Designing Resilient Financial Systems

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ABSTRACT

I study the problem of a policymaker who seeks to improve the resilience of a large and highly interconnected financial system during times of economic stress. I show that under some conditions, the policymaker cannot improve the system's resilience. In other cases, she can significantly improve the system's resilience by imposing ex-ante restrictions on a set of companies. The size of such a set depends on both the uncertainty faced by the policymaker and the ease of implementing restrictions.

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I. Introduction

The resilience of modern financial systems has received considerable attention since the recent financial crisis. However, the literature provides limited theoretical guidance on how a policymaker can improve a financial system's resilience when she is uncertain how losses propagate among related companies during times of economic stress. This paper fills this gap by developing a simple model to study the behavior of such a policymaker.

I show that under some conditions such a policymaker is not able to improve the system's resilience. In other cases, however, she can significantly improve the system's resilience by imposing restrictions—e.g. capital, reserve, or liquidity requirements—on a relatively small fraction of companies. The fraction of companies restricted depends on the nature of the uncertainty faced by the policymaker and the ease of implementing restrictions

The model is motivated by a financial system in which companies are highly interconnected. Within the model, cascading failures occur during times of economic stress due to contagion; the distress of one company may cause distress to the company's counterparties, which, in turn, may cause distress to the counterparties' counterparties, and so on. The policymaker is uncertain about the connectivity structure of the system during times of economic stress.

I first analyze the behavior of the policymaker when she has no information about the set of companies that will play an important role in propagating distress during times of economic stress. I show that if too many companies are well connected in times of economic stress, the policymaker may not be able to improve the system's resilience, as restricting companies at random does not necessarily decrease the likelihood of large cascading failures. However, the policymaker can improve the system's resilience as long as not too many companies are well connected in stressful conditions.

I then analyze the behavior of the policymaker when she knows the set of companies that will play an important role in propagating distress during times of economic stress. In this case, the system's resilience can always be improved, as the policymaker is able to keep distress locally confined by restricting well connected companies.

This paper contributes to two strands of the literature. First, this paper adds to a body of work that explores how network features of financial systems affect the likelihood of contagion (see, for example, Allen and Gale (2000), Eisenberg and Noe (2001), Watts (2002), Dasgupta (2004), Amini et al. (2013), Elliott et al. (2014), Glasserman and Young (2015), and Acemoglu et al. (2015)). Unlike these papers, this paper explicitly models the problem of a policymaker who seeks to improve the system's resilience. Second, this paper adds to recent research that explores how regulations affect the likelihood of large cascading failures (see, for example, Gai et al. (2011), Battiston et al. (2012), Erol and Ordonez (2017), and Aldasoro et al. (2017)). This paper contributes to this literature by developing a tractable model of a large and interconnected system in which optimal policies are obtained in closed form.

II. Baseline Model

The baseline model is motivated by a large financial system in which companies are highly interconnected. The model has two main features. First, the network of inter-company relationships may function as a distress propagation mechanism, which potentially generates large cascading failures. Second, a policymaker, who seeks to minimize the likelihood of large cascading failures, is uncertain which companies are more likely to propagate distress during times of economic stress.

A. The environment

Consider a highly interconnected financial system inhabited by n different companies, with n being potentially large. Let $\mathcal{G}_n$ denote the network of relationships among n companies, where edges represent relationships and nodes represent companies.[1]

Time is discrete and indexed by $t = \{0, 1\}$. At $t = 0$, a policymaker designs and

[1]Relationships may summarize complex linkages associated with common ownership in various asset classes or credit relationships.

implements a policy to minimize the likelihood of large cascading failures at $t = 1$. At $t = 1$, cascading failures may occur if the distress of one company causes distress to that company's neighbors, which may cause distress to the neighbors' neighbors, and so on. Large cascading failures occur when a finite fraction of companies in an infinite network face distress as a result of the distress of one company.[2]

For tractability, the propagation of distress at $t = 1$ is determined by the following stochastic process. First, one randomly chosen company faces distress. Second, distress spreads via randomly selected relationships, henceforth referred to as susceptible links. Every company connected to the first company that faces distress via a sequence of susceptible links faces distress as well. I assume that the resulting distribution of susceptible links over companies follows a power law, and hence, the probability that a company in $\mathcal{G}_n$ has k susceptible links, $\mathbb{P}_n(k)$, is proportional to $k^{-\alpha}$,

$$\mathbb{P}_n(k) \ \propto \ k^{-\alpha} \ , \ k = \{1, 2, \cdots, n-1\} \, , \tag{1}$$

with $\alpha > 2$. Thus, the distribution of the number of susceptible links per company has fat right tails, as there tend to be more companies with a large number of susceptible links than there would be if susceptible links were selected uniformly at random.[3]

B. *The policymaker's problem*

To minimize the likelihood of large cascading failures, the policymaker imposes restrictions—which may represent capital, liquidity, or reserve requirements—on a fraction p of companies at $t = 0$, taking into account that imposing restrictions is costly. Because those restrictions aim to allow companies to absorb losses during times of economic stress, restricted companies

[2] In practice, this idea means that a fixed fraction of companies in a large but finite network face distress as a result of the distress of one company.

[3] Power law distributions appear in a variety of situations involving physical, biological, technological, economical, and social systems (see, for example, Newman (2005) and Gabaix (2009, 2016)).

are assumed to become resilient to distress at $t = 1$.[4] Thus, the policymaker solves

$$\begin{aligned} \min_{p} \quad & \beta \times \mathbb{P}\left[\text{Large cascading failures occur}|p\right] + (1-\beta) \times C(p) \\ \text{s.t.} \quad & 0 \leq p \leq 1 \end{aligned} \tag{2}$$

where $C(p) = p^{\gamma}$, with $\gamma > 0$ for $0 \leq p < 1$, denotes the total cost of implementing the aforementioned policy. I assume that imposing restrictions on all companies is not feasible, and thus, $C(1) = +\infty$. Parameter $0 < \beta < 1$ captures the importance of the system's vulnerability to large cascading failures relative to the ease of implementing restrictions.

III. The Rise of Large Cascading Failures

I next analyze the limiting behavior of a sequence of financial systems and characterize how the limiting link structure of such a sequence affects the likelihood of large cascading failures. Let $\{\mathcal{G}_n\}_{n\in\mathbb{N}}$ denote a sequence of financial systems, indexed by the number of companies n. For any given distribution of susceptible links, let $\mathcal{S}\left(\mathcal{G}_n\right)$ denote the largest subset of companies in $\mathcal{G}_n$ in which any two companies are connected via at least one sequence of susceptible links, and let $|\mathcal{S}\left(\mathcal{G}_n\right)|$ denote the cardinality of such a set. To determine when large cascading failures occur, one can use the following idea, similar to the one proposed by Cohen et al. (2000). Let n_0 denote a sufficiently large natural number. Take the subsequence $\{\mathcal{G}_n\}_{n\geq n_0}$ and assume there is a susceptible link between companies i and j in any given $\mathcal{G}_n \in \{\mathcal{G}_n\}_{n\geq n_0}$, with both companies belonging to each element in the sequence $\{\mathcal{S}\left(\mathcal{G}_n\right)\}_{n\geq n_0}$. If there is at least one susceptible link between i and another company—and loops of susceptible links can be ignored—then large cascading failures occur; otherwise, each number in the sequence $\{\mathbb{E}|\mathcal{S}\left(\mathcal{G}_n\right)|\}_{n\geq n_0}$ is finite and becomes negligible as n grows large.[5]

[4] As a consequence, susceptible links of restricted companies no longer propagate distress. Therefore, one can assume that those links are removed from the link structure that allows distress to propagate at $t = 1$.

[5] As n grows large, loops of susceptible links can be ignored for $\frac{\mathbb{E}_n(k^2)}{\mathbb{E}_n(k)} < 2$. For more details, see Cohen et al. (2000).

This condition can be written as

$$\lim_{n\to\infty} \mathbb{E}_n \left[k_i | i \leftrightarrow j\right] \;=\; \lim_{n\to\infty} \sum_{k_i} k_i \mathbb{P}_n \left[k_i | i \leftrightarrow j\right] \;=\; 2, \tag{3}$$

where $\mathbb{P}_n \left[k_i | i \leftrightarrow j\right]$ denotes the probability that company i has k_i susceptible links, given that i and j are connected via one of those links. It follows from Bayes' rule that equation (3) is equivalent to

$$\lim_{n\to\infty} \frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]} \;=\; 2, \tag{4}$$

which relates the first and second moments of the distribution of susceptible links in the limit. Equation (4) establishes the condition under which large cascading failures arise: If there is enough variation across companies in their number of susceptible links, distress affecting one company almost surely affects a large fraction of companies in the limit.

IV. Optimal Policy

A. *If the set of most connected companies at $t = 1$ is unknown.*

Because $\mathbb{P}_n(k)$ has fat right tails, the policymaker can effectively fragment the largest sequence of susceptible links by imposing restrictions on companies with the largest number of susceptible links at $t = 1$. However, the policymaker may have no information about such companies. In this case, she can solve problem (2) by imposing restrictions on a fraction p of companies uniformly at random.

To see how the distribution of susceptible links changes after implementing this policy, consider a company with k_0 susceptible links. After imposing restrictions, such a company may have k susceptible links, with $k \leq k_0$. In addition, the probability that a subset of k neighbors is not resilient to distress is $(1-p)^k$, whereas the probability that the remaining neighbors are resilient to distress is p^{k_0-k}. Because there are $\binom{k_0}{k}$ different subsets of k neighbors, the new distribution of susceptible links is

$$\mathbb{P}'_n(k) \;=\; \sum_{k\geq k_0} \mathbb{P}_n(k_0)\binom{k_0}{k}(1-p)^k p^{k_0-k}. \tag{5}$$

Thus, the probability that a company with k_0 susceptible links ends up with k susceptible links after the above policy is implemented follows a Binomial distribution. As a consequence,

$$\mathbb{E}'_n[k] = \mathbb{E}_n[k](1-p) \quad \text{and} \quad \mathbb{E}'_n[k^2] = \mathbb{E}_n[k^2](1-p)^2 + \mathbb{E}_n[k]p(1-p),$$

where quantities with superscript prime denote quantities after implementing restrictions. Then, the condition that determines the rise of large cascading failures can be rewritten as $\lim_{n\to\infty} \frac{\mathbb{E}'_n[k^2]}{\mathbb{E}'_n[k]} = 2$, which implies

$$1-p \;=\; \frac{1}{\lim_{n\to\infty}\left(\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]}\right)-1}. \tag{6}$$

If $\alpha \leq 3$, p tends to one as n grows large.[6] In this case, large cascading failures almost surely occur because many companies are still connected to companies with a large number of susceptible links even after a large fraction of companies is restricted at random. However, if $\alpha > 3$ then $\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]}$ does not diverge as n grows large.[7] In this case, $\mathbb{P}\left[\text{Large cascading failures occur}|p\right] = 0$ if a fraction $p \geq p_c$ of companies are restricted, as the largest sequence of susceptible links is fragmented by resilient companies. The threshold p_c is determined by substituting equation (7) into equation (6), which leads to $p_c = 1 - \frac{1}{\left|\frac{2-\alpha}{3-\alpha}\right|-1}$. The threshold p_c is well-defined if $\alpha \leq 4$.

[6]Because $\mathbb{P}_n(k)$ follows a power-law, $\mathbb{E}_n[k^2] \propto \left(\frac{1}{3-\alpha}\right)\left((n-1)^{3-\alpha}-1\right)$. Thus, $\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]}$ diverges as n grows large.

[7]Note

$$\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]} = \left(\frac{2-\alpha}{3-\alpha}\right)\left(\frac{(n-1)^{3-\alpha}-1}{(n-1)^{2-\alpha}-1}\right) \;\approx\; \left|\frac{2-\alpha}{3-\alpha}\right|, \text{ for sufficiently large } n. \tag{7}$$

PROPOSITION 1: *If the policymaker has no information about the set of companies with the largest number of susceptible links at $t = 1$, the solution of problem (2) is given by*

$$p = \begin{cases} 1 - \frac{1}{\left|\frac{2-\alpha}{3-\alpha}\right| - 1} & \textit{if} \quad 3 < \alpha \leq 4 \ \textit{and} \ (1-\beta)\, C(p_c) < \beta \\ 0 & \textit{otherwise.} \end{cases}$$

Proof. If $\alpha \leq 3$, then $p = 0$ solves (2) as $C(1) = +\infty$. If $3 < \alpha \leq 4$, then $p = p_c$ solves (2) if $(1-\beta)C(p_c) < \beta$. Otherwise, $p = 0$ solves (2). □

B. *If the set of most connected companies at $t = 1$ is known.*

When solving (2), suppose the policymaker recognizes whether companies will have more than K susceptible links at $t = 1$, with K being a fixed number. If the policymaker restricts all companies with more than K susceptible links, the maximum number of susceptible links per company is K, and the distribution of susceptible links per company changes as a large number of susceptible links are removed. Because all companies with more than K susceptible links are restricted, the fraction of restricted companies in the limit, p_K, equals $\lim_{n\to\infty} \sum_{k=K}^{n-1} \mathbb{P}_n(k) = p_K$, which implies that

$$K = p_K^{1/(1-\alpha)}. \tag{8}$$

Restricting a fraction p_K of companies results in an approximate random removal of susceptible links from non-restricted companies. In the limit, the probability $\widetilde{p}$ that a susceptible link leads to a restricted company equals the ratio of the number of susceptible links of restricted companies to the total number of susceptible links. Therefore,

$$\widetilde{p} = \lim_{n\to\infty} \left(\frac{1}{\mathbb{E}_n[k]}\right) \left(\sum_{k=K}^{n-1} k\mathbb{P}_n(k)\right) = K^{2-\alpha}. \tag{9}$$

Because the distribution of susceptible links that remains after implementing the above policy is equivalent to the distribution of susceptible links that remains after imposing restrictions to a fraction $\widetilde{p}$ of companies at random—see Cohen et al. (2001)—results in section IV.A can be used to compute the threshold from which large cascading failures arise. It follows from equations (8) and (9) that $\widetilde{p} = p_K^{(2-\alpha)/(1-\alpha)}$. Substituting $(n-1)$ by $K = p_K^{1/(1-\alpha)}$ in equation (7) yields $\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]} = \left(\frac{2-\alpha}{3-\alpha}\right)\left(\frac{p_K^{(3-\alpha)/(1-\alpha)}-1}{p_K^{(2-\alpha)/(1-\alpha)}-1}\right)$. The threshold that determines the rise of large cascading failures is determined from rewriting equation (6) using $\widetilde{p}$ instead of p and the new expression for $\frac{\mathbb{E}_n[k^2]}{\mathbb{E}_n[k]}$. It then follows that

$$p_K^{\frac{2-\alpha}{1-\alpha}} - \left(\frac{2-\alpha}{3-\alpha}\right) p_K^{\frac{3-\alpha}{1-\alpha}} + \left(\frac{2-\alpha}{3-\alpha}\right) - 2 \quad = \quad 0. \tag{10}$$

Equation (10) can be solved numerically for different values of α. In this case, if the policymaker imposes restrictions on a fraction $p > p_K$ of companies then the probability of large cascading failures is zero. It is worth noting that the solution of (10) is strictly smaller than 1. Thus, the system's resilience can always be improved as the policymaker keeps distress locally confined by imposing restrictions on the most connected companies at $t = 1$. This result is in sharp contrast with the case when the set of most connected companies is unknown, as large cascading failures almost surely occur if $\alpha \leq 3$.

PROPOSITION 2: *If the policymaker knows the set of most connected companies at $t = 1$, then the solution of problem (2) is given by*

$$p \quad = \quad \begin{cases} p_K & \text{if } \beta > (1-\beta)\, C(p_K) \\ 0 & \text{otherwise,} \end{cases}$$

where p_K is the solution of equation (10).

Proof. It follows directly from the previous discussion and Proposition 1. $\square$

V. Concluding Remarks

I develop a simple model to study the problem of a policymaker who wants to improve the resilience of a large, highly interconnected financial system. As in reality, the policymaker is uncertain how distress propagates among related companies during times of economic stress. Although the model may not capture the economic incentives underlying the formation of relationships among companies and the reasons some companies may be more prone to propagating distress, the model provides a first approximation of the problem faced by such a policymaker. As a first approximation, the model provides a benchmark to which other models can be compared.

Propositions 1 and 2 underscore that the ability of a policymaker to improve system resilience depends on her information. In particular, if the policymaker does not know the set of most connected companies during times of economic stress, she cannot always improve system resilience. When she is able to improve system resilience, her optimal policy depends on the ease of implementing restrictions.

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