Proposta de resolução

1.

$$r = \frac{a_2}{a_1} = \frac{-\frac{2}{5}}{-\frac{3}{5}} = \frac{2}{3}$$

$$a_n = a_1 \times r^{n-1} \Leftrightarrow a_n = -\frac{3}{5} \times \left(\frac{2}{3}\right)^{n-1}$$

$$\lim (a_n) = \lim \left[-\frac{3}{5} \times \left(\frac{2}{3}\right)^{n-1}\right] = -\frac{3}{5} \times \left(\frac{2}{3}\right)^{+\infty} = -\frac{3}{5} \times 0^+ = 0^-$$

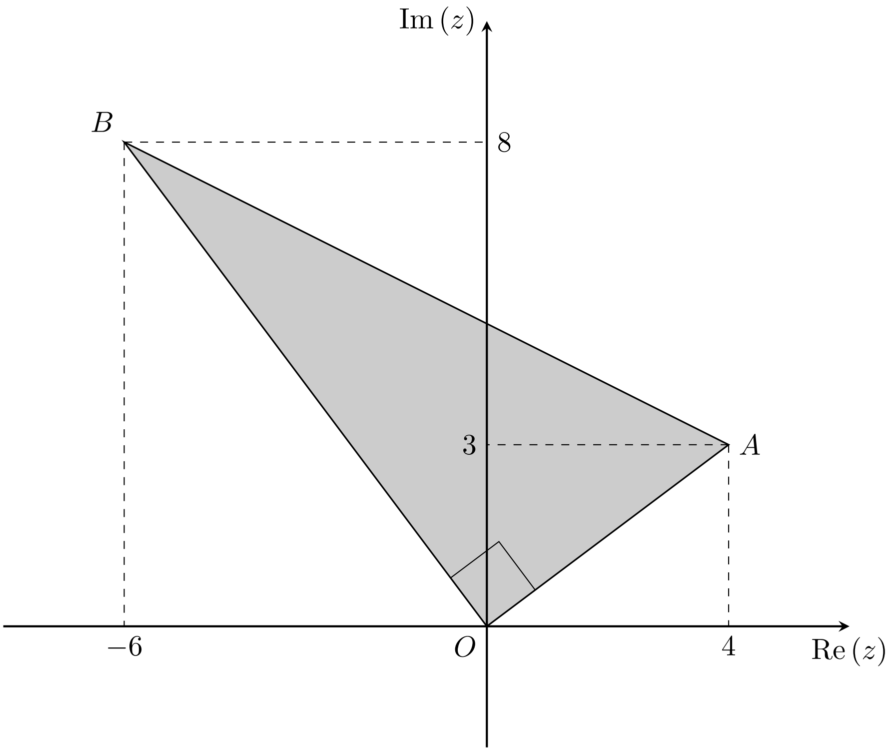
$$\lim f(a_n) = f\left(0^-\right) = 1$$

Opção **(A)**

2.

$$z_1 = \frac{(1+2i)^2}{\left[e^{i\left(\frac{\pi}{4}\right)}\right]^{10}} = \frac{1+4i-4}{e^{i\left(\frac{5\pi}{2}\right)}} = \frac{-3+4i}{i} = \frac{(-3+4i)\,i}{-1} = 4+3i$$

$$z_2 = 2i \times z_1 = 2i\,(4+3i) = 8i + 6i^2 = -6+8i$$



$$A_{[OAB]} = \frac{\overline{OA} \times \overline{OB}}{2} = \frac{\sqrt{4^2+3^2} \times \sqrt{(-6)^2+8^2}}{2} = \frac{5 \times 10}{2} = 25$$

3.

$$g(-2) = -g(2) = \frac{1}{2}$$

$$\lim_{x\to -2} \frac{g(x)+g(2)}{x+2} = 3 \Leftrightarrow \underbrace{\lim_{x\to -2} \frac{g(x)-g(-2)}{x-(-2)}}_{\text{definição de derivada}} = 3 \Leftrightarrow g'(-2) = 3$$

$$f(x) = e-4 \Leftrightarrow e^{2x-1} - 4 = e-4 \Leftrightarrow e^{2x-1} = e \Leftrightarrow 2x-1 = 1 \Leftrightarrow x = 1$$

Assim, $f^{-1}(e-4) = 1$

$$\therefore g'(-2) + f^{-1}(e-4) = 3+1 = 4$$

Opção **(C)**

4.

4.1. Sejam os acontecimentos:

- A: "ser rapaz"
- B: "pretende seguir um curso de engenharia"

$$P\left(\overline{A}\right) = 0.4$$

$$P(A\cap B) = 0.3$$

$$P\left(\overline{B}|\overline{A}\right) = \frac{2}{3} \Leftrightarrow P\left(\overline{A}\cap\overline{B}\right) = \frac{2}{3}\times 0.4 = \frac{4}{15} \Leftrightarrow P\left(\overline{A\cup B}\right) = \frac{4}{15} \Leftrightarrow P(A\cup B) = \frac{11}{15} \Leftrightarrow$$

$$P(A) + P(B) - P(A\cap B) = \frac{11}{15} \Leftrightarrow 0.6 + P(B) - 0.3 = \frac{11}{15} \Leftrightarrow P(B) = \frac{13}{30}$$

$$\therefore P\left(A|B\right) = \frac{P(A\cap B)}{P(B)} = \frac{0.3}{\frac{13}{30}} = \frac{9}{13}$$

4.2.

$$30\times 0.4 = 12 \text{ raparigas}$$

$$30 - 12 = 18 \text{ rapazes}$$

$$\therefore {}^{30}C_4 - {}^{18}C_4 = 24345 \text{ comissões nas condições pretendidas.}$$

5. f é contínua em $x=0$ se e só se $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x) = f(0)$.

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{e^{2x}+x-1}{x-\sqrt{2x}} = \lim_{x\to 0^+} \frac{e^{2x}+x-1}{x} \times \lim_{x\to 0^+} \frac{x}{x-\sqrt{2x}} =$$

$$= \left(2\underbrace{\lim_{2x\to 0^+} \frac{e^{2x}-1}{2x}}_{\text{limite notável}} + \lim_{x\to 0^+} \frac{x}{x}\right) \times \lim_{x\to 0^+} \frac{x\left(x+\sqrt{2x}\right)}{x^2-2x} = (2\times 1+1)\times \lim_{x\to 0^+} \frac{x+\sqrt{2x}}{x-2} = 3\times\frac{0+0}{0-2} = 0$$

$$f(0) = \cos(3\times 0 - k) = \cos(-k)$$

$$\cos(-k) = 0 \Leftrightarrow -k = \frac{\pi}{2} + \lambda\pi, \lambda \in \mathbb{Z} \Leftrightarrow k = -\frac{\pi}{2} - \lambda\pi, \lambda \in \mathbb{Z}$$

$$\text{Se } \lambda = -1 \Rightarrow k = \frac{\pi}{2}$$

Opção **(C)**

6.

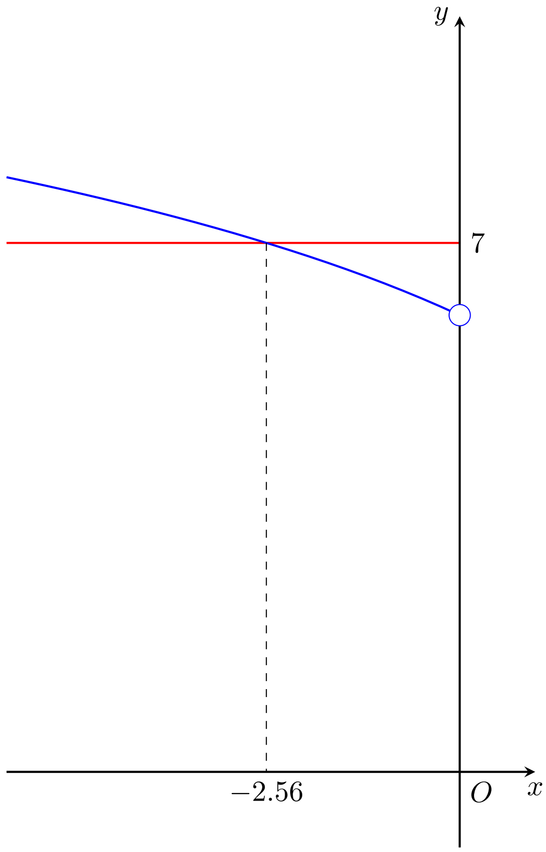
$$f(x) = 1 \Leftrightarrow -\ln(5-x) = 1 \Leftrightarrow 5 - x = e^{-1} \Leftrightarrow x = 5 - \frac{1}{e} \Leftrightarrow x = \frac{5e-1}{e} = \overline{AB}$$

Seja x a abcissa do ponto C, então:

$$h = 1 + \left|f(x)\right| = 1 + \left|-\ln(5-x)\right| = 1 + \ln(5-x) \quad (x < 0)$$

$$A_{[ABC]} = \frac{\overline{AB} \times h}{2} = \frac{\frac{5e-1}{e} \times \big(1 + \ln(5-x)\big)}{2} = \frac{5e-1}{2e}\big(1 + \ln(5-x)\big)$$

$$A_{[ABC]} = 7 \Leftrightarrow \frac{5e-1}{2e}\big(1 + \ln(5-x)\big) = 7$$



$$f(-2.56) = -\ln(5 + 2.56) \approx -2.02$$

$$\therefore C(-2.56, -2.02)$$

7.

$$P\left(A\cap B\right)=0$$

$$P\left(B\right)=P\left(\overline{A}\cap B\right)+P\left(A\cap B\right)=0.55+0=0.55$$

$$P\left(\overline{A}\right)=P\left(\overline{A}\cap B\right)+P\left(\overline{A}\cap\overline{B}\right)\Leftrightarrow 0.7=0.55+P\left(\overline{A}\cap\overline{B}\right)\Leftrightarrow P\left(\overline{A}\cap\overline{B}\right)=0.15$$

$$\therefore P\left(\overline{A}|\overline{B}\right)=\frac{P\left(\overline{A}\cap\overline{B}\right)}{P\left(\overline{B}\right)}=\frac{0.15}{1-0.55}=\frac{1}{3}$$

Opção **(D)**

8.

$$^{2013}C_{100}+^{2013}C_{1912}+a=1\Leftrightarrow^{2013}C_{100}+\underbrace{^{2013}C_{101}}_{^nC_p=^nC_{n-p}}+a=^{2015}C_{102}\Leftrightarrow\underbrace{^{2014}C_{101}}_{^nC_p+^nC_{p+1}=^{n+1}C_{p+1}}+a=^{2015}C_{102}$$

Assim, $a=^{2014}C_{102}$, pois $^nC_p+^nC_{p+1}=^{n+1}C_{p+1}$.

Opção **(D)**

9.

9.1.

$$\lim_{x\to\frac{3\pi}{2}}\frac{g\left(\frac{3\pi}{2}\right)-g\left(x\right)}{2x-3\pi}=-\frac{1}{2}\underbrace{\lim_{x\to\frac{3\pi}{2}}\frac{g\left(x\right)-g\left(\frac{3\pi}{2}\right)}{x-\frac{3\pi}{2}}}_{\text{definição de derivada}}=-\frac{1}{2}\times g'\left(\frac{3\pi}{2}\right)=-\frac{1}{2}\left(e^{\sin\left(\frac{3\pi}{2}\right)}+\cos\left(3\pi\right)\right)=$$

$$=-\frac{1}{2}\left(e^{-1}-1\right)=\frac{1}{2}\left(1-\frac{1}{e}\right)=\frac{e-1}{2e}$$

9.2.

$$g''\left(x\right)=\cos xe^{\sin x}-2\sin\left(2x\right)$$

g'' é contínua em $\left[\frac{3\pi}{4},\pi\right]$ pois é a diferença entre funções contínuas.

$$g''\left(\frac{3\pi}{4}\right)=\cos\frac{3\pi}{4}e^{\sin\frac{3\pi}{4}}-2\sin\frac{3\pi}{2}=-\frac{\sqrt{2}}{2}e^{\frac{\sqrt{2}}{2}}+2>0$$

$$g''\left(\pi\right)=\cos\pi e^{\sin\pi}-2\sin 2\pi=-e^0-2\times 0=-1<0$$

Como $g''\left(\frac{3\pi}{4}\right)\times g''\left(\pi\right)<0$ e g'' é contínua em $\left[\frac{3\pi}{4},\pi\right]$, então pelo corolário do teorema de Bolzano, $g''\left(x\right)=0$ é possível em $\left]\frac{3\pi}{4},\pi\right[$ e como existe mudança de sinal de g'' em $\left[\frac{3\pi}{4},\pi\right]$, então o gráfico de g admite pelo menos um ponto de inflexão com abcissa $a\in\left]\frac{3\pi}{4},\pi\right[$.

10.

$$f\left(a\right)=3$$

$$f'\left(a\right)=m_t=\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$$

$$f''(a) < 0$$

$$f(a) \times f'(a) + f''(a) = 3 \times \frac{\sqrt{3}}{3} + \underbrace{f''(a)}_{<0} < \sqrt{3}$$

Opção **(B)**

11.

11.1.

$$\tan\left(\alpha - \frac{\pi}{2}\right) = \frac{\overline{BC}}{1} \Leftrightarrow \overline{BC} = -\frac{1}{\tan\alpha}$$

$$A_{\text{Sombreada}} = A_{[OBC]} - A_{\text{Setor circular}} = \frac{\overline{OB} \times \overline{BC}}{2} - \frac{1}{2}\left(\alpha - \frac{\pi}{2}\right) \times 1^2 = \frac{1 \times \frac{-1}{\tan\alpha}}{2} - \frac{1}{2}\alpha + \frac{\pi}{4} =$$

$$= -\frac{1}{\tan\alpha} - \frac{1}{2}\alpha + \frac{\pi}{4} = \frac{1}{2}\left(\frac{\pi}{2} - \alpha - \frac{1}{\tan\alpha}\right)$$

11.2.

$$A'(\alpha) = \frac{1}{2}\left(0 - 1 - \frac{0 - 1 \times (\tan\alpha)'}{\tan^2\alpha}\right) = \frac{1}{2}\left(-1 + \frac{\frac{1}{\cos^2\alpha}}{\frac{\sin^2\alpha}{\cos^2\alpha}}\right) = \frac{1}{2}\left(-1 + \frac{1}{\sin^2\alpha}\right)$$

$$A'(\alpha) = 0 \Leftrightarrow \frac{1}{2}\left(-1 + \frac{1}{\sin^2\alpha}\right) = 0 \Leftrightarrow \frac{1}{\sin^2\alpha} = 1 \Leftrightarrow \sin\alpha = \pm 1 \ \wedge \ \sin\alpha \neq 0 \Leftrightarrow$$

$$\alpha = \frac{\pi}{2} + k\pi \ \wedge \ \alpha \neq k\pi, \ k \in \mathbb{Z}$$

Não existem soluções em $\left]\frac{\pi}{2}, \pi\right[$.

α	$\frac{\pi}{2}$		π
$A'(\alpha)$		$+$	
A		$\nearrow$	

Portanto A é estritamente crescente em $\left]\frac{\pi}{2}, \pi\right[$.

12.

$$\lim_{x\to+\infty}\left[g(-x) \times \frac{x}{g(x)} + x - g(x)\right] = \lim_{x\to+\infty} g(-x) \times \lim_{x\to+\infty}\frac{x}{g(x)} + \lim_{x\to+\infty}\big(x - g(x)\big) =$$

$$= g(-\infty) \times \frac{1}{\underbrace{\lim_{x\to+\infty}\frac{g(x)}{x}}_{\text{declive a.o.}}} - \underbrace{\lim_{x\to+\infty}\big(g(x) - x\big)}_{\text{ordenada na origem a.o.}} = 1 \times \frac{1}{1} - 1 = 0$$

Opção **(C)**

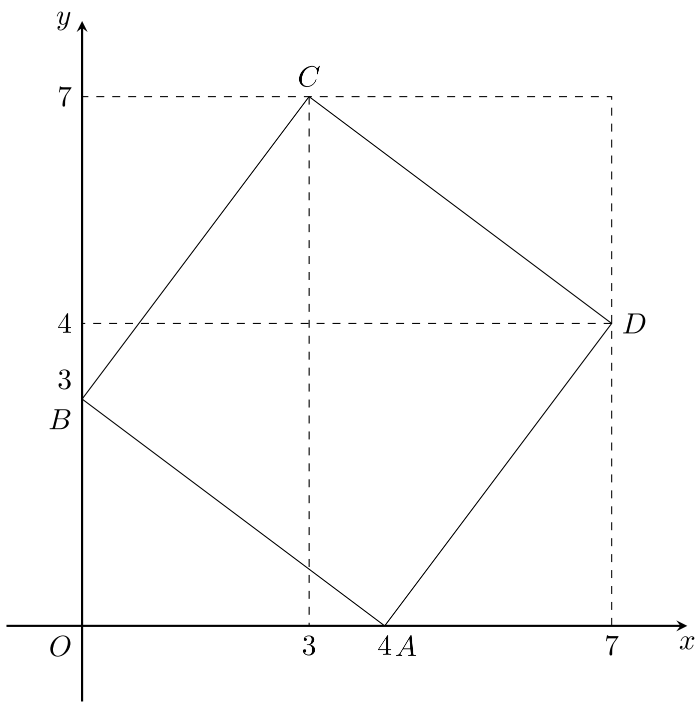
13.

13.1.

$$A\,(4,0,0)$$

$$B\,(0,3,0)$$

$$\overline{AB}=\sqrt{4^2+3^2+0^2}=5$$



$$\overrightarrow{AD}=(a,b,0)$$

$$\overrightarrow{AB}=(-4,3,0)$$

$$\overrightarrow{AD}\cdot\overrightarrow{AB}=0\Leftrightarrow -4a+3b=0\Leftrightarrow b=\frac{4}{3}a$$

$$\overline{AD}=5\Leftrightarrow\sqrt{a^2+b^2+0^2}=5\Rightarrow a^2+\frac{16}{9}a^2=25\Leftrightarrow a^2=9\underbrace{\Rightarrow}_{a>0}a=3$$

Assim, $\overrightarrow{AD}=(3,4,0)$

$$D=A+\overrightarrow{AD}=(4,0,0)+(3,4,0)=(7,4,0)$$

$$C=D+\overrightarrow{AB}=(7,4,0)+(-4,3,0)=(3,7,0)$$

$$\therefore G\,(3,7,5)$$

$$\overrightarrow{DE}=(-3,-4,5)$$

$$\overrightarrow{DG}=(-4,3,5)$$

$$\therefore\overrightarrow{DE}\cdot\overrightarrow{DG}=-3\times(-4)+(-4)\times3+5\times5=25$$

13.2. Seja $\overrightarrow{n}=(a,b,c)$ um vetor normal ao plano α.

$$\begin{cases} \overrightarrow{n}\cdot\overrightarrow{DE}=0 \\ \overrightarrow{n}\cdot\overrightarrow{DG}=0 \end{cases} \Leftrightarrow \begin{cases} -3a-4b+5c=0 \\ -4a+3b+5c=0 \end{cases} \Leftrightarrow \begin{cases} -3a-4b+4a-3b=0 \\ 5c=4a-3b \end{cases} \Leftrightarrow \begin{cases} a=7b \\ c=5b \end{cases}$$

Fazendo $b=1$, por exemplo, tem-se $\overrightarrow{n}=(7,1,5)$

$$\therefore 7(x-7)+1(y-4)+5(z-0)=0 \Leftrightarrow 7x+y+5z=53$$

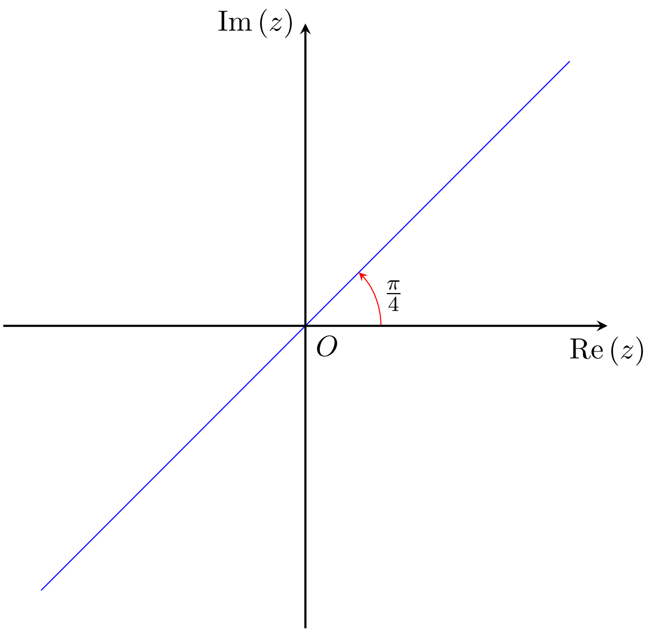
14.

$$i^{4k+3}=i^3=-i=e^{i\left(-\frac{\pi}{2}\right)}, k\in\mathbb{N}_0$$

$$|1+i|=\sqrt{2}$$

$$\operatorname{Arg}(1+i)=\arctan(1)=\frac{\pi}{4}$$

$$z=(1+i)^4\times\left[\frac{i^{4k+3}}{e^{i\alpha}}\right]^2=\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^4\times\left[\frac{e^{i\left(\frac{\pi}{2}\right)}}{e^{i\alpha}}\right]^4=4e^{i\pi}\times\left[e^{i\left(-\frac{\pi}{2}-\alpha\right)}\right]^2=4e^{i\pi}\times e^{i(\pi-2\alpha)}=4e^{i(-2\alpha)}$$



Para a imagem geométrica de z pertencer à bissetriz dos quadrantes ímpares, tem que:

$$-2\alpha=\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}\Leftrightarrow\alpha=-\frac{\pi}{8}-k\frac{\pi}{2},\ k\in\mathbb{Z}$$

$$\text{Se } k=-1\Rightarrow\alpha=\frac{3\pi}{8}$$

Opção **(A)**