Proposta de resolução

1. (b_n) é uma progressão aritmética, pois:

$$b_{n+1} - b_n = \ln(a_{n+1}) - \ln(a_n) = \ln\left(\underbrace{\frac{a_{n+1}}{a_n}}_{(a_n)\text{ p.g.}}\right) = \ln(e) = 1$$

A razão da progressão aritmética é igual a 1

$$b_1 = \ln 2$$

$$b_n = b_1 + (n-1) \times r \Leftrightarrow b_n = \ln 2 + n - 1$$

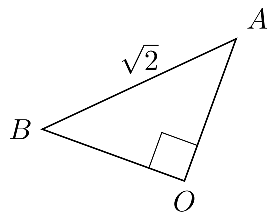
$$b_{10} = \ln 2 + 9$$

$$S_{10} = \frac{b_1 + b_{10}}{2} \times 10 = \frac{\ln 2 + \ln 2 + 9}{2} \times 10 = \left(\ln 4 + \ln e^9\right) \times 5 = 5\ln\left(4 \cdot e^9\right)$$

Opção **(B)**

2.

$$\overline{AB} = \frac{\sqrt{2}}{4} = \sqrt{2}$$



$$\overline{OA}^2 + \overline{OB}^2 = \sqrt{2}^2 \Leftrightarrow 2\overline{OA}^2 = 2 \Leftrightarrow \overline{OA} = \pm 1 \Rightarrow \overline{AB} = 1 \Leftrightarrow |z_A| = 1$$

$$z_A = e^{i\left(\frac{7\pi}{18}\right)}$$

$$z = (z_A)^4 = \left[e^{i\left(\frac{7\pi}{18}\right)}\right]^4 = e^{i\frac{14\pi}{9}} = e^{i\left(-\frac{4\pi}{9}\right)}$$

$$z_B = z_A \times i = \left(e^{i\frac{7\pi}{18}}\right) \times e^{i\frac{\pi}{2}} = e^{i\left(\frac{7\pi}{18}+\frac{\pi}{2}\right)} = e^{i\frac{8\pi}{9}}$$

$$w = (z_B)^3 = \left(e^{i\frac{8\pi}{9}}\right)^3 = e^{i\frac{8\pi}{3}} = e^{i\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z^9 + w = \left[e^{i\left(-\frac{4\pi}{9}\right)}\right]^9 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = e^{i(-4\pi)} + -\frac{1}{2} + \frac{\sqrt{3}}{2}i = 1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\left|\frac{1}{2} + \frac{\sqrt{3}}{2}i\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\text{Arg}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \arctan\sqrt{3} = \frac{\pi}{3}$$

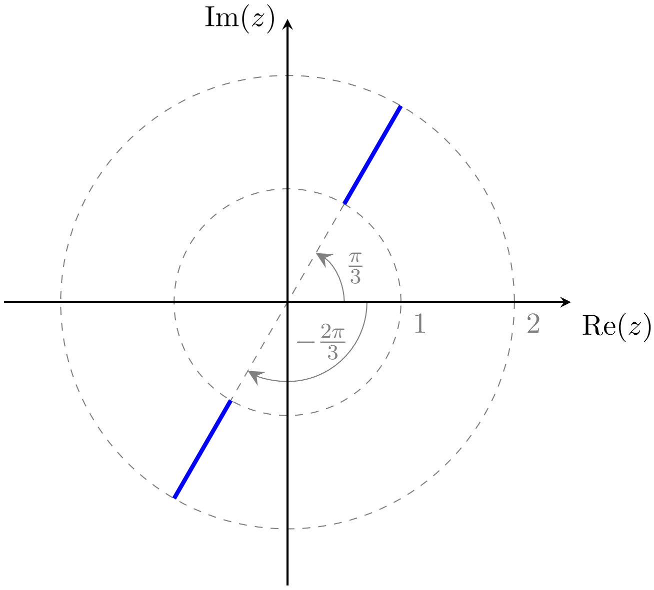
$$\therefore z^9 + w = e^{i\frac{\pi}{3}}$$

3.

$$\left|\frac{\pi}{3}+\operatorname{Arg}(z)\right|=\pi \Leftrightarrow \frac{\pi}{3}+2\operatorname{Arg}(z)=\pi \ \vee \ \frac{\pi}{3}+2\operatorname{Arg}(z)=-\pi \Leftrightarrow$$

$$\Leftrightarrow 2\operatorname{Arg}(z)=\frac{2\pi}{3} \ \vee \ 2\operatorname{Arg}(z)=-\frac{4\pi}{3} \Leftrightarrow \operatorname{Arg}(z)=\frac{\pi}{3} \ \vee \ \operatorname{Arg}(z)=-\frac{2\pi}{3}$$

$$1 \leq |z| \leq 2 \ \wedge \ \left(\operatorname{Arg}(z)=\frac{\pi}{3} \ \vee \ \operatorname{Arg}(z)=-\frac{2\pi}{3}\right)$$



Opção**(B)**

4.

$$P\left(B|A\right)=\frac{3\times 1+3\times 3}{6\times 4-3\times 1}=\frac{4}{7}$$

$P\left(B|A\right)$ é a probabilidade do produto dos valores obtidos nos dois dados ser negativo, sabendo que o ponto de coordenadas (a,b) não pertence ao primeiro quadrante.

Se o ponto de coordenadas (a,b) não pertence ao primeiro quadrante, então os valores obtidos nos dois dado não são positivos simultaneamente, ou seja, existem $6\times 4-3\times 1=21$ casos possíveis, já que 6×4 representa o total de possibilidades e 3×1 representa o número de casos em que os valores obtidos são ambos positivos.

Para que o produto dos valores obtidos nos dados seja negativo, tem que sair dois números com sinais contrários. Assim, terá que sair um valor negativo no dado cúbico e um valor positivo no dado tetraédrico (3×1 possibilidades) ou sair um valor positivo no dado cúbico e um valor negativo no dado tetraédrico (3×3 possibilidades). Então, o número de casos favoráveis é $3\times 1+3\times 3=12$.

Assim, o valor de $P\left(B|A\right)=\dfrac{12}{21}=\dfrac{4}{7}$.

5.

$$\frac{P_{[OABC]}}{4}=\overline{OC} \Leftrightarrow \overline{OC}=4$$

$$C\left(-4,0\right)$$

Seja $\alpha=B\hat{C}D$, então $2\alpha+2\times\dfrac{2\pi}{3}=2\pi \Leftrightarrow \alpha=\dfrac{\pi}{3}$

$$A\hat{C}O = \frac{\alpha}{2} = \frac{\pi}{6}$$

$$m_{AC} = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\therefore y - 0 = \frac{\sqrt{3}}{3}(x+4) \Leftrightarrow 3y = \sqrt{3}x + 4\sqrt{3} \Leftrightarrow \sqrt{3}x - 3y + 4\sqrt{3} = 0$$

Opção **(A)**

6.

$$m = \frac{1-0}{0+3} = \frac{1}{3} = \lim_{x\to+\infty}\frac{g(x)}{x}$$

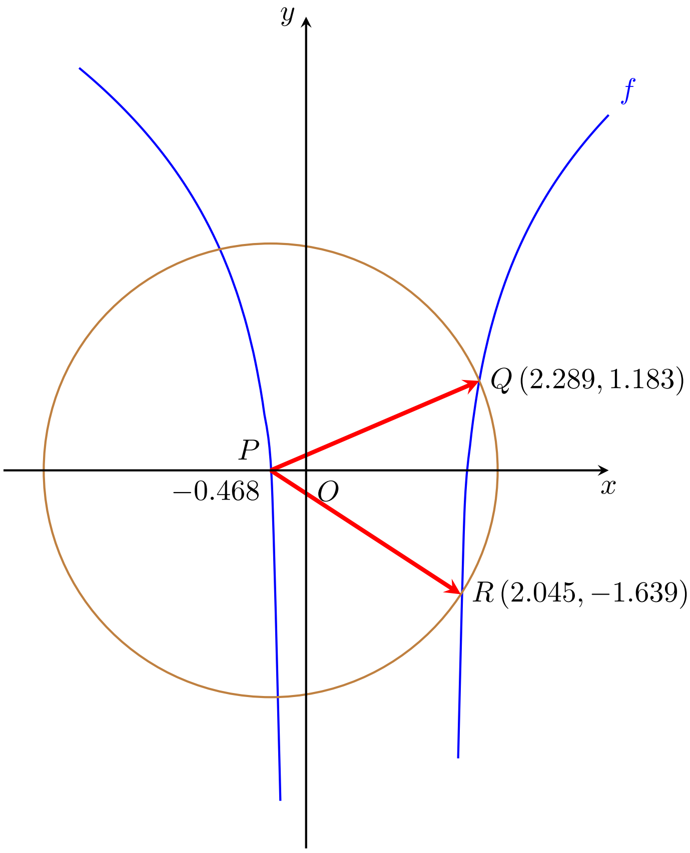
$$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty}\left(\frac{3x}{g(x)} + 1\right) = \frac{3}{\underbrace{\lim_{x\to+\infty}\frac{g(x)}{x}}_{\text{declive a.o.}}} + 1 = \frac{3}{\frac{1}{3}} + 1 = 9 + 1 = 10$$

A reta de equação $y = 10$ é assíntota horizontal ao gráfico de h.

Opção **(B)**

7.

7.1.



$$(x+0.468)^2 + y^2 = 9 \Leftrightarrow y = \pm\sqrt{9-(x+0.468)^2}$$

$$\overrightarrow{PQ} = Q - P = (2.757, 1.183)$$

$$\overrightarrow{PR} = R - P = (2.513, -1.639)$$

$$\overline{PQ} = \overline{PR} = 3$$

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 2.757 \times 2.513 - 1.183 \times 1.639 = 4.989404$$

$$\therefore \cos\left(Q\hat{P}R\right) = \frac{4.989404}{3 \times 3} \Rightarrow Q\hat{P}R = \arccos\left(\frac{4.898404}{9}\right) \approx 0.98 \text{ rad}$$

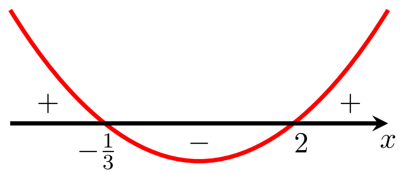
7.2.

$$D = \left\{x \in \mathbb{R} : x - 2 > 0 \ \wedge \ 3x^2 - 5x - 2 > 0 \ \wedge \ x^2 > 0\right\} =$$

$$D = \left\{x \in \mathbb{R} : x > 2 \ \wedge \ \left(x > 2 \ \vee \ x < -\frac{1}{3}\right) \ \wedge \ x \neq 0\right\} =]2, +\infty[$$

$$3x^2 - 5x - 2 = 0 \Leftrightarrow x = \frac{5 \pm \sqrt{25 - 4 \times 3 \times (-2)}}{6} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{5 \pm 7}{6} \Leftrightarrow x = 2 \ \vee \ x = -\frac{1}{3}$$



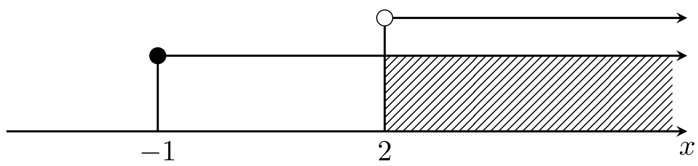
$$3x^2 - 5x - 2 = 3\left(x + \frac{1}{3}\right)(x - 2) = (3x + 1)(x - 2)$$

$$1 + \log_2(x - 2) \leq \log_2\left(3x^2 - 5x - 2\right) - \log_4\left(x^2\right) \Leftrightarrow$$

$$\Leftrightarrow \log_2 2 + \log_2(x - 2) \leq \log_2\left[(3x + 1)(x - 2)\right] - 2 \times \frac{\log_2 x}{\log_2 4} \Leftrightarrow$$

$$\Leftrightarrow \log_2 2 + \log_2 x + \log_2(x - 2) \leq \log_2(3x + 1) + \log_2(x - 2) \Leftrightarrow$$

$$\Leftrightarrow \log_2(2x) \leq \log_2(3x + 1) \Leftrightarrow 2x \leq 3x + 1 \Leftrightarrow -x \leq 1 \Leftrightarrow x \geq -1$$



$$\therefore C.S =]2, +\infty[$$

8.

$$f(\pi) = k + \ln(2 + \cos\pi) = k + \ln(2 - 1) = k$$

$$\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^-} \frac{\tan x}{\pi - x} \underbrace{=}_{y = \pi - x} \lim_{y \to 0} \frac{\tan(\pi - y)}{y} = \lim_{y \to 0} \frac{-\tan y}{y} =$$

$$= -\underbrace{\lim_{y\to 0}\frac{\sin y}{y}}_{\text{limite notável}} \times \lim_{y\to 0}\frac{1}{\cos y} = -1\times\frac{1}{1} = -1$$

$$\therefore k = -1$$

Opção **(A)**

9. Para serem extraídas todas as bolas, apenas existem duas possibilidades em termos de sequencia de cores:

- Sair ABABA
- Sair BABAA

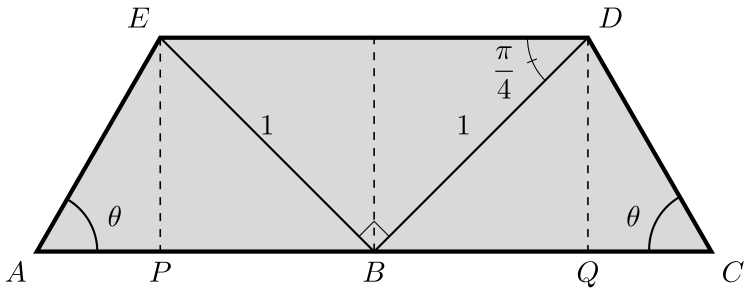
Assim,

$$p = \frac{3\times 2\times 2\times 1\times 1 + 2\times 3\times 1\times 2\times 1}{5!} = \frac{1}{5}$$

Opção **(C)**

10.

10.1.



$$\sin\frac{\pi}{4} = \frac{\overline{DQ}}{1} \Leftrightarrow \overline{DQ} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{\frac{\sqrt{2}}{2}}{\overline{QC}} \Leftrightarrow \overline{QC} = \frac{\sqrt{2}}{2\tan\theta}$$

$$\therefore A_{[ACDE]} = 2\times\left[\left(\frac{\sqrt{2}}{2}\right)^2 + \frac{\frac{\sqrt{2}}{2\tan\theta}\times\frac{\sqrt{2}}{2}}{2}\right] = 2\left(\frac{1}{2}+\frac{1}{4\tan\theta}\right) = 1 + \frac{1}{2\tan\theta}$$

10.2.

$$A'(\theta) = \left(1+\frac{1}{2\theta}\right)' = 0 + \frac{0 - 1\times\frac{2}{\cos^2\theta}}{4\tan^2\theta} = -\frac{\frac{2}{\cos^2\theta}}{4\times\frac{\sin^2\theta}{\cos^2\theta}} = -\frac{1}{2\sin^2\theta}$$

Como $\sin^2\theta > 0,\ \forall\,\theta\in\left]0,\frac{\pi}{2}\right[$, então $A'(\theta) = -\frac{1}{2\sin\theta} < 0,\ \forall\,\theta\in\left]0,\frac{\pi}{2}\right[$.

Portanto, A é estritamente decrescente em $\left]0,\frac{\pi}{2}\right[$.

11.

$${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + {}^{n+1}C_{n-1} + {}^{n+1}C_n + {}^{n+1}C_{n+1} = 258 \Leftrightarrow 1 + n + \frac{n(n-1)}{2} + \frac{(n+1)n}{2} + n + 1 + 1 = 258 \Leftrightarrow$$

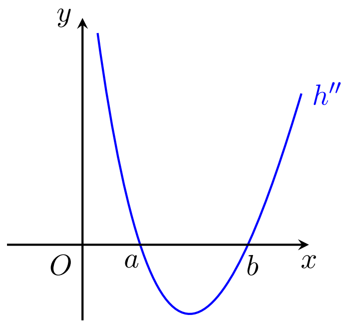
$$3 + 2n + \frac{n^2}{2} - \frac{n}{2} + \frac{n^2}{2} + \frac{n}{2} = 258 \Leftrightarrow n^2 + 2n - 255 = 0 \Leftrightarrow n = 15 \ \vee \ n = -17 \Rightarrow n = 15$$

1 15 105 455 1365 $\cdots$ 1365 455 105 15 1

$$\therefore (15 + 1) - 8 = 8$$

Opção **(C)**

12.



x	0		a		b	$+\infty$
$h''(x)$		$+$	0	$-$	0	$+$
h'		$\nearrow$	Max.	$\searrow$	Min.	$\nearrow$

Opção **(D)**

13.

13.1. $\overrightarrow{n}(5, -1, 3)$ é um vetor diretor da reta.

$$V(0, 0, z)$$
$$5 \times 0 - 0 + 3z = 12 \Leftrightarrow z = 4$$
$$V(0, 0, 4)$$

$$\therefore (x, y, z) = (0, 0, 4) + k(5, -1, 3), \ k \in \mathbb{R}$$

13.2.

$$A(x, -2, 0)$$
$$5x + 2 + 3 \times 0 = 12 \Leftrightarrow x = 2$$
$$A(2, -2, 0)$$
$$B(x, 3, 0)$$
$$5x - 3 + 3 \times 0 = 12 \Leftrightarrow x = 3$$
$$B(3, 3, 0)$$
$$\therefore V = \frac{1}{3} A_b \times h = \frac{1}{3} \times \frac{3 + 2}{2} \times 5 \times 4 = \frac{50}{3}$$

14.

14.1.

$$g'(1)=\lim_{x\to 1}\frac{g(x)-g(1)}{x-1}=\lim_{x\to 1}\frac{x^2+6e^{-x}-1-6e^{-1}}{x-1}=\lim_{x\to 1}\frac{x^2-1}{x-1}+6\lim_{x\to 1}\frac{e^{-x}-e^{-1}}{x-1}=$$

$$=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}-\frac{6}{e}\underbrace{\lim_{-x+1\to 0}\frac{e^{-x+1}}{-x+1}}_{\text{limite notável}}=2-\frac{6}{e}=\frac{2e-6}{e}$$

14.2.

$$D=\mathbb{R}$$

$$x^2+e^x=x^2+6e^{-x}+1\Leftrightarrow e^x-1-\frac{6}{e^x}=0\Leftrightarrow e^{2x}-e^x-6=0\Leftrightarrow e^x=\frac{1\pm\sqrt{1-4\times\times 1\times(-6)}}{2}\Leftrightarrow$$

$$\Leftrightarrow e^x=\frac{1\pm 5}{2}\Leftrightarrow e^x=3\ \vee\ \underbrace{e^x=-2}_{\text{eq. imp.}}\Leftrightarrow x=\ln 3$$

$$\therefore S=\{\ln 3\}$$