REGULATING FINANCIAL NETWORKS: A FLYING BLIND PROBLEM

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ABSTRACT

I develop a model for studying the role that uncertainty about the susceptibility of a financial network to contagion plays in the behavior of its member institutions and the design of preemptive interventions. As uncertainty affects the perception of contagion risk, it can reshape market equilibrium inefficiencies, altering the scope of welfare-improving interventions. The socially optimal level of uncertainty depends on a delicate balance between the information technologies available to policymakers and structural features of the network.

Keywords: Financial networks, network uncertainty, contagion, macroprudential regulation, systemic risk.
JEL classification: C6, E61, G01.

> *"Governments are thus faced with economies that are based on an enormous and complex web of interdependencies and exposures that is constantly evolving and hard to monitor. Governments are also faced with behemoth enterprises that they explicitly and implicitly insure, who exude externalities, and who have incentives that are not aligned with a country's long-run prosperity."*
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> — Matthew O. Jackson, The Human Network (2019, p. 89)

The highly intertwined nature of financial markets has been considered an important factor of instability since the 2007–09 financial crisis.[1] Although many policy interventions since then have considered measures of interconnectedness to promote financial stability, policymakers are confronted with an inconvenient truth when designing such interventions: Lack of detailed information, coupled with complex and opaque interactions among financial institutions, makes it hard to determine the susceptibility of the financial system to contagion. In view of this critical challenge, I provide a conceptual framework for studying the role that such uncertainty plays in equilibrium outcomes and preemptive policy interventions.

The proposed framework generates new predictions on how uncertainty reshapes the self-interested behavior of interrelated institutions, their collective response in equilibrium, and the design of interventions. I show that uncertainty can constitute both a curse and a blessing. As uncertainty alters the perception of contagion risk, it has the potential to not only compound market equilibrium inefficiencies, but also serve as a deterrence against excessive risk-taking.

Uncertainty can also reshape the design of preemptive interventions. Interventions are beneficial, as they curb risk-taking, but they are also costly, as they force institutions to forgo profitable investments. Uncertainty effectively introduces noise regarding the ex post benefits and costs associated with any given intervention. As, from an ex ante perspective, interventions might be less effective in more uncertain environments, more stringent interventions might be warranted only when contagion risk is perceived as sufficiently likely. Because excessively stringent interventions can lead to resource misallocation, reducing uncertainty before designing interventions might be worth the cost. I characterize how the socially optimal level of uncertainty strikes the right balance between the benefits and costs of increasing the ex ante effectiveness of interventions.

I develop the aforementioned ideas in the context of a model in which profit-maximizing institutions are linked through an exogenous network of exposures prone to contagion. These institutions (banks, for short) are born with endowments and select their portfolios without knowing the susceptibility of the network to contagion. Banks can choose between two assets:

[1] See Bank for International Settlements (2009, 2010, 2011), International Monetary Fund (2010), Financial Crisis Inquiry Commission (2011), Yellen (2013), and Tarullo (2019).

(1) a less productive but safer asset and (2) a more productive but riskier asset. Although the network architecture is known and each exposure is assumed to propagate adverse shocks with probability p, the precise value of p is unknown. Before banks select their portfolios, a social planner restricts their choices to maximize expected output. Before designing interventions, the planner can choose to learn more about the precise value of p through a costly information technology. Hence, the planner's intervention design problem is preceded by an information choice. Aside from characterizing the effect of uncertainty on banks' preemptive actions, I study the problem of choosing the optimal level of uncertainty and how to restrict banks' portfolios with such information.

Even without uncertainty, it is reasonable to expect that, under certain conditions, banks underweight the externality they impose on others. At the core of this problem lies the fact that banks fail to internalize that their actions can reshape contagion that starts when they themselves initially fail. The main message of this paper is that uncertainty about p can considerably reshape such inefficiencies as well as alter the scope of interventions.

The model features two channels through which uncertainty compounds market equilibrium inefficiencies. The first channel relates to how uncertainty alters the perceived likelihood of cascades of failures of varying sizes. In the face of higher uncertainty, larger values of p are perceived as relatively more likely. Thus, larger cascades of failures are expected to occur more frequently. Because banks underweight the effect of their decisions on the extent of contagion, they fail to internalize that cascades originating from their initial failures are expected to be larger. Therefore, under certain conditions, an increase in uncertainty increases the wedge between banks' equilibrium choice and the choice of a social planner.

The second channel relates to how uncertainty reshapes the extent of strategic substitutability among banks' actions. In the face of higher uncertainty, banks pay more attention to banks farther away in the network. This is because larger cascades of failures are perceived as more likely and the emergence of such cascades relies more on the actions of banks farther away. Consequently, banks fail to fully internalize the behavior of their closest neighbors, potentially increasing market equilibrium inefficiencies.

Besides exploring the effect of uncertainty on banks' preemptive actions, I also explore how optimal interventions are reshaped by the interplay between uncertainty and structural characteristics of the network. At a basic level, optimal interventions are about forcing banks to internalize their expected systemic footprint—defined as the expected number of banks that fail as a result of cascades of failures that originate when a bank fails. Importantly, the optimal policy considers that more stringent interventions can lead to resource misallocation. Under certain conditions, uncertainty unequivocally pushes the planner toward more caution as the planner takes into account that contagion might occur more frequently than expected.

As stringent interventions can be socially costly, it might be worth reducing uncertainty before designing interventions; reducing uncertainty could be costly as gathering precise bank information is costly for banks and policymakers. I show that at a basic level, the socially optimal level of uncertainty strikes the right balance between the social costs of reducing uncertainty and the benefits of increasing the ex ante effectiveness of interventions.

My results inform the ongoing debate regarding the design of macroprudential regulations. Though post–financial crisis reforms have focused mostly on large institutions, my results underscore that uncertainty can play a key role in the strategic behavior of profit-seeking institutions, their collective action in equilibrium, and the design of preemptive interventions. My results also highlight that an appropriate regulatory framework must be mindful of the benefits and costs associated with reducing uncertainty.

Related Literature.—My paper bridges four strands of the literature. The first strand of literature explores how the architecture of the financial system can reshape contagion.[2] Though my paper and this literature share an emphasis on how the network alters contagion, my focus is markedly different. I focus on the effect of uncertainty on institutions' preemptive actions and how the extent of such uncertainty can reshape optimal preemptive interventions.

The second strand of literature explores how policy interventions can affect contagion.[3] While my paper also focuses on how contagion varies with different interventions, it provides a tractable framework in which optimal policies can be determined in the face of uncertainty. To the best of my knowledge, the characterizations of market equilibria and the socially optimal level of uncertainty are new to this literature.

The third strand of literature explores the implications of model uncertainty in finance.[4] Within this literature, Caballero and Simsek (2013)'s study is most closely related to my work. Although they explore the effects of model uncertainty in banks' portfolio decisions, their analysis focuses on how banks' lack of knowledge about the financial network alters fire sales. Their model is silent, however, about the interplay between structural characteristics of the network and market participants' beliefs in non-regular network architectures. Their model is also silent about the socially optimal level of uncertainty and the design of network-based policy interventions.

[2]An incomplete list includes Rochet and Tirole (1996), Allen and Gale (2000), Freixas et al. (2000), Eisenberg and Noe (2001), Lagunoff and Schreft (2001), Dasgupta (2004), Leitner (2005), Nier et al. (2007), Allen and Babus (2009), Haldane and May (2011), Allen et al. (2012), Amini et al. (2013), Cont et al. (2013), Georg (2013), Zawadowski (2013), Cabrales et al. (2017), Elliott et al. (2014), Glasserman and Young (2015, 2016), Acemoglu et al. (2015), and Castiglionesi et al. (2019); see Capponi (2016) and Jackson and Pernoud (2021) for recent surveys on this topic.

[3]An incomplete list includes Beale et al. (2011), Gai et al. (2011), Battiston et al. (2012), Goyal and Vigier (2014), Aldasoro et al. (2017), Erol and Ordoñez (2017), Gofman (2017), Galeotti et al. (2020), Jackson and Pernoud (2019), Kanik (2019) and Ramírez (2019).

[4]See, for example, Easley and O'Hara (2009) and Routledge and Zin (2009).

The fourth strand of literature explores transparency in the banking system.[5] Within this literature, Alvarez and Barlevy (2021)'s paper is most closely related to my work. They explore how disclosure policies can forestall contagion.[6] Their focus is, however, on characterizing the conditions under which mandatory disclosure can improve welfare rather than on determining how much uncertainty is socially optimal and how to regulate banks' portfolios with such information.

Outline.—The remainder of my paper is organized as follows. Section I presents the baseline model. To build intuition, section II characterizes the market equilibrium and socially optimal portfolios when p is known. Section III explores how uncertainty alters equilibrium outcomes, socially optimal portfolios, and market equilibrium inefficiencies. Section IV explores how much uncertainty is socially optimal. Section V discusses the role of several assumptions and extensions of the baseline model. Section VI concludes. Proofs are deferred to the Online Appendix.

I. Baseline Model

Consider a two-period economy consisting of n risk-neutral banks whose payoffs are linked through an exogenous network of exposures. Exposures are on either the asset side or the liability side and cannot be mitigated via contractual protections. Although banks may differ in their number of exposures, they are ex ante identical in other respects. The network of exposures can be conveniently described via a graph, denoted by $\mathcal{G}_n$. For simplicity, $\mathcal{G}_n$ is simple and undirected. That is, there is at most one exposure between any given pair of banks, no bank is exposed to itself, and exposures lack directionality. Time is indexed by $t \in \{0, 1\}$ and banks are indexed by $i \in \{1, 2, \cdots, n\}$, with n being potentially large but finite.

Assets and Timing.—There are two assets, an illiquid asset and cash, whose payoffs are realized at $t = 1$. The payoff of cash is normalized to 1, whereas the payoff of the illiquid asset is $(1 + \beta)$, with $\beta > 0$. Cash can be broadly interpreted as liquid securities that retain their value in times of economic stress but command lower returns. The illiquid asset can be interpreted as securities that command higher returns but are likely to decrease their value in times of economic stress.

[5]See Goldstein and Sapra (2013), Leitner (2014), and Goldstein and Yang (2017) for literature reviews.

[6]Alvarez and Barlevy (2021) show that when contagion is severe, mandatory disclosure of balance sheet information can be welfare improving, as banks do not completely internalize the social value of the information they reveal about themselves and thus disclose less than is socially optimal. In the spirit of Hirshleifer (1971), they also show that forcing banks to disclose information can be welfare reducing, as secrecy can support socially beneficial risk-sharing among banks.

Every bank is endowed with one dollar and chooses its portfolio at $t = 0$ to maximize its expected payoffs. Just before payoffs are realized, economic conditions deteriorate. When economic conditions deteriorate, banks become more vulnerable to distress affecting related banks, and exposures can function as propagation mechanisms of bank failures. As a result, cascades of failures might occur at $t = 1$.

Propagation Mechanism.—To gain tractability, the propagation of failures among banks is determined by the following stochastic process:

(A1) One bank (chosen uniformly at random) is hit by an adverse liquidity shock $\varepsilon \sim U[0, 1]$. Because it is difficult to sell the illiquid asset when economic conditions deteriorate, such a bank fails if its cash holdings are smaller than ε.

(A2) Though bank i might not be initially affected by ε, i might still fail if there is a sequence of contagious exposures between i and the initially affected bank (assuming such a bank fails). Each exposure is contagious (independently of others) with probability p.

The random selection of contagious exposures serves as a metaphor for banks' and policymakers' difficulty assessing how exposures react in times of economic stress. At a fundamental level, cascades of failures can be broadly interpreted as liquidity-driven crises in which liquidity shocks affecting certain banks induce liquidity shocks for some of their neighbors. When economic conditions deteriorate, those neighbors may face a run because of solvency concerns, which, in turn, might cause solvency concerns about some of the neighbors' neighbors, possibly generating cascades of runs, as in Diamond and Rajan (2011), Caballero and Simsek (2013), and Stein (2013). Thus, cascades of failures could also be interpreted as crises of confidence, as in Zhou (2018). Another example of cascades relates to situations in which liquidity shocks affecting certain banks might lead to write-downs in the balance sheets of some of their neighbors. If resulting losses exceed the capital of such neighbors, those neighbors could fail, which could subsequently cause other banks to fail as well, as in Elliott et al. (2014).

Banks' Tradeoff.—Within this environment, banks face a simple tradeoff. Though banks with more liquid portfolios—that is, portfolios with a higher proportion of cash—exhibit higher resilience to ε, they generate lower payoffs when banks are not affected by ε. Hereafter, I assume that $1/n < \beta$. This assumption ensures that banks have incentives to hold the illiquid asset in equilibrium.

Information.—All features and parameters of the economy are common knowledge, with the exception of p, whose precise value is unknown. Beyond conceptually introducing uncertainty about the susceptibility of the network to contagion, allowing p to be unknown (rather than the network architecture) provides analytical tractability.[7]

Notation.—Throughout the paper, I use the following notation. Let $\mathbb{I}$ denote the $n \times n$ identity matrix, $\mathbb{1}$ denote the $n \times 1$ vector of ones, and $\mathbb{e}_i$ denote a $1 \times n$ (selector) vector with a one in element i and zeros elsewhere. Let $\mathcal{A}_n$ denote the adjacency matrix of $\mathcal{G}_n$. That is, $\mathcal{A}_n$ is a $n \times n$ matrix of zeros and ones where $\mathcal{A}_n(i,j) = 1$ if there exists a direct exposure between banks i and j. I use $\vec{\mathbf{x}} \equiv (x_1, \cdots, x_n)'$ to denote the $n \times 1$ vector representing banks' collective investment choice, where x_i denotes the fraction of bank i's endowment invested in the illiquid asset. In addition, $||\cdot||$ denotes the Euclidean norm operator, and $\mathcal{R}_+$ denotes the set of non-negative real numbers.

To keep track of the propagation of failures, it is important to consider paths along $\mathcal{G}_n$. A path of length k from bank i to j is a sequence of k exposures starting at i and ending at j wherein no bank or exposure is repeated in the sequence. For $k \geq 1$, let $\mathbf{P}_k$ denote the $n \times n$ matrix whose (i,j) element equals the number of paths of length k between i and j. Because $\mathcal{G}_n$ is undirected and simple, $\mathbf{P}_k$ is a symmetric matrix whose diagonal elements equal zero.

II. The Benchmark Case: p Is Known

To provide a benchmark for the main results with uncertainty, this section characterizes equilibrium outcomes and socially optimal portfolios when p is known. This analysis helps better understand how structural features of $\mathcal{G}_n$ and the precise value of p can reshape market equilibrium inefficiencies.

Let l_n denote the length of the largest sequence of exposures in $\mathcal{G}_n$. For a given value of p and a vector representing banks' collective investment choice $\vec{\mathbf{x}}$, the probability that bank i fails at $t=1$ is given by

$$\mathbb{P}_p\,(\text{bank } i \text{ fails}) \;=\; \left(\frac{1}{n}\right) x_i + \left(\frac{1}{n}\right) \mathbb{e}_i \left(\sum_{k=1}^{l_n} p^k \mathbf{P}_k \vec{\mathbf{x}}\right). \tag{1}$$

The two terms on the right-hand side of equation (1) can be interpreted as follows. The first term captures the likelihood that bank i fails as a result of being directly hit by ε; recall that each bank is hit by ε with probability $1/n$. The second term captures the likelihood that bank i fails as a result of being indirectly hit by ε. This term effectively captures the likelihood that i fails as a result of contagion because matrix $\mathbf{P}_k$ keeps track of the number of paths of length k between any two banks. To ensure that the probabilities in (1) are

[7] See Ramírez (2019) for a complementary conceptual framework in which optimal interventions can be derived under uncertainty about the precise architecture of the network.

well-defined, I assume that $\mathcal{G}_n$ and the value of p are such that the following n inequalities are satisfied: $\mathbb{e}_i \left(\sum_{k=1}^{l_n} p^k \mathbf{P}_k \mathbb{1}\right) \leq (n-1), \forall i$.

Taking p and other banks' decisions as given, bank i's optimal investment choice, x_i^* , is selected to maximize bank i's expected payoffs,

$$\pi_i(\vec{\mathbf{x}}, p) \equiv ((1-x_i) + (1+\beta)x_i)\left(1 - \mathbb{P}_p\left(\text{bank } i \text{ fails}\right)\right). \tag{2}$$

Then, the first order condition of bank i can be rewritten as

$$x_i^* = \frac{n}{2}\left(1 - \frac{\mathbb{e}_i}{n}\sum_{k=1}^{l_n} p^k \mathbf{P}_k \vec{\mathbf{x}}_e\right) - \frac{1}{2\beta}, \tag{3}$$

where $\frac{\mathbb{e}_i}{n}\sum_{k=1}^{l_n} p^k \mathbf{P}_k \vec{\mathbf{x}}_e$ captures bank i's exposure to contagion risk—which is endogenously determined in equilibrium as vector $\vec{\mathbf{x}}_e \equiv (x_1^*, \cdots, x_n^*)'$ is an equilibrium outcome.

Equation (3) highlights that banks' actions can be strategic substitutes. Should banks i and j be (directly or indirectly) connected, an increase in x_j^* triggers a downward shift in x_i^*. The higher x_j^*, the more likely j fails if initially hit by ε and, hence, the higher the likelihood that i is affected by contagion. To counteract this likelihood increase, i preemptively shifts to a more liquid portfolio. Notably, the smaller the length of the sequence of exposures connecting i and j, the more likely it is that failures propagate from j to i (or i to j) and, thus, the higher the extent of strategic substitutability between their actions. In other words, banks closer to each other exhibit a higher degree of strategic substitutability relative to what banks farther from each other exhibit.

A. Market Equilibrium

To facilitate the analysis, assume matrices $\Omega_p \equiv \left(\sum_{k=0}^{l_n} p^k \mathbf{P}_k\right)$ and $(\mathbb{I} + \Omega_p)$ are invertible. In equilibrium, all banks optimally choose their portfolios and no bank has unilateral incentives to change its decision. The next proposition provides a convenient characterization of the unique equilibrium of the simultaneous-move n-bank game with payoffs described in (2).

PROPOSITION 1 (Market Equilibrium)**.** *For each $i \in \{1, \cdots, n\}$, define the function $\nu_i : [0,1] \rightarrow \mathcal{R}_+$, where $\nu_i(p) = \frac{\mathbb{e}_i}{n}\left(n - \frac{1}{\beta}\right)\sum_{k=1}^{l_n} p^k \boldsymbol{P}_k \left(\mathbb{I} + \Omega_p\right)^{-1}\mathbb{1}$. Let $\underline{p_i^e}$ denote the solution of $\nu_i(p) = \left(1 - \frac{1}{\beta n} - \frac{2}{n}\right)$ and $\overline{p_i^e}$ denote the solution of $\nu_i(p) = \left(1 - \frac{1}{\beta n}\right)$. Define $\underline{p^e} \equiv \max_{1\leq i \leq n}\left\{\underline{p_i^e}\right\}$ and $\overline{p^e} \equiv \min_{1\leq i\leq n}\left\{\overline{p_i^e}\right\}$. If $\underline{p^e} \leq p \leq \overline{p^e}$, then*

$$\vec{\boldsymbol{x}}_e = \left(n - \frac{1}{\beta}\right)\left(\mathbb{I} + \Omega_p\right)^{-1}\mathbb{1} \tag{4}$$

is the unique market equilibrium.

That is, banks' equilibrium behavior is intimately linked to the architecture of $\mathcal{G}_n$, the precise value of p, and the difference between asset payoffs, β. Proposition 1 underscores that a bank's equilibrium behavior depends not only on the decisions of its direct neighbors, but also on the decisions of its neighbors' neighbors (and so on). The reason is that failures can propagate via sequences of exposures of varying length. As a result, banks consider their exposure to failures of every other bank to which they are (directly or indirectly) connected. The next corollary highlights useful properties of the aforementioned equilibrium.

COROLLARY 1. $x_i^* \equiv \mathbb{e}_i \vec{\boldsymbol{x}}_e$ *is increasing in* β *and decreasing in* p *and the number of paths passing through bank* i.

Intuitively, the higher the β, the higher are the expected payoffs from investing in the illiquid asset. Thus, the lower are the incentives to hold cash. The higher the p, or the number of paths passing through i, the more likely i is affected by contagion, so i preemptively holds more cash at $t = 0$.

B. *Socially Optimal Portfolios*

Consider the problem faced by a risk-neutral planner who understands the role that both the architecture of $\mathcal{G}_n$ and the precise value of p play in how failures propagate. Let $\mathcal{S}_n \equiv [0,1]^n$ denote the n-simplex. The planner selects $\vec{\mathbf{x}}_s \equiv (x_1^s, \cdots, x_n^s) \in \mathcal{S}_n$ to maximize welfare, $W(\vec{\mathbf{x}}, p)$, defined as

$$W(\vec{\mathbf{x}}, p) \;\; \equiv \;\; \sum_{j=1}^{n} \pi_j (\vec{\mathbf{x}}, p) \, . \tag{5}$$

The next proposition provides a useful characterization of the socially optimal investment choice.

PROPOSITION 2 (Socially Optimal Portfolios)**.** *For each* $i \in \{1, \cdots, n\}$, *define the function* $m_i : [0,1] \to \mathcal{R}_+$, *where* $m_i(p) = \frac{\mathbb{e}_i}{2}\left(\left(\frac{1}{n}\Omega_p\right)^{-1} - \left(\frac{1}{\beta}\right)\mathbb{I}\right)\mathbb{1}$. *Let* $\underline{p_i^s}$ *denote the solution of* $m_i(p) = 0$ *and* $\overline{p_i^s}$ *denote the solution of* $m_i(p) = 1$. *Define* $\underline{p^s} \equiv \max_{1 \le i \le n}\left\{\underline{p_i^s}\right\}$ *and* $\overline{p^s} \equiv \min_{1 \le i \le n}\left\{\overline{p_i^s}\right\}$. *If* $\underline{p^s} \le p \le \overline{p^s}$, *then*

$$\vec{\boldsymbol{x}}_s \;\; = \;\; \frac{n}{2}\left(\Omega_p^{-1} - \left(\frac{1}{\beta n}\right)\mathbb{I}\right)\mathbb{1} \tag{6}$$

is the unique solution of the planner's problem.

At the fundamental level, $\vec{\mathbf{x}}_s$ is deliberately selected so that the benefits and costs of forcing banks to hold more liquid portfolios balance each other. Costs arise because forcing banks to allocate more funds toward cash implies forgoing profitable investments. Benefits arise because forcing bank i to hold a more liquid portfolio not only increases i's resilience to ε, but also decreases the likelihood that its (direct and indirect) neighbors fail because of cascades of failures that originate from i's initial failure. The next corollary highlights useful properties of the socially optimal portfolio.

COROLLARY 2. $x_i^s \equiv \mathbb{e}_i \vec{\boldsymbol{x}}_s$ *is an increasing function of* β *and a decreasing function of* p *and the number of paths passing through bank* i.

Put simply, an increase in β increases the resource misallocation associated with forcing banks to hold more liquid portfolios, making socially optimal portfolios more illiquid. An increase in p increases the likelihood of contagion, enhancing the planner's incentives to make every bank more resilient to ε. An increase in the number of paths passing through i increases the system-wide repercussions associated with i's initial failure, boosting the planner's incentives to increase the resilience of i. In sum, proposition 2 and corollary 2 underscore that the socially optimal portfolio ensures that the benefits and costs of forcing banks to internalize the system-wide effect of their individual portfolio choices balance each other.

C. *Market Equilibrium Inefficiencies*

The juxtaposition of propositions 1 and 2 shows that the market equilibrium can be socially inefficient. To facilitate exposition assume $\max\{\underline{p^e}, \underline{p^s}\} \leq p \leq \min\{\overline{p^e}, \overline{p^s}\}$. The wedge between both allocations is then given by

$$\vec{\mathbf{x}}_e - \vec{\mathbf{x}}_s \;=\; \left(n - \frac{1}{\beta}\right)(\mathbb{I} + \Omega_p)^{-1}\,\mathbb{1} - \frac{n}{2}\left(\Omega_p^{-1} - \left(\frac{1}{\beta n}\right)\mathbb{I}\right)\mathbb{1}, \tag{7}$$

which, when $||p^2\mathbf{P}_2||$ is small, can be approximated by

$$\vec{\mathbf{x}}_e - \vec{\mathbf{x}}_s \;\approx\; \left(\left(\frac{n}{2} - \frac{1}{\beta}\right)\Omega_p^{-1} + \frac{1}{2\beta}\mathbb{I}\right)\mathbb{1}. \tag{8}$$

Inspecting equation (8) provides several insights. When β is sufficiently small, stringent interventions are less costly to implement. The more likely it is that failures propagate along $\mathcal{G}_n$, the more stringent interventions become, and, thus, the higher the wedge $(\vec{\mathbf{x}}_e - \vec{\mathbf{x}}_s)$. Although certain banks preemptively shift to more liquid portfolios when facing higher contagion risk, their increase in portfolio liquidity is less than socially optimal. The reason is that banks do not take into consideration the failures that might occur as a consequence of contagion originating from their initial failures. However, when β is sufficient large, the opposite can happen as stringent interventions become costly. That is, the more likely it is that failures propagate along $\mathcal{G}_n$, the lower the wedge $(\vec{\mathbf{x}}_e - \vec{\mathbf{x}}_s)$. While banks fail to internalize the system-wide consequences of their individual portfolio choices, their preemptive shifts to more liquid portfolios more than compensate for the portfolio changes that a social planner would require in the face of higher contagion risk. In this case, contagion risk effectively serves as a deterrence of excessive risk-taking.

The next proposition illustrates an important property related to how market equilibrium inefficiencies vary across banks.

PROPOSITION 3. *For each* $i \in \{1, \cdots, n\}$, *define the function* $q_i : [0,1] \rightarrow \mathcal{R}_+$, *where* $q_i(p) = \left(1 + \frac{1}{1+p\lambda_i}\right)^2 - \frac{2}{n}\left(n - \frac{1}{\beta}\right)$. *Let* λ_i *denote the* i *eigenvalue of* $\mathcal{A}_n$ *and* $\lambda_M \equiv \max_{1\leq i\leq n}\{\lambda_i\}$. *Define the function* $g : \mathcal{R} \rightarrow \mathcal{R}$, *where* $g(\lambda_i) = \left(n - \frac{1}{\beta}\right)\left(\frac{1}{1+(1+p\lambda_i)}\right) - \frac{1}{2}\left(\frac{n}{(1+p\lambda_i)} - \frac{1}{\beta}\right)$. *Let* $\overline{p^{\lambda_i}}$ *denote the solution of* $q_i(p) = 0$ *and define* $\overline{p^{\lambda}} \equiv \min_{1\leq i\leq n}\left\{\overline{p^{\lambda_i}}\right\}$. *If* $||p^2\boldsymbol{P}_2||$ *is small and* $\max\{\underline{p^e}, \underline{p^s}\} \leq p \leq \min\left\{\overline{p^e}, \overline{p^s}, \overline{p^{\lambda}}\right\}$, *then*

$$\mathbb{e}_i(\vec{\boldsymbol{x}}_e - \vec{\boldsymbol{x}}_s) \;\equiv\; x_i^* - x_i^s \;\approx\; \rho + g(\lambda_M)\left(\sum_{j=1}^{n}\chi_j\right)\chi_i, \tag{9}$$

where χ_i *denotes bank* i*'s eigenvector centrality and* ρ *is a real number. Moreover, if* $\mathcal{A}_n$ *is positive semi-definite and* $g(\lambda_M) \geq 0$, *then* $(x_i^* - x_i^s)$ *is an increasing function of* χ_i.

As banks with higher eigenvector centrality can be thought of as banks that play a more important role in the propagation of failures, proposition 3 underscores a simple yet important point. Under certain conditions, banks with a higher systemic footprint are precisely the ones that fail to internalize the negative consequences of their actions the most.

In sum, the very existence of interdependencies among banks can engender a wedge between the market equilibrium and the socially optimal portfolio. Notably, such a wedge can be altered by the architecture of $\mathcal{G}_n$ and the precise value of p as well as the difference between asset payoffs, β. Consequently, these dimensions can reshape the scope of welfare-improving interventions.

III. The Flying Blind Case: *p* Is Unknown

Although the previous analysis highlights the desirability of interventions in financial networks, it misses a fundamental point. Limited and imprecise information not only besets the regulation of financial institutions, but also can reshape their strategic behavior. As Jackson (2019, p. 92) simply puts it: "Central banks, and other national and international government branches and agencies, not to mention financial institutions themselves, are essentially flying jets without instruments. They are making rapid decisions that steer complex machinery based on limited information." This section shows how uncertainty can alter the scope of interventions and constitute both a curse and a blessing as it might compound market equilibrium inefficiencies or help curb excessive risk taking.

I now introduce the key ingredient to the model: uncertainty about the precise value of p. Assume $p \sim F_{\mu_1,\phi} \equiv F[\mu_1 - \frac{\phi}{2}, \mu_1 + \frac{\phi}{2}]$, where $0 \leq \mu_1 \leq 1/2$, $0 \leq \phi \leq 2\mu_1$, and $F_{\mu_1,\cdot}$ denotes a continuous distribution satisfying $\mathbb{E}[p] = \mu_1$. In what follows, I purposely maintain fixed μ_1 as I intend to highlight the relevance of uncertainty to banks' equilibrium behavior and optimal interventions. To gain tractability, assume that the sequence of distributions $\{F_{\mu_1,\phi}\}_{0\leq\phi\leq 2\mu_1}$ generated by (marginal) changes in ϕ satisfies the following property: $F_{\mu_1,\phi+\epsilon}$ is a mean-preserving spread of $F_{\mu_1,\phi}$, with $\epsilon > 0$ arbitrarily small. That is, $F_{\mu_1,\phi+\epsilon}$ can be thought of as being formed by spreading out $F_{\mu_1,\phi}$ while keeping the same average.

This setup allows me to use parameter ϕ as a proxy for uncertainty when performing comparative statics. For $k \geq 1$, let $\mu_k \equiv \mathbb{E}[p^k]$ denote the k^{th} raw moment of $F_{\mu_1,\phi}$. Because of the way failures propagate along $\mathcal{G}_n$, μ_k effectively captures the perceived likelihood of an arbitrary cascade of failures of size k. Given values for μ_1 and ϕ, define the $n \times n$ matrix

$$\begin{aligned}\Psi_\phi &\equiv \mathbb{I} + \mathbb{E}[p]\mathbf{P}_1 + \mathbb{E}[p^2]\mathbf{P}_2 + \mathbb{E}[p^3]\mathbf{P}_3 + \cdots + \mathbb{E}[p^{l_n}]\mathbf{P}_{l_n}, \\ &= \mathbb{I} + \sum_{k=1}^{l_n} \mu_k \mathbf{P}_k.\end{aligned} \tag{10}$$

As Ψ_ϕ takes into account the likelihood of cascades of failures of arbitrary sizes in conjunction with all paths along $\mathcal{G}_n$, Ψ_ϕ captures the perceived susceptibility of the network to contagion. Finally, I make the following parametric assumptions: (i) $\max\{\underline{p^e}, \underline{p^s}\} \leq \mu_1 - \phi/2$, (ii) $\mu_1 + \phi/2 \leq \min\{\overline{p^e}, \overline{p^s}\}$, and (iii) matrices Ψ_ϕ and $(\mathbb{I} + \Psi_\phi)$ are invertible. The first two assumptions ensure that the propositions of section II can be of use in this section, while the third assumption helps provide closed-form solutions.

A. *Uncertainty Alters Banks' Equilibrium Behavior*

Banks choose their portfolios while facing uncertainty about the extent of contagion $\mathcal{G}_n$ might bring about. Taking μ_1, ϕ, and other banks' decisions as given, bank i selects its investment in the illiquid asset, $x^*_{i,u}$, to maximize $\mathbb{E}\left[\pi_i(\vec{\mathbf{x}}, p) \middle| \mu_1, \phi\right]$. Thus, besides being risk-neutral, banks exhibit ambiguity-neutral preferences.[8]

By offering the counterpart of proposition 1, the next proposition characterizes the market equilibrium when banks are uncertain about the precise value of p.

PROPOSITION 4 (Market Equilibrium When Flying Blind)**.** *When banks are uncertain about the precise value of p, the unique equilibrium of the simultaneous-move n-bank game is given by*

$$\vec{\boldsymbol{x}}_e^u = \left(n - \frac{1}{\beta}\right)(\mathbb{I} + \Psi_\phi)^{-1}\,\mathbb{1}, \tag{11}$$

with $\vec{\boldsymbol{x}}_e^u = (x_{1,u}^*, x_{2,u}^*, \cdots, x_{n,u}^*)'$.

When choosing their portfolio, banks consider their exposure to contagion risk under every plausible value of p. The higher the perceived exposure to contagion risk, the higher is banks' portfolio liquidity in equilibrium. Because contagion can propagate via sequences of exposures of varying lengths, banks keep track of not only $\mathbb{E}[p]$, but also higher moments. Proposition 4 underscores that banks' equilibrium behavior is determined by the interplay between the perceived susceptibility of the network to contagion, captured by matrix Ψ_ϕ, and the difference between asset payoffs, β.

As the sequence $\{\mu_k\}_{k=2}^{l_n}$ depends on parameter ϕ, uncertainty can reshape banks' equilibrium behavior. To better understand the mechanisms through which uncertainty alters banks' preemptive response, it is illustrative to analyze banks' optimality conditions. The first order condition of bank i can be rewritten as

$$x_{i,u}^* = \frac{n}{2}\left(1 - \frac{\mathbb{e}_i}{n}\sum_{k=1}^{l_n}\mu_k \mathbf{P}_k \vec{\mathbf{x}}_e^u\right) - \frac{1}{2\beta}. \tag{12}$$

In the face of uncertainty about p, bank i's strategic behavior is reshaped by its perceived exposure to contagion risk, captured by $\frac{\mathbb{e}_i}{n}\sum_{k=1}^{l_n}\mu_k \mathbf{P}_k \vec{\mathbf{x}}_e^u$. It follows from differentiating equation (12) with respect to ϕ that

$$\frac{\partial \vec{\mathbf{x}}_e^u}{\partial \phi} = -(\mathbb{I} + \Psi_\phi)^{-1}\Lambda \vec{\mathbf{x}}_e^u, \tag{13}$$

with $\Lambda \equiv \left(\sum_{k=2}^{l_n}\frac{\partial \mu_k}{\partial \phi}\mathbf{P}_k\right)$. That is, the extent to which variations in uncertainty alter banks' equilibrium behavior depends on the interplay between the expected susceptibility of $\mathcal{G}_n$ to contagion, captured by matrix Ψ_ϕ, and the way changes in uncertainty alter the likelihood of

[8] I deliberately impose that all agents exhibit ambiguity-neutral preferences to make sure my results are not driven by their attitudes toward model uncertainty. For a more detailed discussion, see section V.D.

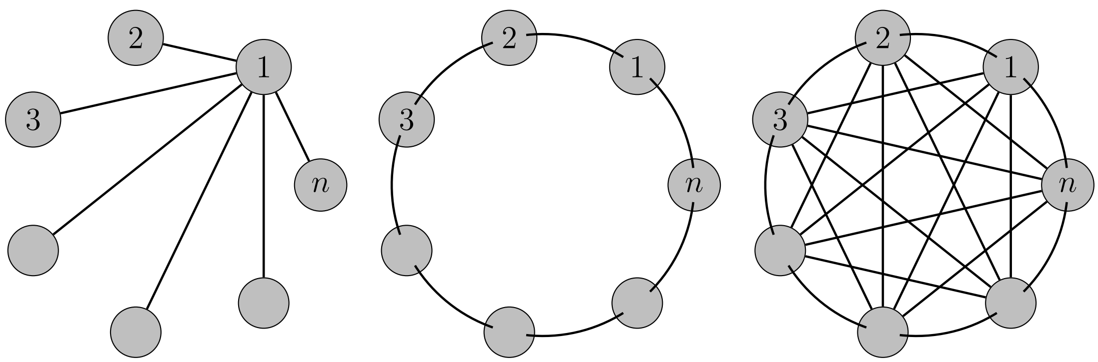
cascades of failures of varying sizes, captured by matrix Λ. As $\mathcal{G}_n$ alters these two dimensions, different network architectures react differently to changes in uncertainty.

Importantly, differentiating equation (12) with respect to $x^*_{i,u}$ and ϕ yields

$$\frac{\partial^2 \vec{\mathbf{x}}^u_e}{\partial\phi\partial x^*_{i,u}} \quad = \quad -\left(\mathbb{I}+\Psi_\phi\right)^{-1}\Lambda\frac{\partial\vec{\mathbf{x}}^u_e}{\partial x^*_{i,u}}, \tag{14}$$

which captures how the strategic substitutability between bank i's and other banks' actions can be affected by changes in uncertainty. As banks' perceived exposure to contagion risk depends on their position in $\mathcal{G}_n$, shocks to uncertainty can command heterogenous changes in strategic substitutability across different pairs of banks.

By focusing on the networks depicted in figure 1, example 1 helps appreciate the mechanisms through which uncertainty can alter banks' preemptive actions.



(a) Star (b) Ring (c) Complete

Figure 1. Three network architectures among n banks, with $n > 3$.

EXAMPLE 1. *Consider an economy with $n > 3$ banks whose payoffs are linked according to figure 1(a), 1(b), or 1(c). When banks are linked according to the star network, their system of first order conditions is characterized by*

$$x^*_{1,u} = \frac{n}{2}\left(1-\mu_1\frac{(n-1)}{n}x^*_{j,u}\right)-\frac{1}{2\beta}, \quad \textit{and} \quad x^*_{j,u} = \frac{\beta(n-\mu_1 x^*_{1,u})-1}{\beta(2+(n-2)\mu_2)}, \tag{15}$$

with $j \in \{2,3,\cdots,n\}$. As a result,

$$x^*_{1,u} = \frac{(2-(n-1)\mu_1+(n-2)\mu_2)(n\beta-1)}{(4+(n-3)\mu_2)\beta}, \quad \textit{and} \quad x^*_{j,u} = \frac{(2-\mu_1)(n\beta-1)}{(4+(n-3)\mu_2)\beta}. \tag{16}$$

Define $\mathcal{S} \equiv \sum_{k=2}^{n-1}\mu_k$. When banks are linked according to the ring or complete networks, the *equilibrium is characterized by*

$$x^*_{i,u} = \frac{1}{2}\left(n - \frac{1}{\beta}\right)\left(1 + \mu_1 + \mathcal{S}\right)^{-1}, \tag{17}$$

$$x^*_{i,u} = \frac{1}{2}\left(n - \frac{1}{\beta}\right)\left(1 + \frac{(n-1)}{2}\mu_1 + \frac{(n-1)(n-2)}{2}\mathcal{S}\right)^{-1}, \tag{18}$$

where (17) describes banks' behavior in the ring network, while (18) describes their behavior in the complete network.

Example 1 highlights three important points. First, uncertainty can alter banks differently if banks are heterogenous in their number of exposures. Take the star network as an example. When assessing its perceived exposure to contagion, bank 1 considers only paths of length one, whereas other banks consider paths of length two. Because μ_2 is an increasing function of ϕ, everyone but bank 1 preemptively holds more liquid portfolios when facing higher uncertainty; note that $\frac{\partial x^*_{1,u}}{\partial \phi} \geq 0$ and $\frac{\partial x^*_{j,u}}{\partial \phi} \leq 0$.

Second, uncertainty can alter the extent of strategic substitutability between banks' actions. Take the star network as an example. In the face of higher uncertainty, peripheral banks become less sensitive to variations in bank 1's portfolio liquidity; note that $\frac{\partial^2 x^*_{j,u}}{\partial \phi \partial x^*_{1,u}} < 0$. Intuitively, an increase in uncertainty makes larger values of p seem more likely. This, in turn, increases the perceived likelihood of larger cascades of failures. As peripheral banks might originate such cascades, banks pay more attention to their behavior.

Third, even if banks are homogenous in their number of exposures, the architecture of $\mathcal{G}_n$ can alter the way uncertainty reshapes banks' equilibrium behavior. Take the ring and the complete networks as examples. In both networks, uncertainty unequivocally serves as a deterrence against banks incentives to hold more illiquid portfolios—note that $\frac{\partial x^*_{i,u}}{\partial \phi} \leq 0$, $\forall\, i$, as $\frac{\partial \mathcal{S}}{\partial \phi} \geq 0$. Notably, there are more paths through which failures propagate in the complete than in the ring network. As a result, in the face of higher uncertainty, banks face stronger incentives to increase their portfolio liquidity in the former than in the latter network.

B. Uncertainty Alters Socially Optimal Portfolios

I now study socially optimal interventions wherein the planner restricts banks' portfolios at $t = 0$.[9] For simplicity, I assume the planner can force banks to hold their socially optimal

[9] An alternative intervention would be to introduce the possibility of a bailout fund. In a bailout, the planner requires banks to contribute some of their endowment into a fund. The fund is then used to pay for ε, aiming to avoid the failure of the initially affected bank. Instead of focusing on bailout policies, however, I focus on the design of preemptive interventions that restrict banks' portfolio decisions. In doing so, I am motivated by the fact that most regulation in recent years has focused on the design of capital and liquidity

portfolios at no extra cost. Consequently, studying the socially optimal investment choice is equivalent to studying the socially optimal intervention. Given μ_1 and ϕ, the planner selects $\vec{\mathbf{x}}_s^u = (x_{1,u}^s, x_{2,u}^s, \cdots, x_{n,u}^s)' \in \Delta_n$ to maximize

$$\mathbb{E}\left[W\left(\vec{\mathbf{x}}, p\right)|\mu_1, \phi\right] \equiv \int_{\mu_1-\phi/2}^{\mu_1+\phi/2} W\left(\vec{\mathbf{x}}, p\right) dF_{\mu_1,\phi}(p), \tag{19}$$

where $W\left(\vec{\mathbf{x}}, p\right)$ is defined as in equation (5). The next proposition characterizes the optimal preemptive intervention when the planner is uncertain about p.

PROPOSITION 5 (Optimal Interventions When Flying Blind)**.** *When the planner is uncertain about the precise value of p, the unique solution of the planner's problem is given by*

$$\vec{\boldsymbol{x}}_s^u = \frac{n}{2}\left(\Psi_\phi^{-1} - \left(\frac{1}{\beta n}\right)\mathbb{I}\right)\mathbb{1}. \tag{20}$$

Before illustrating the implications of proposition 5, it is useful to interpret the term

$$\begin{aligned} \mathbb{e}_i\Psi_\phi\mathbb{1} &= \mathbb{e}_i\left(\mathbb{I} + \mu_1\mathbf{P}_1 + \mu_2\mathbf{P}_2 + \mu_3\mathbf{P}_3 + \cdots + \mu_{l_n}\mathbf{P}_{l_n}\right)\mathbb{1} \\ &= 1 + \sum_{k=1}^{l_n}\sum_{j\neq i}\mu_k\mathbf{P}_k(i,j), \end{aligned} \tag{21}$$

where $\mathbf{P}_k(i,j)$ denotes the number of paths of length k between banks i and j. That is, $\mathbb{e}_i\Psi_\phi\mathbb{1}$ captures the expected number of banks that fail as a result of cascades of failures that start when i initially fails—hereafter, I refer to $\mathbb{e}_i\Psi_\phi\mathbb{1}$ as bank i's expected systemic footprint. With this interpretation in hand, proposition 5 highlights an important point. The optimal intervention balances the benefits and costs of forcing banks to internalize their expected systemic footprint. In particular, equation (20) shows that banks with a higher expected systemic footprint should hold more liquid portfolios, as the system-wide consequences associated with their failures are larger. In addition, $\vec{\mathbf{x}}_s^u$ is an increasing function of β. An increase in β increases the cost of forgoing profitable investments, which, in turn, justifies making optimal interventions less stringent from an ex ante perspective.

As the sequence $\{\mu_k\}_{k=2}^{l_n}$ depends on parameter ϕ, uncertainty about p can reshape the planner's optimal choice despite the fact that she is risk- and ambiguity-neutral. It follows from differentiating equation (20) with respect to ϕ that

$$\frac{\partial\vec{\mathbf{x}}_s^u}{\partial\phi} = -\frac{n}{2}\Psi_\phi^{-1}\Lambda\Psi_\phi^{-1}\mathbb{1}. \tag{22}$$

requirements. Hence, analyzing preemptive policies that restrict banks' portfolio decisions seems more timely. For completeness, section V.F discusses (private and public) bailouts.

Similarly to the market equilibrium, the way uncertainty alters the planner's behavior is reshaped by the interplay between matrices Ψ_ϕ and Λ. By focusing on the networks depicted in figure 1, example 2 illustrates how uncertainty alters optimal interventions.

EXAMPLE 2. *Consider an economy with $n > 3$ banks whose payoffs are linked according to figure 1(a), 1(b), or 1(c). When banks are linked according to the star network, the socially optimal portfolio is characterized by*

$$\begin{aligned} x^s_{1,u} &= \frac{n}{2}\left(\frac{1-(n-1)\mu_1+(n-2)\mu_2}{1-(n-1)\mu_1^2+(n-2)\mu_2}\right)-\frac{1}{2\beta} \\ x^s_{j,u} &= \frac{n}{2}\left(\frac{1-\mu_1}{1-(n-1)\mu_1^2+(n-2)\mu_2}\right)-\frac{1}{2\beta}, \end{aligned} \tag{23}$$

with $j \in \{2,3,\cdots,n\}$. When banks are linked according to the ring or complete networks, the socially optimal portfolio is characterized by

$$x^s_{i,u} = \frac{n}{2}\left(1+2\mu_1+2\mathcal{S}\right)^{-1}-\frac{1}{2\beta}, \tag{24}$$

$$x^s_{i,u} = \frac{n}{2}\left(1+(n-1)\mu_1+(n-1)(n-2)\mathcal{S}\right)^{-1}-\frac{1}{2\beta}, \tag{25}$$

where (24) describes the optimal portfolio in the ring network, while (25) describes the optimal portfolio in the complete network; $\mathcal{S}$ is defined as in example 1.

Example 2 highlights two important observations. First, higher uncertainty commands tailored interventions when banks are heterogenous in their number of exposures. Take the star network as an example. Because μ_2 is an increasing function of ϕ, it is optimal to force everyone but bank 1 to hold more liquid portfolios when facing higher uncertainty; note that $\frac{\partial x^s_{1,u}}{\partial \phi} \geq 0$ and $\frac{\partial x^s_{j,u}}{\partial \phi} \leq 0$, with $j \neq 1$. As higher uncertainty makes larger values of p seem more likely, curbing the risk-taking of banks with higher exposure to larger paths becomes optimal. This is because the emergence of larger cascades of failures relies relatively more on their individual behavior. To reduce the costs of implementing such an intervention, the planner allows banks with less exposure to larger paths to increase their holdings in the illiquid asset.

Second, even when banks are homogenous in their number of exposures, the architecture of $\mathcal{G}_n$ can play an important role in the design of the optimal intervention. Take the ring and the complete networks as examples. In both networks, uncertainty unequivocally pushes the planner towards more caution; note that $\frac{\partial x^s_{i,u}}{\partial \phi} \leq 0$ as $\frac{\partial \mathcal{S}}{\partial \phi} \geq 0$. Intuitively, the higher the ϕ, the less precise is the knowledge available to the planner. Consequently, increasing the portfolio liquidity of every bank becomes optimal to preclude contagion in case p is larger than expected. The juxtaposition of equations (24) and (25) highlights that such an increase in portfolio liquidity is especially important in networks more prone to contagion. Note that an increase in ϕ increases the liquidity of socially optimal portfolios relatively more in the complete than in the ring network. From an ex ante perspective, this is justified because ineffective interventions can have larger negative effects in the former than in the latter network as the complete network exhibits higher susceptibility to contagion.

The next corollary underscores that matrix Λ plays a key role in determining whether uncertainty pushes the planner toward more or less caution despite the fact that she is risk- and ambiguity- neutral.

COROLLARY 3. *If Λ is a positive (negative) semi-definite matrix, then an increase in ϕ makes the planner more (less) cautious.*

That is, if Λ is a positive (negative) semi-definite matrix, then the planner implements more (less) stringent interventions in the face of higher uncertainty. Intuitively, when Λ is a positive semi-definite matrix, an increase in ϕ commands a more than proportional increase in the perceived susceptibility of the network to contagion. From an ex ante perspective, this increment, in turn, justifies more stringent interventions to reduce the likelihood of excessively large cascades of failures.

C. Uncertainty Reshapes Market Equilibrium Inefficiencies

The previous analysis emphasizes that uncertainty can alter banks' preemptive behavior as well as socially optimal portfolios. In doing so, uncertainty can reshape market equilibrium inefficiencies. The next proposition highlights an important observation. Uncertainty can have non-monotonic effects on market equilibrium inefficiencies.

PROPOSITION 6 (Uncertainty Alters Market Equilibrium Inefficiencies)**.** *Let $\Delta x_{i,u} \equiv (x^*_{i,u} - x^s_{i,u})$ denote the wedge between the market equilibrium and the socially optimal portfolio at bank i's level. Let Θ_{ij} denote the (i,j) element of matrix $\Psi^{-1}\Lambda\Psi^{-1}$ and Π_{ij} denote the (i,j) element of matrix $(\mathbb{I}+\Psi)^{-1}\Lambda(\mathbb{I}+\Psi)^{-1}$. Define $\beta^*_i \equiv \frac{2}{n}\left(2 - \frac{\sum_{j\neq i}\Theta_{ij}}{\sum_{j\neq i}\Pi_{ij}}\right)^{-1}$, $\overline{\beta} \equiv \max_{1\leq i\leq n}\beta^*_i$, and $\underline{\beta} \equiv \min_{1\leq i\leq n}\beta^*_i$,*

- *If $\beta \leq \underline{\beta}$, then $\Delta x_{i,u}$ is increasing in ϕ, $\forall$ i.*
- *If $\beta \geq \overline{\beta}$, then $\Delta x_{i,u}$ is decreasing in ϕ, $\forall$ i.*
- *If $\beta \leq \beta^*_i$, then $\Delta x_{i,u}$ is increasing in ϕ. Otherwise, $\Delta x_{i,u}$ is decreasing in ϕ.*

That is, the spread between asset payoffs, β, plays a key role in determining how variations in uncertainty alter market equilibrium inefficiencies. When β is sufficiently small, higher uncertainty unequivocally increases the wedge between the market equilibrium and the socially optimal portfolio. In short, uncertainty constitutes a curse. This wedge increase has two sources within the model. First, forgoing profitable investments is less costly when β is small. Therefore, uncertainty pushes the planner toward more caution as more stringent interventions are less costly to implement. Second, while banks understand contagion might occur more frequently, they fail to internalize that their own failures are more likely to have far-reaching implications. As a result, banks' preemptive shift in portfolio liquidity in the face of higher uncertainty is less than socially optimal. Consequently, $\partial \Delta x_{i,u}/\partial \phi \geq 0$.

When β is sufficiently large, the opposite happens. That is, higher uncertainty unequivocally decreases the wedge between the market equilibrium and the socially optimal portfolio. In short, uncertainty constitutes a blessing. Here uncertainty is beneficial as it amplifies the perception of contagion risk, serving as a deterrence against banks' incentives to hold more illiquid portfolios. On the one hand, stringent interventions are costly to implement when β is large. Thus, uncertainty does not necessarily push the planner toward more caution. On the other hand, banks have more incentives to hold the illiquid asset when β is large, making their choices less susceptible to their perception of contagion risk. While banks continue to fail to internalize that their own failures are more likely to have far-reaching implications, they still understand that contagion might occur more frequently in the face of higher uncertainty. The joint work of these two forces ensures that banks' preemptive shifts in portfolio liquidity are sufficiently large to decrease the wedge between socially optimal and market equilibrium allocations. As a result, $\partial \Delta x_{i,u}/\partial \phi \leq 0$.

Proposition 6 also shows that β-thresholds vary not only across banks, but also across different network architectures. By focusing on networks wherein banks exhibit the same number of exposures, to abstract from variation of β-thresholds across banks, the next example helps illustrate the latter observation.

EXAMPLE 3. *Consider an economy with $n > 3$ banks whose payoffs are linked according to either figure 1(b) or 1(c).*

- *Ring network. If $\beta \leq \beta_r \equiv \frac{1}{2n}\left(\frac{1}{2} - \left(\frac{1+\mu_1+\mathcal{S}}{1+2\mu_1+2\mathcal{S}}\right)^2\right)^{-1}$, then $\Delta x_{i,u}$ increases with ϕ. Otherwise, $\Delta x_{i,u}$ decreases with ϕ.*
- *Complete network. If $\beta \leq \beta_c \equiv \frac{1}{2n}\left(\frac{1}{2} - \left(\frac{1+\frac{(n-1)}{2}\mu_1+\frac{(n-1)(n-2)}{2}\mathcal{S}}{1+(n-1)\mu_1+(n-1)(n-2)\mathcal{S}}\right)^2\right)^{-1}$, then $\Delta x_{i,u}$ increases with ϕ. Otherwise, $\Delta x_{i,u}$ decreases with ϕ.*

That is, the ring and the complete networks exhibit different β-thresholds. Notably, for a given value of μ_1 and $\mathcal{S}$, $\beta_c < \beta_r$. Intuitively, in the face of higher uncertainty, banks in the complete network perceive higher contagion risk than banks in the ring network. This is because the complete network is more prone to contagion than the ring network for any given value of p. Therefore, when ϕ increases, banks in the former architecture preemptively shift to more liquid portfolios more actively than banks in the latter architecture. This behavior, in turn, helps reduce the parameter region wherein β ensures that market equilibrium inefficiencies increase in the face of higher uncertainty. As a consequence, $\beta_c < \beta_r$.

Reducing Uncertainty Is Not Always Welfare-Improving.—Example 3 underscores another important observation. Decreasing uncertainty can be welfare-reducing if not accompanied by regulation. To see this point more clearly, suppose that $\beta_c < \beta < \beta_r$ and the planner can vary parameter ϕ at no cost before banks select their portfolios. Assume further that the planner varies only ϕ without imposing restrictions on banks' portfolios and that the new value of ϕ is publicly communicated to banks. If banks are linked according to the ring network, then reducing ϕ decreases market equilibrium inefficiencies. While banks increase their risk taking as ϕ decreases, reducing ϕ also decreases the network externality associated with banks not internalizing that their failures might engender larger cascades. Here the latter channel overcompensates for the former one. However, if banks are linked according to the complete network, then reducing uncertainty increases market equilibrium inefficiencies. In this case, the main role of uncertainty is to deter banks from choosing excessively illiquid portfolios. As a result, reducing ϕ increases banks' risk-taking incentives more than is socially desirable. In sum, while lowering uncertainty is beneficial when banks are linked according to the ring network, it is not when banks are linked according to the complete network.

IV. Pricing Uncertainty

The previous analysis demonstrates that reducing uncertainty could be detrimental. Yet, reducing uncertainty can also improve the effectiveness of interventions from an ex ante perspective. Though policymakers might have access to limited and imprecise information, it is reasonable to expect that they can gather detailed information before designing interventions. For instance, in the United States, banks are required to file comprehensive reports containing balance sheet information while large banks also have on-site exams conducted at least once every year. In addition, there are two important examples in which policymakers assess banks' financial soundness: the Comprehensive Liquidity Assessment and Review and the Dodd-Frank Act supervisory stress test, both of which are run by the Federal Reserve. In these programs, policymakers evaluate the liquidity risk profile of bank holding companies (BHCs) through a range of metrics and project whether BHCs would be vulnerable should economic conditions deteriorate.[10]

Before exploring how much uncertainty is socially optimal, the next section provides a formal definition of the social value of reducing uncertainty within the model.

A. Social Value of Uncertainty

What are the welfare effects of reducing uncertainty? Suppose the planner could vary parameter ϕ before designing interventions. The social value of reducing uncertainty is then equivalent to the value of reducing uncertainty to the planner. Notably, this value depends on how useful such reduced uncertainty is expected to be for increasing the effectiveness of interventions. Let ϕ_c denote the current value of ϕ. All else being equal, the social value of shifting parameter ϕ from ϕ_c to $(\phi_c - \epsilon)$ is defined as

$$\begin{aligned} V(\phi_c, \phi_c - \epsilon) &\equiv \max_{\vec{\mathbf{x}} \in \Delta_n} \mathbb{E}_{\phi_c - \epsilon} \left(W(\vec{\mathbf{x}}, p) - W(\vec{\mathbf{x}}^*_{\phi_c}, p) \right) \qquad (26) \\ &= \max_{\vec{\mathbf{x}} \in \Delta_n} \int_{\mu_1 - \frac{\phi_c - \epsilon}{2}}^{\mu_1 + \frac{\phi_c - \epsilon}{2}} \left(W(\vec{\mathbf{x}}, p) - W(\vec{\mathbf{x}}^*_{\phi_c}, p) \right) dF_{\mu_1, \phi_c - \epsilon}, \end{aligned}$$

with $\epsilon > 0$, $\vec{\mathbf{x}}^*_{\phi_c} \equiv \arg\max_{\vec{\mathbf{x}} \in \Delta_n} \mathbb{E}_{\phi_c} W(\vec{\mathbf{x}}, p)$, and $\mathbb{E}_{\phi_c} W(\vec{\mathbf{x}}, p) \equiv \int_{\mu_1 - \frac{\phi_c}{2}}^{\mu_1 + \frac{\phi_c}{2}} W(\vec{\mathbf{x}}, p) dF_{\mu_1, \phi_c}$.

That is, the value function $V(\cdot)$ captures the welfare gains of reducing uncertainty, as reduced uncertainty potentially allows the planner to damp contagion more effectively. These welfare gains have two sources within the model. First, more effective interventions decrease the average losses from forcing banks to hold excessively liquid portfolios, decreasing resource misallocation. Second, more effective interventions decrease the average losses from failing to forestall contagion as a result of imposing laxer portfolio restrictions.

B. How Much Uncertainty Is Socially Optimal?

Although reducing uncertainty generates benefits, it can also be costly. Some costs can be direct as gathering and processing detailed balance sheet information is expensive for both policymakers and banks. Some costs can also be indirect as reduced uncertainty might decrease banks' confidentiality. As confidentiality is valuable to banks, reducing uncertainty could compromise their market position, potentially decreasing market efficiency. Reduced uncertainty can also give rise to other issues. For instance, it can decrease policymakers' ability

[10] Examples applicable to other financial institutions include programs implemented by the Securities and Exchange Commission, such as forms N-MFP and PF. Form N-MFP requires registered money market funds to report their portfolio holdings and other information on a monthly basis, while form PF requires private funds to report assets under management.

to collect information in the future (see, Prescott (2008) and Leitner (2012)), promote window dressing or lead market participants to pay too much attention to public signals (see, Morris and Shin (2002) and Angeletos and Pavan (2007)), reduce the ability of policymakers to learn from asset prices (see, Bond and Goldstein (2015)), decrease banks' ability to produce money-like safe liquidity (see, Dang et al. (2017)), or reduce future risk-sharing opportunities (see, Hirshleifer (1971) and Goldstein and Leitner (2018)).

How much uncertainty is then socially optimal? To answer this question, assume the planner has access to a costly information technology that improves the precision of the current information about p. Here the information technology is jointly captured by the sequence of distributions $\{F_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}$ (generated by marginal changes in ϕ) as well as how costly it is to vary parameter ϕ from ϕ_c to ϕ, with $\phi < \phi_c$. Assume the planner can vary parameter ϕ from ϕ_c to $(\phi_c - \epsilon)$ after paying $\frac{\partial c}{\partial \phi}\big|_{\phi_c}$, where $\epsilon > 0$ is arbitrarily small and $c(\cdot)$ is a decreasing convex function of ϕ. That is, lower levels of uncertainty (in other words, higher levels of transparency) are more costly to attain.

The marginal benefit of reducing uncertainty is then given by

$$\frac{\partial}{\partial \phi} V(\phi_c, \phi) \quad = \quad \frac{\partial}{\partial \phi} \left(\max_{\vec{\mathbf{x}} \in \Delta_n} \mathbb{E}_\phi \left(W(\vec{\mathbf{x}}, p) - W(\vec{\mathbf{x}}^*_{\phi_c}, p) \right) \right). \tag{27}$$

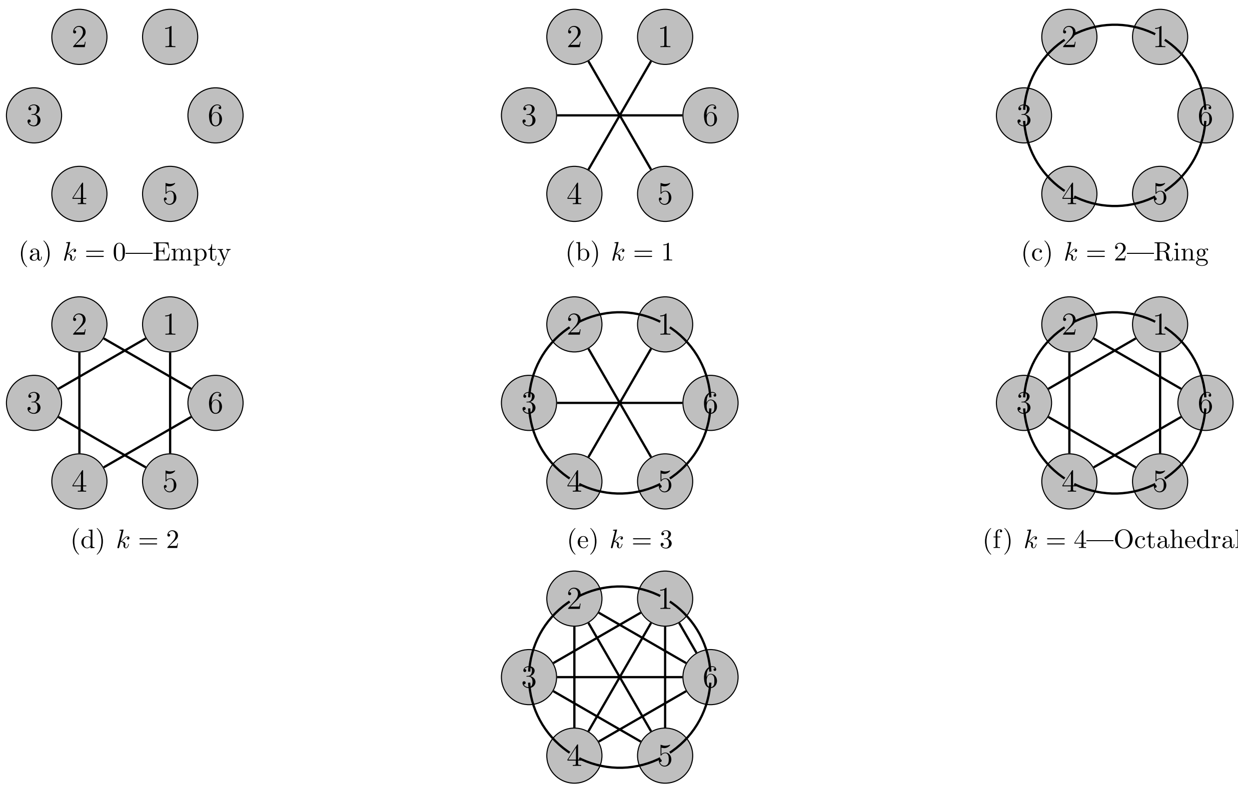
Under standard regularity assumptions, the condition that pins down the socially optimal level of uncertainty, ϕ^*, is then given by

$$\left. \frac{\partial V}{\partial \phi} \right|_{\phi=\phi^*} \quad = \quad \left. \frac{\partial c}{\partial \phi} \right|_{\phi=\phi^*}. \tag{28}$$

Here variable ϕ^* is not given. It is the choice variable that summarizes the planner's information decision. Intuitively, ϕ^* critically depends on a delicate balance. Although reducing uncertainty is costly, not reducing uncertainty is also costly, as it results in welfare losses associated with implementing less effective interventions. The socially optimal level of uncertainty ensures these two forces balance each other. As will become clear in the next paragraphs, the aforementioned tradeoff can be reshaped by the network architecture and the information technologies available to policymakers.

Comparative Statics.—Comparative statics on the solution of equation (28) provides several insights on the role that both the network architecture and the information technologies available to the planner play in determining ϕ^*. By focusing on symmetric networks, the next proposition characterizes how these two dimensions can reshape the socially optimal level of uncertainty. For illustration, figure 2 depicts all symmetric networks among six banks.[11]

[11] A network $\mathcal{G}_n$ is said to be symmetric if, given any two pairs of adjacent banks (i, j) and (k, l) there



(a) $k = 0$—Empty (b) $k = 1$ (c) $k = 2$—Ring (d) $k = 2$ (e) $k = 3$ (f) $k = 4$—Octahedral (g) $k = 5$—Complete

Figure 2. Symmetric networks among 6 banks. Parameter k denotes the number of exposures per bank.

PROPOSITION 7 (Comparative Statics)**.** *Given μ_1, ϕ_c, a sequence $\{F_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}$, and a network $\mathcal{G}_n$ define the function $\alpha_{\mathcal{G},F}(\phi) : [0,\phi_c] \to \mathcal{R}_+$ as*

$$\alpha_{\mathcal{G},F}(\phi) \equiv \frac{1}{n}\left(1 + \sum_{k=1}^{l_n}\sum_{s\neq 1}\mu_k^F(\phi)\boldsymbol{P}_k(1,s)\right), \tag{29}$$

with $\mu_k^F(\phi) \equiv \int_{\mu_1-\frac{\phi}{2}}^{\mu_1+\frac{\phi}{2}} p^k dF_{\mu_1,\phi}$.

- *Let $\mathcal{N}_n$ and $\mathcal{M}_n$ denote two symmetric networks defined over n banks. If*

$$\frac{\partial\alpha_{\mathcal{M},F}(\phi)}{\partial\phi}\left(\frac{1}{\alpha^2_{\mathcal{M},F}(\phi)} - \frac{1}{\alpha^2_{\mathcal{M},F}(\phi_c)}\right) \quad \geq \quad \frac{\partial\alpha_{\mathcal{N},F}(\phi)}{\partial\phi}\left(\frac{1}{\alpha^2_{\mathcal{N},F}(\phi)} - \frac{1}{\alpha^2_{\mathcal{N},F}(\phi_c)}\right), \tag{30}$$

  for all $0 \leq \phi \leq \phi_c$, then $\phi^*(\mathcal{M}_n) \leq \phi^*(\mathcal{N}_n)$.

exist an automorphism $h : [1,n] \to [1,n]$ such that $h(i) = k$ and $h(j) = l$. That is, a symmetric network is both exposure- and bank- transitive. While every symmetric network is regular, not every regular network is symmetric.

- *Suppose $\mathcal{G}_n$ is symmetric and consider two information technologies characterized by $\{H_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}$ and $\{J_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}$, with $H_{\mu_1,\phi_c} = J_{\mu_1,\phi_c}$. If*

$$\frac{\partial\alpha_{\mathcal{G},H}(\phi)}{\partial\phi}\left(\frac{1}{\alpha^2_{\mathcal{G},H}(\phi)} - \frac{1}{\alpha^2_{\mathcal{G},H}(\phi_c)}\right) \;\geq\; \frac{\partial\alpha_{\mathcal{G},J}(\phi)}{\partial\phi}\left(\frac{1}{\alpha^2_{\mathcal{G},J}(\phi)} - \frac{1}{\alpha^2_{\mathcal{G},J}(\phi_c)}\right), \quad (31)$$

  for all $0 \leq \phi \leq \phi_c$, then $\phi^*(\{H_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}) \leq \phi^*(\{J_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c})$.

- *ϕ^* is a decreasing function of β.*

Proposition 7 underscores that the network architecture and the information technology available to the planner can play a key role in determining ϕ^*. The first result establishes that, for a given information technology, it is optimal to achieve higher transparency levels under architecture $\mathcal{M}_n$ when higher transparency is expected to be more cost-effective for forestalling contagion under $\mathcal{M}_n$ than under $\mathcal{N}_n$.

At the core of the above result lies that fact that the value of reducing uncertainty can be reshaped by the network architecture. Figure 3 highlights this point by focusing on symmetric networks whose payoffs are linked according to either figure 1(b) or 1(c). Under the assumption that $\mu_1 = 1/2$, $\beta = 3/10$, $\phi_c = 1$, and $F_{\mu_1,\phi} = U\left[\frac{1}{2} - \frac{(1-\epsilon)}{2}, \frac{1}{2} + \frac{(1-\epsilon)}{2}\right]$, figure 3 depicts the value function $V(\phi_c = 1, \epsilon)$ where ϵ captures the extent of reduced uncertainty. Solid lines depict the ring while dashed lines depict the complete network. The left panel considers $n = 4$ while the right panel considers $n = 8$. As the left panel shows, reducing uncertainty is perceived as more valuable in the complete than in the ring network when n is sufficiently small. Because the complete network exhibits higher susceptibility to contagion, reducing uncertainty is expected to be more beneficial in the complete than in the ring network from an ex ante perspective. As the right panel shows, however, the opposite can happen when n is larger. Unless uncertainty is reduced above a certain threshold, interventions might not allow the planner to effectively prevent contagion in the complete network. When only small reductions of uncertainty are considered, the susceptibility to contagion of the complete network could be perceived as excessively large. Consequently, the less beneficial interventions are expected to be in the complete than in the ring network. While higher connectivity implies a higher susceptibility to contagion, networks expected to be too susceptible to contagion might not necessarily benefit from preemptive regulation as contagion might be too costly to prevent from an ex ante perspective.

By reinterpreting condition (30), the second result highlights the relevance of the information technology available to the planner. Intuitively, for a given network architecture, the more information is perceived to be gained by using one technology, the higher the perceived effectiveness of interventions associated with the use of such technology. Hence, the higher the

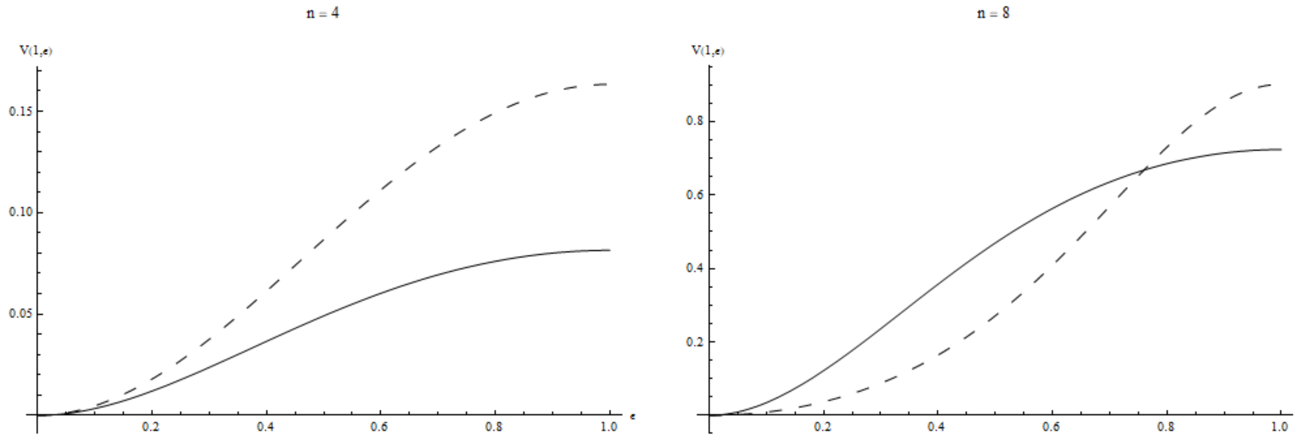


Figure 3. Value of reducing uncertainty, $V(1,\epsilon)$. $\beta = 0.3$, $\mu_1 = 1/2$, $\phi_c = 1$, and $F_{\mu_1,\phi} = U\left[\frac{1}{2} - \frac{(1-\epsilon)}{2}, \frac{1}{2} + \frac{(1-\epsilon)}{2}\right]$.

level of transparency attained. Figure 4 further illustrates the importance of how the sequence $\{F_{\mu_1,\phi}\}_{0\leq\phi\leq\phi_c}$ changes with ϕ. Under the assumption that banks are linked according to either figure 1(b) or 1(c), $n = 4$, $\mu_1 = 1/2$, $\beta = 3/10$, $\phi_c = 1$ and $F_{\mu_1,\phi_c-\epsilon}$ can be either $U\left[\frac{1}{2} - \frac{(1-\epsilon)}{2}, \frac{1}{2} + \frac{(1-\epsilon)}{2}\right]$ or $\mathcal{N}\left(\frac{1}{2}, \frac{(1-\epsilon)^2}{36}\right)$, figure 4 depicts the value function $V(\phi_c = 1, \epsilon)$. As before, ϵ captures the extent of reduced uncertainty.[12] The left panel depicts the ring network, while the right panel depicts the complete network. Blue lines consider that the sequence $\{F_{\mu_1,1-\epsilon}\}_\epsilon$ follows the uniform distribution, while purple lines consider the normal distribution case. As the figure shows, irrespective of the network architecture, reducing uncertainty is perceived as more valuable when the information technology follows the uniform distribution than when it follows the normal distribution. When priors are normally distributed, they are more precise to begin with, relative to what is implied by uniform priors. Intuitively, this difference in precision happens because all values of p are equally likely in the uniform case, whereas, values closer to the extremes are considerably less likely in the normal case. That is why the perceived benefits of reducing uncertainty are smaller in the normal relative to the uniform case.

The final result demonstrates a simple idea. The higher the β, the more transparency needed. The higher the β, the higher are the costs of forgoing profitable investments. Because interventions are warranted only if they generate a sufficiently large drop in the likelihood of contagion, the higher the β, the higher are the planner's incentives to reduce uncertainty.

[12]The variance of the normal distribution is selected to ensure that the probability that p lies within the interval $\left[\frac{1}{2} - \frac{(1-\epsilon)}{2}, \frac{1}{2} + \frac{(1-\epsilon)}{2}\right]$ is close to one (≈ 0.997).

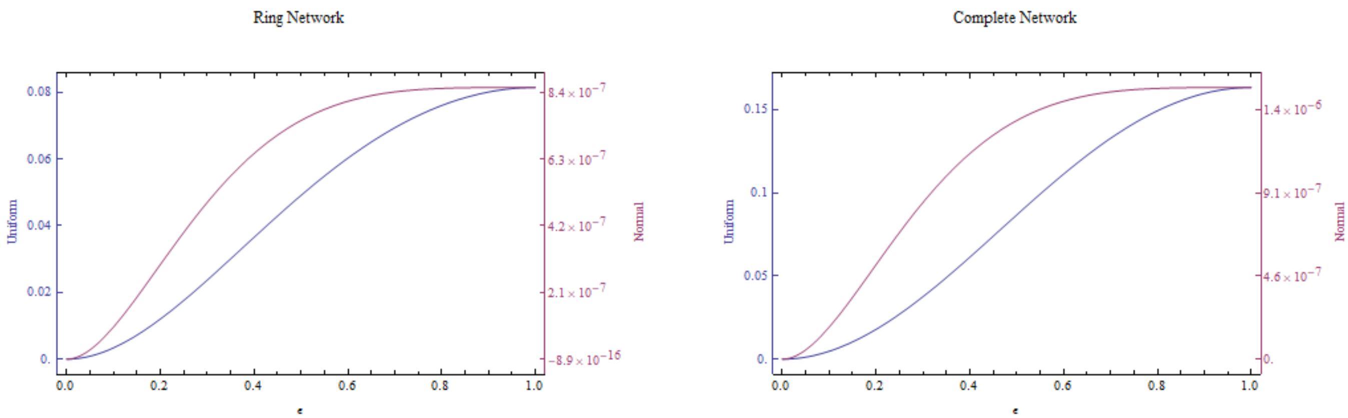


Figure 4. Value of reducing uncertainty. $n = 4$, $\beta = 0.3$, $\mu_1 = 1/2$, $\phi_c = 1$, and $F \in \left\{ U\left[\frac{1}{2} - \frac{(1-\epsilon)}{2}, \frac{1}{2} + \frac{(1-\epsilon)}{2}\right], \mathcal{N}\left(\frac{1}{2}, \frac{(1-\epsilon)^2}{36}\right) \right\}$.

V. Discussion

The previous sections show that uncertainty can reshape market equilibrium inefficiencies engendered by the very existence of interdependencies among banks' payoffs. They also show that uncertainty can alter the extent of welfare-improving interventions. To illustrate the main mechanisms, the baseline model makes a number of simplifying assumptions. This section discusses and supports these assumptions as well as explores extensions of the baseline model.

A. Contagion Mechanism

Although assumption (A2) imposes considerable structure, it provides tractability without failing to incorporate a key strategic consideration into the modeling. While p is an exogenous parameter, a bank's portfolio decision continues to be reshaped by the decisions of others. Take the following example. Suppose there is a sequence of exposures between banks i and j (with $j \neq i$). An increase in the portfolio liquidity of bank i reduces the likelihood of cascades of failures originating from i, thereby reducing j's incentives to hold cash. In turn, j's portfolio rebalancing increases the likelihood that i is affected by cascades of failures originating from j. To counteract this likelihood increase, bank i preemptively shifts to a more liquid portfolio.

That said, the simplicity engendered by assumption (A2) is not cost free. For example, in the baseline model, a bank can be affected by contagion even if it holds a perfectly liquid portfolio. Also, one would expect that the extent of the shortfall at the initially affected bank would affect the severity of contagion. In part, that is because banks that are affected by contagion might have the capacity to absorb small losses. And, of course, a bank's ability to absorb losses will depend both on its own portfolio choice and on the size of the loss to which it is exposed. At the expense of additional notation, the Online Appendix shows that the properties derived in the baseline model continue to hold in a more general setting wherein these concerns are taken into consideration.

B. Bank Heterogeneity

With the exception of their number of exposures, banks are ex ante identical within the baseline model. Because this assumption is at odds with reality, consider an augmented version of the model wherein different banks impose distinct externalities when failing. For example, assume that if bank i fails, society suffers an exogenous loss $l(d_i)$, where d_i denotes the number of exposures of i and $l(\cdot)$ is a non-decreasing function. After including $l(\cdot)$ in the planner's objective function, optimal interventions can be analytically derived. Two reasons explain why the planner now has stronger motives to increase the liquidity of banks with a higher expected systemic footprint. First, the failure of a bank with a high expected systemic footprint not only exposes many other banks to failure, but also increases the losses of economic agents outside the banking sector. Second, in the face of higher uncertainty, the planner cares about the likelihood of events in which the failure of a single bank can have sizable economic consequences. Thus, faced with higher uncertainty, the planner would force banks with a higher expected systemic footprint to hold even more liquid portfolios.

C. Costs of Reducing Uncertainty

Though the costs of reducing uncertainty are exogenously determined, the nature of my results remains the same as long as lower levels of uncertainty are more costly to attain. The particular functional form of $c(\phi)$ clearly varies with the underlying mechanism, but higher levels of transparency would also be more costly to attain in the following environments:

- Consider an economy wherein policymakers need banks' cooperation to receive private information. As illustrated in Prescott (2008), lower uncertainty increases the cost of cooperation for banks, potentially reducing the quality of information policymakers receive, thereby increasing the amount of resources policymakers spend on collecting information.

- Consider an economy wherein (1) policymakers use information not only about the susceptibility of the network to contagion, but also about banks' stock prices, and (2) changes in uncertainty alter the incentives of private investors to gather detailed bank information. As illustrated in Bond and Goldstein (2015) and Goldstein and Yang (2017), less uncertainty can weaken the incentives of investors to acquire more precise bank information. Intuitively, as both less uncertainty about p and detailed bank information produced by investors can be thought of as two pieces of information about banks' fundamentals, they are substitutes. Consequently, less uncertainty about p motivates investors to spend less on generating more precise bank information. In short, less uncertainty crowds out the production of private information. As less information is produced by private investors, banks' asset prices become less informative and, as a consequence, less useful for policymakers, increasing the overall costs of reducing uncertainty.

- Consider an economy similar to the one just described in which changes in uncertainty alter the incentives of private investors to produce detailed bank information. Rather than assuming that policymakers use information from banks' stock prices, assume that their decision-making is based only on information about p. Assume further that the informativeness of asset prices guides corporate decisions of agents outside the banking sector. For example, firms might pay attention to variation in asset prices to evaluate the optimality of their investment decisions. As before, more transparency can decrease the informativeness of asset prices as transparency crowds out the production of private information. As firms base their corporate decisions on less precise information, the resource misallocation outside the banking sector increases. In sum, lower levels of uncertainty are more costly to attain.

D. *Attitudes Toward Ambiguity*

When the precise value of p is unknown, economic agents effectively face model uncertainty. They must consider the range of possible values for p and choose actions to maximize their objective functions. I deliberately consider expected utility with risk-neutrality rather than preferences that explicitly capture agents' attitudes towards ambiguity to make sure my results are not driven by agents' attitudes toward model uncertainty. Both the maxmin expected utility representation (see, Gilboa and Schmeidler (1989)) and the smooth ambiguity model (see, Klibanoff et al. (2005)) add an extra layer of complexity to the analysis. Should the environment exhibit ambiguity aversion, all of my results would be amplified.

E. Bankruptcy Costs and Limited Liability

Introducing bankruptcy costs unequivocally increases market equilibrium inefficiencies. Consider an augmented version of the model in which there is a cost associated with each failure, $\kappa > 0$. Parameter κ captures inefficiencies that arise because of bankruptcy proceedings.[13] Here market equilibrium inefficiencies would increase. This is because banks fail to internalize the social costs associated with bankruptcies as they do not bear these costs when failing. Also, policymakers facing higher uncertainty have a stronger motive to increase the liquidity of banks with a high expected systemic footprint, as cascades of failures now have more severe economic consequences.

Introducing limited liability has similar effects on my results as limited liability further increases banks' risk-taking incentives. Besides the fact that banks are not held accountable for the losses they impose on others, banks might now strategically decide to increase their portfolio illiquidity. As before, market equilibrium inefficiencies unequivocally increase in such an environment.

F. Bailouts

Preemptive interventions can also materialize through private or public bailouts. This section analyzes two simple bailouts.

- *Private Bailouts.*—Suppose that at $t = 0$ banks must decide to contribute to a fund, whose money is then given to the initially affected bank at $t = 1$. Thus, the initially affected bank effectively faces a smaller adverse shock at $t = 1$ while contributing banks hold fewer dollars to invest in the illiquid asset at $t = 0$. Here banks continue to underweight the system-wide effect of their individual decisions, failing to internalize the consequences of their contributions in forestalling contagion. To see this more clearly, assume n is large and each bank has a binary option: Contribute either zero or $1/n$ dollars to the bailout fund. There is an equilibrium in which banks do not contribute to the bailout fund. Suppose only one bank contributes. For such a bank, contributing to the fund is costly and does not generate benefits, as its contribution does not change the likelihood of contagion. Hence, in such an environment, it is optimal not to contribute to the bailout fund.

- *Public Bailouts.*—Suppose the planner requires all banks to contribute $1/n$ dollars into a bailout fund; unused money is then returned to banks at the end of $t = 1$. This

[13]For example, creditors may not immediately receive payment during bankruptcy, interrupting their ability to run their businesses, thereby leading to resource misallocation.

policy is clearly a Pareto-improvement, as contagion disappears, allowing banks to invest their remaining endowment in an asset with higher payoffs and no risk. The same idea applies to a policy in which the planner requires banks to contribute an amount proportional to their expected systemic footprint.

G. Network Endogeneity

Although the baseline model does not capture the incentives underlying the formation of exposures among financial institutions, it provides a tractable approximation of the problem faced by policymakers nowadays. In doing so, this framework provides a benchmark to which other models can be compared. Importantly, the focus here is on how uncertainty alters equilibrium outcomes and optimal interventions rather than on explaining how financial institutions became interconnected to begin with. I do not seek to provide foundations of the mechanism through which adverse shocks can generate contagion. Instead, I take a simple propagation mechanism and explore how uncertainty about such a mechanism can alter banks' preemptive actions as well as policymakers' interventions.

H. Pandemics: The COVID-19 Case

Though exposures aim to capture interdependencies among financial institutions, the proposed framework can also be used to study other settings wherein an individual's actions can affect many others. An important example is the COVID-19 pandemic. At a very basic level, the reason that people's incentives to vaccinate or wear masks are too weak can be related to the reason that banks' incentives to hold more liquid portfolios are too weak within the model. While people might understand the contagion risk associated to social interactions, they fail to internalize the system-wide effects of their individual decisions on the spread of the virus. One of the reasons is that people are not held accountable for spreading the virus. Importantly, as highlighted by my results, uncertainty about how easily the virus spreads can play a key role in reshaping people's actions, generating even weaker incentives to vaccinate or wear masks.

VI. Conclusion

This paper develops a conceptual framework for studying the role that uncertainty about the susceptibility of a financial network to contagion plays in the strategic behavior of profit-seeking institutions, their collective equilibrium response, and preemptive policy interventions. I show that such uncertainty can reshape market equilibrium inefficiencies engendered by the very existence of interdependencies among financial institutions. I also demonstrate that optimal preemptive interventions are about ensuring institutions internalize their expected systemic footprint. I then explore how reducing uncertainty can alter optimal preemptive interventions. Though reducing uncertainty can be costly, it might be worth the cost as it increases the ex ante effectiveness of interventions. Importantly, the social value of uncertainty, which dictates how much uncertainty is socially optimal, is reshaped by structural characteristics of the network and the precision of information technologies available to policymakers.

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