal Interventions under Ambiguity

PROPOSITION 8 (Existence of the maximum): *Given $\mathcal{A}$ and f, suppose the policymaker considers only interventions in which $e_{\bar{\alpha}}(x)$ and $\int_{\alpha\in\mathcal{A}}\left(e_\alpha(x) - e_{\bar{\alpha}}(x)\right)^2 df(\alpha)$ are lower semi-continuous functions of x. Then, there exists a point $\frac{1}{n} \leq x^* \leq 1$ that solves*

$$
\begin{aligned}
\max_x \quad & \mathbb{E}_{\bar{\alpha}}\left(\mathcal{C}_\alpha|x\right) - \left(\frac{\theta}{2}\right) \times \mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}_\alpha|x\right)\right) - \kappa \times \mathbb{1}_\kappa \\
s.t. \quad & \frac{1}{n} \leq |x| \leq 1,
\end{aligned}
$$

Proof. If $e_{\bar{\alpha}}(x)$ and $\int_{\alpha\in\mathcal{A}}(e_\alpha(x)-e_{\bar{\alpha}}(x))^2\,df(\alpha)$ are lower semi-continuous functions of x, then the objective function of the aforementioned optimization problem is an upper semi-continuous function of x. Baire's generalization of the Weierstrass' theorem ensures that such an objective function always attains its maximum over a compact set; see (Ok, 2007, Chapter 4). □

PROPOSITION 9 (Interior Solution): *Given $\mathcal{A}$ and f, suppose*

$$\mathbb{E}_{\bar{\alpha}}\left(\mathcal{C}_\alpha \middle| x\right) - \left(\frac{\theta}{2}\right)\times\mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}_\alpha \middle| x\right)\right)$$

is a strictly concave function of x within the interval $\left[\frac{1}{n},1\right]$. Then, the optimal intervention, x^*, gets arbitrarily close to the solution of the following equation as n get large,

$$\begin{aligned}\eta\left(e_{\bar{\alpha}}(x^*)-(1-x^*)\frac{\partial e_{\bar{\alpha}}(x)}{\partial x}\bigg|_{x=x^*}\right) &= \Delta\delta \\ &+ \left(\frac{\theta}{2}\right)\eta^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}(e_\alpha(x)-e_{\bar{\alpha}}(x))^2\,df(\alpha)\right)\bigg|_{x=x^*}.\end{aligned}$$

Proof. If

$$\mathbb{E}_{\bar{\alpha}}\left(\mathcal{C}_\alpha \middle| x\right) - \left(\frac{\theta}{2}\right)\times\mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}_\alpha \middle| x\right)\right)$$

is a strictly concave function of x within the interval $\left[\frac{1}{n},1\right]$, then the solution of the optimization problem is interior. Notably,

$$\begin{aligned}\mathbb{E}_{\bar{\alpha}}\left(\mathcal{C}_\alpha \middle| x\right) &= \eta-(1-x)\eta e_{\bar{\alpha}}(x)-x\Delta\delta \\ \mathbb{V}_f\left(\mathbb{E}_\alpha\left(\mathcal{C}_\alpha \middle| x\right)\right) &= (1-x)^2\eta^2\int_{\alpha\in\mathcal{A}}(e_\alpha(x)-e_{\bar{\alpha}}(x))^2\,df(\alpha).\end{aligned}$$

As a result, the first order condition of the policymaker's problem can be rewritten as

$$\begin{aligned}\eta\left(e_{\bar{\alpha}}(x^*)-(1-x^*)\frac{\partial e_{\bar{\alpha}}(x)}{\partial x}\bigg|_{x=x^*}\right) &= \Delta\delta \\ &+ \left(\frac{\theta}{2}\right)\eta^2\frac{\partial}{\partial x}\left((1-x)^2\int_{\alpha\in\mathcal{A}}(e_\alpha(x)-e_{\bar{\alpha}}(x))^2\,df(\alpha)\right)\bigg|_{x=x^*}.\end{aligned}$$

Because the fraction of restricted banks must be a rational number, x^* gets arbitrarily close to the solution of the above equation as n gets large. □

PROPOSITION 10 (Selection of Restricted Banks): *Given $\mathcal{A}$ and f, define*

$$\begin{aligned}\Delta_{1/n} &\equiv \eta-\eta\left(1-\frac{1}{n}\right)e_{\bar{\alpha}}(1/n)-\delta\Delta-\frac{\theta}{2}\eta^2\left(1-\frac{1}{n}\right)^2\int_{\alpha\in\mathcal{A}}(e_\alpha(1/n)-e_{\bar{\alpha}}(1/n))^2\,df(\alpha), \\ \Delta_1 &\equiv \eta-\Delta\delta,\end{aligned}$$

$$\Delta_{\mathcal{I}} \equiv \max_{x \in (1/n,1)} \left\{ \eta - \eta (1-x) e_{\bar{\alpha}}(x) - x\Delta\delta - \frac{\theta}{2}\eta^2(1-x)^2 \int_{\alpha\in\mathcal{A}} (e_\alpha(x) - e_{\bar{\alpha}}(x))^2 \, df(\alpha) \right\},$$

The optimal fraction of restricted banks, $x^*_{\mathcal{I}}$, is given by

$$x^*_{\mathcal{I}} = \begin{cases} 1/n \ \textit{if} \ \ \Delta_{1/n} > \max\{\Delta_1, \Delta_{\mathcal{I}}\} \\ 1 \ \textit{if} \ \ \Delta_1 > \max\{\Delta_{1/n}, \Delta_{\mathcal{I}}\} \\ x^*_{\mathcal{I}} \ \textit{if} \ \ \Delta_{\mathcal{I}} > \max\{\Delta_1, \Delta_{1/n}\}, \end{cases}$$

where $x^*_{\mathcal{I}} \in (1/n, 1)$ gets arbitrarily close to the solution of the following equation, x^*,

$$\eta \left(e_{\bar{\alpha}}(x^*) - (1-x^*) \left. \frac{\partial e_{\bar{\alpha}}(x)}{\partial x} \right|_{x=x^*} \right) = \Delta\delta + \left(\frac{\theta}{2}\right) \eta^2 \frac{\partial}{\partial x} \left((1-x)^2 \int_{\alpha\in\mathcal{A}} (e_\alpha(x) - e_{\bar{\alpha}}(x))^2 \, df(\alpha) \right) \Bigg|_{x=x^*}.$$

as n grows large.

Proof. If $\Delta_{1/n} > \max\{\Delta_1, \Delta_{\mathcal{I}}\}$, then $x^*_{\mathcal{I}} = 1/n$, as expected consumption is maximized with almost no regulation. If $\Delta_{\mathcal{I}} > \max\{\Delta_{1/n}, \Delta_1\}$, then the policymaker's problem has an interior solution—that is, $x^*_{\mathcal{I}} \in (1/n, 1)$. In this case, as n grows large, $x^*_{\mathcal{I}}$ gets arbitrarily close to the solution of the first order condition of the policymaker's problem. Finally, if $\Delta_1 > \max\{\Delta_{1/n}, \Delta_{\mathcal{I}}\}$, then $x^*_{\mathcal{I}} = 1$, as expected consumption is maximized by restricting as many banks as possible. □