Designing Resilient Economies

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ABSTRACT

I study the problem of a policymaker who wants to improve the resilience of an economy during times of economic stress but she is unsure of how distress propagates among entities in those conditions. In particular, I explore how the resilience of an economy is improved if the policymaker ex-ante improves the resilience of a fraction of the most likely connected entities during stressful conditions. The model provides a novel link between the uncertainty faced by the policymaker and the identification of systemically important entities.

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The question of how stable are modern economies to stressful conditions has received a lot of attention recently. Given the highly interconnectedness of modern financial systems, an entity-specific shock during stressful conditions may spread to a non-negligible fraction of entities in the broad economy and potentially generate an economic crisis. Despite the importance of this question, there is limited theoretical guidance on what a policymaker should do to improve the resilience of an economy, acknowledging the fact that she may be unsure of how distress propagates among entities during stressful conditions. This paper fills this gap by developing a model that links the resilience of a network economy during stressful conditions and the intrinsic uncertainty faced by the policymaker when designing her policies.

I. Baseline Model

The baseline model provides a tractable framework for modeling how distress propagates among entities during times of economic stress. For tractability, the model assumes that during times of economic stress the link structure that potentially enables an entity-specific shock to spread over the broad economy is randomly generated. Despite the fact that the model fails to capture the economic incentives that underlie the formation of inter-entity links, it helps to better understand how different policies may generate large changes in the resilience of an economy to stressful conditions.

A. *The environment*

Consider an economy inhabited by n entities, with n being potentially large. Each entity is connected to every other entity in the economy via a link—which may summarize a variety of linkages among entities such as credit relationships, strategic alliances, partnerships, and common holdings to name a few. During times of economic stress, entities are in one of two states, either distressed and non-distressed, and inter-entity links are either prone to propagate distress or not. In particular, in times of economic stress, the probability that a randomly chosen entity has k links prone to propagate distress follows a power-law distribution,

$$\mathbb{P}(k) = ck^{-\alpha} \ , \ k = \{1, 2, \cdots, n-1\} \, , \tag{1}$$

with $c \geq 0$ and $\alpha \geq 2$. Thus, the distribution of the number of links prone to propagate distress per entity has fat tails as there tend to be more entities with a very small and very large numbers of links prone to propagate distress than there would be if those links were selected uniformly at random.[1]

During times of economic stress, the propagation of distress from one entity to another is determined by the following stochastic process. First, some links are randomly selected to be prone to propagate distress so that equation (1) is satisfied. Second, a randomly chosen entity faces distress. Immediately after, distress spreads from such an entity via those links selected to be prone to propagate distress. In particular, every entity connected to the entity that initially faces distress via a sequence of links prone to propagate distress is considered to face distress as well.

There is a policymaker who wants to improve the resilience of the economy during times of economic stress, but she is unsure of how distress spreads over the economy in those conditions. In particular, the policymaker knows the stochastic process that determines the propagation of distress from one entity to another, but she is unsure about the true value of α in equation (1).

B. *The policymaker's problem*

When n is sufficiently large, the probability that the economy collapses during times of economic stress equals the probability of the existence of a spanning cluster, which is a cluster of entities whose size is proportional to the size of the economy in which any two entities are connected via a sequence of links prone to propagate distress. This is because the entity that initially faces distress in times of economic stress is almost surely connected to one entity within the spanning cluster. As a result, the policymaker can improve the resilience of the economy to stressful conditions if she is able to decrease the probability of existence of a spanning cluster.

To decrease the probability of the existence of a spanning cluster, the policymaker imposes ex-ante restrictions on a set of entities so that such entities become resilient to distress, and in doing so, potentially stop the propagation of distress over the economy in times of economic stress. In particular, the policymaker follows two strategies:

(S1) impose restrictions on a fraction p of randomly chosen entities, or
(S2) impose restrictions on a fraction p of the most likely connected entities in times of economic stress.

In both strategies, the probability that the economy collapses during times of economic stress

[1]Power law distributions occur in a large variety of situations involving physical, biological, technological, economical and social systems of various kinds; see Newman (2005), Gabaix (2009, 2016).

becomes a weakly decreasing function of p, denoted by $G(p)$, where

$$G(p) \quad = \quad \mathbb{P}\left[\text{spanning cluster exists}|p\right]. \tag{2}$$

Because the more entities are subject to ex-ante restrictions, the harder it is for the policymaker to implement her policy, the cost of making a fraction p of entities in the economy resilient to distress is captured by the function

$$C(p) \quad = \quad p^{\gamma}\ , \quad \text{with } \gamma\ \geq 0. \tag{3}$$

When designing her policies, the policymaker balances the likelihood that the economy collapses during stressful conditions and the implementation costs, and thus, she solves

$$\min \quad \left\{ \underbrace{\min_{p}\left[\beta G(p) + (1-\beta) C(p)\right]}_{\text{Strategy 1}},\ \underbrace{\min_{p}\left[\beta G(p) + (1-\beta) C(p)\right]}_{\text{Strategy 2}} \right\} \tag{4}$$

for a given $\beta \in (0, 1)$.

II. Designing Policies

A. α is known

Consider the policymaker knows α.

A.1. Strategy 1

LEMMA 1: *If the policymaker follows Strategy 1 then p is given by*

$$p \quad = \quad \begin{cases} p_r & \textit{if} \quad 3 < \alpha < 4 \ \textit{and} \ (1-\beta)\, p_r^{\gamma} < \beta \\ 0 & \textit{otherwise} \end{cases}$$

with

$$p_r \quad = \quad 1 - \frac{1}{\left|\frac{2-\alpha}{3-\alpha}\right| - 1} \tag{5}$$

for n sufficiently large.

A.2. Strategy 2

LEMMA 2: *Let θ denote the fraction of entities in the economy that are most highly connected in times of economic stress. If the policymaker follows Strategy 2 then p is given by*

$$p = \begin{cases} p_h & \text{if } \beta > (1-\beta)\left(\theta p_h\right)^{\gamma} \\ 0 & \text{otherwise} \end{cases}$$

for n sufficiently large, where p_h solves

$$p_h^{\frac{2-\alpha}{1-\alpha}} - \left(\frac{2-\alpha}{3-\alpha}\right) p_h^{\frac{3-\alpha}{1-\alpha}} + \left(\frac{2-\alpha}{3-\alpha}\right) - 2 = 0. \quad (6)$$

A.3. Optimal Strategy

PROPOSITION 1: *For n sufficiently large and any given α, strategy 2 solves problem 4.*

B. α is unknown

As α is a random variable, p_h is also random. In particular, equation (6) defines the probability density function for p_h as a function of the distribution of α. If $f_{p(\alpha)}$ denotes the probability density function of p_h then

$$\mathbb{P}\left[p_h \leq p\right] = \int_0^p f_{p(\alpha)}(x)dx \quad (7)$$

The first order conditions of problem 4 can be written as

$$\beta \frac{\partial}{\partial p}\left(1 - \int_0^p f_{p(\alpha)}(x)dx\right) + (1-\beta)\frac{\partial}{\partial p}\left(\theta^{\gamma} p^{\gamma}\right) = 0 \quad (8)$$

Using Leibniz's rule, it directly follows from (6) that the smallest fraction of most highly connected entities the policymaker needs to make resilient to distress so that the economy does not collapse in stressful conditions, which I denoted by p^*, solves the following equation:

$$\beta f_{p(\alpha)}(p^*) = \begin{cases} (1-\beta)\gamma\theta^{\gamma}(p^*)^{\gamma-1} & \text{if } \gamma \neq 1 \\ (1-\beta)\theta & \text{otherwise} \end{cases} \quad (9)$$

III. Comparative Statics

IV. Conclusion

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Appendix: Derivation of formulas

This section contains the derivations of formulas, propositions and corollaries in the body of the paper. The following computations consider four assumptions:

ASSUMPTION 1: *During stressful conditions, distress affecting one entity also affects another entity if there is at least one sequence of relationships between the two entities.*

ASSUMPTION 2: *During stressful conditions, the probability that an entity chosen at random is connected with k other entities follows a power-law distribution*

$$\mathbb{P}(k) = ck^{\alpha} \ , \ \ k = \{1, 2, \cdots, n-1\} \, , \tag{1}$$

where c is a constant and n represents the number of entities in the economy.

ASSUMPTION 3: *Parameter α in assumption 2 is a random variable with a continuous distribution G with finite support $[\underline{\alpha}, \overline{\alpha}]$.*

ASSUMPTION 4: *Increasing the fraction of entities that are resilient to distress during stressful conditions is costly. In particular, if the policymaker decides to ensure that a fraction p of the most connected firms are resilient, she needs to pay $C(p) = p^{\gamma}$, with $\gamma \in (0, 1)$.*

Policymaker's problem

Suppose the policymaker knows with certainty the set of most highly connected entities during stressful conditions. Given $\beta \in (0, 1)$, the policymaker solves

$$\min_{p} \qquad \beta \times \mathbb{P}\left[\text{economy collapses during stressful conditions}|p\right] + (1-\beta) \times p^{\gamma} \tag{2}$$

where p denotes the fraction of most highly connected entities the policymaker imposes restrictions to, so that these entities become resilient to distress during stressful conditions.

REMARK 1: $\mathbb{P}\left[\textit{economy collapses during stressful conditions}|p\right]$ *is a decreasing function of p.*

REMARK 2: p^{γ} *is an increasing function of p.*

Provided how distress spreads during stressful conditions, the economy collapses as long as a giant component, which is the unique largest component that contains a non-trivial fraction of entities in the economy, arises during stressful conditions.

For a given α, let p_{α} denote the smallest fraction of most highly connected entities the policymaker needs to make resilient to distress, so that the economy does not collapse during stressful conditions. Then

$$\begin{aligned} \mathbb{P}\left[\text{economy collapses during stressful conditions}|\ p \wedge \alpha\right] &= \mathbb{P}\left[p < p_{\alpha}\right] \\ &= 1 - \mathbb{P}\left[p_{\alpha} \leq p\right] \end{aligned} \tag{3}$$

If n is sufficiently large and α is known, Cohen et al. (2001) show that p_{α} solves:

$$p_{\alpha}^{\frac{2-\alpha}{1-\alpha}} - \left(\frac{2-\alpha}{3-\alpha}\right) p_{\alpha}^{\frac{3-\alpha}{1-\alpha}} + \left(\frac{2-\alpha}{3-\alpha}\right) - 2 \ = \ 0 \tag{4}$$

which implies that p_α can be written as a function of α by the implicit function theorem. Provided that α is a random variable, equation (4) defines the probability density function for p_α. If $f_{p(\alpha)}$ denotes the probability density function of p_α then

$$\mathbb{P}\left[p_\alpha \leq p\right] \quad = \quad \int_0^p f_{p(\alpha)}(x)dx \tag{5}$$

The first order conditions of problem 2 can be written as

$$\beta \frac{\partial}{\partial p}\left(1 - \int_0^p f_{p(\alpha)}(x)dx\right) + (1-\beta)\frac{\partial}{\partial p}\left(p^\gamma\right) \quad = \quad 0 \tag{6}$$

Using Leibniz's rule, it directly follows from (6) that the smallest fraction of most highly connected entities the policymaker needs to make resilient to distress so that the economy does not collapse in stressful conditions, which I denoted by p^*, solves the following equation:

$$\beta f_{p(\alpha)}(p^*) \quad = \quad (1-\beta)\gamma(p^*)^{\gamma-1} \tag{7}$$