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Firm Networks and Asset Returns

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Changes in the propagation of shocks along firm networks are important to understanding aggregate and cross-sectional features of stock returns. When calibrated to match key characteristics of supplier–customer networks in the United States, a model in which firms are interlinked via enduring relationships generates long-run consumption risks, high and volatile risk premiums, and a small and stable risk-free rate. The model also matches cross-sectional patterns of portfolio returns sorted by firm centrality, a feature unaccounted for by standard asset pricing models. (*JEL* C67, E30, G12, L14)

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Firms are far from functioning as isolated entities. Instead, they are deeply intertwined through a web of material connections, including strategic alliances, joint ventures, research and development (R&D) partnerships, and supplier–customer relationships. As shown by a growing body of literature—and underscored by a multitude of supply chain disruptions during the COVID-19 pandemic—these relationships can serve as transmission channels of shocks to individual firms and, in doing so, potentially alter stock returns.[1] Despite this evidence, the impact of such shock propagation on asset pricing

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[1] See Hertzel et al. (2008), Jorion and Zhang (2009), Boone and Ivanov (2012), Boyarchenko and Costello (2015), Todo, Nakajima, and Matous (2015), Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019), and Carvalho et al. (2020), among others. Using French firm-level data, Di Giovanni, Levchenko, and Mejean (2014) provide evidence of the importance of firm-specific shocks in generating aggregate fluctuations.

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remains, at best, imperfectly understood. In this paper, I develop a model to study the asset pricing properties that stem from the propagation of shocks along firm networks and the extent to which such shock propagation quantitatively explains asset market phenomena.

I show that changes in the propagation of shocks along production networks are important to understanding variations in asset returns both in the aggregate and in the cross-section. In particular, the model generates long-run risks when calibrated to match key structural characteristics of supplier–customer networks in the United States. As a result, the model replicates prime features of asset market data, such as a high and volatile risk premium and a small and stable risk-free rate. Additionally, the model matches cross-sectional patterns of portfolio returns sorted by network centrality.

The model has two main features. First, firm-level shocks propagate via long-lasting relationships. Consequently, firms' cash-flow growth rates are related via a firm network. Second, investors have a preference for early resolution of uncertainty and, thus, care about uncertainty regarding firms' long-term growth prospects.

Aside from aggregate shocks, the distribution of aggregate consumption growth is shaped by two characteristics within my model: (a) the architecture of the firm network and (b) the propensities of relationships to transmit shocks, henceforth referred to as propensities. Propensities vary over time. Such variation aims to capture temporal changes in relationship specific characteristics that make firms more susceptible to shocks affecting their neighbors. As propensities vary over time, the connectivity of the network also varies over time. This variation introduces a time-varying correlation structure between firms' cash-flow growth rates, which, in equilibrium, generates stochastic volatility in consumption growth.

In calibrated models, changes in network connectivity are infrequent because firms engage in sustained and stable relationships with their customers. These enduring relationships create lasting interdependencies between firms' cash-flow growth rates. In these economies, a shock to one firm can modify the current cash flows of every neighboring firm as well as alter the long-term growth prospects of all such firms. These infrequent changes in network connectivity are what fundamentally drive low-frequency movements in aggregate output growth, which, in equilibrium, generate a persistent component in expected aggregate consumption growth. As a result of investors having preferences for early resolution of uncertainty, the model generates long-run risks. The model accounts for sizable risk premiums because investors fear that extended periods of low economic growth coincide with low asset prices. The model generates a small and stable risk-free rate as a result of investors saving for long periods of low economic growth.

In addition to generating long-run risks, calibrated models match cross-sectional patterns of portfolio returns sorted by firm centrality. Central firms command lower risk premiums than peripheral firms because, in the data, relationships of peripheral firms exhibit higher propensities than relationships of central firms. The reason is that peripheral firms tend to rely more heavily on their suppliers. As a consequence, central firms are less exposed than peripheral firms to problems affecting their neighbors, and, thus, they command lower risk premiums. Calibrated models generate a realistic monthly return spread between firms in the lowest and in the highest deciles of centrality. This economically and statistically significant spread arises naturally in equilibrium as compensation for contagion risk, a feature unaccounted for by standard asset pricing models.

The small and persistent component in expected consumption growth generated by low-frequency movements in the connectivity of production networks provides an economic rationale for long-run risk models in the spirit of Bansal and Yaron (2004). Moreover, the model helps explain the cross-section of expected returns, as it provides a mapping between firms' importance in the network and their contagion risk. Overall, these results suggest that extending standard asset pricing models to take into account how shocks propagate along production networks can make significant progress toward generating a unifying framework that simultaneously captures aggregate and cross-sectional features of stock returns.

This paper contributes to three strands of the literature. First, this paper develops a new theoretical framework that adds to a growing body of work focused on understanding the effects of economic linkages in asset pricing (see, e.g., Cohen and Frazzini 2008, Buraschi and Porchia 2012; Ahern 2013; Herskovic 2018; Branger et al. 2020; Gofman, Segal, and Wu 2020; Buraschi and Tebaldi 2024).[2] Unlike most of these papers, however, my model emphasizes relationships at the firm level to explore the asset pricing properties that stem from the propagation of shocks along production networks.

Second, this paper adds to a body of work that explores how granular shocks may lead to aggregate fluctuations in the presence of linkages between different economic sectors (see, e.g., Carvalho 2010; Gabaix 2011; Acemoglu et al. 2012; Blume et al. 2013; Carvalho and Gabaix 2013; Chaney 2014, 2018; Elliott, Golub, and Jackson 2014; Acemoglu, Ozdaglar, and Tahbaz-Salehi 2017; Lim 2017; Oberfield 2018; Elliott, Golub, and Leduc 2022; Elliott and Golub 2022; Dew-Becker 2023). This paper contributes to this literature by exploring the asset pricing implications of linkages at the firm level and studying how changes in the propagation of firm-specific shocks can alter not only macroeconomic variables but also asset returns and aggregate risk premiums.

[2] Cohen and Frazzini (2008) show that stock prices take some time to include news about economically related firms, generating return predictability across assets. Buraschi and Porchia (2012) show that more central firms in a market-based network have lower price dividend ratios and higher expected returns. Using the intersectoral trade network, Ahern (2013) provides evidence that firms in more central industries are more exposed to systematic risk. Herskovic (2018) focuses on the efficiency gains that come from changes in the input-output network and how those changes are priced in equilibrium. Branger et al. (2020) highlight that links' directedness matters for pricing. Gofman, Segal, and Wu (2020) underscore the importance of multiple intermediary stages in supply chains for asset pricing.

Third, this paper adds to a literature that examines the potential sources of long-run risks (see, e.g., Kaltenbrunner and Lochstoer 2010; Kung and Schmid 2015; Bidder and Dew-Becker 2016; Collin-Dufresne, Johannes, and Lochstoer 2016).[3] This paper contributes to this literature by showing that changes in the propagation of shocks along production networks can be an important source of long-run risks.

1. Baseline Model

Though stylized, the baseline model conveys the main intuition for how changes in the propagation of shocks along firm networks, in combination with recursive preferences, can generate long-run risks and matter for the aggregate and the cross-section of stock returns. To facilitate exposition, the baseline model abstracts from firms' production decisions and considers a single-good economy in which firms' outputs are related via a network of long-lasting relationships. Section I of the Internet Appendix shows that, under certain conditions, the main intuition continues to hold within an equilibrium context wherein firms' production decisions are explicitly modeled.

1.1 The environment

Consider an economy with one perishable good and an infinite time horizon. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. In each period, n infinitely lived firms—with n being potentially large—produce the single good. Firms' outputs, henceforth referred to as cash flows, are related via a network of long-lasting relationships. This network can be conveniently described by a graph consisting of a set of nodes, which represent firms, together with (directed) edges joining certain pairs of nodes, which represent relationships. To fix notation, let $\mathcal{G}_n$ denote the network between n firms. Because I focus on the effect of interfirm relationships on asset returns rather than on strategic network formation, relationships are assumed to be exogenously determined and fixed before $t=0$.[4] Besides firms, a large number of identical, infinitely lived individuals are aggregated into a representative investor with Epstein-Zin-Weil preferences who owns all assets in the economy.

[3] Kaltenbrunner and Lochstoer (2010) show that long-run risks endogenously arise in a standard production economy model, even when technology growth is i.i.d., because of consumption smoothing. Kung and Schmid (2015) show that a model of endogenous innovation and R&D is able to generate long-run risks, while Bidder and Dew-Becker (2016) show that long-run risks can also arise in an economy in which investors are pessimistic and not sure about the true model driving the economy. Collin-Dufresne, Johannes, and Lochstoer (2016) show that parameter learning generates long-lasting risks that help explain standard asset pricing puzzles in an economy in which investors are uncertain about the structural parameters governing the model economy.

[4] See Demange and Wooders (2005), Goyal (2007), and Jackson (2008) for a detailed description of network formation models. For models of endogenous formation of production networks, see Lim (2017), Oberfield (2018), and Acemoglu and Azar (2020), among others.

1.2 Firms' cash flows

Firms' cash flows vary stochastically over time and depend on aggregate and firm-level shocks. Input, labor, and capital decisions are deliberately normalized to one. Firm i's cash flow at $t+1$, $y_{i,t+1}$, follows

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \equiv \bar{y}+\varphi_a\sigma\varepsilon^a_{t+1}-\alpha f(z_{i,t+1}),\ \ i\in\{1,\cdots,n\}, \tag{1}$$

where $\bar{y}$, φ_a, σ, and α are nonnegative parameters. Variable $Y_t\equiv\prod_{i=1}^n y_{i,t}^{1/n}$ denotes the aggregate output of the economy at t, $\varepsilon^a_{t+1}\overset{\text{i.i.d.}}{\sim}\mathcal{N}(0,1)$ is an aggregate productivity shock that affects all firms at $t+1$, and $z_{i,t+1}$ is a shock that affects only i at $t+1$ and whose impact on i's cash flow is captured by $f(z_{i,t+1})$. Because I focus on the influence of distress-like events on firms' cash flows, only the magnitude of $z_{i,t+1}$, $|z_{i,t+1}|$, matters in Equation (1). Hence, $f(z_{i,t+1})$ is assumed to be an increasing continuous function of $|z_{i,t+1}|$ with a nonnegative image.

A key feature of my model is that the propagation of shocks along $\mathcal{G}_n$ determines the dependence structure between shocks $\{z_{i,t+1}\}_{i=1}^n$. While enduring relationships may increase firms' growth opportunities via efficiency gains, they may also have unintended consequences, as they increase firms' reliance on their neighbors and, in doing so, increase their exposure to negative shocks affecting other firms in the economy.[5] To capture such unintended consequences in a simple way, $z_{i,t+1}$ is assumed to follow

$$z_{i,t+1} = \vec{z}_t\mathbf{P}_t e_i+\varphi_\eta\sigma\eta_{i,t+1}, \tag{2}$$

where $\eta_{i,t+1}\overset{\text{i.i.d}}{\sim}\mathcal{N}(0,1)$ is an idiosyncratic shock to firm i at $t+1$, φ_η is a positive parameter, $\mathbf{P}_t$ is a $n\times n$ matrix, and $\vec{z}_t\equiv[z_{1,t},z_{2,t},\cdots,z_{n,t}]$ denotes the $1\times n$ (row) vector of shocks at t. Here, e_i is a $n\times 1$ selector vector that extracts the i-th element of row vectors. In the same spirit as Long and Plosser (1983), shocks affecting firms at t can also alter their output at $t+1$, and, as a result, $z_{i,t+1}$ exhibits an autoregressive structure.[6]

To capture shock propagation along $\mathcal{G}_n$, matrix $\mathbf{P}_t$ is defined as

$$\mathbf{P}_t=\vartheta\mathbb{I}+\sum_{k=1}^{l_n}P_t^k, \tag{3}$$

where $\mathbb{I}$ is the $n\times n$ identity matrix and P_t^k is the k-th power of matrix $P_t\equiv\left[p_{ij,t}\right]_{ij}$ whose (i,j) element measures the propensity of relationship (i,j)

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5 Long-lasting relationships potentially allow firms to circumvent difficulties in contracting due to unforeseen contingencies, asymmetries of information, and specificity on firms' investments (see, e.g., Williamson 1979, 1983).

6 The assumption that shocks at t can also affect firms' output at $t+1$ is also consistent with the Kydland and Prescott (1982) time-to-build idea, wherein multiple periods are required to build new goods and only finished goods can be used in the production of other goods.

to transmit shocks from i to j at period t. If there is no direct relationship between i and j, then $p_{ij,t}=0, \forall t$. Parameter $\vartheta \in [0,1)$ can be interpreted as a lower bound of the persistence of firm-level shocks. Consequently, even in the absence of relationships, shocks $z_{i,t+1}$ can exhibit an autoregressive structure.[7]

At a fundamental level, the precise value of $p_{ij,t}$ captures interdependencies between the cash flows of firms i and j at t. Such interdependencies—which cannot be mitigated through contractual protections—may be driven by specific characteristics of the relationship between i and j. Intuitively, the higher $p_{ij,t}$, the higher the likelihood that disruptions affecting i also affect j at t. In the context of production networks, $p_{ij,t}$ might capture restrictions firm j faces when trying to switch from using firm i as a supplier at t. The higher $p_{ij,t}$, the higher the switching costs j might face at t, and, thus, the higher the likelihood that an adverse shock affecting i also affects j—as j might not be able to restructure its production sufficiently fast to overcome i's disruption in production.

The first term in the right-hand side of Equation (3) allows firm-level shocks to have persistent effects on firms' cash flows. The second term captures shock propagation along $\mathcal{G}_n$. This is because the (i,j) element of matrix P_t^k keeps track of the (propensity-weighted) number of paths of length k from firm i to firm j. A path of length k between firms i and j is a sequence of (directed) relationships from i to j wherein no relationship is repeated. As a result, the term $\vec{z}_t\left(\sum_{k=1}^{l_n} P_t^k\right)e_i$—where l_n denotes the length of the longest path in $\mathcal{G}_n$—effectively captures the expected size of the (cumulative) shock that i faces due to the shocks affecting its (direct and indirect) neighbors. As a clear illustration of this observation, note that $\vec{z}_t\left(\sum_{k=1}^{l_n} P_t^k\right)e_i$ can be rewritten as

$$\vec{z}_t\left(\sum_{k=1}^{l_n} P_t^k\right)e_i = \underbrace{\vec{z}_t P_t e_i}_{\substack{\text{shocks to } i \text{ from} \\ \text{neighbors one step away}}} + \cdots + \underbrace{\vec{z}_t P_t^{l_n} e_i}_{\substack{\text{shocks to } i \text{ from} \\ \text{neighbors } l_n \text{ steps away}}}. \tag{4}$$

As Equation (4) shows, all potential paths—through which shocks might propagate along $\mathcal{G}_n$—are considered. And directionality matters as paths are directed.

1.3 Distribution of $\{z_{i,t+1}\}_{i=1}^n$

Given the dynamics of $z_{i,t+1}$, the joint distribution of $\{z_{i,t+1}\}_{i=1}^n$ is determined by two features: the precise architecture of $\mathcal{G}_n$ and the sequence of matrices $\{\mathbf{P}_t\}_{t\geq 0}$. Two special cases help better appreciate how these features reshape such distribution. First, as propensities approach zero, $z_{i,t+1}$ approaches the

[7] Given a network architecture, the stationarity of firm-level shocks can be achieved by imposing an upper limit on the magnitude of ϑ.

first-order autoregressive process

$$z_{i,t+1} = \vartheta z_{i,t} + \varphi_\eta \sigma \eta_{i,t+1}. \tag{5}$$

Hence, firm-level shocks become uncorrelated across firms and their innovations normally distributed. Second, as parameter ϑ approaches zero, $z_{i,t}$ approaches

$$z_{i,t+1} = \vec{z}_t \left(\sum_{k=1}^{l_n} P_t^k \right) e_i + \varphi_\eta \sigma \eta_{i,t+1}, \tag{6}$$

and, thus, the persistence of firm-level shocks $z_{i,t+1}$ is fully determined by the time series behavior of the sequence of matrices $\{P_t\}_{t \geq 0}$. As a consequence, if only one sequence of relationships exists between two firms, the longer the sequence, the smaller the correlation between their firm-level shocks.

1.4 Temporal changes in shock propagation

To capture temporal changes in relationship-specific characteristics, propensities $\left\{p_{ij,t}\right\}_{(i,j)}$ are allowed to vary over time. Such temporal variation may arise from changes in complementarities between firms' activities or the arrival of new technologies that reshape the economy's long-term growth prospects. To facilitate the computation of equilibrium prices, I assume propensities follow a first-order autoregressive Markov process. Hence, the sequence of matrices $\{\mathbf{P}_t\}_{t \geq 0}$ is also Markovian.

1.5 Discussion of modeling assumptions

The baseline model provides a parsimonious and tractable framework to capture cascades of distress-like events. I focus on such cascades motivated by a growing body of empirical work that shows that interfirm relationships can serve as a propagation mechanism of adverse shocks, especially in times of economic stress.[8]

Intuitively, the dependence structure between shocks $\left\{z_{i,t+1}\right\}_{i=1}^{n}$—captured by Equations (2) and (3)—is consistent with the outcome of a variation of a network reliability or percolation model wherein firm-level shocks propagate from one firm to another in a stochastic manner. In a typical network reliability model, edges are independently removed with some probability. Remaining edges transmit a message. A message from i to j is transmitted as long as there is at least one path from i to j after edge removal. Similarly, in a percolation model, edges are removed at random with some probability. And remaining edges percolate a liquid. The question in percolation is whether or not the liquid

8 An incomplete list of papers includes Hertzel et al. (2008), Boone and Ivanov (2012), Todo, Nakajima, and Matous (2015), Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019), and Carvalho et al. (2020).

percolates from one specific node to another—which is similar to the problem of transmitting a message in a reliability context.[9]

The dependence structure between shocks can also be generated as the outcome of input-output models with roundabout production à la Long and Plosser (1983). Within these models, any given good might be used as an input in the production of any other good. Consequently, shocks affecting firm i can also affect the output of any other firm whose production uses (directly or indirectly) i's output as an input.[10]

2. Aggregate Consumption Growth

Aside from aggregate shocks, two features of the model are important to understanding the distribution of aggregate consumption growth: (a) the architecture of $\mathcal{G}_n$ and (b) how firm-level shocks propagate along $\mathcal{G}_n$, captured by $\mathbf{P}_t$ and its dynamics. In this section, I study how changes in these features affect the distribution of aggregate consumption growth and, thus, reshape the distribution of the pricing kernel.

Let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{C_{t+1}}{C_t}\right)$ and $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ denote log consumption and output growth at $t+1$, respectively. To prevent the model from generating the counterfactual implication that dividends and consumption growth are perfectly correlated, $\Delta\widetilde{c}_{t+1}$ and $\widetilde{x}_{t+1}$ follow

$$\widetilde{x}_{t+1} = \mu_x + \phi\,\Delta\widetilde{c}_{t+1} + \varphi_x \sigma\,\varepsilon^x_{t+1}, \tag{7}$$

where μ_x, ϕ, and φ_x are parameters and $\varepsilon^x_{t+1} \overset{\text{i.i.d}}{\sim} \mathcal{N}(0,1)$ is a shock to aggregate dividend growth that is mutually independent from shocks ε^a_{t+1} and $\left\{\eta_{i,t+1}\right\}_{i=1}^n$. As in Campbell (1986), Cecchetti, Lam, and Mark (1993), and Abel (1999), ϕ represents the leverage ratio on equity. If $\phi=1$, then the market portfolio is approximately a claim to total wealth. For parsimony, consider $\alpha=\phi$ and $\varphi_a^2 = \phi^2 + \varphi_x^2$. It then follows from Equations (1) and (7) that

$$\Delta\widetilde{c}_{t+1} = \mu_c - \underbrace{\frac{1}{n}\left(\sum_{i=1}^{n} f(z_{i,t+1})\right)}_{W_{t+1}} + \sigma\,\varepsilon^c_{t+1}, \tag{8}$$

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[9] For more details about network reliability and percolation models, see Colbourn (1987), Grimmett (1989), Stauffer and Aharony (1994), and Newman (2010, chap. 16.1).

[10] Many of these models build on the Long and Plosser (1983) multisectoral model of business cycles and are commonly used to study how microeconomic shocks translate into aggregate fluctuations. An incomplete list of papers using such a framework includes Horvath (1998, 2000), Dupor (1999), Shea (2002), Conley and Dupor (2003), Carvalho (2010), Foerster, Sarte, and Watson (2011), Acemoglu et al. (2012), Carvalho and Gabaix (2013), Carvalho (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017), Herskovic (2018), and Dew-Becker (2023). A complementary approach to this literature explores how interfirm relationships are determined as the endogenous outcome of a stochastic process by which customers and suppliers are matched (see, e.g., Lim 2017; Oberfield 2018; Acemoglu and Azar 2020). Although equilibrium outcomes are derived using different techniques, the way that shocks propagate across trading partners is similar in both approaches once the matching mapping has been determined.

where parameters satisfy $\bar{y}=\mu_x+\phi\mu_c$. And shocks satisfy $\varphi_a\varepsilon^a_{t+1}=\phi\varepsilon^c_{t+1}+\varphi_x\varepsilon^x_{t+1}$ and $\mathrm{var}_t\left(\varepsilon^c_{t+1}\right)=1$. Intuitively, W_{t+1} captures the influence of cascades of distress on the average firm at $t+1$. It follows from Equation (8) that the distribution of $\Delta\widetilde{c}_{t+1}$ critically depends on W_{t+1}. Given the dynamics of firm-level shocks, W_{t+1} is affected by matrix $\mathbf{P}_t$ and the architecture of $\mathcal{G}_n$. As a result, these two features reshape the distribution of $\Delta\widetilde{c}_{t+1}$.

As an illustration of the importance of $\mathbf{P}_t$ and $\mathcal{G}_n$ in determining the distribution of $\Delta\widetilde{c}_{t+1}$, consider a simplified version of the baseline model in which variables $\left\{f(z_{i,t+1})\right\}_{i=1}^n$ are Bernoulli distributed. And whose states depend on whether firms are affected by negative shocks. Suppose that in each period, every firm can face a negative shock independently of others with probability $0\leq q\leq 1$. A negative shock to firm i also affects firm j, and, thus, $f(z_{i,t+1})=f(z_{j,t+1})=1$ if two things happen: (1) there exists a sequence of relationships between i and j in $\mathcal{G}_n$, and (2) every relationship in that sequence transmits shocks at $t+1$. In each period, every relationship either transmits shocks or does not, independently of all others, with probability $0\leq p\leq 1$.

Within the context of the simplified model, now consider two cases. First, suppose there are no relationships. In this case, $\left\{f(z_{i,t+1})\right\}_{i=1}^n$ is a sequence of i.i.d. Bernoulli random variables and nW_{t+1} follows a Binomial distribution. By the Central Limit Theorem, $\sqrt{n}(W_{t+1}-q)$ is normally distributed as n grows large. Provided the absence of relationships, matrix $\mathbf{P}_t$ is irrelevant to determining the distribution of $\Delta\widetilde{c}_{t+1}$—as the unconditional mean and variance of $\sqrt{n}W_{t+1}$ are q and $\frac{q(1-q)}{n}$, respectively. Second, suppose every firm has two relationships. In this case, $\left\{f(z_{i,t+1})\right\}_{i=1}^n$ is a sequence of dependent Bernoulli random variables and nW_{t+1} approximately follows a Binomial distribution if p is sufficiently small. Here, the precise value of p—which determines matrix $\mathbf{P}_t$—affects the distribution of consumption growth, as the unconditional mean and variance of W_{t+1} are approximately π and $\frac{\pi(1-\pi)}{n}$, respectively, where $\pi\in[0,1]$ solves $\pi=q+(1-q)\pi p(\pi p+2[p(1-\pi)+\pi(1-p)])$.

At a fundamental level, the idea is simple. Although W_{t+1} is the aggregation of shocks to individual firms, there is no guarantee that $\Delta\widetilde{c}_{t+1}$ is normally distributed. This is because, in the presence of relationships, $\left\{f(z_{i,t+1})\right\}_{i=1}^n$ is a sequence of dependent random variables. Figure 1 illustrates this observation numerically within the context of the simplified model. The left panel depicts an economy with five firms along a star network. The right panel depicts the density function of $\sqrt{n}W_{t+1}$ for such architecture. As the right panel shows, the distribution of $\sqrt{n}W_{t+1}$ may differ from a normal distribution if p is sufficiently close to one. In particular, as p tends toward one, the distribution of $\sqrt{n}W_{t+1}$ tends to be bimodal.[11]

[11] Within the context of this simplified model, Section III.A in the Internet Appendix explores the distribution of W_{t+1} for finite n. Simulations show that the distribution of $\Delta\widetilde{c}_{t+1}$ may differ from a normal distribution for a large variety of network architectures. In particular, within the baseline model, if some elements of matrix $\mathbf{P}_t$

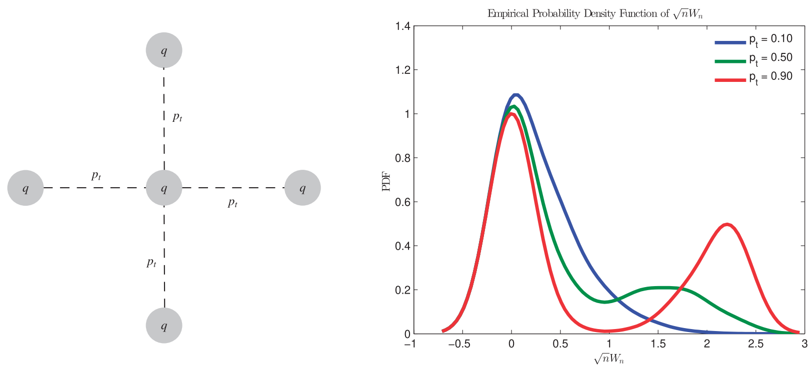

Figure 1
Contagion and distribution of W_{t+1}.
This figure illustrates how changes in p affect the distribution of W_{t+1} within a simplified version of the baseline model. The left panel depicts an economy with five firms connected along a star network. The right panel depicts estimates of the density function of $\sqrt{n}W_{t+1}$ when $p \in \{0.1, 0.5, 0.9\}$.

Despite the existence of relationships—and the convoluted dependencies they may generate between firm-level shocks—the architecture of $\mathcal{G}_n$ and sequence $\{\mathbf{P}_t\}_{t\geq 0}$ can be restricted so that (1) the distribution of W_{t+1} can be approximated by well-known distributions, and (2) $\Delta\widetilde{c}_{t+1}$ is normally distributed as the economy grows large. If $\Delta\widetilde{c}_{t+1}$ is normally distributed, keeping track of temporal changes in the distribution of $\Delta\widetilde{c}_{t+1}$ is equivalent to keeping track of temporal changes in averages and standard deviations. As a result, the dynamics of consumption growth can be recast as a version of Hamilton (1989) Markov-switching model. Section III of the Internet Appendix provides conditions under which W_{t+1} follows a normal distribution as n grows large.

3. Asset Pricing

To appreciate how the propagation of shocks along the network can reshape asset returns, this section embeds the output correlation structure, jointly determined by $\mathcal{G}_n$ and the sequence of matrices $\{\mathbf{P}_t\}_{t\geq 0}$, into a standard asset pricing framework. For ease of exposition, I now show the mechanisms working at equilibrium via approximate analytical solutions. See the appendix for the derivation of these solutions.

To account for asset pricing phenomena that are challenging to address with power utility preferences, the representative investor exhibits Epstein-Zin-Weil

are sufficiently close to one and $\mathcal{G}_n$ is locally connected—that is, there is at least one sequence of relationships between any two firms in an arbitrarily large neighborhood around any given firm—then a nonnegligible fraction of firms in the economy are almost surely affected by negative shocks. Therefore, the distribution of $\Delta\widetilde{c}_{t+1}$ may exhibit thicker tails than a normal distribution would.

preferences and, hence,

$$U_t = \left[(1-\beta)\mathcal{C}_t^{1-\frac{1}{\psi}} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\rho}} \tag{9}$$

represents her utility at period t. Parameter $\psi > 0$, $\psi \neq 1$, represents the intertemporal elasticity of substitution (IES), $\gamma > 0$ is the coefficient of relative-risk aversion for static gambles, and $\beta > 0$ measures the subjective discount factor under certainty. Consequently, the logarithm of the pricing kernel is given by

$$m_{t+1} \equiv \theta \log(\beta) - \frac{\theta}{\psi} \Delta \widetilde{c}_{t+1} + (\theta - 1) r_{a,t+1}, \tag{10}$$

where $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $r_{a,t+1}$ denotes the log gross return on aggregate wealth at $t+1$.[12] The stock return of firm i can be determined using the pricing kernel and the representative investor's first-order condition

$$\mathbb{E}_t \left[\exp\left(m_{t+1} + r_{i,t+1} \right) \right] = 1, \tag{11}$$

where $r_{i,t+1}$ denotes the log gross return of firm i at $t+1$.

3.1 Aggregate asset pricing implications

As the economy grows large, W_{t+1} approximately follows

$$W_{t+1} \approx \mu_w + \rho_w W_t + \sigma_{w,t} \varepsilon_{t+1}^W, \tag{12}$$

where $\sigma_{w,t} \varepsilon_{t+1}^W = (W_{t+1} - \mathbb{E}_t [W_{t+1}])$ is an innovation to the susceptibility of the average firm to cascades of distress—which is normally distributed and mutually independent of ε_{t+1}^a and ε_{t+1}^x. Given how shocks propagate along $\mathcal{G}_n$, ε_{t+1}^W can be thought of as shocks to network connectivity. Because propensities and, thus, matrices $\{\mathbf{P}_t\}_{t \geq 0}$ are time-varying, variable $\sigma_{w,t}$ varies over time. As a result, innovations to the pricing kernel can be approximated by

$$m_{t+1} - \mathbb{E}_t(m_{t+1}) \approx \chi_a \varepsilon_{t+1}^a + \chi_x \varepsilon_{t+1}^x + \chi_{W_t} \varepsilon_{t+1}^W, \tag{13}$$

where χ_a, χ_x, and χ_{W_t} capture the exposure of the pricing kernel to aggregate productivity shocks, aggregate dividends shocks, and shocks to network connectivity, respectively.

Intuitively, a positive shock to network connectivity increases the likelihood of contagion, decreasing aggregate output and consumption, and, in doing so,

12 I use standard terminology to describe γ and ψ. If $\gamma = \frac{1}{\psi}$, these recursive preferences collapse to the standard case of Von Neumann-Morgenstern time-additive expected utility. The functional form of the pricing kernel when $\psi = 1$ is different from the one shown above (for more details, see Weil 1989, appendix A).

increasing marginal utility. Consequently, contagion risk is being priced and innovations in network connectivity carry a negative price of risk.

Because $\sigma_{w,t}$ varies over time, variable χ_{W_t} is time-varying as well. Thus, the conditional volatility of the pricing kernel is not constant. Because innovations ε^a_{t+1}, ε^x_{t+1}, and ε^W_{t+1} are mutually independent of each other,

$$\mathrm{var}_t(m_{t+1}) \approx \chi_a^2+\chi_x^2+\chi_{W_t}^2. \tag{14}$$

Therefore, time variation in the volatility of the pricing kernel is solely driven by temporal changes in network connectivity.

3.2 Cross-sectional asset pricing implications

To better understand how contagion risk is priced in the cross-section, I now study firms' betas with respect to contagion risk. The first-order condition of the representative investor can be rewritten as a beta pricing model,

$$\mathbb{E}_t\left(e^{r_{i,t+1}}-e^{r_{f,t}}\right) = \underbrace{\left(\frac{\mathrm{cov}_t(e^{r_{i,t+1}},e^{m_{t+1}})}{\mathrm{var}_t(e^{m_{t+1}})}\right)}_{\beta_{i,t}}\underbrace{\left(\frac{-\mathrm{var}_t(e^{m_{t+1}})}{\mathbb{E}_t(e^{m_{t+1}})}\right)}_{\lambda_{\widetilde{M}_t}}, \tag{15}$$

where $r_{f,t}$ denotes the logarithm of the risk-free rate at t, $\beta_{i,t}$ denotes firm i's quantity of risk at t, and $\lambda_{\widetilde{M}_t}$ denotes the conditional price of risk at t. Notably, within the model,

$$\beta_{i,t} \propto \mathrm{cov}_t\left(m_{t+1},r_{i,t+1}\right) \approx \lambda_a+\underbrace{\lambda^{NC}_{i,t}}_{\left(\lambda_{W_t}+\varrho_{i,t}+\kappa_{i,t}\right)}. \tag{16}$$

That is, a firm's quantity of risk is jointly determined by (a) innovations in aggregate productivity, captured by λ_a, and (b) innovations in network connectivity, captured by $\lambda^{NC}_{i,t}$.

Importantly, $\lambda^{NC}_{i,t}$ encompasses three sources of variation on firms' betas. The first term, λ_{W_t}, captures the exposure to innovations in the susceptibility of the average firm to cascades of distress, W_{t+1}—and it is equal across firms. Because firms exhibit a significant decrease in cash flow growth when shocks hit, consumption growth is altered by W_{t+1}. Consequently, when network connectivity increases, firms become more vulnerable to shocks affecting others. As a result, firms command higher average returns, as contagion risk commands a negative price of risk and W_{t+1} is negatively correlated with consumption growth.

The second and third terms within $\lambda^{NC}_{i,t}$, $\varrho_{i,t}$ and $\kappa_{i,t}$, are the sole sources of heterogeneity in quantities of risk in the cross-section. First, $\varrho_{i,t}$ captures the exposure to innovations in firm i's susceptibility to shocks affecting its (direct and indirect) neighbors. From a shock propagation perspective, this term captures how isolated firm i is from its neighbors. When firm i is less susceptible to shocks affecting its neighbors, i serves investors as a hedge against shocks to neighboring firms and, thus, commands lower average returns than its neighbors. Second, $\kappa_{i,t}$ captures the equilibrium price effects associated with contagion and relates to the comovement between two different sets of firms: i's neighboring and nonneighboring firms. When i's neighboring firms are less susceptible to shocks affecting other firms, they serve investors as a hedge against shocks affecting i's nonneighboring firms. Because equilibrium prices react to this comovement, the quantity of risk of i and its neighbors decreases as long as this hedging effect dominates.

In sum, the quantity of risk associated with innovations to network connectivity is jointly shaped by three elements within the model: (a) innovations in W_{t+1}, (b) the comovement between i and its neighbors (within-group covariation), and (c) the comovement between i's neighboring and nonneighboring firms (across-group covariation).

4. Calibration

So far, the model illustrates how changes in the propagation of shocks along the network can reshape equilibrium returns. I now calibrate the model to match several key structural features of supplier–customer networks in the United States and explore the extent to which such model quantitatively explains asset market phenomena. Following Campbell and Cochrane (1999) and Bansal and Yaron (2004), among many others, I assume that the decision interval of the representative investor is monthly but the target data to match are annual. Throughout the calibration exercise I assume $f(z)=z^2$. Section 4.1 describes the data. Section 4.2 describes how I calibrate $\mathcal{G}_n$ and the sequence of matrices $\{P_t\}_{t\geq 0}$. Section 4.3 describes the selection of the rest of parameters.

4.1 Data

4.1.1 Material relationships between U.S. public firms I use annual data on relationships between U.S. public firms and their major customers to identify material relationships. The Statement of Financial Accounting Standards (SFAS) No. 131 requires public firms to report information about customers that represent more than 10% of their annual revenues or sales; firms sometimes report customers below the 10% threshold. Reported customers' information is available on the COMPUSTAT Segment files. However, sometimes customers' names are abbreviated inconsistently over time. For these cases, I use a string-matching algorithm, similar to the one used by Atalay et al. (2011), which generates a list of potential customers in COMPUSTAT.[13] I then select the best match by inspecting a firm's name and industry information.

The data set spans from 1980 to 2019 and consists of 8,238 public firms. Similar to Barrot and Sauvagnat (2016), I consider firms i and j to be connected

[13] I thank Enghin Atalay for providing his code.

Table 1
Firm statistics: Industry groups and most connected firms

A. Industry groups

Industry	Firms
Construction	78
Finance, insurance, and real estate	525
Manufacturing	4,082
Mining	588
Retail	304
Service	1,478
Transportation, communications, electric, gas, and sanitary	811
Wholesale	271
Nonclassifiable establishments	101
Total	8,238

B. Most connected firms

1980 to 1999

1980–1984		1985–1989		1990–1994		1995–1999	
Name	*N*	*Name*	*N*	*Name*	*N*	*Name*	*N*
GM	430	GM	503	Walmart	468	Walmart	702
Sears	254	AT&T	427	AT&T	456	GM	435
Ford	238	IBM	345	GM	434	IBM	385
IBM	205	Ford	292	IBM	376	Ford	376
AT&T	146	Sears	198	Ford	367	AT&T	367

2000 to 2019

2000–2004		2005–2009		2010–2014		2015–2019	
Name	*N*	*Name*	*N*	*Name*	*N*	*Name*	*N*
Walmart	686	Walmart	684	Walmart	599	Walmart	511
GM	352	Cardinal	205	Cardinal	204	McKesson	182
Ford	281	GM	190	McKesson	182	AB	173
IBM	182	McKesson	188	AB	173	Cardinal	166
Motorola	161	Ford	168	AT&T	165	Ford	136

This table reports firm statistics for the baseline sample. The sample contains 8,238 public firms. Suppliers are spread across 67 (SIC2) industries, whereas customers are spread across 65 (SIC2) industries. Panel A reports the distribution of firms across major industry groups. Panel B reports the total number of annual relationships—considering both customers and suppliers—of the five most connected firms in the following 5-year intervals: 1980–1984, 1985–1989, 1990–1994, 1995–1999, 2000–2004, 2005–2009, 2010–2014, and 2015–2019. GM, General Motors; Cardinal, Cardinal Health; AB, AmerisourceBergen.

in all years ranging from the first to the last year that i reports j as one of its major customers. This assumption yields 57,464 unique annual supplier–customer relationships. Table 1 reports statistics for firms in the sample. Panel A reports the distribution of firms across major industry groups. About 67% of companies are classified as either manufacturing or service firms. Panel B reports the evolution of the set of most connected firms. Large manufacturers and department stores, such as General Motors, Ford, and Sears, dominated the early eighties. By the end of the sample, the shift in activity from manufacturing to retail and healthcare services is widespread, with Walmart, McKesson, AmerisourceBergen, and Cardinal Health being the most connected firms. Although the size distribution of firms' customers is tilted toward large companies, the distribution of firms' sizes still resembles the size distribution of the CRSP universe.

4.1.2 Propensities Pivotal for my analysis is identifying the propensity of relationships to transmit firm-level shocks. Unfortunately, propensities of supplier–customer relationships are unobservable. To deal with this challenge, I rely on a composite of two measures.[14] The first measure is the percentage of annual sales that customers represent for their suppliers. The higher this percentage, the more likely it is that the relationship is important for both the supplier and the customer. The second measure uses information about the specificity of suppliers, as evidence documented by Barrot and Sauvagnat (2016) suggests that input specificity is a key driver in the propagation of idiosyncratic shocks along production networks. Their idea is simple: the more specific supplier i's output, the higher the likelihood that adverse shocks affecting i also affect its customers.

With these measures in hand, I proxy for p_{ijt} as

$$\begin{aligned} p_{ijt} &= \bar{p} + (\% \text{ company } i\text{'s sales accounted for by } j \text{ at } t) \\ &\quad \times (\text{ specificity of } i \text{ at } t), \end{aligned} \tag{17}$$

where $\bar{p} = 0.18$ represents the average percentage of total sales customers represent from their suppliers in the sample. Having $\bar{p}$ in Equation (17) ensures contagion risk plays a significant role in the calibrated economy without changing the model's cross-sectional or time-series properties.[15]

Percentages of annual sales are obtained from COMPUSTAT. To measure the specificity of suppliers, I construct a composite of three measures of input specificity from the replication files of Barrot and Sauvagnat (2016) and Kogan et al. (2017) as well as COMPUSTAT. Following Barrot and Sauvagnat (2016), I assume that firms are more likely to produce specific goods if they (a) operate in industries producing differentiated goods, (b) have high levels of R&D, or (c) hold a large number of patents. In particular, I define three indicator variables for each supplier-year: $\mathbb{1}^i_{s,t}$ equals one if supplier i is considered to produce specific outputs at t along dimension s, and zero otherwise—where $s \in \mathcal{S} \equiv$

[14] Using the model to uncover propensities from market data is possible only under restrictive assumptions regarding the model parameters. Thus, I resort to the empirical literature for guidance on how to build reasonable proxies. Although the baseline model allows me to capture a fairly general family of dependencies between firms' cash flows, such flexibility comes at a cost. Because the shock propagation mechanism is combinatorial in nature, uncovering propensities from market data tends to be intractable. In particular, as the economy grows large, it becomes difficult to pin down propensities from asset returns (or a function of them) unless one imposes (unrealistic) restrictions on the network architecture or the value of certain parameters.

[15] Instead of trying to impose a specific functional form on $p_{ij,t}$, this parameterization intends to capture a basic idea in the simplest way possible. The higher the percentage of company i's sales accounted for by j or the higher the specificity of i, the higher the likelihood that shocks affecting i also affect j. As both of these dimensions can change over time, the propensity of relationship (i, j) to transmit shocks can also vary over time.

Table 2
Relationship statistics

A. Annual level

	Obs	Mean	25th	Median	75th	Min	Max
% sales	57,464	18.53	9.29	13.63	22.00	0.19	100
Specificity	57,464	29.70	0.00	33.33	33.33	0.00	100
Rauch	55,669	55.93	0.00	100	100	0.00	100
R&D	57,464	5.23	0.00	0.00	4.44	0.00	99.86
Patent	57,464	20.98	0.00	0.00	2.00	0.00	8,629
p_{ijt}	57,464	23.88	18.06	21.56	26.17	18.06	100
Duration	57,464	9.32	4.00	7.00	13.00	1.00	40

B. Relationship level

	Obs	Mean	25th	Median	75th	Min	Max
|AC1| % sales	6,771	32.36	12.50	28.94	50.00	0.00	94.07
|AC2| % sales	6,771	26.54	11.42	24.62	41.75	0.00	89.36
|AC1| specificity	1,985	29.50	16.70	25.00	44.30	0.00	88.30
|AC2| specificity	1,985	23.70	10.00	21.70	33.30	0.00	83.30
|AC1| p_{ijt}	4,327	30.64	12.24	26.32	48.07	0.00	92.92
|AC2| p_{ijt}	4,327	26.29	11.74	25.11	40.47	0.00	85.68

This table reports statistics of supplier-customer relationships in the baseline sample. The sample contains 13,863 relationships between different pairs of firms from 1980 to 2019, representing 57,464 different relationship-year observations. Panel A reports summary statistics at the annual level for (a) the percentage of sales that customers represent for their suppliers, (b) the specificity of suppliers, (c) suppliers' Rauch (1999) score, (d) suppliers' R&D score, (e) suppliers' patent score (which represents the number of patents issued by suppliers in the past 3 years), (f) p_{ijt}, and (g) the duration of relationships in years. Panel B reports summary statistics at the relationship level for relationships that last at least 3 years. |AC1| and |AC2| report the absolute value of the first and second autocorrelation coefficients for (a) the percentage of sales that customers represent for their suppliers, (b) the specificity of suppliers, and (c) the propensity of relationships. Columns Obs denote the number of nonmissing observations used to compute summary statistics. Summary statistics are in percent with the exception of the number of patents and duration in panel A.

$\{\text{Differentiated Goods}, \text{R\&D}, \text{Patents}\}$. Supplier i's specificity at t is defined as the average between variables $\left\{\mathbb{1}^{i}_{s,t}\right\}_{s\in\mathcal{S}}$.[16]

4.1.3 Firm-level financial data Monthly returns, prices, and shares outstanding are obtained from CRSP and the CRSP COMPUSTAT Merged Database. All continuous variables are winsorized at the 1st and 99th percentiles of their distributions.

4.1.4 Summary statistics The sample contains 13,863 relationships between different pairs of firms. Table 2 reports summary statistics. Panel A presents

[16] I use Rauch scores to determine industries producing differentiated goods. Rauch (1999) classifies inputs into differentiated or homogeneous depending on whether goods are traded on an organized exchange. Each industry is coded as being sold on an exchange, reference priced, or homogeneous. Rauch scores capture the share of differentiated goods produced in a given industry. If a supplier belongs to an industry with higher scores, it is more likely such a supplier produces specific outputs. R&D scores capture the ratio of R&D expenses over sales at the firm-year level. This ratio aims to capture the importance of relationship specific investments. In the same spirit as Barrot and Sauvagnat (2016), firm i's R&D score at t is determined by i's score at $t-2$. This is because the benefits of R&D might take some time to materialize, and, thus, current R&D expenditure might not be necessarily informative about the current specificity of a firm. Following Barrot and Sauvagnat (2016), I determine supplier i's patent score at t by computing the cumulative number of patents firm i issued from $t-3$ to $t-1$. This number aims to capture restrictions on alternative sources of inputs. For each dimension, a supplier is considered to produce specific outputs at t if its score is greater than or equal to the average score of suppliers in year t.

Table 3
Structural characteristics of supplier–customer networks

Characteristic	Mean	S.D.	Benchmark
Number of firms	1,326.77	309.28	1,340
Number of relationships	1,436.60	352.90	1,470
Average number of suppliers per firm	1.07	0.04	1.10
Average number of suppliers and customers per firm	2.15	0.09	2.20
Number of connected components	153.37	29.31	153

This table reports structural characteristics of supplier–customer networks generated at an annual frequency from the baseline sample as well as characteristics of the benchmark economy. Firms i and j are connected in the network of year t if (1) firm i reports j as a major customer, and (2) information to construct proxies for $p_{ij,t}$ is available. The number of connected components per network is computed via a depth-first search algorithm as in Tarjan (1972). Column Benchmark reports structural characteristics of the network in the benchmark economy (depicted in Figure 2). The benchmark economy considers relationships that last 10 years or more.

statistics at the annual level. The average and median percentages of sales that customers represent for their suppliers are 18.53% and 13.63%, respectively, whereas the average and median for suppliers' specificity scores are 29.70% and 33.33%. The main variable of interest is the propensity of relationships to transmit adverse shocks. The average and median for this variable are 23.88% and 21.56%, respectively. On average, there are 9.3 years between the first and the last year a firm reports another firm as a major customer. That is, material relationships are long-lasting in the sample.

To examine the persistence of the above variables, panel B presents statistics regarding autocorrelation coefficients computed at the relationship level. The average magnitude of the first and second autocorrelation coefficients for the percentage of sales that customers represent for their suppliers are 32.36% and 26.54%, respectively, and their medians are 28.94% and 24.62%. The average magnitude of the first and second autocorrelation coefficients for suppliers' specificity scores are 29.50% and 23.70%, respectively, with medians of 25% and 21.70%. Propensities are also fairly persistent as the average magnitude of the first and second autocorrelation coefficients are 30.64% and 26.49%, respectively, with medians of 26.32% and 25.11%.

4.2 Uncovering $\mathcal{G}_n$ and propensity matrices $\{P_t\}_{t\geq 0}$

4.2.1 Uncovering $\mathcal{G}_n$. To calibrate $\mathcal{G}_n$, I first construct firm networks at an annual frequency over the sample; nodes represent firms and links represent supplier–customer relationships. Table 3 reports averages and standard deviations for key structural characteristics of U.S. supplier–customer networks. In an average year, there are 1,436 relationships between 1,326 firms. And the average supplier–customer network exhibits 153 different connected components.[17]

[17] Figures 1, 2, and 3 in Section X of the Internet Appendix help illustrate the fact that U.S. production networks are highly asymmetric, meaning that the majority of firms have either one or, at most, two connections, while only a few firms are connected to many others. As a result, their degree distributions—which measure the frequency of firms with a given number of customers and suppliers—are highly skewed to the right.

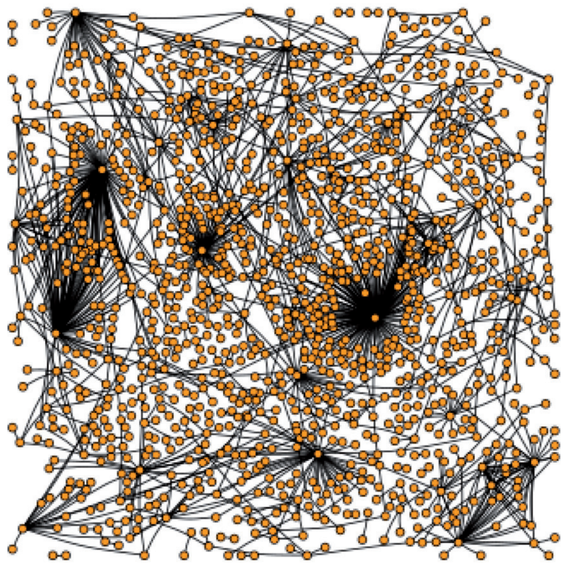

Figure 2
Benchmark $\mathcal{G}_n$
This figure illustrates the network of the benchmark economy.

Although production networks vary over time, my model assumes that the architecture of $\mathcal{G}_n$ is fixed. To help reconcile my model with the data, I use the duration skeleton of the above time series of networks as a proxy for $\mathcal{G}_n$. Instead of relying on the architecture of any given year, the duration skeleton aims to extract essential structural features from the time series of annual networks that are typically concealed by their size and dynamic nature. Here, the duration skeleton consists of all pairs of firms whose relationships last, at least, 10 years—which is about the average duration of relationships in the sample.[18]

Figure 2 depicts the network architecture of the benchmark economy, which contains 1,470 relationships among 1,340 firms and 153 connected components. Notably, as Table 3 shows, this architecture is largely consistent with several structural moments of the time series of U.S. production networks.

18 Intuitively, the duration skeleton captures a simple idea: the more lasting the relationship, the higher the likelihood such a relationship is part of the essential network architecture through which shocks are potentially propagated. Consequently, the duration skeleton better reflects the importance of long-term relationships than other methods often used in the literature, such as community clusters or backbones. At the basic level, the concept behind the duration skeleton is similar to that of the high-salience skeletons. To reduce large-scale networks to their core components, Grady, Thiemann, and Brockmann (2012) use links with high salience to define the skeleton network. Because the salience of a network is computed as a linear superposition of all shortest-path trees, links with high salience are basically those that help connect most of the nodes in the network and, thus, better capture the set of links along which information could potentially flow within a large and complex network. Although the analogy is not perfect here, the notion of the duration skeleton aims to capture a similar idea.

4.2.2 Uncovering $\{P_t\}_{t\geq 0}$ To appreciate the relevance of heterogeneity in propensities across relationships, I calibrate the model under two distinct environments. Model I helps keep parsimony by assuming that all propensities are equal across relationships and determined by a single stochastic process,

$$p_{t+1} = \mu_p + \theta_p p_t + \sigma_p \epsilon_{t+1}, \tag{18}$$

where $\epsilon_{t+1} \overset{\text{i.i.d}}{\sim} \mathcal{N}(0,1)$. I set $\mu_p = 0.0345$, $\theta_p = 0.8550$, and $\sigma_p = 0.0021$ so their values match the dynamics of the average relationship in the sample.

Because differences in institutional features across economic sectors are likely to generate differences in propensities in the cross-section, Model II allows propensities to vary across relationships. In particular, Model II assumes that the propensity of relationship (i, j) follows

$$p_{ij,t+1} = \mu_{ij} + \theta_{ij} p_{ij,t} + \sigma_{ij} \epsilon^{ij}_{t+1}, \tag{19}$$

where innovations $\epsilon^{ij}_{t+1} \overset{\text{i.i.d}}{\sim} \mathcal{N}(0,1)$ and are mutually independent across relationships. Parameters μ_{ij}, θ_{ij}, and σ_{ij} are set equal to their estimates from running a regression akin to Equation (19) with data at the relationship (i, j)-level. Although introducing this type of heterogeneity comes at the cost of losing parsimony—as there is one set of parameters $(\mu_{ij}, \theta_{ij}, \sigma_{ij})$ per relationship (i, j)—Model II continues to exhibit a Markovian structure as in Model I, which allows me to solve for firms' stock prices and returns in equilibrium.

4.3 Selecting the rest of parameters

Because the first two moments of W_{t+1} depend on how shocks propagate along $\mathcal{G}_n$—which, in turn, depends on the dynamics of propensities—the precise value of certain parameters can differ between Models I and II. The choice of the rest of parameters is designed to match observed annual data on consumption and dividend growth and, at the same time, allow both models to produce realistic asset pricing features.

For ease of exposition, these parameters can be separated into four groups. Parameters in the first group define the preferences of the representative investor. I set $\beta = 0.9985$ and $\gamma = 10$ in Model I and $\beta = 0.9975$ and $\gamma = 2.5$ in Model II. And I set $\psi = 1.5$ in both models. Parameters in the second group define the process for aggregate consumption growth. To generate an unconditional mean and volatility of consumption growth similar to the ones found in the data, I set $\mu_c = 0.0041$ in Model I and $\mu_c = 0.0020$ in Model II. And I set $\sigma = 0.0032$ in both models. Parameters in the third group define the process for dividends and its correlation with consumption. To generate a realistic unconditional mean and volatility of dividend growth as well as its correlation with consumption growth, I set $\bar{y} = 0.0102$, $\phi = 3$, and $\varphi_a = 14.279$ in Model I. And $\bar{y} = 0.0043$, $\phi = 4$, and $\varphi_a = 14.2$ in Model II. Finally, to allow both models to generate realistic equity premiums, I set $\vartheta = 0.85$ and $\varphi_\eta = 4.133$ in Model I. And $\vartheta = 0.72$ and $\varphi_\eta = 2.057$ in Model II. Table 4 summarizes the key parameter values in both models.

Table 4
Benchmark parameterizations

Parameter	Model I	Model II
Preferences:		
β	0.9985	0.9975
γ	10	2.5
ψ	1.5	1.5
Consumption:		
μ_C	0.0041	0.0020
σ	0.0032	0.0032
Dividends:		
$\bar{y}$	0.0102	0.0043
ϕ	3	4
φ_a	14.279	14.200
Shock propagation:		
ϑ	0.850	0.720
φ_η	4.133	2.057

This table reports the list of parameter values in the benchmark parameterizations of Models I and II. The first group of parameters defines the preferences of the representative investor: β represents the time discount factor, γ represents the coefficient of relative-risk aversion for static gambles, and ψ represents the intertemporal elasticity of substitution. The second group defines log aggregate consumption growth in Equation (8). The third group defines aggregate output and its correlation with consumption growth in Equation (7). The fourth group defines the dynamics of distress-like events in Equation (2).

5. Implications of Calibrated Models

This section quantitatively evaluates the ability of calibrated models to rationalize features of stock returns. It shows that changes in the propagation of shocks along networks that replicate key structural characteristics of U.S. supply chains are important to understanding variations in stock returns in both the aggregate and the cross-section. Section 5.1 shows that both models generate long-run consumption risks. Section 5.2 shows that both models also generate realistic cross-sectional patterns of portfolio returns sorted by firm-level centrality. Section 5.3 shows that parameter ϑ plays a smaller role in models wherein propensities vary across relationships as in the data. Section 5.4 emphasizes the relevance of downstream propagation (where shocks travel from suppliers to customers) for the cross-sectional results.

5.1 Aggregate implications

Table 5 reports the aggregate moments generated under benchmark parameterizations. Models I and II generate moments largely consistent with those found in the data. In particular, both models deliver realistic averages and volatilities of consumption and dividend growth. Although both models generate a somewhat small first autocorrelation coefficient of consumption growth, they both deliver a small and stable risk-free rate and realistic values for the average market return, its volatility, and the average equity risk premium.

In addition to generating realistic values for the above moments, both models generate a persistent component in expected consumption growth and stochastic consumption volatility that are largely consistent with those assumed by the long-run risks (LRR) model of Bansal and Yaron (2004). As Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) show, these two features,

Table 5
Aggregate moments under benchmark parameterizations

	Data	Model I						Model II					
Variable	Estimate	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
Avg. log consumption	1.97	1.97	1.57	1.78	1.98	2.17	2.33	1.97	1.60	1.81	2.00	2.16	2.28
Vol. log consumption	1.17	1.45	1.19	1.29	1.42	1.57	1.73	1.35	1.04	1.12	1.25	1.42	1.72
AC(1) log consumption	0.56	0.15	−0.05	0.04	0.15	0.25	0.34	0.07	−0.14	−0.05	0.05	0.20	0.30
Avg. log dividend	3.29	3.29	0.05	1.48	3.32	5.02	6.63	3.29	−0.20	1.41	3.33	5.10	6.67
Vol. log dividend	16.19	16.33	14.01	15.08	16.23	17.50	18.72	16.42	14.06	15.15	16.38	17.56	18.93
Avg. market return	10.00	14.11	10.19	11.96	14.07	16.00	17.89	9.18	5.17	7.09	9.23	11.19	12.93
Vol. market return	16.93	22.98	18.91	20.55	22.45	25.03	27.53	19.93	15.18	16.55	18.62	21.58	26.64
Avg. risk-free rate	1.47	2.95	2.71	2.84	2.96	3.07	3.15	1.73	1.50	1.68	1.78	1.85	1.89
Vol. risk-free rate	2.24	0.65	0.44	0.52	0.62	0.75	0.90	0.44	0.08	0.15	0.28	0.54	0.96
Avg. equity risk premium	8.53	11.15	7.21	9.05	11.16	13.04	14.91	7.39	3.40	5.28	7.45	9.43	11.18

This table reports key moments for consumption growth, dividend growth, and asset markets. It also reports moments from simulated data generated with Models I and II. To facilitate comparison between both models, which are calibrated to a monthly decision interval, and the data, reported values are time-aggregated to an annual frequency. Data on consumption are real nondurables and services from the U.S. Bureau of Economic Analysis (BEA), and dividends are from the CRSP (NYSE/AMEX/NASDAQ) value-weighted portfolio; all nominal quantities are deflated using the PCE. Market returns are approximated by the difference between the CRSP value-weighted portfolio and PCE inflation; returns on the risk-free asset are approximated by the difference between the yield on 3-month Treasury bills and PCE inflation. The statistics for the data are based on annual observations from 1980 to 2019 and reported in column Data. The statistics of both models are based on 1,000 Monte Carlo simulations, each with 540 monthly observations. Because the first 60 observations in each simulation are disregarded to eliminate bias from the initial condition, each simulation is comparable to the data with their 40 years. Column Mean reports the average statistics across simulations. Columns 10th, 25th, 50th, 75th, and 90th report the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the distribution of statistics across simulations. All values are in percentage points except for AC(1) coefficients.

together with Epstein-Zin-Weil preferences, help quantitatively explain a variety of important asset market phenomena.[19] Table 6 reports summary statistics of several similarity measures of time series generated with Model I (or II) and the LRR model. To compute averages and standard deviations, I sample from both calibrated models and the LRR model to construct two distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t[\Delta\widetilde{c}_{t+1}]$, and one for the conditional volatility of consumption growth, $\mathrm{Vol}_t[\Delta\widetilde{c}_{t+1}]$. As Table 6 suggests, both models generate similar time series for conditional expected consumption growth and conditional consumption volatility. Taken together, these results are consistent with the idea that shock propagation along production networks represents an important source of long-run risks.

Calibrated models generate a persistent component in expected consumption growth and stochastic consumption volatility for two reasons. First, the architecture of $\mathcal{G}_n$ is fixed, and, thus, relationships are long-lasting. Second, network connectivity is fairly stable over time. Consequently, the connectivity of the production network—and, thus, the shock propagation mechanism—changes infrequently in both benchmark economies. These infrequent changes generate low-frequency movements in firms' growth prospects, which, in turn, generate a persistent component in aggregate output and expected consumption growth.

Changes in the shock propagation mechanism are infrequent because firms engage in enduring and stable relationships with their major customers. For instance, in the data, these relationships last more than 9 years on average. In doing so, such relationships generate long-term interdependencies among firms' cash flow growth rates, driving low-frequency movements in aggregate output growth. In turn, these low-frequency movements generate persistent changes in aggregate consumption growth in equilibrium. In such an economy, an adverse shock to an individual firm can affect not only the current cash flows of all its neighboring firms but also the long-term growth prospects of all such firms, thereby enhancing the temporal effect of adverse shocks to individual firms.

5.1.1 Empirical support for the aggregate model predictions Because my model emphasizes the long-run implications of low-frequency changes in the propagation of shocks along production networks, it is important to provide support for its aggregate predictions.

[19] Since Bansal and Yaron (2004), several authors have used the long-run risk framework to explain an array of market phenomena. For instance, Kiku (2006) provides an explanation of the value premium within the long-run risks framework. Drechsler and Yaron (2011) show that a calibrated long-run risks model generates a variance premium with time variation and return predictability that is consistent with the data. Bansal and Shaliastovich (2013) develop a long-run risks model that accounts for bond return predictability and violations of uncovered interest parity in currency markets.

Table 6
Similarities between Models I and II and the long-run risk model

A. Model I

Similarity	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$						$\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$					
Measure	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
ED	0.96	0.95	0.95	0.96	0.96	0.97	0.85	0.84	0.84	0.85	0.85	0.86
DTW	0.72	0.64	0.69	0.73	0.77	0.79	0.13	0.12	0.12	0.13	0.14	0.14
PACF	0.56	0.52	0.53	0.55	0.57	0.61	0.57	0.53	0.55	0.57	0.59	0.62
ED in AR	0.59	0.52	0.55	0.58	0.62	0.68	0.54	0.45	0.49	0.53	0.58	0.62
LP in ARIMA	0.60	0.55	0.56	0.58	0.62	0.67	0.47	0.41	0.42	0.43	0.46	0.63

B. Model II

Similarity	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$						$\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$					
Measure	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
ED	0.96	0.95	0.95	0.96	0.96	0.97	0.85	0.84	0.84	0.85	0.85	0.86
DTW	0.67	0.58	0.63	0.68	0.72	0.76	0.13	0.12	0.12	0.13	0.14	0.14
PACF	0.55	0.48	0.50	0.54	0.59	0.65	0.57	0.49	0.52	0.56	0.63	0.67
ED in AR	0.52	0.36	0.41	0.49	0.61	0.73	0.52	0.33	0.39	0.50	0.64	0.74
LP in ARIMA	0.50	0.38	0.40	0.44	0.60	0.72	0.47	0.38	0.40	0.42	0.47	0.71

This table provides evidence that Models I and II generate processes for expected consumption growth and conditional consumption growth volatility largely consistent with those generated by the long-run risk (LRR) model of Bansal and Yaron (2004). This table reports statistics (Mean, 10th, 25th, 50th, 75th, and 90th percentiles) of the distribution of similarity measures between time series generated with either (1) one calibrated model or (2) the benchmark parameterization of the LRR model (see Bansal and Yaron 2004, table IV). To compute statistics, I sample from both models to construct two distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for conditional consumption volatility, $\mathrm{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 1,000 simulations, each with 480 observations. All similarity measures report scores computed as $\frac{1}{1+distance}$, where *distance* is defined according to each similarity measure. Similarity measures are defined as follows. Let $\mathbf{X}_T=(X_1,\cdots,X_T)$ and $\mathbf{Y}_T=(Y_1,\cdots,Y_T)$ denote realizations from two time series, $X=\{X_t\}$ and $Y=\{Y_t\}$. The first and second similarity measures focus on the proximity between X and Y at specific points of time. The Euclidean distance (ED) is defined as $\sqrt{\sum_{t=1}^{T}(X_t-Y_t)^2}$, whereas the dynamic time warping (DTW) distance is defined as $\min_r\left(\sum_{i=1}^{m}\left|X_{a_i}-Y_{b_i}\right|\right)$, where $r=\left((X_{a_1},Y_{b_1}),\cdots,(X_{a_m},Y_{b_m})\right)$ is a sequence of m pairs that preserves the order of observations. By finding a mapping such that the distance between X and Y is minimized, DTW allows two time series that are similar but locally out of phase to align in a nonlinear manner. The third measure focuses on correlation-based distances. It uses the partial autocorrelation function (PACF) to define the distance between X and Y; here, distance is defined as $\sqrt{(\hat{\rho}_{X_t}-\hat{\rho}_{Y_t})'\Omega(\hat{\rho}_{X_t}-\hat{\rho}_{Y_t})}$, where Ω is a matrix of weights, whereas $\hat{\rho}_{X_t}$ and $\hat{\rho}_{Y_t}$ are the estimated partial autocorrelations of X and Y, respectively. The fourth and fifth measures assume that a specific model generates both X and Y. The idea is to fit such a model to each time series and then measure the dissimilarity between fitted models. The fourth measure computes the distance between X and Y as the ED between their truncated autoregressive (AR) operators. The fifth measure computes dissimilarity between X and Y in terms of their linear predictive (LP) coding in ARIMA processes as in Kalpakis, Gada, and Puttagunta (2001). Panel A reports values from simulating Model I, and panel B reports values from simulating Model II. Table 4 reports the configuration of parameters within both models.

Table 7
Predictability of consumption growth and consumption growth volatility

A. Consumption growth

	A.1: Actual data			A.2: Simulated data ∣ Model I						A.3: Simulated data ∣ Model II					
Variable	Coef.	S.E.	R^2	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
$B(1)$	−0.17	0.07	0.08	−0.06	−0.29	−0.18	−0.06	0.05	0.17	−0.11	−0.25	−0.17	−0.10	−0.04	0.00
$B(3)$	−0.53	0.24	0.14	−0.15	−0.82	−0.48	−0.13	0.17	0.51	−0.36	−0.80	−0.54	−0.32	−0.13	0.02
$B(5)$	−0.62	0.38	0.09	−0.19	−1.21	−0.70	−0.17	0.34	0.78	−0.64	−1.43	−0.98	−0.57	−0.21	0.08

B. Consumption growth volatility

	B.1: Actual data			B.2: Simulated data ∣ Model I						B.3: Simulated data ∣ Model II					
Variable	Coef.	S.E.	R^2	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
$B(1)$	0.08	0.01	0.06	0.00	−0.12	−0.05	0.00	0.06	0.13	0.02	−0.02	−0.00	0.01	0.04	0.08
$B(3)$	0.09	0.02	0.10	0.00	−0.12	−0.06	0.00	0.06	0.13	0.03	−0.03	−0.00	0.02	0.05	0.10
$B(5)$	0.07	0.03	0.06	0.00	−0.12	−0.05	0.00	0.06	0.12	0.03	−0.04	−0.00	0.02	0.06	0.12

This table provides evidence on predictability of (1) consumption growth and (2) consumption growth volatility by the average propensity across relationships (a proxy for network connectivity). Data on consumption are real nondurables and services from the U.S. Bureau of Economic Analysis (BEA) and dividends are from the CRSP (NYSE/AMEX/NASDAQ) value-weighted portfolio; all nominal quantities are deflated using the PCE deflator. The statistics for the data are based on annual observations from 1980 to 2019. Standard errors (S.E.) are Newey and West (1987) corrected using 10 lags. Columns R^2 report the regression's coefficient of determination, R^2. The statistics for the simulated data in both models are based on 1,000 simulations, each comparable to the data with its 40 years once observations are time-aggregated to an annual frequency. Column Mean reports the average estimated coefficient across simulations. Columns 10th, 25th, 50th, 75th, and 90th report the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the distribution of estimated coefficients across simulations. Panel A reports results from regressing $g^a_{t+1}+g^a_{t+2}+\cdots+g^a_{t+j}=\alpha(j)+B(j)|\log(\bar{p}_t)|+\upsilon_{t+j}$, where g^a is annualized consumption growth, $\bar{p}_t$ is the average propensity across relationships in the supplier-customer network of year t, and j denotes the forecast horizon in years. Panel B reports results from regressing $|\epsilon^{g^a}_{t+j}|=\alpha(j)+B(j)\log(\bar{p}_t)+\upsilon_{t+j}$, where $|\epsilon^{g^a}_t|$ is the volatility of consumption growth. Following Bansal and Yaron (2004), the volatility of consumption growth is defined as the absolute value of the residual of $g^a_t=\sum_{j=1}^{5} A_j g^a_{t-j}+\epsilon^{g^a}_t$, where g^a is annualized consumption growth.

The model predicts that persistent changes in network connectivity are an important source of low-frequency fluctuations in consumption growth—which is important for generating a sizable unconditional equity premium. Panel A of Table 7 provides support for this prediction. Panel A shows that the average propensity across relationships (an indirect proxy for network connectivity) can forecast consumption growth at 1-, 3-, and 5-year horizons. Column A.1 reports predictability regressions using the sample data. Columns A.2 and A.3 report the corresponding evidence from the perspectives of Models I and II, respectively. Both models capture the negative association between consumption growth and network connectivity. And, as in the data, the magnitude of (average and median) coefficients increases with the horizon. While the magnitude of (average) coefficients estimated from simulated data tends to be somewhat smaller than the magnitude of those estimated from actual data, the average estimated coefficient across simulations tends to be within two standard errors of those estimated from the data.

The model also predicts that persistent changes in the shock propagation mechanism are a source of time-varying consumption growth volatility. Panel B of Table 7 lends support for this result by showing that consumption growth volatility can be predicted by the average propensity. Column B.1 reports predictability regressions of consumption growth volatility in the data. Columns B.2 and B.3 report the corresponding evidence from the perspectives of Models I and II, respectively. Somewhat aligned with results from data, both models capture the nonnegative association between consumption growth volatility and the average propensity. When compared with the data, Model I tends to produce smaller estimates than Model II, whose average estimated coefficient tends to be within two standard errors of those estimated from data.

Conditional heteroscedasticity in consumption growth has important implications for generating predictable variation in the conditional equity premium. Panel A of Table 8 lends additional support for the model mechanism and the idea that changes in network connectivity are a source of conditional heteroscedasticity in consumption growth. In particular, panel A shows that, besides price-dividend ratios, network connectivity predicts excess stock returns. As panel A shows, the model captures the positive association between excess stock returns and network connectivity. And consistent with the model mechanism, panel B of Table 8 shows that stock market volatility can also be predicted by the average propensity of relationships.

Overall, these results suggest that persistent variation in the connectivity of production networks is an important source of long-run consumption risks.[20]

20 As in Bansal and Yaron (2004), predictability results are estimated with considerable sampling error. This fact, in conjunction with the high persistence in network connectivity within both benchmark economies, suggests that these results should be interpreted with caution.

Table 8
Predictability of excess stock returns and stock market volatility

A. Excess stock returns

	A.1: Actual data			A.2: Simulated data | Model I						A.3: Simulated data | Model II					
Variable	Coef.	S.E.	R^2	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
$B(1)$	1.40	0.73	0.04	0.38	−2.91	−1.34	0.45	1.97	3.56	2.16	−0.27	0.64	1.88	3.36	5.00
$B(3)$	3.93	0.92	0.15	0.14	−7.77	−3.98	0.26	4.21	7.89	2.06	−4.02	−0.86	2.05	5.27	8.01
$B(5)$	6.58	2.18	0.27	0.00	−11.55	−6.33	0.15	6.12	11.65	2.35	−8.62	−3.13	2.66	7.98	12.66

B. Stock market volatility

	B.1: Actual data			B.2: Simulated data | Model I						B.3: Simulated data | Model II					
Variable	Coef.	S.E.	R^2	Mean	10th	25th	50th	75th	90th	Mean	10th	25th	50th	75th	90th
$B(1)$	1.65	0.41	0.12	0.19	−1.89	−0.83	0.12	1.22	2.15	0.86	−0.38	0.10	0.61	1.30	2.42
$B(3)$	1.14	0.31	0.06	0.10	−1.91	−0.85	0.08	1.03	1.97	1.03	−0.43	0.12	0.72	1.55	2.94
$B(5)$	0.52	0.63	0.01	−0.00	−2.05	−0.97	−0.01	1.02	1.88	1.21	−0.50	0.16	0.84	1.83	3.39

This table provides evidence on predictability of (1) excess stock returns by price dividend ratios and network connectivity and (2) stock market volatility by network connectivity. Asset market data are from CRSP. Dividends are from the CRSP (NYSE/AMEX/NASDAQ) value-weighted portfolio; all nominal quantities are deflated using the PCE deflator. The statistics for the data are based on annual observations from 1980 to 2019. Standard errors (S.E.) are Newey and West (1987) corrected using 10 lags. Columns R^2 report the regression's coefficient of determination, R^2. The statistics for the simulated data in both models are based on 1,000 simulations, each comparable to the data with their 40 years once observations are time-aggregated to an annual frequency. Column Mean reports the average estimated coefficient across simulations. Columns 10th, 25th, 50th, 75th, and 90th report the 10^{th}, 25^{th}, 50^{th}, 75^{th}, and 90^{th} percentiles of the distribution of estimated coefficients across simulations. Panel A reports results from regressing $r^e_{t+1}+r^e_{t+2}+\cdots+r^e_{t+j}=\alpha(j)+A(j)\log(P_t/D_t)+B(j)|\log(\bar{p}_t)|+\upsilon_{t+j}$, where r^e_{t+j} is the excess return of the CRSP value-weighted portfolio j years ahead, $\log(P_t/D_t)$ is the log price dividend ratio of the CRSP value-weighted portfolio, and $\bar{p}_t$ is the average propensity across relationships in the supplier-customer network of year t. Panel B reports results from regressing $|\epsilon^{mkt}_{t+j}|=\alpha(j)+B(j)\log(\bar{p}_t)+\upsilon_{t+j}$, where $|\epsilon^{mkt}_t|$ is the stock market volatility, defined as the absolute value of the residual of the regression $r^m_t=\sum_{j=1}^{5}A_j r^m_{t-j}+\epsilon^{mkt}_t$, where r^m_t is the CRSP value-weighted return at t.

5.1.2 Robustness tests Because uncovering propensities from data is critical for the above analysis, one might be concerned that my results are driven by the measure used to capture firms' specificity. To alleviate this concern, I use the asset redeployability measure of Kim and Kung (2017) as another proxy for firms' specificity. The idea is simple: the extent to which a firm's output has alternative uses—"asset redeployability" in the terms of Kim and Kung (2017)—has a direct connection with a firm's specificity. As the usability of an asset increases, the more likely it is that firms can use that asset as an input. Thus, firms with higher input specificity scores are likely to produce assets with lower asset redeployability scores. Panel A of Tables I and II in the Internet Appendix show that the predictability results of consumption growth are largely consistent with what they would be if I were to use the inverse of the squared asset redeployability score as a proxy for input specificity in Equation (17).[21]

Another concern is the sample biases associated with the COMPUSTAT Segment files. Although some firms report customers even when they are below the 10% threshold, such customers might not be reported consistently over time. In principle, the fact that I am missing customers introduces noise, which is likely to bias my results against finding any effect of shock propagation on consumption growth. Yet, to make sure this selection issue is not driving my results, I go one step further and replicate my analysis using alternative data (FACTSET Supply Chain Relationships) that are not prone to the selection issues highlighted above. Section VI of the Internet Appendix provides more details about these data. Consistent with the findings of Atalay et al. (2011) and Barrot and Sauvagnat (2016), panels A and B of Table III in the Internet Appendix show that the selection issue is not a first-order concern for the aggregate model predictions. In sum, the unobserved mass to the left of the 10% threshold in the COMPUSTAT Segment files does not significantly alter my results.[22]

5.2 Cross-sectional implications

Besides helping to understand aggregate asset market phenomena, my model helps with understanding the cross-section of stock returns, as it provides a mapping between firms' quantities of priced risk and their exposure to contagion risk.

I resort to PageRank centrality—a variation of eigenvector centrality—to proxy for firms' exposure to contagion risk. At a basic level, firm i's

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[21] Data on firm-level redeployability come from the replication files of Kim and Kung (2017). Because data on redeployability are missing for many supplier-year observations in my sample, Table II in the Internet Appendix explores how results change when one fills missing specificity information with zeros.

[22] For example, Atalay et al. (2011) find that the fraction of suppliers of each customer that one misses because of the 10% threshold is similar for customers with many or few suppliers. Similarly, Barrot and Sauvagnat (2016), using firm-to-firm relationships from Capital IQ find that the truncation issue in COMPUSTAT is not of first-order importance for their results.

PageRank centrality is proportional to the fraction of the time i is reached by a shock that travels along the network in a random walk fashion for a long period. Consequently, it is reasonable to think that PageRank provides an appropriate proxy for firms' exposure to contagion risk within my model.[23] Importantly, because PageRank takes into consideration the architecture of $\mathcal{G}_n$ as well as the precise value of propensities, firm-level centrality scores are time-varying.

To quantitatively assess the impact of contagion risk on firms' risk–return trade-off, I simulate both benchmark economies at a monthly frequency and construct three (value-weighted) portfolios based on firm-level centrality. A portfolio of firms within the 10th percentile of centrality (Low), a portfolio of firms within the 90th percentile of centrality (High), and a portfolio that buys firms in the Low portfolio and sells firms in the High portfolio (henceforth referred to as LMH). Firms are assigned into portfolios once per year. And portfolios are not rebalanced for the next 12 months. This exercise reveals that the LMH portfolio generates a statistically and economically significant monthly average return. In particular, such a portfolio generates a 0.209% in Model I and a 0.701% in Model II.

The above result is explained by the fact that relationships of peripheral firms in both calibrated models, as in the data, tend to exhibit higher propensities than relationships of central firms. Consequently, peripheral firms have higher exposure to negative shocks affecting their direct neighbors. On average, such a contagion risk outweighs the potential benefits peripheral firms receive from their few relationships, and, thus, they command higher risk premiums than central firms. Central firms, however, benefit from diversification of their direct neighbors as their relationships exhibit, on average, small propensities. As a result, their contagion risk is outweighed by the potential benefits generated by their many relationships.

Table 9 shows that both calibrated models generate a realistic spread between the Low and the High firm-level centrality portfolios, as the average monthly return difference between these portfolios is 0.457% in the data. Table 9 also reports monthly average raw returns, alphas, and loadings from the five-factor

[23] I select PageRank centrality not only because it closely reflects how shocks potentially propagate within my model but also because it exhibits desirable features when taking the model to the data. Section IV of the Internet Appendix provides a brief overview of commonly used centrality measures and the benefits of using PageRank to capture contagion risk within my framework. In particular, if $\widetilde{\chi}_{i,t}$ denotes the PageRank centrality of firm i at t, then $\widetilde{\chi}_{i,t}$ satisfies

$$\widetilde{\chi}_{i,t} = \alpha' \left(\sum_{j \in \mathcal{G}_n^i} p_{ij,t} \left(\frac{\widetilde{\chi}_{j,t}}{d_j} \right) \right) + \beta', \tag{20}$$

where α' and β' are positive constants, d_j is the number of relationships of firm j, and $\mathcal{G}_n^i$ denotes the set of neighbors of firm i. The first term in Equation (20) represents eigenvector centrality while the second term is an additional score that all firms receive so that firms that do not belong to strongly connected components of two or more still generate nonzero centrality scores.

Table 9
Performance of (firm-level) centrality portfolios

		Simulation		5-factor model (actual data)					
Portfolio	Raw return	M.I	M.II	Alpha	MKT	SMB	HML	RMW	CMA
Low	1.89	0.93	1.07	0.99	0.98	0.31	−0.23	−0.27	0.06
High	1.43	0.72	0.37	0.42	0.97	−0.19	−0.06	−0.00	0.17
Low-High	0.45	0.20	0.70	0.23	0.01	0.51	−0.17	−0.27	−0.10
S.E.	0.12	0.05	0.10	0.09	0.02	0.03	0.04	0.04	0.06

This table provides evidence that (firm-level) centrality matters in the cross-section of expected returns. This table reports monthly average raw returns, model-implied returns (via simulation), alphas, and loadings from the five-factor model of Fama and French (2015) for three (value-weighted) portfolios constructed by sorting stocks based on (firm-level) centrality: a portfolio of stocks within the 10th percentile of centrality (Low), a portfolio of stocks within the 90th percentile of centrality (High), and a portfolio that buys stocks in the Low portfolio and sells stocks in the High portfolio (Low - High). In the spirit of Fama and French (1993), stocks are assigned into portfolios in July every year. And these portfolios are not rebalanced for the next 12 months. The sample is from July 1980 to December 2019. Columns raw returns, (model-implied returns in) simulated data, and alphas are in percent. For ease of comparison, the statistics for the simulated data are based on 1,000 simulations, each comparable to the data with its 480 observations. M.I (Model I) reports average statistics of the portfolio return generated from the model calibrated as in the first column of Table 4—wherein propensities *do not* vary across relationships. M.II (Model II) reports average statistics of the portfolio return generated from the model calibrated as in the second column of Table 4—wherein propensities vary across relationships. The bottom row provides standard errors (S.E.) for the coefficients of the (Low - High) portfolio.

model of Fama and French (2015) for the Low, the High, and the LMH portfolios. As Table 9 suggests, the return of the LMH portfolio cannot be fully explained by standard asset pricing models, such as the five-factor model. Firms in the Low portfolio command an average monthly return of 1.890%, whereas firms in the High portfolio command an average monthly return of 1.432%. The 0.457% difference between these two portfolios is economically and statistically significant and appears naturally within an equilibrium context as a compensation for contagion risk.

5.2.1 Industry composition and centrality Ahern (2013) also explores the implications of shock propagation along networks for the cross-section of stock returns. To do so, he represents the economy as a network of intersectoral trade and shows that stocks in more central industries command higher returns than stocks in more peripheral industries. He reconciles this finding with a simple idea. Stocks in more central industries have greater exposure to sectoral shocks that transmit from one industry to another. As a result, stocks in central industries covary more closely with the market return and future consumption growth.

Why, then, do I obtain an opposite relationship between centrality and stock returns? Although representing the economy as a network of industries (rather than as a network of firms) can make it easier to take models to the data, this representation ignores subtle but important heterogeneity at the firm level. When representing the economy as a network of industries, the quantity of risk associated with innovations in network connectivity is not altered by within-industry covariation—that is, the comovement between the cash flows of firms within the same industry. This is because all firms within an industry are assumed to be equal. While this assumption can help keep models simple, it fails to recognize that within certain industries firms' exposure to contagion risk could exhibit significant variation.

Tables 10 and 11 lend support to this idea. Table 10 shows that the LMH portfolio generates a significant alpha even after controlling for an industry centrality factor similar to the Ahern (2013) Central Minus Peripheral (CMP) factor. Table 11 shows that the LMH portfolio generates a significant return even when considering firms within the same industry. If only industry-level information matters, such portfolios should generate returns that are economically and statistically insignificant. Section VII in the Internet Appendix provides additional evidence that the aforementioned finding is robust to industry composition.

Taken together, these results highlight the relevance of looking at firm-level centrality measures and the first-order importance of within-industry covariation.

5.2.2 Robustness tests This section studies the robustness of the cross-sectional result to concerns regarding the measure used to capture input specificity or truncation issues inherent to the COMPUSTAT data set. Panels B and C in Tables I and II in the Internet Appendix show that results reported in Tables 9 and 10 are robust to changes in the proxy for input specificity. Panels B and C of Table III in the Internet Appendix show that these results are also not affected by the 10% truncation issue of the COMPUSTAT data set. Overall, these results ensure that the cross-sectional finding is unlikely to be driven by how input specificity is measured or the existence of sample biases in the customer-supplier data set.

5.3 Relevance of heterogeneity in propensities

High persistence in network connectivity plays a pivotal role in the ability of calibrated models to replicate aggregate features of stocks returns. The mechanism through which both models generate such high persistence is markedly different. In Model I—wherein propensities are equal across relationships—high values for parameters ϑ and θ_p ensure high persistence in network connectivity. In Model II—wherein propensities vary across relationships—smaller values for ϑ can generate a sufficiently high persistence in network connectivity. And, thus, parameter ϑ plays a smaller role when propensities can vary across relationships. In Model II, shocks to firms whose relationships exhibit propensities more persistent than the propensity of the average relationship have a more enduring impact on W_{t+1} and, in doing so, on consumption growth. Therefore, when propensities can vary across relationships as in the data, high persistence in network connectivity can be generated with smaller values for parameter ϑ. By considering several alternative parameterizations to Models I and II, Section IX in the Internet Appendix helps illustrate this observation.

Table 10
Performance of centrality portfolios after controlling for industry centrality

		5-factor model						Industry
Portfolio	Raw return	Alpha	MKT	SMB	HML	RMW	CMA	CMP
Low	1.79	1.05	1.00	0.38	−0.26	−0.27	0.06	−0.09
High	1.19	0.44	0.96	−0.15	−0.06	0.01	0.15	0.03
Low-High	0.60	0.44	0.03	0.54	−0.20	−0.29	−0.08	−0.12
S.E.	0.19	0.12	0.03	0.04	0.05	0.06	0.07	0.03

This table provides evidence that results reported in Table 9 continue to hold even after controlling for industry centrality. This table reports monthly average raw returns, alphas, loadings from the five-factor model of Fama and French (2015), and loadings on a portfolio of central minus peripheral (CMP) industries for the three (value-weighted) portfolios considered in Table 9. In the spirit of Ahern (2013), the CMP factor is formed by subtracting the monthly returns of stocks in the lowest tercile of industry centrality from the returns of stocks in the highest tercile of industry centrality. And within these terciles, returns are value-weighted. Data to construct the CMP portfolio come from the U.S. Bureau of Economic Analysis (BEA) Input-Ouput tables. The bottom row provides standard errors (S.E.) for the coefficients of the (Low - High) portfolio. Columns raw returns and alphas are in percent. As detailed and consistent Input-Output tables are available from 1997, reported numbers are based on data from July 1997 to December 2019.

Table 11
Performance of centrality portfolios among manufacturing and service firms

A. Manufacturing firms

		5-factor model					
Portfolio	Raw return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.00	1.06	1.03	0.45	−0.33	−0.24	0.09
High	1.46	0.45	0.95	−0.16	−0.17	−0.01	0.30
Low-High	0.54	0.27	0.08	0.62	−0.16	−0.23	−0.21
S.E.	0.15	0.10	0.02	0.03	0.04	0.04	0.07

B. Service firms

		5-factor model					
Portfolio	Raw return	Alpha	MKT	SMB	HML	RMW	CMA
Low	2.08	1.09	1.17	0.41	−0.34	−0.24	−0.06
High	1.42	0.74	0.87	−0.12	−0.19	−0.28	−0.34
Low-High	0.66	0.01	0.30	0.53	−0.15	0.03	0.28
S.E.	0.23	0.23	0.05	0.08	0.10	0.10	0.15

This table provides evidence that within industry covariation is of first-order importance for my results. Panel A focuses on manufacturing firms, and panel B focuses on service firms. Similar to Tables 9 and 10, this table reports monthly average raw returns, alphas, and loadings from the five-factor model of Fama and French (2015) for the three (value-weighted) portfolios considered in Table 9. The sample is from July 1980 to December 2019. Columns raw returns and alphas are in percent. The bottom row provides standard errors (S.E.) for the coefficients of the (Low - High) portfolio.

5.4 Relevance of directionality of relationships

In light of a literature that emphasizes the relevance of directionality in shock propagation, I explore whether my cross-sectional findings are consistent with downstream propagation (where shocks travel from suppliers to customers) or upstream propagation (where shocks travel from customers to suppliers).

Table IV in the Internet Appendix shows that the LMH portfolio continues to generate a positive and statistically significant return when firms are sorted based on in-degree centrality. Yet the sign of the LMH portfolio's return flips when firms are sorted based on out-degree centrality. In-degree centrality captures a firm's exposure to contagion risk coming from its suppliers, as it is the sum of propensities of a firm's relationships with its direct suppliers. Yet out-degree centrality captures the amount of contagion risk to which one firm exposes its customers, as it is the sum of propensities of a firm's relationships with its direct customers. And consistent with the idea that directionality matters, the LMH portfolio does not generate a statistically significant return when firms are sorted based on the sum of in- and out-degree centralities.

Taken together, these results support the idea that downstream propagation can have first-order effects on stock returns.

6. Conclusion

This paper studies the asset pricing properties that stem from the propagation of firm-level shocks along production networks. The fundamental insight is that persistent variation in shock propagation along such networks can generate long-run risks, and it is important for generating both time-series and cross-sectional predictability in stock returns.

Calibrated models that match key structural features of supplier–customer networks in the United States can generate long-run consumption risks, high and volatile risk premiums, and a low and stable risk-free rate. Low-frequency changes in network connectivity generate persistence in firms' growth prospects, which, in turn, drives a small but persistent component in expected aggregate consumption growth in equilibrium. With investors with preference for early resolution of uncertainty, sizable risk premiums arise because investors fear that extended periods of low economic growth coincide with low asset prices. A small risk-free rate is driven by investors saving for long periods of low economic growth.

The proposed conceptual framework also helps with understanding the cross-section of stock returns, as it provides a mapping between firms' quantities of priced risk and their exposure to contagion risk. As in the data, firms that are more central command lower risk premiums than firms that are less central. Central firms tend to benefit from the diversification of their direct neighbors, and, thus, they mitigate contagion risk better than peripheral firms.

Appendix. Approximate Solutions

To emphasize the mechanisms working at equilibrium, this appendix derives approximate expressions for equilibrium asset returns using log-linear approximations as in Campbell and Shiller (1988) and Bansal and Yaron (2004). Intuitively, to solve the model I look for equilibrium prices so that price–dividend ratios are stationary, as in Mehra and Prescott (1985), Weil (1989), and Kandel and Stambaugh (1991), among many others. Because the network architecture is fixed, the state of matrix $\mathbf{P}_t$ determines the equilibrium distribution of $\Delta\widetilde{c}_{t+1}$. Because $\mathbf{P}_t$ follows a Markov process, the distribution of $\Delta\widetilde{c}_{t+1}$ varies over time and its dynamics satisfies the Markov property.

A Relevant Equations

Before solving for equilibrium returns, it is useful to consider the system of equations that characterizes the benchmark economy:

$$\Delta\widetilde{c}_{t+1} = \mu_c - W_{t+1} + \sigma\varepsilon^c_{t+1}, \tag{A1}$$

$$\widetilde{x}_{t+1} = \bar{y} - \alpha W_{t+1} + \varphi_a \sigma \varepsilon^a_{t+1}, \tag{A2}$$

$$W_{t+1} \approx \mu_w + \rho_w W_t + \sigma_{w,t}\varepsilon^W_{t+1}, \tag{A3}$$

where $\varepsilon^c_{t+1} \equiv (\frac{1}{\phi})(\varphi_a \varepsilon^a_{t+1} - \varphi_x \varepsilon^x_{t+1})$, and shocks $\varepsilon^a_{t+1}, \varepsilon^x_{t+1}$, and ε^W_{t+1} are mutually independent of each other. Equation (A1) directly follows from expression (8), whereas Equation (A2) follows from aggregating firms output using expression (1) and $Y_t \equiv \prod_{i=1}^n y_{i,t}^{1/n}$. Equation (A3) follows from an important observation: within the benchmark economy, the dynamics of W_{t+1} can be approximated by an autoregressive process as the economy grows large. Because $f(\cdot)$ is a continuous mapping, W_{t+1} can be rewritten as

$$\begin{aligned} W_{t+1} &= \frac{1}{n}\left(\sum_{i=1}^n f(z_{i,t+1})\right) \\ &= \frac{1}{n}\left(\sum_{i=1}^n f(z_{i,t}) + f'(z^*_{i,t})(z_{i,t+1} - z_{i,t})\right), \end{aligned} \tag{A4}$$

where $z^*_{i,t} \in [z_{i,t}, z_{i,t+1}]$ if $z_{i,t} < z_{i,t+1}$; or $z^*_{i,t} \in [z_{i,t+1}, z_{i,t}]$ if $z_{i,t+1} < z_{i,t}$. And the second equality in (A4) follows from the mean value theorem. Now, define $\mathcal{Z}_{i,t+1} \equiv f'(z^*_{i,t})(z_{i,t+1} - z_{i,t})$ and $\mathcal{S}_{t+1} \equiv \frac{1}{n}\sum_{i=1}^n \mathcal{Z}_{i,t+1}$. It then directly follows from Equation (A4) that

$$W_{t+1} = W_t + \mathcal{S}_{t+1}. \tag{A5}$$

As Table 3 and Figure 2 show, the network architecture of the benchmark economy is sparse. Consequently, within both calibrated models, the order of variables in the sequence $\{\mathcal{Z}_{i,t+1}\}_{i=1}^n$ can be selected so that $\mathcal{Z}_{i,t+1}$ and $\mathcal{Z}_{i+m,t+1}$ are approximately independent for large m. That is, the sequence $\{\mathcal{Z}_{i,t+1}\}_{i=1}^n$ is α-mixing as random variables far apart from one another are nearly independent. And because innovations to $z_{i,t}$ are normally distributed, the sequence $\{\mathcal{Z}_{i,t+1}\}_{i=1}^n$ is also stationary.

It then follows from Billingsley (1995, theorem 27.4) that $\sqrt{n}\mathcal{S}_{t+1}$ has a limiting normal distribution as n grows large. Therefore, one can always find constants μ_w and ρ_w such that W_{t+1} can be rewritten as in expression (A3) where $\sigma_{w,t}\varepsilon^W_{t+1}$ is normally distributed. Here, variable $\sigma^2_{w,t} \equiv \mathrm{var}_t[W_{t+1}]$ is time-varying and its precise value is determined by propensities $\{p_{ij,t}\}_{(i,j)}$ and shocks $\{\eta_{i,t+1}\}_{i=1}^n$.

B Pricing Kernel

The pricing kernel is given by

$$m_{t+1} \equiv \theta\log(\beta) - \frac{\theta}{\psi}\Delta\widetilde{c}_{t+1} + (\theta - 1)r_{c,t+1}. \tag{B1}$$

with $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $r_{c,t+1}$ denotes the (endogenous) log return of aggregate wealth—an asset that delivers aggregate consumption as its dividend each period. The return of any firm i can be determined using the pricing kernel and the representative investor's first-order condition

$$\mathbb{E}_t\left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1. \tag{B2}$$

In what follows, I first solve for the price and return of aggregate wealth. With these solutions in hand, I solve for the pricing kernel, the return on the market portfolio, and the risk-free rate. These expressions are then used to solve for equilibrium returns in the cross-section.

To solve for the log return of aggregate wealth $r_{c,t+1}$, I substitute $r_{i,t+1}$ for $r_{c,t+1}$ in (B2). In the same spirit as Bansal and Yaron (2004), I conjecture that the (unobservable) log price to consumption ratio, $d_t^c \equiv \log\left(\frac{P_t}{C_t}\right)$, follows

$$d_t^c = A_0^c + A_1^c W_t. \tag{B3}$$

With the (yet-to-be-solved) endogenous variable d_t^c in hand, I can determined $r_{c,t+1}$ using the log-linear approximation

$$r_{c,t+1} \;\approx\; \kappa_0^c + \Delta\widetilde{c}_{t+1} + \kappa_1^c d_{t+1}^c - d_t^c, \tag{B4}$$

with

$$\kappa_1^c = \frac{\exp(\bar{d}^c)}{1+\exp(\bar{d}^c)} \quad \text{and} \quad \kappa_0^c = \log(1+\exp(\bar{d}^c)) - \kappa_1^c \bar{d}^c, \tag{B5}$$

where $\bar{d}^c$ denotes the mean log price to consumption ratio.

To solve for constants A_0^c and A_1^c, I plug the conjectured expressions for d_{t+1}^c and $r_{c,t+1}$ into the Euler equation and group terms by state variables. The following observation is critical. If $(m_{t+1} + r_{c,t+1})$ is (approximately) normally distributed, then the above first-order condition is equivalent to

$$\mathbb{E}_t\left[m_{t+1} + r_{c,t+1}\right] + \frac{1}{2}\text{var}_t\left[m_{t+1} + r_{c,t+1}\right] \;\approx\; 0 \tag{B6}$$

By separating terms associated with state variables, I obtain a system of equations whose solution is given by:

$$A_0^c \;=\; \frac{\log(\beta) + \left(1-\frac{1}{\psi}\right)(\mu_c - \mu_w) + \kappa_0^c + \kappa_1^c \mu_w A_1^c + \frac{\theta}{2}\sigma^2(1-\frac{1}{\psi})^2}{1-\kappa_1^c}, \tag{B7}$$

$$A_1^c \;=\; -\frac{\rho_w}{1-\kappa_1^c \rho_w}\left(1 - \frac{1}{\psi}\right). \tag{B8}$$

The above solutions depend on the approximating constants, κ_0^c and κ_1^c, which, in turn, depend on the unknown unconditional mean of the log price to consumption ratio, $\bar{d}^c$. The precise value of $\bar{d}^c$ can be found by solving the (nonlinear) equation

$$\bar{d}^c \;=\; \mathbb{E}[d_{t+1}^c] \;=\; \mathbb{E}[A_0^c(\bar{d}^c) + A_1^c(\bar{d}^c) W_{t+1}]. \tag{B9}$$

B.1. Innovations to the Pricing Kernel. Substituting the above solutions, it can be shown that innovations to the pricing kernel are given by

$$m_{t+1} - \mathbb{E}_t(m_{t+1}) \;=\; \left(\theta - 1 - \frac{\theta}{\psi}\right)\sigma\varepsilon_{t+1}^c + \left((\theta-1)(\kappa_1^c A_1^c - 1) + \frac{\theta}{\psi}\right)\sigma_{w,t}\varepsilon_{t+1}^W. \tag{B10}$$

Because $\varepsilon_{t+1}^c \equiv (1/\phi)\left[\varphi_a \varepsilon_{t+1}^a - \varphi_x \varepsilon_{t+1}^x\right]$, Equation (B10) can be written as

$$\begin{aligned} m_{t+1} - \mathbb{E}_t(m_{t+1}) \;=\; & \left(\theta - 1 - \frac{\theta}{\psi}\right)\frac{\varphi_a}{\phi}\sigma\varepsilon_{t+1}^a - \left(\theta - 1 - \frac{\theta}{\psi}\right)\frac{\varphi_x}{\phi}\sigma\varepsilon_{t+1}^x \\ & + \left((\theta-1)(\kappa_1^c A_1^c - 1) + \frac{\theta}{\psi}\right)\sigma_{w,t}\varepsilon_{t+1}^W, \end{aligned} \tag{B11}$$

which is equivalent to expression (13). That is, innovations to the pricing kernel are driven by aggregate productivity shocks, aggregate dividend growth shocks, and innovations to network connectivity, ε_{t+1}^W. Here, an increase in the persistence of W_t, ρ_w, raises $|A_1^c|$, increasing the exposure of the pricing kernel to innovations to contagion as $\psi > 1$ and $\theta < 0$.

B.2. Conditional Volatility of the Pricing Kernel. Using the above expressions it can be shown that

$$\begin{aligned}
\mathrm{var}_t(m_{t+1}) &= \mathrm{var}_t\left(\theta\log(\beta)-\frac{\theta}{\psi}\Delta\widetilde{c}_{t+1}+(\theta-1)r_{c,t+1}\right) \\
&= \frac{\theta^2}{\psi^2}\mathrm{var}_t(\Delta\widetilde{c}_{t+1})+(\theta-1)^2\mathrm{var}_t(r_{c,t+1})-2\frac{\theta}{\psi}(\theta-1)\mathrm{cov}_t(\Delta\widetilde{c}_{t+1},r_{c,t+1}), \\
&= \sigma^2_{w,t}\left((\theta-1)(\kappa_1^c A_1^c-1)+\frac{\theta}{\psi}\right)^2+\left(\theta-1-\frac{\theta}{\psi}\right)^2\sigma^2,
\end{aligned} \tag{B12}$$

which is equivalent to expression (14). Therefore, the conditional volatility of the pricing kernel is time-varying. And its temporal changes are solely driven by changes in network connectivity.

C Risk-Free Rate

The risk-free rate is intimately linked with the pricing kernel. If m_{t+1} is normally distributed, it follows from the representative investor's first-order condition that

$$r_{f,t}+\mathbb{E}_t(m_{t+1})+\frac{1}{2}\mathrm{var}_t(m_{t+1}) = 0, \tag{C1}$$

with $r_{f,t}=\log(R_{f,t})$. Here,

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \theta\log(\beta)-\frac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1})+(\theta-1)\mathbb{E}_t(r_{c,t+1}) \\
&= \theta\log(\beta)+\left((\theta-1)-\frac{\theta}{\psi}\right)(\mu_c-(\mu_w+\rho_w W_t)) \\
&\quad + (\theta-1)\left(\kappa_0^c+\kappa_1^c(A_0^c+A_1^c(\mu_w+\rho_w W_t))-(A_0^c+A_1^c W_t)\right).
\end{aligned} \tag{C2}$$

And the conditional variance of the pricing kernel is reported in Equation (B12). It is then clear that $r_{f,t}=A_0^f+A_1^f W_t+A_2^f\sigma^2_{w,t}$, where constants $\left\{A_0^f,A_1^f,A_2^f\right\}$ can be found by grouping terms associated with the state variables using Equation (C1).

D Market Portfolio

The solution coefficients for the valuation ratio of the market portfolio—an asset that delivers aggregate output each period—can be derived in a similar fashion to those for the asset that delivers aggregate wealth. The (observable) log price-dividend ratio for the market portfolio, $d_t^m\equiv\log\left(\frac{P_t^m}{Y_t}\right)$, is conjectured to follow

$$d_t^m=A_0^m+A_1^m W_t, \tag{D1}$$

where constants A_0^m and A_1^m can be solved for. With this expression in hand, I can solve for the log market return, $r_{m,t+1}$, using the log-linear approximation

$$r_{m,t+1} \approx \kappa_0^m+x_{t+1}+\kappa_1^m d_{t+1}^m-d_t^m \tag{D2}$$

with

$$\kappa_1^m=\frac{\exp(\bar{d^m})}{1+\exp(\bar{d^m})} \quad\text{and}\quad \kappa_0^m=\log(1+\exp(\bar{d^m}))-\kappa_1^m\bar{d^m}, \tag{D3}$$

and $\bar{d^m}$ denotes the mean log price-dividend ratio of the market portfolio.

As before, I plug the conjectured expressions for d^m_{t+1} and $r_{m,t+1}$ into the Euler equation to solve for constants A^m_0 and A^m_1. The key underlying assumption is that $(m_{t+1}+r_{m,t+1})$ is (approximately) normally distributed, and, thus, $\mathbb{E}_t(m_{t+1}+r_{m,t+1})+\frac{1}{2}\mathrm{var}_t(m_{t+1}+r_{m,t+1})\approx 0$. This process yields

$$A^m_1 = \frac{-\rho_w\left(\alpha+(\theta-1)(1-\kappa^c_1 A^c_1)-\frac{\theta}{\psi}\right)-(\theta-1)A^c_1}{1-\kappa^m_1\rho_w} = \frac{-\rho_w\left(\alpha-\frac{1}{\psi}\right)}{1-\kappa^m_1\rho_w}, \tag{D4}$$

$$A^m_0 = \frac{\frac{1}{2}\delta^2_1+\delta_2+\kappa^m_1 A^m_1\mu_w}{1-\kappa^m_1}, \tag{D5}$$

with

$$\delta^2_1 = \left[\sigma^2\left(\theta-1-\frac{\theta}{\psi}\right)^2+\varphi^2_a\sigma^2+\frac{2}{\phi}\left(\theta-1-\frac{\theta}{\psi}\right)\varphi^2_a\sigma^2\right], \tag{D6}$$

$$\delta_2 = \theta\log(\beta)-\frac{\theta}{\psi}(\mu_c-\mu_w)+(\theta-1)(\kappa^c_0+\mu_c-\mu_w+\kappa^c_1(A^c_0+A^c_1\mu_w)-A^c_0)+\kappa^m_0+\bar{y}-\alpha\mu_w.$$

And the unknown unconditional mean of the log price dividend ratio of the market portfolio, $\bar{d^m}$, can be found by solving the (nonlinear) equation

$$\bar{d^m}=\mathbb{E}[d^m_{t+1}] = \mathbb{E}[A^m_0(\bar{d^m})+A^m_1(\bar{d^m})W_{t+1}]. \tag{D7}$$

D.1. Innovations to the Market Return. Substituting the above solutions, it can be shown that innovations to the return of the market portfolio are given by

$$r_{m,t+1}-\mathbb{E}_t(r_{m,t+1}) = \varphi_a\sigma\varepsilon^a_{t+1}+\left(\kappa^m_1 A^m_1-\alpha\right)\sigma_{w,t}\varepsilon^W_{t+1}. \tag{D8}$$

That is, innovations to the market return are driven by aggregate productivity shocks and innovations to network connectivity, ε^W_{t+1}. Notably, an increase in ρ_w raises $|A^m_1|$, increasing the exposure of the market return to innovations to network connectivity.

E Equity Premium

Under the above assumptions, both the market portfolio and the pricing kernel are (approximately) lognormally distributed. Hence, the continuous equity risk premium can be expressed as

$$\mathbb{E}_t[r_{m,t+1}-r_{f,t}] = -\mathrm{cov}_t(m_{t+1},r_{m,t+1})-\frac{1}{2}\mathrm{var}_t(r_{m,t+1}), \tag{E1}$$

where

$$\mathrm{cov}_t\left[m_{t+1},r_{m,t+1}\right] = \left(\theta-1-\frac{\theta}{\psi}\right)\frac{\varphi^2_a}{\phi}\sigma^2+\left((\theta-1)(\kappa^c_1 A^c_1-1)+\frac{\theta}{\psi}\right)(\kappa^m_1 A^m_1-\alpha)\sigma^2_{w,t}, \tag{E2}$$

$$\mathrm{var}_t(r_{m,t+1}) = \varphi^2_a\sigma^2+(\kappa^m_1 A^m_1-\alpha)^2\sigma^2_{w,t}. \tag{E3}$$

Notably, an increase in ρ_w raises both $|A^c_1|$ and $|A^m_1|$, increasing the covariance between the pricing kernel and the market return, and, thus, increasing the equity premium.

F Firms' Return in the Cross-Section

The solution coefficients for the valuation ratio of asset i—an asset that delivers firm i's output each period—can be derived in a similar fashion. In particular, the (observable) log price-dividend ratio for asset i, $d_t^i \equiv \log\left(\frac{P_t^i}{y_{i,t}}\right)$, is conjectured to follow

$$d_t^i = A_0^i + A_1^i W_t, \tag{F1}$$

where constants A_0^i and A_1^i can be solved for. With this expression in hand, I can solve for the log return of asset i, $r_{i,t+1}$, using the log-linear approximation

$$r_{i,t+1} \approx \kappa_0^i + x_{i,t+1} + \kappa_1^i d_{t+1}^i - d_t^i \tag{F2}$$

with

$$\kappa_1^i = \frac{\exp(\bar{d}^i)}{1+\exp(\bar{d}^i)} \quad \text{and} \quad \kappa_0^i = \log(1+\exp(\bar{d}^i)) - \kappa_1^i \bar{d}^i, \tag{F3}$$

and $\bar{d}^i$ denotes the mean log price-dividend ratio of asset i and $x_{i,t+1} = \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right)$. It is worth noting that $x_{i,t+1}$ can also be expressed in terms of variable W_t, aggregate productivity shocks, ε_{t+1}^a, and innovations to firm-level shocks $z_{i,t}$ as

$$\begin{aligned} x_{i,t+1} &= \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) \qquad \text{(F4)} \\ &= \log\left(\frac{y_{i,t+1}}{Y_t}\right) + \log\left(\frac{Y_t}{y_{i,t}}\right) \\ &= \log\left(\frac{y_{i,t+1}}{Y_t}\right) + \underbrace{\left(\left\{\frac{1}{n}\sum_{j=1}^{n}\log(y_{j,t})\right\} - \log(y_{i,t})\right)}_{\substack{\text{output difference between} \\ \text{firm } i \text{ and the average firm}}} \\ &= \bar{y} + \varphi_a \sigma \varepsilon_{t+1}^a - \alpha W_t - \alpha \underbrace{\left(f(z_{i,t+1}) - f(z_{i,t})\right)}_{f'(z_{i,t}^*)(z_{i,t+1}-z_{i,t})}. \end{aligned}$$

To better understand the dynamics of $x_{i,t+1}$, it is illustrative to express $\mathcal{G}_n$ as a union of connected components,

$$\mathcal{G}_n \equiv \bigcup_{i \in \mathcal{G}_n} \mathcal{G}_n^i, \tag{F5}$$

where $\mathcal{G}_n^i$ denotes the connected component that firm i belongs to—that is, the set of (direct and indirect) neighbors of firm i. Then, define the following averages

$$\begin{aligned} \mathcal{S}_{t+1}^i &\equiv \frac{1}{n}\left(\sum_{j \in \mathcal{G}_n^i, j \neq i} f'(z_{j,t}^*)(z_{j,t+1} - z_{j,t})\right) \qquad \text{(F6)} \\ \mathcal{S}_{t+1}^{-i} &\equiv \frac{1}{n}\left(\sum_{j \in \mathcal{G}_n \setminus \mathcal{G}_n^i} f'(z_{j,t}^*)(z_{j,t+1} - z_{j,t})\right), \end{aligned}$$

where $\mathcal{S}^{i}_{t+1}$ relates to the average innovation faced by firm i's (direct and indirect) neighboring firms at $t+1$ whereas $\mathcal{S}^{-i}_{t+1}$ relates to the average innovation faced by firm i's nonneighboring firms at $t+1$. Using the above definitions, it follows from Equation (A5) that

$$W_{t+1} \;=\; W_t+\underbrace{\frac{1}{n}\times f'(z^*_{i,t})(z_{i,t+1}-z_{i,t})+\mathcal{S}^{i}_{t+1}+\mathcal{S}^{-i}_{t+1}}_{\mathcal{S}_{t+1}}, \tag{F7}$$

which implies that

$$f'(z^*_{i,t})(z_{i,t+1}-z_{i,t}) \;=\; n((W_{t+1}-W_t)-\mathcal{S}^{i}_{t+1}-\mathcal{S}^{-i}_{t+1}). \tag{F8}$$

It then directly follows from Equations (F4) and (F8) that

$$x_{i,t+1} \;=\; \bar{y}+\varphi_a\sigma\varepsilon^{a}_{t+1}-\alpha nW_{t+1}-\alpha(1-n)W_t+\alpha n(\mathcal{S}^{i}_{t+1}+\mathcal{S}^{-i}_{t+1}). \tag{F9}$$

Namely, firm i's output growth approximately follows[24]

$$x_{i,t+1} \;\approx\; B^i_0+B^i_1W_{t+1}+\varphi_a\sigma\varepsilon^{a}_{t+1}+\underbrace{\sigma_i\varepsilon^{i}_{t+1}}_{\substack{\text{term unexplained by}\\ \text{innovations to } W_{t+1} \text{ or } \varepsilon^{a}_{t+1}}}, \tag{F10}$$

where ε^{i}_{t+1} is independent from ε^{a}_{t+1} and ε^{W}_{t+1}. And $\sigma^2_i\equiv\text{var}[\varepsilon^{i}_{t+1}]$. Here, parameters $\left\{B^i_0, B^i_1, \sigma_i\right\}$ can be estimated from simulated data. With these estimates in hand, I plug the conjectured expressions for $x_{i,t+1}$, d^i_{t+1} and $r_{i,t+1}$ into the Euler equation to solve for constants A^i_0 and A^i_1. This process yields

$$A^i_1 \;=\; \frac{\rho_w\left\{\frac{\theta}{\psi}-(\theta-1)+B^i_1\right\}-(\theta-1)A^c_1(1-\kappa^c_1\rho_w)}{1-\kappa^i_1\rho_w}, \tag{F11}$$

$$A^i_0 \;=\; \frac{\frac{1}{2}\left\{\delta^2_1+\sigma^2_i\right\}+L_i}{1-\kappa^i_1}, \tag{F12}$$

with

$$L_i \;=\; \theta\log(\beta)+(\mu_c-\mu_w)\left(\theta-1-\frac{\theta}{\psi}\right)+(\theta-1)(\kappa^c_0+\kappa^c_1(A^c_0+A^c_1\mu_w)-A^c_0) \tag{F13}$$

$$+\;\kappa^i_0+B^i_0+B^i_1\mu_w+\kappa^i_1A^i_1\mu_w.$$

With these expressions in hand, the unknown unconditional mean of asset i's log price dividend ratio, $\bar{d^i}$, is found by solving the (nonlinear) equation

$$\bar{d^i}=\mathbb{E}[d^i_{t+1}] \;=\; \mathbb{E}[A^i_0(\bar{d^i})+A^i_1(\bar{d^i})W_{t+1}]. \tag{F14}$$

F.1. Innovations to Firm i's Return. Using the above expressions, it can be shown that innovations to firm i's return are given by

$$\begin{aligned} r_{i,t+1}-\mathbb{E}_t[r_{i,t+1}] \;&\approx\; \varphi_a\sigma\varepsilon^{a}_{t+1}+(\kappa^i_1A^i_1-\alpha n)(W_{t+1}-\mathbb{E}_t[W_{t+1}]) \\ &\quad+\alpha n\left(\mathcal{S}^{i}_{t+1}-\mathbb{E}_t[\mathcal{S}^{i}_{t+1}]\right)+\alpha n\left(\mathcal{S}^{-i}_{t+1}-\mathbb{E}_t[\mathcal{S}^{-i}_{t+1}]\right) \\ &=\; \varphi_a\sigma\varepsilon^{a}_{t+1}+(\kappa^i_1A^i_1-\alpha n)\sigma_{w,t}\varepsilon^{W}_{t+1}+\alpha n\upsilon^{i}_{t+1}+\alpha n\upsilon^{-i}_{t+1} \end{aligned} \tag{F15}$$

where $\upsilon^{i}_{t+1}\equiv(\mathcal{S}^{i}_{t+1}-\mathbb{E}_t[\mathcal{S}^{i}_{t+1}])$ and $\upsilon^{-i}_{t+1}\equiv\mathcal{S}^{-i}_{t+1}-\mathbb{E}_t[\mathcal{S}^{-i}_{t+1}]$ capture innovations in variables $\mathcal{S}^{i}_{t+1}$ and $\mathcal{S}^{-i}_{t+1}$, respectively.

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[24] Although considering that $\text{var}_t[\varepsilon^{i}_{t+1}]$ is constant might be a strong assumption, this assumption helps present ideas in a clearer way. One can modify Equation (F10) so $x_{i,t+1}$ also depends on more state variables and approximation (F10) is potentially more accurate. Section II in the Internet Appendix explores such an extension.

F.2. Firms' Quantities of Risk in the Cross-Section. Taking into consideration expressions (B11) and (F15) yields

$$\mathrm{cov}_t\left(m_{t+1}, r_{i,t+1}\right) = \underbrace{\left(\theta - 1 - \frac{\theta}{\psi}\right)\frac{\varphi_a^2}{\phi}\sigma^2}_{\lambda_a} + \underbrace{\left((\theta-1)(\kappa_1^c A_1^c - 1) + \frac{\theta}{\psi}\right)(\kappa_1^i A_1^i - \alpha n)\sigma_{w,t}^2}_{\lambda_{W_t}} \quad \text{(F16)}$$

$$+ \underbrace{\alpha n\left((\theta-1)(\kappa_1^c A_1^c - 1) + \frac{\theta}{\psi}\right) \times \mathrm{cov}_t(\sigma_{w,t}\varepsilon_{t+1}^W, \upsilon_{t+1}^i)}_{\varrho_{i,t}}$$

$$+ \underbrace{\alpha n\left((\theta-1)(\kappa_1^c A_1^c - 1) + \frac{\theta}{\psi}\right) \times \mathrm{cov}_t(\sigma_{w,t}\varepsilon_{t+1}^W, \upsilon_{t+1}^{-i})}_{\kappa_{i,t}},$$

which is equivalent to expression (16).

Equation (F16) helps illustrate how contagion risk is priced across firms. It shows that a firm's quantity of risk is simultaneously determined by its exposure to the only two sources of risk within the model: (a) innovations to aggregate shocks, captured by λ_a, and (b) innovations to network connectivity, captured by the sum $\lambda_{W_t} + \varrho_{i,t} + \kappa_{i,t}$.

Code Availability: The replication code is available in the Harvard Dataverse, https://doi.org/10.7910/DVN/VJKTMQ.

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