The Unruh effect and it's generalization to curved spacetimes

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Unruh effect in Minkowski

In the formulation of quantum field theory in curved spacetimes, the particle concept is observer-dependent. For stationary spacetimes, it can be built with the aid of timelike Killing vector fields. The high degree of symmetry in Minkowski spacetime time, described by the metric

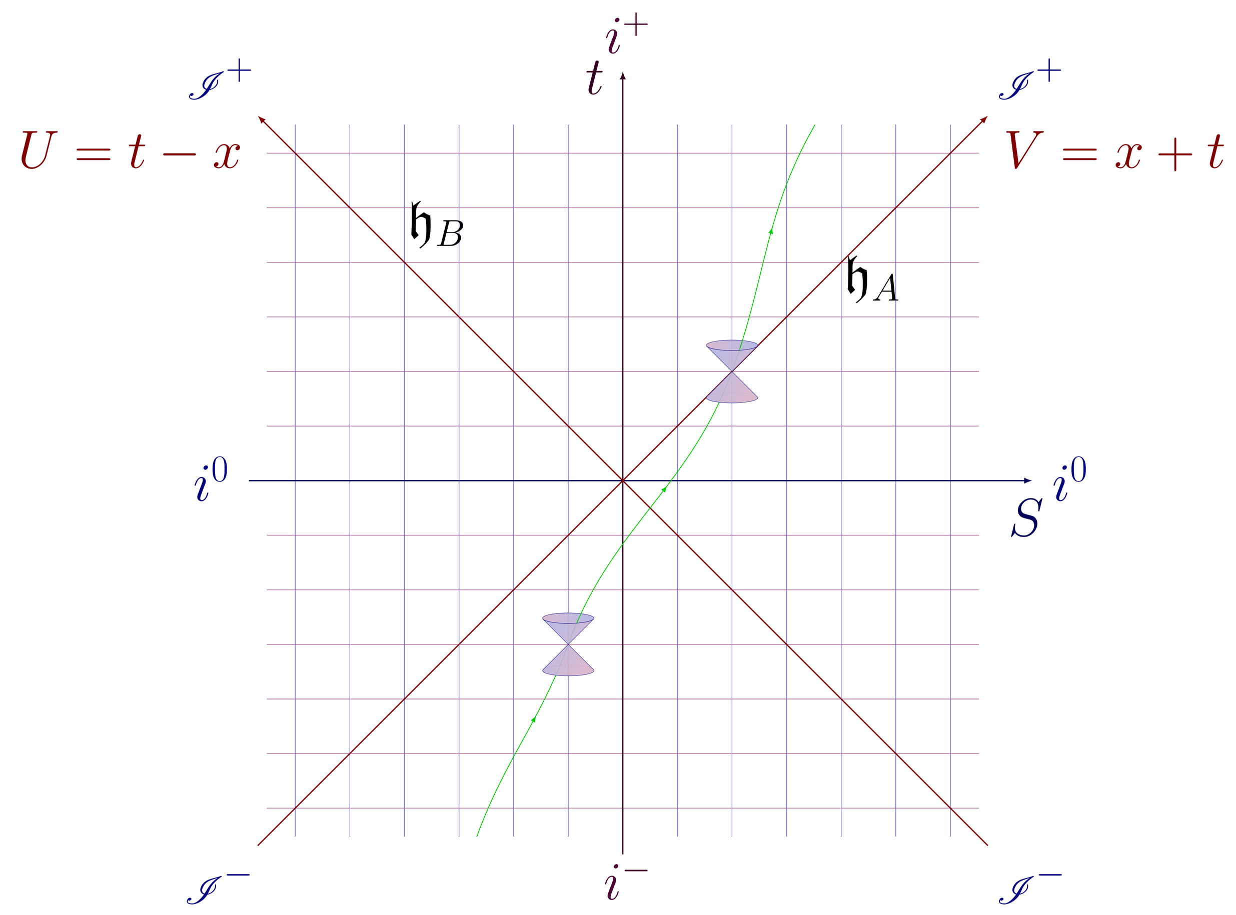
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \tag{1}$$

allows many possible Killing fields, consider the following two

$$\xi^a = (\partial_t)^a \quad \text{and} \quad \chi^a = a\left[x\,(\partial_t)^a + t\,(\partial_x)^a\right]. \tag{2}$$

Observers that follows the orbits of χ^a can be interpreted as being uniformly accelerated. The second Killing field separates the spacetime in four regions, with the boundaries being the surfaces where it is null, $\mathfrak{h}_A$ and $\mathfrak{h}_B$:

$$\begin{array}{ll} \text{Region I} = I^-(\mathfrak{h}_A) \cap I^+(\mathfrak{h}_B) & \text{Region II} = I^+(\mathfrak{h}_A) \cap I^-(\mathfrak{h}_B) \\ \text{Region III} = J^-(S) & \text{Region IV} = J^+(S) \end{array}. \tag{3}$$



On $\mathfrak{h}_A$ ($\mathfrak{h}_B$), modes are positive-frequency with respect to retarded (advanced) time V (U) if, and only if, they are positive-frequency with respect to the inertial one t. Moreover, the Killing time v (u) induced by χ^a are related to the inertial ones on $\mathfrak{h}_A$ ($\mathfrak{h}_B$) by

$$v = \frac{1}{a}\ln|V| \quad \text{and} \quad u = -\frac{1}{a}\ln|U|. \tag{4}$$

Analyzing pure positive frequencies on the horizon, allows us to find a Bogoliubov transformation S that relate the two possible constructions. If $|0\rangle_M$ represents the Minkowski inertial vacuum, we have

$$S\,|0\rangle_M = \prod_j \left(\sum_{n=0}^{\infty} e^{-\frac{\pi n \omega_j}{a}} |n_j\rangle_I \otimes |n_j\rangle_{II}\right), \tag{5}$$

where $|n_j\rangle_I$ represents a state with n particles in the basis mode $\{\psi_j^I\}$ of region I, the same stands for region II. The density matrix restricted to region I is

$$\rho^I = \prod_j \left(\sum_{n=0}^{\infty} e^{-\frac{2n\omega_j}{a}} |n_j\rangle_I \langle n_j|_I\right). \tag{6}$$

This result is compatible with the statement that uniformly accelerated observers interpret the Minskowski vacuum as a thermal bath of particles at a temperature

$$T = \frac{a}{2\pi} \cong \frac{a}{10^{19}\text{m/s}^2}\text{K}. \tag{7}$$

Unruh effect in curved spacetimes

The main feature used to derive the Unruh effect is the existence of the surfaces $\mathfrak{h}_A$ and $\mathfrak{h}_B$. Such a structure can be generalized by bifurcate Killing horizon, associated with a Killing field ξ^a, that induce a spacetime separation as in the previous description. In this case, the surface gravity of the horizon, κ, plays a key role. It is given by

$$\kappa = \lim_{\mathfrak{h}}(a\xi), \tag{8}$$

where a is the norm of the 4-acceleration of a observer whose worldline is one orbit of ξ^a. Moreover, we can use it, together with the induced Killing time v (u), to obtain an affine parameter V (U) for the null geodesics that generate the horizons. The expressions obtained are

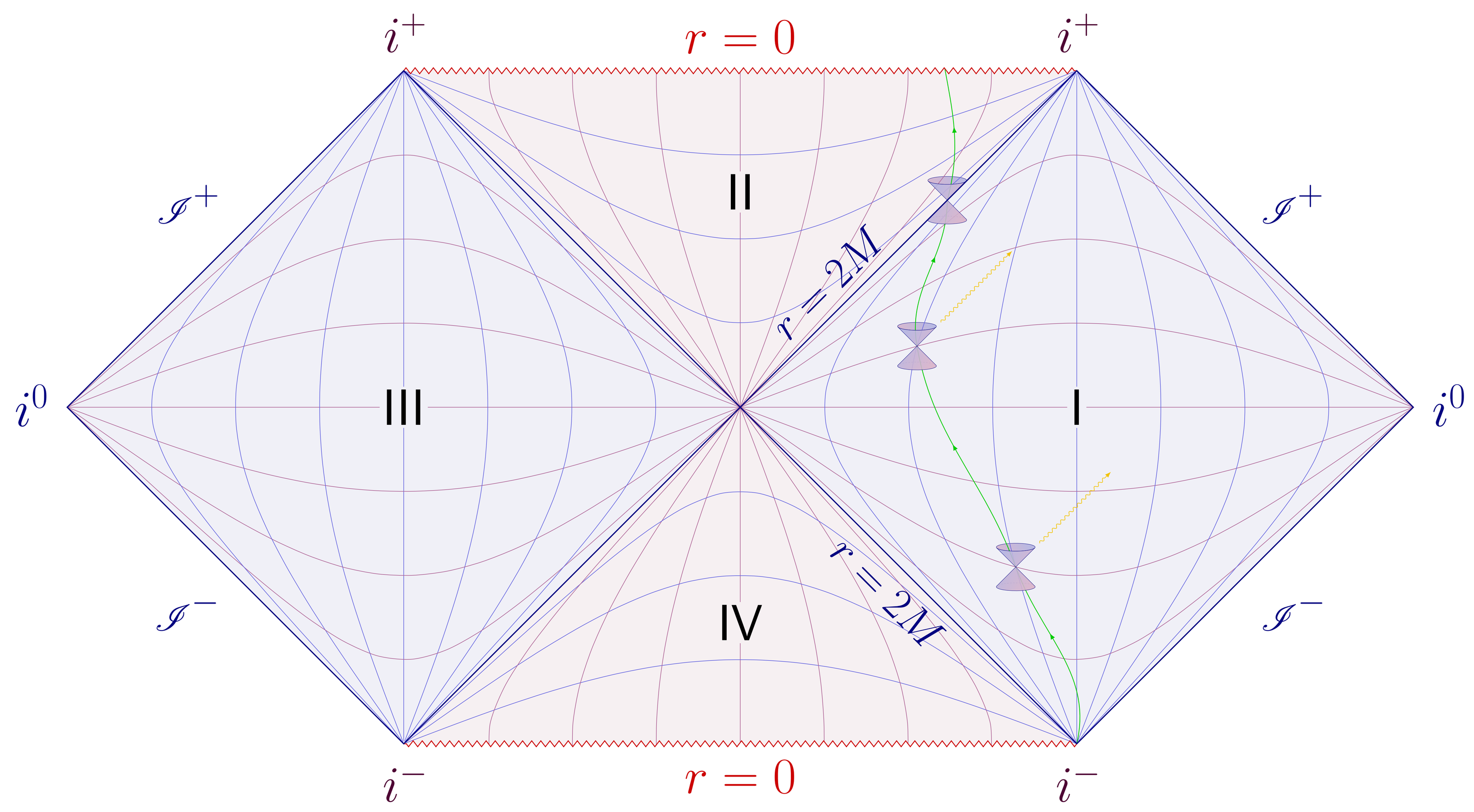
$$v = \frac{1}{\kappa}\ln|V| \quad \text{and} \quad u = -\frac{1}{\kappa}\ln|U|. \tag{9}$$

Henceforth, we pick a pure, quasifree state that is non-singular and invariant under the isometries generated by ξ^a to be the vacuum state. Then, due to the same procedure done in Minkowski, we find that observers that follows the orbits of the Killing field perceive this vacuum state as a particles thermal bath at a temperature

$$T = \frac{\kappa}{2\pi}. \tag{10}$$

For example, consider the extended Schwarschild spacetime whose metric is

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{11}$$



We pick the Killing field $\xi^a = (\partial_t)^a$ that leads to an acceleration $a^\mu = M/r^2\,(\partial_r)^\mu$. Therefore, we find

$$a\xi = \frac{M}{r^2}, \tag{12}$$

then, the surface gravity of the event horizon is

$$\kappa = \lim_{r\to 2M}\frac{M}{r^2} = \frac{1}{4M}. \tag{13}$$

Thus, if the field state is in the Hartle-Hawking vacuum (a state that satisfies all the required conditions), observers in the orbits of ξ^a ought to see a particle thermal bath at temperature

$$T = \frac{1}{8\pi M} \cong \frac{6\times 10^{-8} M_\odot}{M}\text{K}. \tag{14}$$

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References

[1] R. M. Wald. *Quantum field theory in curved spacetimes and black hole thermodynamics.* 1994.